Does competition for multiple investors mitigate the hold-up problem?

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Abstract

Two symmetric countries compete over two-period under a non-preferential (preferential) taxation regime to attract multiple investors where investors are strategic and investments are sunk once invested. We compare tax revenues of competing countries when one investor enters the market with a scenario where two investors enter the market during the initial period keeping the amount of capital fixed. We show that the combined tax revenue of competing countries is higher when two investors enter the market during the initial period, that is, the presence of multiple investors mitigates the hold-up problem. We also show that the equilibrium tax revenues under non-preferential regimes are strictly higher than that under preferential regimes.

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1 Introduction

Returns on investments can be fully expropriated when investments are sunk once invested. This deters foreign investments. The hold-up problem due to

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the threat of expropriation has been well studied[1]. In particular, Kehoe (1989) shows that competition between governments can solve the hold-up problem. Konrad and Lommerud (2001) show that incomplete information may partially solve this problem. In the literature on tax competition, Bond and Samuelson (1986) show that when investors do not know the productivity of competing countries, a country with higher productivity offers tax holidays during the initial period to signal its higher productivity. Moreover, governments also face dynamic inconsistency problems when they cannot commit to future tax rates[2]. Janeba (2000) shows that firms may build capacity in more than one country when a country cannot commit to future tax rates. Having production capacity in more than one country allows a firm owner to shift production to a country with lower tax rates. Countries compete to attract production to their jurisdiction by lowering the tax rate and that mitigates the hold-up problem. Kishore and Roy (2014) show that a non-preferential taxation regime can solve the hold-up as well as dynamic inconsistency problems when a single country wishes to attract investors that differ in their cost of mobility[3]. Under a preferential taxation regime, a country has an incentive to lower taxes during the later period to attract less willing investors. Therefore, an investor has an incentive to wait for a lower tax rate before making investments. Under a non-preferential regime, a country is committed to not lowering the tax rate during the later period. In this paper, we look at the competition between two symmetric countries for large strategic investors. We show that the presence of multiple investors mitigates the hold-up problem. When both investors invest in the same country during the initial period, the expected tax payments during the later period are higher compared to the case when investors invest in different countries. Therefore, a country has to undercut its competitor by a discrete margin to attract both investors. At the same time, a country can attract one investor even when it sets a tax rate higher than its competitor. This reduces competition during the initial period and increases tax revenues.

The paper is also related to a huge literature on "tax competition" [4]. Authors argue that the cost of capital relocation is falling over time. Large multinationals choose their location based on tax rebates offered by host governments. This leads to *"race to the bottom"* effect where tax rates on many forms of capital are close to zero. Authors argue that countries are increasingly adopting preferential taxation regimes where they set a lower tax rate on footloose foreign capital compared to immobile domestic capital or capital with older vintages. This leads to competition among nations to attract footloose foreign investments. Various measures have been taken through supranational agencies such as OECD to promote cooperation on international taxation and encourage

[1]See for example; Eaton and Gersowitz(1983), Thomas and Worrall (1994), Konrad and Lommerud (2001)

[2]See for example; Janeba (2000), Kishore and Roy (2014), Kishore (2017)

[3]Kishore (2017) show the opposite is true when investors are large or the cost of capital relocation decreases over time

[4]See Wilson (1999) for a survey on tax competition

countries to adopt non-preferential taxation regimes. [5] Moreover, new measures and rules have been decided upon to reduce profit shifting and base erosion[6]. The global minimum tax under pillar two establishes a floor on corporate tax competition which will ensure a multinational enterprise (MNE) is subject to tax in each jurisdiction at a 15 percent effective minimum tax rate regardless of where it operates. Over 50 jurisdictions are taking steps to implement the global minimum tax rate.

Authors compare tax revenues of competing countries under two different taxation regimes in a static setting: (i) competing countries jointly adopt non-preferential taxation regimes that restrict them from setting different tax rates based on different mobility, vintages, nationality, etc., and (ii) competing countries adopt preferential taxation regime where they are free to set different tax rates for different capital bases[7]. Authors find that non-preferential taxation regimes are desirable under certain circumstances.

Haupt and Peters (2005) show that equilibrium tax revenues are higher when competing countries adopt non-preferential regimes. In this paper, countries compete over two independent tax bases. One of the capital bases has home bias for one country, and opposite is true for the other capital base. When a capital base has home bias for one country, more than half of capital is invested in the country. Under preferential regimes, competition for one capital base is independent of the other capital base. Therefore, competition is higher because a smaller country is more willing to lower the tax rate to attract investments. In this scenario, non-preferential regime bring symmetry between two countries and that reduces competition. Bucovetsky and Haufler (2007) show that when countries differ in their size of domestic capital bases, non-preferential regimes generate greater tax revenues. The larger of the two countries obtains an equal tax revenues under two regimes, while the smaller country obtains higher tax revenues. Therefore, the combined tax revenue of two countries is larger under non-preferential regimes. In particular, Janeba and Peters (1999) show that a country has unilateral incentives to commit to non-preferential regimes even when the competitor adopts preferential taxation. This paper differs from the existing literature in many ways. While the papers discussed above are static, we look at two-period dynamic competition where investments from the initial period are sunk. Moreover, we look at competition for large strategic investors who take into account the effect of their investment decision on future tax rates. We show that even when two countries are ex-ante symmetric, equilibrium tax revenues of competing countries are strictly large under non-preferential regimes. Under non-preferential regimes, both investors invest in the same country during the initial period. This reduces competition and increases tax revenues of the

[5] See OECD (2004): The OECD's project on harmful tax practices.

[6] OECD/G20 Base Erosion and Profit Shifting Project: Outcome Statement on the Two-Pillar Solution to Address the Tax Challenges Arising from the Digitalisation of the Economy, 11 July 2023.

[7] See for example; Haupt and Krieger (2020), Haupt and Peters (2005), Janeba and Peters (1999), Janeba and Smart (2003), Keen (2001), Kishore (2019), Mongrain and Wilson (2018), Wilson (2005), Bucovetsky and Haufler (2007), Merceau, Mongrain, and Wilson (2010), Wang (2004)

country that does not attract investments during the initial period as well. Therefore, a country has to offer a smaller tax holidays during the initial period to attract investments.

The paper is also related to the literature on "switching cost", and competition between firms in the presence of loyal consumer bases. The nature of equilibria we analyze has been studied in Bertrand type competition between firms with loyal customers (See Narasimhan (1988)), or when customers have switching costs (Farrell and Klemperer (2007)).

