Input Tariff in Oligopoly: Entry, Heterogeneity, and Demand Curvature *

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Abstract

How does an increase in tariff on intermediate input affect different margins of trade and what in turn are consequences for optimal tariff? We address this question in a setting with vertical specialization where oligopolistic, downstream Home firms procure input from perfectly competitive, Foreign upstream firms. Our key focus is to understand how Home optimal tariff departs from the competitive benchmark (inverse of foreign export supply elasticity). While underproduction in oligopoly puts a downward pressure on tariff, welfare improvement arising from rationalization (in presence of entry) and possible reallocation (in presence of firm heterogeneity) can put an upward pressure on tariff. Thus, in general, optimal tariff can be higher or lower than the competitive benchmark.

In absence of firm heterogeneity, optimal tariff is lower than competitive benchmark *if and only if* intensive margin decreases with tariff. Intensive margin always decreases with tariff in the short run irrespective of the demand function. Thus, the optimal short run tariff is always lower than the benchmark. However, in the long run, intensive margin decreases with tariff, and consequently, the long run optimal tariff is strictly lower than the competitive benchmark if and only if the demand function is strictly convex. Starting from the short run optimal tariff, an increase in tariff improves long run welfare as it mitigates the excessive entry in a homogeneous product setting like ours. In presence of firm heterogeneity, optimal tariff is lower than competitive benchmark if and only if a share-weighted sum of intensive margin decreases with tariff. Share weighted intensive margin decreases with tariff for linear and strictly convex demand functions – implying that optimal tariff continues to be lower than competitive benchmark under heterogeneity as long as the demand functions are strictly convex. However, share weighted intensive margin can increase with tariff, and hence optimal tariff can exceed competitive benchmark for strictly concave demand functions (provided Herfindahl is sufficiently high). We conclude our discussion with optimal tariffs in presence of vertical oligopoly (i.e., oligopoly in both upstream and downstream sectors), product differentiation, and multiple instruments (i.e. competition policy along with tariffs).

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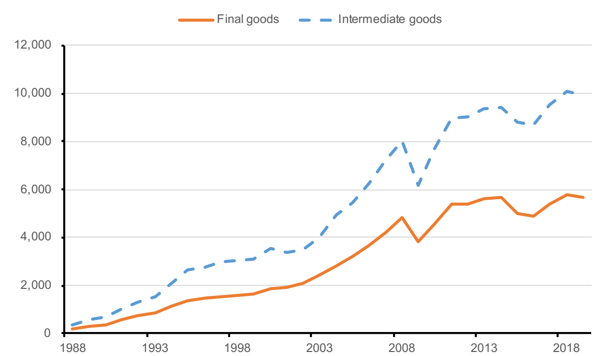
1 Introduction

Trade in the modern economy is largely in car parts, not cars (Krugman, 2018). A significant fraction of world trade – nearly two-thirds according to the estimates of Johnson and Noguera (2012) – is in intermediate goods. A reduction in tariffs across the globe has led to tremendous growth in intermediate goods trade prompted by vertical specialization across countries (Hummels, Ishii, and Yi, 2001; Yi, 2003; Hanson, Mataloni, and Slaughter 2005). While trade in final goods has also increased, trade in intermediate goods has grown at a faster rate over the last three decades (see Figure 1). Increased importance of intermediate goods trade has led to development of new models that embed vertical specialization across countries. Recently, attention has turned to trade policy in such multi-layered production settings. See for example, Caliendo, Feenstra, Romalis, and Taylor (2021), Lashkaripour and Lugovskyy (2022), and Antras, Fort, Gutierrez and Tintelnot (2022). We contribute to this growing literature.

Figure 1 – World imports, 1988-2019



Source: Authors' own compilation based on the World Integrated Trade Solution (WITS) database.

Notes: We divide the world import value by applying the Broad Economic Categories (BEC) classification which distinguishes between intermediate, capital, and consumption goods. Following Arkolakis et al. (2008), intermediate goods are defined as any imports of intermediate and capital goods.

How do tariff reductions on intermediate goods imports affect the number of importers (extensive margin), average volume of imports per firm (intensive margin), and welfare, and what, in turn, are the implications for optimal tariff? We address this question in an oligopoly environment with free entry where firms in one country (say Home) specialize in final goods and import intermediate goods from the rest of the world. Interplay between demand features (e.g., curvature of demand, elasticity of slope) and entry environment—i.e., exogenous versus endogenous market structure—play an important role in determining these effects. Mrazova and Neary (2017) demonstrate how demand conditions shape comparative statics results in presence of monopoly and monopolistic competition. In our oligopoly framework—where strategic interaction among firms is explicitly modeled—demand side conditions like slope and curvature play a critical role too. Our focus however is on the interaction: how the supply-side environment (endogenous versus exogenous market structure) interacts with demand-side features in determining the effects of tariff reduction on imports and welfare. The nuanced nature of this interaction is best reflected in our findings in intensive margin and optimal tariff which we discuss below.

When the market structure is exogenously given, intensive margin always increases with a reduction in tariff irrespective of the demand function. This demand-invariant nature of the result goes away when market structure is determined endogenously. With free entry, intensive margin increases *if and only if* the demand function is strictly convex. For constant elasticity demand— a prominent, strictly convex demand function—each firm imports more as tariffs decline. However, for linear demand, firm-level imports is invariant to tariff as the incentive to import more (due to lower tariff) is exactly offset by incentive to import less due to business-stealing effect arising from new entry. Thus, the response of intensive margin (to tariff cuts) depends not only on demand conditions but also on whether the market structure is exogenously or endogenously determined.

Like intensive margin, optimal tariff displays a demand invariant feature when the market structure is exogenous. Irrespective of the demand function, optimal tariff is strictly lower than the inverse of export supply elasticity. Imposing tariffs on intermediate goods improves terms-of-trade (i.e., reduces input price) and increases welfare at Home. Counteracting this welfare gain from terms-of-trade improvement is welfare loss from exacerbating underproduction in oligopoly. This lowers optimal tariff below the traditional competitive benchmark of inverse of export supply elasticity. As Lashkaripour and Lugovskyy (2022) note, tariff plays two roles in a second-best world like ours where subsidies on consumption and production are not available. It improves terms-of-trade and corrects misallocation. Under oligopolistic final goods sector, this misallocation manifests in the form of underproduction which is partially corrected by setting tariff at a lower level (than the competitive benchmark). In fact, irrespective of the demand function, optimal tariff could be negative if the final goods sector is too concentrated. However, this result—intermediate input tariffs are lower—does not necessarily hold once entry is allowed.

When market structure is endogenous, an additional distortion/misallocation is present: *excessive entry*—too many firms enter in a free-entry equilibrium in a homogeneous products oligopoly (Mankiw and Whinston, 1986). Tariffs exacerbate welfare loss from underproduction but also mitigate the welfare loss from excessive entry. These two effects exactly offset each other under linear demand, and hence optimal tariffs equal the inverse of export supply elasticity for linear demand as it would be under perfect competition. However, due to this additional welfare improving role of tariffs—curbing excessive entry—optimal tariff can be strictly higher than the inverse of export supply elasticity with endogenous market structure. Indeed, optimal tariff is strictly higher (lower) than the competitive benchmark for strictly concave (convex) demand functions.

Discussions in sections 2 and 3 focus on symmetric environments where all firms have identical costs. We introduce cost heterogeneity among firms in section 4. Heterogeneity gives rise to two new features. First, even for the same demand function, intensive margin could increase or decrease with an increase in tariffs. Second, in addition to affecting allocative efficiency, tariffs impact production efficiency. However, unlike allocative efficiency which necessarily worsens with tariffs, production efficiency could improve or worsen due to reallocation of production from inefficient to efficient firms. Once again, intensive margin plays a crucial role in determining whether optimal tariff exceeds or falls short of competitive benchmark. Optimal tariff is strictly lower than competitive benchmark if and only if a weighted intensive margin—where output shares act as weights—decrease with tariffs. This condition holds for linear as well as all convex demand functions functions. Strictly concave demand functions also satisfy this condition when Herfindahl index is high.

A few remarks regarding our modeling choices are in order. First, we focus primarily on market interactions (rather than bilateral negotiations) as a large fraction of intermediate goods is internationally traded in markets among anonymous final goods producers and intermediate goods suppliers. Second, market-based interactions guide us to consider homogeneous (rather than differentiated) intermediate goods as homogeneous intermediate goods tend to be exchanged through markets whereas differentiated intermediate goods tend to be exchanged through non-market mechanisms (Nunn, 2007). Third, we work with a perfectly competitive setting for input production, but our results go through for imperfectly competitive setting too—see Section 5.1 for extension to vertical oligopoly framework. Fourth, we assume that Home industry is oligopolistic and Home firms produce a homogeneous final good. In doing so, we acknowledge the empirical regularity that a small fraction of firms dominates international trade and follow the literature in trade theory where differentiated good is typically paired with monopolistic competition and homogeneous goods with oligopolistic competition.[1] Our analysis can be easily extended to accommodate product differentiation (Section 5.2). Nevertheless, we focus on a homogeneous-product setting as it provides a nuanced effect of tariffs via entry which in turn influences the sign (and the magnitude) of optimal tariffs in an important way. Finally, to highlight the rich interaction between entry and demand conditions we made several simplifying assumptions including no intermediate goods producers at Home, no Foreign consumers, and absence of policy instruments (other than tariff). In Section 3.1 (see Remarks) we discuss how the presence of Foreign consumers and intermediate goods producers would affect our results. We also discuss the role of multiple instruments.

Our paper is naturally related to early literature on vertical oligopoly and trade policy (e.g., Ishikawa and Spencer, 1999). This literature focuses on a strategic interaction among firms in vertically-related markets and examines the effect of export subsidies on intermediate goods and welfare, treating the market structure as (typically) exogenous. In contrast, the market structure is endogenous in our framework which in turn affects the optimal trade policy through entry/exit (extensive margin) induced by tariff changes. In single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985), and Etro (2011) have shown that entry/exit considerations can alter optimal trade policy. Bagwell and Staiger (2012) have shown that restraining export subsidies – an often used feature of trade agreements – can potentially benefit consumers when free entry is invoked in linear oligopoly. Our work complements this body of work by introducing multiple stages of production and general demand which generates a rich set of possibilities.

Blanchard (2007) provides an early analysis of trade policy with multiple production stages. In the presence of vertical foreign direct investment (FDI), she finds that governments of source countries have an incentive to improve market access for imported inputs from foreign affiliates of source country firms.[2] Our primary focus is, however, on outsourcing rather than FDI. Antràs and Staiger (2012), Ornelas and Turner (2008, 2012), and Grossman and Helpman (2020) investigate trade policy in the context of intermediate goods trade and international outsourcing. Antràs and Staiger (2012) and Ornelas and Turner (2008, 2012) study trade agreements and organizational structure respectively, while Grossman and Helpman (2020) study the effect of unanticipated tariffs on intermediate goods in a setting with firm-to-firm supply relationships. At the heart of these papers is non-market interactions such as matching and bargaining, associated with contractual incompleteness. This paper instead highlights vertical linkages of production through market interactions. Regarding bilateral negotiations versus market interactions, the suitability of modeling approaches depends on a particular application (e.g., a particular industry or a particular policy question). Inderst (2010) notes that bilateral negotiations subject to contract incompleteness might be more applicable in tight bilateral oligopoly with differentiated products, whereas market interactions with uniform contractual terms are more applicable for a homogeneous product.[3]

We contribute to the literature on optimal tariffs under imperfect competition. In monopolistic competition models with homogeneous firms, Gros (1987) and Flam and Helpman (1987) show that optimal tariff is strictly positive even in a small open economy. Venables (1987) and Ossa (2011) demonstrate that import tariffs can induce welfare-inducing entry at Home by prompting consumers to switch from imported varieties to Home varieties. In models of monopolistic competition with heterogeneous firms, Demidova and Rodríguez-Clare

[1]See Neary (2016) for a comprehensive treatment of oligopoly in general equilibrium where firms are large in their own markets but small in the economy. See also Head and Spencer (2017) for usefulness of oligopoly to analyze the role of large firms in globalization as well as the welfare effects of trade policy.

