Marital Stability With Committed Couples: A Revealed Preference Analysis*

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September 8, 2023

Abstract

We present a revealed preference characterization of marital stability where some couples are committed. A couple is committed if they can only divorce upon mutual consent. We provide theoretical insights into the potential of the characterization for identifying intrahousehold consumption patterns. We show that when there is no price variation for private goods between potential couples, it is only possible to identify intrahousehold resource allocations for non-committed couples. Simulation exercises using household data drawn from the Longitudinal Internet Studies for the Social Sciences (LISS) panel support our theoretical findings. Our results show that in the presence of price variation, the empirical implications of marital stability can be used for identifying household consumption allocations for both committed and non-committed couples.

JEL classifications: C14, D11, C78

Keywords: household consumption, marital stability, divorce laws, revealed preference analysis, intrahousehold allocation

1 Introduction

Households consist of multiple decision-makers with potentially different preferences. Over the past few decades, there has been a notable increase in the use of structural frameworks for analyzing intrahousehold allocations of time and resources (see Chiappori and Mazzocco, 2017). In the absence of direct observation on "who gets what" in the household, such

*We thank Laurens Cherchye, Thomas Demuynck, Bram De Rock, and seminar participants in the RES Annual Conference, the Spring Meeting of Young Economists, the York-Durham Workshop, University of Antwerp, Bristol, Essex, Glasgow, Leuven, and Louvain-la-Neuve for many helpful suggestions.

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models help in understanding intrahousehold consumption patterns. Following Becker, 1973, researchers have often combined household bargaining analysis with a marital matching framework to exploit individuals' outside options as threat points. A series of recent studies have used a combination of the two theories of "who marries whom" and "who gets what" as a framework for empirical work (see, e.g., Cherchye et al., 2017; Goussé et al., 2017; Weber, 2017).

While most countries in the world allow their residents to divorce, the conditions under which divorce is permitted vary. The existing literature on matching models implicitly assumes a unilateral divorce rule, wherein any individual can end their marriage without requiring the spouse's consent. However, from a practical perspective, it is more likely to be the case that some (or all) couples are effectively governed by a mutual consent divorce rule. For instance, mutual consent for divorce may be required directly by law. Many societies consider marriage as means of solemn commitment and generally discourage divorce, either through law or through tradition, by only permitting divorce if both parties are willing to consent. This norm is still widely practiced around the world in countries such as Hungary, Turkey, India, the Czech Republic, Japan, South Korea, Paraguay, Slovenia, Taiwan and Vietnam. In these markets, legally married couples are committed as they need mutual consent for divorce while cohabitating couples are non-committed as they can dissolve their partnership without undergoing a formal divorce procedure. Even in countries that follow a unilateral divorce rule, mutual consent may be effectively required because of family laws related to children or the division of assets. For example, Russia allows unilateral divorce but requires mutual consent for couple with infants under one year old. In many countries, joint custody of children necessitates agreement between both parents on a co-parenting schedule. In several jurisdictions, there is no legal default if parents propose conflicting schedules. Thus, mutual agreement and consent for a co-parenting schedule are necessary before the couple can officially divorce. Similarly, the division of shared assets might be disputed if no default splitting rule exists.

Unlike much of the previous research, we focus on the case where some couples are committed.[1] We present a nonparametric structural characterization of household consumption under the assumption of marital stability with committed couples. We assume that committed couples can divorce only if both partners agree while non-committed couples can divorce unilaterally. In providing the empirical implications of marital stability, we consider two cases depending on whether committed couples can promise transfer to their current partners to ensure their agreement to divorce. Next, we provide theoretical insights into

[1] A notable exception is Sun and Yang, 2021, who consider a more general equilibrium concept in the context of non-transferable utility matching in the labor market.

the empirical bite of our characterizations. We start by investigating the issue of rationalizability of a dataset. Rationalizability corresponds to the necessary conditions such that a model could potentially be rejected by data. We find that if there is no price variation across counterfactual matches and if individual incomes are the same inside and outside marriages, then violations of the model can only come from non-committed couples. We also study whether the characterizations can be of use for identifying intrahousehold allocations. We show that indentifiability of intrahousehold allocations of committed couples requires the dataset to have price variation for private goods across potential matches.

To support our theoretical results, we conduct simulation exercises using a sample of Dutch household data drawn from the LISS panel. Since our theoretical findings suggest that the presence of committed couples can lead to low empirical bite of our stability conditions, our simulations focus on the case where all couples are committed. The simulations show that the theoretically necessary conditions for identification are also sufficient for the model with transfers. That is, if there is some variation in prices for private goods, our characterization of marital stability with transfers can obtain informative bounds on the intrahousehold allocations of committed couples.

A key ingredient of our method is the use of a revealed preference framework following the tradition of Afriat, 1967; Diewert, 1973, and Varian, 1982. Our revealed preference characterization of marital stability is nonparametric in nature: it does not need a prior functional specification of individual utilities. As such, this nonparametric approach avoids potentially erroneous conclusions due to wrongly specified parametric forms and, thus, allows for individual-level heterogeneity in preferences. Our results complement the revealed preference method developed by Cherchye et al., 2017, who derived the implications of stable marriages for household consumption under the implicit assumption of unilateral divorce. We extend this earlier work by characterizing marital stability with committed couples.

We emphasize two specific features of our framework. First, our model uses an imperfectly transferable utility (ITU) framework. We account for intrahousehold consumption transfers but do not assume perfectly transferable utility (TU). The latter assumption (TU) would require strict conditions on utility functions such as generalized quasilinear form. While appealing from a theoretical perspective, the TU framework imposes substantial restrictions on individuals' preferences which may not hold in general (see Chiappori and Gugl, 2020). Our framework contributes to the growing strand of literature that uses an imperfectly transferable utility case to model marriage markets (see Legros and Newman, 2007; Choo and Seitz, 2013; Chiappori, 2017; Galichon et al., 2019).[2] Second, we focus on static equilibrium

[2]There is also important literature on revealed preferences of matching in the non-transferable utility (NTU) and transferable utility (TU) cases (see Echenique, 2008; Echenique et al., 2013; Demuynck and

conditions for marital stability in a competitive frictionless marriage market (see Shapley and Shubik, 1971; Becker, 1973). The static nature of our model is a substantial simplification of real-world marital behavior. Intertemporal considerations and the ease with which one can meet potential partners are particularly relevant when analyzing household decisions with long-term consequences (such as fertility). Nonetheless, the equilibrium concept of marital stability that we consider provides a natural starting point from which to analyze individuals' marital and consumption behavior. It can be used as a building block for more advanced dynamic models (for a review, see Chiappori and Mazzocco, 2017).

The remainder of the paper is organized as follows. Section 2 introduces the structural components of the model and defines the notions of stable matching. Section 3 presents our main results. We provide revealed preference characterization of marital stability with and without transfers. We also provide theoretical results concerning the empirical content of the models. Section 4 presents the simulation analysis. Section 5 concludes. All proofs omitted in the text are collected in the Appendix.

2 Structural Components and Marital Stability

We begin by introducing the matching and household consumption setting that we consider for a given marriage market. Next, we defines stability of marriages.

Matching. Let M be a set of men and W be a set of women. We refer to a man as $m \in M$ and to a woman as $w \in W$. Denote by $\emptyset$ the option of staying alone as single. Let $\sigma : M \cup W \to M \cup W \cup \{\emptyset\}$ be a matching function describing who is married to whom and satisfying the following properties:

- $\sigma(m) \in W \cup \{\emptyset\}$ for every $m \in M$,
- $\sigma(w) \in M \cup \{\emptyset\}$ for every $w \in W$,
- $w = \sigma(m)$ if and only if $m = \sigma(w)$.

In what follows, we focus on married couples ($|M| = |W|$). This assumption of $|M| = |W|$ is not critical for our results but simplifies the exposition.

Data. Following the structure of typical datasets, we assume that aggregate consumption parameters in the observed households are known, while individual parameters are not.

Salman, 2022). However, this literature primarily focuses on testing the stability of the matching market rather than identifying the intrahousehold resource allocation.

Admittedly, some datasets like the Danish and Japanese data used by Bonke and Browning, 2009 and Lise and Yamada, 2019, contain detailed information at the individual level (e.g., assignable private consumption). Such information can be easily included in our framework through appropriately defined constraints.

Consumption. We consider two types of goods. The first type consists of goods consumed privately by the individual members. For a given couple (m, w), let $q_{m,w} \in \mathbb{R}^n_+$ be the vector of total private consumption and $q^m_{m,w}, q^w_{m,w} \in \mathbb{R}^n_+$ be the vectors of private consumption of man m and woman w, respectively. The second type of consumption consists of goods consumed publicly by household members. Let $Q_{m,w} \in \mathbb{R}^N_+$ denote the vector of public consumption. We assume that the vectors $q_{m,w}$ and $Q_{m,w}$ are known for the observed couples (i.e., when $w = \sigma(m)$), but not for the potential couples. Further, for all observed couples $(m, \sigma(m))$, we treat their individual shares of private consumption, $q^m_{m,\sigma(m)}$ and $q^{\sigma(m)}_{m,\sigma(m)}$, as unknowns that satisfy the adding up constraint,

$$q^m_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)} = q_{m,\sigma(m)}.$$

Incomes and Prices. Consumption decisions are made under budget constraints defined by prices and income. We assume that prices and income are observed for all potential matches, allowing us to establish the budget constraints for any potential outside option $(m, w) \in M \cup \{\emptyset\} \times W \cup \{\emptyset\}$. The income of a couple (m, w) is denoted by $y_{m,w} \in \mathbb{R}_{++}$, while the incomes of man m and woman w when single are denoted by $y_{m,\emptyset} \in \mathbb{R}_{++}$ and $y_{\emptyset,w} \in \mathbb{R}_{++}$, respectively. Next, let $p_{m,w} \in \mathbb{R}^n_{++}$ be the price vector of the private goods and $P_{m,w} \in \mathbb{R}^N_{++}$ be the price vector of the public goods faced by the couple (m, w).

Preferences. Individuals enjoy their private consumption and public consumption. We assume that every individual $i \in M \cup W$ is endowed with a continuous, monotone, and concave utility function $u^i : \mathbb{R}^n_+ \times \mathbb{R}^N_+ \to \mathbb{R}$.