We consider a two-period model where two symmetric countries compete to attract foreign investments and investments in any of the two countries are fully immobile. Similar to Konrad and Kovenock (2009), we assume that competing countries commit to non-preferential taxation regimes for the entire duration of the game. We compare the equilibrium outcomes under two scenarios: (i) one investor with two units of indivisible capital enters the market in each period, and (ii) two investor each having one unit of indivisible capital enter the market in each market. We also compare the outcomes under non-preferential regimes with that of preferential taxation regimes. We show that the equilibrium tax revenues is strictly higher under non-preferential regimes compared to preferential taxation regimes. Moreover, the equilibrium tax revenues under non-preferential taxation regimes when two investors enter the market during the initial period are strictly greater than the case when one investor enters the market in each period. When only one investor enters the market during the initial period, the investor faces an equal tax rate irrespective of which country it chooses to invest. When two investors enter the market during the initial period, the tax rate during the later period depends on the investment decisions made by investors during the initial period. When both investors invest in different country during the initial period then both faces an equal tax rate during the later period. On the other hand, when both investors invest in the same country, the tax rate during the later period is higher. Therefore, a country has to undercut its competitor by a discrete margin to attract both investors. This reduces competition during the initial period and increases tax revenues.

2 Model

We consider two identical countries/jurisdictions indexed by $i \in (A, B)$. The economy lasts for two periods, 1 and 2. Countries (A, B) compete to attract investments from outside their jurisdictions. Competing countries have no domestic capital at the beginning of period 1. At the beginning of period 1, two investors (each owning one unit of capital) enter the market outside of the jurisdictions of competing countries. At the beginning of period 2, an investor with 2 units of capital enters the market outside of the jurisdictions of countries A and B. For simplicity, we assume that outside the two competing countries, the return on invested capital is equal to 0. The return on investments in the country A and B is equal to 1 in each period.

We analyze this dynamic tax competition between two symmetric countries when at the beginning of the game countries commit to a non-preferential taxation regime for the entire duration of the game. If a country commits to a non-preferential taxation regime, it cannot set discriminating tax rates depending on the origin of the capital (domestic and foreign) or capital bases of different vintages (old investments and potential new investments). We assume that governments maximize tax revenues, and investors maximize net returns on investments after-tax payments. We further assume neither governments nor investors discount future income. The stages of the game can be described below:

Stage 1: Both countries jointly adopt non-preferential (preferential) taxation regimes for the entire duration of the game.

Stage 2: At the beginning of period 1, competing countries simultaneously announce the tax rates applicable in period 1. The investors observe the prevailing tax rates and make an investment in the country A or country B. The maximum tax rate a country can set is equal to 1. The tax rate can be negative, that is, a government can offer tax holidays during the initial period.

Stage 3: At the beginning of period 2, competing countries announce tax rates applicable for period 2. The new investor observes prevailing tax rates and makes an investment in the country A or country B. Governments receive taxes at the end of period 2.

Note that the investments decision of investors during period two is completely determined by tax rates chosen by the two countries. We look at the subgame-perfect Nash equilibrium of this two-stage dynamic game. We provide a formal analysis of the competition when both countries adopt non-preferential taxation regimes. Analysis of the competition under a preferential taxation regime is trivial and is left for discussion at the end of the paper. First, we consider a baseline scenario where one investor with one unit of capital enters the market in each period.

3 Case I: One Investor in Each Period

The paper aims to highlight the difference in the nature of tax competition when one investor enters the market in each period compared to the case when two investors enter the market in each period. The case where one investor enters the market in each period serves as a basis for comparison. When one investor enters the market in each period, investors own two units of non-divisible capital. When two investors enter the market in each period, each investor owns one unit of capital. Therefore, the total available capital in each period under two scenarios is equal, and this assumption makes the comparison easier. First, we look at the competition for investments in period 2.

3.1 Case I: Tax Competition in Period Two

The tax revenue of country A in period 2, $TR1_2^A$, can be represented as:

$$TR1_2^A = \begin{cases} 4t_a, & \text{if } t_a \leq t_b \\ 2t_a, & \text{if } t_a > t_b \end{cases} \tag{1}$$

When the tax rate of country A, t_a, is lower than that of country B, t_b, country A receives tax revenues of $2t_a$ from the domestic capital base, and it also attract the new investor and obtains tax revenues of $2t_a$ from new investments. The tax revenue of country B, $TR1_2^B$, can be represented as:

$$TR1_2^B = \begin{cases} 2t_b, & \text{if } t_b < t_a \\ 0, & \text{if } t_b \geq t_a \end{cases} \tag{2}$$

When country B sets a tax rate, t_b, that is strictly lower than the tax rate of country A, t_a, it attracts the new investor and obtains tax revenues of $2t_b$ from new investments. When the tax rate of country B is higher than that of country A, i.e., $t_b \geq t_a$, it fails to attract investments and does not receive a non-negative amount of tax revenue.

It is easy to argue that a pure strategy Nash equilibrium does not exist. As long as country B sets a tax rate greater than $\frac{1}{2}$, country A has an incentive to undercut the tax rate to attract new investments. When country B sets a tax rate lower than $\frac{1}{2}$, country A has an incentive to set the tax rate equal to 1 to maximize tax revenues from its domestic capital base and forgo new investments. In response, country B also increases the tax rate.

Lemma 1. *A pure strategy Nash equilibrium does not exist in period 2.*
Proof. See Appendix.

Given a pure strategy Nash equilibrium does not exist, we find a mixed strategy Nash equilibrium. Without a loss of generality suppose country A attracts the investor in period 1. Country A is guaranteed a tax revenue of 2 if it sets the tax rate of 1 and receives taxes from immobile domestic investments only. From (1), the tax revenue of country A is $4t_a$ when it sets a tax rate lower than that of country B. Therefore, the *minimum* tax rate country A sets in any equilibrium, t_m^a, is such that, $4t_m^a = 2$. From the equality we obtain

$$t_m^a = \frac{1}{2}. \tag{3}$$

Now consider a mixed strategy Nash equilibrium where competing countries randomize over the set $[\frac{1}{2}, 1]$. The equilibrium tax revenues of country A and country B are 2 and 1, respectively. Let F_a and F_b denote distributions of taxes over the support of country A and country B. Suppose country A sets $t_a \in (\frac{1}{2}, 1)$. With a probability of $[1 - F_b(t_a)]$, it attract the new investor as well and it receives $4t_i[1 - F_b(t_a)]$ in tax revenues. With a probability of $F_b(t_a)$ it does not attract the new investor and it receives tax revenues of $2t_iF_b(t_a)$ from the domestic capital base. The total tax revenue of country A is equal to $4t_i[1 - F_b(t_a)] + 2t_iF_b(t_a)$. A country receives an equal tax revenue everywhere over the support. Therefore, the following equality holds:

$$4t_i[1 - F_b(t_a)] + 2t_i F_b(t_a) = 2$$

Rearranging the above equality we obtain

$$F_b(t) = 2 - \frac{1}{t} \tag{4}$$

Note that $F_b(\frac{1}{2}) = 0$, and $F_b(1) = 1$, i.e., there is no probability mass over the support of country B.

