Unraveling Credibility in Stress Test Result Disclosures*

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ABSTRACT

We model the credibility problems financial regulators often face in disclosing the stress test result. The regulator has motives to lie about the result. The motives come from the desire to guide agents' actions by influencing their beliefs. We show that the regulator can disclose some information credibly by making an imprecise announcement: revealing only the range (or, interval) in which the result lies. In the interest of credibility, the regulator needs to reveal information with less precision when the stress test result is either too good or too bad. As the result moves away from both extremes, information can be revealed credibly with more precision.

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Introduction

Since the 2008 financial crisis, financial regulators worldwide have been conducting periodic stress tests to assess the banking sector's resilience under hypothetically adverse economic conditions. While stress tests play a vital role in safeguarding financial stability, the matter concerning the disclosure of stress test results remains controversial and has consistently plagued the regulatory landscape. Questions surrounding the credibility of financial regulators in disclosing stress test results have emerged time and again. To name a few, the stress test conducted by the Committee of European Banking Supervisors (CEBS) in 2010 significantly underestimated the capital need for European banks compared to estimates by independent consultants. In 2016 EBA's stress test, the reactions of investors gauged from the 8% drop in the share price of UniCredit, Italy's biggest bank, and the 3% fall in bank shares across Europe are in contrast to the result that showed improvement[1]. Such inconsistencies have raised doubts about the regulators' credibility in providing stress test results.

This paper is a theoretical exploration of the credibility challenges that arise from the natural features of the banking system in disclosing stress test results and how financial regulators can disclose some information credibly without any commitment to speak the truth. We set up a stylized model where a financial regulator conducts stress tests and learns about the fraction of banks resilient to adverse economic scenarios. The regulator, privately informed about the stress test result, sends a message. After receiving the message, depositors and banks act on it, and the pay-offs from their actions matter to the regulator. This simple yet powerful set-up allows us to delve into the conflicts of interest the regulator faces with both depositors and banks, and how these dynamics influence the regulator's ability to disclose the result credibly.

Conflict of interest with Depositors: Stress test results inform depositors [2] about the banking sector's financial health and influence their withdrawal decisions. If they are skeptical about the health of banks, they ask for early withdrawal (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005). To fulfill depositors' demands, the bank needs to liquidate its assets, which entails a fraction of liquidated assets as deadweight loss. To boost depositors' confidence and prevent early withdrawals,

[1] See Schuermann (2014) for a discussion on stress tests conducted by CEBS. For reference to the 2016 EBA stress test, see https://www.theguardian.com/business/economics-blog/2016/aug/01/all.

[2] Although we mention only depositors, the meaning extends to holders of all types of short-term financial instruments such as the repo, commercial paper, etc., that involve rollover risk

the regulator is *inclined to lie by inflating the test result in its message*.

Conflict of interests with Banks: Similarly, stress test results guide banks to decide on the optimal level of risk they should take in their investments. Since banks only consider the private costs of failure and ignore the negative externalities, they tend to invest in riskier projects than what the regulator would consider optimal. To persuade banks to take less risk, the regulator is *inclined to lie by understating the test result in its message*.

Addressing Credibility issues with Cheap Talk: We apply the concept of cheap talk introduced in Crawford and Sobel (1982) to resolve the above credibility problem. Instead of announcing the result, the regulator reveals a range (or, an interval) that contains the result. The ranges are constructed such that the cost of lying about the range outweighs the benefit, thus disciplining the regulator and *eliminating the need for regulator's commitment to telling the truth*. Hence, the disclosure mechanism does not require the strong commitment assumption, that has been prevalent in this literature which we discuss below.

The possible reason for the lack of studies on applying cheap talk in the disclosure of stress test results could be the focus of existing literature on only depositors. Cheap talk cannot be informative if the regulator would prefer to send the same message irrespective of the test result. In the case of depositors, we show that the regulator always prefers no withdrawal to avoid any liquidation loss and sends the same message to achieve a 'no withdrawal' outcome. Its preference for depositors' actions, (thus its message) is invariant to the realization of the test result. However, in the case of banks, the regulator wants to understate the result to persuade them to pick a less risky project, but not too much so that the level of riskiness of their investments matches the regulator's preference given the actual result. When we consider both depositors and banks as receivers of the message in a single set-up, the regulator's preference for their actions (thus its message) varies with the stress test result. It sends different messages depending on the result, making cheap talk informative and enabling the regulator to communicate. The presence of banks in the model disciplines the regulator's relationship with depositors and allows the possibility of cheap talk.

Quality of Communication in Cheap Talk Equilibrium: We use precision as a measure of the quality of communication and examine the precision with which information can be revealed credibly through cheap talk. When the test result gets close to either the lower or upper extreme (for e.g., when the result is revealed in terms of the fraction of banks passing the test, the lower and upper extremes are 0 and 1), information needs to be revealed with less precision to maintain credibility. The reason for this is the regulator's increasing conflict of interest with depositors and banks. For instance, banks' preferred project deviates more (or becomes riskier) from the regulator's choice as the stress test result moves close to the lower extremity. Similarly, the marginal cost of early withdrawal by depositors increases with the test result. The result improves with an improvement in the banks' asset quality. When the result gets close to the upper extreme, early withdrawal by depositors leads to liquidation of more good-quality assets[3]. More misalignment of the regulator's interests with banks and depositors at both ends incentivizes the regulator to manipulate more. To sound credible, cheap talk requires the regulator to send the message after introducing more noise or uncertainty to the result. As the result moves to the intermediate region, the extent of conflict with either group recedes, and information can be revealed credibly with more precision.

From a theoretical perspective, we also argue that depositors and banks (as potential receivers of stress test results) should be jointly studied because this approach uncovers a mutual disciplining effect. In our set-up, the regulator's incentive to manipulate the result lies in opposite directions for the two groups. While the regulator wants to lie to depositors by *inflating the test result* for preventing early withdrawals, it is inclined to lie to banks by *understating the test result* for curbing excessive risk-taking. If the cost of banks taking excessive risk is high, the regulator is less likely to overstate the result, which lessens its credibility problem with depositors. Similarly, the regulator's concern for early withdrawal by depositors and the resultant liquidation loss prevents it from understating the result too much, which lessens its credibility problem with banks.

Related Literature: We contribute to the literature on stress test disclosure. Bouvard et al. (2015) study a model where the disclosure policy is state-contingent. When the regulator is privately informed about aggregate and bank-specific results, it releases aggregate result after a moderate shock and bank-specific results after a large negative shock. In Faria-e-Castro et al. (2017) and Goldstein

[3] Note that the above concern of the regulator regarding early withdrawal by depositors is different from the one studied in the literature (Bouvard et al., 2015; Goldstein and Leitner, 2018): when the stress test result is low, revealing only aggregate result can cause systemic run as depositors cannot distinguish good banks from bad ones. To prevent this, the regulator should disclose bank-specific information. In Appendix B, we consider the disclosure of bank-specific information along with the aggregate result. We focus on the disclosure of aggregate results and the credibility issues associated with such policy. For regions where the regulator prefers to disclose only aggregate results, it must communicate through cheap talk to remain credible.

and Leitner (2018), the optimal disclosure rule takes the form of Bayesian persuasion where no disclosure (equivalent to disclosing aggregate result only) is optimal in normal times, and some disclosure (disclosing bank-specific information) is required during critical times to prevent a market freeze. Shapiro and Skeie (2015) is another work where the regulator communicates information to two audiences, depositors and banks. However, the information transmitted in their model relates to the regulator's type, specifically the cost of bailing out banks. The regulator signals its type to the market through costly actions, such as injecting capital, liquidating the bank, or taking no action (forbearance). Inostroza (2023) explores disclosure to multiple audiences belonging to the same category. The decision of a group of depositors/investors to provide credit to the bank strategically depends on both the regulatory disclosure and each other's actions. Their objective is to choose a disclosure policy that minimizes bank defaults. Unlike our study, Inostroza (2023) does not investigate the influence of stress test result disclosure on the risk-taking decisions of banks.

Primarily, this strand of the literature studies optimal disclosure policy under different economic environments and assumes that the regulator can commit to disclose the result truthfully under each disclosure policy[4]. However, the unique aspect of our model lies in the regulator's inability to commit to truthfully disclosing stress test results, which we justify based on the conflicts of interest it faces with the receivers of the stress test results (i.e., depositors and banks). Commitment, defined as the ability to make credible and enforceable promises, is questionable in the context of stress tests for two reasons. Firstly, once the receivers believe in the regulator, it gains incentives to manipulate the results, which defies credibility. Secondly, enforcing financial regulators to reveal the results credibly is practically implausible. Moreover, distinguishing the truth from the regulator's announcements is challenging, as the information used for conducting stress tests is not disclosed, even the models used for such tests are kept secret to prevent banks from gaming them (Leitner and Williams, 2022). Thus, we focus on studying a disclosure policy that allows credible revelation of results in an environment where the regulator lacks commitment to truthfully revealing the results.

