Government Size and Risk Premium

Abhishek Kumar*
Sushanta Mallick†

November 29, 2023

Abstract

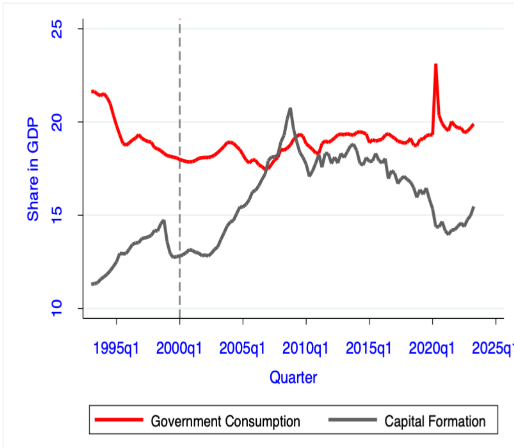
Given the rise in government debts in recent times, this paper aims to understand the effect of an increase in government size on risk premium and its transmission in the economy. We jointly identify the term spread shock (originating at the short and the long end) and government size shocks, using max share identification. Term spread shock originating at the long end is driven by higher risk premium unlike the shock originating at the short end, and increases inflation and reduces growth. Results suggest that the increase in the share of government expenditure in GDP (size) increases long-term rates by increasing the risk (term premium) and hence obstructs the transmission of monetary policy. As expected the effect of government size on risk premium is more pronounced during recessions compared to expansions. By including a news shock about future economic activity, we rule out that the effect of government size shock on term premium is not driven by news shock. We estimate the parameters of a new Keynesian model with term premium by matching the responses in data with responses from the model. The model can generate a similar rise in risk premium due to the increase in government size and estimated parameters suggest that the coefficient of risk aversion in the recession is more than twice of in expansion.

*University of Southampton, UK . E-mail: a.kumar@soton.ac.uk
†Queen Mary University of London, London, UK. E-mail: s.k.mallick@qmul.ac.uk

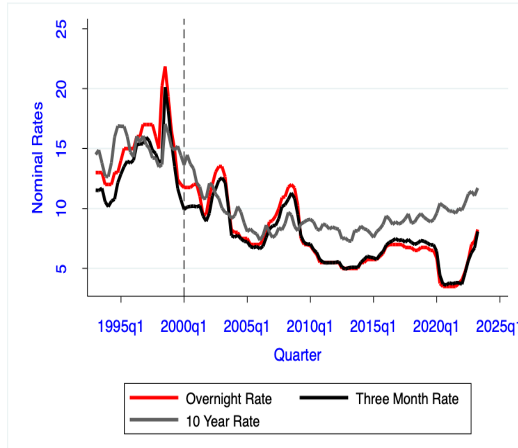
1 Introduction

After a sustained period of decline, government debt to GDP ratio increased very sharply in South Africa in the 2010s. Like other countries, South Africa also implemented fiscal stimulus during the financial crisis, and the economy recovered quickly but it never achieved the pre-crisis growth that further kept decelerating through out the last decade. The share of government expenditure in GDP (*government size*) kept increasing and the debt to GDP ratio increased by more than 40 percentage points in the last decade. A large part of the increase in government debt to GDP ratio is due to the increase in external debt which went up almost three times during the same period. The increase in government size clearly led to crowding out of private investment and has been accompanied by a decline in the share of investment to GDP ratio and the GDP growth rate. This suggests that the debt driven fiscal expansion has not done good to the growth rate and has also increased unemployment in South Africa (Havemann and Hollander, 2022).

The rate of inflation was moderate in the last decade and the overnight policy rate kept declining, but the long rates (ten year sovereign bond yield) were increasing, leading to a substantial increase in the term spread (Figure 1). The simultaneous decline in growth with an increase in term spread is puzzling, as the existing literature suggests that the increase in term spread is associated with economic expansion (Benzoni et al. (2014)). The term spread can increase because of the increase in future average short term rates driven by anticipation of future activities or decline in short term rate due to anticipated favorable supply shock as in Kurmann and Otrok (2013). It can also increase due to an increase in risk premium (term-premium) arising due to higher size of the government. Erasmus and Steenkamp (2022) decompose term spread into expectations of future average short term rates and a risk premium and find that a large part of the increase in term spread in South Africa is driven by increase in risk premium (term premium). The increase in risk premium has ensured that the long rates have kept increasing despite the lowering of short term policy rate. Since, the usual monetary transmission channel is broken due to the increase in risk premium, it is clear that monetary policy alone cannot support growth by keeping the policy rate lower.



(a) Share in GDP



(b) Interest Rates

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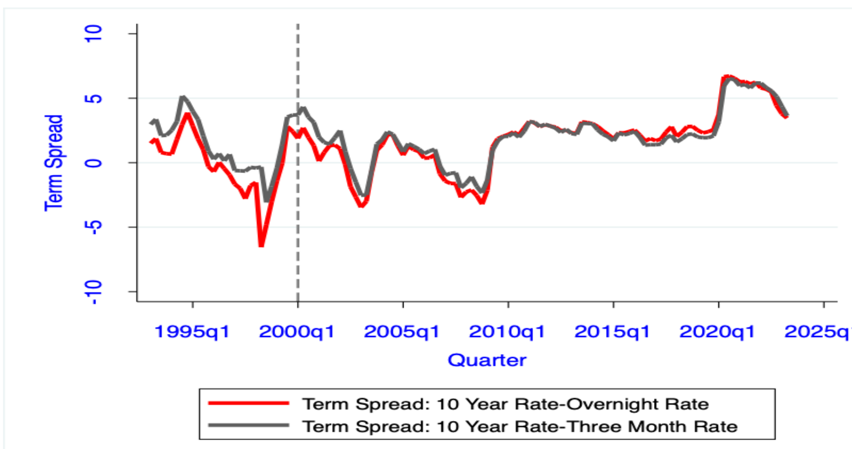


(c) Term Spreads

Figure 1: Share of Capital Formation and Government Consumption in GDP; Interest Rates and Term Spreads. Source: Saint Louis Fed

This is because monetary policy aims to move long term rates that determine investment and durable consumption in a desired direction by moving short term rates, but it has failed to do so. Also by keeping the policy rate lower, monetary policy is not able to support government borrowing and stabilize government debt, as the long term rates are determined in the market and monetary policy has little control over that.

The macro-finance models of term structure and term premium used in Rudebusch & Swanson (2012), Bretscher et al. (2020) and Hovrath et al. (2022), suggest that both government expenditure and the level of debt are important determinants of term-premium. Bretscher et al.(2020) estimate the empirical and model-implied responses due to the shock to government spending level and government spending uncertainty and suggest that the model-implied responses are similar to the empirical responses in the data. On the other hand, Hovrath et al. (2022) estimate the parameter of a similar model with generalised method of moments, and the simulated data from the model generates a level of term premium which is comparable to the term premium in the data. The inflation risk premium plays an important role in generating the risk premium in Hovrath et al. (2022). But, both Bretscher et al. (2020) and Hovrath et al. (2022) consider the level of government spending and implement a shock to the level of government spending.

Based on these, in this paper we explore the effect of increase in the size of the government in South African economy. Especially we focus on the increase in the term spread and the deceleration in the growth rate. We choose the size of the government instead of government expenditure, because there has been a noticeable increase in the size of the government in South Africa in the last decade (see figure 1). Size of the government has been widely used in New Keynesian models to evaluate fiscal policies (Justiniano et al. (2010)). Also the previous studies focus on the level of term premium whereas the focus in this paper is on the response of term premium due to an exogenous change in the size of the government.

Since the size of the government and term premium are related, in this paper we first estimate a term spread shock similar to Kurmann and Otrok (2013)[1]. This shock

[1] This serves two purposes. First it helps us in analysing the effect of term spread shock in the South African economy. Second when we extend the model to include the size of the government and identifies two shocks (term spread and government size shocks), it ensures that government size shock is not contaminated with term spread shock.

explains the maximum share in the forecast error variance of the term spread. Kurmann and Otrok (2013) argue that the term spread shock resembles a shock to future productivity where the Federal Reserve lowers the interest rate and the shock increases output and decreases inflation. This makes the shock as a favorable supply shock as in Kurmann and Otrok (2013). But, in the case of South Africa, our results suggest that this shock decreases growth, short-term interest rate and inflation. This is counter-intuitive and could be result of incorrect identification as the term spread can increase due to increase in the long end of the rate or decrease in short end of the rate.

Hence we further consider the term spread shocks originating at the short and the long end of the interest rates which is missing in Kurmann and Otrok (2013). We estimate a model with term spread, log GDP, log consumer prices and overnight rate and restrict the contemporaneous response of the overnight rate due to term spread shock to zero and obtain the **term spread shock originating at the long end**. Similarly, we estimate another model with term spread, log GDP, log consumer prices, and ten year rate and restrict the contemporaneous response of the ten year rate due to term spread shock to zero and obtain the **term spread shock originating at the short end**. Although, we do not observe the term premium data, the response of term spread and short term rates gives us the response of the term premium. Results suggest that the term spread shock originating at the long end is driven by increase in term premium whereas the term spread shock originating at the short end does not lead to significant effect on the term premium in the beginning. This is as expected and gives us confidence that these two shocks are correctly identified.

Moreover the term spread shock originating at the long end increases inflation and reduces growth- induces negative covariance between growth and inflation- unlike unrestricted and term spread shock originating at the short end. Rudebusch & Swanson (2012), Bretscher et al. (2020), and Horvath et al. (2022) argue that it is essential for a shock to cause a negative covariance between inflation and growth/consumption to cause term premium. This is because higher inflation reduces the price of bond and investor would demand premium for holding assets whose value is decreasing during the periods of decreasing consumption. Results obtained in this paper suggest that only the term spread shock originating at the long end produces the negative covariance between growth and inflation and is theoretically consistent. The focus in this paper is on the government size shock that is expected to increase term premium based on the literature. Since the shock at the long end is driving term premium, the identification of government size shock in the absence of this shock could lead to bias.

Hence, we jointly identify two orthogonal shocks: term spread shock originating at the long end, and a government size shock. Government size is measured as ratio of government final consumption expenditure to GDP. It is important to jointly identify these shocks, as the term spread shock originating at the long end leads to an increase in term premium; excluding that shock may lead to mis-identification of government size shock as the government size shock is expected to increase term premium as well. The identification strategy also helps to disentangle the exogenous increase in term premium from the increase in term premium driven by government size. These shocks explain the maximum share in forecast error variance of the term spread and government size. It is important to mention that in the case of one such shock there is analytical solution available which is there in Kurmann and Otrok (2013). But in the case of two (joint) shocks, there is no analytical solution available and hence we use optimization to obtain these two shocks jointly. This is a contribution to the literature that aims to identify multiple shocks based on share in forecast error variance which is also known as max share identification in SVAR literature.

We find that the government size shock increases term spread and decreases growth. Further the government size shock generates the negative covariance between inflation and output (inflation and consumption), we obtain the response of term premium due to a shock to government size. The response of the term premium suggests that most of the increase in term spread due to government size shock is driven by the increase in term premium. Since the government size shock leads to higher risk premium, this is the likely reason that growth rate has been declining in South Africa in the last decade.

We further identify expansionary, neutral and recessionary government size shocks by restricting the response of output due to government size shock to be $>= 0$, $= 0$ and $<= 0$ respectively for two time periods $t = 0$ and $t = 1$. The neutral government size shock is effectively a government expenditure shock as this is change in government size $\frac{G}{Y}$ without any change in Y. These additional estimations helps us to explore the non-linearity in response of term premium due to government size shock. As expected we find that recessionary government size shock induces highest increase in term premium due to government size shock among all these four government size shock estimated in this paper; unrestricted, expansionary, neutral and recessionary. Further, the recessionary government size shock also induces highest negative correlation between the response of output and inflation due to government size shock among all these four models.

Andreasen et al. (2023) argues that uncertainty shocks have more pronounced effects in recession and they conclude that risk matters more in recession. A higher size of the government is a risk for the household as it reduces the household ability to smooth adverse shock. For example higher government size may imply lower after tax wage and that reduces the ability of household to smooth shocks by utilizing intensive labour margin. Similar to Andreasen et al. (2023), we find that these risks are higher in recession. We also find that unrestricted and recessionary shocks produce identical response of term premium. This is expected because unrestricted shock leads to substantial reduction in output and hence we conclude that government size shock is essentially a recessionary shock but different from typical demand shock as it does not cause decline in inflation.

Since the neutral government size shock is identified by restricting the response of output to zero, this is effectively a shock to government expenditure and response of term premium from this model can be compared with the response of term premium in the literature. Rudebusch & Swanson (2012) using a theoretical model obtain a response of 0.17 basis points response of term premium which is substantially lower than a 20 basis point response of term premium due to government expenditure shock in this paper. It is important to mention that the objective in Rudebusch & Swanson (2012) was to estimate obtain a level of term premium in the model which is comparable to the level of term premium in the data. But in this paper we are interested in the response of the term premium due to government size and expenditure shocks.

The changes in the government size could be also driven by the news about future productivity. A news about increasing future productivity may allow government to consolidate and vice-versa. This news shock may also influence the term spread as in Kurmann and Otrok (2013). Hence we bring a news shock to productivity similar to Barsky and Sims (2011). The identification is same but we use GDP instead of TFP and the shock maximizes the forecast error variance of GDP but does not affect the GDP contemporaneously. We estimate a model with three orthogonal shocks, exogenous term spread shock originating at long end, government size shock and a news shock. The response of term premium due to government size shock remains very similar to previous model with two structural shocks. These results suggest that the larger size of the government is increasing the market risk, keeping long term rates high due to higher term premium despite lowering of rates by the Central Bank. In other words, the large size of government is not allowing the usual monetary transmission.

In the next step, we write a macro-finance model that is similar to the model in Rudebusch & Swanson (2012) and Hovrath et al. (2022) to explain the responses obtained from structural vector auto regression. The focus in Rudebusch & Swanson (2012) and Hovrath et al. (2022) is to generate a level of term premium that is comparable to the term premium in the data. Rudebusch & Swanson (2012) do estimate the response of term premium due to government spending shock from the theoretical model but that shock generates very low response of term premium compared to the response of term premiumd due to government expenditure and size shock.

This paper aims to analyse the response of the term premium due to a shock to the size of the government which differs from Rudebusch & Swanson (2012). Hence we introduce a shock to the size of the government that is similar to Justiniano et al. (2010). We estimate the parameters of the model by minimizing the distance between responses of the term premium due to the expansionary and recessionary government size shocks from the structural vector auto regression model. This is similar to the approach in Basu and Bundick (2017). The estimate coefficients from these two estimates are able to identify the recessionary and expansionary period as we explain in detail in the paper. The model is able to generate a similar response of the term premium due to the expansionary and recessionary government size shocks although it requires a higher value of risk aversion. This is expected given the significantly higher response of term premium obtained in this paper using the data. Most importantly, we find that risk aversion in recession is more than twice of the value in expansion. This is another contribution to the literature on term premium in macro-finance models.

The plan of the paper is as follows. Section 2 explains the identification of shocks from structural vector autoregression, and the estimation of the response of term premium using the response of term spread and short-term rate, and gives a brief overview of the data from the South African Economy. Section 3 presents the responses and share in forecast error variance of the model variables due to term spread government size and news shocks. Section 4 presents the New-Keynesian model of term structure. Section 5 presents the results from the model, followed by concluding remarks.

2 Empirical Framework and Data

2.1 Empirical Framework

A general structural vector auto-regression model is given by:

$$A_0 y_t = a + \sum_{j=1}^{p} A_j y_{t-j} + \epsilon_t$$

The reduced form model is given by:

$$y_t = b + \sum_{j=1}^{p} B_j y_{t-j} + u_t$$

Where $b = A_0^{-1} a$, $B_j = A_0^{-1} A_j$ and $u_t = A_0^{-1} \epsilon_t$. The covariance matrix of the reduced form shocks $E(u_t, u_t') = \sum = \left(A_0^{-1}\right)\left(A_0^{-1}\right)'$ is known. We assume that $E\left(\epsilon_t \epsilon_t'\right) = I$

We can write the impulse response matrix at horizon h

$$IR^h = C(h)A_0^{-1}$$

Where $C(h)$ is the hth element in the expansion if $\left[I_n - \sum_{j=1}^{p} B_j L^j\right]^{-1}$ where $h = 0, 1, H$ and $C(0) = I_n$. The element in row (i) and column (j) denotes the response of ith variable due to shock associated with jth variable. The matrix A_0^{-1} is unknown and needs to be estimated to calculate the structural impulse response IR^h. The reduced form covariance matrix is known and one can do Cholesky decomposition of the same to estimate the A_0^{-1} as given below:

$$\sum = PP' = \left(A_0^{-1}\right)\left(A_0^{-1}\right)'$$

This implies $A_0^{-1} = P$. But as shown in Uhlig (2004), the matrix P obtained by Cholesky decomposition is not the only matrix that satisfies the above as we can write:

$$\sum = PQQ'P'$$

For any orthonormal matrix Q $(QQ' = I)$. This gives us $A_0^{-1} = PQ$ and hence the structural impulse response can be written as:

$$IR^h = C(h)PQ$$

The response of the i^{th} variable due to a shock associated with j^{th}variable is given by:

$$IR^h(i,j) = e_i^{'}C(h)PQe_j = e_i^{'}C(h)Pq_j = c_{ih}^{'}q_j$$

Where q_j is j^{th} column of Q and $c_{ih}^{'}$ is i^{th} row of $C(h)P$. The important point is that $Q = I_n$ gives the identification based on Cholesky decomposition and additional identification such as sign restrictions can be achieved by imposing restrictions on Q.

The forecast error variance of the i^{th} variable due to a shock associated with j^{th} variable at horizon h is given by

$$\sum_{h=0}^{h=h} IR^h(i,j)' IR^h(i,j) = \sum_{h=0}^{h=h} q_j' c_{ih} c_{ih}' q_j = q_j' \left(\sum_{h=0}^{h=h} c_{ih} c_{ih}' \right) q_j$$

The diagonal elements of $\sum_{h=0}^{h=h} c_{ih} c_{ih}'$ contain the forecast error variance of ith variable due to given shocks. The forecast error variance of the i^{th} variable due to all shocks is given by $\sum_{h=0}^{h=h} c_{ih}' c_{ih}$. Hence the share of j^{th} variable in the forecast error variance of i^{th} variable is given by

$$FEV(i,j,h) = \frac{q_j' \left(\sum_{h=0}^{h=h} c_{ih} c_{ih}' \right) q_j}{\sum_{h=0}^{h=h} c_{ih}' c_{ih}}$$

We define

$$FEV(i,h) = \frac{\left(\sum_{h=0}^{h=h} c_{ih} c_{ih}' \right)}{\sum_{h=0}^{h=h} c_{ih}' c_{ih}}$$

2.2 Term Spread Shock

We identify the term spread shock based on the share of forecast error variance decomposition. This is purely agnostic and driven by data. We put very minimal restrictions on identification which is hard to disagree with. We order term spread as the first variable in the VAR model and identify the first column of Q using the following optimization problem

$$q_1^* = \arg\max_{q_1} q_1' FEV(1,h) q_1$$

Subject to

$$q_1' FEV(1,h) q_1 \geq q_1' FEV(j,h) q_1 \quad \text{for } j = 2,3,4$$

The objective function maximizes the share of the first shock which we refer to as term spread shock in forecast error variance of term spread. In literature, this type of identification is known as max share identification. The constraint implies that the share of the variance explained by the term spread shock of term spread is higher than the share of term spread shock in the forecast error variance of other variables. The identification is intuitive. One can choose variables which are important for term spread determination and to forecast term spread. The identified shock is the one which explains the maximum share of the forecast error variance. In other words, this is the source of variation which is driving term spread away from its predicted value based on the variables in the model and hence is an exogenous shock.