Now suppose country B sets $t_b \in (\frac{1}{2}, 1)$. With a probability of $[1 - F_a(t_b)]$, it attracts the new investor and it receives $2t_i[1 - F_a(t_b)]$ in taxes. With a probability of $F_a(t_b)$ it does not attract investments in period 2 and it does not receive a positive tax revenues in period 2. Therefore, the total tax revenue is $2t_i[1 - F_a(t_b)]$. The equilibrium tax revenue of country B is 1, and a country obtains an equal tax revenue everywhere over the support. Therefore, the following equality holds:

$$2t_i[1 - F_a(t_b)] = 1$$

Using the above equality, we obtain the distribution of taxes over the support of country A, $F_a(t)$.

$$F_a(t) = \begin{cases} 1 - \frac{1}{2t}, & t \in [\frac{1}{2}, 1] \\ \frac{1}{2}, & t = 1 \end{cases} \tag{5}$$

Note that $F_a(\frac{1}{2}) = 0$, and $F_a(1) = \frac{1}{2}$. Therefore, there is a probability mass of $(1 - F_a(1)) \equiv \frac{1}{2}$ at the *supremum* of the support of country A.

We argue that the proposed strategies described by (4) and (5) constitute a mixed strategy Nash equilibrium. Note that if country A sets a tax rate lower than $\frac{1}{2}$, its tax revenue is less than 2 even though it attracts the new investor with certainty. Given country A does not set a tax rate lower than $\frac{1}{2}$, and there is no probability mass at the *infimum* of support of country A, country B has no incentive to set a tax rate lower than $\frac{1}{2}$. Because a country earns an equal tax revenue everywhere over the support, it cannot gain from redistributing taxes over the support. Therefore, we argue that no country can do better from a unilateral deviation. Lemma 2 describes the outcome formally.

Lemma 2. *In a scenario where country A attracts the investor in period 1, a unique Mixed strategy Nash equilibrium exists in period 2. The equilibrium tax revenues of country A and country B are* 2 *and* 1*, respectively. Competing countries randomize over the common support* $[\frac{1}{2}, 1]$*. Distributions of taxes over the supports of country A and country B are described by (5) and (4), respectively.*

Proof. Proof of Lemma 2 is similar to Lemma 4. See Appendix for proof of Lemma 4.

3.2 Case I: Tax Competition in Period One

Now we analyze the outcome of period 1. Note that the expected tax payments in period 2 are the same whether an investor invests in the country A or country B in period 1. Therefore, an investor invests in a country that offers a lower tax rate in period 1. From Lemma 2, we observe that the country that attracts the investor in period 1 obtains a tax revenue of 2 in period 2, while the country that does not attract the investor in period 1 receives 1 in tax revenues in period 2. The revenue gain during the later period from attracting the investor in period 1 is equal to 1. Therefore, the *maximum* tax subsidy a country is willing to offer in period 1 is $\frac{1}{2}$ for one unit of investment, or subsidy of 1 for two units of investment. From Lemma 2, We already know that there is a unique Nash equilibrium in mixed strategies in period 2. Therefore, there is a unique subgame perfect Nash equilibrium of the game. Lemma 3 describes the outcome formally.

Lemma 3. *A unique subgame perfect Nash equilibrium exists. In the unique Nash equilibrium in period 1, competing countries set the tax rate equal to* $-(\frac{1}{2})$*, i.e., competing countries offer a subsidy of* $\frac{1}{2}$ *per unit of investments. The equilibrium tax revenue of competing countries is equal to* 1.

Proof. Proof is obvious.

As both countries set an equal tax rate, the investor invests in either of the two countries. The country that attracts the investor offers a tax subsidy of 1 in period 1, and it receives tax revenues of amount 2 in period 2. The total tax revenue over two periods is equal to 1. The country that does not attract the investor in period 1 receives no positive amount of tax revenues in period 1, and it receives a tax revenue of amount 1 in period 2. Therefore, the two countries receive an equal tax revenue over two periods whether or not a country attracts an investor in period 1.

4 Case I: Minimum Tax Rule

4.1 Non-preferential

Suppose competing countries cannot set a tax rate lower than t_{min}, where $t_{min} \geq 0$. When $t_{min} \leq \frac{1}{2}$, the minimum tax will have no effect on the equilibrium outcome in period 2. On the other hand, it will have an effect on the equilibrium outcome in period 1.

First, we analyze the case when $t_{min} \leq \frac{1}{2}$. Lemma 2 describes the equilibrium outcome in period 2 in this case. The country that attracts the investor in period 1 receives tax revenue of 2, while the other country receives tax revenue of 1. The revenue gain in period 2 from attracting the investor in period 1 is 1. Therefore, a country is willing to offer a tax subsidy of amount 1 in period 1 to attract the investor. As described in Lemma 3, a country offers a tax rate of $-(\frac{1}{2})$ in period 1 to attract the investor. This is the equilibrium outcome when competing countries do not follow the minimum tax rule. In the presence of a minimum tax rule, the minimum tax rate a country can set is t_{min}. Therefore, both countries set the tax rate of t_{min} in period 1 and attract the investor with a probability of $\frac{1}{2}$. The expected tax revenue of a country is $\frac{1}{2} + \frac{1}{2}(2t_{min} + 2) \equiv \frac{3}{2} + t_{min}$. Therefore, additional revenue gain from the minimum tax rule is equal to $\frac{1}{2} + t_{min}$.

