Multi-market contact, cartel and product quality

Neelanjan Sen*
Madras School of Economics
Gandhi Mandapam Road, Chennai-600025, Tamil Nadu, India

Uday Bhanu Sinha†
Department of Economics, Delhi School of Economics,
University of Delhi, Delhi-110007, India‡

September 9, 2023

Abstract

This paper develops a model of multi-market contact and collusion where product quality is chosen by firms in different markets. We show how multi-market contact and collusion can affect the quality choices adversely in the markets such that the availability of high-quality product reduces and the availability of low-quality product increases. There are two markets with identical demand which are geographically separated but served by both firms. Firms produce goods which are horizontally and vertically differentiated. It is shown that the firms can coordinate and collude during the quality choice stage, and as a result of this collusion in quality choice, the firms will under-invest in product quality improvement (and thereby save the fixed cost), leading to the availability of more low-quality products and the availability of less high-quality products in the market. We also show that when the fixed cost is low so that producing only the high-quality product is the unique Nash equilibrium, then welfare may increase if the firms form the cartel and either of the firms switch to the lower quality product (instead of producing, the higher quality product). Otherwise, after collusion, welfare always reduces.

JEL classification: L 24, L 13, D 43, D 12
Keywords: Collusion; Multi-market contact; Product Differentiation; Quality; Cournot

No conflict of interest statement: The authors declare that there is no conflict of interest.
Funding statement: No funding is received from any sources.

*Corresponding author, email address: neelu.sen@gmail.com; neelanjan@mse.ac.in
†email address: : uday@econdse.org; sinha.uday@gmail.com
‡The authors thank Rajit Biswas and Aditya Bhattacharjea for the valuable suggestions and comments. The usual disclaimer applies.

1 Introduction

Belleflamme and Peitz (2015) argues that although most competition laws forbid price-fixing agreements, explicit cartels continue to form and operate in a vast array of industries. In most of the cartel cases reported (detected), the firms involved had tried to reduce their joint output or had tried to increase the prices of the products or were involved in market-sharing agreements or bid rigging. There are very few cartel cases reported and thereby analyzed in the literature where the firms had colluded and fixed the quality of the product to be supplied by each firm in different markets. However firms may choose to form cartels that has bearing on the quality. OECD (2013) states "At the level of competition enforcement, the role of quality as a factor in assessing the legality of horizontal and vertical restraints is well established. Co-ordinated efforts between competitors to limit quality improvements or to degrade existing quality are generally most appropriately treated as equivalent to a cartel. ..... Typically, horizontal agreements to restrict innovation, limit product choice or degrade existing quality are treated as hard-core cartel offenses equivalent to price-fixing or market sharing." Although the majority of detected cartels have eliminated price competition, firms can occasionally decide to degrade the quality of the product and thereby reduce competition.[1] OECD (2013) reports two cases relevant in this context: i) Ordre des Pharmaciens of the Belgian Pharmacists Association (BPA)[2] and ii) Adanim Mortgage Bank, Ltd.[3] Moreover, OECD (2013) reports about Australia's express freight cartel, where three of Australia's transport companies and their senior executives colluded for approximately 20 years to fix prices and share the Australian express freight market. Interestingly, three of Australia's transport companies involved deliberately tried to lose or damage freight to encourage the customers to return to their original carrier. Therefore, the objective of the present paper is to model this behaviour and analyze the incentives of the firms in degrading the product quality and thereby forming a quality-fixing cartel when the firms have multi-market contacts. We show that even if the firms avoid price-fixing or quantity controls and/or market sharing, they can still collude and choose the quality of the products to be supplied by each firm in multiple markets.[4] This helps the firms to acquire more profits, though, it will hurt the consumers. On the policy front, we suggest that anti-trust authorities, should not only consider traditional forms of collusion but also should be vigilant about collusion in terms of "quality choice".

We explain this phenomenon using a model of multi-market contact and collusion, when only at the time of determining the product quality the firms collude. Thus, the objective of

[1] OECD (2013) also mentions that "Cartelists might agree not to introduce new products, for example, or they could agree not to offer certain services to their customers. Another possibility is that they could agree to restrict the hours during which their businesses are open."

[2] BPA maintained a set of rules with which its members were expected to comply regarding the schedule showing which pharmacies were to remain open (and therefore which ones were to be closed, as well) on any given weekend in various areas within Belgium.

[3] In Israel some banks tried to collude and close their branches on Fridays and undertake not to compete with each other in serving the public by opening the branches on that day.

[4] Mishra and Rao (2015) finds that anti-competitive practices are prevalent in the Indian telecom industry, with the leading operators engaging in practices such as predatory pricing, cartelization, and abuse of dominance. The study notes that these practices have negative effects on consumers, including higher prices, reduced choice, and poor quality of service.

the paper is to highlight collusion only via the choice of the quality of the product of each firm in different markets, even if the firms don't collude at the time of setting the quantities or the prices. It is shown in the paper, that due to multi-market contacts and collusion, the availability of high-quality products will reduce and the availability of low-quality products will increase. Thus, the firms may deliberately produce low-quality products (instead of producing high-quality products) to soften the competition in the market or the firms will under-invest in product quality improvement (and thereby save the fixed cost), which ultimately fetches more profits for the firms.

Our paper is related to the literature on multi-market contacts. Multi-market contacts arise when the same set of rival firms compete in multiple markets and these contacts are very common in the current business environment. Edwards (1955) observed that firms having multi-market contact will compete less aggressively as they will fear retaliation by rivals in all other markets. There have been various studies on multi-market contracts in different industries.[5] Many of these studies find evidence of multi-market contact leading to collusion and higher prices. In a seminal contribution, Bernheim and Whinston (1990) showed that multi-market contact might help firms sustain collusive outcomes in the context of a repeated game. This is possible as the firms can pool the incentive compatibility constraints for sustaining collusion across markets. Following Bernheim and Whinston (1990), the role of multi-market contact has been analyzed to understand the behaviour of international cartels.[6] Bhattacharjea and Sinha (2015) provide a general structure to analyze the international cartel involving the sphere of influence. Byford and Gans (2014) further extends the model of Bernheim and Whinston (1990) to include costly entry decisions of firms in different markets. Choi and Gerlach (2012) analyzes international cartel formation with multi-market contact in the presence of multinational firms. Sinha (2018) analyses how foreign direct investment may also facilitate collusion in an international context. All the above papers on multi-market contact and collusion have focused on price or quantity as the strategic variable, and their main finding is that multi-market contact could be anti-competitive and can raise prices.

An important question that arises in this context is: if multi-market contact and, thereby, collusion raises prices in different markets, should it not also affect the quality of products supplied by the firms in different markets? Some empirical works support the hypothesis that multi-market contact harms the quality of the products in the markets. Prince and Simon (2009) measures the on-time performance of airline as service quality and finds that multi-market contact increases delays. Lin and McCarthy (2023) examines the effects of multi-market contact on health insurance prices and quality using comprehensive data on the Medicare Advantage (MA) market from 2008 through 2015.[7] They show that the prices are significantly higher and

[5]Some well-known empirical studies on multi-market contacts are related to airlines (Evans and Kessides (1994) and Ciliberto and Williams (2010)); cement (Jans and Rosenbaum (1997)); mobile telephone (Parker and Röller (1997), Busse (2000)); radio advertisement (Waldfogel and Wulf (2006)); hotels (Fernandez and Marin, 1998); movie (Feinberg (2014)) and automobile market (Leheyda (2008)).

[6]See Lommerud and Sørgard (2001), Bond and Syropoulos (2008, 2012), Ashournia et al. (2013), Akinbosoye et al. (2012) and Colombo and Labrecciosa (2007). Before Bernheim and Whinston (1990), Pinto (1986) considered an infinite horizon model to show the possibility of collusion within the "sphere of influence" in the context of the international market.