The baseline model is estimated with term spread (difference between ten year rate and three three-month rate), log GDP , log consumer prices and one of the interest rates, overnight or 10-year yield (long-term rate). The choice of the variable is based on the new Keynesian paradigm; growth, inflation and interest rate represent a reasonable set of variables for policy analysis (Ireland, 2013). The central bank in South Africa targets inflation, and most of the inflation-targeting central banks have an interest rate reaction function that can be obtained using inflation and growth. These inflation-targeting central banks respond to deviation of inflation from the target level of inflation, and deviation of output from steady state/potential output. The new Keynesian model in section 4 contains this type of reaction function which is also known as the Taylor rule. We estimate all models with four lags which is reasonable given that we use quarterly data.

2.3 Term Spread Shocks Originating at Short and Long End

We make a distinction in the term spread movements caused by the movement at the short end and the long end of the rate. The term spread change at the short end of the rate is likely to be driven by policy changes as the central bank has reasonable control over the interest rate at the short end. Such policy-driven rate can change if the central bank lowers interest rates anticipating a productivity shock as in Kurmann and Otrok (2013) or it could be interest rate hikes by the central bank due to an adverse markup/ cost-push shock. The term spread at the long end of the rate is driven by a change in the risk perception i.e. term premium. The change in risk premium can arise due to higher inflation expectations or higher government debt which increases the probability of insolvency of the government. If the government debt is only issued in the domestic currency then inflation risk and insolvency risk are the same as government can always inflate away the debt in case of an insolvency-like scenario. But this is unlikely to be the case in South Africa with a substantial amount of foreign debt.

Kurmann and Otrok (2013) identify a term spread shock using a similar approach but do not make a distinction between term spread shock arising at the long end and the short end of the term structure. The earlier literature on term spread summarized by Wheelock and Wohar (2009) generally finds that the yield spread is positively associated with future GDP growth even when a short-term interest rate is included but they do not make an explicit distinction between term spread caused by movement at the short- and the long-end as undertaken in this paper. Hamilton and Kim (2002) decompose the yield spread into an expected interest rate component and a term premium component, finding that both have predictive power for future economic activity, but the term spread shock is likely to be caused by both ends of the market. We identify shocks originating at the short- and long-end, and the long end shock may arise purely due to a change in term premium. The conflicting result in the literature on the effect of term spread shock could be partly driven by the fact that the existing literature does not make a distinction between term spread caused by movement at the long or short end. In case of four variable models, the optimization problem is given by:

$$q_1^* = \arg\max_{q_1} q_1' FEV(1,h) q_1$$

Subject to

$$q_1' FEV(1,h) q_1 \geq q_1' FEV(j,h) q_1 \text{for } j = 2,3,4$$

$$e_4' C(h) P Q e_1 = e_4' C(h) P q_1 = 0$$

The first constraint is same as before. The last constraint implies that the term spread shock does not lead to change in long term rate (overnight rate) contemporaneously and hence term spread shock is driven by the short end (long end) of the rate. We order term spread as first variable and long-term rate (overnight) rate as fourth variable.

2.4 Term Spread and Government Size Shocks

We estimate another set of models in which we include share of government expenditure in GDP which is called government size in this paper. Share of government expenditure is likely to influence the term spread by causing long term rates to rise due to higher inflation expectations. It can also raise long term rates due to increase in risk premium associated with long term bonds. The optimization problem is given by:

$$q_1^*, q_2^* = \arg\max_{q_1,q_2} q_1' FEV(1,h)q_1 + q_2' FEV(2,h)q_2$$

Subject to

$$q_1' FEV(1,h)q_1 \geq q_1' FEV(j,h)q_1 \text{for } j = 2,3,4,5$$

$$q_2' FEV(2,h)q_2 \geq q_2' FEV(j,h)q_2 \text{for } j = 1,3,4,5$$

$$q_1' q_2 = 0$$

where the objective function is to maximize the sum of the share of term spread explained by term spread shock and the share of government size explained by government size shock. The first constraint implies that the share of the variance explained by the term spread shock of term spread is higher than the share of term spread shock in the forecast error variance of other variables. The second constraint implies that the share of the variance explained by the government size shock of government size is higher than the share of government size shock in the forecast error variance of other variables. The third constraint implies that these two shocks are orthogonal (structural). Similar to four variable case, we make distinction between term spread driven by long or short end of the rate and add one additional constraint which implies that term spread shock does not affect long-term (overnight) rate contemporaneously. We do not restrict contemporaneous response of interest rate due to a shock to the size of the government. We order term spread as first variable and long-term rate (overnight) as fifth variable in the VAR model. The optimization problem is given by:

$$q_1^*, q_2^* = \arg\max_{q_1,q_2} q_1' FEV(1,h) q_1 + q_2' FEV(2,h) q_2$$

Subject to

$$q_1' FEV(1,h) q_1 \geq q_1' FEV(j,h) q_1 \text{for } j = 2,3,4,5$$

$$q_2' FEV(2,h) q_2 \geq q_2' FEV(j,h) q_2 \text{for } j = 1,3,4,5$$

$$q_1' q_2 = 0$$

$$e_5' C(0) PQe_1 = e_5' C(0) Pq_1 = 0$$

where the first three constraints are same as before. The last constraint implies that the term spread shock originates at either short or long end depending upon the model.

2.5 Expansionary, Recessionary and Neutral Government Size Shock

Andreasen et al. (2023) argues that uncertainty shocks have more pronounced effects in recession and they conclude that risk matters more in recession. The government size in this paper is defined as $\frac{G}{Y}$. The shock arises in case of the government expenditure multiplier is lower than 1 and in that case only $\frac{G}{Y}$ increases. A government expenditure multiplier greater than one will raise Y more than G and hence will not be causing a government size shock. The government size shock can cause three scenarios for output, increase, decrease and no affect on output. The third one is also a shock to the government expenditure because it changes $\frac{G}{Y}$ without affecting y. This allows us to compare our results with existing result in the literature which obtains the response of term premium due to government expenditure shock such as Rudebusch & Swanson (2012). These three identification also allows us to compare the response of term premium due to government size originating in different scenarios as in Andreasen et al. (2023). Our estimation allows us to estimate the nonlinear responses by separating estimating models with inequality constraints on the behaviour of output due to government size shock as we explain below. Based on these three scenarios, we create expansionary, recessionary and neutral government size shock. This helps in evaluating the internal consistency of the model that the recessionary government size shock must create higher increase in term premium. The optimization problem is given by:

$$q_1^*, q_2^* = \arg\max_{q_1,q_2} q_1' FEV(1,h) q_1 + q_2' FEV(2,h) q_2$$

Subject to

$$q_1' FEV(1,h) q_1 \geq q_1' FEV(j,h) q_1 \text{for } j = 2,3,4,5$$

$$q_2' FEV(2,h) q_2 \geq q_2' FEV(j,h) q_2 \text{for } j = 1,3,4,5$$

$$q_1' q_2 = 0$$

$$e_5' C(0) P Q e_1 = e_5' C(0) P q_1 = 0$$

where the above constraints are same as before. We use additional constraints to make distinction between these three shocks.

Recession

$$e_3' C(0) P q_2 \leq 0 \qquad e_3' C(1) P q_2 \leq 0$$

Expansion

$$e_3' C(0) P q_2 \geq 0 \qquad e_3' C(1) P q_2 \geq 0$$

Neutral

$$e_3' C(0) P q_2 = 0 \qquad e_3' C(1) P q_2 = 0$$

where $e_3' C(0) P q_2$ and $e_3' C(1) P q_2$ gives the response of output due to government size shock at time 0 and 1 and two consecutive periods of decline in output is defines as recession.

2.6 Term Spread, Government Size and News Shocks

We further extend the empirical setting to bring a news shock about productivity. This is important because Kurmann and Otrok (2013) argues that news about future productivity leads to large swings in term spread. The optimization problem is given by:

$$q_1^*, q_2^*, q_{23}^* = \arg\max_{q_1, q_2} q_1' FEV(1,h) q_1 + q_2' FEV(2,h) q_2 + +q_3' FEV(3,h) q_3$$

Subject to

$$q_1' FEV(1,h) q_1 \geq q_1' FEV(j,h) q_1 \text{for } j = 2,3,4,5$$

$$q_2' FEV(2,h) q_2 \geq q_2' FEV(j,h) q_2 \text{for } j = 1,3,4,5$$

$$q_3' FEV(3,h) q_3 \geq q_3' FEV(j,h) q_3 \text{for } j = 1,2,4,5$$

$$q_1' q_2 = 0$$

$$q_1' q_3 = 0$$

$$q_2' q_3 = 0$$

$$e_5^{'} C(0) P Q e_1 = e_5^{'} C(0) P q_1 = 0$$

$$e_3^{'} C(0) P Q e_3 = e_5^{'} C(0) P q_3 = 0$$

Where the first two constraints are same as before. The third constraint implies that the share of the variance explained by the news shock of GDP is higher than the share of news shock in the forecast error variance of other variables. The next three constraints implies that these three shocks are orthogonal. The 7th constraint is same as before and the last constraint implies that the news shock does not affect GDP contemporaneously. In case of five variables also, we replace GDP growth with investment growth and consumption and estimate two additional five variable models. We also use alternative measure of size of the government and term spread and estimate two more models. These results are similar to the results reported here and are given in online appendix. We do not have data on term premium to estimate a model with term premium and obtain its response directly. But the empirical setting used in this paper allows the estimation of the response of the term premium due to the shocks and we explain that in the next section.

2.7 From Term Spread to Term Premium

In general, expectations hypothesis gives the long rates as expected value of future short term rate. We write the long rates as sum of expected value of future short term rates and term premium ϕ_t^m.

$$i_t^m = E_t \frac{1}{m} \left\{ \sum_{j=0}^{m-1} i_{t+j} \right\} + \phi_t^m$$

We can write the spread, i.e., the difference between long term rates and short term rates (slope) as:

$$\underbrace{(i_t^m - i_t)}_{\text{Slope}} = \underbrace{\left(\frac{1}{m} E_t \left\{ \sum_{j=0}^{m-1} i_{t+j} \right\} - i_t \right)}_{\text{Avergae Expected Future Short Rate - Short Rate}} + \underbrace{\left[i_t^m - \frac{1}{m} E_t \left\{ \sum_{j=0}^{m-1} i_{t+j} \right\} \right]}_{\text{Term Premium}}$$

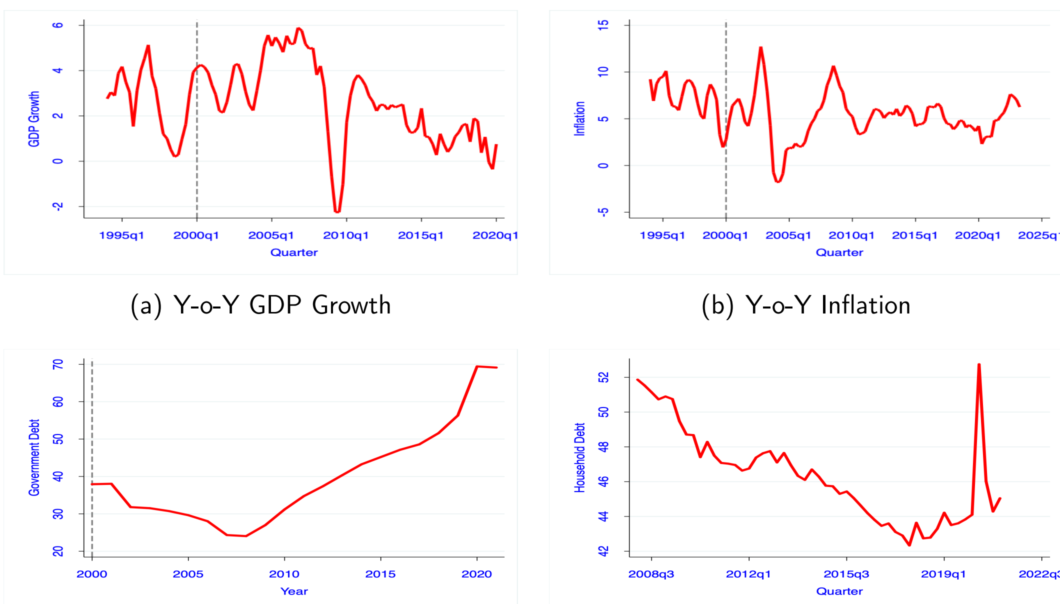
where the slope is made up of term premium and difference between average of expected future short rates and current short rate. The term premium can be written as

$$\phi_t^m = \left(i_t^m - \frac{1}{m} E_t \left\{ \sum_{j=0}^{m-1} i_{t+j} \right\} = (i_t^m - i_t) - \left(\frac{1}{m} E_t \left\{ \sum_{j=0}^{m-1} i_{t+j} \right\} - i_t \right) \right)$$

The difference between slope and excess of average of expected future short rates compared to current short term rates gives the term premium. We do not observe term premium and hence cannot estimate the response of term premium directly using it in the VAR model. But we can estimate the response of term premium indirectly by estimating the response of all the items on the right hand side. We know the response of the spread from the model containing overnight rate. This give us the response of $(i_t^m - i_t)$ at each point of time. From the same model, we obtain the response of $\frac{1}{m} E_t \left\{ \sum_{j=0}^{m-1} i_{t+j} \right\}$ using 40 quarter moving average of the response of i_t and the same model gives us the response of i_t. This way we obtain responses of the term premium due to a shock to the term spread shock and shock to the size of the government. We use the term spread $(i_t^m - i_t)$ as difference between ten year and three month rate and overnight rate as i_t. In the above specification three month and overnight rates are denoted by i_t. Since these two rates are very similar but not the same, this allows us to estimate the response of term premium.

2.8 Data

The national accounts data is obtained from Federal Reserve Bank of Saint Louis and is for the time period 1993Q1 to 2023Q2.



(a) Y-o-Y GDP Growth

(b) Y-o-Y Inflation

(c) Central Government Debt (% of GDP)

(d) Household Debt (% of GDP)

Figure 2: Growth, Inflation, Government and Household Debt
Source: Saint Louis Fed

We use consumer prices, overnight interest rate, three month rate (yield), ten year rate (yield), government final consumption expenditure, private final consumption expenditure and gross domestic product. The interest rate and government debt data have been also obtained from Federal Reserve Bank of Saint Louis. As we can see from figure 2, there has been a substantial increase in the debt to GDP ratio in the last decade. It increased from 25% to 70%. This has been partly driven by rising share of the government expenditure in gross domestic product as shown in figure 1. The leveraging by government has been accompanied by a deleveraging of the private sector, and the share of household debt to GDP was declining in most of the last decade followed by an increase towards the end of the decade which jumped during COVID-19. South African economy did not recover completely from the great financial crisis of 2008, and despite a V-shape recovery, the growth has been declining steadily in the last decade (figure 3). The inflation was benign during the last decade despite the buildup in government debt, and the very recent rise in inflation is partly driven by higher global commodity prices that led to an increase in inflation in most part of the world. Against this background, we aim to understand the reasons for the increase in term spread and its transmission in the South African economy[2].

3 Results: SVAR

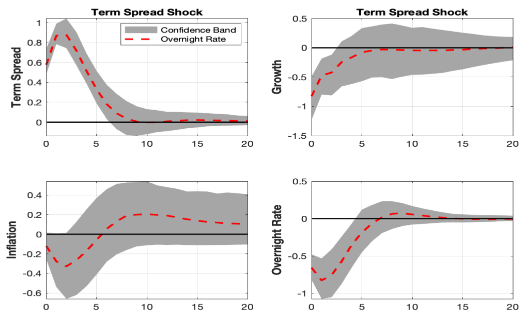
3.1 Unrestricted, Short End and Long End Term Spread Shock

We estimate the baseline models along the lines of Kurmann and Ortok (2013). The base line model is estimate with term spread (difference between ten year and three month rate), log of GDP, log of consumer price, and either overnight or ten-year rate. We use two lags as all information criterion suggest two lags and the VAR satisfies stability condition with two lags to do meaningful impulse response analysis[3]. We use $h = 20$ i.e. the share in forecast error variance over 5 years. Kurmann and Ortok (2013) argue that term spread is driven by accommodating monetary policy which is reacting to positive news shock and this shock leads to an increase in output and a decline in inflation. Hence the

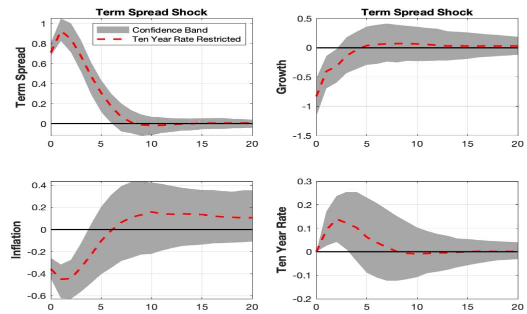
[2]Online appendix gives the link to the data sources used in this paper.

[3]Results related to lag length selection and stability tests are provided in appendix A.5

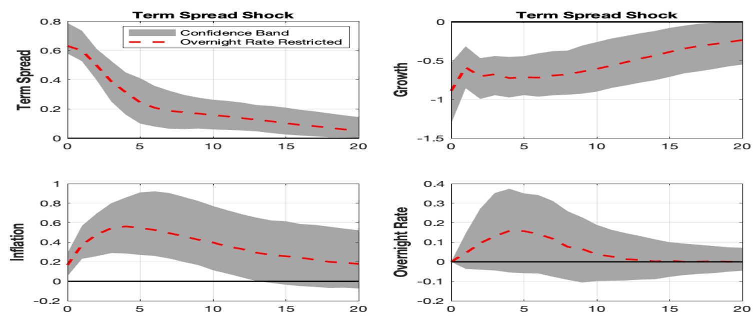
term spread shock behaves as a favorable supply shock.