In a related work, Pereira (2021) studies information disclosure when a financial regulator lacks the commitment to truthfully report stress test results. Her focus lies in examining the effects of po-

[4] An important assumption underlying the Bayesian persuasion framework is commitment on the regulator's part to reveal the results truthfully (Kamenica and Gentzkow, 2011). This assumption can be questioned in the context of stress tests. Besides, the manner in which the result is disclosed under Bayesian persuasion may be unrealistic for financial regulators to implement in practice.

tential manipulation of stress test results on disclosure policies and financial stability, rather than exploring the origins of the credibility problem and ways to communicate results credibly. In contrast, our paper unravels the endogenous emergence of credibility issues, arising from the regulator's conflicts of interest with both depositors and banks, which motivates manipulation in our model. Even in regions where the regulator refrains from manipulation in Pereira (2021), it can still benefit from manipulating results in our model. Furthermore, our study stands out in considering the regulator's credibility problem with both depositors and banks. The existing literature has only studied one or the other [5]. By doing so, we identify a mutual disciplining effect where the presence of banks (depositors) makes the regulator's credibility problem with depositors (banks) less severe. In a slightly allied context, Galvão and Shalders (2023) explores central bank communication with or without commitment, using a model of speculative currency attack similar to Morris and Shin (1998). However, their model does not encompass concerns about the regulator revealing the results truthfully, as in our study.

The remainder of the paper is organized as follows. Section 1 introduces the model. Section 2 describes the credibility problems faced by the regulator while disclosing the stress test result. Section 3 studies disclosure through cheap talk, presents the main results and describes the properties of cheap talk equilibrium. Section 4 concludes. We provide all proofs and additional results in the appendix.

1. The Model Setup

1.1. Technology and Preferences

Consider a banking industry with a continuum of banks indexed by $b \in (0,1)$. Each bank holds a legacy asset financed by a continuum of depositors indexed by $j \in (0,1)$. There are three time periods, $t = 0,\ 1,\ 2$ and one good. Bank promises its depositors an amount $D > 1$ at $t = 2$. However, depositors can withdraw before period 2, which entitles them to 1 unit of the good.

Each bank is of either H (high quality) or L (low quality) type, and accordingly, the legacy asset will generate a payoff of A_H or A_L, with $A_H > A_L$. Nature draws the fraction of H type banks $p \in (0,1)$ from the distribution $\pi(p)$ and each bank has the same probability (p) of being H type. In

[5] Primarily, the literature studies only depositors and the associated problems arising from early withdrawals. One exception is Corona et al. (2019), which studies how stress test results influence the risk-taking decisions of banks.

period 1, each risk neutral bank also receives an endowment of 1 unit and they have access to an investment opportunity denoted by project $V_k \in [\underline{V}, \overline{V}]$ with $\underline{V} \geq 1$. For simplicity, we assume that banks have limited liability which means depositors do not have recourse to the returns generated from investment in this new project if they cannot recover D from the legacy assets[6]. A project V_k generates a payoff of V_k with probability $q(V_k)$ where $\frac{\partial q(V_k)}{\partial V_k} < 0$ and $\frac{\partial^2 q(V_k)}{\partial V_k^2} \leq 0$. We assume $q(\underline{V}) = 1, q(\overline{V}) = 0$. The higher the return, the higher the risk and lower is the probability of success.

The project fails with probability $1 - q(V_k)$ and generates no return. A bank's survival after the project's failure is conditional on its type. H type banks can survive, but L type banks cannot[7]. A bank's failure is costly for the bank and the economy. We assume the private cost of bank failure (loss of jobs, future rents, etc.) to be C_P. In addition, bank failure has negative externalities such as loss of valuable relationships with borrowers, reduced intermediation activities, disruptions in payment infrastructure, etc. C_S denotes the aggregate (or social) cost of bank failure. The difference between the two, $C_S - C_P = \eta > 0$, captures the negative externalities. Our structure on the risk-return trade-off of the projects and conditions for survival following the project's failure follows Corona et al. (2019).

t = 0	t = 1	t = 2
• Nature decides fraction of H type banks. • Regulator runs stress test and observes p. • Regulator sends message $\hat{p}$	• Depositors decide whether to run or wait. • Banks choose project V_k	• Pay-off from the projects and legacy assets are realised. • Remaining depositors are paid.

Figure 1: Timeline

1.2. Stress Test

At $t = 0$, a financial regulator conducts stress tests to ascertain the banking sector's health and finds out the fraction of H type banks or p[8]. Then it publicly sends a message $(\widehat{p})$ that communicates

[6] This assumption is equivalent to off-balance sheet funding as in Segura and Zeng (2020), where banks have the option of providing debtors access to returns generated from existing assets. In our model, it is never optimal for banks to exercise this option.

[7] The survival condition upon failure of the project can be justified by assuming that a bank needs to hold a minimum amount of capital to operate or a minimum amount of liquidity to survive. Legacy assets of H type banks generate surplus payoff to fulfill this requirement.

[8] We model the result of the stress test as an aggregate measure for the entire banking system rather being bank specific and study the credibility of the financial regulator in disclosing the result. If macroprudential stability is the primary

its private information about p, where $\widehat{p}$ may not be equal to p. We make the assumption of public disclosure to stay close to the practice currently followed by financial regulators. For instance a Federal Reserve supervisory staff report by Clark and Ryu (2013), writes: "Given a widely held view among supervisors and most third-party observers that the public disclosure of stress testing results enhances available information and supports market discipline, it will continue and it is perhaps even likely to be expanded over time."

We further assume that banks do not observe their types; thus, the regulator has an information advantage over banks. This can be justified if the stress test involves an assessment of the bank's exposure to the states of other banks, which is known only to the regulator (Goldstein and Leitner, 2018). In our set-up, it can be modelled by making the performance of banks' legacy assets dependent. For the sake of simplicity, we abstract from explicitly modelling such dependence.

After observing the message sent by the regulator, depositors decide whether to withdraw at $t = 1$ or wait till $t = 2$. Banks decide on the project for investing the endowment. Figure 1 summarizes the timing of the events described in the model.

1.3. Early Withdrawal and Liquidation

Assumption 1: Legacy Asset Payoff and Liquidity

1. $A_H > D > A_L > 1$

2. $A_H > \frac{1}{1-\delta} > A_L$

The first part states that the legacy assets of H type banks generate sufficient payoff to pay depositors the promised amount at $t = 2$, whereas depositors of L type banks cannot be paid in full. The second part relates to the option of withdrawal at $t = 1$. If depositors decide to withdraw early, the bank needs to liquidate the legacy asset. Liquidation produces only $(1 - \delta)$ fraction of the asset being liquidated, where $0 < \delta < 1$ captures the loss from liquidation. From the second condition, H type banks are liquid as they can fulfill the promised payment of 1 at $t = 1$, but L type banks cannot. Our assumptions on the bank's legacy asset follow Faria-e-Castro et al. (2017).

goal, the academic literature supports disclosing aggregate results only. As argued by Goldstein, Sapra, et al. (2014), publishing bank-specific results are costly as banks tend to take sub-optimal actions and are more reluctant to share information with the regulator. Aggregating the results also eliminates idiosyncratic noise and measurement errors across individual banks and reduces the destabilizing effects of the information. Nonetheless, we consider disclosing bank-specific information in Appendix B and present our results.

If a bank is believed to be of H type, depositors do not demand early withdrawal. Because withdrawing early pays 1, while waiting till $t = 2$ pays D > 1. For a bank of L type, there always exists an early withdrawal equilibrium. Since the bank is not liquid, it will not have any asset left after a run in period 1. Anticipating this, depositors would decide to withdraw early.

The decision to withdraw early depends on the belief a depositor holds about the bank being of H type. Let s_r is the probability threshold such that if depositors believe that a bank is of H type with a probability above s_r, they prefer to wait. At s_r, depositors are indifferent between withdrawing early and waiting, or $s_r + (1 - s_r)(1 - \delta)A_L = s_r D$, and

$$s_r = \frac{(1-\delta)A_L}{(1-\delta)A_L + D - 1} \tag{1}$$

If depositors believe that a bank is of H type with probability in the set of $[0, s_r]$, they prefer to withdraw early. Although multiple equilibria exist for beliefs in $[0, s_r]$, we select the run equilibrium for simplicity à la Faria-e-Castro et al. (2017).

1.4. Bank's choice of project

At $t = 1$, each bank chooses a project V_k to invest the endowment of 1 unit by maximizing the final payoff net off the private cost of failure.

$$\underset{V_k}{argmax}\; q(V_k)V_k - (1 - q(V_k))(1 - p)C_P$$

Solving for the first order condition, the bank's choice of project is given by,

$$V_k^b = -\frac{q(V_k)}{q'(V_k)} - (1-p)C_P \tag{2}$$

where $q'(V_k) = \partial q / \partial V_k$. Banks do not consider the negative externalities $(\eta = C_s - C_p)$ arising from their failure. If the regulator is instead asked to choose a project on behalf of banks, it would consider the social cost of failure C_S and prefer a project where

$$V_k^r = -\frac{q(V_k)}{q'(V_k)} - (1-p)C_S \tag{3}$$

Note, in our model, if the stress test result reveals the presence of a large number of L type banks (p is low), banks infer a higher likelihood of failure. Thus, banks invest in a less risky project as it stands to incur a private cost of C_P upon failure. The decision of banks to take less risk when the regulator reveals a low p, could also be justified through other channels such as fear of banks from suffering contagion and a run by depositors[9]. This modelling assumption is also consistent with Corona et al. (2019)[10].