[7]Lin and McCarthy (2023) measure quality based on the plan's overall star rating introduced by CMS in

high-quality plans less pervasive as multi-market contact increases. Moreover, it is shown by Lin and McCarthy (2023) that increasing multi-market contact reduces the prevalence of high-quality contracts, where a one standard deviation increase in multi-market contact leads to as much as a 10 per cent reduction in the percentage of high-rated contracts in a market (where high-rated is defined as receiving a star rating of 4 or more). Therefore, it is important to study theoretically how multi-market contacts facilitate collusion and thereby adversely affect the quality of the products available in all the markets where the firms meet.

Thus, in this article we try to understand how in the presence of product differentiation and multi-market contacts firms may under supply the high-quality products in the markets. In this paper, we develop a model of multi-market contact and collusion where product quality is chosen by firms in different markets. It is assumed that the firms will neither form any cartel in the quantity competition stage nor get involved in any market-sharing agreements. This is because the probability of detecting a cartel (quantity/price cartel) by the antitrust authority is extremely high or very high penalty to be imposed after detection (most restrictive regimes)[8]. In contrast, the firms can form cartels while deciding the quality of the products, as it is often less likely to be detected.[9] We show that multi-market contact and collusion can affect the quality choices adversely in the markets such that the availability of high-quality products reduces and the availability of low-quality products increases. To the best of our knowledge, this is the first paper to show that the product quality supplied by firms may be affected due to multi-market contact as it opens up the possibility of coordination and collusion between firms. In this model, there are two identical markets which are geographically separated. Each firm operates in both markets. Demand is identical in all markets, and goods are horizontally and vertically differentiated.

We consider the two-stage game, wherein in the first stage, the firms decide on the quality of goods that they want to supply in each market. Firms have the option of choosing either a low or a high-quality product based on the available technologies. The firm producing the high-quality product incurs a higher fixed cost than the other firm that produces the low-quality product (which is assumed to be zero in the model for simplicity). This may be due to the more advanced technology the high-quality product demands as well as for marketing/brand promotion of the high-quality product. In the second stage, they compete in quantities in both markets. In this simple setting, multi-market contact may play a crucial role in the quality choice stage. Firms might collude in the quality choice game even though they compete in quantity in the second stage. Thus, the firms can coordinate and collude during the quality choice in the first stage, and as a result of this collusion in quality choice, the firms will under-invest in product quality improvement (and thereby save the fixed cost), leading to the availability of more low-quality products and the availability of less high-quality products in the market.

In case the firms decide not to collude in the first stage, then: i) both the firms will produce the higher quality product in both markets if the fixed cost is relatively low or the quality

2009.

[8]See Hylton and Deng (2007).

[9]See OECD (2013) on the role and measurement of quality in competition analysis.

difference (net of cost) between the products is very high, ii) both the firms will produce the lower quality product in both the markets if the fixed cost is relatively high or the quality difference (net of cost) is very low and iii) the firms will produce different types of products in each market if the fixed cost is neither very high nor very low or the quality difference (net of cost) is neither very high nor very low.

It is shown that when the fixed cost is low so that producing only the high-quality product is the unique Nash equilibrium, if the horizontal product differentiation is low, then irrespective of the quality difference (net of cost), the firms will form the cartel and either of the firms should switch to the lower quality product (instead of producing, the higher quality product). Interestingly, it is also shown that there also exist cases where the firms will collude and produce only the low-quality product in both markets, when producing only the high-quality product is the unique Nash equilibrium under independent (strategic) quality choice. On the other hand, if the fixed cost is neither very high nor very low so that one firm produces the high-quality product and the other firm produces the low-quality product when the product quality is chosen independently (vice-verse in the other market), the firms can also collude and change the outcome. In such a situation if the quality difference (net of cost) is low, then the firms will form the cartel and produce only the low-quality product. Thus, due to multi-market contact, if the firms collude and choose the quality of the product jointly, the production of the lower-quality product will increase, and the production of the high-quality product will decrease. It is also observed that when the fixed cost is low so that producing only the high-quality product is the unique Nash equilibrium; then welfare will increase when the firms form the cartel and either of the firms switches to the lower quality product (instead of producing, the higher quality product) if the horizontal product differentiation is very low. Otherwise, after collusion, welfare always reduces.

The structure of the paper is as follows. Section 2 discusses the basic setup and the market structure to be used throughout this paper. Section 3 incorporates the Cournot-Nash equilibrium outcomes (when the firms compete in quantities). Section 4 discusses the strategic quality choice of the firms, followed by the choice of the quality of the products under collusion in the next Section 5. In Section 6, we summarize the final outcomes and thereby discuss when the firms will collude. We discuss the impact of the collusion on welfare in Section 7, and finally, we conclude.

2 Basic set-up

There are two markets, market A and market B. A and B can also be called two distinct geographical locations. In these markets, there are two Cournot competitive firms (firm i, i=1,2): firm 1 and firm 2, selling good 1 and good 2, respectively, in both markets. The goods are both horizontally and vertically differentiated. Moreover, we assume that the goods are non-tradable from one region to the other region.[10] This means that each firm should have two plants, one in region A and another in region B if it wants to operate in both markets. Thus, output produced

[10]It may be due to the nature of the good (service or perishable goods) or some regional barriers.

by firm i in region j is denoted as q_{ji}, where $j = A, B$, and the firms have multi-market contact.

The utility function of the representative consumers (consumer A for market A and consumer B for market B) in both markets is given by equation (1) following Hackner (2000) and Sen et al. (2021).

$$U_j(q_{j1}, q_{j2}, I_j) = \alpha_{j1}q_{j1} + \alpha_{j2}q_{j2} - \frac{1}{2}[q_{j1}^2 + q_{j2}^2 + 2\gamma q_{j1}q_{j2}] + I_j, j = A, B. \quad (1)$$

The consumer j buys two goods, 1 and 2 ($q_{ji}, i = 1, 2$) at price p_{ji} and a composite good marked as I_j. As in Symeonidis (1999), each firm's variety is described by a 'quality' index α_{ji}, which can represent either some physical characteristic or brand image. Hence, $\alpha_{j1} \neq \alpha_{j2}$ represents that there exists vertical product differentiation (or quality difference). The parameter $\gamma \in [0, 1]$ measures the substitutability between the products horizontally in each market (the lower the γ, the higher the degree of differentiation). If $\gamma = 0$, each firm has monopolistic market power, while if $\gamma = 1$, the products are perfect substitutes (provided $\alpha_{j1} = \alpha_{j2}$ also holds).

Consumer j maximizes utility subject to the budget constraint $\Sigma p_{ji}q_{ji} + I_j \leq M_j$, where M_j denotes income and the price of the composite good is normalized to one. The first-order condition determining the optimal consumption of good i in market j is,

$$\frac{\partial U_j}{\partial q_{ji}} = \alpha_{ji} - q_{ji} - \gamma q_{jk} - p_{ji} = 0, \ k \neq i, k = 1, 2. \quad (2)$$

There are two technologies available to the firms irrespective of the regions. They can either produce a higher quality product (good H) or a lower quality product (good L). Cost function of firm i is given by $C_H(q_{ji}) = c_H q_{ji} + F$ if the higher quality product is produced and $C_L(q_{ji}) = c_L q_{ji}$ if the lower quality product is produced, where $i = 1, 2$ and $c_H > c_L > 0$ (as well as $\alpha_{ji} > c_H$). This means that the marginal cost incurred for producing the higher (lower) quality product is more (less). The firm producing the higher quality product incurs a fixed-cost F. Without any loss of generality, we assume that the fixed cost is zero for the lower-quality product. Moreover, for simplicity, it is considered that the firms produce only one product in a single market, i.e. either the higher or the lower quality product in a specific market. This may be due to the quality check by some agency (e.g. Mobile connectivity and thereby the quality of voice calls, Safety ratings for cars) so that the firm can maintain a specific brand value, as selling products of different quality in the same market may create confusion among the consumers. The firms disclose the quality of the product publicly before deciding how much output to produce in each market via some acknowledged rating agency. Further, after announcing the quality of the product the firm wants to supply to the consumers, it can't deviate from the quality it has set as it violates the laws of the land and may face legal sanctions, which is too costly. Thus, quality announcement is publicly verifiable.[11]

The quality (α_{ji}) associated with the higher quality product is measured as α_H and the quality (α_{ji}) associated with the lower quality product is measured as α_L in the utility function (irrespective of the firms and the regions). It is also assumed that $0 < \theta = \frac{(\alpha_L - c_L)}{(\alpha_H - c_H)} < 1$, where

[11]There is no information asymmetry regarding product qualities in our model.

the ratio "θ" expresses the disadvantage of the firm that produces the low quality-product (as in Sen at al. (2021)), such that more is θ less is the disadvantage (ignoring the fixed-cost). A higher value of θ implies that the quality difference (net of cost) between the products is low (See Sen et al. (2021) for details).