(a) Unrestricted Term Spread Shock



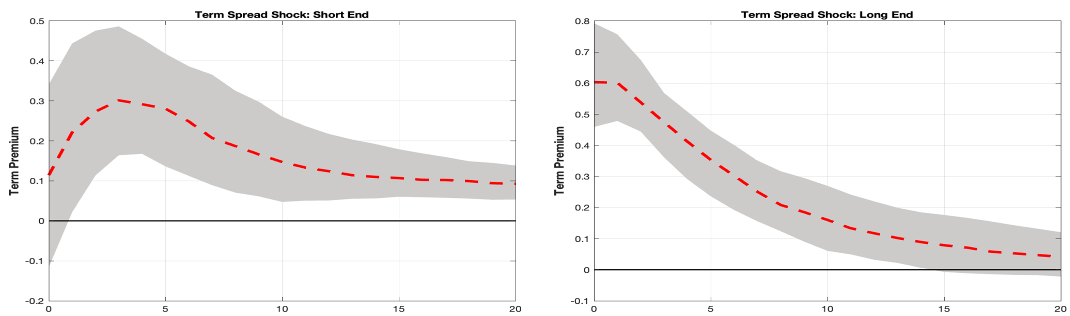
(b) Term Spread Shock at Short End



(c) Term Spread Shock at Long End

Figure 3: Notes: We use four variables (term spread, log GDP , log consumer price and either overnight or ten-year rate) in the SVAR and estimate three models. The term spread is the difference between a ten-year rate and a three-month rate. We identify the term spread shock that explains the maximum forecast error variance of term spread and also explains the higher variance of term spread compared to other variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the responses of the variable due to term spread shock from a model including the overnight rate. b) gives the responses of the variable due to term spread shock from a model including the ten-year rate but the contemporaneous response of the ten-year rate due to term spread shock is restricted to zero. (c) gives the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

The baseline model (unrestricted term spread shock) for South Africa with overnight rate which is comparable to Kurmann and Ortok (2013) indeed suggests that the term spread increase is driven by the decline in short term rate (figure 3 a). But unlike Kurmann and Ortok (2013), we find that it leads to a decline in GDP and inflation although the effect on inflation is not statistically significant. Since we use log GDP, the results suggest that the term spread shock cause a negative growth of 1% at impact and the negative effect persists for four quarters. The term spread shock driven by lowering of short term rates behaves as a negative demand shock which is puzzling. This is the core issue in the South African economy where the lowering of short term rate has failed to stimulate growth and has been accompanied by decreasing growth rate.



(a) Unrestricted Term Spread Shock

(b) Term Spread Shock at Long End

Figure 4: Notes: We use four variables (term spread, GDP , consumer price and either overnight or ten-year rate) in the SVAR and estimate three models. The term spread is the difference between a ten-year rate and a three-month rate. We identify the term spread shock that explains the maximum forecast error variance of term spread and also explains the higher variance of term spread compared to other variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the response of the term premium due to term spread shock from a model including the overnight rate. (b) gives the responses of the term premium due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

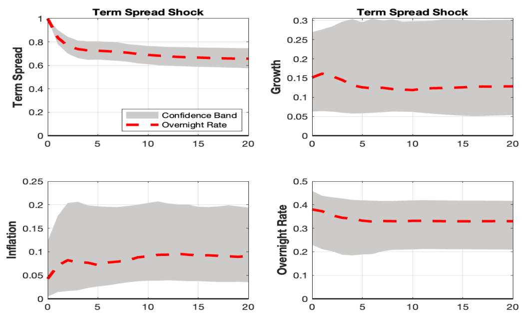
To resolve this puzzle, we allow the term spread shock to originate either by movement in the short end of the rate or long end of the rate. This is the novelty in the identification used in this paper and we achieve that by restricting the contemporaneous response of overnight rate and long term rate due to term spread shock. If we restrict the contemporaneous response of ten year rate due to term spread shock then the **shock**

originates at the short end of the rate and responses are given in (figure 3 b). These responses (price and GDP) are very similar to the responses due to the term spread shock from unrestricted model. Hence we conclude that the term spread shock from the unrestricted model is originating at the short end which is also clear from the movement of overnight rate due to this shock (figure 3a). One important point to note about these two sets of responses is that they produce positive covariance between growth and inflation and hence even they raise term premium, that would not be theoretically consistent. This is because in theory a shock that induces negative covariance between growth and inflation is likely to cause higher term premium, Rudebusch & Swanson (2012).

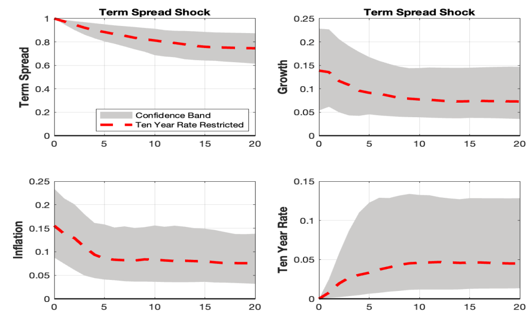
If we restrict the contemporaneous response of overnight rate due to term spread shock then the **shock originates at the long end** of the rate and responses are given in (figure 3 c). The term spread shock originating at the long end leads to an increase in inflation and decrease in growth and hence induces a negative covariance between growth and inflation. Although this shock also induces similar reduction in GDP at impact, the negative effect is persistent and lasts up to five years. The maximum increase in price due to this shock is 0.5% and occurs by the end of fourth quarter. We conclude that out of these three models only the term spread shock originating at the long end can cause increase in term premium that is theoretically consistent.

We show the response of the term premium due to the two shocks (unrestricted and terms spread shock originating at long end) in figure 4. This is because in only these two models we can estimate the response of term premium. These two shock leads to increase in term premium. The shock originating at the short end[4] does not lead to to an increase in term premium in the beginning but it has minor effect on risk premium after a few quarters but it does not induce the theoretically consistent response of inflation and growth as mentioned before. The shock originating at the long end is driven by a change in risk premium, induces significant increase in term premium (figure 4 b) and reduces growth and increases inflation (figure 3 c).

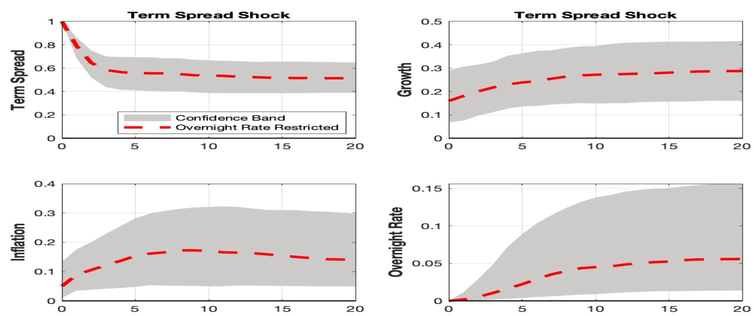
[4]The unrestricted shock and shock originating at short end produces similar response of output and prices



(a) Unrestricted Term Spread Shock



(b) Term Spread Shock at Short End



(c) Term Spread Shock at Long End

Figure 5: Notes: We use four variables (term spread, GDP , consumer price and either overnight or ten-year rate) in the SVAR and estimate three models. The term spread is the difference between a ten-year rate and a three-month rate. We identify the term spread shock that explains the maximum forecast error variance of term spread and also explains the higher variance of term spread compared to other variables. The shaded areas represent the one standard deviation confidence band of the share of term spread shock in forecast error variance of variables. (a) gives the share of term spread shock in forecast error variance of variables from a model including overnight rate. b) gives share of term spread shock in forecast error variance of variables from a model including the ten-year rate but the contemporaneous response of the ten-year rate due to term spread shock is restricted to zero. (c) gives the share of term spread shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

Hence, only the term spread shock originating at the long end is meaningful shock to

understand the behavior of term premium[5].
Figure 5 gives the share of term spread shock in the forecast error variance of the model variables. As expected, the term spread shocks in three models explain the entire variance of the term spread in the beginning, but this decrease with time. The unrestricted term spread shock explains around 40% of the variation in overnight rate suggesting that it is indeed driven by movements at the short end. It explains 15% and 10% of the forecast error variance of output and inflation respectively by the fifth year. The term spread shock originating at short end explains 5% of the forecast error variance of ten year rate by fifth year. It also explains around 10% of the forecast error variance of output and inflation by fifth year.

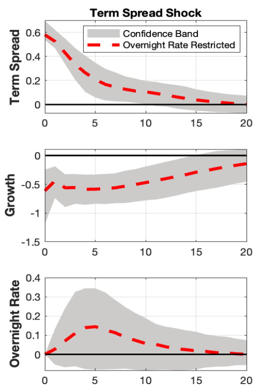
The share of term spread shock originating at long end in forecast error variance of term spread reduces to 50% by the end of year five. The term spread shock originating at long end explains around 30% of the forecast error variance of output by fifth year. The term spread shock originating at long end explains around 20% of the forecast error variance of inflation by third year, and this increases to 15% by the end of fifth year. It is important to mention that term spread shock originating at the long end explains higher proportion of forecast error variance of output and inflation compared to the unrestricted term spread shock and the term spread shock originating at the short end.

One obvious shock driving these co-movements, increase in term spread and inflation and decrease in growth, could be the increasing size of the government in South Africa and we explore that in the next section. Hence we keep the term spread shock originating at the long end in the next section, as this shock increases term premium which is theoretically consistent. This is because in the absence of this shock, the shock to the size of the government can be confounded with the term spread shock at long end, as the government size shock is also expected to an increase term premium.

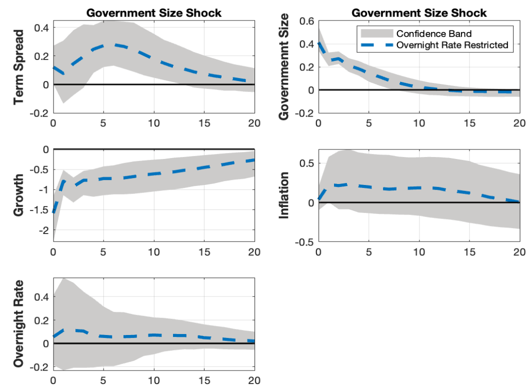
[5]We replace log GDP with log consumption and log investment and estimate two additional four variable models. These results are similar to the reported results in the paper and available on request.

3.2 Term Spread and Government Size Shocks

We extend the model with size of the government and identify two shocks simultaneously, a term spread shock originating at the long end which is endogenously driving term premium and the government size shock that is likely to influence term premium.



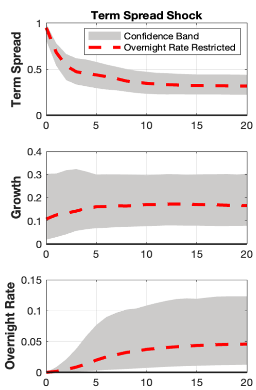
(a) Term Spread Shock at Long End



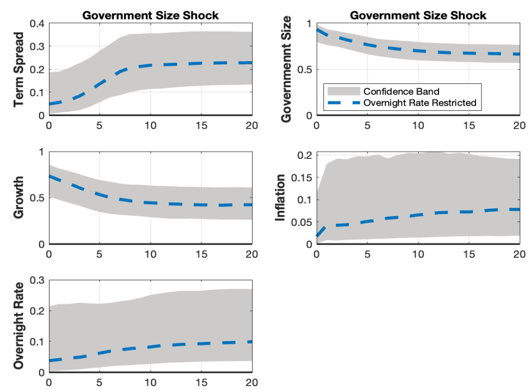
(b) Government Size Shock

Figure 6: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.71 and is statistically significant. The sample period is 1993Q1-2023Q2.

As argued before, we implement joint estimation of these two shocks, as in absence of the term spread shock originating at the long end, the government size shock may not be identified. Government size shock is likely to increase risk premium but some of the increase in risk premium could be purely exogenous which is captured by the term spread shock originating at the long end. *This strategy helps us in estimating the unbiased estimate of the effect of government size shock on risk premium which is one of the main objectives of this paper.*The size of the government is defined as share of government expenditure to GDP and we use national accounts data for the same. As mentioned before we obtain these data from Federal Reserve Bank of Saint Louis and the link for the same is provided in online appendix. The response of model variable due to term spread and government size shocks are given in figure 6.



(a) Term Spread Shock at Long End

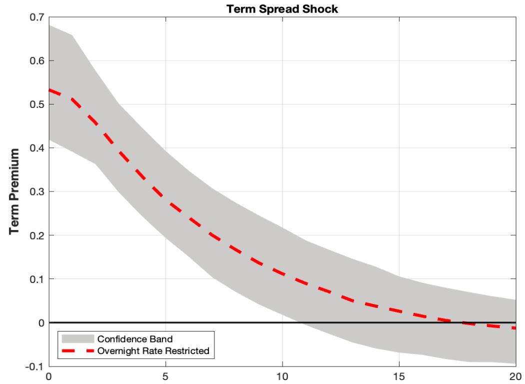


(b) Government Size Shock

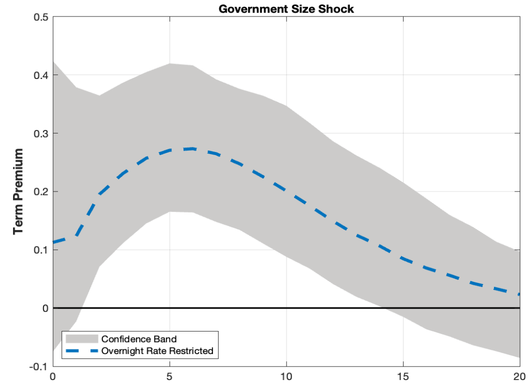
Figure 7: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

The term spread shock originating at long end produces similar response of variables as given in figure 3 (c) and this gives us confidence that the strategy is able to identify the shocks correctly. The term spread shock originating at long end leads to an increase in the government size but the effect is short lived. Government size shock leads to a permanent reduction in growth rate and the impact effect is almost thrice of the term spread shock originating at long end and is more persistent. It is important to mention that we do not restrict the contemporaneous response of overnight rate and long term rate due to shock to the size of the government. A shock to the size of the government leads to higher term spread in the medium run. Figure 7 gives the share of term spread and government size shock in the forecast error variance of the model variables. The shares of term spread shock in the forecast error variance of growth, inflation and interest rates are similar to the ones reported in figure 5(c). The term spread shock explains around 5% of the forecast error variance of the size of the government. Government size shock explains very little variance of the term spread in the beginning, but by the fifth year this becomes almost 20%. This suggests that the changes in term spread are driven by changes in the size of the government and this is also evident in the data.

This shock explains almost 100% of the forecast error variance of the size of the government in the beginning but in the medium run the share declines. Most importantly, this shock explains around 50% of the forecast error variance of growth and 10% of inflation. The shock explains around 10% of forecast error variance of the short term rates by fifth year. This suggests that a shock to the size of the government raises term spread and decreases growth. We estimate additional five variable models for South Africa in which we replace log GDP with log consumption and log investment one by one and also use alternative measures of government size and term spread. As expected the government size shock has more persistent negative effect on capital formation compared to the consumption. Higher size of the government creates risk and this should matter more for investment. These results are given on the online appendix and are similar to the one reported here.



(a) Term Spread Shock at Long End



(b) Government Size Shock

Figure 8: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

Results presented in this section suggest that a shock to the size of the government increases term spread and that too via movement in long term rates in medium run. The long term rates could increase because of higher expected value of the future rates or it can increase due to increase in risk or term premium. A shock to the size of the government is likely to influence term premium by generating a negative covariance between consumption and inflation which is essential for increase in term premium (Rudebusch & Swanson (2012) and Horvath et al. (2020) and Bretscher et al.(2020)). The shock to the size of the government generates a negative covariance between inflation and growth (figure 6). Although the effect on the inflation is not statistically significant in model with log GDP, we compute the correlation between the response of log GDP and log consumer prices due to the government size shock and that turns out -0.71 and is statistically significant[6].

This also generates a negative covariance between consumption growth and inflation as shown in online appendix. Although, we do not have explicit data on term premium to estimate a model with term premium and obtain its response, the empirical setting used in this paper allows the estimation of the response of the term premium due to the shocks as done in the previous sections. Figure 8 presents the response of the term premium due to term spread shock originating at long end and government size shock[7]. Bringing the size of the government does not influence the response of term premium due to the term spread shock. The term spread shock at the long end continues to be driven by an increase in risk premium unlike the term spread shock originating at the short end as shown in the previous section. Further, we see that the increase in term spread due to the shock to government size shock is mostly driven by increase in term premium. This is the reason that the term spread shock is contractionary in South Africa[8].

3.3 Expansionary, Recessionary and Neutral Government Size Shock

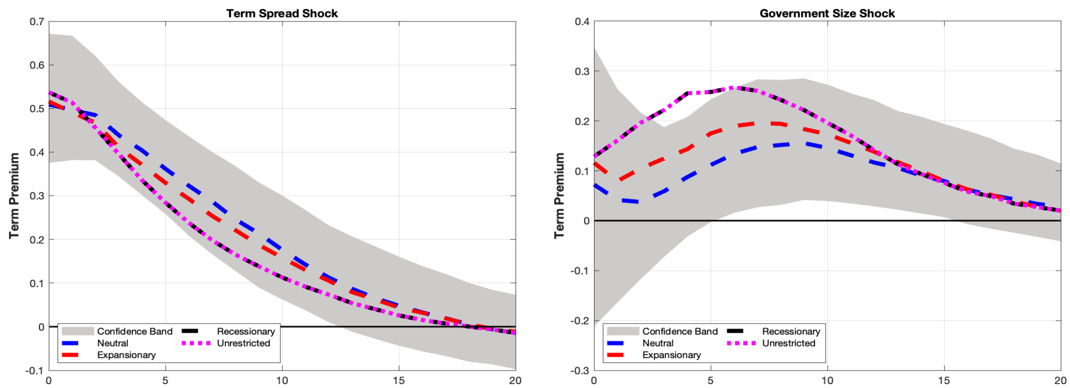
In the previous section we identified unrestricted government size shock. In this section, We estimate additional five variable models for South Africa in to identify expansionary, neutral and recessionary government size shocks. Expansionary government size shock is identified by restricting the response of output due to government size shock to be $>= 0$

[6]The correlation between the response of log private final consumption expenditure and log consumer prices due to the government size shock is -0.73 and the correlation between the response of log gross fixed capital formation and log consumer prices due to the government size shock is -0.72. Both these correlations are statistically significant as well.

[7]These responses are also significant at 95% confidence interval and these results are available on request.

[8]Kurmann and Ortok (2013) suggest that the term spread shock in the US is driven by the news about future productivity which allows federal reserve to lower the interest rates. This increases output and decreases inflation unlike what we see in South Africa as explained in the previous section. Cascaldi-Garcia (2017) refutes the claims in Kurmann and Ortok (2013), and in response to that, Kurmann and Ortok (2017) argue that Cascaldi-Garcia (2017) does not make any distinction between positive and negative news and that is problematic, and as the distinction between positive and negative news is made, the claims of Kurmann and Ortok (2013) remain true.

for two time periods $t = 0$ and $t = 1$.