Second, consider the case when $t_{min} > \frac{1}{2}$. In this case, the equilibrium tax rate in period 2 is t_{min}. The country that attracts the investor in period 1 receives tax revenues of amount $2t_{min}$ with certainty. Moreover, it also receives tax revenues of $2t_{min}$ with probability $\frac{1}{2}$. Therefore, the expected tax revenue in period 2 of the country that attracts the investor in period 1 is $2t_{min} + \frac{1}{2}(2t_{min}) \equiv 3t_{min}$. The expected tax revenue in period 2 of a country that does not attracts investments in period 1 is t_{min}. The revenue gain in period 2 from attracting the investor in period 1 is $2t_{min}$. A country is willing to offer a tax subsidy of $2t_{min}$ in period 1 to attract the investor in period 1. Therefore, the equilibrium tax rate in period 1 is t_{min}. Both countries are equally likely to attract the investor. With a probability of $\frac{1}{2}$, a country attracts the investor in period 1. In this case, it receives tax revenues of $2t_{min}$ in period 1 and $3t_{min}$ in period 2. When a country does not attract investments in period 1 then it receives tax revenues of t_{min}. Therefore, the expected tax revenue of a country is $\frac{1}{2}(5t_{min} + t_{min}) \equiv 3t_{min}$.

4.2 Preferential

When competing countries do not adopt the minimum tax rule, the country that attracts the investor in period 1 receives tax revenues of 2 in period 2, while the country does not receive positive tax revenue. The revenue gain to a country from attracting the investor in period 1 is 2. Therefore, a country is willing to offer a tax subsidy of amount 2 in period 1 to attract an investor. Competing countries set a tax rate of -1 in period 1. The equilibrium tax revenue of competing countries is equal to 0.

When competing countries adopt the minimum tax rule, a country continues to set a tax rate of 1 on its immobile domestic capital base. On the other hand, it cannot set a tax rate lower than t_{min} on new investments. Therefore, a country receives tax revenues of 2 from its domestic capital base in period 2 with certainty when it attracts the investor in period 1. Moreover, it also receives tax revenues of $2t_{min}$ from new investments with a probability of $\frac{1}{2}$. Therefore, the expected tax revenue in period 2 is $2 + t_min$. On the other hand, the country that does not attract investments in period 1 receives t_{min} in tax revenues in period 2. Therefore, the revenue gains in period 2 from attracting investments in period 1 is 2. A country is willing to offer a tax subsidy of 2 to attract the investor in period 1, but it is bound to set a tax rate greater than t_{min}. The equilibrium tax rate in period 1 is t_{min}. A country attracts investments in period 1 with a probability of $\frac{1}{2}$ and receives tax revenues of $2t_{min}$. With a probability of $\frac{1}{2}$ it does not attract investments in period 1 and receives tax revenues of t_{min} in period 2.

5 Case II: Two Investors in Each Period

In this section, we analyze the case of interest where two investors each having one unit of capital enter the market in each period.

5.1 Case II: Tax Competition in Period Two

There can be two possible scenarios in period 2; (i) two investors invest in different countries in period 1, and (ii) two investors invest in the same country in period 1. First, we consider the first case.

5.1.1 Case II: Symmetric Tax Competition in Period Two

Both countries have one unit of capital at the beginning of period 2 that is completely immobile across jurisdictions. There is an investor with two units of capital outside the jurisdictions of competing countries, and it can relocate to either country A or B without incurring a cost. Two countries are committed to non-preferential regimes, therefore, country A and country B set tax rates t_a and t_b, respectively. The tax revenue of country $i \in (A, B)$, is represented as

$$NPST_i^2 = \begin{cases} t_i + 2t_i, & \text{if } t_i \leq t_j \\ t_i, & \text{if } t_i > t_j \end{cases} \tag{6}$$

We assume that when an investor is indifferent between investing in the country A and country B, it invests in the country A. The assumption does not affect the equilibrium. When country sets a tax rate, t_a, such that, $t_a \leq t_b$, it receives t_a in tax revenues from the domestic capital base, and it also receives taxes of amount $2t_a$ from new investments. When the tax rate of country A is larger than that of country B, it only receives t_a in taxes from the domestic capital base. The tax revenue of country B can be analogously defined.

Lemma 4 *When both countries attract an investor each in period 1, a pure strategy Nash equilibrium does not exist in period 2.*

Proof. Proof is similar to Lemma 1.

The intuition for the above result is similar to Lemma 2. Given a pure strategy Nash equilibrium does not exist, we analyze a mixed strategy Nash equilibrium. Consider a symmetric mixed strategy Nash equilibrium where competing countries randomize over the common support $[\frac{1}{3}, 1]$, with the distribution of taxes over the support denoted by F. The distribution function F is convex and there is no probability mass over the support. Note that when a country sets $t_i = 1$, it attracts new investments with zero probability and receives taxes only from the domestic capital base. Therefore, the equilibrium tax revenue is equal to 1. When a country sets a tax rate of $\frac{1}{3}$ then it attracts the new investor with probability 1. It is easy to argue that in no equilibrium a country sets a tax rate lower than $\frac{1}{3}$. Suppose country $i \in (A, B)$ sets $t_i \in (\frac{1}{3}, 1)$. With probability $1 - F_j(t_i)$ it undercuts the tax rate of country j and receives a tax revenue of

$2t_i(1-F_j(t_i))$ from new investments. It also receives a tax revenue of t_i from the domestic capital base. Because a country receives an equal revenue everywhere on the support, the following equality holds:

$$t_i + 2t_i(1 - F_j(t_i)) = 1.$$

Rearranging the above equation we obtain

$$F(t) = 1 + \frac{1}{2}(1 - \frac{1}{t}),\ t \in [\frac{1}{3}, 1]. \tag{7}$$

Lemma 5 describes the characterization of the mixed strategy Nash equilibrium.

Lemma 5 *A unique symmetric mixed strategy Nash equilibrium exists in period 2 when both countries attract one investor each in period 1. The equilibrium tax revenue of competing countries is equal to* 1*. Both countries randomize over the common support* $[\frac{1}{3}, 1]$*. There is no probability mass over the support. The distribution of taxes over the support is given by (7).*
Proof. See Appendix.

A mixed strategy Nash equilibrium of this type has been analyzed before in the literature on tax competition. Wang (2004), Kishore (2019), Anderson and Konrad (2001), Wilson (2005), Marceau, Mongrain and Wilson (2007) consider tax competition in a static model with mobile and immobile capital bases, resulting in equilibria similar to the one described here. Similar equilibria also arise in the literature on switching cost (Farell and Klemperer (2007)). Narasimhan (1988) analyzed competition between consumers where a fraction of consumers are loyal to a particular firm, resulting in similar equilibria.

5.1.2 Case II: Asymmetric Tax Competition in Period Two

Now we look at the outcomes in period 2 when one country attracts both investors in period 1. Without loss of generality we assume that country A attracts both investors in period 1.