2.1 Sequence of the game

The sequence of the game is as follows. In stage one, the firms decide which quality of product to produce in different markets, i.e. what quality in market A and market B separately. We call this stage as the "Quality selection stage". The firms may form a cartel in the quality selection stage. This we call as "Collusive quality choice". Otherwise, they may not collude and independently decide the quality of the products, which is called as "Strategic Quality choice". In the second stage, the firms compete, i.e. the Cournot game is played, and the profits are then realized. We call this stage the "Quantity competition stage". The game is solved using backward induction. Specifically, it is assumed that the firms will neither form any cartel in the quantity competition stage nor get involved in any market-sharing agreements, as in the present model the firms chose that quantities non-cooperatively in the second stage and determine the Nash-equilibrium quantities. We thus assume that the probability of detecting a cartel by the antitrust authority is extremely high, such that the firms do not form a cartel at the time of determining the output in each market as well as not forming any market-sharing agreements. The possibility of the cartel in the quantity competition stage can be considered in the future as an extension of this paper, and that may strengthen our results. However, in this paper, it is shown that even if firms can not collude at the time of determining the output, they may still collude at the time of determining the product quality in the first stage. We show that due to the presence of multi-market contact, collusive quality choice may occur and thus the supply of high-quality products will decrease, and the supply of low-quality products will increase after collusion. This is similar to the findings of Prince and Simon (2009) and Lin and McCarthy (2023) in the context of airline services and health insurance, respectively.

3 Quantity competition

Let us begin the analysis with the quantity competition game that is played in the second stage. In a single market (in market A as well as in market B), we can have either of the four outcomes specified as follows: i) both the firms produce the better quality product, ii) firm 1 produces the better quality product and firm 2 produces the lower quality product, iii) firm 2 produces the better quality product and firm 1 produces the lower quality product and iv) both the firms produce the lower quality products. We discuss each of these cases in the following sections.

The profit of firm i earned in market j, $j = A, B$, $i = 1, 2$, is defined as follows:

a) Π^j_{iHH}: if firm 1 produces good H and firm 2 produces good H,

b) Π^j_{iHL}: if firm 1 produces good H and firm 2 produces good L,

c) Π^j_{iLH}: if firm 1 produces good L and firm 2 produces good H and

d) Π^j_{iLL}: if firm 1 produces good L and firm 2 produces good L.

3.1 Only higher quality products are produced in a single market

Assume that only the better quality products are produced by both the firms in market A. Then the inverse demand function for good i can directly be determined from the equation (2) as

$$p_{Ai}(q_{Ai}, q_{Ak}) = \alpha_H - q_{Ai} - \gamma q_{Ak};\ k \neq i, k = 1, 2. \tag{3}$$

Therefore, the profit function of firm 1 is $\Pi^A_1 = [\alpha_H - q_{A1} - \gamma q_{A2} - c_H]q_{A1} - F$ and the profit function of firm 2 is $\Pi^A_2 = [\alpha_H - q_{A2} - \gamma q_{A1} - c_H]q_{A2} - F$.

From the profit functions, we get the reaction functions of firm 1 and firm 2 as

$$q_{A1} = \frac{\alpha_H - \gamma q_{A2} - c_H}{2} \quad and \quad q_{A2} = \frac{\alpha_H - \gamma q_{A1} - c_H}{2}$$

respectively. Solving the reaction functions, we find that the equilibrium quantities are

$$q^*_{A1} = q^*_{A2} = \frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}. \tag{4}$$

Thus, the firms' equilibrium profits in market A are

$$\Pi^A_{1HH} = \Pi^A_{2HH} = \left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F. \tag{5}$$

A similar result is observed in market B if the better quality products are produced by both the firms, i.e. $\Pi^B_{1HH} = \Pi^B_{2HH} = \Pi^A_{1HH} = \Pi^A_{2HH}$ and $q^*_{B1} = q^*_{B2} = q^*_{A1} = q^*_{A2}$ as shown in equation (4). We also assume that $\Pi^A_{1HH} = \Pi^A_{2HH} > 0$, thus throughout the paper we assume that

$$\left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 > F. \tag{6}$$

3.2 Both higher and lower quality products are produced in a single market

Let us assume that in region A, firm 1 sells the higher quality product and firm 2 sells the lower quality product. The inverse demand function for good 1 and good 2 in market A can directly be determined from the equation (2) as

$$p_{A1} = \alpha_H - q_{A1} - \gamma q_{A2} \quad and \quad p_{A2} = \alpha_L - q_{A2} - \gamma q_{A1} \tag{7}$$

respectively. The profit function of the firm 1 is $\Pi^A_1 = [\alpha_H - q_{A1} - \gamma q_{A2} - c_H]q_{A1} - F$ and the profit function of the firm 2 is $\Pi^A_2 = [\alpha_L - q_{A2} - \gamma q_{A1} - c_L]q_{A2}$.

The reaction functions of firm 1 and firm 2 are

$$q_{A1} = \frac{\alpha_H - \gamma q_{A2} - c_H}{2} \quad and \quad q_{A2} = \frac{\alpha_L - \gamma q_{A1} - c_L}{2}$$

respectively. Solving the reaction functions, we find that the equilibrium quantities are

$$q^*_{A1} = \frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)} \quad and \quad q^*_{A2} = \frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}. \tag{8}$$

We also assume, as in Sen et al. (2021) that

$$2(\alpha_L - c_L) - \gamma(\alpha_H - c_H) > 0 \quad or \quad 2\theta > \gamma \tag{9}$$

such that $q^*_{A2} > 0$. Thus, here $q^*_{A1} > q^*_{A2}$ and the firms' equilibrium profits in market A are[12]

$$\Pi^A_{1HL} = \left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 - F \quad and \quad \Pi^A_{2HL} = \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2. \tag{10}$$

Moreover, in market B, if firm 1 produces the higher quality product and firm 2 produces the lower quality product, then the profits are as follows: $\Pi^B_{1HL} = \Pi^A_{1HL}$ and $\Pi^B_{2HL} = \Pi^A_{2HL}$ and the quantities are $q^*_{B1} = q^*_{A1}$ and $q^*_{B2} = q^*_{A2}$ as shown in equation (8).

Similarly, if instead of firm 1, firm 2 produces the higher quality product in market A and firm 1 produces the lower quality product, we will have the equilibrium quantities as

$$q^*_{A2} = \frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)} \quad and \quad q^*_{A1} = \frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}. \tag{11}$$

Thus, the firms' equilibrium profits in market A are

$$\Pi^A_{2LH} = \left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 - F \quad and \quad \Pi^A_{1LH} = \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 \tag{12}$$

where $\Pi^A_{2LH} = \Pi^A_{1HL}$ and $\Pi^A_{1LH} = \Pi^A_{2HL}$. Moreover, in market B, if firm 2 produces the higher quality product and firm 1 produces the lower quality product, then the profits are as follows: $\Pi^B_{2LH} = \Pi^A_{2LH}$ and $\Pi^B_{1LH} = \Pi^A_{1LH}$ and the quantities are $q^*_{B2} = q^*_{A2}$ and $q^*_{B1} = q^*_{A1}$ as shown in equation (11).