(a) Term Spread Shock at Long End

(b) Government Size Shock

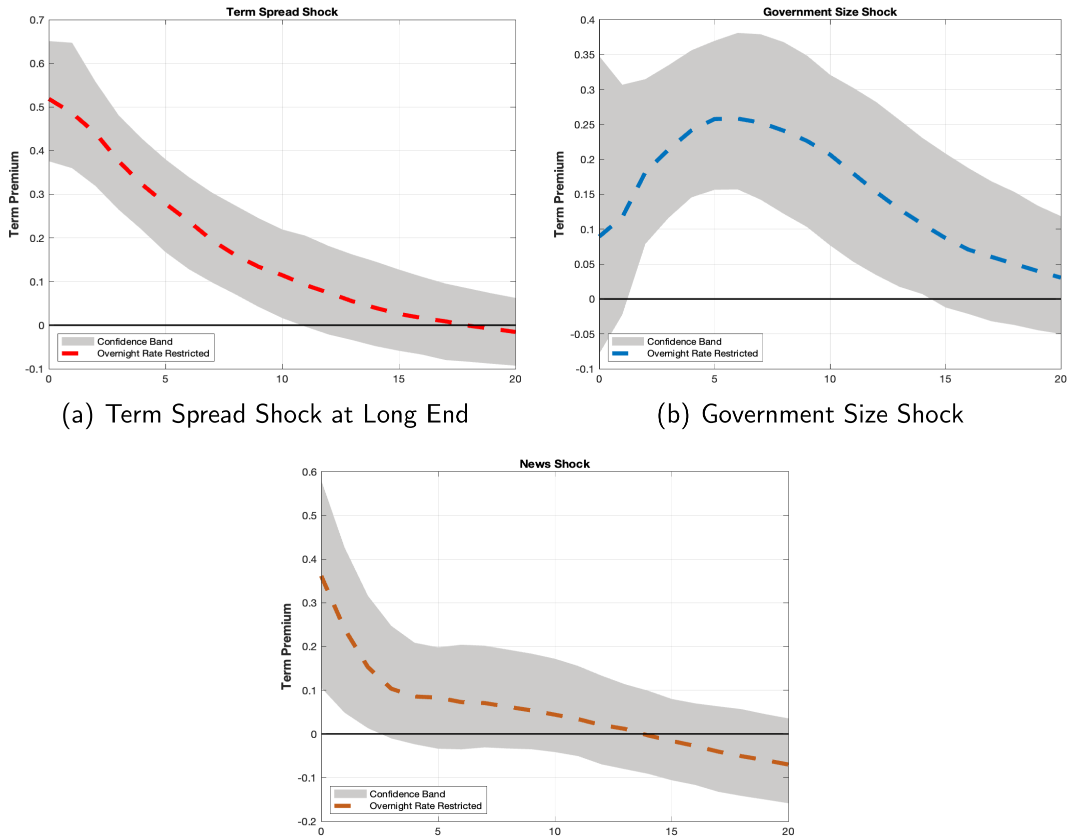
Figure 9: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The (a) and (b) give responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The shaded areas represent the one standard deviation confidence band of response due to the respective shocks obtained from the model having neutral government size shock. The neutral government size shock is identified using additional restriction that government size shock does not affect the output for two time periods $t = 0$ and $t = 1$. The expansionary government size shock is identified using additional restriction that government size shock increases the output for two time periods $t = 0$ and $t = 1$.The recessionary government size shock is identified using additional restriction that government size shock decreases the output for two time periods $t = 0$ and $t = 1$. The sample period is 1993Q1-2023Q2.

Neutral government size shock is identified by restricting the response of output due to government size shock to be $= 0$ for two time periods $t = 0$ and $t = 1$. Recessionary government size shock is identified by restricting the response of output due to government size shock to be $<= 0$ for two time periods $t = 0$ and $t = 1$. These restrictions have been explained in the section 2.5. The responses of model variables and forecast error variance decomposition from these models are given in online appendix. Here we only report the response of term premium due to the two identified shocks.

As we can see from the figure 9 both the recessionary and unrestricted government size shock produces identical response of the term premium and hence it confirms that unrestricted shock is a recessionary shock in the model. But both expansionary and neutral government size shock also leads to higher term premium. The neutral government size shock is identified by restricting the response of output due to government size shock to zero and hence this is a shock to the government expenditure. Rudebush and Swanson (2012) are able to generate a term premium response of 0.17 basis points only compared to a 20 basis points response of term premium in this paper. It is important to mention that Rudebush and Swanson (2012) use a theoretical model and their objective is to generate the level of term premium in the model which is compared to the level of term premium and data. Unlike us they do not aim to estimate the response of term premium due to government expenditure or size shock. The confidence band in the figure is given for the response of term premium due to the neutral government size shock and we conclude that recessionary government size shock produces statistically higher term premium than the neutral and expansionary government size shock. Hence we can say that increase in the size of the government leads to significant risk in the bond market and these risks are even higher in times of recession. We also find that the term spread shock originating at long end produces identical response of term premium in the models estimated with unrestricted and recessionary government size shock. Also, the recessionary government size shock produces highest negative correlation between output and prices and the expansionary government size shock produces smallest negative correlation between output and inflation among all the four government size shock estimated in this paper. These are provided in the online appendix.

3.4 Term Spread, Government Size and News Shocks

Kurmann and Ortok (2013) argues that term spread could be driven by the news about future productivity. The government size is also likely to be influenced by the news about future productivity. Expected slowdown in the economy may lead to increase in the size of the government and vice-versa. To rule out this channel, We extend the model to include a news shock similar to Barsky and Sims (2011).



(a) Term Spread Shock at Long End

(b) Government Size Shock

(c) News Shock

Figure 10: We use five variables (term spread, size of the government, GDP , consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify three shocks, term spread shock originating at the long end, government size shock and new shock that maximize the sum of share of these three orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red, blue and orange lines give the responses of the term premium due to term spread, government size and news shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

The responses of model variables and forecast error variance decomposition from the model is given in online appendix. Here we only report the response of the term premium due to these three shocks. The news shock is expected increase in output observed in advance. The news shock explains the maximum share in forecast error variance of log GDP but does not affect the log GDP contemporaneously. The response of model variables and forecast error variance decomposition due to the three shocks are given in appendix. The response of model variables due to the term spread shock originating at long end and government size shock are similar to figure 6.

The news shock leads to reduction in overnight rate which is similar to the findings in Kurmann and Ortok (2013). The news shock leads to increase in output with lag as expected and decreases inflation but these effects are not statistically significant. The news shock also leads to fiscal consolidation as expected. The share of term spread shock originating at long end and government size shock in forecast error variance of model variables are given in online appendix. These are similar to the results shown in figure 7 except that we find that government size shock becomes more important in explaining the variation in output. Figure 10 gives the response of term premium due to these three shocks. The response of term premium due to government size shock and term spread shock originating at the long end is similar to figure 8. The news shock increases term premium but the effect is short lived.

In the next section we present a New Keynesian model with term premium. In literature this type of models have been used to generate the level of the term premium comparable to the data. In this paper we aim to generate response of term premium in this model which is comparable to the response of term premium due to government size in the data. We estimate the parameters of the model by impulse response matching. In other words, the parameters of the model are identified using the condition that it produces similar response of these variables due to a shock to the size of the government. The model is similar to Horvath et al. (2022) except that we introduce a shock to the size of the government which is our focus.

4 Model

4.1 Household

The model is based on the New Keynesian DSGE model of Rudebusch & Swanson (2012), Kisacikoglu (2020) and Horvath et al. (2022). The household continuation value of utility (V_t) is of Epstein–Zin form and is given by

$$V_t = U\left(C_t, L_t\right) + \beta\left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \text{ if } U\left(C_t, L_t\right) \geq 0$$

$$V_t = U\left(C_t, L_t\right) - \beta\left[E_t\left(-V_{t+1}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \text{ if } U\left(C_t, L_t\right) < 0$$

where C_t is household consumption and L_t is the labour hours. The Epstein-Zin preferences allows the risk aversion to be separated from intertemporal elasticity of substitution by having an additional parameter α. A higher degree of risk aversion is required to generate reasonable term premium in the data, and Epstein–Zin preferences allow that without reducing the intertemporal elasticity of substitution. To be consistent with balance growth path the following functional form is used for $U\left(C_t, L_t\right)$ as argued in Rudebusch and Swanson (2012):[9].

$$U\left(C_t, L_t\right) = \frac{C_t^{1-\phi}}{1-\phi} + \chi_0 Z_t^{1-\phi} \frac{\left(1-L_t\right)^{1-\eta}}{1-\eta}$$

χ_0 is calibrated to give steady state ratio of leisure to labor as 2. The nominal budget constraint is given by:

$$\left(1-\tau_t\right) W_t L_t + R_{t-1} B_{t-1} = B_t + P_t C_t$$

where W_t is the nominal wage in time period t, τ_t is the labour income tax rate in time t and B_t is the nominal bond holding at time t. The availability of nominal bonds creates inter-temporal market and allows the households to smooth consumption. In real terms, the budget constraint is given by:

[9] Andreasen (2012) use a non-separable utility function in labour and consumption that is consistent with balanced growth path too.

$$(1-\tau_t)\, w_t L_t + R_{t-1}\frac{b_{t-1}}{\pi_t} = b_t + C_t$$

Where Z_t is a deterministic productivity trend. The Lagrangian of the household problem is given by:

$$\mathcal{L} = V_0 - E_0 \sum_{t=0}^{\infty} \omega_t \left(\frac{C_t^{1-\phi}}{1-\phi} + \chi_0 Z_t^{1-\phi} \frac{(1-L_t)^{1-\eta}}{1-\eta} + \beta \left[E_t V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} - V_t \right) +$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left((1-\tau_t)\, w_t L_t + R_{t-1}\frac{b_{t-1}}{\pi_t} - b_t - C_t \right)$$

where ω_t and λ_t are Lagrange multipliers. First order condition with respect to consumption is given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \omega_t C_t^{-\phi} - \beta^t \lambda_t = 0 \implies \lambda_t = \frac{\omega_t C_t^{-\phi}}{\beta^t}$$

First order condition with respect to labour is given by:

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\omega_t \chi_0 Z_t^{1-\phi} (1-L_t)^{-\eta} + \beta^t \lambda_t (1-\tau_t)\, W_t = 0$$

Combining first order condition with respect to consumption and labour, we obtain household's intra-temporal substitution condition given by:

$$Z_t^{1-\phi} \chi_0 (1-L_t)^{-\eta} = C_t^{-\phi} (1-\tau_t)\, W_t$$

First order condition with respect to bond is given by:

$$\frac{\partial \mathcal{L}}{\partial B_t} = -\lambda_t + \beta \lambda_{t+1} \frac{R_t}{\pi_{t+1}} = 0$$

Using household first order condition with respect to consumption, we obtain:

$$\frac{\omega_t C_t^{-\phi}}{\beta^t} = \beta \frac{\omega_{t+1} C_{t+1}^{-\phi}}{\beta^{t+1}} \frac{R_t}{\pi_{t+1}}$$

First order condition with respect to the value function is given by:

$$\frac{\partial \mathcal{L}}{\partial V_t} = \omega_{t-1}\beta \left[E_t V_t^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}} V_t^{-\alpha} = E_{t-1}w_t$$

This gives us $E_t w_{t+1}$ and using it we obtain the household consumption Euler equation:

$$1 = \beta \left(\frac{C_t}{C_{t+1}}\right)^{\phi} \left(\frac{V_{t+1}}{\left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} \frac{R_t}{\pi_{t+1}}$$

Where $\Lambda_{t,t+1} = \frac{\pi_{t+1}}{R_t} = \beta \left(\frac{C_t}{C_{t+1}}\right)^{\phi} \left(\frac{V_{t+1}}{\left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}$ is the real stochastic discount factor. Since we have a trend in the model, we make the value function stationary by dividing it both sides with $Z_t^{1-\phi}$. We need two auxiliary equation to write the value function as in Rudebusch and Swanson (2012) and Hovrath et al. (2022). This is required to improve the numerical accuracy of the model.

$$\frac{\left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}}{Z_t^{1-\phi}} = \frac{Vk_t}{Z_t^{1-\phi}} = \bar{V}\gamma^{1-\phi}\left(\frac{Ve_t}{Z_t^{1-\phi}}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{Ve_t}{Z_t^{1-\phi}} = \frac{E_t V_{t+1}^{1-\alpha}}{\left[\bar{V}\gamma^{1-\phi}\right]^{1-\alpha}\left(Z_t^{1-\phi}\right)^{1-\alpha}}$$

Where $\bar{V}$ is the steady state value of the value function and is given by $\bar{V} = \frac{\frac{c^{1-\phi}}{1-\phi} + \chi_0 \frac{(1-L)^{1-\eta}}{1-\eta}}{1-\beta}$. c is the steady state value of stationary variable $c_t = \frac{C_t^{1-\phi}}{Z_t^{1-\phi}}$. γ is the steady state value of $\frac{Z_t}{Z_{t-1}}$ and we assume it to be 1. We derive the household's consumption-only coefficient of relative risk aversion as in Swanson (2012) in presence of labour income tax and it is given by[10]:

[10] The consumption only coefficient of risk aversion is curvature of household value function respect to the assets. It is related to the Arrow (1965) and Pratt (965) measure of risk aversion which is defined as the curvature of period utility with respect to the consumption. It is basically the one shot payment that household is willing to make today to avoid a risk of size σ in the next period when σ becomes very small.The detailed derivation is given in appendix C.

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{(1-L)}{L}\frac{\phi}{\eta}} + \alpha\frac{(1-\phi)}{1 + \frac{(1-L)(1-\phi)}{L(1-\eta)}}$$

This is a steady state measure. Although the steady state tax rate or steady state share of the government expenditure in GDP does not enter the risk aversion, it does not mean that these do not influence risk aversion. A higher government expenditure is like a wealth shock to the households, and the households respond to that by increasing the labour hours. Hence a higher government expenditure in the steady state implies higher steady state labour hours and that implies higher values of consumption-only coefficient of relative risk aversion for reasonable values of α, θ and η. We can write the consumption-only coefficient of relative risk aversion as:

$$R^c(a;\theta) = \frac{1}{\frac{1}{\phi} + \frac{(1-L)}{L\eta}} + \alpha\frac{1}{\frac{1}{(1-\phi)} + \frac{(1-L)}{L(1-\eta)}}$$

Which further becomes

$$R^c(a;\theta) = \frac{1}{IES + \text{Frisch Elasticity}} + \alpha\frac{1}{\frac{1}{(1-\phi)} + \frac{\eta}{(1-\eta)}}$$

where we use $\frac{(1-L)}{L(1-\eta)}$ as Frisch elasticity of labor supply. A model with usual CRRA preferences will give consumption-only coefficient of relative risk aversion as:

$$R^c(a;\theta) = \frac{1}{IES + \text{Frisch elasticity}}$$

In this case, the higher values of IES and Frisch Elasticity imply lower value of consumption-only coefficient of relative risk aversion. This is intuitive as higher value of both these allows higher smoothing of consumption arising due to any adverse exogenous shock. In case of Epstein-Zin preferences, the relationship of consumption-only coefficient of relative risk aversion with IES is dependent on Frisch elasticity. In figure 11, we show the relationship between coefficient of risk aversion and IES for two values of Frisch elasticity. Despite the dependence on Frisch elasticity, we find that the coefficient of risk aversion declines with higher value of IES. The relationship of consumption-only coefficient of relative risk aversion with Frisch elasticity is dependent on IES too. As expected, a IES value of 1 implies very low values for the coefficient or risk aversion for any value of Frisch elasticity. Similar to the usual preferences, we find that coefficient of risk aversion declines with higher value of Frisch elasticity but for $IES = 1$, it marginally increases with higher value of Frisch elasticity.

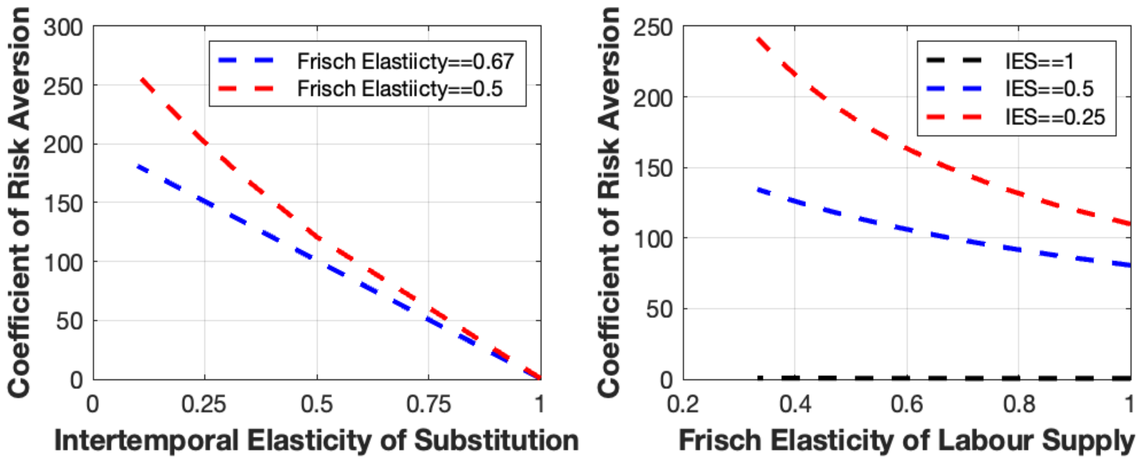


Figure 11: Coefficient of risk aversion. We assume $\alpha == -200$

The final goods producers, intermediate goods producers and the conduct of monetary policy are similar to the standard new Keynesian model, and hence we have provided these in online appendix.