[2]See Blanchard and Matschke (2015) for empirical support for this argument.

[3]As mentioned earlier in the introduction, we consider product differentiation in section 4.2.

(2009) were the first one to study optimal tariffs in a small economy. Felbermayr et al. (2013) extend the analysis to a large economy. Despite a negative impact of productivity, they find that import tariff improves welfare. Costinot et al. (2020) provide a strict generalization of their results and find that optimal tariffs can be lower (and even be negative) with general preferences and technology, but they do not examine input-output linkages.

Presence of input-output linkages can lead to low optimal tariff in a second-best world. Caliendo et al. (2021) offers a second-best argument for low optimal tariff in the presence of roundabout production. In absence of other instruments, tariff is suitably adjusted downward to mitigate welfare loss from double marginalisation that arise in roundabout production structure. Antràs et al. (2021) provides a rationale for lower tariff on inputs (compared to final goods) on efficiency grounds in the presence of scale economies. Unilateral tariff on either final goods or inputs enlarges each sector and raises welfare. However, input tariff raise final-good producers' costs, limiting their potential benefits leading to lower input tariffs.[4] In a fairly general multi-industry, multi-country trade setting, Lashkaripour and Lugovskyy (2022) show that optimal tariff is independent of the input-output linkages, but only when optimal production subsidies are available.

Rich in structure and input-output linkages, most of these works assume either perfect competition or monopolistic competition with CES preferences.[5] Under monopolistic competition (with CES preferences), firm level output is typically optimal and free entry number of firms is insufficient. In contrast, in a homogeneous products, free-entry oligopoly, firm level output is always insufficient and too many firms enter in equilibrium. The nature of output and entry misallocation differ between monopolistic competition and oligopoly and so does the nature of optimal tariffs. By focusing on one vertically related industry, we sacrifice some of the richness in Caliendo et al. (2021), Antràs et al. (2022), and Lashkaripour and Lugovskyy (2022). However, our focus (albeit narrow) allows us to provide a comprehensive account of the impact of tariffs for an important class of imperfect competition environment: free-entry oligopoly with vertical relationships and general demand.

As mentioned earlier, the importance of demand conditions—in particular elasticity of demand and elasticity of slope of demand—has been succinctly shown by Mrazova and Neary (2017). While elasticity is important for determining price levels, understanding change in price levels requires characterizing how elasticity changes, which depends on both elasticity of demand as well as its slope. In addition to focus on oligopoly, our work differ from theirs in that we work with input demand instead of demand for final goods. This difference however does not feature prominently since one-to-one production technology enables smooth translation of demand for final good to demand for inputs at least when the market structure is exogenously given. To facilitate this translation under endogenous market structure, we introduce entry-augmented input demand curve which exploits a positive association between aggregate output and number of firms—a condition that holds unless the demand for final good is, loosely speaking, too log-convex. Despite working with input demand curve, our characterization are in terms of curvature of final demand both under exogenous and endogenous market structure. This is in part due to the production technology, and perfect competition in the upstream sector. Elasticity of slope of input demand function appears in vertical oligopoly—where both final goods and intermediate goods sector are oligopolistic. We discuss vertical oligopoly comprehensively in section 5.1(with details in Appendix).

The plan of the paper is as follows. Section 2 presents the basic model and examine the impact of tariffs on intensive and extensive margin. Analyzing welfare effects of tariffs, Section 3 characterizes optimal tariff with and without entry. Section 4 derives optimal input tariff in presence of cost heterogeneity. In all these sections, we assume that (a) producers of final goods and intermediate produce and sell homogeneous products, (b) final

[4] Also see Blanchard, Bown, and Johnson (2017). In a perfectly competitive setting, they find that governments set lower tariffs on goods where global value chain linkages are stronger. While their finding is similar in spirit, the value added approach taken in their paper is more appropriate for tariff on final goods than tariff on inputs

[5] Antràs et al. (2021) notes that monopolistic competition with CES preferences is isomorphic to perfect competition with external economies to scale.

goods sector is oligopolistic and the intermediate goods sector is perfectly competitive, and (c) finally, tariff is the only available policy instrument. We relax most of these assumptions in section 5.

2 Model

We develop a model of vertical specialization where firms from one country (say Home) produce a final good by using an intermediate good imported from the rest of the world. Home government imposes ad valorem tariffs on the intermediate good. Taking the input price and the tariff rate as given, Home firms compete in the final good market as Cournot competitors. First, we provide some details of our model (Section 2.1) and analyze a short-run setting where the number of Home firms is fixed (Section 2.2). Subsequently, we allow for entry and exit of firms which leads to endogenous determination of market structure (Section 2.3). Throughout this section we focus on the effect of tariffs. We examine how reduction in tariffs affect aggregate imports, firm-level imports (intensive margin), and the number of importers (extensive margin) of the intermediate good. Equipped with these findings, in Section 3, we turn to characterizing welfare-maximizing optimal tariffs for both settings – with and without free entry.

2.1 Basics

Home comprises a unit mass of identical individuals each with $\bar{L}$ units of labor endowment. One unit of labor is required for producing one unit of numeraire good y which is freely traded and produced under perfect competition. As the price of y is 1 and each individual has $\bar{L}$ units of labor, the wage rate and labor income are unity and $\bar{L}$ respectively.

Each individual consumes the numeraire good y and a final good Q. Preferences are given by a quasi-linear utility function $V = U(Q) + y$ where $U(0) = 0, U'(Q) > 0$, and $U''(Q) < 0$ for $Q > 0$. Assuming $\bar{L}$ to be high enough, utility maximization subject to budget constraint yields the inverse demand function: $P = P(Q)(\equiv U'(Q))$, where P is the price of final good. We assume that $P(Q)$ is twice continuously differentiable, $P'(Q) < 0$, and the following inequality holds for all $Q > 0$:

$$2P'(Q) + QP''(Q) < 0. \tag{1}$$

A large class of demand functions—including linear, constant-elasticity (with elasticity greater than unity), and all logconcave demand functions—satisfy (1). This condition implies that industry marginal revenue is downward sloping and it rules out demand functions that are, loosely speaking, too convex. The condition specified in (1) can be restated as

$$\eta(Q) > -2.$$

where $\eta(Q) \equiv \frac{QP''(Q)}{P'(Q)}$ denotes the elasticity of slope of demand. This is constant for a class of demand functions including linear and constant-elasticity demand. Mrázová and Neary (2017) use this condition extensively in their work on demand structure and firm behavior (see section 4 for a detailed discussion on this assumption).

On the production side, there is a large number of identical Home firms which produce the final good Q. Each Home firm first decides whether to incur the entry cost K prior to entering the downstream sector and engage in production of the final good Q. One unit of the intermediate good X is required to produce one unit of the final good Q. There are no intermediate good producers at Home. Upon entry, Home firms import the intermediate good from the rest of the world and transform it into the final good without incurring any additional costs.

The upstream sector comprises a large number of price-taking firms (in the rest of the world) producing the intermediate good under a strictly convex cost function. This gives rise to upward sloping supply for intermediate imports:

$$r = h(X), \tag{2}$$

where r denotes the per-unit world price of X and $h'(X) > 0$ for all $X > 0$. We assume $0 \leq h(0) < \lim_{Q\to 0} P(Q)$ which ensures an interior equilibrium. Home government imposes tariffs on intermediate imports. Let t denote the ad-valorem tariff rate chosen by the Home government. Throughout this section we assume that t is given. Tariff revenues are rebated back to Home consumers.[6]

2.2 Short-run Analysis

We begin with the short-run analysis where the number of Home firms is fixed. Let M denote the number of Home firms producing the final good. Given the input price r, ad valorem tariff t, and one-to-one production technology, each Home firm's unit cost is $r(1+t)$. Home firm i chooses q_i to maximize its profits

$$\pi_i \equiv \left(P\left(q_i + \sum_{j\neq i}^{M} q_j\right) - r(1+t)\right)q_i.$$

Solving the first-order conditions corresponding to Home firms' profit maximization problem yields the symmetric equilibrium quantity for each Home firm $q_1 = q_2 = ... = q_M \equiv q(>0)$:

$$q = -\frac{P(Q) - r(1+t)}{P'(Q)}, \tag{3}$$

where Q uniquely solves the following:

$$MP(Q) + QP'(Q) - Mr(1+t) = 0. \tag{4}$$

One unit of the final good Q requires one unit of the intermediate good X. Substituting Q with X in (4) and rearranging yields the inverse demand function for intermediate imports:

$$r = \frac{P(X) + \frac{XP'(X)}{M}}{1+t} \equiv g(X, M, t), \tag{5}$$

where

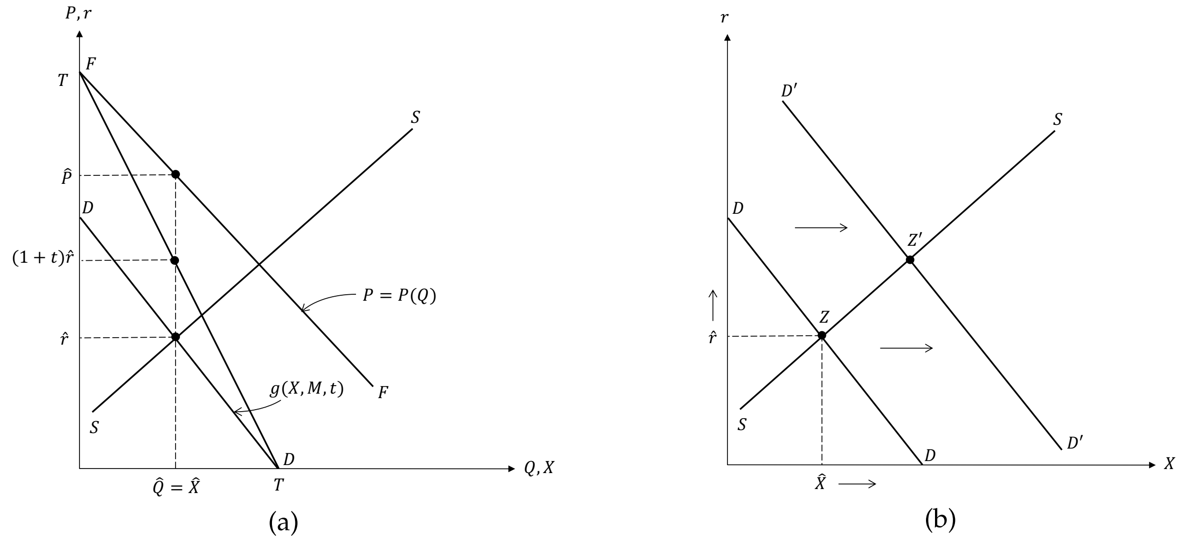
$$g_X = \frac{(M+1)P'(X) + XP''(X)}{M(1+t)} < 0, \quad g_M = -\frac{XP'(X)}{M^2(1+t)} > 0, \quad g_t = -\frac{g(.)}{1+t} < 0. \tag{6}$$

From the signs of the partials in (6), it follows immediately that the demand for intermediate imports is downward-sloping. Furthermore, an increase in the number of importers raises the demand for intermediate imports. Similarly, a reduction in ad-valorem tariffs acts as a reduction in unit costs and boosts up the demand for intermediate imports.