3.3 Only lower quality products are produced in a single market

Assume that only the lower quality products are produced by both firms in market A. The inverse demand function for good i can directly be determined from the equation (2) as

$$p_{Ai}(q_{Ai}, q_{Ak}) = \alpha_L - q_{Ai} - \gamma q_{Ak};\ k \neq i, k = 1, 2. \tag{13}$$

The profit function of firm 1 is $\Pi^A_1 = [\alpha_L - q_{A1} - \gamma q_{A2} - c_L]q_{A1}$ and the profit function of firm 2 is $\Pi^A_2 = [\alpha_L - q_{A2} - \gamma q_{A1} - c_L]q_{A2}$.

[12] Assumption (6) also guarantees that $\Pi^A_{1HL} > 0$.

The reaction functions of firm 1 and firm 2 are

$$q_{A1} = \frac{\alpha_L - \gamma q_{A2} - c_L}{2} \quad and \quad q_{A2} = \frac{\alpha_L - \gamma q_{A1} - c_L}{2}$$

respectively. Solving the reaction functions, we find that the equilibrium quantities are

$$q^*_{A1} = q^*_{A2} = \frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}. \tag{14}$$

Thus, the firms' equilibrium profits in market A are

$$\Pi^A_{1LL} = \Pi^A_{2LL} = \left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2. \tag{15}$$

A similar result is observed in market B if the lower quality products are produced by both the firms, i.e. $\Pi^B_{1LL} = \Pi^B_{2LL} = \Pi^A_{1LL} = \Pi^A_{2LL}$ and the quantities are $q^*_{B1} = q^*_{B2} = q^*_{A1} = q^*_{A2}$ as shown in equation (14).

This ends the discussion of the quantity-competition game played in stage 2. The profits derived in this section, is now used in the following two sections to determine the final outcomes for the "Strategic Quality choice" as well as for the "Collusive quality choice". In the following section (Section 4) we first discuss about the "Strategic Quality choice" and later in section 5, the "Collusive quality choice" is discussed.

4 Strategic Quality choice

In the first stage of game firms can either collude or not collude. In this section, we discuss the selection of the quality of the products by the firms in the first stage of the game independently. Thus, they do not collude and strategically decide the quality of the products. In the following tables, the outcomes and the payoffs of the firms are depicted for market A and market B, respectively. Note that in each market, the firms can choose the better quality product (good H) or the lower quality product (good L) either non-cooperatively or by forming a cartel. Thus, the firms can form a cartel and decide the type of goods to be produced by both firms in each market jointly. Now in the present section, we determine the Nash equilibrium under independent (strategic) quality choice, and later in the following section (Section 5), we determine the outcome under collusive quality choice separately.

Profits

Market A		Firm 2	
		H	L
Firm 1	H	Π^A_{1HH}, Π^A_{2HH}	Π^A_{1HL}, Π^A_{2HL}
	L	Π^A_{1LH}, Π^A_{2LH}	Π^A_{1LL}, Π^A_{2LL}

Market B		Firm 2	
		H	L
Firm 1	H	Π^B_{1HH}, Π^B_{2HH}	Π^B_{1HL}, Π^B_{2HL}
	L	Π^B_{1LH}, Π^B_{2LH}	Π^B_{1LL}, Π^B_{2LL}

4.1 HH as the Nash equilibrium in both markets

We observe that in this stage, HH is the Nash equilibrium if

i) $\Pi^j_{1HH} > \Pi^j_{1LH}$ for $j = A, B$ and

ii) $\Pi^j_{2HH} > \Pi^j_{2HL}$ for $j = A, B$.

This is true if

$$\left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F > \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2. \tag{16}$$

This means that for HH to be the Nash equilibrium, we should have

$$\underline{f} = (2-\gamma)^2 - (2\theta - \gamma)^2 > f \tag{17}$$

where $f = F(4-\gamma^2)^2/(\alpha_H - c_H)^2$.

In this paper, our objective is to explain everything in terms of the horizontal product differentiation and quality difference (net of cost), where γ and θ are the inverse measures of horizontal product differentiation and quality difference (net of cost) respectively. Thus, without any loss of generality, we have assumed $\alpha_H - c_H = 1$ and $F = 0.1$ for drawing the diagrams, where we measure θ in the x-axis and γ in the y-axis, but all the results are proved using the equations as shown in equation (17). The above condition (17) holds only in zone HH of Figure 1, such that θ is very low or the quality difference (net of cost) is very high. Thus, HH is the equilibrium if the relative inefficiency in terms of producing the lower quality product is very high (θ is very low) and that gives more incentives to the firms to produce the better quality product, even if the production of the higher quality product involves fixed-cost (F). From condition (17), this also means that for any F, if θ increases or the quality difference (net of cost) between the two products is reduced, then $\underline{f}$ will fall. Thus, for higher θ so that we are either in zone HL or in zone LL of Figure 1, where $\underline{f} \leq f$, HH will not be the Nash equilibrium. This happens as then the relative inefficiency in terms of producing the lower quality product reduces, and that gives more incentives to the firms to produce the lower quality product. We thus argue that HH is the Nash equilibrium so that both the firms produce the higher quality product in market A and market B if the fixed cost is relatively low ($\underline{f} > f$ using equation (17)) or the quality difference (net of cost) is very high.

4.2 LL as the Nash equilibrium in both markets

We observe that in this stage, LL is the Nash equilibrium if

i) $\Pi^j_{1HL} < \Pi^j_{1LL}$ for $j = A, B$

ii) $\Pi^j_{2LH} < \Pi^j_{2LL}$ for $j = A, B$.

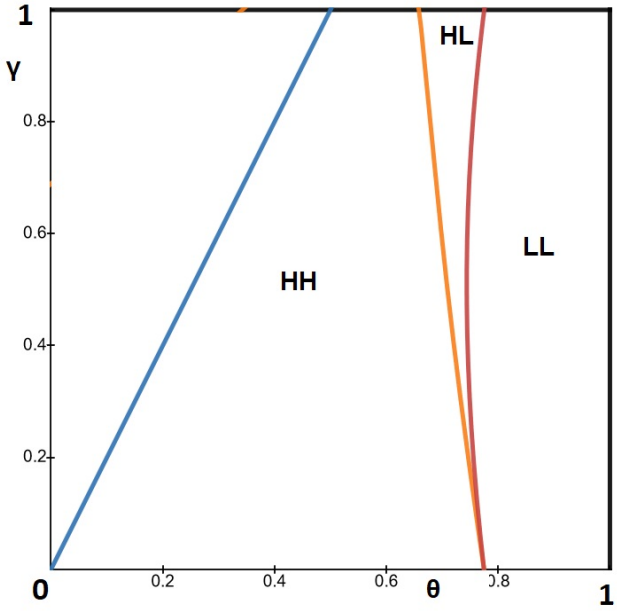

Figure 1: Nash Equilibrium: Strategic Quality choice

This is true if

$$\left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 - F < \left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2. \tag{18}$$

This means that for LL to be the Nash equilibrium, we should have

$$\overline{f} = (2-\gamma\theta)^2 - (2-\gamma)^2\theta^2 < f \tag{19}$$

as $\underline{f} < \overline{f}$ and $f = F(4-\gamma^2)^2/(\alpha_H - c_H)^2$. The above condition (19) holds only in zone LL of Figure 1. This is true for any F if θ is very high or the quality difference (net of cost) between the two products is very low, as $\overline{f}$ falls in θ. This happens as then the relative inefficiency in terms of producing the lower quality product becomes relatively very insignificant, and that gives more incentives to the firms to produce the lower quality product, as the firms can save the fixed cost of producing the better quality product. We therefore argue that LL is the Nash equilibrium so that both the firms produce the lower quality product in market A and market B if the fixed cost is relatively high ($\overline{f} < f$ using equation (19)) or the quality difference (net of cost) is very low.