Marital Stability. Next, we define the concept of marital stability with committed couples. Committed couples can divorce only upon the mutual agreement of both partners, while other couples can divorce unilaterally. Let us denote by $\mathcal{V} \subseteq M \times W$, where $m = \sigma(w)$, the set of **committed couples** (i.e., the set of couples who can only divorce upon mutual consent). Next, we define a **coalition** as a tuple $(S, \hat{\sigma})$, where $S \subseteq M \cup W \cup \{\emptyset\}$ is the set of members of the coalition and $\hat{\sigma} : S \to S$ is the matching function among the members of the coalition. We say that a coalition $(S, \hat{\sigma})$ is **permissible** if for every $i \in S$, if $(i, \sigma(i)) \in \mathcal{V}$, then $\sigma(i) \in S$. Intuitively, if a member of the set of committed couple is part of the coalition, then the spouse should also be a member of the coalition. This notion of permissible coalition is in close spirit to cooperative games on graphs (see Myerson, 1977).

Given a set of committed couples, we say that an observed matching is stable if there are no permissible blocking coalitions. Intuitively, a coalition is blocking if the members of the coalition can improve on the current matching allocation. To define this concept more formally, we start by defining blocking pairs. A potential couple (m, w) is called a weakly blocking pair if both m and w weakly prefer to marry each other than stay in their current marriages. Formally, $(m, w) \in (M \cup \{\emptyset\}) \times (W \cup \{\emptyset\})$ is a **weakly blocking pair** if there is a consumption bundle $(q^m_{m,w}, q^w_{m,w}, Q_{m,w})$ such that

$$p_{m,w}(q^m_{m,w} + q^w_{m,w}) + P_{m,w}Q_{m,w} \leq y_{m,w},$$

and

$$\begin{aligned} u^m(q^m_{m,w}, Q_{m,w}) &\geq u^m(q^m_{m,\sigma(m)}, Q_{m,\sigma(m)}) \\ u^w(q^w_{m,w}, Q_{m,w}) &\geq u^w(q^w_{\sigma(w),w}, Q_{\sigma(w),w}). \end{aligned}$$

A pair $(m, w) \in (M \cup \{\emptyset\}) \times (W \cup \{\emptyset\})$ is a **blocking pair** if it is weakly blocking and at least one of the partners (m or w) is strictly better off in the new match. That is, at least one of the two inequalities above is strict. A coalition $(S, \hat{\sigma})$ is **blocking** if every rematched couple $(m, \hat{\sigma}(m))$ is weakly blocking and at least one is blocking. One can think about the set of all stable matchings with committed couples as the core of the corresponding game with restricted coalition structure. That is, matchings that belong to the core are stable. We say that a matching is in the core if there exist well-defined individual utilities such that there is no permissible coalition that is blocking. Definition 1 states this formally.

Definition 1. *Given the set of committed couples $\mathcal{V}$, a matching σ is in the* **core** $(C^{\mathcal{V}})$ *if there are continuous, monotone and concave individual utilities*

$$u^m : \mathbb{R}^n_+ \times \mathbb{R}^N_+ \to \mathbb{R} \text{ and } u^w : \mathbb{R}^n_+ \times \mathbb{R}^N_+ \to \mathbb{R},$$

for every $m \in M$ and $w \in W$ such that there are no permissible blocking coalitions.

We remark that Pareto efficiency of within-household resource allocations follows directly from the fact that the matching is in the core. This is because the requirement of no permissible blocking coalitions applied on the observed couples implies that there cannot be another feasible household allocation that results in both spouses being better off and at least one spouse being strictly better off than the allocation $(q^m_{m,\sigma(m)}, q^{\sigma(m)}_{m,\sigma(m)}, Q_{m,\sigma(m)})$. Indeed, if an observed couple is not Pareto efficient, they would always be capable of forming a permissible coalition and would, therefore, block themselves. This aligns our model within the collective household literature, which was first introduced by Chiappori, 1988, 1992.

3 Revealed Preference Characterization

In this section, we present our main results. We start by discussing the dataset available to the researcher and the notion of rationalizability of the dataset. We also present a graph representation of the marriage market, which is used to express the revealed preference characterization. Next, we present necessary and sufficient revealed preference conditions for stable matching with committed couples. Finally, we discuss the empirical content of the revealed preference conditions.

Data and Rationalizability. We consider a dataset $\mathcal{D}$ with the following information,

- a matching function σ,
- a set of committed couples $\mathcal{V}$,
- household consumption for all matched couples $(q_{m,\sigma(m)}, Q_{m,\sigma(m)})$,
- prices and income of all potential pairs $(p_{m,w}, P_{m,w})$ and $y_{m,w}$.

Given dataset $\mathcal{D}$, we say that it is rationalizable by a stable matching if there exists a nonempty feasible set to a system of inequalities that are implied by the stability of the marriage market. The existence of a feasible solution is both necessary and sufficient for the observed household behavior and the economic framework to be consistent with each other.

Graph Representation. We introduce a description of the marriage market in terms of graph theory.[3] We consider finite, directed graphs of the form $G = (V, E)$, where V is the set of vertices and E is the set of edges. Given a dataset $\mathcal{D}$, we can present the marriage market as a directed graph as follows. Each observed couple in the data is a vertex and a directed edge in this graph represents a potential pair where the male of the outgoing vertex is matched with the female of the incoming vertex. For example, the edge from vertex $(m, \sigma(m))$ to vertex $(\sigma(w), w)$ would represent the potential pair (m, w). Next, we define a weight function, $A(\mathcal{D})$, for the edges representing the potential pairs. The weight of an edge

[3]Our graph theoretic interpretation differs from that used in the matching literature as we start under existent matching and look for potential blocking coalitions.

going from $(m, \sigma(m))$ to $(\sigma(w), w)$, which represents the potential pair (m, w), is denoted by $a_{m,w} \in A(\mathcal{D})$. We define the weight of each edge $a_{m,w}$ as follows:

$$a_{m,w} = p_{m,w}(q^m_{m,\sigma(m)} + q^w_{\sigma(w),w}) + P^m_{m,w} Q_{m,\sigma(m)} + P^w_{m,w} Q_{\sigma(w),w} - y_{m,w},$$

with

$$P^m_{m,w} + P^w_{m,w} = P_{m,w}.$$

The weight function $A(\mathcal{D})$ can be easily interpreted based on revealed preferences. First, the edge weight $a_{m,w}$ defines individual prices $P^m_{m,w} \in \mathbb{R}^N_+$ and $P^w_{m,w} \in \mathbb{R}^N_+$ which reflect the willingness-to-pay of m and w, respectively, for the public consumption in the allocation $(q^m_{m,w}, q^w_{m,w}, Q_{m,w})$. These prices must satisfy the Pareto efficiency condition of intrahousehold allocations, meaning that $P^m_{m,w} + P^w_{m,w} = P_{m,w}$, where $P_{m,w}$ is the market price of the public good. These prices can be seen as Lindahl prices that support the efficient consumption of the public good. Next, the edge weight $a_{m,w}$ specifies the income that the potential pair (m, w) would be left with if they would buy the bundle they consume in their current matches $(q^m_{m,\sigma(m)}, Q_{m,\sigma(m)})$ and $(q^w_{\sigma(w),w}, Q_{\sigma(w),w})$ with their budget conditions (prices $(p_{m,w}, P_{m,w})$ and income $y_{m,w}$). As we will show below, our revealed preference characterization of marital stability will impose specific restrictions on these edge weights $a_{m,w}$.

Finally, we introduce the notion of a path of remarriages and represent it as a path in the directed graph defined above. A path in a directed graph is a sequence of edges, which connects a sequence of vertices. A **path of remarriages** is defined by a set of agents $S = \{m_1, \ldots, m_{n-1}, \sigma(m_2), \ldots, \sigma(m_n)\}$ and a matching $\hat{\sigma} : S \to S$ such that $\hat{\sigma}(m_j) = \sigma(m_{j+1})$ for every $j = 1, \ldots, n-1$. We can represent this path of remarriages as a path in the above directed graph in which the sequence of vertices is $((m_1, \sigma(m_1)), (m_2, \sigma(m_2)), \ldots, (m_n, \sigma(m_n)))$ and the sequences of edges is $((m_1, \sigma(m_2)), (m_2, \sigma(m_3)), \ldots, (m_{n-1}, \sigma(m_n)))$. The set of edges in this path specify who is remarrying whom. In what follows, we will denote such a path as $P = ((m_1, (m_2, \sigma(m_2)), \cdots, (m_{n-1}, \sigma(m_{n-1})), \sigma(m_n))$. In the Appendix (Lemma 2), we demonstrate that every (permissible) path of remarriages corresponds to a permissible coalition and every permissible coalition contains a path of remarriages.

3.1 Revealed Preference Conditions

We now examine the rationalizability conditions for marital stability in the presence of committed couples. An individual may find outside options which are preferable than their current marriage, but the same may not be true for their current spouse. Thus, if the couple is committed there is an incentive for the individual who prefers to divorce to compensate the partner in order to make them to agree to divorce. Thus, it is important to consider the possibility that committed partners can promise transfers to one another in a case of divorce. We consider two cases depending on whether individuals can commit to post-divorce transfers. First, we present the rationalizability conditions when no transfers are allowed between current partners post-divorce. We refer to this case as *core without transfers*. Next, we examine the case when transfers between partners are allowed. This case is referred to as *core with transfers*.

Core without transfers. Core without transfers requires that there is no permissible blocking coalition, (i.e. a permissible coalition endowed with a rematching such that every pair is weakly blocking and at least one is blocking). Let us focus on a potential pair (m, w) in a coalition. Suppose that for a given consumption allocations $q^m_{m,\sigma(m)}$ and $q^w_{\sigma(w),w}$, and individual prices $P^m_{m,w}$ and $P^w_{m,w}$ (with $P^m_{m,w} + P^w_{m,w} = P_{m,w}$), we have that $a_{m,w} < 0$. This implies that,

$$p_{m,w}(q^m_{m,\sigma(m)} + q^w_{\sigma(w),w}) + P^m_{m,w}Q_{m,\sigma(m)} + P^w_{m,w}Q_{\sigma(w),w} < y_{m,w}.$$

In this scenario, if m and w were to form a couple, they could obtain a bundle of goods that costs less than their total potential income, $y_{m,w}$, and is at least as good as the bundles they are currently consuming. Therefore, (m, w) would be a blocking pair, as the match would result in both individuals being better off than they are in their current marriages.