Equating demand and supply of intermediate imports—given by (5) and (2)—respectively, we obtain the

[6] Our results hold for specific tariffs as well. These two forms of tariffs are equivalent in a framework like ours where the upstream sector is perfectly competitive.

Figure 3 – Equilibrium with fixed number of firms



equilibrium values of variables $(\hat{X}, \hat{r})$ where

$$\hat{r} = h(\hat{X}) = g(\hat{X}, M, t). \tag{7}$$

Each Home firm imports $\hat{x} = \frac{\hat{X}}{M}$ units of the intermediate good to produce $\hat{q} = \frac{\hat{Q}}{M}$ units of the final good where $\hat{Q} = \hat{X}$ and $\hat{P} = P(\hat{Q})$ respectively denote the equilibrium values of aggregate output and price of the final good.

Figure 3a describes the equilibrium. FF and DD denote the demand function for the final good and the intermediate good respectively. TT plots $(1+t)g(X, M, t) = P(X) + \frac{XP'(X)}{M}$ against X. SS denotes the supply function $r = h(X)$. The intersection between DD and SS gives the equilibrium values $(\hat{X}, \hat{r})$. The tariff-inclusive input price $\hat{r}(1+t)$ and the final good price $\hat{P}$ are then read off TT and FF respectively at $X = \hat{X}$. Proposition 1 describes how these equilibrium outcomes vary with tariffs and the number of importers (see Appendix A.1.1 for proof).

Proposition 1

For a given number of importers (M), a reduction in a tariff rate (t) raises aggregate imports ($\hat{X}$) and the firm-level import of the intermediate good ($\hat{x}$). For a given tariff rate t, an increase in the number of importers raises aggregate imports but lowers firm-level imports of the intermediate good. Input price ($\hat{r}$) increases with both a reduction in the tariff rate and an increase in the number of importers.

Consider Figure 3b which illustrates in isolation the demand and supply for intermediate imports. As noted earlier, both a reduction in tariff rate t and an increase in the number of importers M lead to higher aggregate demand for intermediate imports at a given r. Demand curve DD shifts outward to $D'D'$ in Figure 3b. The supply curve SS remains unchanged. Consequently, the equilibrium point moves from Z to Z'. Both aggregate volume of imports and the input price increase with tariff reductions. While not stated separately in Proposition 1, the effect of changes in t or M on the final good market are immediate from the effect on $\hat{X}$. Whenever $\hat{X}$ increases, aggregate production of the final good $\hat{Q}(= \hat{X})$ increases, whereas the final good price $\hat{P} = P(\hat{Q})$

declines. Thus, both lower tariffs and increased competition lead to larger aggregate output and lower price in the final goods market.

What about the effect on the intensive margin, i.e., $\hat{x} = \frac{\hat{X}}{M}$? As aggregate imports increase with tariff reduction, firm-level imports $\hat{x}$ increases too (since M is fixed). To examine the effect an increase in M on firm-level imports, decompose $\frac{d\hat{x}}{dM}$ as

$$\frac{d\hat{x}}{dM} = \frac{\partial \hat{x}}{\partial M} + \frac{\partial \hat{x}}{\partial r}\frac{d\hat{r}}{dM}.$$

As long as quantities are strategic substitutes, entry in oligopoly gives rise to business-stealing effect: an increase in the number of firms reduces each incumbent firm's output The lower the production of the final good, the lower the imports of the intermediate good and thus $\frac{\partial \hat{x}}{\partial M} < 0$. With an increase in the input price—induced by an increase in M—greater competition reduces the demand for intermediate good even further and thus $\frac{d\hat{x}}{dM} < \frac{\partial \hat{x}}{\partial M} < 0$.

To summarize the effect of tariff reduction, all three—input price, aggregate imports, firm-level imports (intensive margin)—increase with a reduction in tariff. The first two continue to hold as we endogenize M. However, the effect on intensive margin becomes ambiguous as a reduction in tariffs not only reduces cost for each importer but also lead to higher number of importers. Cost reduction raises firm-level imports while increased competition lowers it. Below, we develop the long run analysis with free entry of firms and show how demand curvature play an important role in resolving this ambiguity. To be clear, the ambiguity remains—intensive margin can increase or decrease with a reduction in tariffs—but we provide a necessary and sufficient condition for the direction of change which depends crticially on demand curvature.

2.3 Long-run Analysis

The key feature of the long-run analysis is that the number of importers, M, is endogenously determined. To avoid notational clutter, we continue to use "hat" to denote equilibrium values of variable for an arbitrary tariff rate t. For example, $\hat{X}$ and $\hat{x}$ respectively denote aggregate imports and firm-level imports. The only new notation is $\hat{M}$—the number of importers which is endogenously determined (via free entry).

Home firms incur entry cost $K(> 0)$ to enter the downstream sector. Assuming K is low enough such that at least one Home firm engages in the final good production, the free entry number of Home firms is determined implicitly by the value of M that satisfies the zero-profits condition:

$$(P(Q) - r(1 + t))q - K = 0,$$

where q and Q are as defined in (3) and (4) respectively. Rewriting (3) as $P(Q) - r(1+t) = -P'(Q)q$ and replacing q with $\frac{Q}{M}$, we can express the zero-profits condition as

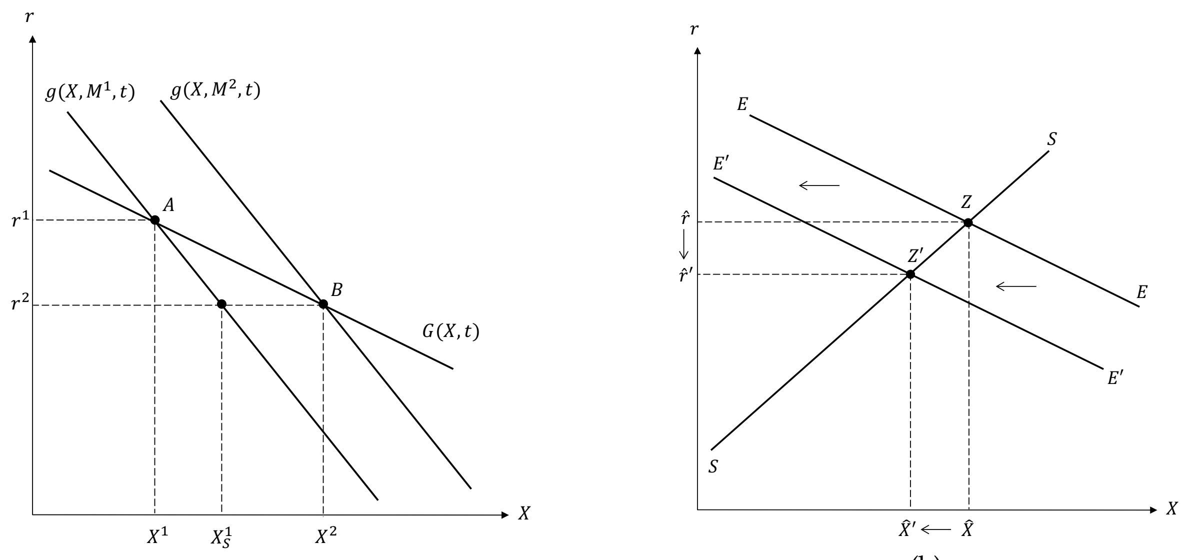
$$-\frac{P'(Q)Q^2}{M^2} - K = 0. \tag{8}$$

Differentiating (8) and applying (1) we find that the free entry level of M and Q has a monotone relationship:

$$\frac{dM}{dQ} = -\frac{q(2P'(Q) + QP''(Q))}{2K} > 0.$$

Let $M(Q)$ denote the unique value of M that is compatible with the free entry equilibrium level of Q. Then

Figure 4 – Equilibrium with free entry



we can express each Home firm's Cournot output as

$$\hat{q} = -\frac{P(Q) - r(1+t)}{P'(Q)} \tag{9}$$

where Q uniquely solves

$$M(Q)P(Q) + QP'(Q) - M(Q)r(1+t) = 0. \tag{10}$$

As in the short run, (1) guarantees uniqueness of Q in the long run (see Appendix A.1.2). Note that (10) is effectively the same as (4) except for one substitution: $M(Q)$ in place of M. This substitution preserves the analogy with the short-run analysis and simplifies the derivation of the aggregate demand curve for intermediate imports in the presence of entry. Substituting Q with X in (10) and rearranging yields the inverse demand function for intermediate imports:

$$r = \frac{P(X) + \frac{XP'(X)}{M(X)}}{1+t} \equiv g(X, M(X), t) \equiv G(X, t), \tag{11}$$

where

$$G_X = g_X + g_M M'(X) = \frac{2M(X)P'(X) + XP''(X)}{2M(X)(1+t)} < 0, \quad G_t = -\frac{G(.)}{1+t} < 0.$$

We refer to $G(X,t)$ in as entry-augmented demand for import. Demand for intermediate imports is downward-sloping ($G_X < 0$). Furthermore, it increases with a reduction in tariff ($G_t < 0$).

Figure 4a illustrates the relationship between the long-run entry-augmented demand curve $G(X,t)$ and the short-run demand curves $g(X, M, t)$. Suppose the input price is r^1. Let X^1 and M^1 denote aggregate demand and the number of importers that enter the downstream sector corresponding to $r = r_1$. Point A lies on the short-run demand curve D^1D^1 as depicted by $g(X, M^1, t)$. As the input price decreases to r_2, aggregate demand increases to X_S^1 if the number of importers is fixed at M^1. With free entry, however, as the input price decreases, entry becomes more profitable, and the number of importers increases to $M^2 (> M^1)$. The relevant short-run demand curve D^2D^2 is given by $g(X, M^2, t)$ which lies to the right of D^1D^1. Entry increases aggregate demand from X_S^1 to X^2. Thus the long-run demand for intermediate imports $G(X, t)$ passes through points A and B.

Equating (11) and (2) yields the equilibrium values $(\hat{X}, \hat{r})$ where

$$\hat{r} = h(\hat{X}) = G(\hat{X}, t). \tag{12}$$

Each Home firm imports $\hat{x} = \frac{\hat{X}}{\hat{M}}$ units of the intermediate good to produce $\hat{q} = \frac{\hat{Q}}{\hat{M}}$ units of the final good where $\hat{Q} = \hat{X}$ and $\hat{P} = P(\hat{Q})$ respectively denote the equilibrium values of aggregate output and the price of the final good. The equilibrium is depicted in Figure 4b where EE denotes the long run demand for the intermediate good $G(X, t)$, SS denotes the supply function $r = h(X)$, and the intersection point Z denotes the equilibrium: $(\hat{X}, \hat{r})$. Proposition 2 describes how these equilibrium outcomes vary with tariff reductions under free entry (see Appendix A.1.3 for proof).

Proposition 2

A reduction in ad-valorem tariff rate t raises aggregate imports ($\hat{X}$), the number of importers ($\hat{M}$) and the input price ($\hat{r}$). Firm-level imports ($\hat{x}$) increase with tariff reduction if and only if the demand function is strictly convex.