4.3 HL and LH as Nash equilibria in both markets

We observe that in this stage, there is also the possibility of multiple equilibria of the form HL and LH if

i) $\Pi^j_{1HL} > \Pi^j_{1LL}$ and $\Pi^j_{2HL} > \Pi^j_{2HH}$, $j = A, B$ and

ii) $\Pi^j_{1LH} > \Pi^j_{1HH}$ and $\Pi^j_{2LH} > \Pi^j_{2LL},\ j = A, B.$

This is true if

$$\left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 - F > \left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 \tag{20}$$

and

$$\left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 > \left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F. \tag{21}$$

After simplifying, we observe that it is true if

$$\underline{f} = (2-\gamma)^2 - (2\theta - \gamma)^2 < f \ \ and \ \ \overline{f} = (2-\gamma\theta)^2 - (2-\gamma)^2\theta^2 > f. \tag{22}$$

This means that HL and LH are the Nash equilibria in both markets if

$$f \in [\underline{f}, \overline{f}] \ \ as \ \ \underline{f} < \overline{f}. \tag{23}$$

The above condition (23) holds only in zone HL of Figure 1. This is true for any F if θ is moderate or the quality difference (net of cost) between the two products is neither very low nor very high. This happens as then the relative inefficiency in terms of producing the lower quality product is neither very low nor very high, and that gives more incentives to the firms to produce the lower quality product, corroborated by the fact that then one of the firms can save the fixed-cost of producing the better quality product. On the other hand, it also gives higher incentives to the other firm in the market to produce better quality product. Thus, if the fixed-cost (F) is neither very high nor very low, i.e. $f \in [\underline{f}, \overline{f}]$, then in one market say market A firm 1 will produce the high-quality good and firm 2 will produce the low-quality good. Thus, in these circumstances, both firms will neither produce the high-quality product nor will they produce the low-quality good.

However, as $\Pi^j_{1HL} > \Pi^j_{1LH}$ and $\Pi^j_{2HL} < \Pi^j_{2LH},\ j = A, B$, each firm will get more profit if it produces the better quality product and the other firm produces the lower quality product in a single market if $f \in [\underline{f}, \overline{f}]$. Thus using the idea of focal equilibrium, we can argue that the firms in this stage will assign the products in the following way if $f \in [\underline{f}, \overline{f}]$:

i) Firm 1 will produce good H and firm 2 will produce good L in market A and firm 1 will produce good L, and firm 2 will produce good H in market B or

ii) Firm 2 will produce good H and firm 1 will produce good L in market A and firm 2 will produce good L, and firm 1 will produce good H in market B.

4.4 Results

From the above discussion, we come to the following lemma.

Lemma 1
i) HH is the Nash equilibrium so that both the firms produce the higher quality product in market A and market B if the fixed cost is relatively low ($\underline{f} > f$) or the quality difference (net of cost) is very high.

ii) LL is the Nash equilibrium so that both the firms produce the lower quality product in market A and market B if the fixed cost is relatively high ($\overline{f} < f$) or the quality difference (net of cost) is very low.

iii) HL and LH are the Nash equilibria in both markets so that the firms produce different types of product in each market if the fixed cost is neither very high nor very low ($f \in [\underline{f}, \overline{f}]$) or the quality difference (net of cost) is neither very high nor very low.

5 Collusive quality choice

In the first stage firms may or may not form the cartel. In this section it is shown that rather if the firms collude and decide the type of goods to be produced by both of them in each market jointly, then they can change the outcome as determined in the previous section (Section 4) where they determine the product qualities strategically. In the following sub-sections the role of horizontal product differentiation or γ and quality difference (net of cost) or θ in determining the possibility of collusive quality choice is discussed.

5.1 Explicit Cartel: HL/LH chosen instead of HH

In this section, we show that if $\underline{f} > f$ so that HH is the unique Nash equilibrium, i.e. we are in zone HH of Figure 1, firms can form a cartel and not produce the better quality product (good H) in both the market. Thus for the firms, it is sometimes optimal to decide the product jointly (thus forming a cartel) in the following way if $\underline{f} > f$: i) Firm 1 will produce good H and firm 2 will produce good L in market A and firm 1 will produce good L and firm 2 will produce good H in market B or ii) Firm 2 will produce good H and firm 1 will produce good L in market A and firm 2 will produce good L and firm 1 will produce good H in market B. Throughout the paper, we also assume that if either of the firms deviates from this specific strategy at the time of announcing the product quality, then in that period itself, they immediately come back to the Nash-equilibrium strategy.

For this to hold, we should have the following respectively:
i) $\Pi^A_{1HH} + \Pi^B_{1HH} < \Pi^A_{1HL} + \Pi^B_{1LH}$ and $\Pi^A_{2HH} + \Pi^B_{2HH} < \Pi^A_{2HL} + \Pi^B_{2LH}$ or
ii) $\Pi^A_{1HH} + \Pi^B_{1HH} < \Pi^A_{1LH} + \Pi^B_{1HL}$ and $\Pi^A_{2HH} + \Pi^B_{2HH} < \Pi^A_{2LH} + \Pi^B_{2HL}$.

We argue, using equation (5) and equation (10), that it is possible if

$$2\left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - 2F < \left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 + \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F. \tag{24}$$

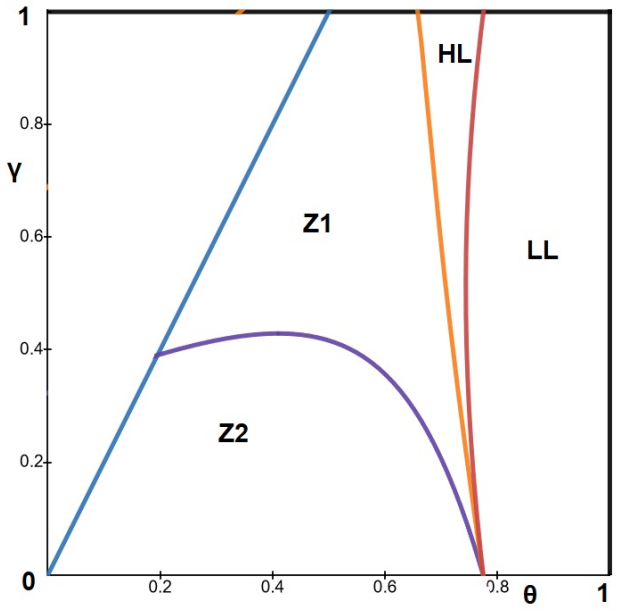

Figure 2: Quality Cartel HL or LH

or

$$2(2-\gamma)^2 - (2-\gamma\theta)^2 - (2\theta-\gamma)^2 < f. \tag{25}$$

given $\underline{f} = (2-\gamma)^2 - (2\theta-\gamma)^2 > f = F(4-\gamma^2)^2/(\alpha_H - c_H)^2$.

Let us discuss how the possibility of the cartel depends on the level of horizontal and vertical product differentiation, i.e. on γ and θ, respectively. Thus, we concentrate on condition (25), and without any loss of generality, assume $\alpha_H - c_H = 1$ as stated before. Therefore, condition (25) can be written as

$$Z = \frac{2(2-\gamma)^2 - (2-\gamma\theta)^2 - (2\theta-\gamma)^2}{(4-\gamma^2)^2} < F. \tag{26}$$

The above condition (26) holds only in zone $Z1$ of Figure 2, which is a subset of zone HH of Figure 1 for $F = 0.1$ as stated before. However, it is not possible in zone $Z2$ of Figure 2, which is also a subset of zone HH of Figure 1. The L.H.S. of condition (26) i.e. $Z = \frac{(1-\theta)(4+4\theta-8\gamma+\gamma^2+\gamma^2\theta)}{(4-\gamma^2)^2}$, falls in γ.[13] This means that when γ is high or the horizontal product differentiation is less, then producing the high-quality product by both the firms in the same market is less profitable. This is because when the horizontal product differentiation is less, the high-quality products produced by the two firms face more competition, and the firms have to incur the fixed cost (F) separately. Thus, when γ is high or the horizontal product differentiation is less, either of the firms should switch to the lower quality product (instead of producing the higher quality product) which also allows the firm to save fixed costs. It also allows the other firm that produces the higher quality product to have more profit in comparison to the situation when

[13] We observe that $\frac{dZ}{d\gamma} = \frac{2(12\gamma+12\gamma\theta-16+\gamma^3-\gamma^2\theta-12\gamma^2+3\gamma^3\theta)}{(4-\gamma^2)^3} < 0$.

both the firms produce the higher quality products. Thus, if γ is relatively high or the horizontal product differentiation is low (in zone $Z1$), then irrespective of θ or the quality difference (net of cost) the firms will form the cartel and choose HL or LH as the collusive strategy. Since there are two markets they coordinate on HL and LH outcomes in two markets and hence their overall payoff are greater than producing the high-quality product (HH) in both markets. Two firms earn the same total profit by this coordination.