1.5. Welfare

The first best level of allocation will have the following features:

1. No early withdrawal by depositors so that deadweight loss arising from liquidation of legacy assets can be avoided.

2. Banks consider negative externalities of their failure and choose the socially optimal project V_k^r as per Equation (3).

Given the fraction of H type banks (p) in the economy, the first best level of welfare is given by

$$Y(p) = \overline{a(p)} + \left[q(V_k^r)V_k^r - (1-q(V_k^r))(1-p)C_S\right] \tag{4}$$

where $\overline{a(p)} = pA_H + (1-p)A_L$, is the value of legacy asset of all banks.

Departure from the above occurs due to (i) liquidation loss on the legacy asset $A_H,\ or\ A_L$ when depositors decide to withdraw early, and (ii) banks' disregard for the negative externalities of their failure; picking a riskier project V_k^b instead of V_k^r.

It can be questioned, why the regulator would benefit from having an insolvent and illiquid bank avoiding default. In our model, the failure of a bank imposes a cost of C_s, irrespective of the bank's solvency and liquidity condition. So, it is in the interest of the regulator to avoid such default[11].

2. Credibility of Disclosure

The regulator is inclined to lie about the stress test result to influence depositors' and banks' actions and minimize the deviation from the first best level of welfare. The desire to lie creates *credibility problem*. In what follows, we, first, discuss the credibility problem of the regulator with banks and depositors, and then we propose a disclosure policy using the concept of cheap talk.

[9] For simplicity, we abstract from modelling these channels explicitly.

[10] Although the authors consider bail out decisions later, we abstract from such regulatory intervention and focus primarily on communication

[11] We agree that there could be questions on whether this could give rise to a moral hazard problem. To keep our analysis sharp on regulatory communication and credibility issue, we abstract away from complications arising out of moral hazard and base our modelling assumption on the evidence that bank failure imposes significant economic costs (Kupiec and Ramirez, 2013).

2.1. Depositors only

We start by describing the credibility problem if depositors are considered to be the only receivers of the message sent by the regulator. After conducting the stress test, the regulator sends a message $\widehat{p}$, which may not be equal to the actual result p. We assume depositors have different means to access the message sent by the regulator and accordingly observe a signal s_j based on $\widehat{p}$. We denote the cumulative distribution function and density function of the signal s conditional on $\widehat{p}$ by $F(s|\widehat{p})$ and $f(s|\widehat{p})$ respectively. We make this assumption[12] to smooth the function that maps the revealed stress test result $(\widehat{p})$ to the fraction of depositors who decide to withdraw early $(F(s_r|\widehat{p}))$. It can also be formulated in another manner where depositors have different degrees of risk aversion. The withdrawal threshold s_r would then vary for each depositor, monotonically increasing with the degree of risk aversion. Both assumptions would lead to similar outcomes.

Note, in case the regulator does not reveal any message or the message turns out to be completely uninformative about p, we assume the worst possible outcome. Since the depositors cannot distinguish between H and L type banks, they prefer to run. We can also model this by introducing some prior probability p_0 that depositors hold about the banking sector. In case of no message or completely uninformative message by the regulator, depositors act based on the signals received conditional on p_0. This would only complicate our analysis later without adding any significant value to our objective.

Assumption 2: Monotone Likelihood Ratio Property: If $\widehat{p_1} > \widehat{p_2}$, the ratio $\dfrac{f(s|\widehat{p_1})}{f(s|\widehat{p_2})}$ is strictly increasing in s. The above assumption implies that $F(s|\widehat{p})$ is decreasing in $\widehat{p}$, or in other words less depositors are likely to observe a signal below s when the revealed $\widehat{p}$ is higher.

Depositors decide to withdraw early if they receive a signal below the threshold s_r. The fraction of depositors who decides to do so and force the bank to liquidate legacy assets is given by $F(s_r|\widehat{p})$. This leads to a liquidation loss of,

$$Loss(p, \widehat{p}, s_r) = \delta \overline{a(p)} F(s_r|\widehat{p})$$

where $\overline{a(p)} = p\, A_H + (1-p)\, A_L$. Since liquidation leads to destruction of δ fraction of the legacy assets, the regulator would always want to prevent it irrespective of the realization of p. After ob-

[12] The assumption should not be interpreted as depositors evaluating the message differently, or depositors are irrational and making wrong inferences.

serving the stress test result p, the regulator would reveal $\widehat{p}$ so that the liquidation loss is minimized,

$$\underset{\widehat{p}}{argmin}\ \delta\overline{a(p)}F(s_r|\widehat{p}) \tag{5}$$

2.1.1. Communicating through Cheap talk

Proposition 1. *Communication through cheap talk is not possible when depositors are the only receivers of the message.*

Proof. Consider the objective of the regulator in Equation (5). It can minimize the liquidation loss by choosing $\hat{p}$ that minimizes the fraction of depositors asking for early withdrawal, or $F(s_r|\widehat{p})$. The regulator's objective is independent of its private information about the result (p). So, irrespective of the realization of the stress test result (p), it would want to reveal the maximum possible $\widehat{p}$. For cheap talk to be informative, the regulator should send a different message $(\widehat{p})$ given different realizations of the result (p), which does not hold in this case. The regulator's preference over depositors' actions does not vary with the realized test result; thus, its message is independent of p. This denies the possibility of communicating through cheap talk. □

2.2. Banks only

Now consider banks as the only receivers. After the regulator reveals the result, banks pick a project V_k^b as per Equation (2). Since $C_S - C_P = \eta > 0$, the project, V_k^b is riskier than the regulator's choice, V_k^r as shown in Equation (3).

2.2.1. Communicating through Cheap talk

Proposition 2. *Communication through cheap talk is possible when banks are the receivers of the message.*

Proof. Theorem 1 in Crawford and Sobel (1982) ||

After obtaining the result p from the stress test, the regulator would send a message that induces banks to choose a project closest to the one it prefers. Since $V_k^b > V_k^r$ and $\partial V_k^b/\partial\widehat{p} > 0$, the regulator will understate the result of the stress test $(\widehat{p} < p)$ to persuade banks to pick a less risky project or one that is close to V_k^r. Note that, here, there is a limit to this desire for understatement. Unlike the previous case, the regulator's preference for banks' action is not indifferent to the test result; the regulator will not always wish to claim the lowest type possible. The message it wants to send $(\widehat{p})$

is a function of the test result (p). Thus, it allows the possibility of cheap talk, where banks can rely on the regulator for some truthful information. In the following section, we jointly model banks and depositors and characterize the cheap talk equilibrium. $\square$

3. Disclosure through Cheap talk

While cheap talk is not possible with depositors alone, publicly communicating with banks and depositors can make it work. The regulator's preference over their actions is not independent of the actual realization of the stress test result (p), unlike the case with only depositors. The presence of banks disciplines the regulator's relationship with depositors, thus allowing the possibility of cheap talk. The situation is similar to what Farrell and Gibbons (1989) call "One-Sided Discipline". There is a separating equilibrium in private with one receiver (banks) but not with the other (depositors), and there is a separating equilibrium in public (where both banks and depositors are receivers).

The regulator's conflicts with depositors and banks lie in opposite directions. It wants to overstate the result to depositors to minimize the instances of early withdrawal. At the same time, it intends to understate the result to banks, expecting them to pick a less risky project. Below, we consider cheap talk as a mean for the regulator to publicly communicate its private information (p) in a credible manner.

Cheap Talk: After conducting stress test, the regulator privately observes p, which is uniformly distributed over $(0, 1)$. Communication through cheap talk is characterised by a partition $(0, p_1 .... p_n, 1)$ of the interval $(0, 1)$. In equilibrium, the regulator announces the subinterval where the result (p) lies. Thus in equilibrium, the regulator's message is noisy - instead of revealing the result, it gives a range. Once the regulator announces the subinterval, receivers guess the best estimate of the test result conditional on the revealed subinterval and accordingly take actions.

For truth-telling, a regulator must prefer to reveal the subinterval containing the result rather than any other subinterval. This requirement is satisfied if the regulator is indifferent between revealing two adjacent subintervals (p_{i-1}, p_i), and (p_i, p_{i+1}) when the result falls on the boundary $(p = p_i)$ or on the partition element that creates the two subintervals.