The effect of tariff reduction on $\hat{X}$ and $\hat{r}$ is immediate from Figure 4b. Lower tariffs always lead to higher aggregate demand for intermediate imports at a given r. The entry-augmented demand curve EE shifts outward to $E'E'$ in Figure 4b. The supply curve SS remains unchanged. As the equilibrium point moves from Z to Z', both aggregate imports $\hat{X}$ and input price $\hat{r}$ increase with a reduction in tariff. As $\hat{X}(= \hat{M}\hat{x})$ increases with lower tariffs, at least one of the two—$\hat{M}$ or $\hat{x}$—must increase. Consider first the extensive margin. Differentiating (8) with respect to t and rearranging, we get:

$$\frac{d\hat{M}}{dt} = -\frac{\hat{x}(2P'(\hat{X}) + \hat{X}P''(\hat{X}))}{2K}\frac{d\hat{X}}{dt} = -\frac{P'(\hat{X})\hat{x}(1 + \frac{\eta(\hat{X})}{2})}{K}\frac{d\hat{X}}{dt} > 0, \tag{13}$$

where $\eta(\hat{X}) \equiv \frac{XP''(\hat{X})}{P'(\hat{X})}$ is the elasticity of slope of the inverse demand function for the final good and the inequality in (13) follows from noting that $d\hat{X}/dt < 0$ and $2P'(.) + XP''(.) < 0$ (recall equation 1). If the demand function is not too convex, the number of importers increases with reduction in tariffs.

The effect of tariff reduction on the intensive margin depends on demand curvature. When M is fixed, aggregate imports $\hat{X}$ and firm-level imports $\hat{x}(= \frac{\hat{X}}{M})$ move in the same direction. That is not necessarily the case under free entry. Rewriting (8) as

$$-P'(\hat{X})\hat{x}^2 - K = 0,$$

differentiating, and rearranging subsequently gives

$$2\hat{x}\frac{d\hat{x}}{dt} = \frac{KP''(\hat{X})}{[P'(\hat{X})]^2}\frac{d\hat{X}}{dt}, \tag{14}$$

which implies that the intensive margin and aggregate imports move in the same direction if and only if $P''(.) > 0$. Lower tariffs prompt each Home firm to scale up production which in turn raises the demand for intermediate good. However, entry (induced by lower tariffs) prompts existing firms to produce less and import less. These two opposing effects exactly cancel out for linear demand. For strictly convex demand, $\hat{x}$ increases with a reduction in tariffs while the opposite is true for strictly concave demand function.

Here is a summary of our findings. Both input price and aggregate imports are decreasing in tariffs irrespective of the entry environment. However, firm-level imports may increase or decrease with tariffs. In the short run, firms always cut back their imports in response to higher tariffs. However, in the long run, firm-level imports decline in response to higher tariffs if and only if the demand is strictly convex. As we show below, the effect of tariff on intensive margin plays a crucial role in determining whether optimal tariff in our setting falls below or stays above the competitive benchmark: inverse of export supply elasticity.

3 Welfare and Optimal Tariff

The traditional terms-of-trade motive for tariffs—which manifests in the form of input price reduction—exists for both short-run and long-run settings. Counteracting this in the short run is underproduction in oligopoly, which makes optimal tariffs lower compared to what it would have been under a perfectly competitive downstream sector: the inverse of export supply elasticity. In the long run, however, optimal tariffs could be higher since in a homogeneous-product setting like ours tariffs improve welfare by mitigating excess entry in the downstream sector. First we show that a necessary and sufficient condition for tariffs to be lower than the competitive benchmark is that intensive margin declines with an increase in tariffs or equivalently tariffs lower the scale of production at the firm level - a condition that holds if and only if the demand curve is strictly convex. Subsequently we characterize optimal tariffs in the short run and long run. Negative optimal tariffs can arise irrespective of the demand curvature in the short run. In the long run, only strictly convex demand functions admit this possibility.

Let us start with the welfare expression. Quasilinear preferences together with the assumption that all tariff revenues are rebated back to consumers allow us to express Home welfare (W) as

$$W \equiv \underbrace{\left[\int_0^{\hat{Q}} P(y)dy - P(\hat{Q})\hat{Q}\right]}_{\text{Consumer surplus}} + \underbrace{\left[P(\hat{Q})\hat{Q} - \hat{r}(1+t)\hat{X} - \hat{M}K\right]}_{\text{Home profits}} + \underbrace{\hat{r}t\hat{X}}_{\text{Tariff revenues}} + \underbrace{\bar{L}}_{\text{Labor income}},$$

which can be simplified further to:

$$W = \int_0^{\hat{X}} P(y)dy - \hat{r}\hat{X} - \hat{M}K + \bar{L}.$$

Differentiating welfare with respect to t we get (upon some rearrangement):

$$\frac{dW}{dt} = \left(((\hat{P} - \hat{r}(1+t))x - K\right)\frac{d\hat{M}}{dt} + M\left(\hat{P} - \hat{r}(1+t)\right)\frac{d\hat{x}}{dt} + \hat{r}\left(t - \frac{1}{e_s}\right)\frac{d\hat{X}}{dt}$$

In the short run M is fixed. Hence $\frac{d\hat{M}}{dt} = 0$. In the long run, profits dissipate due to entry. Thus $(\hat{P} - \hat{r}(1+t))x - K = 0$. Since $((\hat{P} - \hat{r}(1+t))x - K)\frac{d\hat{M}}{dt} = 0$, the following must hold at welfare maximizing value of t irrespective of whether M is fixed or endogenously determined via free entry.

$$M(\hat{P} - \hat{r}(1+t))\frac{d\hat{x}}{dt} + \hat{r}\left(t - \frac{1}{e_s}\right)\frac{d\hat{X}}{dt} = 0 \tag{15}$$

Oligopolistic market structure involves pricing above marginal cost, i.e., $\hat{P} - \hat{r}(1+t) > 0$. Furthermore, $\frac{d\hat{X}}{dt} < 0$. These inequalities hold both in the short run as well as in the long run. Then (15) implies

$$\operatorname{sgn}(t - \frac{1}{e_s}) = \operatorname{sgn}\frac{d\hat{x}}{dt} \tag{16}$$

must hold at the welfare-maximizing tariff level. Proposition 3 (a) restates 16 in words. Part (b) follows from combining part (a) and Proposition 2.

Proposition 3

Let t^* denote the optimal ad-valorem tariff rate and e_s^* denote the elasticity of supply for the intermediate good respectively, both of which are evaluated at $t = t^*$ and $(X^*, r^*) = (\hat{X}, \hat{r})|_{t=t^*}$. Then
(a) Optimal tariff is strictly lower than inverse of supply elasticity if and only if intensive margin is increasing in tariff.
(b) Optimal tariff is strictly lower than inverse of supply elasticity (i) for all demand functions in the short run, (ii) and for all strictly convex demand functions in the long run.

At the heart of the short run result is underproduction in oligopoly which requires downward adjustment of tariff from the competitive benchmark. Long run production exhibits scale economies as reflected in the industry average cost (of production):

$$\frac{r(1+t)X + MK}{X} = r(1+t) + \frac{K}{x}$$

Whether tariff increases the scale of production (x) or not play a crucial role in determining whether optimal tariff exceeds or falls short of the competitive benchmark. The result follows from noting that an increase (decrease) in tariff leads to higher x if and only if the demand function is strictly concave (convex).

In what follows, we proceed in two stages. First, treating the number of importers as fixed, we examine how Home welfare varies with tariffs and determine optimal tariff in the short-run setting (Section 3.1). Subsequently, we derive optimal tariffs for the long-run setting in which the number of importers is endogenously determined via free entry (Section 3.2).

3.1 Short Run

Consider the effect of tariffs on Home welfare in the short run where the number of importers M is assumed to be fixed. Differentiating W with respect to t and rearranging yields:

$$\frac{dW}{dt} = (\hat{P} - \hat{r})\frac{d\hat{X}}{dt} - \hat{X}\frac{d\hat{r}}{dt}. \tag{17}$$

where $\hat{P} = P(\hat{X})$.

From Proposition 1 we know that an increase in a tariff rate t lowers aggregate output $\hat{X}$ as well as the input price $\hat{r}$. The first term, $(\hat{P} - \hat{r})\frac{d\hat{X}}{dt}$, captures a welfare loss due to a tariff-induced output reduction. Home consumers value the good at $\hat{P}$ while it costs $\hat{r}(< \hat{P})$ to produce the final good (from Home's perspective). This price-cost margin, $\hat{P} - \hat{r}$, multiplied by the amount of output lost, $\frac{d\hat{X}}{dt}$, is the magnitude of the welfare loss. The second term, $\hat{X}\frac{d\hat{r}}{dt}$, captures a welfare gain due to a lower input price which can be thought of as the terms-of-trade improvement for Home. The optimal tariff rate strikes a balance between the two – the welfare loss from the reduction in output and the welfare gain from the terms-of-trade improvement.

To simplify (17) further, note that

$$\frac{d\hat{r}}{dt} = h'(\hat{X})\frac{d\hat{X}}{dt}$$

which follows from differentiating (2). Furthermore, (3) yields the markup pricing formula in downstream oligopoly: the equilibrium price $\hat{P}$ equals the markup $(\frac{1}{1-\frac{1}{Me_d}})$ times a constant marginal cost $\hat{r}(1+t)$ for Home firms, i.e.,

$$\hat{P} = \frac{\hat{r}(1+t)}{1-\frac{1}{Me_d}},$$

where $e_d \equiv -\frac{\hat{P}}{\hat{Q}P'(\hat{Q})}(>0)$ is the elasticity of inverse demand for the final good at $P = \hat{P}$. Substituting the expressions for $\frac{d\hat{r}}{dt}$ and $\hat{p}$ into (17) and simplifying subsequently we get:

$$\frac{dW}{dt} = \left(\left(\frac{t+\frac{1}{Me_d}}{1-\frac{1}{Me_d}}\right)e_s - 1\right)\hat{X}h'(\hat{X})\frac{d\hat{X}}{dt}, \tag{18}$$

where $e_s \equiv \frac{\hat{r}}{\hat{X}h'(\hat{X})}(>0)$ is the inverse of the foreign supply elasticity of intermediate imports at $X = \hat{X}$. Optimal tariff is given by the value of tariff that solves $\frac{dW}{dt} = 0$.

Proposition 4

Optimal ad-valorem tariff rate is implicitly given by:

$$t^* = \frac{1}{e_s^*} - \frac{1}{Me_d^*}\left(1+\frac{1}{e_s^*}\right), \tag{19}$$

where e_d^* and e_s^* denote demand elasticity for the final good and supply elasticity for the intermediate imports respectively, both of which are evaluated at $t = t^*$ and $(X^*, r^*) = (\hat{X}, \hat{r})|_{t=t^*}$.

(i) $\lim_{M\to\infty} t^* = \frac{1}{e_s^*} > 0$, $\lim_{e_s\to\infty} t^* = -\frac{1}{Me_d^*} < 0$.

(ii) Suppose e_s is constant or strictly decreasing in r. There exists a unique M^* such that optimal tariff rate is strictly positive if and only if $M > M^*$.

When the downstream sector is perfectly competitive (i.e., $M = \infty$) we get the familiar result:

$$t^* = \frac{1}{e_s^*} > 0.$$

Optimal tariff is strictly positive and its value equals the inverse of export supply elasticity for intermediate imports. Imposing tariff on intermediate imports improves terms-of-trade and increases welfare. Terms-of-trade improvement however comes at the cost of exacerbating underproduction in oligopoly. Tariffs reduce output and create a welfare loss as the price-cost margin, $P(\hat{X}) - \hat{r}(1+t)$, is strictly positive when M is finite. Hence, in the presence of the oligopolistic downstream sector, t^* is strictly lower than $\frac{1}{e_s^*}$. Nevertheless, since $\lim_{M\to\infty} t^* > 0$, the possibility of positive optimal tariff remains as long as M is larger than a threshold value, M^* say. Part (ii) of Proposition 4 says that a sharp characterization exists when either export supply elasticity is constant or decreasing in r. This condition ensures that the optimal tariff increases as competition increases in the downstream sector which in turn implies[7]

$$M > M^* \quad \Leftrightarrow \quad t^* > 0.$$

[7] See Appendix A.1.4 for a formal proof of the statement.