Proposition 1 *For $\underline{f} > f$ so that HH is the unique Nash equilibrium, if γ is relatively high or the horizontal product differentiation is low, then irrespective of θ or the quality difference (net of cost) the firms will form the cartel and chose HL or LH as the collusive strategy.*

5.2 Explicit Cartel: HL/LH chosen instead of LL

In this section, we discuss that if $\overline{f} < f$ so that LL is the unique Nash equilibrium, i.e. we are in zone LL of Figure 1, whether the firms will form a cartel and not produce the lower quality product (good L) in both the market. Thus for the firms, it may be sometimes optimal to decide the product jointly (thus forming a cartel) in the following way even if $\overline{f} < f$:
i) Firm 1 will produce good L and firm 2 will produce good H in market A and firm 1 will produce good H and firm 2 will produce good L in market B or
ii) Firm 2 will produce good L and firm 1 will produce good H in market A and firm 2 will produce good H and firm 1 will produce good L in market B.

For this to hold, we should have the following respectively:
i) $\Pi^A_{1LL} + \Pi^B_{1LL} < \Pi^A_{1LH} + \Pi^B_{1HL}$ and $\Pi^A_{2LL} + \Pi^B_{2LL} < \Pi^A_{2LH} + \Pi^B_{2HL}$ or
ii) $\Pi^A_{1LL} + \Pi^B_{1LL} < \Pi^A_{1HL} + \Pi^B_{1LH}$ and $\Pi^A_{2LL} + \Pi^B_{2LL} < \Pi^A_{2HL} + \Pi^B_{2LH}$.

We argue, using equation (10) and equation (15), that it is possible if

$$2\left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 < \left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 + \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F. \tag{27}$$

or

$$F < \frac{(2\theta-\gamma)^2 + (2-\gamma\theta)^2 - 2(2-\gamma)^2\theta^2}{(4-\gamma^2)^2}. \tag{28}$$

given $\overline{f} = (2-\gamma\theta)^2 - (2-\gamma)^2\theta^2 < f = F(4-\gamma^2)^2/(\alpha_H - c_H)^2$ or

$$\frac{(2-\gamma\theta)^2 - (2-\gamma)^2\theta^2}{(4-\gamma^2)^2} < F \tag{29}$$

Thus combining the above two conditions for the cartel to be possible, we should have

$$\frac{(2-\gamma\theta)^2 - (2-\gamma)^2\theta^2}{(4-\gamma^2)^2} < F < \frac{(2\theta-\gamma)^2 + (2-\gamma\theta)^2 - 2(2-\gamma)^2\theta^2}{(4-\gamma^2)^2}. \tag{30}$$

However, this is not possible.

Proposition 2 *Given $\overline{f} < f$ so that LL is the unique Nash equilibrium, then the firms will not collude and choose HL or LH as the strategy.*

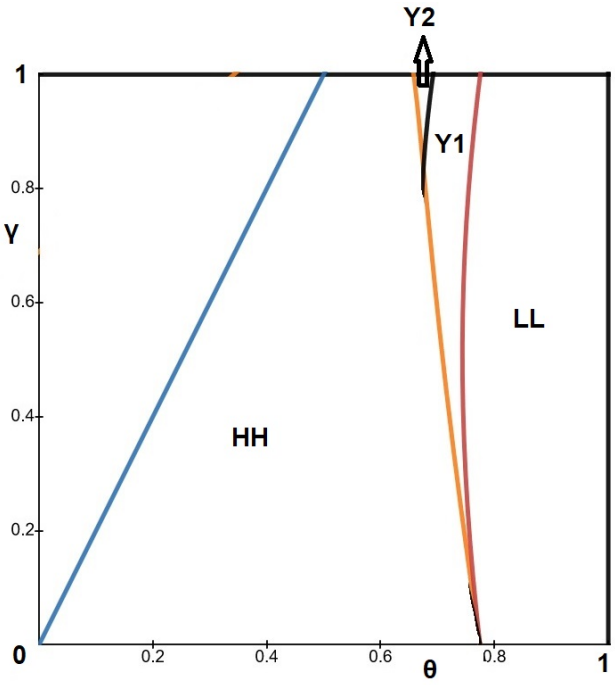

Figure 3: Quality Cartel LL

5.3 Explicit Cartel: LL chosen instead of HL/LH

In this section, we discuss that if $f \in [\underline{f}, \overline{f}]$ so that HL or LH are the equilibrium, i.e. we are in zone HL of Figure 1, whether the firms will form a cartel and only produce the lower quality product (good L) in both the market. For this to hold, we should have the following respectively:
i) $\Pi^A_{1LL} + \Pi^B_{1LL} > \Pi^A_{1LH} + \Pi^B_{1HL}$ and $\Pi^A_{2LL} + \Pi^B_{2LL} > \Pi^A_{2LH} + \Pi^B_{2HL}$ or
ii) $\Pi^A_{1LL} + \Pi^B_{1LL} > \Pi^A_{1HL} + \Pi^B_{1LH}$ and $\Pi^A_{2LL} + \Pi^B_{2LL} > \Pi^A_{2HL} + \Pi^B_{2LH}$.

We argue, using equation (15) and equation (10), that it is possible if

$$2\left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 > \left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 + \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F. \tag{31}$$

or

$$F > \frac{(2\theta - \gamma)^2 + (2 - \gamma\theta)^2 - 2(2-\gamma)^2\theta^2}{(4-\gamma^2)^2} = Y. \tag{32}$$

given $f \in [\underline{f}, \overline{f}]$ where $f = F(4-\gamma^2)^2/(\alpha_H - c_H)^2$ or

$$\frac{(2-\gamma\theta)^2 - (2-\gamma)^2\theta^2}{(4-\gamma^2)^2} > F > \frac{(2-\gamma)^2 - (2\theta - \gamma)^2}{(4-\gamma^2)^2}. \tag{33}$$

We observe that the condition (32) holds only in zone $Y1$ of Figure 3, which is a subset of zone HL of Figure 1 for $F = 0.1$ as stated before. However, it is not possible in zone $Y2$ of Figure 3, which is also a subset of zone HL of Figure 1. The R.H.S. of condition (32) i.e.

$Y = \frac{(2\theta-\gamma)^2+(2-\gamma\theta)^2-2(2-\gamma)^2\theta^2}{(4-\gamma^2)^2}$, falls in θ.[14] If θ is relatively high or the quality difference (net of cost) is low between the high-quality product and the low-quality product, then for the firms that produce the high-quality product it is better to produce only the low-quality products in both markets. As then the loss of profit from switching to the low-quality product is less, and this also helps the firms to save the fixed costs. Thus, if θ is relatively high or the quality difference (net of cost) is low, then the firms will form the cartel and choose LL as the collusive strategy.