If the regulator reveals $p_i \in (p_{i-1}, p_i)$, let's call the result inferred by receivers from the message $\widehat{p}(p_{i-1}, p_i)$ as $\widehat{p}_{-i}$ for notational convenience. Banks choose a project where $V_k^b(\widehat{p}_{-i}) = -\frac{q(V_k)}{q'(V_k)} -$

$(1-\widehat{p}_{-i})C_P$. Early withdrawal by a subset of depositors cause $F(s_r|\widehat{p}_{-i})$ fraction of legacy assets getting liquidated. This results in a level of welfare given by,

$$Y(p_i,\widehat{p}_{-i}) = \overline{a(p_i)}\Big\{1-\delta F(s_r|\widehat{p}_{-i})\Big\} + \Big[q(V_k^b)V_k^b(\widehat{p}_{-i}) - (1-q(V_k^b))(1-p_i)C_S\Big] \qquad (6)$$

The expression inside the square bracket is the pay-offs generated from the project net of the social cost of failure. The first term accounts for the pay-offs generated from the legacy assets net of loss due to early liquidation.

Similarly, if the regulator reveals $p_i \in (p_i, p_{i+1})$, let's call the inferred test result by receivers from the message $\widehat{p}(p_i, p_{i+1})$ as $\widehat{p}_{+i}$ for notational convenience. Banks choose a project where $V_k^b(\widehat{p}_{+i}) = -\frac{q(V_k)}{q'(V_k)} - (1-\widehat{p}_{+i})C_P$. Early withdrawal by a subset of depositors cause $F(s_r|\widehat{p}_{+i})$ fraction of legacy assets getting liquidated. This results in a level of welfare given by,

$$Y(p_i,\widehat{p}_{+i}) = \overline{a(p_i)}\Big\{1-\delta F(s_r|\widehat{p}_{+i})\Big\} + \Big[q(V_k^b)V_k^b(\widehat{p}_{+i}) - (1-q(V_k^b))(1-p_i)C_S\Big] \qquad (7)$$

To make the regulator with a result of p_i indifferent between the subintervals (p_{i-1}, p_i) and (p_i, p_{i+1}), the arbitrage condition $Y(p_i,\widehat{p}_{-i}) = Y(p_i,\widehat{p}_{+i})$ must hold or,

$$\begin{aligned} \delta\overline{a(p_i)}\Big\{F(s_r|\widehat{p}_{-i} - F(s_r|\widehat{p}_{+i})\Big\} + \Big[q(V_k^b)V_k^b(\widehat{p}_{+i}) - q(V_k^b)V_k^b(\widehat{p}_{-i})\Big] \\ = \Big\{q(V_k^b,\widehat{p}_{-i}) - q(V_k^b,\widehat{p}_{+i})\Big\}(1-p_i)C_S \end{aligned} \qquad (8)$$

The above arbitrage condition implies that for a given p_i, the loss of welfare due to revealing the lower subinterval (p_{i-1}, p_i) over the upper subinterval (p_i, p_{i+1}) must be compensated by the gain. The terms on the left hand side presents the loss arising out of revealing the lower subinterval. The first term describes the increase in liquidation loss due to excess withdrawal when $p_i \in (p_{i-1}, p_i)$ is revealed (since $F(s_r|\widehat{p}_{-i}) > F(s_r|\widehat{p}_{+i})$). The second term presents the loss due to banks picking a project with low return. Since $q(V_k)V_k$ is increasing and reaches maximum at V_k^* when $p = 1$, banks pick a project $V_k \in (\underline{V}, V_k^*)$ for $p < 1$. So there will be loss when the regulator reveals the lower interval and banks pick $V_k^b(\widehat{p}_{-i})$ instead of $V_k^b(\widehat{p}_{+i})$ as $q(V_k^b(\widehat{p}_{+i}))V_k^b(\widehat{p}_{+i}) > q(V_k^b(\widehat{p}_{-i}))V_k^b(\widehat{p}_{-i})$. These losses are compensated by the gain which is the reduction in the probability of incurring the social cost of bank failure as banks pick a less risky project or, $q(V_k^b,\widehat{p}_{+i}) < q(V_k^b,\widehat{p}_{-i})$.

3.1. Cheap talk Equilibria

The arbitrage condition in Equation (8) results in the following second order linear difference equation that describes the subintervals.

$$p_{i+1} = \alpha p_i - p_{i-1} + \beta \tag{9}$$

where α and β are constants. Details are given in the appendix under Lemma (2). The subintervals are constructed such that the regulator prefers to report the subinterval containing the result instead of reporting any other subinterval.

If there are n number of partitions, or $n+1$ number of subintervals between 0 and 1, the boundary conditions that need to be satisfied are,

$$p_0 = 0 \qquad p_{n+1} = 1$$

With the above linear difference equation and boundary conditions, we describe the properties of the subintervals below.

3.2. Properties of Cheap talk Equilibria

Proposition 3. *An infinite number of equilibria exists; the number of possible subintervals between* $(0,1)$ *can vary from one to infinity with one being the case with no partition. As the number of partitions increases, the sizes of subintervals converge to a constant as per the following rule:*

1. *The first subinterval* (p_0, p_1) *and the last subinterval* (p_n, p_{n+1}) *contains* λ_1 *and* λ_2 *fraction of the entire interval respectively.*

2. *The second subinterval* (p_1, p_2) *and second last subinterval* (p_{n-1}, p_n) *contains* λ_1 *and* λ_2 *fraction of the remaining* $(1 - \lambda_1 - \lambda_2)$ *part respectively and so on.*

   λ_1*, and* λ_2 *are constants; independent of the stress test result* p*.*

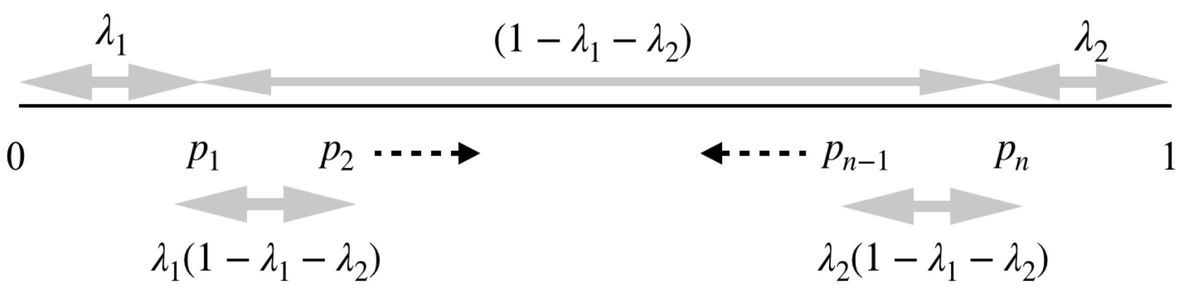

Figure 2

The first part of the proposition says that a partition equilibrium is possible with every non-negative integer n; the number of subintervals can vary from one to infinity. In Crawford and Sobel (1982), the number of possible equilibria is finite, because the conflict between the sender and the receiver is an exogenous constant. Unlike that, the extent of regulator's conflict with both depositors and banks varies with the test result (p). With depositors, the conflict arises due to early withdrawal. The consequent liquidation loss is $\delta\overline{a(p)}F(s_r|\widehat{p})$, where $\overline{a(p)} = pA_H + (1-p)A_L$. In addition to the exogenous parameter (δ), the conflict also depends on the test result p. With banks, the conflict arises due to the presence of negative externalities associated with bank failure. From Equation (2) and 3, it can be seen that the difference between the regulator's (V_k^r) and banks' preferred project (V_k^b) also depends on p in addition to the exogenous constant $(\eta = C_S - C_P)$.

Next, the proposition says that as the number of partitions increases, the length of the subintervals starts stabilizing from both ends. Figure 2 illustrates the dynamics of the subintervals. For any number of partitions n, the first subinterval $(0, p_1)$ would contain approximately λ_1 fraction of the entire interval. The last subinterval $(p_n, 1)$ would contain λ_2 fraction of the entire interval. The second subinterval and second to last subinterval will contain λ_1 and λ_2 fraction of the remaining $1-(\lambda_1+\lambda_2)$. As the number of partitions (n) increases, the intervals will get finer and finer from the lower and upper ends in λ_1 and λ_2 proportion, respectively. Increasing the number of partitions by one more unit will make the intermediate subintervals finer, leaving the subintervals at both ends approximately of the same size.

The intuition behind the size of the subintervals is related to the conflict of the regulator with banks and depositors. With depositors, the conflict arises due to liquidation loss which is $\delta\overline{a(p)}F(s_r|\widehat{p})$, where $\overline{a(p)} = pA_H + (1-p)A_L$. When one more depositor decides to withdraw early, $\delta\overline{a(p)}$ fraction of the legacy asset is lost. This marginal cost of liquidation increases with p as more high-quality assets are getting liquidated, thus making the conflict between the regulator and the depositors severe for high values of p. With banks, the regulator's conflict is severe for low values of p. For a given η, the choice of banks diverges more from the regulator's choice when p is small, which incentivizes the regulator to lie more.