Consider a path of remarriages $P = ((m_1, (m_2, \sigma(m_2)), \cdots, (m_{n-1}, \sigma(m_{n-1})), \sigma(m_n))$. If $a_{m_r,\sigma(m_{r+1})} \leq 0$ for all $r \in \{1, \cdots, n-1\}$, with at least one inequality being strict, then the path of remarriages P would specify a blocking coalition. Furthermore, this path of remarriages would correspond to a permissible coalition if either $m_1 = m_n$ or both $(m_1, \sigma(m_1))$ and $(m_n, \sigma(m_n)) \notin \mathcal{V}$. That is, if either the path of remarriages is a cycle of remarriages ($m_1 = m_n$) or couples at both ends of the path are not committed ($(m_1, \sigma(m_1))$ and $(m_n, \sigma(m_n)) \notin \mathcal{V}$). Both of these cases ensure that both spouses of any committed couple is in the coalition. Therefore, given a dataset $\mathcal{D}$, if the observed marriages are stable, there must exist feasible intrahousehold allocations within current marriages and individual prices such that if the edge weights along any path of remarriages P are negative and at least one is strictly negative, it must not be permissible. Following this logic, Definition 2 defines the concept of *path consistency*, and Theorem 1 states that path consistency is a necessary and sufficient condition for rationalizability with core without transfers.

Definition 2. *Given a dataset $\mathcal{D}$, $A(\mathcal{D})$ satisfies* **path consistency** *if, for all $m \in M$ and*

$w \in W$, there are

$$q^m_{m,\sigma(m)}, q^w_{\sigma(w),w} \in \mathbb{R}^n_+ \text{ and } P^m_{m,w}, P^w_{m,w} \in \mathbb{R}^N_{++}$$

with

$$q^m_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)} = q_{m,\sigma(m)} \text{ and } P^m_{m,w} + P^w_{m,w} = P_{m,w},$$

such that for every path of remarriages $P = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1}), \sigma(m_n))$ if

$$a_{m_j,\sigma(m_{j+1})} \leq 0 \text{ for all } 1 \leq j \leq n-1$$

with at least one inequality being strict then,

(i) $(m_1, \sigma(m_1)) \in \mathcal{V}$ or $(m_n, \sigma(m_n)) \in \mathcal{V}$, and

(ii) $m_1 \neq m_n$.

Theorem 1. *A dataset $\mathcal{D}$ is rationalizable with core without transfers if and only if $A(\mathcal{D})$ satisfies path consistency.*

Intuitively, the path consistency condition ensures that the observed data is rationalizable by a stable matching by requiring any path of remarriages that is blocking to be non-permissible. Condition (i) guarantees that the path of remarriages $P = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1}), \sigma(m_n))$ cannot start and end with uncommitted couples. Condition (ii) guarantees that it cannot be a cycle. If either of the conditions are violated, then the corresponding path of remarriages would correspond to a permissible blocking coalition.

Core with transfers. Next, we consider the situation where individuals can commit to transferring money to their current partners, thereby providing incentives to consent to divorce. Let us denote by $t_m \in \mathbb{R}$ the transfer from m to $\sigma(m)$ upon divorce. If t_m is positive (negative), m pays (receives) some money to (from) his current partner $\sigma(m)$ after divorce. Thus, the disposable income of a potential match (m, w) is $\hat{y}_{m,w} = y_{m,w} - t_m + t_{\sigma(w)}$. We assume that transfers (if any) are made only between spouses who belong to the set of committed couples. This is because if a couple can divorce unilaterally, there is no need to incentivize the partner to divorce. This implies that for any permissible coalition S, if $m \notin S$ or $\sigma(m) \notin S$, then $t_m = 0$.

Fix a set of consumption allocations and individual prices. Consider a path of remarriages $P = (m_1, (m_2, \sigma(m_2)) \cdots, (m_{n-1}, \sigma(m_{n-1})), \sigma(m_n))$ and a set of transfers t_{m_r} for all $r \in \{1, \cdots, n\}$. If, for the given set of transfers and path of remarriages, the following inequality holds for all $r \in \{1, \cdots, n-1\}$, with at least one being strict,

$$a_{m_r,\sigma(m_{r+1})} - t_{m_r} + t_{m_{r+1}} \leq 0,$$

then the path corresponds to a blocking coalition. Furthermore, the path would correspond to a permissible coalition if either $m_1 = m_n$ or $(m_1, \sigma(m_1))$ and $(m_n, \sigma(m_n)) \notin \mathcal{V}$. Summing the edge weights along the path implies

$$\sum_{r=1}^{n-1} a_{m_r,\sigma(m_{r+1})} < 0.$$

The transfers cancel out because for every committed couple their partner is also in the path. That is, if $-t_{m_r}$ appears in the sum corresponding to the potential pair $(m_r, \sigma(m_{r+1}))$, then $+t_{m_r}$ appears in the sum corresponding to the potential pair $(m_{r-1}, \sigma(m_r))$.

Given a dataset $\mathcal{D}$, if the observed marriages are stable, there must exist feasible intra-household allocations within current marriages and individual prices such that if the sum of edge weights along any path of remarriages P is strictly negative, it must not be permissible. Definition 3 defines this condition formally as path monotonicity. Clearly, path monotonicity is a necessary condition for rationalizability with core with transfers. Theorem 2 states that it is also sufficient.

Definition 3. *Given a dataset $\mathcal{D}$, $A(\mathcal{D})$ satisfies* **path monotonicity** *if, for all $m \in M$ and $w \in W$, there are*

$$q^m_{m,\sigma(m)}, q^w_{\sigma(w),w} \in \mathbb{R}^n_+ \text{ and } P^m_{m,w}, P^w_{m,w} \in \mathbb{R}^N_{++}$$

with

$$q^m_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)} = q_{m,\sigma(m)} \text{ and } P^m_{m,w} + P^w_{m,w} = P_{m,w},$$

such that for every path of remarriages $P = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1})), \sigma(m_n))$ if

$$\sum_{r=1}^{n-1} a_{m_r,\sigma(m_{r+1})} < 0$$

then,

(i) $(m_1, \sigma(m_1)) \in \mathcal{V}$ *or* $(m_n, \sigma(m_n)) \in \mathcal{V}$, *and*

(ii) $m_1 \neq m_n$.

Theorem 2. *A dataset $\mathcal{D}$ is rationalizable with core with transfers if and only if $A(\mathcal{D})$ satisfies path monotonicity.*

Remarks. Before moving on to the empirical content of the rationalizability conditions, three remarks are in order. First, the conditions presented in Definitions 2 and 3 are the path counterparts of two well-known conditions in the revealed preference literature: cyclical consistency (Afriat, 1967) and cyclical monotonicity (Rockafellar, 1970). In particular, the path consistency and path monotonicity conditions collapse to their cyclical counterparts if $\mathcal{V} = M \times W$ (i.e., when all couples are governed by mutual consent divorce). Second, the results of Cherchye et al., 2017 can be obtained as a corollary of Theorems 1 by setting $\mathcal{V} = \emptyset$ (i.e., when all individuals can divorce unilaterally). Third, the stability conditions and corresponding revealed preference conditions can be trivially modified if one wants to limit the size of blocking coalitions (or the length of the path of remarriages).

3.2 Empirical Content

We now focus on the empirical tractability of the models. In particular, we discuss the empirical bite of the revealed preference conditions to detect violations of marital stability and to identify the intrahousehold consumption allocations.

Rationalizability. Before we proceed, we discuss the nested structure of the stability conditions. If the observed household behavior is consistent with the rationality conditions with transfers, it would also be consistent with the rationality conditions without transfers. This means that data consistency with path consistency is a necessary condition for consistency with path monotonicity. Thus, for the discussion below, we focus on the marital stability conditions that allow transfers.

As discussed above, our setup considers a dataset in which the matching function, household consumption for all matched couples, and prices and income for all potential pairs are observed. While it is easy to observe the matching function and household consumption of matched couples in household surveys, prices and income for potential outside options are typically unknown. Therefore, empirical applications require making some assumptions about the prices and income that individuals would face in counterfactual matches.

Suppose we assume that individuals face the same prices for private and public goods across all potential matches and that household income is the sum of individual incomes, which are independent of their partner's income. For example, we can consider the household income to be the sum of individuals' labor income where individual wages are independent of whom one is matched with. Imputation of prices for every potential couple can be approached in a similar way. Such assumptions are consistent with the empirical setups used in existing applications of such frameworks (see e.g. Cherchye et al., 2017, 2020; Browning et al., 2021). We show that, under such a data setting, violation of marital stability cannot be due to coalitions formed only of committed couples.

To present the formal result, we define the notion of a **blocking cycle**. A path of remarriages $P = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1})), \sigma(m_n))$ forms a blocking cycle if $m_1 = m_n$ and every pair is weakly blocking and at least one is blocking. When transfers between committed spouses are allowed, a blocking cycle corresponds to case (ii) of Definition 3. Corollary 1 shows that, under the data structure defined above, any observed household behavior cannot imply blocking cycles of remarriages. That is, violation of marital stability cannot be due to blocking coalitions formed solely by committed couples. Thus, if the observed household behavior is rejected, it will be due to the presence of permissible blocking coalitions involving non-committed couples. As we show through simulations in Section 4, if prices and/or income vary across outside options, a dataset can create violations of the rationalizability conditions by generating blocking cycles.

Corollary 1. *Suppose transfers between partners are allowed. If, for every $m, m' \in M$ and $w, w' \in W$,*

(i) $p_{m,w} = p_{m',w'}$,

(ii) $P_{m,w} = P_{m',w'}$, *and*

(iii) $y_{m,w} = y_m + y_w$,

then no dataset $\mathcal{D}$ can contain a blocking cycle.

Identifiability. Revealed preference characterization of the collective model can be a useful tool for the identification of intrahousehold allocation. As the rationalizability conditions are linear in nature, they can be used to identify the unobserved parameters of household allocation (such as individual private consumption, Lindahl prices, or sharing rule). This is usually in the form of set identification where the identified set contains all the feasible values of the unobserved parameter that are consistent with the stability conditions. For example, we can identify female's private consumption by defining an upper (lower) bound by maximizing (minimizing) the linear function (q^w) subject to the rationalizability conditions. Cherchye et al., 2017 have demonstrated that the stability condition with $\mathcal{V} = \emptyset$ can tightly identify household allocations. Here we discuss the identifying power of the conditions in more general cases.