Lemma 6 *When one country attracts both investors in period 1, a pure strategy Nash equilibrium does not exist in period 2.*
Proof. Same as Lemma 1. See Appendix for the proof of Lemma 1.

The intuition for Lemma 6 is similar to Lemma 1. As before, we analyze a mixed strategy Nash equilibrium of the game in period 2. Without loss of generality suppose country A attracts both investors in period 1. Consider a candidate for mixed strategy Nash equilibrium where country A and country B randomizes over the supports $[\frac{1}{2}, 1]$ and $[\frac{1}{2}, 1)$, respectively with distribution functions F_a and F_b. When country A sets t_{min} and attracts the new investor with probability one, its tax revenues is equal to $4t_{min}$. When country A sets a tax rate of 1 it receives a tax revenue of 2 with certainty. Therefore, the minimum tax revenue country A receives in any equilibrium is 2. Therefore, the *minimum* tax rate country A sets in equilibrium is equal to $\frac{1}{2}$. In the proposed equilibrium, country B sets a tax rate of 1 with probability zero. When country A sets the tax rate equal to 1, it attracts the new investor with probability zero and receives a tax revenue of 2. Therefore, the equilibrium tax revenue of country A is equal to 2. When country B sets the tax rate of $\frac{1}{2}$, it attracts the new investor with probability one, and receives a tax revenue of 1. In Appendix we show that no country has a probability mass at the *infimum* of the support. Suppose country A sets a tax rate t_a such that $\frac{1}{2} < t_a < 1$. It undercuts the tax rate of country B with probability $(1 - F_b(t_a))$ and receives revenues of amount $2t_a$ from the new investor. Country A also receives tax revenues of $2t_a$ with certainty from the domestic capital base. Therefore, the following equality holds:

$$2t_a + 2t_a(1 - F_b(t_a)) = 2.$$

Rearranging the above equality we obtain

$$F_b(t) = 2(1 - \frac{1}{2t}),\ t \in [\frac{1}{2}, 1) \tag{8}$$

Now suppose country B sets a tax rate t_b such that $\frac{1}{2} < t_b < 1$. Country B undercuts the tax rate of country A with probability $(1 - F_a(t_b))$ and receives tax revenues of amount $2t_b$ from the new investor. Therefore, the following equality holds:

$$2t_a(1 - F_a(t_b)) = 1.$$

Rearranging the above equality we obtain

$$F_a(t) = 1 - \frac{1}{2t},\ t \in [\frac{1}{2}, 1] \tag{9}$$

Note that $F_a(1) = \frac{1}{2}$. There is a probability mass, m_a, at the *supremum* of the support of country A, where

$$m_a = \frac{1}{2} \tag{10}$$

Lemma 7 describes the characterization of the mixed strategy Nash equilibrium and the equilibrium tax revenues of competing countries.

Lemma 7 *When one country attracts both investors in period 1, a unique mixed strategy Nash equilibrium exists in period 2. The equilibrium tax revenues of country A (that has 2 units of domestic capital base) and country B are* 2 *and* 1*, respectively. Both countries randomize over the common support* $[\frac{1}{2}, 1]$*, with country B setting the tax rate of* 1 *with zero probability. There is a probability mass,* $m_a \equiv \frac{1}{2}$*, at the supremum of the support of country A. The distributions of taxes over the supports of country A and country B are* F_a *and* F_b*, respectively.* F_a *and* F_b *are given by (9) and (8), respectively.*
Proof. Proof is similar to Lemma 5.

A mixed strategy equilibrium of this type arises when a firm with a loyal consumer base compete with a firm that has no loyal consumer base in Narasimhan (1988). In equilibrium, the tax revenue of the country with a domestic capital base (large country) is equal to that it can receive by completely expropriating the return of its domestic capital. On the other hand, the tax revenue of the country without a domestic capital base (small country) is considerably greater. Authors argue that one of the rationals for having non-preferential taxation regimes is asymmetry of capital bases of competing countries (See Bucovetsky and Haufler(2007)). Wilson (2005), and Marceau, Mongrain, and Wilson (2010) also find similar results.

5.2 Case II: Expected Tax Payments in Period Two

In section (3.1) and (3.2) we look at the outcomes in period 2 depending on whether two countries attract one investor each, or one country attracts both investors in period 1. While making an investment in period 1, an investor needs to look at the prevailing tax rates in period 1, and the expected tax payments in period 2. Therefore, it is important to compare the expected tax rates in period 2 under two scenarios.

Firstly, suppose two investors invest in different countries. From Lemma 5, the distribution of taxes in period 2 is $F \equiv 1 + \frac{1}{2}(1 - \frac{1}{t})$. There is no probability mass over the support is $f \equiv \frac{1}{2t^2}$. Therefore, when two investors invest in different countries, the expected amount an investor pays in period 2, E^2, is equal to:

$$E^2 \equiv \int_{\frac{1}{3}}^{1} tf(t)dt \equiv \frac{1}{2}log3. \tag{11}$$

Suppose both investors invest in one country. We have assumed that country A attracts both investors in period 1. From Lemma 7, the distribution of taxes over the support of country A in period 2 is given by $F_a \equiv 1 - \frac{1}{2t}$, $t \in [\frac{1}{2}, 1]$, with a probability mass of $\frac{1}{2}$ at 1. The density function is, $f_a \equiv \frac{1}{2t^2}$. Therefore, when both investors invest in one country, the expected amount an investor pays in period 2, E_a^2 is equal to,

$$E_a^2 \equiv \int_{\frac{1}{2}}^{1} tf(t)dt + \frac{1}{2} \equiv \frac{1}{2}(1 + log2) \tag{12}$$

From (8), the distribution of taxes over the support of country B is, $F_b(t) \equiv 2(1 - \frac{1}{2t}), t \in [\frac{1}{2}, 1]$. Therefore, the density function of taxes over the support of country B, $f_b(t) \equiv \frac{1}{t^2}, t \in [\frac{1}{2}, 1]$. Therefore, the expected tax rate in country B, E_b^2, is

$$E_b^2 = \int_{1/2}^{1} tf_b(t)dt = log2 \tag{13}$$

Note that investors from the previous period cannot relocate to another country in period 2. Therefore, the expected tax payments in period 2 for investors from the previous period is given by (12). Lemma 8 describes the outcome formally. Proof follows from the above discussion.