To see why optimal tariff could be negative, suppose e_s is constant and let $e_s \to \infty$. As the supply function becomes perfectly elastic, terms-of-trade motive for tariffs disappears. In this case, an import subsidy raises aggregate output which increases Home welfare. This setting is similar to a single-stage, Cournot oligopoly with M domestic firms and constant marginal cost. In such settings, an import subsidy increases welfare by mitigating underproduction in oligopoly. Optimality of an import subsidy is not restricted to the case when e_s is large. Even for arbitrarily small e_s there exists suitable demand functions and market structure such that $t^* < 0$.

Remarks

To keep the analysis sharp, we have assumed that (a) all intermediate input suppliers locate in the rest of the world and (b) all consumers reside at Home, i.e., Home firms do not export the final good to the rest of the world. Profits of Home intermediate input suppliers increase with tariffs which increases Home welfare. Adverse effect of tariffs on consumer surplus is lower if some consumers reside in the rest of the world, which also increases Home welfare. Both these effects work towards increasing optimal tariffs. Allowing for these effects discussed will affect both the sign and the magnitude of the optimal tariffs but no substantive changes are expected in terms of the analysis.[8]

We have assumed a simple one-to-one production technology. In particular, we have considered a single intermediate input, c units of which is used to produce one unit of final good. By focusing on single input we have abstracted away from factor price distortion and its related effect on production efficiency. By design, we also ignore feedback effects which arise naturally in Beshkar and Lashkaripour (2020) as they consider an explicit input-output structure in a neoclassical trade framework. If the final good were produced using labor and intermediate input, optimal tariff would have been higher as part of the adverse effect could have been moderated by substituting input with labor. Similar effects would arise if there were multiple inputs which were substitutable to some degree.

3.2 Long Run

Consider next the effect of tariffs on Home welfare in the long run where the number of importers is endogenous. However, here, equilibrium values refer to the ones obtained in section 3.3 where the number of Home firms, $\hat{M}$, is endogenously determined. Differentiating W with respect to t yields

$$\frac{dW}{dt} = (\hat{P} - \hat{r})\frac{d\hat{X}}{dt} - \hat{X}\frac{d\hat{r}}{dt} - \frac{d\hat{M}}{dt}K. \tag{20}$$

As in the short run in (17), the first two terms in (20) respectively capture the welfare loss from a reduction in aggregate output and the welfare gains from a reduction in the input price, although changes in these terms due to tariffs are different from the case where M is fixed. The new term, $-\frac{d\hat{M}}{dt}K$, captures the welfare gain from fixed cost savings due to tariff-induced exit of Home firms, which occurs in only free-entry equilibrium. In the long run, the optimal tariff rate strikes a balance between the three – the welfare loss from the reduction in output and the welfare gain from the terms-of-trade improvement as well as tariff-induced exit of Home firms.

[8]Note that this expectation is based on the premise that the Home government maximizes welfare. The presence of firms in both the upstream and downstream sectors can bring up important political economy considerations because the interests of firms in these two sectors do not necessarily coincide. We abstract from such political economy considerations in this paper. See, for example, Blanchard et al. (2017) for an analysis along these lines.

Simplify the expression $\frac{d\hat{M}}{dt}$ in (13) as:

$$\frac{d\hat{M}}{dt} = -\frac{P'(\hat{X})\hat{x}(1+\frac{\eta(\hat{X})}{2})}{K}\frac{d\hat{X}}{dt} = \frac{(\hat{P}-\hat{r}(1+t))(1+\frac{\eta(\hat{X})}{2})}{K}\frac{d\hat{X}}{dt},$$

where the last equality follows from rewriting (3) as $\hat{P} - \hat{r}(1+t) = -P'(\hat{X})\hat{x}$. Using this equality and noting that $\hat{X}\frac{d\hat{r}}{dt} = \hat{X}h'(\hat{X})\frac{d\hat{X}}{dt}$, we can express (20) as:

$$\frac{dW}{dt} = \left(\left(\frac{t\left(1-\frac{1}{\hat{M}e_d}-\frac{\hat{\eta}}{2\hat{M}e_d}\right)-\frac{\hat{\eta}}{2\hat{M}e_d}}{1-\frac{1}{\hat{M}e_d}}\right)e_s - 1\right)\hat{X}h'(\hat{X})\frac{d\hat{X}}{dt}, \tag{21}$$

where $\hat{\eta} = \eta(\hat{X}) = \frac{XP''(\hat{X})}{P'(\hat{X})}$. Setting (21) to zero and solving for t gives the expression for the optimal tariff.

Proposition 5

The optimal ad-valorem tariff rate t^* is implicitly given by

$$t^* = \frac{1}{e_s^*} + \left(\frac{\frac{\eta^*}{2M^*e_d^*}}{1-\frac{1}{M^*e_d^*}-\frac{\eta^*}{2M^*e_d^*}}\right)\left(1+\frac{1}{e_s^*}\right), \tag{22}$$

where e_d^*, e_s^*, and η^* denote the elasticity of demand for the final good, the elasticity of export supply for the intermediate imports, and the elasticity of slope of the inverse demand function respectively, all of which are all evaluated at $t = t^*$ and $(X^*, r^*, M^*) = (\hat{X}, \hat{r}, \hat{M})|_{t=t^*}$. Furthermore,

(i) $t^* \gtreqless \frac{1}{e_s^*} \;\Leftrightarrow\; P''(.) \lesseqgtr 0$,

(ii) $\lim_{K\to 0} t^* = \frac{1}{e_s^*} > 0, \quad \lim_{e_s^*\to\infty} t^* \gtreqless 0 \;\Leftrightarrow\; P''(.) \lesseqgtr 0$,

(iii) For all concave demand functions, optimal tariff is strictly positive. Negative optimal tariff arise only for convex demand functions.

As the entry cost becomes arbitrarily small, the downstream sector becomes almost perfectly competitive. From the optimal tariff formula in (22), it follows immediately that the value of the optimal tariff t^* approaches $\frac{1}{e_s^*}$ – the inverse of the export supply elasticity for intermediate imports. Apart from this terms-of-trade route that exists irrespective of the market structure, tariffs also affect welfare through two additional channels in free-entry oligopoly. As in the fixed M case, an increase in tariff worsens welfare by lowering aggregate output. However, a tariff can improve welfare too by lowering $\hat{M}$ and saving entry costs. The welfare gain from entry cost savings exactly offsets the welfare loss from a output reduction when demand is linear. Thus, for linear demand, $t^* = \frac{1}{e_s^*}$ holds irrespectively of whether there is perfect competition or (free-entry) oligopoly in the downstream sector. However, in general, t^* could be larger or smaller than $\frac{1}{e_s^*}$, depending on the demand curvature. Simple inspection of (22) indeed reveals that

$$t^* \gtreqless \frac{1}{e_s^*} \;\Leftrightarrow\; \eta^* \gtreqless 0 \;\Leftrightarrow\; P''(.) \lesseqgtr 0.$$

When $e_s^* \to \infty$, this translates to part (ii): strictly positive tariff is optimal if and only if the demand function is strictly concave.

To understand why demand curvature plays an important role, let $e_s^* \to \infty$, and consider an infinitesimally small increase in t from $t = 0$. Decomposing $\frac{d\hat{X}}{dt}$ as

$$\frac{d\hat{X}}{dt} = M\frac{d\hat{x}}{dt} + \hat{x}\frac{d\hat{M}}{dt},$$

and noting that

$$(\hat{P} - \hat{r})\hat{x} - K = 0$$

holds in free-entry, free-trade equilibrium, we obtain the following implication from (20);

$$\left.\frac{dW}{dt}\right|_{t=0} \gtreqless 0 \;\;\Leftrightarrow\;\; M(\hat{P} - \hat{r})\frac{d\hat{x}}{dt} - \hat{X}\frac{d\hat{r}}{dt} \gtreqless 0.$$

When $e_s^* \to \infty$, $\frac{d\hat{r}}{dt} \to 0$ and thus

$$\left.\frac{dW}{dt}\right|_{t=0} \gtreqless 0 \;\;\Leftrightarrow\;\; \frac{d\hat{x}}{dt} \gtreqless 0 \;\;\Leftrightarrow\;\; P''(.) \lesseqgtr 0.$$

where the last inequality follows from Proposition 2.

Homogeneous product oligopoly exhibits underproduction and excessive entry, i.e., more firms enter in equilibrium than is socially optimal and produce too little (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). Increase in tariffs reduce $\hat{X}$ which exacerbates underproduction and lowers welfare but it also reduces $\hat{M}$ which mitigates excessive entry and increases welfare. Loosely speaking, which effect dominates depends on the effect of tariffs on $\hat{x} = \frac{\hat{X}}{\hat{M}}$, or equivalently the effect of tariffs on firm-level output $\hat{q}$. In the presence of entry costs K, higher firm-level output implies greater economies of scale and consequently higher welfare. As we have shown in Proposition 2, firm-level output increases (decreases) with tariffs when the demand function is strictly concave (convex). Thus, starting from free trade, a small ad-valorem import tariff (subsidy) improves welfare irrespective of entry costs and market structure if and only if the demand function is strictly concave (convex). With $e_s < \infty$, however, the terms-of-trade motive for tariff resurfaces. Thus, a positive tariff becomes optimal not only for strictly concave demand functions but for a larger range of parameterizations as indicated in part (iii) of Proposition 5.

4 Heterogeneous Firms

The short-run welfare analysis illustrates how oligopolistic *underproduction* tempers the terms-of-trade motive of tariffs while the long-run welfare analysis highlights how tariffs can improve welfare by curbing *excessive entry*. A common assumption in these sections was that firms have identical costs. Here, we relax that assumption and consider cost heterogeneity to focus on a new role of tariffs: reallocation.

In addition to *improving terms of trade* and *exacerbating underproduction*—which are present in both short-run and long-run versions of the model—tariffs *alter production efficiency* by reallocating output among firms with different costs. In particular, tariffs improves production efficiency for all strictly log-concave demand functions by increasing the efficient firms' share of imports while production efficiency worsens for strictly log-convex demand functions.

Change in production efficiency notwithstanding, optimal tariff continues to be lower than the inverse of supply elasticity as long as all firms reduce their imports in response to higher tariffs. However, unlike homogeneous firms setting, some firms might import more in response to increased tariffs if costs are sufficiently dispersed. Qualitatively different firm-level response to tariffs—some firms import more while some import less—is unique to heterogeneous firms setting. Whether optimal tariff t^* lies above or below inverse of supply elasticity $\frac{1}{e_s}$ depends on the share-weighted change in firm-level imports which in turn is influenced by demand curvature and Herfindahl index.

We show that $t^* < \frac{1}{e_s}$ continue to hold for linear demand, constant-elasticity demand, and more generally convex demand functions irrespective of cost differences. However, for strictly concave demand functions, $t^* > \frac{1}{e_s}$ can arise when Herfindahl index is sufficiently large (i.e. industry concentration is sufficiently high). Below, we discuss in details the rich differential impact of tariffs on firms, and consequent implications for intensive margin, welfare, and optimal tariff.