Since the credibility of the regulator diminishes as the test result (p) moves close to either the lower end (where conflict with banks is severe) or the upper end (where conflict with depositors is severe), the result needs to be revealed through larger subintervals. The regulator has to lie about the subinterval if it wants to lie. For large subintervals, lying changes depositors' and banks' inference about the test result (p) by a larger amount, and their actions move further away from the regulator's choice. So it is in the interest of the financial regulator to reveal the subintervals truthfully.

Next, we discuss how the sizes of the subintervals essentially decide the precision with which the regulator can communicate its message.

3.2.1. Credible Disclosure and Quality of Communication

The quality of communication can be measured by residual uncertainty about the test result (p). After the regulator reveals the subinterval (p_{i+1}, p_i) in its message, the uncertainty in the result can be expressed as,

$$var(p) = \frac{(p_{i+1} - p_i)^2}{12} \tag{10}$$

where $var(p)$ denotes the variance. The larger the size of the subintervals, the more the uncertainty; hence lower is the precision or quality of communication.

Proposition (3) suggests that infinite number of equilibria is possible. As we increase the number of partitions (n), the intervals start getting finer. To study the quality of communication, we focus on the finest partition equilibrium $(n \to \infty)$[13]. For any realization of the stress test result (p), the result can be revealed with the highest precision in this equilibrium.

Even in the finest partition equilibrium, there is a limit to the precision with which information can be revealed. We have shown in Proposition (3) that, as the number of partitions increase, the sizes of the subintervals start stabilizing from both ends. The subintervals are finer in the intermediate region, thus enabling the regulator to communicate its message with less uncertainty. At both ends, however, the sizes of subintervals are relatively large, thus limiting the precision with which the result can be revealed. The precision of the revealed information, thus depends on the stress test result (p).

Proposition 4. *For any realization of p, the quality of communication is governed by the liquidation cost parameter (δ) and the negative externality of bank failure (η). While δ and η increase conflicts and lessen the regulator's credibility with depositors and banks respectively, there is a mutual disciplining effect. The*

[13] The idea here is that if the regulator selects an equilibrium before the stress test result (p) is known, it will choose the equilibrium with the finest partition since the equilibrium with the finest partition is pareto-superior (Crawford and Sobel, 1982).

regulator's conflict with depositors (banks) lessens its credibility problem with banks (depositors). In other words,

$$\frac{\partial \lambda_1}{\partial \delta} < 0\,, \quad \frac{\partial \lambda_2}{\partial \eta} < 0 \quad for\ any\ p$$

The constants λ_1 and λ_2 in Proposition (3) determine the size of the subintervals, thus the quality of information that can be revealed in the finest partition equilibrium.

An increase in the liquidation cost parameter δ increases the regulator's conflict with depositors. Thus, λ_2, which governs the size of the subinterval from the upper end (the region where the conflict is severe with depositors), increases or $\frac{\partial \lambda_2}{\partial \delta} > 0$. However, an increase in δ also discourages the regulator from understating the result to banks. Note the regulator intends to understate the result to prevent banks from taking excessive risk. In the presence of depositors, understating the result will cause more liquidation as $F(s_r|\hat{p})$ is decreasing in $\hat{p}$. If δ is sufficiently high, the regulator is less likely to do so, thus improving its credibility with banks, or $\frac{\partial \lambda_1}{\partial \delta} < 0$.

Similarly, an increase in the negative externality parameter η increases the regulator's conflict with banks. Thus, λ_1, which governs the size of the subinterval from the lower end (the region where the conflict is severe with banks), increases or $\frac{\partial \lambda_1}{\partial \eta} > 0$. However, an increase in η also discourages the regulator from overstating the result to depositors. The regulator wants to overstate to prevent early withdrawals and consequent liquidation loss. But overstating the result leads banks to pick a more risky project for investment. For a higher η, banks' choice of project differs from the regulator's choice by a higher amount, which diminishes the regulator's desire to overstate. This lessens the regulator's credibility problem with depositors, or $\frac{\partial \lambda_2}{\partial \eta} < 0$.

The disciplining effect, however, is minimal when the test result (p) is very large or small. For small p, the conflict is severe with banks, but the marginal cost of liquidation is low as $\overline{a(p)}$ is less (there is less good quality legacy assets that get to be liquidated), which lowers the cost of understating the test result. Thus the disciplining effect imposed by depositors on the regulator's credibility problem with banks is low. For large p, the conflict is severe with depositors. However, the cost of overstating the test result is low as the regulator's preference is less misaligned with banks when p is large, thus reducing the disciplining effect banks impose on the regulator's credibility problem with depositors.

When the stress test result is close to either extreme, conflict with one group of receivers dominates the disciplining effect imposed by the other group, which results in larger subintervals and lowers precision or the quality of communication. For intermediate values of p, the disciplining effect dominates the conflict, thus reducing the size of the subintervals and enabling the regulator to reveal information with more precision.

4. Discussion and Conclusion

In light of the credibility problems often associated with the disclosure of stress test results, this paper provides a first analysis of the disclosure, particularly in an environment where the financial regulator lacks a commitment to truthfully reveal the results. Our objectives are twofold, aiming to shed light on the inherent credibility problems in stress test result disclosures and exploring whether a credible disclosure policy can be achieved without assuming any commitment on part of the financial regulator to speak the truth.

Within our simple setup, we uncover the regulator's conflicts of interest with banks, and depositors. These conflicts stem from misaligned interests of the regulator with those of the receivers. In order to prevent early withdrawal by depositors, the regulator may be inclined to lie by inflating the test result in its message, whereas to curb excessive risk-taking by banks, it may understate the results. Such actions, though potentially advantageous for short-term gains, erode trust and confidence in the financial system, posing substantial risks to its stability. Next, we propose a credible alternative for disclosure in such a regulatory landscape. To communicate credibly, the regulator should adopt a strategy of revealing a range within which the stress test result lies, instead of disclosing a single value. These ranges are strategically designed to discourage any deceptive behavior by the regulator, as the potential cost of lying far outweighs any perceived benefits.

The credible disclosure policy, however, affects the precision of the revealed information. Suppose the stress test result indicates that a significant fraction of banks either pass or fail the test. In these extreme regions, the regulator's conflicts of interest are severe with either group of receivers (depositors and banks), and it is more inclined to lie. To sound credible, the disclosure rule demands more noise to be introduced in the revealed message resulting in a lower precision of the revealed information. The ranges that contain the results tend to be large. As the stress test result moves away from either extreme, the regulator's conflicts with receivers attenuate, and the ranges that contain the results tend to be small, consequently improving the precision of the revealed information.

The point that the financial regulator can credibly reveal stress test result without assuming any commitment to speak the truth has important implications for regulatory initiatives. The disclosure policy in such an environment determines the precision with which stress test results can be revealed. Depending on the welfare loss that comes from imprecise revelation[14], the regulators should proactively steer the precision with which stress test result can be revealed. It can do so by influencing the two parameters, the liquidation loss and the negative externalities associated with bank failure. For instance, if the welfare loss of imprecise revelation at the upper extreme is higher than the welfare loss of imprecise revelation at the lower extreme, policies should aim to reduce the liquidation loss.

There are several intriguing extensions we foresee to our analysis. The analysis assumes that the only tool at the regulator's disposal to achieve the optimal level of welfare is to reveal the message strategically. Expanding our analysis to explore the interaction between disclosures and complementary policy tools, such as capital restrictions and liquidity support by acting as a lender of last resort, promises valuable insights.

In conclusion, our research highlights a mechanism to address the credibility problems surrounding stress test result disclosures. By adopting a credible disclosure policy, built on the foundation of strategic range revelations, regulators can regain trust, and enhance decision-making within the financial sector.

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[14] As is standard in the cheap talk literature, If the regulator can commit to telling the truth and if it is believed, the welfare would be higher than what is achieved under cheap talk. However, we consider that such commitments are impossible. Further, the loss of welfare increases with the size of the interval in which the results are revealed. A larger interval, although necessary for credible disclosure, introduces more uncertainty about the result, which in turn influences the banks' and depositors' actions to deviate from the first best. The inference follows directly from Crawford and Sobel (1982).