4.2 Government

Real public spending (G_t) evolves as a time varying fraction of real output as in Justiniano et al. (2010);

$$G_t = \left(1 - \frac{1}{\kappa_t}\right) Y_t$$

κ_t determines the size of the government. Higher value of κ_t implies higher value of $\frac{G}{Y}$. This shock is same as the government size shock implemented in the empirical section. We need a similar shock in this model, as we aim to estimate the parameters of the model using impulse response matching. The government size shock (κ_t) follows the stationary stochastic process given by:

$$log(\kappa_t) = (1-\rho_\kappa)log(\kappa) + \rho_\kappa log(\kappa_{t-1}) + \epsilon_{\kappa,t} \quad 0 \leq \rho_\kappa < 1 \qquad \epsilon_{\kappa,t} \sim N(0, \sigma_\kappa^2)$$

$(1 - \frac{1}{\kappa})$ gives steady state value to government consumption to output ratio which we calibrate with the data. The government budget constraint is given by:

$$B_t + W_t L_t \tau_t = R_{t-1} B_{t-1} + G_t$$

which in real terms is given by

$$b_t + w_t L_t \tau_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + G_t$$

The model requires a fiscal rule that sets taxes based on debt level and business cycle.

$$\tau_t - \bar{\tau} = \rho_\tau(\tau_{t-1} - \bar{\tau}) + \rho_{\tau b}\left(\frac{b_t - b}{y}\right) + \rho_{\tau y}\left(\frac{y_t - y}{y}\right) + \epsilon_t^\tau$$
$$\epsilon_t^\tau \sim N\left(0, \sigma_\tau^2\right)$$

where τ_t is the tax rate at time t, $\bar{\tau}$ is the steady state tax rate, ρ_τ is the persistence in the tax rate, $\rho_{\tau b}$ is the sensitivity of tax rate to government debt to GDP ratio, $rho_{\tau y}$ is the sensitivity of tax rate to output gap. The tax rule is similar to the discussion in Leeper, Plante and Traum (2010). The steady state tax rate is calculated using the steady state values of the model parameters. Higher value of debt to GDP ratio and higher size of the government imply higher steady state tax rate in the model.

4.3 Bond Pricing

The price of a default-free n-period zero-coupon bond that pays $1 at maturity can be described with a recursive formula:

$$p_t^{(n)} = E_t\left[\Lambda_{t,t+1} p_{t+1}^{n-1}\right]$$

The continuously compounded return to maturity on the n-period zero-coupon bond is defined to be

$$r_t^n = \frac{-1}{n} log\left(p_t^n\right)$$

where r_t^n is the net rate and log if R_t^n. We define the risk neutral bond price as:

$$\hat{p}_t^n = e^{-r_t} E_t \hat{p}_{t+1}^{n-1}$$

The implied term premium is defined as the difference between the yield expected by the risk-averse investor minus the yield expected by the risk-neutral investor.

4.4 Aggregation

The aggregate output in the economy is given by:

$$Y_t = S_t^{-1} A_t K_t(i)^{1-\theta} \left(Z_t L_t(i)\right)^{\theta}$$

Where $K_t = \bar{k} Z_t$. The aggregate output depends upon the dispersion in price in the economy given by:

$$S_t = \left[\int \left[\frac{P_t(i)}{P_t}\right]^{-\phi_m/\theta} di\right]^{\theta}$$

The aggregate resource constraint in the economy is given by:

$$Y_t = C_t + I_t + G_t$$

$I_t = (1 - \delta + \gamma) Z_t \bar{k}$ is the investment required to keep capital constant on the balanced growth path.

5 Results from Estimated Model

Table 1: Calibrated Parameters

Parameter	Value	Description
β	0.99	Discount Factor
$\frac{L_{max}-L}{L}$	2	Leisure Labour Ratio
δ	0.25	Annual Depreciation 10%
θ	.66	Labour Share
ξ	0.8	Calvo Parameter
ϕ_m	5	Net Markup (25%)
b/y	0.70	Debt to GDP Ratio
κ	$5/4$	$(1-\frac{1}{\kappa})$ Share of Government Expenditure in GDP (20%)

We set Calvo parameter (ξ) to 0.8, which implies an average contract length of about five quarters which is same as in Hovrath et al. (2022). Rudebush and Swanson (2012) calibrate this to 0.78. We calibrate the discount factor to 0.99 which implies a nominal steady state interest rate of 4%. We calibrate the value of L_{max} as 3 and L as 1. This gives $\frac{L_{max}-L}{L}$ as 2. We calibrate the labor share to 0.66 which is standard in the literature. The calibration implies a net markup of 25%. Our calibration implies debt to GDP ratio of 70% and the share of government in GDP at 20% which is consistent with the South African Economy in recent years explained in the data section. We also estimate additional models with debt to GDP ratio of 25%. This is because the debt was at low level in early 2010s and this also allows us to compare the parameters especially risk aversion parameter required on different levels of debt to match the response of term premium due to government size shock. We further calibrate the parameters related to other shocks in the model and these calibrated parameters are given in table 2[11].

We obtain remaining parameter estimates from the model using the following optimization

[11]Initially, we attempt estimating these parameters by matching the response of other variables due to the shock to the size of the government in the model and data. But these estimation attempts give a very poor fit of the response of term premium in the model with the data and hence we further calibrate these parameters which are not essential for the response of the term premium due to the shock to the size of government.

Table 2: Calibrated Parameters

Parameter	Value	Description
		Standard Deviation of Shocks
σ_a	0.04	Standard Deviation of Technology Shock
$\sigma_{\phi m}$	0.04	Standard Deviation of Markup Shock
σ_τ	0.04	Standard Deviation of Tax Shock
σ_i	0.04	Standard Deviation of Interest Rate Shock
		Persistence of Shocks
ρ_a	0.80	Persistence of Technology Shock
$\rho_{\phi m}$	0.80	Persistence of Markup Shock
		Monetary Policy
ρ_i	0.6	Interest Rate Persistence
ρ_π	0.3	Sensitivity of Interest Rate to Inflation
ρ_g	0.2	Sensitivity of Interest Rate to Output Gap

$$\xi^* == \arg\min_{\xi} \left[\hat{\Psi} - \Psi\left(\xi\right)\right]' V^{-1} \left[\hat{\Psi} - \Psi\left(\xi\right)\right]$$

Where $\hat{\Psi}$ contains the response of term premium due to the shock to the size of government from vector autoregression model explained in the previous section. ξ contains the parameter being estimated and $\Psi\left(\xi\right)$ is the response of the same variables due to a shock to the size of government from the model. We take V is an identity matrix. We only estimate parameters related to the shock to the size of the government, tax rule, preference parameters related to households which determine consumption only coefficient of risk aversion that is important for the response of term premium and moving average of inflation. We estimate two sets of parameters, one for expansionary government size shock and another for recessionary government size shock. We only match the response of term premium and government size shock as we focus on the response of term premium.

Table 3 and 4 gives estimated parameters based on the responses due to expansionary and recessionary government size shock. Both these estimates have been obtained with a debt to GDP ratio of 70% and governments size of 20%. The inter-temporal elasticity

Table 3: Estimated Parameters Using Impulse Response Matching: Expansionary Government Size Shock

Parameter	Value	Description
		Standard Deviation of Shocks
σ_κ	0.0437	Standard Deviation of Government Size Shock
		Persistence of Shocks
ρ_κ	0.9065	Persistence of Government Size Shock
		Fiscal Policy
ρ_τ	0.0390	Persistence in Tax Rate
$\rho_{\tau b}$	0.1518	Sensitivity of Tax to Debt Ratio
$\rho_{\tau y}$	-0.0064	Sensitivity of Tax to Output Gap
		Monetary Policy
θ_π	-0.1709	Coefficient of Moving Average Inflation
		Household Preferences
ϕ	9.5189	$1/\phi$ is Inter-temporal Elasticity of Substitution
η	9.8948	$\frac{1-L}{L\eta}$ is Frisch Elasticity of Labor Supply
α	-1515.1	Coefficient Associated with Risk Aversion

Notes: We match the response of the term premium due to expansionary government size shock in the model with the response of the term premium due to government size shock in the structural vector auto-regression model.

of substitution refers to the % change in consumption between present and future due to 1 percentage point change in real interest rate.

The inter-temporal elasticity of substitution is much higher in recession (0.16) compared to the expansion (0.11). Kilponen et al. (2022) show that the equilibrium real interest rate elasticity of output is in the range of 0.05-0.20 in the US. The estimate obtained in this paper lies in the range of estimates provided by Kilponen et al. (2022) and that gives us confidence that the parameters have been correctly identified. The Frisch elasticity of labour supply in recession (0.41) is twice of the expansion (0.20). These are similar to the findings in Attanasio et al. (2018) although we have lower magnitude. Attanasio et al. (2018) argue that, Frisch elasticity of labour supply varies from 1.53 in normal times to 1.61 in the first quarter of a recession to 1.71 after four quarters.

Table 4: Estimated Parameters Using Impulse Response Matching: Recessionary Government Size Shock

Parameter	Value	Description
		Standard Deviation of Shocks
σ_κ	0.0629	Standard Deviation of Government Size Shock
		Persistence of Shocks
ρ_κ	0.8030	Persistence of Government Size Shock
		Fiscal Policy
ρ_τ	0.0455	Persistence in Tax Rate
$\rho_{\tau b}$	-0.0190	Sensitivity of Tax to Debt Ratio
$\rho_{\tau y}$	-0.8157	Sensitivity of Tax to Output Gap
		Monetary Policy
θ_π	0.4302	Coefficient of Moving Average Inflation
		Household Preferences
ϕ	6.2822	$1/\phi$ is Inter-temporal Elasticity of Substitution
η	4.8178	$\frac{1-L}{L\eta}$ is Frisch Elasticity of Labor Supply
α	-6789.0	Coefficient Associated with Risk Aversion

Notes: We match the response of the term premium due to expansionary government size shock in the model with the response of the term premium due to government size shock in the structural vector auto-regression model.

The persistence of tax rate is similar in the two scenarios. As expected the sensitivity of tax to debt ratio is positive in expansion but marginally negative in recession. Most importantly during the expansion tax does not respond to output gap (very low negative value) but during recessions the tax rises. This suggest that tax is not working as a automatic stabiliser as it should. Ideally the tax should have been higher in expansion than recession. As expected the inflation persistence given by θ_π is almost twice of the value in recession. All these suggest that the parameters are able to distinctly identify the expansionary and recessionary periods.

The government expenditure shock has higher variance and lower persistence in recessionary period compared to the expansionary period. Most importantly the coefficient

α determining the risk is much higher in recessionary period compared to expansionary period. The consumption only coefficient of risk aversion in recession (9521.1) is more than twice of the value in expansion (4430.3). This is expected and corroborate the findings of Andreasen et al. (2023) who argue that risk matters more in the recession.

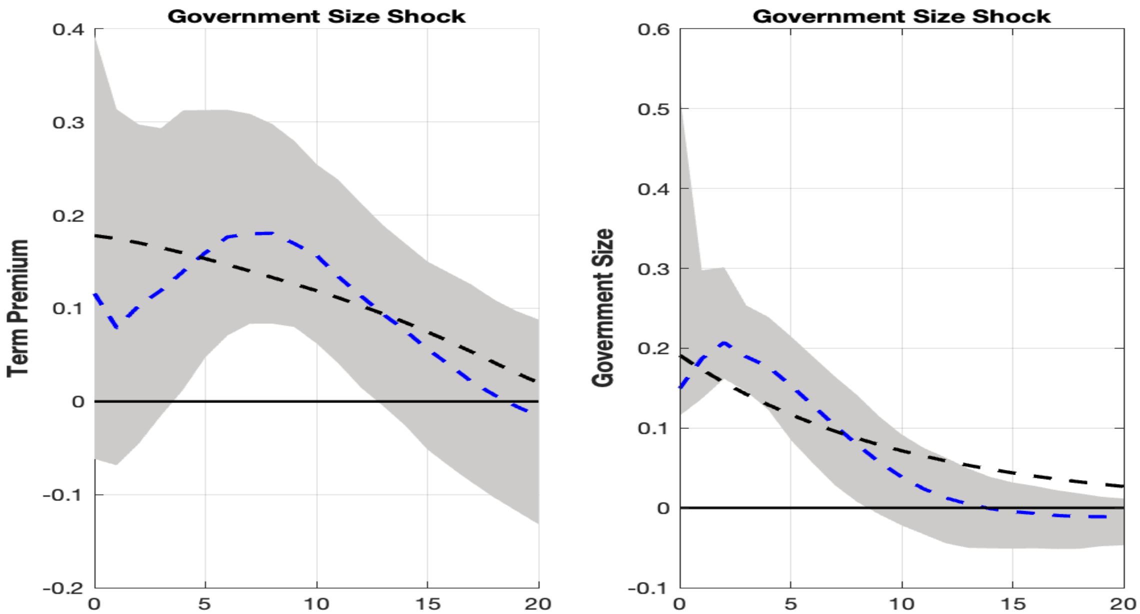


Figure 12: Response of variables due to the recessionary government size shock. The structural vector autoregression (data) is the five variable model with overnight rate. The model is the new Keynesian model with parameters given in tables 1, 2 and 3.

The coefficient of risk aversion estimated in this paper is much higher than in Rudebush and Swanson (2012) and Hovrath et al. (2022). Rudebush and Swanson (2012) are only able to generate a term premium response of 0.17 basis points with a coefficient of risk aversion of 110. Given that the term premium response in this paper is more than 100 times of the term premium response from the theoretical model in Rudebush and Swanson (2012), we do not think higher value of risk aversion in this paper is problematic. If we compare it with a conventional log utility function then that has a risk aversion coefficient of 1 and Rudebush and Swanson (2012) have a risk aversion 110 times higher than that to generate a term premium response of 0.17 basis points only. Moreover the quantification of risk aversion in terms of a number is problematic as it is hard to make sense of a particular number. But our results suggest that the risk aversion is more than twice in recession than expansion that is intuitive and easy to understand.

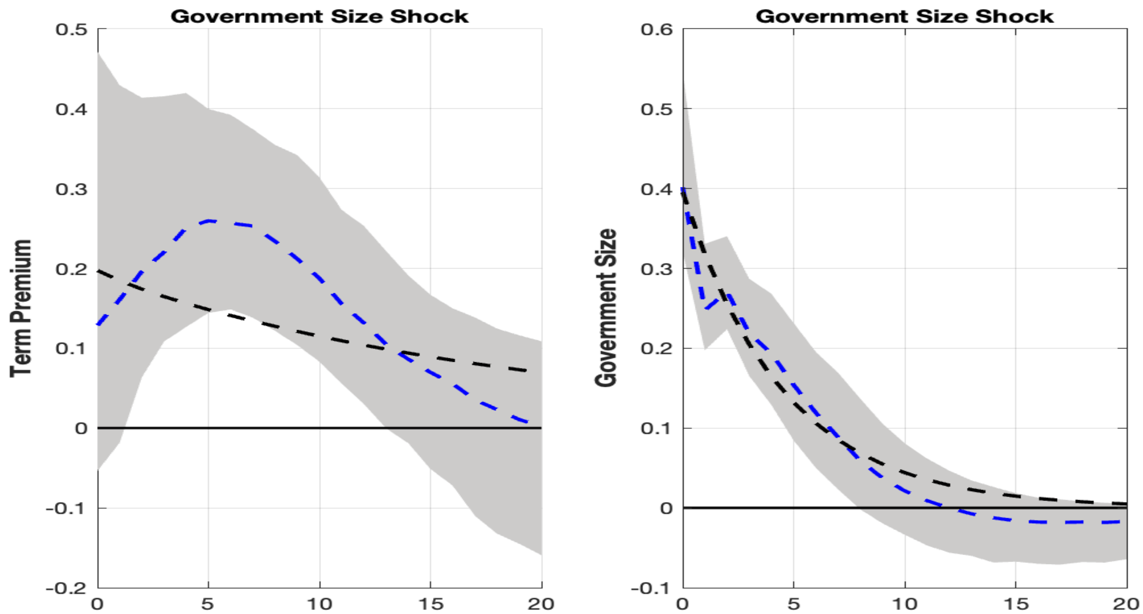


Figure 13: Response of variables due to the recessionary government size shock. The structural vector autoregression (data) is the five variable model with overnight rate. The model is the new Keynesian model with parameters given in tables 1, 2 and 3.

Figures 12 and 13 give the response of term premium and government size shocks and corresponding responses from the model based on the estimated parameters. As we can see the response from the model lie in the confidence band of the response from the data and hence we argue that this new Keynesian model can generate similar response of term premium due to government size shock. This is because the government size shock in the model and the data is almost identical. The above estimates were done assuming the debt to GDP ratio of 70%. We estimate parameters using a debt to GDP ratio of 25% to analyse the effect of debt on risk aversion. The inter-temporal elasticity of substitution is lower and and Frisch elasticity of labour supply is higher. Although the estimates of α is lower but the consumption only coefficient of risk aversion is almost twice of the

Table 5: Estimated Parameters Using Impulse Response Matching: Recessionary Government Size Shock

Parameter	Value	Description
		Standard Deviation of Shocks
σ_κ	0.0639	Standard Deviation of Government Size Shock
		Persistence of Shocks
ρ_κ	0.7940	Persistence of Government Size Shock
		Fiscal Policy
ρ_τ	0.0155	Persistence in Tax Rate
$\rho_{\tau b}$	0.0994	Sensitivity of Tax to Debt Ratio
$\rho_{\tau y}$	-0.8526	Sensitivity of Tax to Output Gap
		Monetary Policy
θ_π	0.2574	Coefficient of Moving Average Inflation
		Household Preferences
ϕ	22.9289	$1/\phi$ is Inter-temporal Elasticity of Substitution
η	3.4636	$\frac{1-L}{L\eta}$ is Frisch Elasticity of Labor Supply
α	-1704.9	Coefficient Associated with Risk Aversion

Notes: We match the response of the term premium due to expansionary government size shock in the model with the response of the term premium due to government size shock in the structural vector auto-regression model.

value of consumption only coefficient of risk aversion obtained with debt to GDP ratio of 70%. This suggest that a lower level of debt a higher risk aversion is required to generate the similar response of term premium. Figures 14 gives the response of term premium and government size shocks and corresponding responses from the model based on the estimated parameters with debt to GDP ratio of 25% and these responses are statistically identical. .

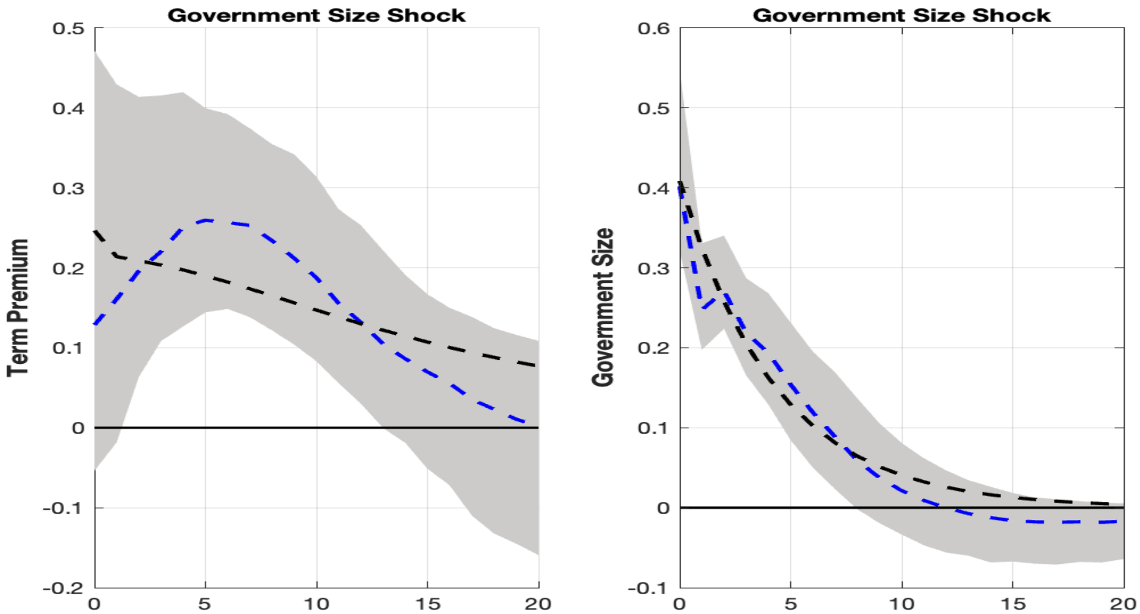


Figure 14: Response of variables due to the recessionary government size shock. The structural vector autoregression (data) is the five variable model with overnight rate. The model is the new Keynesian model with parameters given in tables 1, 2 and 3.