To focus on cost heterogeneity, we abstract from entry-exit considerations and assume that the number of firms, M, is fixed. Firms differ in their labor requirements. Home firm i combines one unit of intermediate input with a_i units of labor to produce one unit of final good:

$$q_i = \min\{\frac{l_i}{a_i}, x_i\}$$

Arrange M firms such that $a_i < a_{i+1}$ where $i \in \{1, 2, ..., M-1\}$. To ensure that all M firms are active, we assume that the ad-valorem tariff rate and cost-difference among the firms are not too large. Leontief technology plus wage of unity imply that Home firm i's marginal cost is $a_i + r(1+t)$. Proceeding as in the short run analysis in section 2, we find that Home firm i's equilibrium output is:

$$q_i = -\frac{P(Q) - a_i - r(1+t)}{P'(Q)}, \tag{23}$$

where Q uniquely solves

$$MP(Q) + QP'(Q) - M(\bar{a} + r(1+t)) = 0 \tag{24}$$

and $\bar{a} = \frac{\sum_i a_i}{M}$. Leontief production technology implies $x_i = q_i$ and consequently $X = \sum_i x_i = \sum_i q_i = Q$. Substituting Q with X in (24) and rearranging yields

$$r = \frac{P(X) + \frac{XP'(X)}{M} - \bar{a}}{1+t} \equiv \tilde{g}(X, M, t)$$

where $\bar{a} = \frac{\sum_i a_i}{M}$ and $\tilde{g}(.)$ denotes the inverse demand function for intermediate inputs. Equating $\tilde{g}(X, M, t) = h(X)$ and solving for X gives $\hat{X}$—aggregate intermediate imports—which upon substitution in appropriate equations gives equilibrium values for other variables. We continue to use $\hat{z}$ to denote equilibrium values of variable z.

Aggregate imports decline with an increase in tariffs. Since M is fixed, average imports per firm decline as well. Using $q_i = x_i$ and $Q = X$, we can rewrite (23) as

$$\hat{x}_i = \frac{\hat{X}}{M} + \frac{a_i - \bar{a}}{P'(\hat{X})}$$

which upon differentiation and some rearrangement gives:

$$\text{sgn}\frac{d\hat{x}_i}{dt} = \text{sgn}\bigg(\eta(s_i - \frac{1}{M}) - \frac{1}{M}\bigg). \tag{25}$$

where $\eta = \frac{\hat{X}P''(\hat{X})}{P(\hat{X})}$ denotes elasticity of slope, and $\hat{x}_i$ and $\hat{s}_i$ respectively denote firm i's volume of imports and share of imports in equilibrium.

Observe that we have $\frac{d\hat{x}_i}{dt} < 0$ when all firms have identical costs (and hence $s_i = \frac{1}{M}$ for all i). Even when import shares are dispersed, all firms cut back their imports when the elasticity of slope $\eta \in [-1, 0]$—a condition satisfied by demand functions which are both convex and logconcave. For example, this condition holds when $P(Q)$ is linear ($\eta = 0$) or log-linear ($\eta = -1$). More generally however, all firms do not necessarily cut back their imports in response to increased tariffs. An increase in ad-valorem tariff rate raises a firm's own cost as well as its rivals' costs. Increase in own cost prompts a firm to import less while an increase in rivals' costs prompts the same firm to import more since imports/quantities are strategic substitutes. In symmetric environment, own-cost effect dominates and each firm imports less as tariffs increase. This dominance does not necessarily hold under cost-heterogeneity when demand functions are strictly concave or strictly log-convex. An implication of (25) is that

$$\frac{d\hat{x}_i}{dt} > (=, <) \frac{d\hat{x}_{i+1}}{dt} \Longleftrightarrow \eta > (=, <) 0$$

Untangling this inequality and analyzing (25) gives us the following result.

Proposition 6 *Irrespective of cost-differences, all firms reduce imports in response to higher tariffs when demand functions are convex but logconcave (i.e.,$\eta \in [0, -1]$). For all other demand functions, a firm i reduce its imports if and only if*

$$\eta(s_i - \frac{1}{M}) - \frac{1}{M} > 0.$$

For strictly concave demand functions ($\eta > 0$), there exists $i^* \in [1, M)$ such that a firm i cuts back its imports if and only if it is less efficient than firm i^*. For strictly log-convex demand functions ($\eta < -1$), there exists $i^{**} \in (1, M]$ such that a a firm i cuts back its imports if and only if it is more efficient than firm i^{**}. In all cases, aggregate imports decline with an increase in tariffs.

Production efficiency naturally improves when efficient firms increase their imports in response to increased tariffs. However, improvement in production efficiency requires much less—increase in efficient firms import shares rather than their absolute volumes. Consider two firms i and j where firm i is more efficient (i.e., $a_i < a_j$). Using (23) we can express the ratio of import volumes as:

$$\frac{s_i}{s_j} = 1 + \frac{a_j - a_i}{\hat{P} - \hat{r}(1+t) - a_j} = 1 + \frac{a_j - a_i}{-\hat{X}P'(\hat{X}) + \bar{a} - a_j} \tag{26}$$

Differentiating (26), noting $\frac{d\hat{X}}{dt} < 0$, and using $\eta = \frac{\hat{X}P''(\hat{X}}{P'(\hat{X})}$ we get:

$$\text{sgn}\frac{d\frac{s_i}{s_j}}{dt} = \text{sgn}(-P'(\hat{X}) - \hat{X}P''(\hat{X}) = \text{sgn}(\eta + 1)$$

As t increases, both P and $\hat{r}(1+t)$ increase irrespective of the demand function but not necessarily by the same amount. Which one increaases more depend on elasticity of slope or equivalently on log-curvature. When demand is log-linear (i.e., $\eta = -1$), $\hat{P}$ and $\hat{r}(1+t)$ increase by the same amount which leaves $\hat{P} - \hat{r}(1+t)$ and consequently relative import shares unchanged. For strictly log-concave demand functions ($\eta > -1$) however $\hat{P}$ increases less than $\hat{r}(1+t)$. As price-cost margin squeezes, output share of relatively more efficient firms increase. The opposite is true for strictly log-convex demand functions.

This production efficiency altering role of tariffs suggest that, for strictly logconcave (logconvex) demand, optimal tariff is higher (lower) with cost-heterogeneity compared to the case when all firms have identical costs ($a_i = \bar{a}$, say). Can the improvement in production efficiency outweigh the predominant concern in oligopoly, namely underproduction, and push t^* above $\frac{1}{e_s}$? To answer that, express the effect of tariffs on welfare as:

$$\frac{dW}{dt} = \sum_{i=1}^{M}(\hat{P} - \hat{r}(1+t) - a_i)\frac{d\hat{x}_i}{dt} + \hat{r}(t - \frac{1}{e_s})\frac{d\hat{X}}{dt}.$$

Substituting q_i with $\hat{x}_i$, Q with $\hat{X}$, and $P(Q)$ with $\hat{P}$ in (23) yields $\hat{P} - \hat{r}(1+t) - a_i = -P'(\hat{X})\hat{x}_i = -\hat{s}_i P'(\hat{X})\hat{X}$ where $\hat{s}_i = \frac{\hat{x}_i}{\hat{X}}$. Expressing $-P'(\hat{X})\hat{X}$ as $\frac{\hat{P}}{e_d}$ we can rewrite $\frac{dW}{dt}$ as:

$$\frac{dW}{dt} = -\frac{\hat{P}}{Me_d}\left(\sum_{i=1}^{M}\hat{s}_i\frac{d\hat{x}_i}{dt}\right) + \hat{r}(t - \frac{1}{e_s})\frac{d\hat{X}}{dt} \tag{27}$$

Since $\frac{d\hat{X}}{dt} < 0$ (27) implies that

$$\text{sgn}(t^* - \frac{1}{e_s}) = \text{sgn}\left(\sum_{i=1}^{M}\hat{s}_i\frac{d\hat{x}_i}{dt}\right) = \text{sgn}\left(\eta(H - \frac{1}{M}) - \frac{1}{M}\right) \tag{28}$$

where $H = \sum_{i=1}^{n}\hat{s}_i^2$ denote Herfindahl index. As firm-level imports decrease for some firms and increase for others, the robust positive association between $t^* - \frac{1}{e_s}$ and average volume of imports in section 3 does not go through as it is. However, as (28) shows, a slight modification, namely, replacing simple average by weighted average restore the association. Unbundling (28) we get our final result.

Proposition 7 *Optimal tariff (t^*) is strictly lower than the inverse of supply elasticity ($\frac{1}{e_s}$) if and only if*

$$\eta(MH - 1) < 1 \tag{29}$$

where H denotes Herfindahl index evaluated at $t = t^*$. The condition holds for linear demand, constant-elasticity demand and indeed for all strictly convex demand functions irrespective of the cost-differences across firms. However, for strictly concave demand, the conditions holds only if the costs are not too dispersed.

The first thing to note is that (1) always holds when firms are identical. In that case, $s_i = \frac{1}{M}$, $H = \frac{1}{M}$ and the left-hand side of (29) is zero. Why does the condition always hold for all convex demand functions ($\eta \leq 0$)? Note that the first equality in (28) implies that a necessary condition for optimal tariff to exceed inverse of supply elasticity is that some firm must increase its imports in response to higher tariffs. From Proposition 6 we know this cannot occur for linear, log-linear, or any demand functions which satisfy $\eta \in [-1, 0]$. While the possibility $\frac{dx_i}{dt} > 0$ exists for $\eta < -1$, the reallocation is, loosely speaking, of the wrong kind, i.e. from efficient firms to inefficient firms which calls for lower rather than higher optimal tariff. Thus, $t^* - \frac{1}{e_s} > 0$ can hold only for strictly concave demand functions provided costs are sufficiently dispersed. For a concrete example, consider the class of inverse demand functions given by

$$P = a - \frac{bQ^{1+\eta}}{1+\eta}$$

where a, b, and η are strictly positive cosntants. For all $\eta > 0$, there exists $M > 1 + \frac{1}{\eta}$ such that $t^* > \frac{1}{e_s}$ hold when firms are sufficiently heterogeneous and consequently H is suitably large.[9]

To summarize, tariffs improve terms-of-trade (reduce input price) and amplify underproduction in oligopoly. These effects underpin all versions of the oligopoly model and work towards reducing optimal tariff (t^*) below the inverse of supply elasticity ($\frac{1}{e_s}$). When market structure is endogenous, tariffs can improve welfare further by curbing excess entry. Under cost-heterogeneity, tariffs can increase (reduce) welfare by reallocating import shares and improving (worsening) production efficiency. Despite welfare improvement from rationalization of the industry and welfare changes from reallocation of import shares, $t^* < \frac{1}{e_s}$ hold for all strictly convex demand functions. For linear demand, $t^* \leq \frac{1}{e_s}$, where the equality holds under both free entry and heterogeneous costs version of the model. For strictly concave demand functions, optimal tariff is sensitive to the details of the oligopoly environment: $t^* < \frac{1}{e_s}$ when M is fixed, $t^* > \frac{1}{e_s}$ when M is endogenously determined via free entry, and finally, under cost heterogeneity, t^* could be higher or lower than $\frac{1}{e_s}$ depending on the degree of heterogeneity and the number of firms. Negative optimal tariff can arise for all demand functions in all versions of the model except when demand function is strictly concave and the market structure is endogenously determined via free entry.

5 Discussion

So far we have assumed that (i) the upstream sector is perfectly competitive, (ii) downstream firms produce homogeneous goods, and (iii) import tariffs are the only policy instrument. We chose the simplest possible market structure in the upstream sector: perfect competition. Simplicity notwithstanding, this choice highlight the importance of *oligopoly* and *free entry* in the downstream sector in determining the effect of tariffs on market structure and welfare. Our choice of homogeneous goods is in line with the literature on imperfect competition in international trade where, typically, oligopoly is paired with homogeneous goods, and monopolistic competition is paired with differentiated goods. Below, in Sections 5.1 and 5.2, we show how our results can be extended to incorporate imperfect competition in the upstream sector as well as product differentiation in the downstream sector. Following the standard practice in the trade policy literature, we have focused exclusively on tariffs as the only policy instrument. Section 5.3 considers an extension where the Home government chooses the number of importers (or equivalently entry taxes) as well as tariffs. We show that the presence of such active competition policy lowers levels of optimal tariffs. In fact, while the issues raised in (i)-(iii) might seem disparate, the analyses in Sections 5.1-5.3 suggest a common theme—optimal tariffs are lower compared to the baseline case.