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Online Appendix

A. Proofs

Lemma 1. *(Auxiliary result): The bank's optimal choice of the project given by Equation (2), is unique and bank would prefer to invest the endowment rather than storing it. The regulator would prefer the banks to invest in a project as given by Equation (3) to storing the endowment.*

Proof. First order condition from bank's optimization problem yields $V_k^b = -\frac{q(V_k)}{q'(V_k)} - \left(1-\widehat{p}\right)C_P$. Given our assumptions $\frac{\partial q}{\partial V_k} < 0$ and $\frac{\partial^2 q}{\partial V_k^2} \leq 0$, the outcome of the project $q(V_k)V_k$ is strictly concave. If we ignore the private cost of failure for the time being or $C_P = 0$, It reaches a maximum at some V_k^*. Upto this point, the $q(V_k)V_k$ is an increasing function of V_k. When private cost of failure C_P or social cost of failure C_S is considered, the optimization exercise would always yield a project $V_k \in \left(\underline{V}, V_k^*\right)$. Hence banks as well as the regulator would always prefer investing the endowment as per their objective function rather than storing it. □

Lemma 2. *(Auxiliary result): In cheap talk, the partitions of the interval $(0,1)$ satisfies the following linear difference equation:*

$$p_{i+1} = \alpha p_i - p_{i-1} + \beta$$

where

$$\alpha = 4\frac{C_S}{C_P} - 2 + \frac{8\delta c\left(A_H - A_L\right)\left(\overline{V} - \underline{V}\right)}{C_P^2} \tag{A.1}$$

and

$$\beta = 4\left(1 - \frac{C_S}{C_P}\right) + \frac{2\delta c A_L\left(\overline{V} - \underline{V}\right)}{C_P^2} \tag{A.2}$$

Proof. Referring to arbitrage condition in Equation (8) from the main text, we have

$$\delta\overline{a(p_i)}\left\{F(s_r|\widehat{p_{-i}} - F(s_r|\widehat{p_{+i}})\right\} + \left[q(V_k^b, \widehat{p_{+i}})V_k^b(\widehat{p_{+i}}) - q(V_k^b, \widehat{p_{-i}})V_k^b(\widehat{p_{-i}})\right]$$

$$= \left\{q(V_k^b, \widehat{p_{-i}}) - q(V_k^b, \widehat{p_{+i}})\right\}(1-p_i)C_S$$

Using the functional form for $q(V_k) = \frac{\overline{V}-V_k}{\overline{V}-\underline{V}}$, the first order condition for bank's optimal choice of the project yields, $V_k^b = \frac{\overline{V} - \left(1-\widehat{p}\right)C_P}{2}$. For the signals received by depositors after regulator reveals $\widehat{p}$, we assume $F(s|\widehat{p}) = 1 - I_s(\omega_1, \omega_2)$, where I_s is the regularized beta function. ω_1, *and* ω_2 are chosen such that $F(s_r|\widehat{p}) = 1 - s_r$. Using Taylor's expansion around $\widehat{p} = 1$ and ignoring higher order terms, $F(s|\widehat{p})$ can be written as $c(1-\widehat{p})$ where c is a function of (s, ω_1, ω_2).

We use above substitutions and replace $\widehat{p}_{-i}$ with $\frac{p_{i-1}+p_i}{2}$ and $\widehat{p}_{+i}$ with $\frac{p_i+p_{i+1}}{2}$ in the arbitrage condition. After simplification,

$$p_{i+1} + p_{i-1} = p_i \left[8 \frac{\delta c(\overline{V} - \underline{V})(A_H - A_L)}{C_P^2} + 4 \frac{C_S}{C_P} - 2 \right] + 4 \left[1 - \frac{C_S}{C_P} + 2 \frac{\delta c(\overline{V} - \underline{V}) A_L}{C_P^2} \right]$$

Rearranging the above as $p_{i+1} = \alpha p_i - p_{i-1} + \beta$, will give the give the required expression for α and β. □

Proposition 3:

Proof. The arbitrage condition results in a second order difference equation of the form

$$p_{i+1} = \alpha p_i - p_{i-1} + \beta$$

The second order difference equation has a solution of the following form.

$$p_i = \gamma_1 r_1^i + \gamma_2 r_2^i + \frac{\beta}{2-\alpha}$$

To reach the solution, we first solve for the homogeneous equation $p_{i+1} = \alpha p_i - p_{i-1}$, which has a solution of the form $p_i = r^i$. Rewriting the homogeneous equation in r leads to $r^2 - \alpha r + 1 = 0$. The quadratic equation has roots r_1 and r_2 given by

$$r_{1/2} = \frac{\alpha \pm \sqrt{\alpha^2 - 4}}{2}$$

The solution for the homogeneous equation can be written as $p_i = \gamma_1 r_1^i + \gamma_2 r_2^i$. The original non-homogeneous difference equation has a particular solution of the form $p_i = \frac{\beta}{2-\alpha}$. Combining these two, we find the solutions for our original non-homogeneous equation as

$$p_i = \gamma_1 \left(\frac{\alpha + \sqrt{\alpha^2 - 4}}{2} \right)^i + \gamma_2 \left(\frac{\alpha - \sqrt{\alpha^2 - 4}}{2} \right)^i + \frac{\beta}{2-\alpha}$$

Let's say there are n number of dividing points between $(0,1)$. Then the solution must satisfy the boundary condition $p_0 = 0$ and $p_{n+1} = 1$. Using the boundary conditions, we find

$$\gamma_1 = \frac{1 - \left(\frac{\beta}{2-\alpha}\right)\left(1 - r_2^{n+1}\right)}{r_1^{n+1} - r_2^{n+1}} \qquad \gamma_2 = -\gamma_1 - \frac{\beta}{2-\alpha}$$

For any non-negative integer n, we can find a solution to γ_1 and γ_1, and find the required intervals. This proves first part of the proposition.

We can write the second part of Proposition (3) as

$$\lim_{n\to\infty} \frac{p_i - p_{i-1}}{p_{(n+1)-(i-1)} - p_{i-1}} = \lambda_1 \qquad \lim_{n\to\infty} \frac{p_{(n+1)-(i-1)} - p_{(n+1)-i}}{p_{(n+1)-(i-1)} - p_{i-1}} = \lambda_2 \tag{A.3}$$

For example if $i = 2$, it says $\lim_{n\to\infty} \frac{p_2-p_1}{p_n-p_1} = \lambda_1$ and $\lim_{n\to\infty} \frac{p_n-p_{n-1}}{p_n-p_1} = \lambda_2$. If there are n number of dividing points between 0 and 1, the second subinterval (p_1, p_2) contains λ_1 fraction of the remaining (p_1, p_n). The remaining interval is determined after accounting for the first subinterval (p_0, p_1) and last subinterval (p_n, p_{n+1}). The second last subinterval contains λ_2 fraction of the remaining. Hence it would suffice to prove the limit conditions in Equation (A.3).

$$\lim_{n\to\infty} \frac{p_i - p_{i-1}}{p_{(n+1)-(i-1)} - p_{i-1}}$$

$$= \lim_{n\to\infty} \frac{\gamma_1 r_1^i + \gamma_2 r_2^i - \gamma_1 r_1^{i-1} - \gamma_2 r_2^{i-1}}{\gamma_1 r_1^{n+1-i-1} + \gamma_2 r_2^{n+1-i-1} - \gamma_1 r_1^{i-1} - \gamma_2 r_2^{i-1}}$$

Replacing γ_2 with -$\gamma_1 - \frac{\beta}{2-\alpha}$,

$$= \lim_{n\to\infty} \frac{\gamma_1 \left[r_1^i - r_2^i - r_1^{i-1} + r_2^{i-1}\right] + \frac{\beta}{2-\alpha}\left[r_2^{i-1} - r_2^i\right]}{\gamma_1 \left[r_1^{n+1-i-1} - r_2^{n+1-i-1} - r_1^{i-1} + r_2^{i-1}\right] + \frac{\beta}{2-\alpha}\left[r_2^{i-1} - r_2^{n+1-i+1}\right]}$$

It can be easily verified that $r_2\left(= \frac{\alpha-\sqrt{\alpha^2-4}}{2}\right) < 1$ and $r_1\left(= \frac{\alpha+\sqrt{\alpha^2-4}}{2}\right) > 1$. For very large n, r_2^{n+1} approaches zero, so it can be ignored, thus $\gamma_1 = \frac{1-\frac{\beta}{2-\alpha}}{r_1^{n+1}}$. Simplifying the above expression, we get

$$= \lim_{n\to\infty} \frac{\left(1 - \frac{\beta}{2-\alpha}\right)\frac{1}{r_1^{n+1}}\left[r_1^i - r_2^i - r_1^{i-1} + r_2^{i-1}\right] + \frac{\beta}{2-\alpha}\left[r_2^{i-1} - r_2^i\right]}{\left(1 - \frac{\beta}{2-\alpha}\right)\left[\frac{1}{r_1^{i-1}} - \frac{1}{r_1^{n+1}}\left(r_2^{n+1-i-1} + r_1^{i-1} - r_2^{i-1}\right)\right] + \frac{\beta}{2-\alpha}\left[r_2^{i-1} - r_2^{n+1-i+1}\right]}$$

Since $r_1 > 1$, as n approaches to ∞, we can ignore $\left(\frac{1}{r_1^{n+1}}\right)$ terms, which simplifies the above to

$$= \frac{\frac{\beta}{2-\alpha}\left(1 - r_2\right) r_2^{i-1}}{\frac{\beta}{2-\alpha} r_2^{i-1} + \frac{1-\frac{\beta}{2-\alpha}}{r_1^{i-1}}}$$

Using $r_1 * r_2 = 1$ (roots of the quadratic equation $r^2 - \alpha r + 1$), we obtain λ_1 to be $\frac{\beta}{2-\alpha}\left(1 - r_2\right)$ or,

$$\lambda_1 = \frac{\beta}{2-\alpha}\left(1 - \frac{\alpha - \sqrt{\alpha^2 - 4}}{2}\right)$$