As previously discussed, if there is variation in prices and/or income across potential matches, it is possible for a dataset to violate revealed preference restrictions (in particular, to generate blocking cycles). Interestingly, however, this assumption is not sufficient to ensure that the model has identifying power. Corollary 2 shows that if prices for private goods are the same across outside options, then the stability conditions with transfers cannot identify intrahousehold allocations for committed couples. That is, even though the model might have empirical content (i.e., there are datasets that reject the model) and it can identify intrahousehold allocations for non-committed couples, it still fails to provide any identification for committed couples. In other words, we can assign any possible intrahousehold allocation for committed couples without violating the rationalizability restrictions. Once again, the nested structure of the models implies that whenever the stability conditions with transfers lack identifying power, the stability conditions without transfers also has no identifying power.

Corollary 2. *If $p_{m,w} = p_{m',w'}$ for every $m, m' \in M$ and $w, w' \in W$ and a dataset $\mathcal{D}$ is rationalizable with core with transfers, then the model has no identifying power for committed couples. Equivalently, if there exist (q^m, q^w, Q, P^m, P^w) such that*

$$q^m_{m,\sigma(m)}, q^w_{\sigma(w),w} \in \mathbb{R}^n_+ \text{ and } P^m_{m,w}, P^w_{m,w} \in \mathbb{R}^n_{++}$$

with

$$q^m_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)} = q_{m,\sigma(m)} \text{ and } P^m_{m,w} + P^w_{m,w} = P_{m,w},$$

and the dataset is rationalizable with core with transfers, then for any committed couple $(i, \sigma(i)) \in \mathcal{V}$, for any allocation $(\bar{q}^i_{i,\sigma(i)}, \bar{q}^{\sigma(i)}_{i,\sigma(i)}, \bar{Q}_{i,\sigma(i)})$ such that

$$\bar{q}^i_{i,\sigma(i)}, \bar{q}^{\sigma(i}_{i,\sigma(i)} \in \mathbb{R}^n_+ \quad \text{with} \quad \bar{q}^i_{i,\sigma(i)} + \bar{q}^{\sigma(i)}_{i,\sigma(i)} = q_{i,\sigma(i)}$$

there exist

$$\bar{q}^m_{m,\sigma(m)}, \bar{q}^w_{\sigma(w),w} \in \mathbb{R}^n_+ \text{ for all } m \neq i \text{ and } \bar{P}^m_{m,w}, \bar{P}^w_{m,w} \in \mathbb{R}^n_{++} \text{ for all } (m,w)$$

with

$$\bar{q}^m_{m,\sigma(m)} + \bar{q}^{\sigma(m)}_{m,\sigma(m)} = q_{m,\sigma(m)} \quad \text{with} \quad \bar{P}^m_{m,w} + \bar{P}^w_{m,w} = P_{m,w},$$

such that the dataset is rationalizable with core with transfers.

Corollary 2 suggests that to use marital stability as the identifying assumption for committed couples, it is necessary to have price variation for private goods across potential matches. In this context, we remark that price variation is typically required in revealed preference methods to gain identifying power (see, e.g., Varian, 1982; Beatty and Crawford, 2011; Cherchye et al., 2015). In the collective household literature, many methods that take a more traditional parametric approach also require price variation for identification (see, e.g., Browning and Chiappori, 1998; Chiappori and Ekeland, 2009; Browning et al., 2013). In our context of marriage markets, identification requires price variations across counterfactual matches. One way to accomplish this is by utilizing regional price differences. Differences in prices faced by different couples could arise in real life situations when individuals relocate for the purpose of marriage and encounter varying prices in different regions. Such price variation can be present, for example, due to differences in housing costs or local market structure (Smith, 2004; Nakamura et al., 2011).

4 Simulation Analysis

Through simulation exercises, we explore the empirical power of the stability conditions. We focus on three limiting cases: (i) when all couples are non-committed (i.e., $\mathcal{V} = \emptyset$); we refer to this case as unilateral divorce, (ii) when all couples are committed (i.e., $\mathcal{V} = M \times W$) and transfers between spouses are permitted; we refer to this case as mutual consent with transfers, and (iii) when all couples are committed (i.e., $\mathcal{V} = M \times W$) but transfers between spouses are not allowed; we refer to this case as mutual consent without transfers.

Data. Our analysis is based on a subset of households drawn from the LISS panel, a representative survey of households in the Netherlands conducted by CentERdata.[4] The survey collects rich data on economics and sociodemographic variables at both individual and household levels. For this application, we use a part of the sample analyzed by Cherchye et al., 2017. Their sample selection criteria consider individuals with or without children, working at least 10 hours per week in the labor market, and aged between 25 and 65. The sample contains 832 individuals comprising of 239 couples, 166 single males, and 188 single females. For the sake of the illustration and due to computational limitations, we focus on couples without children and use randomly selected 30 couples in each simulation.

In terms of the setup, we consider a labor supply setting where individuals choose their leisure, private consumption, and public consumption. In the data, we observe both aggregated household expenditure as well as some assignable expenditure. Following Cherchye et al., 2017, we use all expenditure information to form a Hicksian good with a price normalized to one. We assume that the non-assignable expenditure is equally divided between public and private consumption. For leisure, we take the price to be the individual's hourly wage.

[4] Although the Netherlands has a policy of allowing unilateral divorce, we choose to use this dataset for the illustration as our main aim is to explore the empirical tractability of the rationalizability conditions. Furthermore, this dataset has been utilized by numerous studies that employ the collective household model. In particular, Cherchye et al., 2017 use the same dataset to identify intrahousehold resource allocation in the context of unilateral divorce.

Stability Indices. The rationalizability conditions presented above are strict in nature. The observed household data would either satisfy the constraints or fail to find a feasible solution. In reality, household consumption behavior may not be exactly consistent with the model if, for example, the data contain measurement errors, there are frictions in the marriage market, or other factors, such as match quality, affect marital behavior. In such cases, it is useful to quantify the deviations of observed data from exactly rationalizable behavior. Following Cherchye et al., 2017, we evaluate the goodness-of-fit of a model using stability indices. These indices allow us to quantify the degree to which the observed behavior is consistent with the exactly rationalizable behavior.

Formally, we include a stability index $s_{m,w}$ in the edge weight $a_{m,w}$ corresponding to the outside option $(m, w) \in M \cup \{\emptyset\} \times W \cup \{\emptyset\}$. In particular, we redefine $a_{m,w}$ as

$$a_{m,w} = p_{m,w}(q^m_{m,\sigma(m)} + q^w_{\sigma(w),w}) + P^m_{m,w} Q_{m,\sigma(m)} + P^w_{m,w} Q_{\sigma(w),w} - y_{m,w} s_{m,w}.$$

Further, we add the restriction $0 \leq s_{m,w} \leq 1$. Imposing $s_{m,w} = 1$ results in the original rationalizability restrictions, while imposing $s_{m,w} = 0$ would rationalize any data. Intuitively, the stability indices measure the loss of post-divorce income needed to represent the observed marriages as stable. Generally, a lower stability index corresponds to a higher income loss associated with a particular outside option. This can be interpreted as a greater violation of the underlying model assumptions.

In our simulations, we will use stability indices to evaluate the empirical content of the model. To identify the values of stability indices for a given dataset $\mathcal{D}$, we compute

$$\max \sum_m \sum_w s_{m,w}$$

subject to the rationalizability conditions. This gives a stability index $s_{m,w}$ for each outside option. If the original constraints are satisfied, then there is no need for adjustment, and all stability indices will be equal to one. Otherwise, a strictly smaller index will be required to rationalize the behavior. In what follows, we will summarize the stability indices by computing the average value of all stability indices. Intuitively, the average stability index measures the average income loss required to make the market stable.

Simulation Setup. We consider three types of simulation setup. We randomly draw 30 couples to be used across all the scenarios. In each scenario, we introduce variation in prices and/or income and draw 100 instances of marriage markets from the full sample.

(1) **Prices**. In the first scenario, we introduce variation in prices of private and public goods across potential matches. To capture this, we assume that prices for both private and public Hicksian consumption are equal to one in existing marriages, while prices for potential marriages are randomly assigned as follows:

$$p_{m,f} = 1 + \alpha\epsilon \text{ where } \epsilon \sim U[-1,1] \text{ and } \alpha \in \{0\%, 5\%, 10\%, 15\%, 20\%, 25\%\},$$
$$P_{m,f} = 1 + \alpha\eta \text{ where } \eta \sim U[-1,1] \text{ and } \alpha \in \{0\%, 5\%, 10\%, 15\%, 20\%, 25\%\}.$$

(2) **Income**. In the second scenario, we introduce variation in household incomes, while prices of private and public goods are assumed to be the same. Empirically, such a variation could be present in the data if there is a marriage premium and individual incomes depend on the partners' characteristics (see Korenman and Neumark, 1991). To capture this, we assume that the household incomes in existing marriages are the sum of individual labor incomes, while household incomes in hypothetical marriages are determined as follows:

$$y_{m,f} = (y_m + y_f)(1 + \alpha\epsilon) \quad \text{where,}$$
$$\epsilon \sim U[-1,1] \text{ and } \alpha \in \{0\%, 5\%, 10\%, 15\%, 20\%, 25\%\}.$$

(3) **Prices and Income**. In the final scenario, we introduce variation in both prices and income. This combines the two variations shown above.

4.1 Rationalizability

We start by checking whether and to what extent the simulated household behavior in the three data scenarios would be consistent with the rationalizability conditions. We summarize the results by documenting the average stability indices across the 100 draws. As discussed above, stability indices are important indicators of the empirical content of the models. If a stability index is strictly below one, then the revealed preference conditions do have empirical content as formally the conditions imposed are violated. As we will show later, the presence of empirical content is necessary (though not sufficient) for the identification of household allocation.