6 Conclusion

In the 2010s, the growth of GDP declined significantly in South Africa with increasing size of the government and an increase in term spread which is puzzling. In the literature, the term spread increase has been associated with economic expansion (Benzoni et al. (2014)). But the existing literature does not make the explicit distinction between term spread originating at the short- and long-end. The term spread can change due to movement in policy rate (short-end) or due to change in risk premium (long end). We estimate three term spread shocks, unrestricted and term spread shocks originating at the short- and long-end. Although, we do not have term premium data for South Africa, the SVAR model allows us to estimate the response of term premium due to the shocks using expectations hypothesis of term structure. Results suggest that the term spread shock at the long end is indeed driven by an increase in risk premium (term premium) unlike the term spread shock originating at the short end. Moreover the term spread shock originating at the long end increases inflation and reduced growth generating a negative covariance between these two variables unlike the unrestricted term spread shock and term spread shock originating at short end. Hence we conclude that term spread shock originating at long end generates increase in term premium which is theoretically consistent.

We augment the model with the size of the government and jointly identify exogenous shocks to the size of the government and term spread shocks originating at long-end using max share identification. This is because the size of the government and term premium are related and we do not want the shock to the size of the government to be confounded with term spread shock. We find that the shock to the size of the government increases term spread and inflation and decreases growth in South Africa. Further we find that the increase in term spread due to this shock is driven mostly by an increase in term premium. Hence we argue that the size of the government is a bottleneck in promoting growth as it is increasing market risk, i.e., term premium, which is increasing the long rate, hurting investment and growth. This is also making monetary policy ineffective as the central bank is not able to lower long term rates despite lowering the short rates. Hence fiscal consolidation is the way forward for the South African economy. The right way of achieving fiscal consolidation is still a question that needs to be answered.

We further identify expansionary, neutral and recessionary government size shocks by restricting the response of output due to government size shock. The neutral government size shock is effectively a government expenditure shock as this is arising due to change in G and not Y. We find that recessionary government size shock induces highest increase in term premium and highest negative correlation between output and consumption due to government size shock and among all these four government size shock estimated in this paper; unrestricted, expansionary, neutral and recessionary. A higher size of the government is a risk for the household as it reduces the household ability to smooth adverse shock and as expected these risks are higher in recession.

Both term spread and the government size could be driven by the news about the future economic activity. To rule out the possibility that the observed effect of government size on term premium could be driven by these confounding factors, we estimate a model with three orthogonal shocks, term spread shock originating at long end, government size and news about the future economic activity. The results suggest that the effect of government size shock on term premium is not influenced by the inclusion of new shock. This suggest that the identification strategy used in this paper is able to identify the exogenous movement in the government size and its effect on term premium.

In the next stage we formulate a macro-finance model of term premium and estimate the parameters of the model, matching the response of term premium due to government size shock in the model and data. The model is able to generate a similar response of the term premium as seen in the data but it requires a high value of consumption only coefficient of risk aversion. Most importantly the estimated coefficient of risk aversion is twice during the recession compared to the expansion. These results suggest that an increase in the size of the government in the last decade has obstructed the monetary policy transmission and led to slowdown in the growth rate. Fiscal consolidation is necessary for the effective monetary transmission and to stimulate growth in the medium run.

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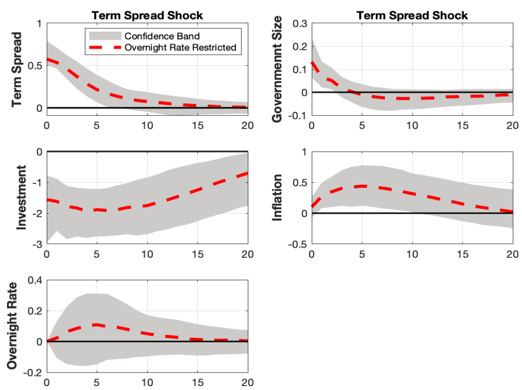
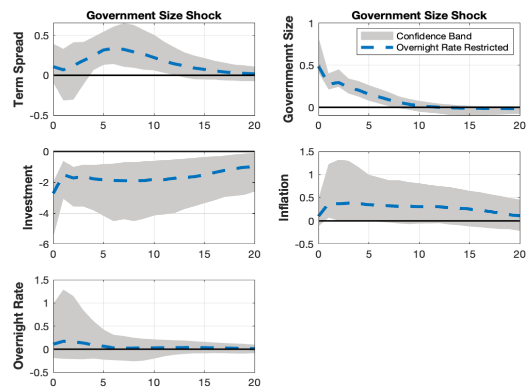
Online Appendix

A Additional Results from Five Variable VAR Models

A.1 Model with Investment Instead of GDP

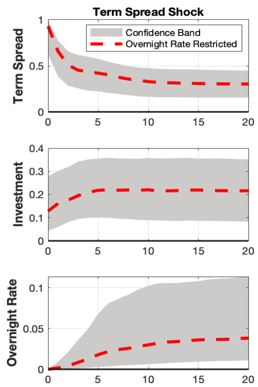


(a) Term Spread Shock at Long End

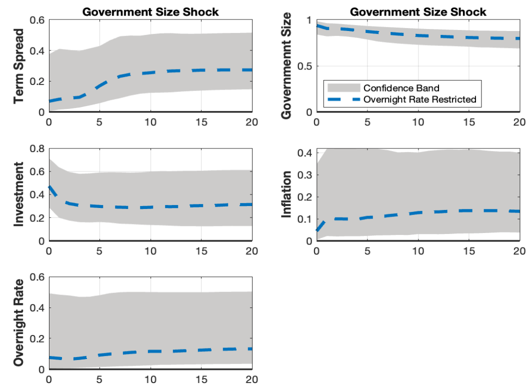


(b) Government Size Shock

Figure A.1: Notes: We use five variables (term spread, government size, gross capital formation, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log gross fixed capital formation and log consumer prices due to the government size shock is -0.72 and is statistically significant. The sample period is 1993Q1-2023Q2.

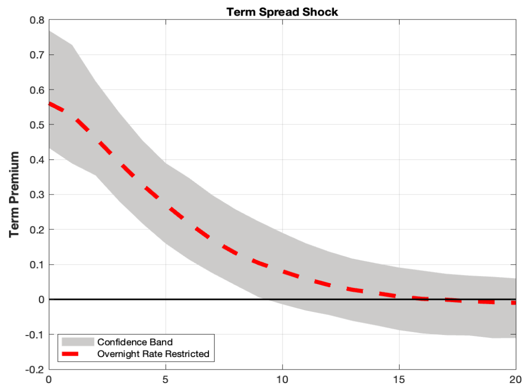


(a) Term Spread Shock at Long End

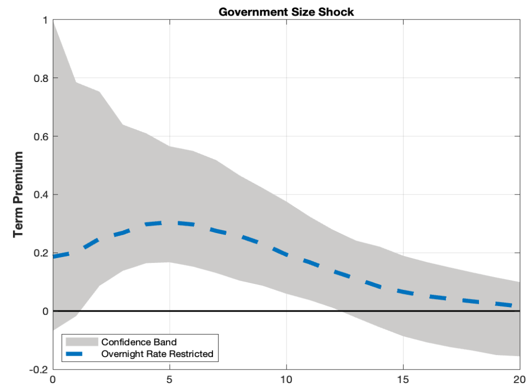


(b) Government Size Shock

Figure A.2: Notes: We use five variables (term spread, government size, gross capital formation, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



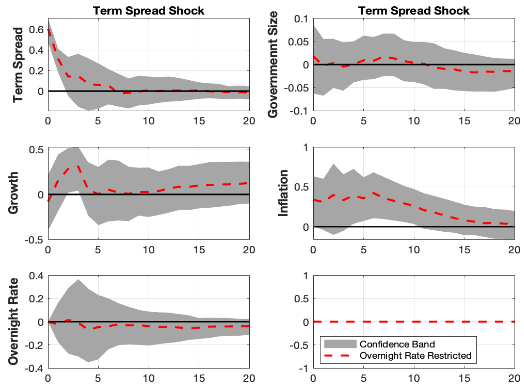
(a) Term Spread Shock at Long End



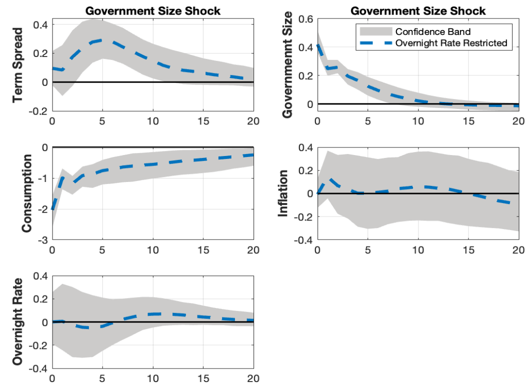
(b) Government Size Shock

Figure A.3: We use five variables (term spread, size of the government, gross capital formation, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.2 Model with Private Consumption Instead of GDP

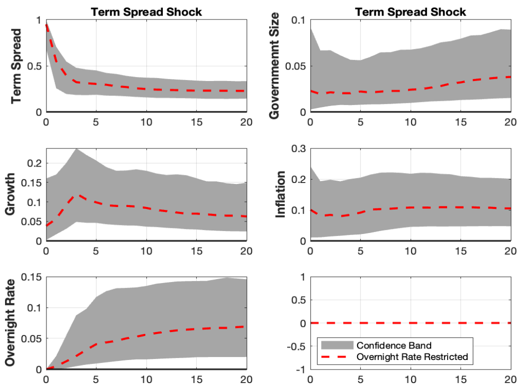


(a) Term Spread Shock at Long End

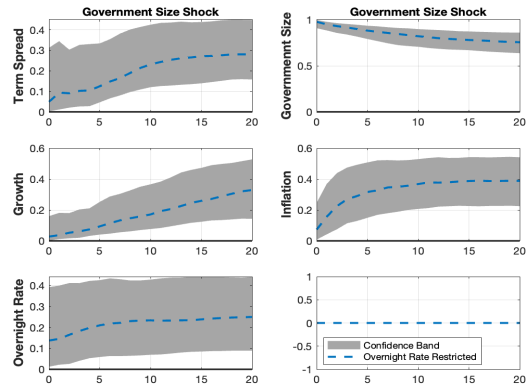


(b) Government Size Shock

Figure A.4: Notes: We use five variables (term spread, government size, private consumption, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log private final consumption expenditure and log consumer prices due to the government size shock is -0.73 and is statistically significant. The sample period is 1993Q1-2023Q2.

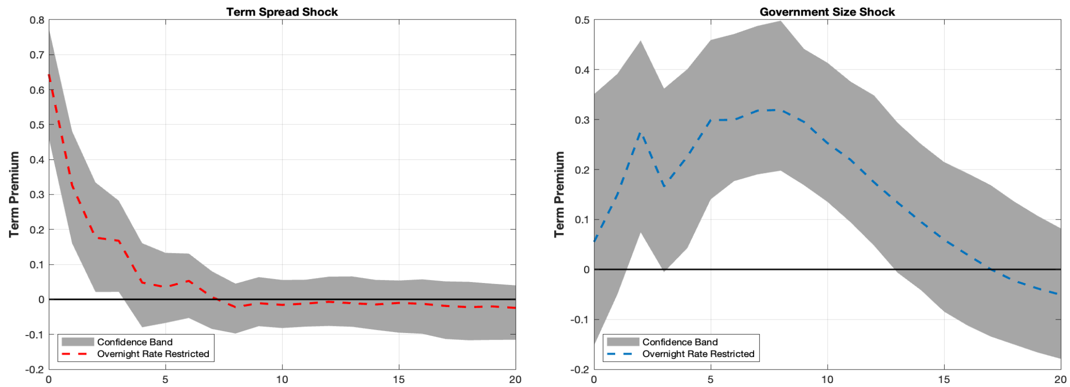


(a) Term Spread Shock at Long End



(b) Government Size Shock

Figure A.5: Notes: We use five variables (term spread, government size, private consumption, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

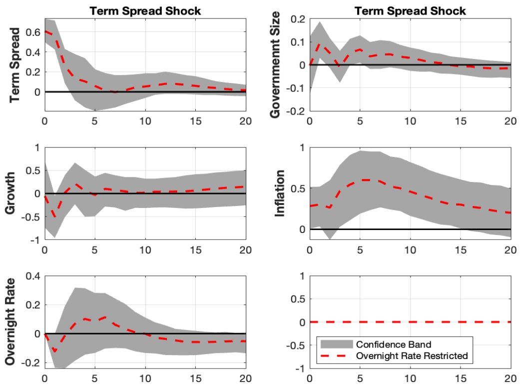


(a) Term Spread Shock at Long End

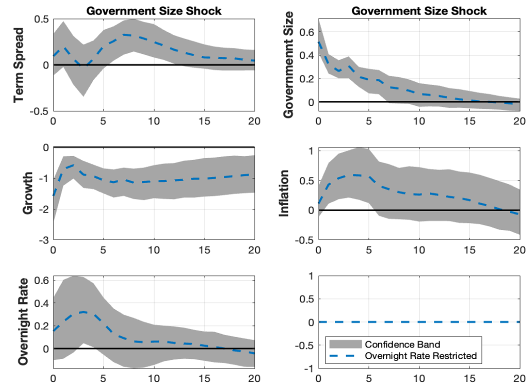
(b) Government Size Shock

Figure A.6: We use five variables (term spread, size of the government, private consumption, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.3 Model with Alternative Term Spread

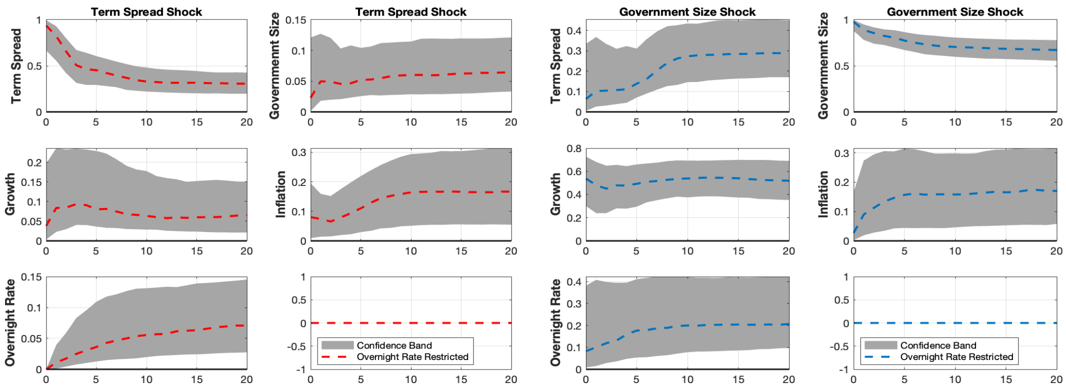


(a) Term Spread Shock at Long End



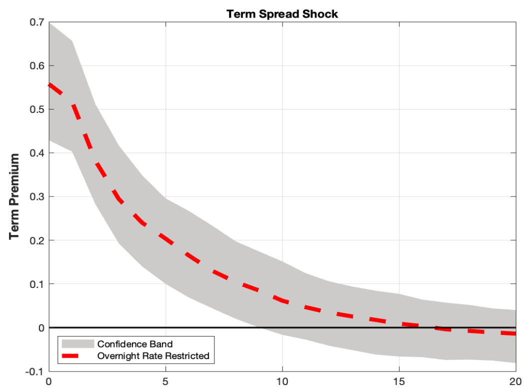
(b) Government Size Shock

Figure A.7: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.72 and is statistically significant. The sample period is 1993Q1-2023Q2.

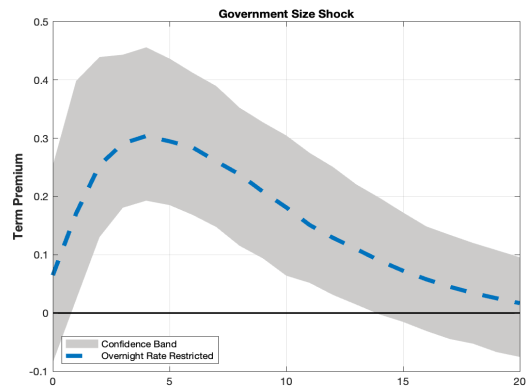


(a) Term Spread Shock at Long End (b) Government Size Shock

Figure A.8: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



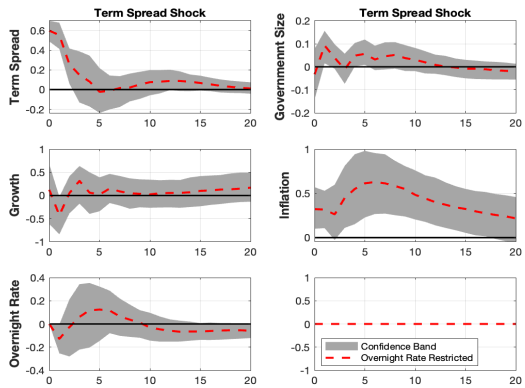
(a) Term Spread Shock at Long End



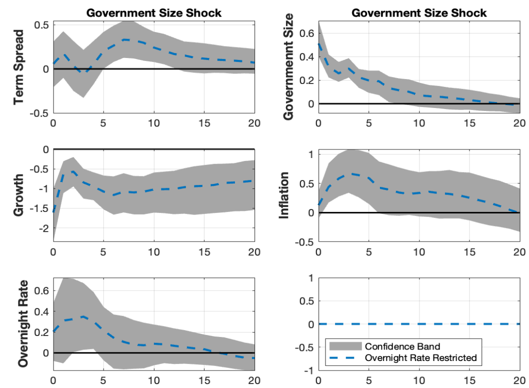
(b) Government Size Shock

Figure A.9: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.4 Model with Alternative Term Spread and Size of the Government

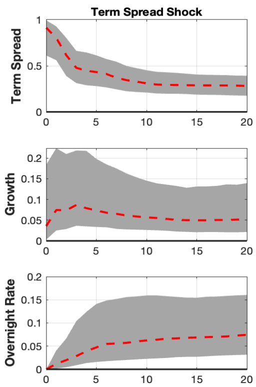


(a) Term Spread Shock at Long End

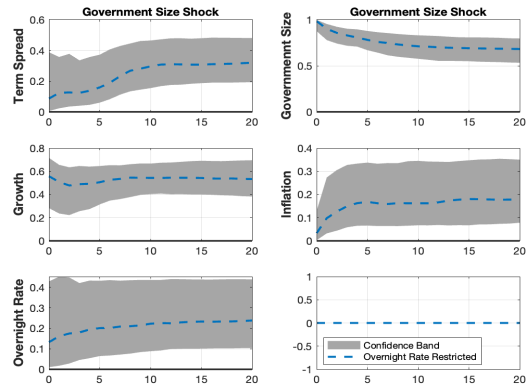


(b) Government Size Shock

Figure A.10: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.69 and is statistically significant. The sample period is 1993Q1-2023Q2.