Lemma 8. *In a scenario where two countries attract one investor each in period 1, the expected tax rate applicable in period 2 is, $E^2 \equiv \frac{1}{2}log3$. When one country attracts both investors in period 1, the expected tax rate in period 2 in the country that attracts both investors in period 1 is, $E_a^2 \equiv \frac{1}{2}(1+log2)$. E_a^2 is also the expected tax payments in period 2 for investors who invested in period 1. The expected tax rate applicable in the country that attracts no investments in period 1 is, $E_b^2 \equiv log2$.*

Note that $\frac{1}{2}(1+log2) > \frac{1}{2}log3$. Therefore, investors from period 1 pay higher taxes in period 2 when they invest in the same country compared to the case when they invest in different countries.

In period 2, an investor invests in a country that sets a lower tax rate. Therefore, it does not matter whether we have one investor with two units of capital, or there are two investors each have one unit of capital in period two.

5.3 Case II: Tax Competition in Period One

From (11) and (12), the expected tax payments in period 2 when both investors invest in different countries is $\frac{1}{2}log3$, and the expected tax payments when both investors invest in the same country is $\frac{1}{2}(1+log2)$. Therefore, investors have to pay an amount $(\frac{1}{2}(1+log2) - \frac{1}{2}log3) \equiv \frac{1}{2}(1+log(\frac{2}{3}))$ higher in taxes in period 2 when they invest in the same country. In period 1, a country chooses strategies that maximize the sum of tax revenues from period 1 and period 2. The tax revenue of country i in period 1, NPT_i, can be represented as:

$$NPT_i = \begin{cases} 2t_i + 2, & \text{if } t_i < t_j - \frac{1}{2}(1+log(\frac{2}{3}) \\ t_i + 1, & \text{if } t_j - \frac{1}{2}(1+log(\frac{2}{3}) \leq t_i \leq t_j + \frac{1}{2}(1+log(\frac{2}{3}) \\ 1, & \text{if } t_i > t_j + \frac{1}{2}(1+log(\frac{2}{3})) \end{cases} \tag{14}$$

When country i undercuts the tax rate of country j by a margin of $\frac{1}{2}(1+log(\frac{2}{3}))$, it attract both investors. The tax revenue in period 1 is equal to $2t_i$. When a country attracts two investors in period 1, it also receives tax revenues of amount 2 in period 2. Therefore, the total tax revenue is $2t_i + 2$. When $t_j - \frac{1}{2}(1+log(\frac{2}{3}) \leq t_i \leq t_j + \frac{1}{2}(1+log(\frac{2}{3})$, both countries attract an investor each in period 1. The tax revenue in period 1 is equal to t_i. The tax revenue in period 2 is equal to 1. Therefore, the total tax revenue is equal to $1 + t_i$. When country i sets t_i such that $t_i > t_j + \frac{1}{2}(1+log(\frac{2}{3}))$, it does not attract an investor in period 1. It receives tax revenues of amount 1 in period 2. Therefore, the total tax revenue is equal to 1.

Proposition 1. *A pure strategy Nash equilibrium exists in period 1. One country sets a tax tax rate of $t^H \equiv 0$, while the other country sets a tax rate of*

$t^L \equiv -\frac{1}{2}(1 + log(\frac{2}{3})$. Both investors invest in the country that sets the tax rate equal of t^L. The equilibrium tax revenue of country that attracts both investors in period 1 is equal to $2 + 2t^L$, and the equilibrium tax revenue of the other country is 1 .

Proof. Suppose without a loss of generality that country B sets 0, and country A sets $-\frac{1}{2}(1 + log(\frac{2}{3})$ in period 1. Country A attracts both investors during the initial period. Fist, we show that country B cannot deviate unilaterally and do better. The outcome remains unchanged when country B deviates and sets a tax rate greater than 0. Suppose country B sets a tax rate, t_b, such that $2t^L < t_b < 0$. In this case, country B attracts one investor in period 1, and obtains tax revenues of amount 1 in period 2. The tax revenue of country B in period 2 remains unchanged while it also offers tax holidays during period 1. Therefore, the tax revenue of country B decreases. Suppose country B deviates and sets a tax rate of $2t^L$. Country B attracts both investors during the initial period, and obtains tax revenues of amount 2 in period 2. The total tax revenue is, $2 + 4t^L \equiv 2log(\frac{3}{2})$, that is strictly less than 1. Therefore, country B cannot deviate unilaterally and do better. It is straight forward to show that country A cannot deviate unilaterally and do better. Therefore, the strategies described in Proposition 1 constitute a pure strategy Nash equilibrium. □

Note that the equilibrium need not be unique. Lemma 9 describes constraints that strategies must satisfy in any pure strategy Nash equilibrium.

Lemma 9 *In any pure strategy Nash equilibrium, the difference between tax rates of two countries is equal to t^L. Moreover, no country sets a strictly positive tax rate.*

Proof. Now, we need to show that the equilibrium described above is unique. Let t_A and t_B be equilibrium tax rates of country A and country B, respectively. Observe that in no pure strategy equilibrium the difference between tax rates of two countries cab be less than t^L. In this case, both countries attract one investor each in period one. Therefore, the country that sets a lower tax rate can marginally increase its tax rate and still attract one investor in period 1. Similarly, there can be no equilibrium where the difference between tax rates of two countries is greater than t^L. In this case, the country that sets a lower tax rate can increase its tax rate and still attract both investors. Therefore, in any pure strategy Nash equilibrium the difference of tax rates of two competing countries is equal to t^L. This proves that the difference between equilibrium tax rates of two countries is t^L. Suppose the equilibrium tax rate of the country with a higher equilibrium tax rate is strictly positive. In this case, the country can lower the tax rate marginally and attract one investor in period 1. The tax revenue from period two remains unchanged but it also receives a positive amount in period one. Contradicting that we have a pure strategy Nash equilibrium. □

We are looking for a set of possible pure strategy Nash equilibria such that both countries set non-positive tax rates, and one country undercuts the tax rate of its competitor by a margin of t^L and attracts both investors. Let t^m be the minimum tax rate a country is willing to set to attract both investors. The equilibrium strategy of the competitor is $t^m - t^L$, where t^L is a negative number. The best possible deviation for the country that attracts both investor is to set a tax rate of, $t^m - 2t^L$, and attract one investor in period one. The total tax revenue in this case is equal to, $1 + (t^m + 1 + log(\frac{2}{2})$, where the first term is tax revenues from period two and the first term is tax revenues from period one. Let us define t^m such that the following equality holds:

$$2 + 2t^m = 1 + (t^m + 1 + log(\frac{2}{2}) \tag{15}$$

From the above equality we obtain, $t^m = log(\frac{2}{3})$. Proposition 2 describes all pure strategy Nash equilibria.