Table 1 presents the results of this exercise. Panels A, B, and C correspond to the scenarios where we introduce variation in prices, income, and both prices and income, respectively. Each panel contains three rows for the three cases under consideration: unilateral divorce, mutual consent divorce without transfers, and mutual consent divorce with transfers. There are six columns corresponding to the different scenarios from the default $\alpha = 0\%$, where

	0%	5%	10%	15%	20%	25%
Panel A: prices						
unilateral	.9995	.9992	.9985	.9975	.9963	.9947
	.97 1 1	.95 1 1	.94 1 1	.93 1 1	.91 1 1	.89 1 1
mutual consent w/o transfers	1	1	1	1	1	1
	1 1 1	.98 1 1	.97 1 1	.97 1 1	.98 1 1	.97 1 1
mutual consent with transfers	1	.9999	.9993	.9984	.9971	.9958
	1 1 1	.97 1 1	.96 1 1	94 1 1	.91 1 1	.89 1 1
Panel B: income						
unilateral	.9995	.9956	.9876	.9787	.9698	.9610
	.97 1 1	.89 1 1	.86 1 1	.82 1 1	.76 1 1	.71 1 1
mutual consent w/o transfers	1	1	1	1	1	1
	1 1 1	1 1 1	.98 1 1	.96 1 1	.94 1 1	.95 1 1
mutual consent with transfers	1	.9962	.9880	.9790	.9702	.9613
	1 1 1	.93 1 1	.88 1 1	.82 1 1	.76 1 1	.73 1 1
Panel C: prices and income						
unilateral	.9995	.9954	.9870	.9776	.9684	.9591
	.97 1 1	.90 1 1	.86 1 1	.80 1 1	.74 1 1	.69 1 1
mutual consent w/o transfers	1	1	1	1	1	1
	1 1 1	.95 1 1	1 1 1	.96 1 1	1 1 1	1 1 1
mutual consent with transfers	1	.9961	.9877	.9783	.9693	.9601
	1 1 1	.92 1 1	.86 1 1	.79 1 1	.73 1 1	.67 1 1

Notes: The number in each cell is the mean value of the average stability indices across all the simulations in the given scenario. The three numbers below correspond to the minimum, median and maximum of average stability indices across simulations.

Table 1: Stability indices

the original prices and wages are used in the computation, to $\alpha = 25\%$ allowing for a large variation in the prices and/or incomes across potential matches.

As expected, Table 1 shows that the stability conditions under unilateral divorce are the most restrictive. In all scenarios, the observed household behavior is not exactly consistent with stability conditions. That is, we require strictly below one stability index to rationalize the observed consumption and marriages. This is true even when we assume that prices are the same across counterfactual matches and individual incomes inside and outside marriage are the same ($\alpha = 0\%$). By contrast, when prices and incomes are assumed to be the same across matches, the stability conditions under mutual consent divorce (with and without transfers) have no empirical power. In this setting, any household behavior is consistent with the path consistency and path monotonicity conditions, as shown by the entire distribution of average stability indices being one. This observation is consistent with the theoretical result shown in Corollary 1.

When prices and/or incomes are allowed to vary across the outside options, the stability conditions under mutual consent divorce do have some empirical content. Moreover, the larger the price and income variation, the harder it becomes for the data to satisfy the exact conditions. This fact is indicated by the lower average values of stability indices along the rows. Finally, given the nested structure of the models, the average stability indices are generally lower under mutual consent divorce without transfers than under mutual consent divorce with transfers. While most of the simulated behavior is consistent with the stability conditions under mutual consent divorce without transfers, a few simulations do not satisfy the exact restrictions. This is indicated by the minimum values in the distribution of average stability indices, which are strictly less than one.

4.2 Identifiability

Next, we consider whether the stability conditions can be used to identify the intrahousehold allocations. As the models are non-parametric in nature, the identification is partial. Specifically, the identified set includes all possible values of the unobserved parameter that are consistent with the stability conditions.

In what follows, we will focus on identifying the conditional sharing rule, which captures the individual private consumption shares $q^{m}_{m,\sigma(m)}$ and $q^{\sigma m}_{m,\sigma(m)}$ conditional upon the public consumption $Q_{m,\sigma(m)}$ (Browning et al., 2014). Formally, the conditional sharing rules of males and females are defined as

$$\eta^{w}_{\sigma(w),w} = p_{\sigma(w),w} q^{w}_{\sigma(w),w} \text{ and } \eta^{\sigma(w)}_{\sigma(w),w} = p_{\sigma(w),w} q^{\sigma(w)}_{\sigma(w),w},$$

where $\eta^{w}_{\sigma(w),w} + \eta^{\sigma(w)}_{\sigma(w),w}$ must add up to the total private expenditures of the household. The conditional sharing rule can be set identified through linear programming techniques. In particular, by maximizing (minimizing) $\eta^{w}_{\sigma(w),w}$ subject to our linear rationalizability conditions, we can obtain upper (lower) bounds on the conditional income shares. For the ease of interpretation, we represent the conditional shares as the fraction of aggregate household expenditure on private goods.

We will summarize the identifying power of the stability conditions by focusing on the difference between the upper and lower bounds. This will indicate the tightness of the identified set. We expect that a model with high identifying power will lead to tighter identification. The LISS panel data provide information on assignable private consumption, which we can call the naive bounds for private consumption shares. These bounds are not based on any model and can be directly inferred from the data. If the model fails to improve upon these naive bounds, then it indicates that the model has no identifying power.

	0%	5%	10%	15%	20%	25%
Panel A: prices						
unilateral	.0869	.0008	.0003	.0001	.0001	.0001
	0 .06 .52	0 0 .14	0 0 .53	0 0 .36	0 0 .24	0 0 .29
mutual consent w/o transfers	.6070	.6070	.6070	.6070	.6070	.6070
	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84
mutual consent with transfers	.6070	.0985	.0384	.0165	.0085	.0036
	.24 .61 .84	0 0 .84	0 0 .63	0 0 .63	0 0 .56	0 0 .56
Panel B: income						
unilateral	.0869	.0905	.0898	.0886	.0854	.0857
	0 .06 .52	0 .07 .63	0 .07 .63	0 .06 .63	0 .06 .63	0 .06 .63
mutual consent w/o transfers	.6070	.6070	.6070	.6070	.6070	.6070
	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84
mutual consent with transfers	.6070	.6070	.6070	.6070	.6070	.6070
	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84
Panel C: prices and income						
unilateral	.0869	.0000	.0000	.0000	.0000	.0000
	0 .06 .52	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
mutual consent w/o transfers	.6070	.6070	.6070	.6070	.6070	.6070
	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84	.24 .61 .84
mutual consent with transfers	.6070	.0000	.0000	.0000	.0000	.0000
	.24 .61 .84	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0

Notes: The number in each cell is the mean value of the difference between upper and lower bounds on female private consumption shares in the given scenario. The three numbers below correspond to the minimum, median and maximum of the distribution.

Table 2: Intrahousehold identification

Table 2 presents the results for the identification of the female shares of private consumption. The structure of the table is similar to Table 1. Panels A, B, and C correspond to the scenarios where we introduce variation in prices, income, and both prices and income, respectively. Each panel contains three rows for the three theories under consideration: unilateral divorce, mutual consent divorce without transfers, and mutual consent divorce with transfers. There are six columns corresponding to the different scenarios from the default $\alpha = 0\%$, where the original prices and incomes are used in the computation, to $\alpha = 25\%$ allowing for a large variation in the prices and/or incomes across potential matches.

In line with the results in Table 1, Table 2 shows that the stability conditions under unilateral divorce exhibit the highest identifying power. The average width of the identified bounds is the smallest from the unilateral divorce model. This is expected as this model is nested within both the mutual consent models. Even when prices and income are assumed to be the same across counterfactual matches, the stability conditions under unilateral divorce lead to tight identification with an average width of 8.69 percentage points. By contrast, the stability conditions under mutual consent divorce (with and without transfers) exhibit no identifying power if there is no variation in prices and incomes. The identified bounds are the same as the naive bounds.[5]

When we introduce variation in prices and/or income, the identification power of the unilateral model improves. With regards to mutual consent divorce, the stability conditions exhibit some identifying power if there is variation in prices and post-divorce transfers are allowed (see Panels A and C in Table 2). For example, we can see a significant improvement in the model's identifying power when there is a small price variation ($\alpha = 5\%$). The average width of the identified bounds reduces from 60.70 percentage points when $\alpha = 0\%$ to 9.85 percentage points when $\alpha = 5\%$. However, the model has no identifying power if there is income but no price variation. This is in line with our theoretical result in Corollary 2. Note that income variation does not significantly improve the identifying power even for the unilateral divorce model (Panel B in Table 2). Finally, having both price and income variation provides point identification for both unilateral divorce and mutual consent divorce with transfer models. This is true even when the degree of variation is small ($\alpha = 5\%$; see Panel C in Table 2).

[5] As noted above, the naive bounds are identified using the available information on assignable private consumption. For the 100 randomly drawn markets, the average width of the naive bounds is 0.6070. Note that the naive bounds are the same for all the scenarios as we use the same marriage markets for all the simulations across the three scenarios.

5 Concluding Remarks

We presented a novel framework to analyze rational household consumption under the assumption of marital stability with committed couples. The existing literature has primarily considered unilateral divorce, which assumes that all individuals can divorce their partners at will. We extended the framework by generalizing the setting and allowing some couple to be able to divorce only by mutual consent. We provided revealed preference characterization for cases when ex-partners are and aren't allowed to transfer money to each other upon divorce. The stability conditions are linear in nature and can be used to (partially) identify the intrahousehold resource allocation. The key features of such identification are that they (i) do not make parametric assumptions about utility functions, (ii) allow for individual heterogeneity of preferences, and (iii) use cross-sectional household data. We presented a theoretical analysis of the empirical content (whether a dataset can reject the model) and identifiability (whether intrahousehold consumption patterns can be non-trivially identified) of the models. Along these lines, we showed the necessary data conditions under which the models have empirical bite for rationalizability and identification of intrahousehold allocation for committed couples.

A Proofs

A.1 Lemmata

We start by showing the equivalence between a potential pair (m, w) being non-blocking and their edge weight $a_{m,w} \geq 0$. We prove this in Lemma 1.