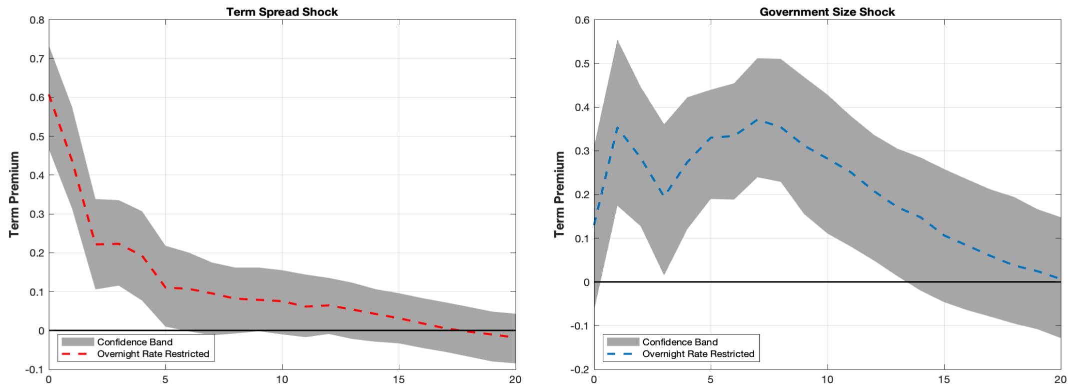


(a) Term Spread Shock at Long End



(b) Government Size Shock

Figure A.11: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

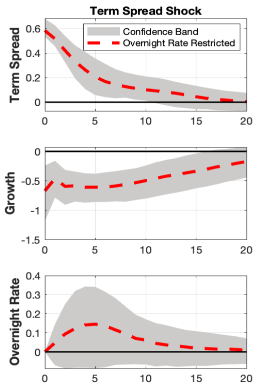


(a) Term Spread Shock at Long End

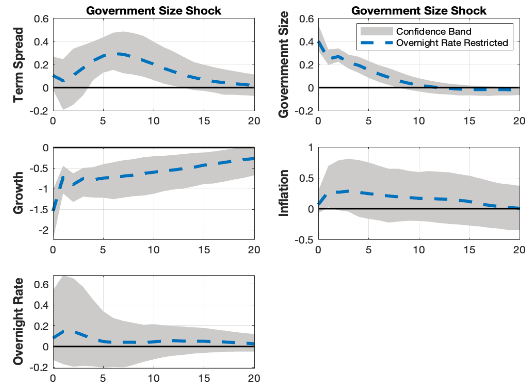
(b) Government Size Shock

Figure A.12: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an overnight rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.5 Recessionary Government Size Shock

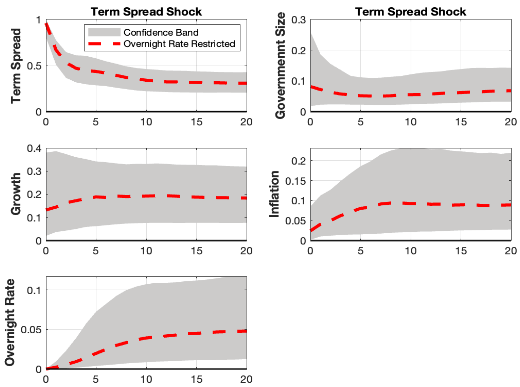


(a) Term Spread Shock at Long End

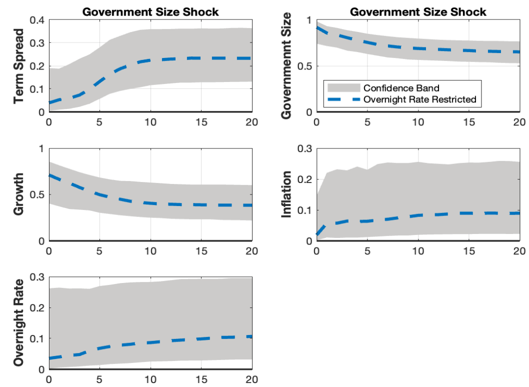


(b) Government Size Shock

Figure A.13: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The recessionary government size shock is identified using additional restriction that government size shock decreases the output for two time periods $t = 0$ and $t = 1$. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.74 and is statistically significant. The sample period is 1993Q1-2023Q2.

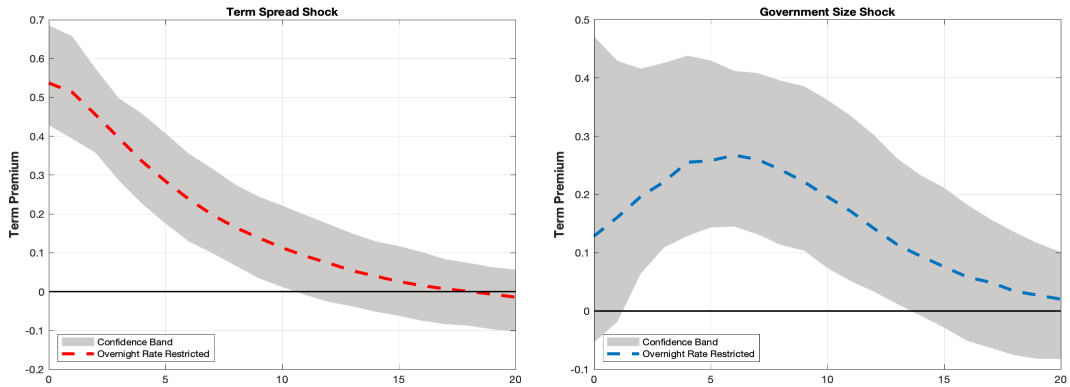


(a) Term Spread Shock at Long End



(b) Government Size Shock

Figure A.14: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

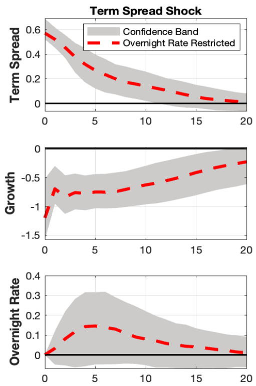


(a) Term Spread Shock at Long End

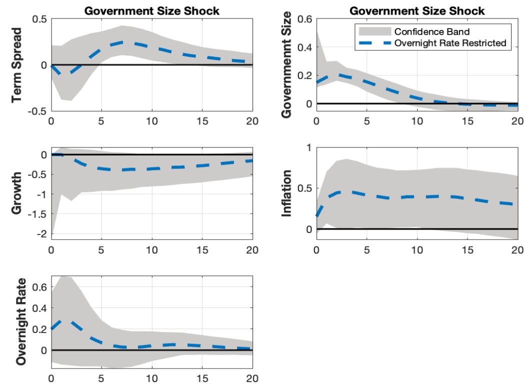
(b) Government Size Shock

Figure A.15: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.6 Expansionary Government Size Shock

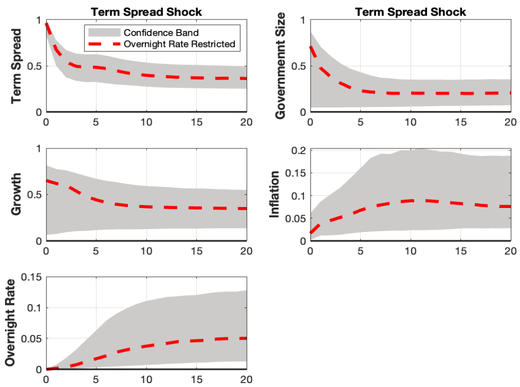


(a) Term Spread Shock at Long End

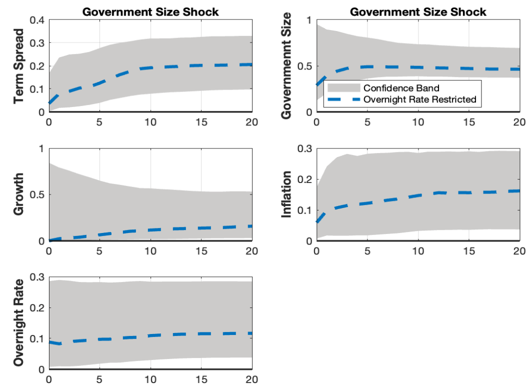


(b) Government Size Shock

Figure A.16: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The expansionary government size shock is identified using additional restriction that government size shock increases the output for two time periods $t = 0$ and $t = 1$.The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.63 and is statistically significant. The sample period is 1993Q1-2023Q2.

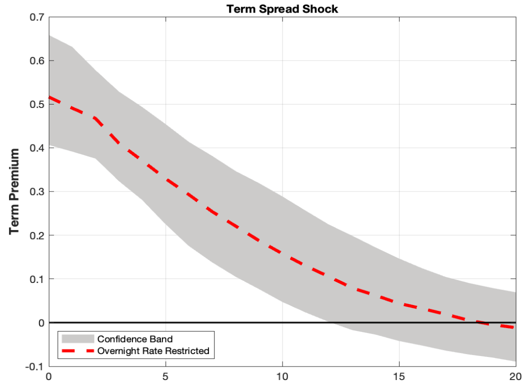


(a) Term Spread Shock at Long End

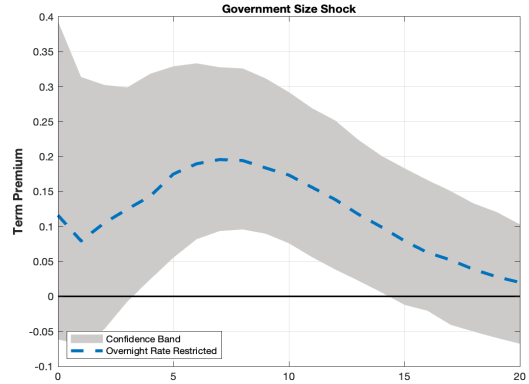


(b) Government Size Shock

Figure A.17: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



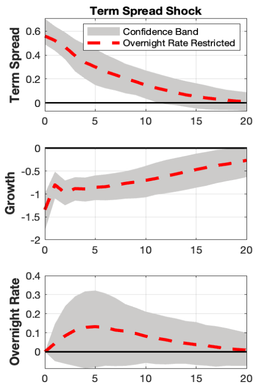
(a) Term Spread Shock at Long End



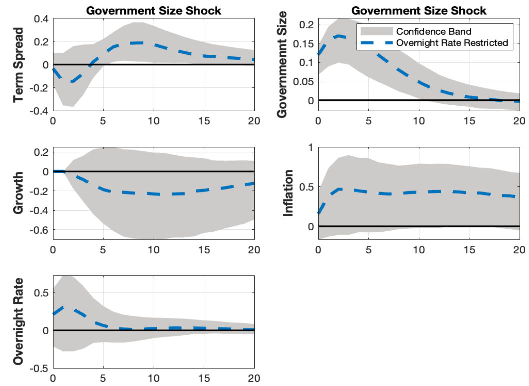
(b) Government Size Shock

Figure A.18: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

A.7 Neutral Government Size or Government Expenditure Shock

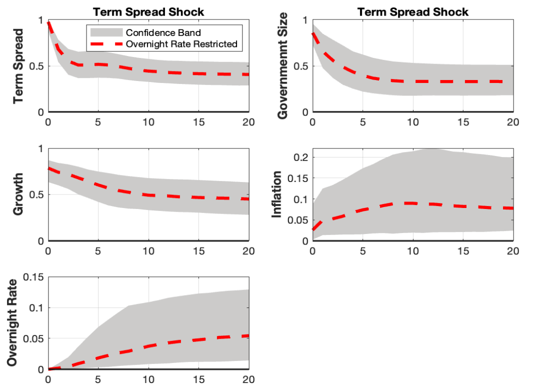


(a) Term Spread Shock at Long End

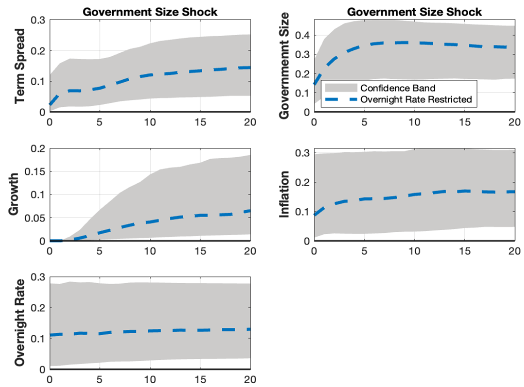


(b) Government Size Shock

Figure A.19: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The neutral government size shock is identified using additional restriction that government size shock does not affect the output for two time periods $t = 0$ and $t = 1$. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.69 and is statistically significant. The sample period is 1993Q1-2023Q2.

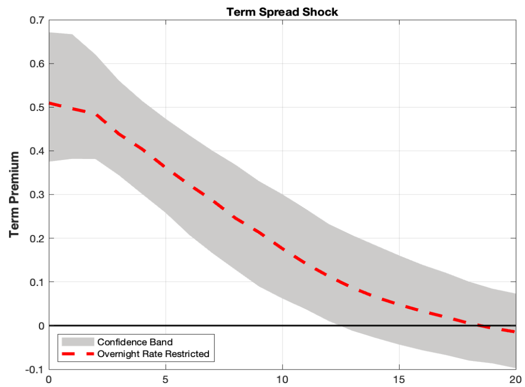


(a) Term Spread Shock at Long End

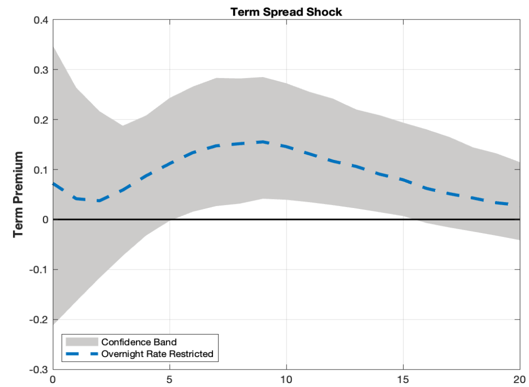


(b) Government Size Shock

Figure A.20: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of share of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.



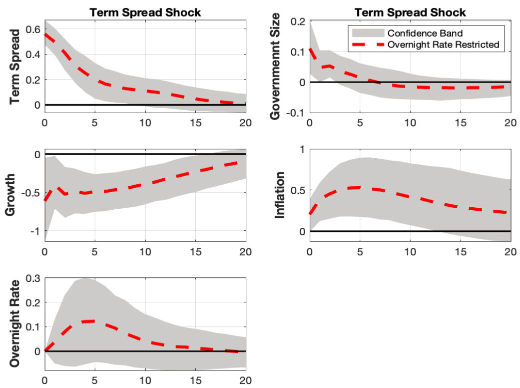
(a) Term Spread Shock at Long End



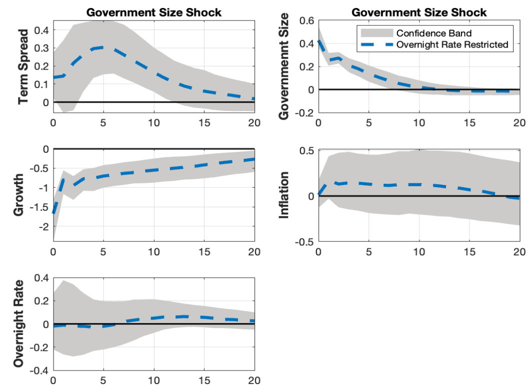
(b) Government Size Shock

Figure A.21: We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three month rate. The size of the government is given by the ratio of government final consumption expenditure to private final consumption expenditure. We jointly identify two shocks, term spread shock originating at the long end and government size shock that maximize the sum of shares of these two orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band. The red and blue lines give the responses of the term premium due to term spread and government size shock respectively from a model including the overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2022Q1.

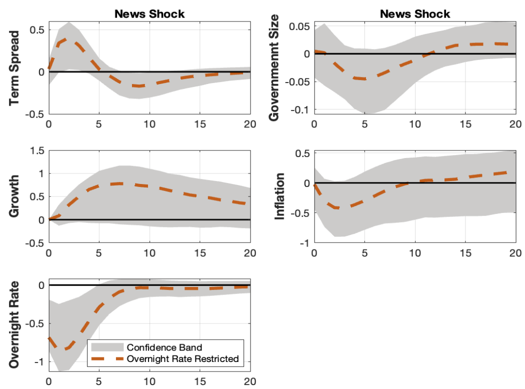
A.8 Term Spread, Government Size and News Shocks



(a) Term Spread Shock at Long End

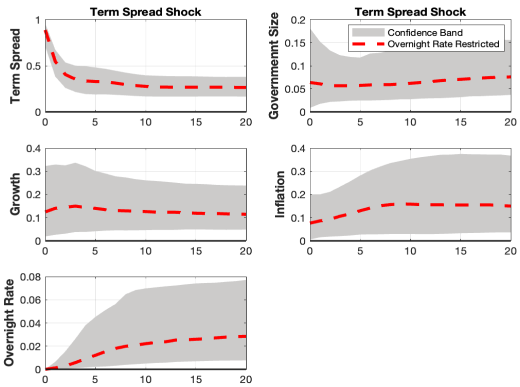


(b) Government Size Shock

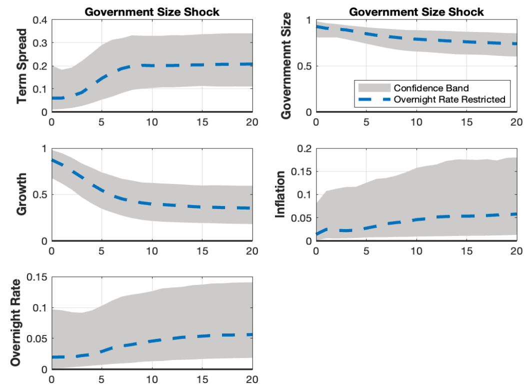


(c) News Shock

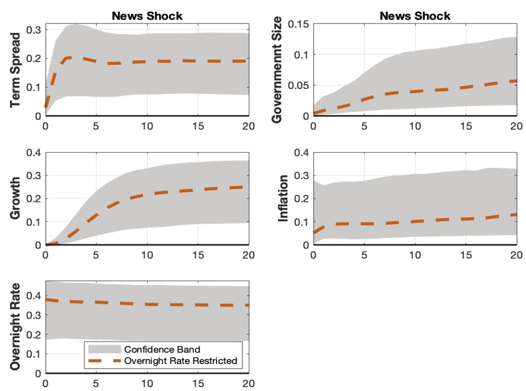
Figure A.22: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify three shocks, term spread shock originating at the long end, government size shock and new shock that maximize the sum of share of these three orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the the responses of the variable due to term spread shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the the responses of the variable due to government size shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (c) gives the the responses of the variable due to news shock from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. The correlation between the response of log GDP and log consumer prices due to the government size shock is -0.69 and is statistically significant. The sample period is 1993Q1-2023Q2.