Proposition 3 *For $f \in [\underline{f}, \overline{f}]$ so that HL or LH are the Nash-equilibria, if θ is relatively high or the quality difference (net of cost) is low, then the firms will form the cartel and choose LL as the collusive strategy.*

5.4 Explicit Cartel: LL chosen instead of HH

Moreover, let us once more focus on zone $Z1$ of Figure 2, where the firms prefer to form a cartel so that HL or LH is the collusive outcome instead of choosing HH as the Nash-equilibrium outcome (See Section 5.1). In this section, we discuss whether the firms will form a cartel and only produce the lower quality product (good L) in both markets or prefer to form a cartel of the form HL or LH where both the qualities are produced by different firms in the same market. As discussed in the previous section (Section 5.3), only producing the lower quality product is better if the following conditions hold:
i) $\Pi^A_{1LL} + \Pi^B_{1LL} > \Pi^A_{1LH} + \Pi^B_{1HL}$ and $\Pi^A_{2LL} + \Pi^B_{2LL} > \Pi^A_{2LH} + \Pi^B_{2HL}$; or
ii) $\Pi^A_{1LL} + \Pi^B_{1LL} > \Pi^A_{1HL} + \Pi^B_{1LH}$ and $\Pi^A_{2LL} + \Pi^B_{2LL} > \Pi^A_{2HL} + \Pi^B_{2LH}$.

Thus, this is true if condition (32), as stated in the previous section (Section 5.3), holds in zone $Z1$ of Figure 2. We observe that it is only possible in zone X of Figure 4, which is a subset of zone $Z1$ of Figure 2. However, it is not possible in zone W of Figure 4, which is also a subset of zone $Z1$ of Figure 2. If θ is relatively high or the quality difference (net of cost) is low between the high-quality product and the low-quality product, then w.r.t. the cartel of the form HL or LH, it is better to produce only the low-quality products in both markets. As then the loss of profit from switching to the low-quality product is less and this also helps the firms to save the fixed costs. Thus, instead of adopting the collusive strategy HL or LH, it is sometimes optimal for the firms to form the cartel and choose LL as the collusive strategy if θ is relatively high or the quality difference (net of cost) is low.

Proposition 4 *For $f < \underline{f}$, instead of adopting the collusive strategy HL or LH, it is sometimes optimal for the firms to form the cartel and choose LL as the collusive strategy, if θ is relatively high or the quality difference (net of cost) is low.*

6 Summary

Thus, as discussed before from the previous proposition, we can conclude that using Figure 4 that the firms will collude in the following way.

[14] We observe that $\frac{dY}{d\theta} = \frac{2(8\gamma\theta-4\theta-4\gamma-\gamma^2\theta)}{(4-\gamma^2)^2} < 0$.

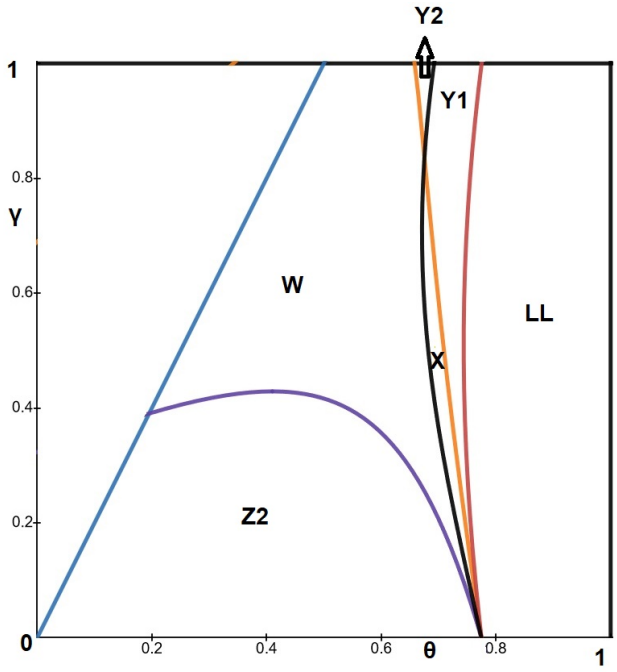

Figure 4: Quality choice

Results:

i) In zone $Y1$ and zone X, the firms will collude and produce only the lower quality product, whereas in zone LL the firms will produce the lower quality products only and not have the incentive to collude and thereby produce the higher quality product.

ii) In zone W the firms will collude and produce a combination of high-quality and low-quality products in one market and reciprocally the opposite qualities in the other market, rather than producing only the high quality product in both markets. It is also to be noted that zone $Y2$ is the region where the firms do not collude and strategically produce different quality products as stated for zone W.

iii) However, in zone $Z2$ the firms will not collude and produce the high quality product in both markets.

Thus, when θ is high or the quality difference (net of cost) between the products is low, i.e. in zone X, zone $Y1$ and zone LL then it is optimal for the firms to produce only the lower quality product. Therefore, in these zones the higher-quality product is never produced. This also means that the higher-quality product is produced only if θ is low or the quality difference (net of cost) between the products is high, i.e. in zone $Z2$, zone W and zone $Y2$. Specifically, in zone $Z2$ only the high-quality product is produced by both the firms in both the markets as given low θ, γ is also low or the horizontal product differentiation between the products is high. On the other hand, in zone W and zone $Y2$ the low-quality product is also produced along with the high-quality product as γ is high or the horizontal product differentiation is low as

producing only the high-quality product by both the firms in the same market is less profitable. This is because when the horizontal product differentiation is less the high-quality products produced by the two firms face more competition and the firms have to incur the fixed-cost (F) separately. Thus, either of the firms should switch to the lower quality product (instead of producing the higher quality product) which also allows the firm to save fixed costs.

7 Welfare analysis

In this section, we discuss the impact of collusion on the welfare of one region, i.e. market A or market B. As the markets are identical, the outcomes are identical both in terms of the outputs and profits (as discussed in Section 4 and Section 5). Thus, we discuss the effect of the collusion for a single market, say market A. Welfare (W) is the sum of the consumer surplus (CS) and industry profits (IP). From the utility function (see equation 1), we get $CS = \frac{1}{2}[{q_{A1}}^2 + 2\gamma q_{A1}q_{A2} + {q_{A2}}^2]$ (See Sen et al. (2021)) and the profits of the firms are defined before in Section 3. We first derive the consumer surplus and welfare for all the outcomes in the following sections and finally examine the effect of the collusion on the welfare.

7.1 Only good H is produced

When only good H is produced in market A, then as defined in Section 3.1, the amount of good H produced by both the firms separately are

$$q^*_{A1} = q^*_{A2} = \frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)} \tag{34}$$

and the firms' profits in market A are

$$\Pi^A_{1HH} = \Pi^A_{2HH} = \left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - F. \tag{35}$$

Therefore, the consumer surplus is $CS^{HH} = \frac{1}{2}[{q^*_{A1}}^2 + {q^*_{A2}}^2 + 2\gamma q^*_{A1}q^*_{A2}]$ or

$$CS^{HH} = (1+\gamma)\left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 \tag{36}$$

and the industry profit is $IP^{HH} = \Pi^A_{1HH} + \Pi^A_{2HH}$ or

$$IP^{HH} = 2\left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - 2F. \tag{37}$$

Thus, the welfare is $W^{HH} = CS^{HH} + IP^{HH}$ or

$$W^{HH} = (3+\gamma)\left[\frac{(2-\gamma)(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2 - 2F. \tag{38}$$

7.2 Only good L is produced

When only good L is produced in market A, then as defined in Section 3.2, the amount of good L produced by both firms separately are

$$q_{A1}^* = q_{A2}^* = \frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)} \tag{39}$$

and the firms' profits in market A are

$$\Pi_{1LL}^A = \Pi_{2LL}^A = \left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2. \tag{40}$$

Therefore, the consumer surplus is $CS^{LL} = \frac{1}{2}[{q_{A1}^*}^2 + {q_{A2}^*}^2 + 2\gamma q_{A1}^* q_{A2}^*]$ or

$$CS^{LL} = (1+\gamma)\left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 \tag{41}$$

and the industry profit is $IP^{LL} = \Pi_{1LL}^A + \Pi_{2LL}^A$ or

$$IP^{LL} = 2\left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2. \tag{42}$$