Lemma 1. *A potential couple $(m, w) \in M \times W$ is (weakly) blocking if and only if for all*

$$q^m_{m,\sigma(m)}, q^w_{\sigma(w),w} \in \mathbb{R}^n_+ \text{ and } P^m_{m,w}, P^w_{m,w} \in \mathbb{R}^N_{++}$$

with

$$q^m_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)} = q_{m,\sigma(m)} \text{ and } P^m_{m,w} + P^w_{m,w} = P_{m,w},$$

we have,

$$a_{m,w} (\leq) < 0.$$

Proof. ($\Leftarrow$) Consider the potential couple $(m, w) \in M \times W$. On the contrary, suppose that for all feasible consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$, we have $a_{m,w} < 0$ but the couple is not a blocking pair. Consider the couple's decision problem.

Given that every couple takes a Pareto efficient decision (otherwise the couple would block themselves), we can restate their problem as follows.

$$\max_{q^m,q^w,Q} u^m(q^m,Q)+\mu u^w(q^w,Q)$$
$$\text{s.t. } p_{m,w}(q^m+q^w)+P_{m,w}Q\leq y_{m,w}$$

Taking the first-order conditions, we obtain

$$\begin{cases} \nabla_{q^m}u^m(q^m,Q)=\lambda_{m,w}p_{m,w} \\ \mu\nabla_{q^w}u^w(q^w,Q)=\lambda_{m,w}p_{m,w} \\ \nabla_Q u^m(q^m,Q)+\mu\nabla_Q u^w(q^w,Q)=\lambda_{m,w}P_{m,w} \end{cases}$$

Let

$$P^w_{m,w}=\mu\frac{\nabla_Q u^w(q^w,Q)}{\lambda_{m,w}} \text{ and } P^m_{m,w}=\frac{\nabla_Q u^m(q^m,Q)}{\lambda_{m,w}}.$$ [6]

We know that in a competitive marriage market setting, if (m,w) is not a blocking pair and the resulting Pareto frontier is continuous and strictly decreasing, then Alkan and Gale, 1990 implies that there is a $\mu>0$ such that the optimal consumption vector (q^m,q^w,Q) satisfies:

$$\begin{cases} u^m(q^m,Q)\leq u^m(q^m_{m,\sigma(m)},Q_{m,\sigma(m)}) \\ \mu u^w(q^w,Q)\leq \mu u^w(q^w_{\sigma(w),w},Q_{\sigma(w),w}) \end{cases}$$

Given that individual utilities are concave and monotone, we can obtain the following inequalities using the supergradients.

$$0\leq u^m(q^m_{m,\sigma(m)},Q_{m,\sigma(m)})-u^m(q^m,Q)\leq \lambda_{m,w}\left(p_{m,w}(q^m_{m,\sigma(m)}-q^m)+P^m_{m,w}(Q_{m,\sigma(m)}-Q)\right)$$

and

$$0\leq \mu u^w(q^w_{\sigma(w),w},Q_{\sigma(w),w})-\mu u^w(q^w,Q)\leq \lambda_{m,w}\left(p_{m,w}(q^w_{\sigma(w),w}-q^w)+P^w_{m,w}(Q_{\sigma(w),w}-Q)\right).$$

The couple's budget constraint implies,

$$p_{m,w}(q^m+q^w)+P^m_{m,w}Q+P^w_{m,w}Q=y_{m,w}.$$

[6]Since the utility functions are monotone (gradients are non-negative), and $\lambda_{m,w}\geq 0$ is a Lagrange multiplier, non-negativity of the constructed Lindahl prices is guaranteed.

Adding the two inequalities above and given the budget constraint, we obtain

$$0 \leq p_{m,w} q^m_{m,\sigma(m)} + P^m_{m,w} Q_{m,\sigma(m)} + p_{m,w} q^w_{\sigma(w),w} + P^w_{m,w} Q_{\sigma(w),w} - y_{m,w} \leq a_{m,w}$$

This is a contradiction.[7]

($\Rightarrow$) We need to show that if a couple (m, w) is blocking then for all feasible consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$, $a_{m,w} < 0$. Equivalently, we need to show that if there exist consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$ such that $a_{m,w} \geq 0$ then the couple (m, w) is not a blocking couple. Consider the consumption vectors $q^m_{m,\sigma(m)}, q^w_{\sigma(w),w}$ and individual prices $P^m_{m,w}, P^w_{m,w}$ such that $a_{m,w} \geq 0$. Following Browning et al., 2021, we will show that there exist utility functions under which the pair would not be blocking.[8] That is, we will construct continuous, monotone, and concave utility functions for every individual such that the observed matching would be stable under unilateral divorce. We present a candidate utility function below. The sufficiency (of the revealed preference conditions) in the proofs for Theorems 1 and 2 would use the very same candidate utility function.

For a vector x, let $[x]_j$ denote the j-th component of the vector. Given the Lindahl prices and individual consumption values that satisfy the no negative edges condition, let the numbers m and M be such that,

$$M > \max_{m \in M, w \in W, j \leq n, J \leq N} \{[p_{m,w}]_j; [P^m_{m,w}]_J; [P^w_{m,w}]_J\} \text{ and}$$
$$m < \min_{m \in M, w \in W, j \leq n, J \leq N} \{[p_{m,w}]_j; [P^m_{m,w}]_J; [P^w_{m,w}]_J\}.$$

Next, we define a supplementary function that we will use to construct individual utilities. Let v be defined as

$$v(x) = \begin{cases} Mx & \text{if } x \leq 0, \\ mx & \text{if } x > 0. \end{cases}$$

[7] If we consider that $a_{m,w} \leq 0$ but the potential couple is not a weakly blocking pair, then we would need to update the implications of Alkan and Gale, 1990 to ensure that at least one inequality should be strict. This is because, if for any $\mu > 0$, the conditions hold with equality signs, then for every point on the Pareto frontier, the utilities of m and w would be as high when they are matched to one another as the utilities they get in the current marriages. Thus, they would form a weakly blocking pair. Given this, one of the supergradient inequalities should also hold with strict inequality. Consequently, the sum of the inequalities should also hold as strict inequality, and therefore the resulting implication would be $0 < a_{m,w}$. This is a contradiction.

[8] In our definition of $a_{m,w}$, we can directly use the notion of "disposable" income ($\hat{y}_{m,w} = y_{m,w} - t_m + t_{\sigma(m)}$) which takes into account the post-divorce transfers if present.

Let the candidate utility function be defined as

$$\hat{u}^i(q, Q) = \sum_{j=1}^{n} v([q]_j - [q^i_{i,\sigma(i)}]_j) + \sum_{J=1}^{N} v([Q]_J - [Q_{i,\sigma(i)}]_J)$$

for every $i \in M \cup W$.

Consider the candidate utility function described above and on the contrary assume that the couple (m, w) is blocking. For this specification of utilities, we have

$$\hat{u}^m(q^m_{m,\sigma(m)}, Q_{m,\sigma(m)}) = 0,\ \hat{u}^w(q^w_{\sigma(w),w}, Q_{\sigma(w),w}) = 0$$

Given the assumption that the couple is blocking, it must be that

$$\begin{aligned} \hat{u}^m(q^m, Q) &\geq \hat{u}^m(q^m_{m,\sigma(m)}, Q_{m,\sigma(m)}) = 0, \\ \hat{u}^w(q^w, Q) &\geq \hat{u}^w(q^w_{\sigma(w),w}, Q_{\sigma(w),w}) = 0, \end{aligned}$$

with at least one strict inequality. For m, if $[q^m]_j > [q^m_{m,\sigma(m)}]_j$ (or $[Q]_J > [Q_{m,\sigma(m)}]_J$) then by construction of number m we know that $m([q^m]_j - [q^m_{m,\sigma(m)}]_j) < p_{m,w}([q^m]_j - [q^m_{m,\sigma(m)}]_j)$ (or, $m([Q]_J - [Q_{m,\sigma(m)}]_J) < P^m_{m,w}([Q]_J - [Q_{m,\sigma(m)}]_J)$). On the other hand, if $[q^m]_j \leq [q^m_{m,\sigma(m)}]_j$ (or, $[Q]_J \leq [Q_{m,\sigma(m)}]_J$), then by construction of M we know that $M([q^m]_j - [q^m_{m,\sigma(m)}]_j) < p_{m,w}([q^m]_j - [q^m_{m,\sigma(m)}]_j)$ (or, $M([Q]_J - [Q_{m,\sigma(m)}]_J) < P^m_{m,w}([Q]_J - [Q_{m,\sigma(m)}]_J)$). We can add up the inequalities over all goods and obtain,

$$p_{m,w}(q^m_{m,w} - q^m_{m,\sigma(m)}) + P^m_{m,w}(Q - Q_{m,\sigma(m)}) > 0.$$

Similarly, we can obtain the inequality for w,

$$p_{m,w}(q^w_{m,w} - q^w_{\sigma(w)}, w) + P^w_{m,w}(Q - Q_{\sigma(w),w}) > 0.$$

Combining the inequalities for m and w, we obtain

$$p_{m,w}(q^m_{m,w} - q^m_{m,\sigma(m)}) + P^m_{m,w}(Q - Q_{m,\sigma(m)}) + p_{m,w}(q^w_{m,w} - q^w_{\sigma(w),w}) + P^w_{m,w}(Q - Q_{\sigma(w),w}) > 0.$$

Given that the Lindahl prices add up to the market price of the public good ($P^m_{m,w} + P^w_{m,w} = P_{m,w}$) and that $p_{m,w}q^m_{m,w} + P_{m,w}Q + p_{m,w}q^w_{m,w} = y_{m,w}$, we can simplify the inequality above to obtain

$$p_{m,w}q^m_{m,\sigma(m)} + P^m_{m,w}Q_{m,\sigma(m)} + p_{m,w}q^w_{\sigma(w),w} + P^w_{m,w}Q_{\sigma(w),w} - y_{m,w} = a_{m,w} < 0,$$

which is a contradiction.[9] $\square$

It should be noted that in the sufficiency proof, the constructed utility for an individual is not specific to any potential blocking pair. This construction is therefore universal and can be applied to more intricate scenarios, such as proving the sufficiency aspect of rationalizing datasets under mutual consent divorce.

Next, we establish a connection between the concept of a permissible coalition and the idea of a path or cycle on the graph. In particular, a sequence of remarriages within a permissible coalition can be depicted as a path or cycle of remarriages. Note that a cycle or path of remarriages, by construction, represents a permissible coalition. We prove in Lemma 2 that the existence of a permissible coalition implies the existence of a permissible subcoalition that can be represented as a cycle or path of remarriages on the graph.