For λ_2, which quantifies the convergence of subintervals from upper extreme,

$$\lim_{n\to\infty} \frac{p_{(n+1)-(i-1)} - p_{(n+1)-i}}{p_{(n+1)-(i-1)} - p_{i-1}}$$

$$= \lim_{n\to\infty} \frac{\gamma_1 r_1^{n+1-i-1} + \gamma_2 r_2^{n+1-i-1} - \gamma_1 r_1^{n+1-i} - \gamma_2 r_2^{n+1-i}}{\gamma_1 r_1^{n+1-i-1} + \gamma_2 r_2^{n+1-i-1} - \gamma_1 r_1^{i-1} - \gamma_2 r_2^{i-1}}$$

For very large n, replacing γ_2 with -$\gamma_1 - \frac{\beta}{2-\alpha}$,

$$= \lim_{n\to\infty} \frac{\gamma_1\left[r_1^{n+1-i+1} - r_2^{n+1-i+1} - r_1^{n+1-i} + r_2^{n+1-i}\right] + \frac{\beta}{2-\alpha}\left[r_2^{n+1-i} - r_2^{n+1-i+1}\right]}{\gamma_1\left[r_1^{n+1-i-1} - r_2^{n+1-i-1} - r_1^{i-1} + r_2^{i-1}\right] + \frac{\beta}{2-\alpha}\left[r_2^{i-1} - r_2^{n+1-i+1}\right]}$$

Replacing γ_1 with $\frac{1-\frac{\beta}{2-\alpha}}{r_1^{n+1}}$ and ignoring $\frac{1}{r_1^{n+1}}$ terms, the above expression simplifies to $\frac{\left(1-\frac{\beta}{2-\alpha}\right)\left(\frac{1}{r_1^{i-1}}-\frac{1}{r_1^{i}}\right)}{\frac{\beta}{2-\alpha}r_2^{i-1}+\left(1-\frac{\beta}{2-\alpha}\right)\left(\frac{1}{r_1^{i-1}}\right)}$.

Using $(r_1 * r_2) = 1$, we obtain,

$$\lambda_2 = \frac{(1-\frac{\beta}{2-\alpha})(r_1 - 1)}{r_1}, \quad where \;\; r_1 = \frac{\alpha + \sqrt{\alpha^2 - 4}}{2}$$

□

Proposition 4:

Proof. The proposition involves four partial derivatives which we prove below. Before we go into the proofs, we need to impose certain restrictions on parameters to make sure $\frac{\beta}{2-\alpha} > 0$, else λ_1 could be negative. From Equation (A.1) and Equation (A.2)

$$2 - \alpha = 4\left(1 - \frac{C_S}{C_P}\right) - \frac{8\delta s_r (A_H - A_L)(\overline{V} - \underline{V})}{C_P^2}$$

and

$$\beta = 4\left(1 - \frac{C_S}{C_P}\right) + \frac{2\delta s_r A_L (\overline{V} - \underline{V})}{C_P^2}$$

Since $\frac{C_S}{C_P} > 1$, $(2-\alpha) < 0$. We need restrictions on parameters such that β is also less than zero which will be satisfied if $\frac{C_S}{C_P} > 1 + \frac{\delta s_r A_L(\overline{V}-\underline{V})}{2C_P^2}$, or $\eta > \frac{\delta s_r A_L(\overline{V}-\underline{V})}{2C_P}$. We proceed to prove the result assuming this constraint is satisfied.

1. $\dfrac{\partial \lambda_1}{\partial \eta} > 0$

$$\lambda_1 = \left(1 - r_2\right)\left(\frac{\beta}{2-\alpha}\right)$$

$$\implies \frac{\partial \lambda_1}{\partial \eta} = \frac{\left[\frac{\partial \beta}{\partial \eta}\left(2-\alpha\right) + \beta\frac{\partial \alpha}{\partial \eta}\right]\left(1 - r_2\right)}{\left(2-\alpha\right)^2} - \left(\frac{\beta}{2-\alpha}\right)\frac{\partial r_2}{\partial \eta}$$

$r_2 = \frac{\alpha-\sqrt{\alpha^2-4}}{2}$, so $\frac{\partial r_2}{\partial \eta} = \frac{\partial \alpha}{\partial \eta}\left[\frac{1}{2} - \frac{\alpha}{\sqrt{\alpha^2-4}}\right]$. It can be verified that the term inside the square bracket is negative and replacing C_S with $C_P + \eta$, it can be shown that $\frac{\partial \alpha}{\partial \eta} > 0$. So the second term in $\frac{\partial \lambda_1}{\partial \eta}$ is less than zero. We need to show the first term is greater than zero, or suffice to show $\left[\frac{\partial \beta}{\partial \eta}(2-\alpha) + \beta\frac{\partial \alpha}{\partial \eta}\right]\rangle 0$. When η increases by 1 unit, $\frac{\partial \beta}{\partial \eta}$ decreases by $(4/C_P)$ and $\frac{\partial \alpha}{\partial \eta}$ increases by $(4/C_P)$. Note that $(2-\alpha)$ and β are both negative. If absolute value of $(2-\alpha)$ is greater than absolute of value of β, the required expression becomes greater than zero. Since $|2-\alpha| = \alpha - 2$ and $|\beta| = -\beta$,

$$\alpha - 2 = 4\left(\frac{C_S}{C_P} - 1\right) + \frac{8\delta s_r(A_H - A_L)(\overline{V} - \underline{V})}{C_P^2} \qquad -\beta = 4\left(\frac{C_S}{C_P} - 1\right) - \frac{2\delta s_r A_L(\overline{V} - \underline{V})}{C_P^2}$$

It is clear from above that $\alpha - 2 > -\beta$, which proves that $\frac{\partial \lambda_1}{\partial \eta} > 0$.

2. $$\frac{\partial \lambda_1}{\partial \delta} < 0$$

$$\implies \frac{\partial \lambda_1}{\partial \delta} = \left[\frac{\partial \beta}{\partial \delta}\left(2-\alpha\right) + \beta\frac{\partial \alpha}{\partial \delta}\right]\frac{(1-r_2)}{(2-\alpha)^2} - \left(\frac{\beta}{2-\alpha}\right)\frac{\partial r_2}{\partial \alpha}\frac{\partial \alpha}{\partial \delta}$$
$$= \frac{\partial \beta}{\partial \delta}\left(\frac{1-r_2}{2-\alpha}\right) + \beta\frac{\partial \alpha}{\partial \delta}\left[\frac{1-r_2}{(2-\alpha)^2} - \frac{\partial r_2}{\partial \alpha}\left(\frac{1}{2-\alpha}\right)\right]$$

Since $r_2 = \frac{\alpha-\sqrt{\alpha^2-4}}{2}$, $\frac{\partial r_2}{\partial \alpha} > 0$[15]. With $\beta < 0$, the second term is negative. Since $\frac{\partial \beta}{\partial \delta} > 0$ and $(2-\alpha) < 0$, the first term is also negative. Hence $\frac{\partial \lambda_1}{\partial \delta} < 0$

3. $$\frac{\partial \lambda_2}{\partial \eta} < 0$$

$$\frac{\partial \lambda_2}{\partial \eta} = \left[\frac{\partial \beta}{\partial \eta}(2-\alpha) + \beta\frac{\partial \alpha}{\partial \eta}\right]\left(\frac{1}{r_1} - 1\right)\left[\frac{1}{(2-\alpha)^2}\right] + \frac{1}{r_1^2}\frac{\partial r_1}{\partial \alpha}\frac{\partial \alpha}{\partial \eta}\left[1 - \frac{\beta}{2-\alpha}\right]$$

While proving $\frac{\partial \lambda_1}{\partial \eta} > 0$, we have showed that $\left[\frac{\partial \beta}{\partial \eta}(2-\alpha) + \beta\frac{\partial \alpha}{\partial \eta}\right]\rangle 0$. Since $\frac{1}{r_1} - 1 < 0$, first term is negative. Both $\frac{\partial r_1}{\partial \alpha} > 0$ and $\frac{\partial \alpha}{\partial \eta} > 0$. Since the second term is positive, for $\frac{\partial \lambda_2}{\partial \eta} < 0$, we need the magnitude of decrease in first term exceeds the magnitude of increase in the second term when η increases by one unit. Replacing $\frac{\partial \alpha}{\partial \eta} = \frac{4}{C_P}$, $\frac{\partial \beta}{\partial \eta} = -\frac{4}{C_P}$ and $\frac{\partial r_1}{\partial \alpha} = \frac{1}{2} + \frac{\alpha}{\alpha^2-4}$, the required condition can be written as,

$$\left(1 - \frac{1}{r_1}\right)\left[\frac{1}{(2-\alpha)^2}\right]\left[\beta - (2-\alpha)\right] > \frac{1}{r_1^2}\left(1 - \frac{\beta}{2-\alpha}\right)\left(\frac{1}{2} + \frac{\alpha}{\sqrt{\alpha^2-4}}\right)$$

[15] Note $\frac{\partial r_2}{\partial \alpha} > 0$ *implies* $\frac{\alpha}{\sqrt{\alpha^2-4}} < 1$. This condition is satisfied as from Equation (A.1), it can be verified that α is always greater than 2.