5.1 Vertical oligopoly

Consider a vertical oligopoly setting, where firms in both upstream and downstream sectors act as Cournot competitors. We continue to assume that there are no intermediate goods suppliers at Home. All final goods producers at Home import intermediate goods from the rest of the world (which we refer to as Foreign in the discussion below). There is free entry of firms in both sectors.

Recall the inverse demand function for the intermediate good faced by Foreign firms:

$$r = \frac{P(X) + \frac{XP'(X)}{M}}{1+t} \equiv g(X, M, t),$$

[9] Note that aggregate imports do not depend on how a_i are distributed as long as their sum and consequently the mean is constant. However any increase in dispersion in a_i causes H to increase as firms' shares get more dispersed.

where $g(X, M, t)$ satisfies (6). Suppose $N(\geq 2)$ Foreign firms enter the upstream sector paying the entry cost K_F. For simplicity, we assume that Foreign firm faces constant marginal cost c. Each Foreign firm $i \in \{1, 2, ..., N\}$ chooses x_i to maximize

$$\pi_{Fi} \equiv \left(g\left(x_i + \sum_{j \neq i}^{N} x_j, M, t\right) - c\right)x_i,$$

taking other Foreign firms' quantities (x_j) as given. Furthermore, we assume that

$$(N + 1)g_X(X, M, t) + Xg_{XX}(X, M, t) < 0,$$

which ensures uniqueness of the upstream Cournot equilibrium. Solving the first-order conditions yields the equilibrium quantity for each Foreign firm:

$$x = -\frac{g(X, M, t) - c}{g_X(X, M, t)},$$

where X uniquely solves

$$Ng(X, M, t) + Xg_X(X, M, t) - Nc = 0.$$

The solution denotes the aggregate volume of imports in the short run, i.e., given market structure (M, N). In addition, the following pair of equations hold in the long run:

$$\frac{(P(Q) - r(1 + t))Q}{M} - K_H = 0,$$
$$\frac{(r - c)X}{N} - K_F = 0,$$

which describe the zero-profits conditions in the downstream and upstream sector respectively. Substituting $Q = X$, using $r = g(X, M, t)$, and solving the above aggregate first-order condition and free entry conditions simultaneously yields implicit solutions of $\hat{X}$, $\hat{M}$, and $\hat{N}$ in a free-entry, vertical oligopoly equilibrium.

As in section 3, a reduction in tariffs increases the aggregate volume of imports (exports) and the number of importers (exporters). The effect on the intensive margin continues to be ambiguous. Firm-level imports increase with tariff reductions if and only if the demand function is strictly convex. While the properties of optimal tariffs are similar as before, imperfect competition in the upstream sector means that there is no explicit export supply curve. When M and N are fixed in the short run, an import subsidy is optimal when the number of Home importers (M) is relatively small. When these numbers are endogenous in the long run, an import subsidy is optimal for a larger range of parameterizations. This is because tariffs on imported inputs discourage entry not only of Home importers ($\hat{M}$) but also Foreign exporters ($\hat{N}$). An increased concentration in the upstream sector puts upward pressure on the input price. This dampens the terms-of-trade motive of tariffs and increases the likelihood of an import subsidy.

Terms-of-trade can worsen, i.e., the input price ($\hat{r}$) can increase, with an increase in tariff. To see why, rewrite the zero-profits condition in the upstream sector as

$$\hat{r} = c + \frac{K_F}{\hat{x}},$$

where $\hat{r}$ and $\hat{x}$ are the input price and quantity of upstream imports respectively. An increase in $\hat{r}$ is equivalent to a reduction in $\hat{x}$. We have that

$$\frac{d\hat{x}}{dt} = \frac{\partial \hat{x}}{\partial t} + \frac{\partial \hat{x}}{\partial N}\frac{d\hat{N}}{dt} + \frac{\partial \hat{x}}{\partial M}\frac{d\hat{M}}{dt}.$$

When M and N are exogenously fixed, each upstream firm cuts back its production in response to increased t. Thus $\frac{\partial \hat{x}}{\partial t} < 0$. There is a further cutback as an increase in t reduces the number of Home firms which in turn reduces the demand for the intermediate good. Thus $\frac{\partial \hat{x}}{\partial M}\frac{d\hat{M}}{dt} < 0$. Counteracting these negative effects (on $\hat{x}$) is a positive effect arising from weaker competition among upstream firms. An increase in t reduces the number of Foreign firms which in turn increases the scale of production for surviving Foreign firms. Thus $\frac{\partial \hat{x}}{\partial N}\frac{d\hat{N}}{dt} > 0$. Combining these effects, we find that $\frac{d\hat{x}}{dt} < 0$ for $\eta (\equiv \frac{QP''(Q)}{P'(Q))}) > 1$. However, for all demand functions satisfying $\eta < 1$ – a class of demand that includes both constant elasticity demand and linear demand – there exist K_H and K_F such that $\frac{d\hat{x}}{dt} < 0$, and consequently $\frac{d\hat{r}}{dt} > 0$.

Our finding $\frac{d\hat{r}}{dt} > 0$ is akin to Lerner paradox which occurs when import tariffs worsen country's terms-of-trade. While rare in perfectly competitive settings, Venables (1987) and more recently Bagwell and Lee (2020a, b) have shown that the paradox can arise in single-stage, monopolistic competition models.[10] As tariffs raise the price of Foreign varieties, Home consumers substitute Foreign varieties with Home varieties, which induces exit of Foreign firms and entry of Home firms. An increased concentration of Foreign firms raises the Foreign price index which in turn can worsen the terms-of-trade. One-to-one production technology implies that the final good produced by Home firms and the intermediate good produced by Foreign firms are complements in production. Thus, an increase in tariffs not only prompts exit of Foreign firms but also exit of Home firms. Exit of Home firms shifts the input demand inward but has no direct effect on the input price for constant elasticity demand or linear demand as the elasticity of input demand is unaffected by the shift. For those demand functions, and more generally for demand functions that are not too concave, the Lerner paradox can arise in our model when the indirect upward pressure (of tariffs) on the input price induced by exit of Foreign firms outweighs the direct downward pressure (of tariffs) on the input price.

5.2 Product differentiation

Incorporating differentiated products in our framework is straightforward. Suppose that there are M Home firms each producing a distinct variety. Let q_i and p_i denote the quantity and price of variety i. Assume that the representative consumer at Home maximizes $U(q_1, q_2, ..., q_M) + y$, where y is the numeraire good and

$$U(q_1, q_2, ..., q_M) = a\sum_{i=1}^{M} q_i - \sum_{i=1}^{M}\frac{q_i^2}{2} - \frac{1}{2}\sum_{j\neq i} q_i q_j.$$

This quadratic utility specification gives rise to a linear demand system:

$$p_i = a - q_i - b\sum_{j\neq i} q_j.$$

The parameter $b \in [0, 1]$ captures the degree of product differentiation. As b increases, varieties become less differentiated. When $b = 1$ we get linear demand for homogeneous goods. As in our baseline case, we assume one-to-one production technology and a perfectly competitive upstream sector.

[10]See also Bagwell and Staigher (2012a) for a similar result in linear Cournot framework with free entry.

Proceeding as in section 2, we find that the short-run demand for the intermediate good for M firms is

$$r = \frac{a - \frac{2+(M-1)b}{M}X}{1+t} = g(X, M, t).$$

Using the zero-profits condition $(p_i - r(1+t))q_i - K = 0$ where K denotes entry costs, we obtain the long-run demand curve for intermediate imports:

$$r = \frac{a - (2-b)\sqrt{K} - bX}{1+t} = G(X, t).$$

The equilibrium value of aggregate imports is given by $\hat{X}$ where $G(\hat{X}, t) = h(\hat{X})$. The number of importers and volume of imports per firm are given by $\hat{M} = \frac{\hat{X}}{\sqrt{K}}$ and $\hat{x} = \frac{\hat{X}}{\hat{M}} = \sqrt{K}$ respectively. Both aggregate volume of imports ($\hat{X}$) and the number of importers ($\hat{M}$) increase with reductions in tariffs. However, as in long run analysis in section 2, firm-level imports ($\hat{x}$) do not vary with tariffs when demand is linear.

Substituting $q_i = \hat{x} = \frac{\hat{X}}{\hat{M}} = \sqrt{K}$ in $U(.)$ and simplifying, we can express Home welfare as

$$W = \hat{X}\left(a - \frac{(1 + b(\hat{M}-1)}{2}\sqrt{K}\right) - \hat{r}\hat{X} - \hat{M}K + \bar{L}.$$

Setting $\frac{dW}{dt} = 0$ and rearranging yields

$$t^* = \frac{1}{e_s^*} - \frac{(1-b)\sqrt{K}}{2r^*}, \tag{30}$$

where $r^* = \hat{r}$ at $t = t^*$. Observe that, as in section 4, optimal tariff equals the inverse of export supply elasticity when products are homogeneous ($b = 1$) or entry costs are vanishingly small ($K \to 0$). Comparing (30) with (22) shows that the optimal tariff is lower in the presence of product differentiation ($b < 1$). Imposing tariff worsens welfare by exacerbating underproduction and increases welfare by improving terms-of-trade and saving entry costs (by prompting exit). These effects are present irrespective of whether the final good is homogeneous or differentiated. However, when products are differentiated, fewer firms (resulting from higher tariffs) also imply fewer varieties which lowers welfare. This additional source of welfare losses lowers the optimal tariff. In fact, it follows immediately from (30) that when the export supply elasticity is large, an import subsidy could be optimal – a possibility that does not arise for linear demand in a homogeneous product setting.

5.3 Policy

Multiple Instruments: We have assumed that the welfare-maximizing Home government has a single policy instrument at its disposal of and that instrument is input tariff. This is in line with tradition in the literature of optimal tariffs.[11] In the presence of per unit consumption subsidy c_y, equation (19) could be expressed as

$$(1+t^*)(1-c_y) = \left(1 - \frac{1}{Me_d^*}\right)\left(1 + \frac{1}{e_s^*}\right).$$

Setting $c_y = \frac{1}{Me_d^*}$ yields the classical result: $t^* = \frac{1}{e_s^*}$. This argument needs to be qualified in the presence of entry/exit consideration. With appropriate per-unit subsidies, only one downstream firm is needed to produce the entire output as the final good is homogeneous. Per-unit instruments alone cannot achieve that. An entry

[11]See Lashkaripour and Lugovskyy (2022) for a detailed discussion of first-best, second-best, and third-best policies in the presence of input-output linkages.

tax is needed. The first-best allocation could be achieved with a entry tax correcting excessive entry, a subsidy correcting underproduction, and an import tariff improving terms of trade. Thus, the absence of other policy instruments (i.e., other than tariffs) play an important role in our environment.

Competition policy: Instead of dwelling more on per unit taxes and subsidies we conclude our discussion by looking at a scenario where such instruments are not available. The only instrument at government's disposal is entry tax which influence the market structure. For simplicity, we work with an equivalent competition policy problem: Home government directly chooses the number of firms (M) to maximize welfare. In reality, a government indirectly affects market structure by entry taxes/subsidies, merger approvals, and ownership restrictions. However, choosing the number of firms directly captures the essence of competition policy in a simple fashion (Horn and Levinsohn, 2001).