Proposition 2. *A strategy pair (S_A, S_B) constitutes a pure strategy Nash equilibrium where $S_B \in [0, t^m - t^L]$, $S_A \in [t^L, t^m]$, $|S_A - S_B| = -t^L$, $t^L = -\frac{1}{2}(1 + log(\frac{2}{3}))$, and t^m is defined by (15).*

Proof. Condition (15) ensures that the country that sets a lower tax rate obtains higher tax revenues when it attracts both investors compared to setting a tax rate higher than the other country and attract one investor. The rest of proof follows from Lemma 9 and Proposition 1. □

Note that equilibrium tax revenues of competing countries decrease when tax rates decrease. The tax revenue of country that does not attract investments during the initial period is equal to 1. The minimum tax revenue in any Nash equilibrium of the country that attracts both investors in period 1 is, $T_{min} \equiv 2 + 2log(\frac{2}{3}) > 1$. The maximum tax revenue of the country that attracts both investors during the initial period is, $T_{max} \equiv 2 + 2t^L \equiv 1 - log(\frac{2}{3})$. Lemma 10 describes the outcome formally.

Lemma 10. *In any pure strategy Nash equilibria described in Proposition 2, the maximum and the minimum tax revenue of the country that attracts both investors during the initial period are $T_{max} \equiv 2 + 2t^L \equiv 1 - log(\frac{2}{3})$ and $T_{min} \equiv 2 + 2log(\frac{2}{3}) > 1$. The equilibrium tax revenue of the country that does not attract investments during the initial period is equal to* 1 .

6 Comparison: Two Investors Vs. One Investor

In this section we compare the outcomes under two cases when both countries adopt non-preferential regimes: (i) one investor enters the market during the initial period, and (ii) two investors enter the market during the initial period. Note that the total amount of capital is equal under both scenarios.

One country attracts both investors when two investors enter the market during the initial period. Therefore, the outcome of period two does not change. The outcomes in period one are different. From Lemma 3, competing countries offer a tax holidays of amount $\frac{1}{2}$ during the initial period. From Proposition 2, the maximum tax holidays offered during the initial period when two investors enter the market is, $t^m \equiv -log(\frac{2}{3})$. Note that $t^m < \frac{1}{2}$. A country has to offer a lower amount of tax holidays during the initial period to attract both investors. Therefore, the equilibrium tax revenue of the country that attracts both investors is strictly larger. Proposition 3 describes the outcome formally.

Proposition 3. *One of the competing countries receives a higher tax revenues when two investors enter the market during the initial period compared to the case when one investor enters the market. The other investor receives an equal tax revenue under two cases. Therefore, the combined tax revenues of two competing countries is higher when two investors enter the marker during the initial period.*

7 Preferential Taxation Regime

It is easy to argue competing countries do not receive a positive amount of tax revenue when both countries adopt preferential taxation regimes. First, consider the case when only one investor enters the market during the initial period. The country that attracts the only investor during the initial period, receives tax revenues of amount two in period 2 from the domestic capital base. The country that has no domestic capital at the beginning of period 2 does not receive a positive amount. Bertrand-type competition leads to both countries setting a tax rate of zero on new investments. Therefore, competing countries are willing to offer tax holidays of the amount of one per unit of invested capital during the initial period. The cost of providing tax holidays to attract the investor during the initial period completely offsets the gains during the later period. Now, we consider the case when two investors each having one unit of capital enter the market during the initial period. Note that returns are fully expropriated during the later period irrespective of whether investors invest in different countries or the same country. As before, competing countries do not receive a positive amount of tax revenues from new investments in period two. Competing countries are willing to offer a tax holiday of the amount of 1 per unit of investments during the initial period. Therefore, the cost of attracting investments during the initial period completely offsets the revenue gains during the later period. Proposition 4 describes the outcome formally.

Proposition 4. *Competing countries do not receive a positive amount of tax revenues when both countries adopt preferential taxation regimes. This is true irrespective of whether two investors enter the market, or one investor enters the market during the initial period.*

It is clear that competing countries obtain higher tax revenues when both countries adopt non-preferential regimes. Moreover, incentives to adopt non-preferential regimes are higher when two investors enter the market during the initial period. Haupt and Peters (2005) also show that tax revenues under non-preferential regimes are higher in the presence of home bias. In this paper, during the initial period investors have no home bias for either country. Investors have home bias during the later period for the country where they initially invested. Therefore, the result holds when home bias is endogenously determined by the initial investment decision made by investors. Bucovetsky and Haufler (2007) show that competing countries earn higher tax revenues under non-preferential regimes when they differ in size, that is, one country has a larger domestic capital base. In this paper as well asymmetry plays a crucial role in determining the outcome. During the later period, the combined tax revenues of competing countries are higher under non-preferential regimes when one country attracts both investors. The country that does not attract investments during the initial period also receives a strictly positive amount during the later period. This allows a country to offer a smaller amount of tax holidays during the initial period to attract both investors. Therefore, the result also holds when asymmetry is endogenously determined.

8 Conclusion

We compare the outcomes of a two periods dynamic tax competition game when (i) one investor enters the market in each period, and (ii) two investors enter the market in each period. When one investor enters the market in each period, the total tax subsidy a country offers during the initial period is equal to the gain from attracting the investor during the later period. This is the standard result when capital is sunk after investments are made and the host government can fully expropriate invested capital. One solution to such hold-up problem discussed in the literature is that the host country offers tax subsidy during the initial period to compensate the investor up-front for possible losses in the future. We show that the hold-up problem is considerably reduced in the presence of multiple investors. Although, the outcome of tax competition during the later period remains the same, the host country offers relatively smaller tax subsidy during the initial period in the presence of multiple strategic investors.

We also show that competing countries earn strictly higher tax revenues when both countries adopt non-preferential regimes. Moreover, the gains from adopting non-preferential regimes are higher in the presence of multiple investors.

In this paper, we have assumed that both countries adopt non-preferential regimes for the entire duration of the game during the initial period. Another interesting question is the kind of taxation regime that will emerge in the presence of multiple strategic investors when competing countries choose either to adopt a non-preferential or a preferential regime non-cooperatively at the beginning of the game. Another interesting question is the nature of tax competition in the presence of multiple strategic investors when attracting more investments leads to agglomeration advantages for the host country during the later period. Moreover, the nature of competition for foreign investments in the presence of more than two countries in the presence of multiple investors is also an interesting question. We leave these interesting questions for future research.