(a) Term Spread Shock at Long End



(b) Government Size Shock



(c) News Shock

Figure A.23: Notes: We use five variables (term spread, government size, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and a three-month rate. The size of the government is given by the ratio of the government final consumption expenditure to GDP. We jointly identify three shocks, term spread shock originating at the long end, government size shock and new shock that maximize the sum of share of these three orthogonal shocks in forecast error variance of respective variables. The shaded areas represent the one standard deviation confidence band of responses of variables due to the term spread shock. (a) gives the share of term spread shock in forecast error variance of variables from a model including the overnight rate but the contemporaneous response of the overnight rate due to term spread shock is restricted to zero. (b) gives the share of government size shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. (c) the share of news shock in forecast error variance of variables from a model including overnight rate but the contemporaneous response of the overnight rate due to the term spread shock is restricted to zero. The sample period is 1993Q1-2023Q2.

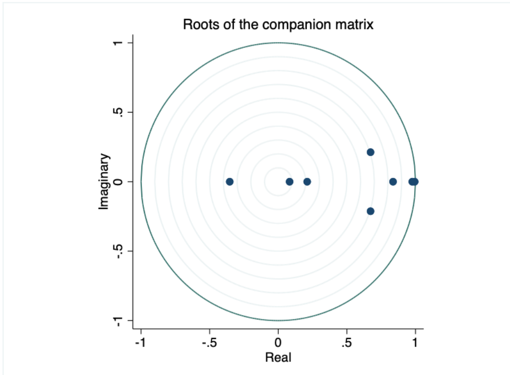
A.9 VAR: Diagnostics

Table A.1: Lag Length Selection: Four Variables VAR

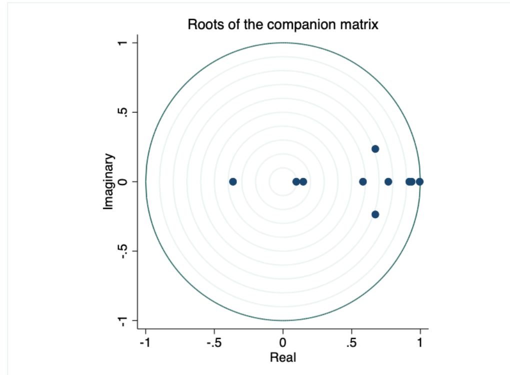
Lag	FPE	AIC	HQIC	SBIC
0	6.13573			
1	8.20E-09	-7.26575	-7.07508	-6.79615
2	4.4e-09*	-7.88946*	-7.54624*	-7.04416*
3	4.90E-09	-7.78783	-7.29208	-6.56685
4	5.60E-09	-7.66095	-7.01265	-6.06428

Table A.2: Lag Length Selection: Five Variables VAR

Lag	FPE	AIC	HQIC	SBIC
0	8.24713			
1	1.90E-10	-8.21213	-7.92612	-7.50772
2	8.8e-11*	-8.97042*	-8.44606*	-7.679*
3	9.60E-11	-8.88118	-8.11848	-7.00275
4	1.10E-10	-8.77899	-7.77794	-6.31355



(a) Four Variable VAR



(b) Five Variable VAR

Figure A.24: Notes: Stability test (a) We use four variables (term spread, GDP, consumer price and overnight rate) in the SVAR. (b) We use five variables (term spread, size of the government, GDP, consumer price and overnight rate) in the SVAR. Term spread is the difference between a ten-year rate and an three month rate. The size of the government is given by the ratio of the government final consumption expenditure to private final consumption expenditure. The sample period is 1993Q1-2023Q2.

B Coefficient of Risk Aversion in Generalized Recursive Preferences With Labor Income Tax

Proposition 12 in Swanson (2009) gives the absolute risk aversion for Epstein-Zin preferences given below:

$$R^a(a;\theta) = \frac{-V_{11}(a,\theta)}{V_1(a,\theta)} + \alpha\frac{V_1(a,\theta)}{V(a,\theta)}$$

The first derivative of the value function is given by:

$$V_1(a_t,\theta_t) = (1+r_t)u_1(c_t^*,l_t^*)$$

where we multiply by $(1+r)$ because assets in the beginning of time period t generate income in time period t. The second derivative of the value function is given by:

$$V_{11}(a_t, \theta_t) = (1 + r_t)\left[u_{11}(c_t^*, l_t^*)\frac{\partial c_t^*}{\partial a_t} + u_{12}(c_t^*, l_t^*)\frac{\partial l_t^*}{\partial a_t}\right]$$

Intra-temporal condition (consumption leisure decision) with labour income tax is given by:

$$-u_2(c_t^*, l_t^*) = w_t(1-\tau)u_1(c_t^*, l_t^*)$$

Differentiating both sides, we obtain:

$$-u_{21}(c_t^*, l_t^*)\frac{\partial c_t^*}{\partial a_t} - u_{22}(c_t^*, l_t^*)\frac{\partial l_t^*}{\partial a_t} = w_t(1-\tau)u_{11}(c_t^*, l_t^*)\frac{\partial c_t^*}{\partial a_t} + w_t(1-\tau)u_{12}(c_t^*, l_t^*)\frac{\partial l_t^*}{\partial a_t}$$

which gives:

$$\frac{\partial l_t^*}{\partial a_t} = -\lambda_t \frac{\partial c_t^*}{\partial a_t}$$

where

$$\lambda_t = \frac{[u_{21}(c_t^*, l_t^*) + w_t(1-\tau)u_{11}(c_t^*, l_t^*)]}{[u_{22}(c_t^*, l_t^*) + w_t(1-\tau)u_{12}(c_t^*, l_t^*)]}$$

Swanson (2009) does not consider labour income tax. Budget constraint with labour income tax is given by

$$a_{t+1} = (1+r)a_t + w_t(1-\tau_t)l_t - c_t$$

Iterating it gives the life time budget constraint given by:

$$a_t = \sum_{s=0}^{\infty} \frac{1}{(1+r)^{s+1}}\left[c_{t+s} - w_t(1-\tau_t)l_{t+s}\right] + \lim_{s\to\infty}\left(\frac{1}{1+r}\right)^{s+1} a_{t+s+1}$$

Using $\lim_{s\to\infty}\left(\frac{1}{1+r}\right)^{s+1} a_{t+s+1} \to 0$ gives:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^{s+1}}\left[c_{t+s}\right] = a_t + \sum_{s=0}^{\infty} \frac{1}{(1+r)^{s+1}}\left[w_t(1-\tau_t)l_{t+s}\right]$$

We multiply both sides by $(1+r)$ and that gives:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} [c_{t+s}] = (1+r)a_t + \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} [w_t(1-\tau_t)l_{t+s}]$$

Using $\frac{\partial l_t^*}{\partial a_t} = \frac{\partial l_{t+1}^*}{\partial a_t}$, see (Swanson (2009)) gives:

$$\frac{1+r}{r} \frac{\partial c_t^*}{\partial a_t} = (1+r) + w(1-\tau) \frac{1+r}{r} \frac{\partial l_t^*}{\partial a_t}$$

Using $\frac{\partial l_t^*}{\partial a_t} = -\lambda_t \frac{\partial c_t^*}{\partial a_t}$

$$\frac{1+r}{r} \frac{\partial c_t^*}{\partial a_t} = (1+r) + w(1-\tau) \frac{1+r}{r} (-\lambda_t) \frac{\partial c_t^*}{\partial a_t}$$

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1+w(1-\tau)\lambda}$$

Hence

$$R^a(a;\theta) = \frac{-(1+r_t)[u_{11} - u_{12}\lambda] \frac{\partial c_t^*}{\partial a_t}}{(1+r_t)u_1} + \alpha \frac{(1+r_t)u_1}{V(a,\theta)}$$

$V(a,\theta) = \frac{u}{1-\beta} = \frac{u}{1-\frac{1}{1+r}} = u(1+r)/r$

$$R^a(a;\theta) = \frac{-(1+r)[u_{11} - u_{12}\lambda] \frac{\partial c_t^*}{\partial a_t}}{(1+r)u_1} + \alpha \frac{(1+r)u_1}{u(1+r)/r}$$

$$R^a(a;\theta) = \frac{-u_{11} + u_{12}\lambda}{u_1} \frac{r}{1+w(1-\tau)\lambda} + \alpha \frac{u_1 r}{u}$$

Using equation 54 in Swanson (2012) and $A = c/r$ gives household's consumption-only coefficient of relative risk aversion given below:

$$R^c(a;\theta) = \frac{-u_{11} + u_{12}\lambda}{u_1} \frac{c}{1+w(1-\tau)\lambda} + \alpha \frac{u_1 c}{u}$$

use $u_{11} = -\phi c^{-\phi-1}$, $u_1 = c^{-\phi}$ and $u_{12} = 0$

$$R^c(a;\theta) = \frac{\phi c^{-\phi-1}}{c^{-\phi}} \frac{c}{1+w(1-\tau)\lambda} + \alpha \frac{c^{-\phi} c}{u}$$

$$R^c(a;\theta) = \frac{\phi}{1 + w(1-\tau)\lambda} + \alpha\frac{c^{-\phi}c}{u}$$

Use $-u_2 = w(1-\tau)u_1$, $\lambda = \frac{w(1-\tau)u_{11}}{u_{22}}$, $u_{22} = (-\eta)\,\chi_0\,(1-L_t)^{-\eta-1}$,$u_2 = -\chi_0\,(1-L_t)^{-\eta}$

$$R^c(a;\theta) = \frac{\phi}{1 - \frac{u_2}{(1-\tau)u_1}(1-\tau)\lambda} + \alpha\frac{c^{-\phi}c}{u}$$

Also $\lambda = \frac{w(1-\tau)u_{11}}{u_{22}}$

$$R^a(a;\theta) = \frac{\phi}{1 - \frac{u_2}{(1-\tau)u_1}(1-\tau)\frac{w(1-\tau)u_{11}}{u_{22}}} + \alpha\frac{c^{-\phi}c}{u}$$

$$R^a(a;\theta) = \frac{\phi}{1 - w(1-\tau)\frac{u_2}{u_1}\frac{u_{11}}{u_{22}}} + \alpha\frac{c^{-\phi}c}{u}$$

$$R^a(a;\theta) = \frac{\phi}{1 - w(1-\tau)\frac{u_{11}}{u_1}\frac{u_2}{u_{22}}} + \alpha\frac{c^{-\phi}c}{u}$$

We have $u_2 = -\chi_0\,(1-L_t)^{-\eta}$, $u_{22} = (-\eta)\,\chi_0\,(1-L_t)^{-\eta-1}$ and $u_{11} = -\phi c^{-\phi-1}$, $u_1 = c^{-\phi}$

$$\frac{u_2}{u_{22}} = \frac{-\chi_0\,(1-L_t)^{-\eta}}{(-\eta)\,\chi_0\,(1-L_t)^{-\eta-1}} = \frac{1-L}{\eta}$$

$$\frac{u_{11}}{u_1} = \frac{-\phi c^{-\phi-1}}{c^{-\phi}} = \frac{-\phi}{c}$$

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{\phi}{c}\frac{(1-L)}{\eta}w(1-\tau)} + \alpha\frac{c^{-\phi}c}{\frac{c^{1-\phi}}{1-\phi} + \chi_0\frac{(1-L)^{1-\eta}}{1-\eta}}$$

Since we have $-u_2 = w(1-\tau)u_1$

$$\chi_0\,(1-L_t)^{-\eta} = w(1-\tau)c^{-\phi}$$

Hence

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{(1-L)}{c}\frac{w(1-\tau)\phi}{\eta}} + \alpha \frac{c^{-\phi}c}{\frac{c^{1-\phi}}{1-\phi} + \frac{w(1-\tau)(1-L)c^{-\phi}}{(1-\eta)}}$$

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{(1-L)}{c}\frac{w(1-\tau)\phi}{\eta}} + \alpha \frac{c^{-\phi}c\,(1-\phi)}{c^{1-\phi} + \frac{w(1-\tau)(1-L)c^{1-\phi}(1-\phi)}{(1-\eta)c}}$$

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{(1-L)}{c}\frac{w(1-\tau)\phi}{\eta}} + \alpha \frac{(1-\phi)}{1 + \frac{w(1-\tau)(1-L)(1-\phi)}{(1-\eta)c}}$$

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{(1-L)}{Lc}\frac{wL(1-\tau)\phi}{\eta}} + \alpha \frac{(1-\phi)}{1 + \frac{wL(1-\tau)(1-L)(1-\phi)}{L(1-\eta)c}}$$

Using $wL(1-\tau) = c$

$$R^c(a;\theta) = \frac{\phi}{1 + \frac{(1-L)}{L}\frac{\phi}{\eta}} + \alpha \frac{(1-\phi)}{1 + \frac{(1-L)}{L}\frac{(1-\phi)}{(1-\eta)}}$$

Which is same as in footnote 23 of Rudebusch and Swanson (2012). ϕ gives intertemporal elasticity of substitution (IES where $\phi = \frac{1}{IES}$. The Frisch elasticity is given by:

$$\text{Frisch Elasticity} = \frac{U'_{L_t}}{L_t\left[U'_{L_t} - \frac{U'_{L_tC_t}}{U'_{L_tL_t}}\right]}$$

$$= \frac{-\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta}}{L_t\left(-\eta\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta-1} - \frac{0}{(-\phi)C_t^{-\phi-1}}\right)}$$

$$= \frac{-\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta-1}(1-L_t)}{L_t\left(-\eta\chi_0 Z_t^{1-\phi}(1-L_t)^{-\eta-1}\right)}$$

$$\text{Frisch Elasticity } = \frac{(1-L_t)}{\eta L_t}$$

$$R^c(a;\theta) = \frac{\phi}{1 + \text{Frisch Elasticity } \times\phi} + \alpha \frac{(1-\phi)}{1 + \text{Frisch Elasticity } \frac{\eta(1-\phi)}{(1-\eta)}}$$

C Model

C.1 Final Good Producer

The final good is produced by a firm in a perfectly competitive market, which combines the intermediate goods using the constant returns to scale technology given by:

$$y_t \leq \left[\int_0^1 y_t(i)^{(\phi_m - 1)/\phi_m} di \right]^{\phi_m/(\phi_m - 1)}$$

Where ϕ_m is the elasticity of substitution between intermediate goods $Y_t(i)$ with given price $P_t(i)$. Profit maximization leads to the following demand for intermediate goods:

$$y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\phi_m} y_t$$

Where

$$P_t = \left[\int_0^1 P_t(i)^{1-\phi_m} di \right]^{1/(1-\phi_m)}$$

$$P_t^{1-\phi_m} = \int_0^1 P_t(i)^{1-\phi_m} di$$

C.2 Intermediate goods producers

The production function of the intermediate goods producer is given by:

$$Y_t(i) = A_t \bar{K_t}(i)^{1-\theta} \left(Z_t L_t(i) \right)^{\theta}$$

where A_t is a stationary productivity shock given by:

$$logA_t = \rho_a logA_{t-1} + \epsilon_{a,t} \quad 0 \leq \rho_a < 1 \quad \epsilon_{a,t} \sim N\left(0, \sigma_a^2\right)$$

In each period there is a probability (ξ) that the firm will not be able to change prices. So whenever the firm changes its price, it maximizes the expected sum of discounted profits. The objective function is given by

$$\sum_{k=0}^{\infty} \xi^k \beta^k \lambda_{t,t+k} \left[P_t(i) - mc_{t+k}\right] Y_{t+k}(i)$$

The first order condition with respect to $P_t(i)$ is given by:

$$P_t(i) = \frac{\phi_m}{\phi_m - 1} \frac{\sum_{k=0}^{\infty} \xi^k \beta^k \Lambda_{t+k} mc_{t+k}(i) Y_{t+k}(i)}{\sum_{k=0}^{\infty} \xi^k \beta^k \Lambda_{t+k} Y_{t+k}}$$

where

$$mc_t(i) = \frac{W_t L_t(i)}{\theta Y_t}$$

We also introduce a markup shock in the model given by:

$$log(\phi_{mt}) = (1-\rho_\phi) log(\phi_m) + \rho_{\phi_m} log(\phi_{mt-1}) + \epsilon_{\phi_m,t} \quad 0 \le \rho_{\phi_m} < 1 \qquad \epsilon_{\phi_m,t} \sim N(0, \sigma^2_{\phi_m})$$

C.3 Monetary policy

Central bank implements a monetary policy based on modified Taylor rule given by:

$$log\left(\frac{i_t}{i}\right) = \rho_i log\left(\frac{i_{t-1}}{i}\right) + (1 - \rho_i)$$

$$\left[log\,(i) + log\left(\pi_t^4\right) + \rho_\pi log\left(\frac{\pi_t^4}{\pi}\right) + \rho_g log\left(\frac{y_t}{y}\right)\right] + \epsilon_{it} \quad \epsilon_{i,t} \sim N(0, \sigma_i^2)$$

where R and π are steady state value of R_t and π_t and y is the steady state value of stationary variable $y_t = \frac{Y_t}{Z_t}$. π_t^4 is the geometric moving average of inflation

$$log\left(\pi_t^4\right) = \theta_\pi log(\pi_{t-1}^4) + (1 - \theta_\pi) log\left(\pi_t\right)$$

We estimate θ_π.

D Data Sources

- 3-Month or 90-day Rates and Yields: Treasury Securities for South Africa
  https://fred.stlouisfed.org/series/IR3TTS01ZAM156N
- Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for South Africa
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- Immediate Rates: Less than 24 Hours: Central Bank Rates for South Africa
  https://fred.stlouisfed.org/series/IRSTCB01ZAM156N
- Private Final Consumption Expenditure in South Africa
  https://fred.stlouisfed.org/series/NAEXKP02ZAQ189S
- Gross Fixed Capital Formation in South Africa
  https://fred.stlouisfed.org/series/NFIRSAXDCZAQ
- Government Final Consumption Expenditure in South Africa
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- Real Gross Domestic Product for South Africa
  https://fred.stlouisfed.org/series/NGDPRSAXDCZAQ
- Consumer Price Index: All Items for South Africa
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- General government gross debt for South Africa
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- Household Debt to GDP for South Africa
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