To highlight the role of competition policy, consider first an environment with free trade (i.e., $t = 0$). Differentiating welfare with respect to M yields:

$$\frac{dW}{dM} = (\hat{P} - \hat{r})\frac{d\hat{X}}{dM} - \hat{X}\frac{d\hat{r}}{dM} - K.$$

Let M_f denote the number of importers in the free-entry equilibrium. Decomposing $\frac{d\hat{X}}{dM}$ as $\hat{x} + M\frac{\partial \hat{x}}{\partial M}$ and noting that $(\hat{P} - \hat{r})\hat{x} - K = 0$ holds in free-entry equilibrium, we get:

$$\left.\frac{dW}{dM}\right|_{M=M_f} = (\hat{P} - \hat{r})\frac{\partial \hat{x}}{\partial M} - \hat{X}\frac{d\hat{r}}{dM}.$$

From Proposition 1 we know that an increase in the number of importers decreases firm-level imports $\hat{x}$ and increases the input price $\hat{r}$. Thus $\frac{\partial \hat{x}}{\partial M} < 0$ and $\frac{d\hat{r}}{dM} > 0$, which in turn implies that

$$\left.\frac{dW}{dM}\right|_{M=M_f} < 0.$$

Restricting the number of importers at margin always improves Home welfare, since there is an excessive number of importers in free-entry equilibrium. Marginal entrant/importer ignores the business stolen from existing firms, leading to excessive entry in homogeneous product oligopoly (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987).[12] In our trade setting, marginal entrants also ignore the negative impact on terms-of-trade which exacerbates excess entry. Thus, for any tariff rate t, the following relationship holds:

$$M_f(t) > M^0(t)$$

where $M_f(t)$ and $M^0(t)$ are free-entry and welfare maximizing number of firms respectively.

Now let us re-introduce tariffs. Suppose a welfare-maximizing Home government chooses both M and t, so that it can use both trade and competition policies at the same time. How does that affect optimal tariff? Observe that

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + \frac{\partial W}{\partial M}\frac{dM^0(t)}{dt}.$$

[12] In a closed economy setting, maintaining the assumption of a homogeneous product, Ghosh and Morita (2007) show that entry might be insufficient in a vertical oligopoly model like the one seen in section 5.1. The trade literature typically focuses on differentiated goods and CES preferences where insufficient entry is the norm. An exception is Bagwell and Lee (2020a, b). Considering quadratic preferences used by Melitz and Ottaviano (2008), they show that entry can be insufficient or excessive in equilibrium depending on demand and cost parameters.

As the number of importers adjusts optimally in equilibrium, solving for optimal tariff, t^*, boils down to choosing the value of t such that $\frac{\partial W}{\partial t} = 0$ or equivalently:

$$(\hat{P} - \hat{r})\frac{\partial \hat{X}}{\partial t} - \hat{X}\frac{\partial \hat{r}}{\partial t} = 0.$$

Using $\hat{P} - \hat{r}(1+t) = -P'(\hat{X})\hat{x}$ and $\hat{X}\frac{\partial \hat{r}}{\partial t} = \hat{X}h'(\hat{X})\frac{\partial \hat{X}}{\partial t}$ in the equation above and rearranging, optimal tariff in this case is expressed as

$$t^* = \frac{1}{e_s^*} - \frac{1}{M^* e_d^*}\left(1 + \frac{1}{e_s^*}\right),$$

where all the expressions are evaluated at $t = t^*, M = M^* = M^0(t^*)$.

Note that the above expression for optimal tariff in the presence of active competition policy is the same as t^* in (19) – short-run optimal tariff – with one difference: the value of $M(= M^*)$ is endogenously determined by a welfare-maximizing Home government. As in the short run, optimal tariff is always less than the inverse of export supply elasticity, and in fact be negative if entry costs are large (and M^* is small). The possibility of t^* exceeding $\frac{1}{e_s}$ arises with free entry as higher tariffs can partially mitigate the welfare loss from excessive entry. With competition policy in place, that welfare enhancing effect of tariffs goes away. However, the welfare worsening effect of tariffs (through exacerbating underproduction) still remains which in turn lowers t^* below $\frac{1}{e_s}$ – the optimal tariff under perfect competition (where only the terms-of-trade motive of tariffs is present).

6 Conclusion

Recent years have witnessed a significant increase in vertical specialization and a concomitant growth in trade in intermediate goods. While there is a burgeoning literature exploring firms' participation in global supply chains, normative aspects (e.g., trade policy) in presence of such production structures have started to receive attention only recently (Grossman and Helpman, 2020; Caliendo et al.,2021; Laksharipour and Lugovosky, 2022). We contribute to that literature by examining optimal input tariff in environments which capture a significant fraction of world trade in intermediate inputs: (a) final goods and intermediate goods producers interact in markets, (b) market structure is oligopoly, and (c) most importantly, there is free entry and exit of oligopolistic firms.

We show that whether optimal input tariff exceeds or fall short of competitive benchmark—i.e., inverse of foreign export supply elasticity—depends on how intensive margin responds to tariff. In absence of entry, firm-level imports always increases with a reduction in tariffs and, thus, optimal tariff is strictly lower than inverse of export supply elasticity. In presence of free entry of foreign firms, intensive margin increases with tariff if and only if the demand function is strictly convex. Accordingly, optimal tariff is strictly lower (higher) than inverse of export supply elasticity if and only if the demand function is strictly convex. In presence of heterogeneity, optimal tariff is unambiguously lower than the competitive benchmark only for linear and strictly convex demand functions. Under strictly concave demand, optimal tariff exceeds competitive benchmark when Herfindahl index is sufficiently high. While oligopoly brings tariff down from the competitive benchmark, entry considerations and heterogeneity puts upward pressure on tariffs, since, in a homogeneous product setting like ours, tariff improves welfare by mitigating excess entry—where the number of firms is endogenously determined—and improves production efficiency (e.g., for log-concave demand functions)—where firms exhibit cost heterogeneity.

An important assumption maintained throughout the paper is a non-strategic rest of the world which does not engage actively in trade policy. Theoretically, a natural next step is to consider vertical specialization with two countries (Home and Foreign say) where consumers reside in both countries that set tariffs strategically—Home on intermediate goods and Foreign on final goods. As in all trade frameworks, the terms-of-trade motive for tariffs would be present here as well. However, complementarity between intermediate goods and final goods might prompt each government to set lower tariffs than it would have in the absence of vertical specialization. In future research, we plan to explore these issues and related implications for trade agreements when countries are specialized in different layers of production.

Appendix: Proofs

A.1 Proof of Proposition 1

Differentiating (7) and rearranging, we get

$$\frac{d\hat{X}}{dt} = \frac{g_t}{h'(X) - g_X} < 0, \quad \frac{d\hat{X}}{dM} = \frac{g_M}{h'(X) - g_X} > 0, \tag{A.1}$$

where the inequalities follow from noting that $h'(X) > 0$ and

$$g_X = \frac{(M+1)P'(X) + XP''(X)}{M(1+t)} < 0, \quad g_M = -\frac{XP'(X)}{M^2(1+t)} > 0, \quad g_t = -\frac{g(.)}{1+t} < 0.$$

Applying (A.1) to the input supply function (2), we get

$$\frac{d\hat{r}}{dt} = h'(X)\frac{d\hat{X}}{dt} < 0, \quad \frac{d\hat{r}}{dM} = h'(X)\frac{d\hat{X}}{dM} > 0,$$

and

$$\frac{d\hat{x}}{dt} = \frac{1}{M}\frac{d\hat{X}}{dt} < 0,$$

where the inequalities follow from noting that $\frac{d\hat{X}}{dt} < 0$. Finally, rearranging (4), we get

$$P(\hat{X}) + P'(\hat{X})\hat{x} = \hat{r}(1+t).$$

Differentiating the above equality yields:

$$P'(\hat{X})\frac{d\hat{x}}{dM} = (1+t)\frac{d\hat{r}}{dM} - (P'(\hat{X}) + \hat{x}P''(\hat{X}))\frac{d\hat{X}}{dM}. \tag{A.2}$$

The right-hand side of (A.2) is positive since $\frac{d\hat{r}}{dM} > 0$, $\frac{d\hat{X}}{dM} > 0$ and $P'(\hat{X}) + \hat{x}P''(\hat{X}) < 0$. Thus, $\frac{d\hat{x}}{dM} < 0$ since $P'(\hat{X}) < 0$.

A.2 Proof of Uniqueness of Q

Consider first the short-run equilibrium. Define the left-hand side of (4) as

$$f(Q) \equiv MP(Q) + QP'(Q) - Mr(1+t).$$

Differentiating $f(Q)$ with respect to Q and rearranging we get

$$f'(Q) = (M+1)P'(Q) + QP''(Q) < 0,$$

where the inequality follows from applying (1). The inequality implies that the left-hand side of (4) is strictly decreasing in Q which in turn ensures that q and Q in the short-run equilibrium are unique.

Consider next the long-run equilibrium. Define the left-hand side of (10) as

$$F(Q) \equiv M(Q)P(Q) + QP'(Q) - M(Q)r(1+t).$$

Differentiating $F(Q)$ with respect to Q and rearranging we get

$$F'(Q) = \frac{2M(Q)P'(Q) + QP''(Q)}{2} < 0,$$

where the inequality follows from applying (1). The inequality implies that q and Q in the long-run equilibrium are also unique.

A.3 Proof of Proposition 2

Differentiating (12) and rearranging, we get

$$\frac{d\hat{X}}{dt} = \frac{G_t}{h'(X) - G_X} < 0, \tag{A.3}$$

where the inequality follows from noting that $h'(X) > 0$ and

$$G_X = g_X + g_M M'(X) = \frac{2M(X)P'(X) + XP''(X)}{2M(X)(1+t)} < 0, \quad G_t = -\frac{G(.)}{1+t} < 0.$$

Applying (A.3) to the input supply function (2), we get

$$\frac{d\hat{r}}{dt} = h'(X)\frac{d\hat{X}}{dt} < 0.$$

From (13) we know that

$$\frac{d\hat{M}}{dt} = -\frac{P'(\hat{X})\hat{x}(1 + \frac{\eta(\hat{X})}{2})}{K}\frac{d\hat{X}}{dt} > 0.$$

From (14) we have

$$\frac{d\hat{x}}{dt} = \frac{1}{2\hat{x}}\frac{KP''(\hat{X})}{[P'(\hat{X})]^2}\frac{d\hat{X}}{dt}.$$

Since $\frac{d\hat{X}}{dt} < 0$, it follows that

$$\frac{d\hat{x}}{dt} < 0 \quad \Longleftrightarrow \quad P''(.) > 0.$$

A.4 Proof of Proposition 4

The expression for t^* in (19) as well as the proof part (i) follows from the text preceding Proposition 4. To prove part (ii) it suffices to show that t^* is strictly increasing in M. Rewrite (17) as

$$\frac{\hat{P}}{\hat{r}} - \left(1 + \frac{1}{e_s}\right) = 0. \tag{A.4}$$

Suppose t remains at t^*. From Proposition 1, we know that as M increases, $\hat{P}$ declines and $\hat{r}$ increases. Thus, $\frac{\hat{P}}{\hat{r}}$ decreases with an increase in M. Furthermore, as $\hat{r}$ increases, e_s decreases. Thus, the left-hand side of (A.4) decreases with an increase in M and falls below zero. An increase in t restores the equality in (A.4), since Proposition 1 implies that effect of an increase on t on $\hat{P}$, $\hat{r}$, and consequently on e_s is exactly the opposite to that of an increase in M. Thus t^* is strictly decreasing in M.

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