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9 Appendix

Proof of Lemma 1. Note that each country can set a tax rate of 1 and receive a tax revenue of 1 from the domestic capital base. Therefore, the equilibrium tax revenue should be at least 1. The proof is easy. Suppose country A and country B set the tax rates t_a and t_b, respectively. Consider a symmetric pure strategy Nash equilibrium such that $t_a = t_b$. Suppose $t_a = t_b > \frac{1}{3}$. Country A can deviate and set a tax rate $t_b - \epsilon$ for arbitrarily small $\epsilon > 0$ and do better. Suppose $t_a = t_b = \frac{1}{3}$. Country B can deviate and set a tax rate marginally below and do better. Now consider an asymmetric pure strategy Nash equilibrium where $p_a > P_b$. Country B can increase its tax to $p_b + \frac{p_a - p_b}{2}$ and does better. Hence, no symmetric or asymmetric pure strategy Nash equilibrium exists. □

Proof of Lemma 5. We follow Propositions 2-5 in Narasimhan (1988) to prove the uniqueness. Let S_a and S_b be equilibrium strategy sets of country A and country B, respectively.

Step 1. The strategy sets S_a and S_b are convex. *Proof.* First, we show that there are no holes in $T = S_a \cap S_b$. Let $\hat{T} = inf(T)$ and $\hat{\hat{T}} = sup(T)$. We show that there is no interval $I = (T^k, T^h)$, such that, for $\hat{T} < T^k < T^h < \hat{\hat{T}}$ and for $T \in I, T \notin T$. This could happen when one of the countries randomizes over the interval I and the other does not or when neither country randomizes over the interval I. We show that neither of these two is possible.

First, we show that if country B sets $t_b \in T$ with zero probability then so does country A. Let us define t^1 and t^2 as

$$t^1 \in S_b,\ and\ t^1 = sup(t|t < T^k)$$
$$t^2 \in S_b,\ and\ t^2 = inf(t|t > T^h)$$

It is cleat that the tax revenue of country A when it charges t^1 and t^2 respectively, are not equal. Contradicting that we have a mixed strategy Nash equilibrium. Similarly, when both countries do not randomize over the set T, the tax revenues at t^1 and t^2 are not equal. This proves that $S_a \cap S_b$ is convex.

Now we show that $T' = S_a - S_a \cap S_b$ is convex. As before define $\hat{T} = inf(T')$ and $\hat{\hat{T}} = sup(T')$. Suppose $inf(T') < inf(S_b)$. This cannot happen because country A can increase its tax revenue by shifting probability to $inf(S_b)$. Therefore, a set where country A randomizes but country B does not should be at the upper end or the lower end. But this cannot happen because country A does not receive an equal tax revenue at the upper end and the lower end of the hole. Similarly, we can show that $T' = S_b - S_a \cap S_b$ is convex.

Step 2. Neither country can have a mass point at the interior of other's support.

Proof. Suppose country B has a probability mass of m at p_b that lies in the interior of country A's support. Given p_b lies in the interior of country A's support, there exist an arbitrarily small $\epsilon > 0$, such that the set $(p_b - \epsilon, p_b + \epsilon)$ lies in the interior of country A's support. Country A can do better by moving probability from $p_b+\epsilon$ to $p_b-\epsilon$ because by reducing its tax rate by an arbitrarily small margin, it undercuts the tax rate of country B by a discrete positive probability. Therefore, we conclude that a country cannot have a mass point at the interior of the other's support.

Step 3. Neither country can have a mass point at the lower end of the other's support.
Proof. The argument is similar to that in *Step 2.* Note that in equilibrium the tax revenue of country A is strictly positive. Therefore, the *infimum* of the support of country A is strictly positive. Suppose country B has a mass point at the lower end of country A's support. Country A can reduce its tax rate by an arbitrarily small margin $\epsilon > 0$, and increase its probability of attracting the investor with discrete positive probability. This increases country A's tax revenue. This contradicts that we have a mixed strategy Nash equilibrium.

Step 4. Neither country can have a mass point at the upper boundary of the other's support when the other country has a mass point at the boundary.
Proof. The argument is simple. When a country has a mass point at the upper boundary of the support, the competitor is better off setting the tax rate at the boundary point with probability zero and lowering its tax rate arbitrarily below the boundary point.

Step 5. Strategy sets S_a and S_b are identical. When one country has a mass point at $sup(S_i), i \in (A, B)$, then country j sets $t_j = sup(S_i)$ with probability zero. *Proof.* Assume to the contrary that S_a and S_b are not identical. Without loss of generality suppose $S_b \subset S_a$. From the earlier discussion we know that a set where country A randomizes but country B does not should be either at the upper end or the lower end of country B's support. Suppose the set lies to the upper end of country B's support. Define $T = S_a - S_a \cap S_b$. Because strategy sets are convex, the set $T \equiv (sup(S_a), T)$ is convex. But country A does better by shifting probability from the set T to $sup(S_a)$. Contradicting that we have a mixed strategy Nash equilibrium. Similarly, we can show that such an interval cannot exist to the lower end of country B's support. Therefore, we conclude that two strategy sets are identical. It is easy to argue that when one country has a mass point at the *supremum* of the support then the other country does better by setting the tax rate with probability zero at the *supremum* and setting a tax rate arbitrarily below.

Step 6. $sup(S_a) = sup(S_b) = 1$.
Proof. The argument is simple. Suppose $sup(S_a) = sup(S_b) = r < 1$. When a country sets the tax rate at the *supremum* of the support, the other country undercuts its tax rate with probability one. Therefore, the tax revenue is equal to $r < 1$. The tax revenue is equal to 1 when a country sets the tax rate equal to 1. Contradicting that we have a mixed strategy Nash equilibrium.

Step 7. Only country A can have a mass point at the *supremum* of its support.
Proof. We have assumed that country A has one unit of domestic capital base, and country B has no domestic capital base. From the previous discussion, the *supremum* of the support is equal to 1. When country A has a probability mass at the *supremum* of the support it receives tax revenues of amount 1 from

the domestic capital base. Now assume to the contrary that country B has a probability mass at the *supremum* of the support. From the earlier discussion, country A cannot have a mass point at the *supremum*. Therefore, country A undercuts the tax rate of country B with probability 1. The tax revenue of country B is zero because it has no domestic capital base. This is a contradiction because equilibrium tax revenues of both countries are strictly positive.

This proves that the mixed strategy Nash equilibrium described in Lemma 4 is unique. $\square$