OR,

$$\frac{r_1(r_1-1)}{(2-\alpha)^2} > (\alpha-2)\left(\frac{1}{2}+\frac{\alpha}{\sqrt{\alpha^2-4}}\right)$$

After substituting r_1 with $\frac{\alpha+\sqrt{\alpha^2-4}}{2}$, it can be shown that above condition is always satisfied for $\alpha > 2$, which holds under Equation (A.1). This proves $\frac{\partial \lambda_2}{\partial \eta} < 0$.

4. $\frac{\partial \lambda_2}{\partial \delta} > 0$

$$\lambda_2 = \left(1-\frac{1}{r_1}\right)\left(1-\frac{\beta}{2-\alpha}\right)$$

$$\implies \frac{\partial \lambda_2}{\partial \delta} = \frac{1}{r_1^2}\frac{\partial r_1}{\partial \alpha}\frac{\partial \alpha}{\partial \delta}\left(1-\frac{\beta}{2-\alpha}\right) + \left[\frac{\partial \beta}{\partial \delta}\left(2-\alpha\right)+\beta\frac{\partial \alpha}{\partial \delta}\right]\frac{\left(\frac{1}{r_1}-1\right)}{(2-\alpha)^2}$$

$r_1 = \frac{\alpha+\sqrt{\alpha^2-4}}{2}$, so $\frac{\partial r_1}{\partial \alpha}\frac{\partial \alpha}{\partial \delta} = \frac{\partial \alpha}{\partial \delta}\left[\frac{1}{2}+\frac{\alpha}{\sqrt{\alpha^2-4}}\right]$. Both $\frac{\partial r_1}{\partial \alpha} > 0$ and $\frac{\partial \alpha}{\partial \delta} > 0$. Together with $\left(1-\frac{\beta}{2-\alpha}\right) > 0$, this makes the first term in $\frac{\partial \lambda_2}{\partial \delta}$ greater than zero. In second term, $\left(\frac{1}{r_1}-1\right)$ is less than zero. We need to only show the term inside the square bracket $\left[\frac{\partial \beta}{\partial \delta}\left(2-\alpha\right)+\beta\frac{\partial \alpha}{\partial \delta}\right] < 0$. Note β and $(2-\alpha)$ are both negative while $\frac{\partial \alpha}{\partial \delta} > 0$ and $\frac{\partial \beta}{\partial \delta} > 0$, making the term inside the square bracket less than zero. Thus $\frac{\partial \lambda_2}{\partial \delta} > 0$.

□

B. Allowing Disclosure of Bank Specific Information

Our main result in Proposition (3) considers that the regulator discloses only aggregate information (the fraction of H type banks or p), not bank-specific information. For credible disclosure, we argue that the regulator needs to reveal a range containing the result instead of the actual result. In this section, we consider the case in which the regulator chooses whether to (i) disclose bank specific information, or (ii) disclose only the range in which aggregate results lie. We assume that the regulator (who still lacks the ability to commit to reveal aggregate results truthfully) can credibly disclose bank specific information.

Proposition 5. *There exists a p^* such that, in equilibrium, the regulator prefers to disclose bank specific information when stress test reveals that the fraction of H type banks are low; $p \in (0, p*)$, and discloses only aggregate information (the ranges that contain the result) if $p \geq p*$. The solution to $p*$ is given by the following expression,*

$$\delta\left[F(s_r|p)pA_H - \{1-F(s_r|p)\}(1-p)A_L\right] + \frac{C_P C_S p(1-p)}{4(\overline{V}-\underline{V})} = 0 \tag{B.1}$$

When the regulator reveals bank specific information, there are two possible sources of gain. First, it is always beneficial for banks to invest in projects after being fully informed about their type than to make investment decisions under uncertainty when only aggregate information is released. The second term $\frac{C_P C_S p(1-p)}{4(\bar{V}-\underline{V})}$ is the corresponding gain in welfare. The right panel on top, Figure 3 plots the same against the stress test result p on the horizontal axis.

Secondly, when bank specific information is released, depositors can distinguish between H and L type banks; hence no H type bank faces early withdrawal from depositors. The corresponding gain is $\delta F(s_r|p)pA_H$, which is the liquidation loss H type banks would have suffered under revelation of aggregate result. The benefit of revealing only aggregate information comes from avoiding a complete liquidation of L type banks' legacy assets, i.e. $\delta\{1-F(s_r|p)\}(1-p)A_L$. In the top panel (left), we plot the net gain in legacy assets payoff from disclosing bank specific information over aggregate result. For low values of p, the difference is positive. Disclosure of bank specific information is beneficial. Since the proportion of depositors getting signal below the run threshold, or $F(sr|p)$ is sufficiently high, disclosing only the aggregate result leads to liquidation loss on all banks (both H and L type). By disclosing bank specific information, liquidation of H type banks' legacy assets can be avoided. As p increases and becomes large enough $(> p^*)$, the proportion of depositors getting signal below the run threshold is low and revealing only aggregate result is beneficial. By pooling H type banks with a few L type banks has little effect on the liquidation loss of the former, while it significantly reduces the liquidation loss of the latter.

As depicted in the bottom panel of Figure 3, for low values of p, it is beneficial to disclose bank specific information. As p increases beyond $> p^*$), disclosure of aggregate information via cheap talk results in a higher level of welfare .

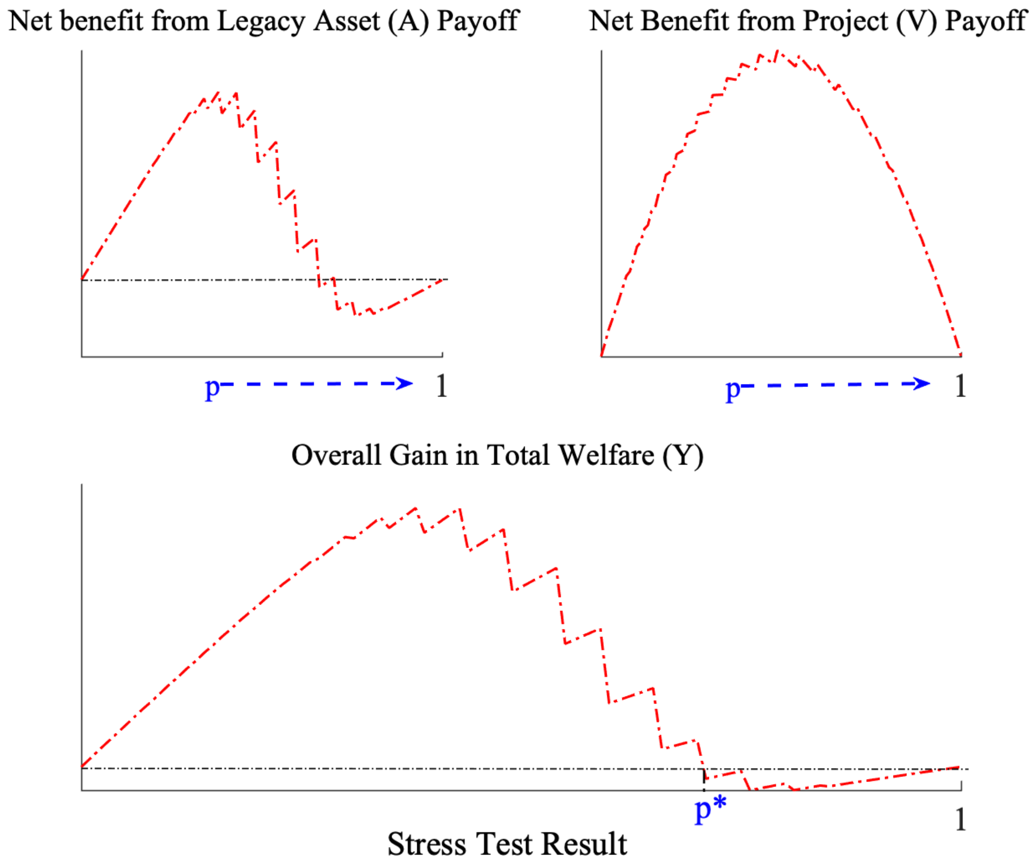

Figure 3: Benefits from disclosing bank specific result over aggregate result by Cheap Talk

The figure plots net gain in welfare from disclosing bank specific information over aggregate stress test result through cheap talk against the stress test result p on the horizontal axis. On the top panel, the left figure plots the benefit resulting from legacy assets $(A_H\ or,\ A_L)$ whose payoff depends on depositors withdrawal decisions. On the right panel, we plot the benefit resulting banks' investment in risky projects (V_k). The figure on the bottom panel plots the overall gain in welfare (Y). The dotted line in all panels indicates the zero line.