Lemma 2. *Let $S \subseteq M \cup W$ be a permissible coalition endowed with rematching $\hat{\sigma}$, then there is a path of remarriages, that is*

$$\hat{S} = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1})), \sigma(m_n))$$

such that

(i) $\hat{\sigma}(m_j) = \sigma(m_{j+1})$ *for all* $j \leq n-1$, *and*

(ii) either $m_n = m_1$ *or* $(m_1, \sigma(m_1)), (m_n, \sigma(m_n)) \notin \mathcal{V}$.

Proof. We prove the Lemma by construction. First, note that since $\hat{\sigma}$ is a matching, $|M \cup \{\emptyset\} \cap S| = |W \cup \{\emptyset\} \cap S|$. Moreover, since S is a permissible coalition, then an individual without their spouse in S cannot be a committed couple. In our construction, we consider two cases, (1) $\exists m \in S$ such that $\sigma(m) \notin S$ and (2) $\neg\exists m \in S$ such that $\sigma(m) \notin S$. The first case would correspond to a path of remarriages and the second one would correspond to a cycle of remarriages. For both cases, we show an algorithm to construct $\hat{S}$.

Case 1: $\exists m \in S$ such that $\sigma(m) \notin S$

Step 1:

Let $m_1 = m \in S$ for some $m \in S$ such that $\sigma(m) \notin S$.
(if there are multiple such m, pick one at random)

[9] A similar proof can be immediately extended to prove that if a couple is weakly blocking then $a_{m,w} \leq 0$, just by substituting appropriate inequalities.

Let $\sigma(m_2) = \hat{\sigma}(m_1) \in S$.
($\sigma(m_2)$ is always in S, otherwise $|M \cap S| \neq |W \cap S|$)

Let $\hat{S}_1 = (m_1, \sigma(m_2))$.

Step i for $i \geq 2$:

if $m_i \in S$, then

Let $\sigma(m_{i+1}) = \hat{\sigma}(m_i)$
($\sigma(m_{i+1})$ is always in S, otherwise $|M \cap S| \neq |W \cap S|$)
Let $\hat{S}_i = (\hat{S}_{i-1}, m_i, \sigma(m_{i+1}))$.

else if $m_i \notin S$

Let $\hat{S} = \hat{S}_{i-1}$.

Let $n = i$.

The algorithm takes finite time (with a finite number of agents) and returns a sequence $\hat{S}$ that satisfies all the conditions for being a path of remarriages.

Case 2: $\neg\exists m \in S$ such that $\sigma(m) \notin S$

Step 1:

Let $m_1 = m \in S$ for some M.

Let $\sigma(m_2) = \hat{\sigma}(m_1)$.
($\sigma(m_2) \in S$, otherwise $|M \cap S| \neq |W \cap S|$)

Step i for $i \geq 2$:

if $m_i \neq m_1$, then

Let $\sigma(m_{i+1}) = \hat{\sigma}(m_i)$
($\sigma(m_{i+1})$ is always in S, otherwise $|M \cap S| \neq |W \cap S|$)
Let $\hat{S}_i = (\hat{S}_{i-1}, m_i, \sigma(m_{i+1}))$.

else ($m_i = m_1$)

Let $\hat{S} = \hat{S}_{i-1}$.

Let $n = i$.

The algorithm takes finite time (with a finite number of agents) and returns a sequence $\hat{S}$ that satisfies all the conditions for being a path of remarriages. $\square$

A.2 Proof of Theorem 1

Proof. ($\Rightarrow$) Assume that the dataset is rationalizable with core without transfers but on the contrary $A(\mathcal{D})$ fails path consistency. That is, there is a path of remarriages $\hat{S} = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1}), \sigma(m_n))$ with either (i) $(m_1, \sigma(m_1)) \notin \mathcal{V}$ and $(m_n, \sigma(m_n)) \notin \mathcal{V}$, or (ii) $m_1 = m_n$, such that for all feasible consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$, $a_{m_j,\sigma(m_{j+1})} \leq 0$ for all $j \leq n-1$ with at least one inequality being strict. First, note that $\hat{S}$ is a permissible coalition. Given that the data is rationalizable with core, any permissible coalition cannot be blocking. This means that either (i) there exists at least one pair (denote by $(m_k, \hat{\sigma}(m_k))$ in $\hat{S}$ which is not weakly blocking, or (ii) none of the pairs in $\hat{S}$ are blocking. For (i), Lemma 1 implies that for the pair $(m_k, \hat{\sigma}(m_k))$, $a_{m_k,\hat{\sigma}(m_k)} > 0$. For (ii), Lemma 1 and the assumption above implies that $a_{m_j,\hat{\sigma}(m_j)} = 0$ for all $j \leq n-1$. Both of these lead to a contradiction.

($\Leftarrow$) Suppose that $A(\mathcal{D})$ satisfies path consistency. We show that the dataset can be rationalized using the candidate utility functions $\hat{u}^i$ described in the proof of Lemma 1. On the contrary, assume that the dataset is not rationalizable with core without transfers. This means that there is a permissible coalition such that every pair is weakly blocking and at least one pair is blocking. Lemma 2 implies that there is a path of remarriages $\hat{S}$, such that every pair is weakly blocking and at least one is blocking. From Lemma 1, we know that if there are consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$ such that

$$a_{m_j,\sigma(m_{j+1})} \geq 0 \text{ for every } m_j \in \hat{S},$$

then none of the pairs are blocking. Thus, given that path consistency is satisfied, the coalition cannot be blocking since there is no blocking pair and all pairs are weakly blocking. $\square$

A.3 Proof of Theorem 2

Proof. ($\Rightarrow$) Consider a dataset that is rationalizable with core with transfers. Assume on the contrary, that the data violate the path monotonicity condition. That is, there is a path of remarriages $\hat{S} = (m_1, (m_2, \sigma(m_2)), \ldots, (m_{n-1}, \sigma(m_{n-1}), \sigma(m_n))$ with either (i) $(m_1, \sigma(m_1)) \notin \mathcal{V}$ and $(m_n, \sigma(m_n)) \notin \mathcal{V}$, or (ii) $m_1 = m_n$, such that for all feasible consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$, $\sum_{j=1}^{n-1} a_{m_j,\sigma(m_{j+1})} < 0$. Note that $\hat{S}$ represents a permissible coalition. We will construct a sequence of transfers such that all rematches in $\hat{S}$ are weakly blocking and at least one is blocking. Let $\hat{t}_{m_1} = 0$

and $\hat{t}_{m_j} = a_{m_{j-1},\sigma(m_j)} + \hat{t}_{m_{j-1}}$ for $2 \leq j \leq n-1$. Given the construction, we have

$$a_{m_{j-1},\sigma(m_j)} + \hat{t}_{m_{j-1}} - \hat{t}_{m_j} = 0$$

for all $2 \leq j \leq n-1$. We need to show that

$$a_{m_{n-1},\sigma(m_n)} + \hat{t}_{m_{n-1}} - \hat{t}_{m_n} < 0.$$

If $(m_1, \sigma(m_1)), (m_n, \sigma(m_n)) \notin \mathcal{V}$, then $\hat{t}_{m_n} = \hat{t}_{m_1} = 0$ since there are no transfers going to and coming from individuals who do not belong to the coalition. Otherwise, if $m_1 = m_n$, then $\hat{t}_{m_1} = \hat{t}_{m_n} = 0$. Given this, showing $a_{m_{n-1},\sigma(m_n)} + \hat{t}_{m_{n-1}} - \hat{t}_{m_n} < 0$ is equivalent to showing

$$a_{m_{n-1},w_n} + \hat{t}_{m_{n-1}} < 0.$$

Recall that by construction

$$\begin{aligned}
a_{m_{n-1},\sigma(m_n)} + \hat{t}_{m_{n-1}} &= a_{m_{n-1},\sigma(m_n)} + a_{m_{n-2},\sigma(m_{n-1})} + \hat{t}_{m_{n-2}} \\
&= a_{m_{n-1},\sigma(m_n)} + a_{m_{n-2},\sigma(m_{n-1})} + a_{m_{n-3},\sigma(m_{n-2})} + \hat{t}_{m_{n-3}} \\
&\vdots \\
&= \sum_{j=1}^{n-1} a_{m_j,\sigma(m_{j+1})} + \hat{t}_1 \\
&= \sum_{j=1}^{n-1} a_{m_j,\sigma(m_{j+1})}
\end{aligned}$$

By violation of path monotonicity, we have

$$a_{m_{n-1},\sigma(m_n)} + \hat{t}_{m_{n-1}} < 0.$$

We have shown that there exist transfers such that, $a_{m_j,\hat{\sigma}(m_j)} + \hat{t}_{m_j} - \hat{t}_{\hat{\sigma}(m_j)} \leq 0$ for all $1 \leq j \leq n$, with at least one inequality being strict. By Lemma 1, this means that we have constructed a permissible coalition and a vector of transfers such that every pair in the coalition is weakly blocking with at least one pair being blocking. This implies that the coalition formed by $\hat{S}$ is a permissible blocking coalition, which is a violation of the fact that the dataset is rationalizable with the core with transfers.

($\Leftarrow$) Assume that path monotonicity is satisfied. We show that the dataset can be rationalized using the candidate utility functions $\hat{u}^i$ described in the proof of Lemma 1. Assume on the contrary that the dataset is not rationalizable. That is, there is a permissible coalition and vector of transfers such that every pair is weakly blocking and at least one pair is blocking. Given Lemmas 1 and 2, we know that there is a path of remarriages and transfers such that for all consumption vectors $(q^m_{m,\sigma(m)}, q^w_{\sigma(w),w})$ and individual prices $(P^m_{m,w}, P^w_{m,w})$,

$$a_{m_j,\sigma(m_{j+1})} + t_{m_j} - t_{m_{j+1}} \leq 0 \text{ for every } j \leq n-1$$

with at least one inequality being strict. Summing up the inequalities and canceling the transfers, we would obtain,

$$\sum_{j=1}^{n-1} a_{m_j,\sigma(m_{j+1})} + t_{m_1} - t_{m_n}.$$

Given that $\hat{S}$ is permissible, then either (i) $(m_1, \sigma(m_1)), (m_n, \sigma(m_n)) \notin \mathcal{V}$ or (ii) $m_1 = m_n$. In case (i), $t_{m_1} = 0$ and $t_{m_n} = 0$ since the transfers do not go out of the coalition. In case (ii), $t_{m_1} = t_{m_n}$ and thus the transfers cancel out. Hence, we obtain that

$$\sum_{j=1}^{n-1} a_{m_j,\sigma(m_{j+1})} + t_{m_1} - t_{m_n} = \sum_{j=1}^{n-1} a_{m_j,\sigma(m_{j+1})} < 0,$$

which is a violation of path monotonicity. □

A.4 Proof of Corollary 1

Proof. Let $p_{m,w} = p$ and $P_{m,w} = P$ as they are assumed to be the same across all couples. Let $P^m_{m,w} = P^m$ for all $m \in M$ and $P^w_{m,w} = P - P^m$. On the contrary, assume that there is a blocking cycle S. Consider the cycle of remarriages and focus on the potential pairs formed by m and $\sigma(m)$ for some $m \in S$. Let us denote the rematches as (m, w') and $(m', \sigma(m))$.

$$\begin{aligned}
\sum_{m \in S} a_{m,\hat{\sigma}(m)} &= \sum_{m* \neq m,m'} a_{m*,\hat{\sigma}(m*)} + a_{m,w'} + a_{m',\sigma(m)} \\
&= \sum_{m* \neq m,m'} a_{m*,\hat{\sigma}(m*)} + \\
&\quad p(q^m_{m,\sigma(m)} + q^{w'}_{\sigma(w'),w'}) + P^m Q_{m,\sigma(m)} + P^w Q_{\sigma(w'),w'} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + q^{\sigma(m)}_{m,\sigma(m)}) + P^m Q_{m',\sigma(m')} + P^w Q_{m,\sigma(m)} - y_{m',\sigma(m)}
\end{aligned}$$

Recall that $y_{m,w} = y_m + y_w$, then we can further rearrange the terms

$$\begin{aligned}
\sum_{m \in S} a_{m,\hat{\sigma}(m)} = &\sum_{m* \neq m,m'} a_{m*,\hat{\sigma}(m*)} + \\
&\quad p(q^m_{m,\sigma(m)} + q^{w'}_{\sigma(w'),w'}) + P^m Q_{m,\sigma(m)} + P^w Q_{\sigma(w'),w'} - y_m - y_{w'} +
\end{aligned}$$

$$
\begin{aligned}
& p(q^{m'}_{m',\sigma(m')} + q^{\sigma(m)}_{m,\sigma(m)}) + P^m Q_{m',\sigma(m')} + P^w Q_{m,\sigma(m)} - y_{m'} - y_{\sigma(m)} \\
= & \sum_{m* \neq m,m'} a_{m*,\hat{\sigma}(m*)} + \\
& p(q^{m}_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)}) + P^m Q_{m,\sigma(m)} + P^w Q_{m,\sigma(m)} - y_{m} - y_{\sigma(m)} + \\
& p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^m Q_{m',\sigma(m')} + P^w Q_{\sigma(w'),w'} - y_{m'} - y_{w'} \\
= & \sum_{m* \neq m,m'} a_{m*,\hat{\sigma}(m*)} + \\
& p(q^{m}_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)}) + P Q_{m,\sigma(m)} - y_{m,\sigma(m)} + \\
& p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^m Q_{m',\sigma(m')} + P^w Q_{\sigma(w'),w'} - y_{m'} - y_{w'} \\
= & \sum_{m* \neq m,m'} a_{m*,\hat{\sigma}(m*)} + \\
& p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^m Q_{m',\sigma(m')} + P^w Q_{\sigma(w'),w'} - y_{m'} - y_{w'}
\end{aligned}
$$

Thus, we have removed the terms corresponding to the observed couple $(m, \sigma(m))$ from the cycle of remarriages. Repeating the same procedure for other observed matches in S, we can obtain that the sum of edge weights along the cycle equals zero for every cycle of remarriages. Thus, path monotonicity along the cycle of remarriages is trivially satisfied, implying that the data cannot contain the blocking cycle. □

A.5 Proof of Corollary 2

Proof. Cherchye et al., 2017 and Browning et al., 2021 have shown that, in a static matching framework, the Lindahl prices $P^m_{m,\sigma(m)}$ and $P^{\sigma(m)}_{m,\sigma(m)}$ cannot be identified even if $\mathcal{V} = \emptyset$. This is because these prices are not constrained by the revealed preference conditions. As such, the identification of sharing rule is driven by the identification of individual private consumption shares. Given the nested structure of the models, the Lindahl prices remain unidentifiable if some couples are committed (i.e., when $\mathcal{V} \neq \emptyset$). We need to show that if prices for private goods are the same across counterfactual matches, the individual shares of private consumption cannot be identified either.

Consider a dataset which is rationalizable with core with transfers. Suppose $p_{m,w} = p$ for every $(m, w) \in M \cup \{\emptyset\} \times W \cup \{\emptyset\}$. As the dataset is rationalizable, there are individual consumption bundles $(q^m_{m,\sigma(m)}, q^{\sigma(m)}_{m,\sigma(m)})$ and Lindahl prices $(P^m_{m,w}, P^w_{m,w})$ such that there is no blocking path of remarriages. Consider a committed couple $(m, \sigma(m))$. Suppose there is another allocation $(\hat{q}^m_{m,\sigma(m)}, \hat{q}^{\sigma(m)}_{m,\sigma(m)})$ which makes the matching unstable. Then, according to Theorem 2, there is a violation of path monotonicity over a path of remarriages S that includes m and $\sigma(m)$. Consider this path of remarriages and focus on the potential pairs formed by m and $\sigma(m)$. Let us denote the rematches as (m, w') and $(m', \sigma(m))$. Violation of path monotonicity over this path implies,

$$
\begin{aligned}
0 &> \sum_{m \in S} a_{m,\hat{\sigma}(m)} \\
&> \sum_{m* \neq m,m'} a_{m*,w*} + \\
&\quad p(\hat{q}^{m}_{m,\sigma(m)} + q^{w'}_{\sigma(w'),w'}) + P^{m}_{m,w'} Q_{m,\sigma(m)} + P^{w'}_{m,w'} Q_{\sigma(w'),w'} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + \hat{q}^{\sigma(m)}_{m,\sigma(m)}) + P^{m'}_{m',\sigma(m)} Q_{m',\sigma(m')} + P^{\sigma(m)}_{m',\sigma(m)} Q_{m,\sigma(m)} - y_{m',\sigma(m)} \\
&> \sum_{m* \neq m,m'} a_{m*,w*} + \\
&\quad p(\hat{q}^{m}_{m,\sigma(m)} + \hat{q}^{\sigma(m)}_{m,\sigma(m)}) + P^{m}_{m,w'} Q_{m,\sigma(m)} + P^{\sigma(m)}_{m',\sigma(m)} Q_{m,\sigma(m)} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^{m'}_{m',\sigma(m)} Q_{m',\sigma(m')} + P^{w'}_{m,w'} Q_{\sigma(w'),w'} - y_{m',\sigma(m)} \\
&> \sum_{m* \neq m,m'} a_{m*,w*} + \\
&\quad p(q_{m,\sigma(m)}) + P^{m}_{m,w'} Q_{m,\sigma(m)} + P^{\sigma(m)}_{m',\sigma(m)} Q_{m,\sigma(m)} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^{m'}_{m',\sigma(m)} Q_{m',\sigma(m')} + P^{w'}_{m,w'} Q_{\sigma(w'),w'} - y_{m',\sigma(m)}
\end{aligned}
$$

At the same time, rationalizability of the data with $q^{m}_{m,\sigma(m)}$ and $q^{\sigma(m)}_{m,\sigma(m)}$ implies that

$$
\begin{aligned}
0 &\leq \sum_{m \in S} a_{m,\hat{\sigma}(m)} \\
&\leq \sum_{m* \neq m,m'} a_{m*,w*} + \\
&\quad p(q^{m}_{m,\sigma(m)} + q^{w'}_{\sigma(w'),w'}) + P^{m}_{m,w'} Q_{m,\sigma(m)} + P^{w'}_{m,w'} Q_{\sigma(w'),w'} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + q^{\sigma(m)}_{m,\sigma(m)}) + P^{m'}_{m',\sigma(m)} Q_{m',\sigma(m')} + P^{\sigma(m)}_{m',\sigma(m)} Q_{m,\sigma(m)} - y_{m',\sigma(m)} \\
&\leq \sum_{m* \neq m,m'} a_{m*,w*} + \\
&\quad p(q^{m}_{m,\sigma(m)} + q^{\sigma(m)}_{m,\sigma(m)}) + P^{m}_{m,w'} Q_{m,\sigma(m)} + P^{\sigma(m)}_{m',\sigma(m)} Q_{m,\sigma(m)} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^{m'}_{m',\sigma(m)} Q_{m',\sigma(m')} + P^{w'}_{m,w'} Q_{\sigma(w'),w'} - y_{m',\sigma(m)} \\
&\leq \sum_{m* \neq m,m'} a_{m*,w*} + \\
&\quad p(q_{m,\sigma(m)}) + P^{m}_{m,w'} Q_{m,\sigma(m)} + P^{\sigma(m)}_{m',\sigma(m)} Q_{m,\sigma(m)} - y_{m,w'} + \\
&\quad p(q^{m'}_{m',\sigma(m')} + q^{w'}_{\sigma(w'),w'}) + P^{m'}_{m',\sigma(m)} Q_{m',\sigma(m')} + P^{w'}_{m,w'} Q_{\sigma(w'),w'} - y_{m',\sigma(m)}
\end{aligned}
$$

That is a contradiction. Thus, the dataset is also rationalizable with the allocation $\hat{q}^{m}_{m,\sigma(m)}$

and $\hat{q}^{\sigma(m)}_{m,\sigma(m)}$ for $(m, \sigma(m))$ and $\hat{q}^{m}_{m,\sigma(m)} = q^{m}_{m,\sigma(m)}$ and $\hat{q}^{\sigma(m)}_{m,\sigma(m)} = q^{\sigma(m)}_{m,\sigma(m)}$ for all other couples. We can repeat this logic for the remaining committed couples in the path. $\square$

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