Thus, the welfare is $W^{LL} = CS^{LL} + IP^{LL}$ or

$$W^{LL} = (3+\gamma)\left[\frac{(2-\gamma)(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2. \tag{43}$$

7.3 Both good H and good L are produced

Let us assume that both good H and good L are produced in market A, then as defined in Section 3.3, the amount of good H and good L produced by the firms are respectively

$$q_{A1}^* = \frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)} \quad and \quad q_{A2}^* = \frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)} \tag{44}$$

and the firms' profits in market A are

$$\Pi_{1HL}^A = \left[\frac{2(\alpha_H - c_H) - \gamma(\alpha_L - c_L)}{(4-\gamma^2)}\right]^2 - F \;\; and \;\; \Pi_{2HL}^A = \left[\frac{2(\alpha_L - c_L) - \gamma(\alpha_H - c_H)}{(4-\gamma^2)}\right]^2. \tag{45}$$

Therefore, the consumer surplus is $CS^{HL} = \frac{1}{2}[{q_{A1}^*}^2 + {q_{A2}^*}^2 + 2\gamma q_{A1}^* q_{A2}^*]$ or

$$CS^{HL} = \frac{(4-3\gamma^2)(\alpha_H - c_H)^2 + (4-3\gamma^2)(\alpha_L - c_L)^2 + 2\gamma^3(\alpha_H - c_H)(\alpha_L - c_L)}{2(4-\gamma^2)^2} \tag{46}$$

and the industry profit is $IP^{HL} = \Pi_{1HL}^A + \Pi_{2HL}^A$ or

$$IP^{HL} = \frac{(4+\gamma^2)(\alpha_H - c_H)^2 + (4+\gamma^2)(\alpha_L - c_L)^2 - 8\gamma(\alpha_H - c_H)(\alpha_L - c_L)}{(4-\gamma^2)^2} - F. \tag{47}$$

Thus, the welfare is $W^{HL} = CS^{HL} + IP^{HL}$ or

$$W^{HL} = \frac{(12-\gamma^2)(\alpha_H - c_H)^2 + (12-\gamma^2)(\alpha_L - c_L)^2 - 2\gamma(8-\gamma^2)(\alpha_H - c_H)(\alpha_L - c_L)}{2(4-\gamma^2)^2} - F. \tag{48}$$

7.4 Effect of collusion on welfare

Finally, we examine here the effect of the collusion on the welfare.

7.4.1 Shift from HH to HL

Let us first consider zone W of Figure 4. In this zone, instead of producing only the high quality product independently, the firms collude and produce both the high and low quality product (HL) separately as discussed in Section 5.1. On the other hand, if the firms do not collude, then they produce only the high quality product (HH) as discussed in Section 4.1. First, after comparing the outputs we observe that if the firms collude, then the total production of the high quality product falls and the production of the low quality product increases.

After comparing the consumer surplus of the two scenarios (HH and HL), using equation (36) and equation (46), we observe that after collusion the consumer surplus will increase or $CS^{HL} > CS^{HH}$ in zone W of Figure 4 if

$$(4-3\gamma^2) + (4-3\gamma^2)\theta^2 + 2\gamma^3\theta > 2(1+\gamma)(2-\gamma)^2. \tag{49}$$

However, it is never possible, thus consumer surplus falls in zone W of Figure 4 if the firms collude as the total production of the high quality product falls and the production of the low quality product increases.

Moreover as $\Pi^A_{1HH} + \Pi^B_{1HH} = \Pi^A_{1HH} + \Pi^A_{2HH} < \Pi^A_{1HL} + \Pi^B_{1LH} = \Pi^A_{1HL} + \Pi^A_{1HL}$, the industry profit increases.

Now after comparing the welfare of the two scenarios, using equation (38) and equation (48), we observe that after collusion the welfare will increase or $W^{HL} > W^{HH}$ in zone W of Figure 4 if

$$(12-\gamma^2) + (12-\gamma^2)\theta^2 - 2\gamma\theta(8-\gamma^2) + 2F(4-\gamma^2)^2 > 2(3+\gamma)(2-\gamma)^2. \tag{50}$$

This is possible in zone W_1 of Figure 5, but the welfare reduces in zone W_0, where zone W_1 and zone W_0 are the subsets of zone W of Figure 4 where $F = 0.1$ as stated before. In zone W_1 as γ is very high or the horizontal product differentiation is very low, producing only the high-quality product by both the firms in the same market is less profitable. This is because when the horizontal product differentiation is less the high-quality products produced by the two firms face more competition and the firms have to incur the fixed-cost (F) separately. Thus,

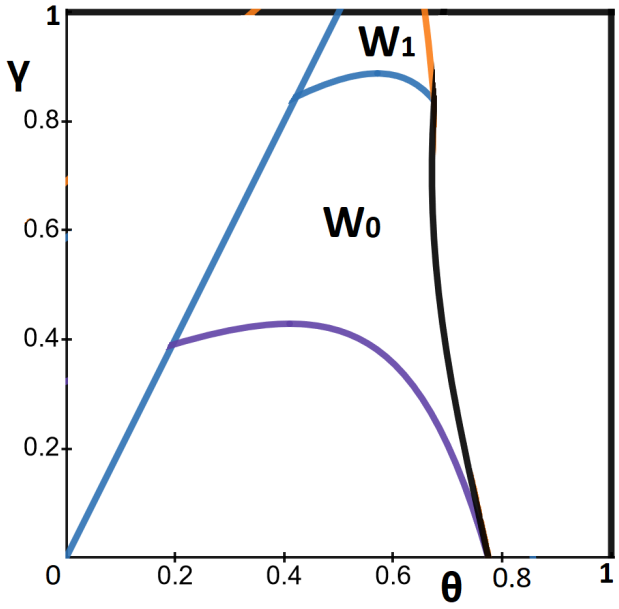

Figure 5: Welfare

when either of the firms switches to the lower quality product (instead of producing the higher quality product) which also allows the firm to save fixed costs, the increase in the industry profit can compensate for the fall in the consumer surplus and thus welfare increases in zone W_1. Therefore, for $\underline{f} > f$ so that HH is the unique Nash equilibrium (as discussed in Section 4.1), if γ is very high or the horizontal product differentiation is very low (as in zone W_1), then welfare will increase if the firms form the cartel and choose HL or LH as the collusive strategy but consumer surplus falls.

7.4.2 Shift from HH to LL

Let us first consider zone X of Figure 4. In this zone instead of producing only the high quality product strategically (non-cooperatively) in the equilibrium, the firms collude and produce only the low quality product (LL) separately as discussed in Section 5.4. On the other hand, if the firms do not collude, then they produce only the high quality product (HH) as discussed in Section 4.1. Comparing the outputs we observe that if the firms collude, then the production of the high quality product falls (which ultimately is not produced) and the production of the low quality product increases.

After comparing the consumer surplus of the two scenarios (HH and LL), using equation (36) and equation (41), we observe that after collusion the consumer surplus will fall as $CS^{LL} < CS^{HH}$ in zone X of Figure 4. Moreover, as $\Pi^A_{1HH} + \Pi^B_{1HH} = \Pi^A_{1HH} + \Pi^A_{2HH} < \Pi^A_{1LL} + \Pi^B_{1LL} = \Pi^A_{1LL} + \Pi^A_{1LL}$, the industry profit will increase.

Now after comparing the welfare of the two scenarios, using equation (38) and equation (48), we observe that after collusion the welfare will increase or $W^{LL} > W^{HH}$ in zone X of

Figure 4 if

$$(3+\gamma)(2-\gamma)^2\theta^2 + 2F(4-\gamma^2)^2 > (3+\gamma)(2-\gamma)^2. \tag{51}$$

However, this is not possible. Thus, after collusion the welfare will fall as $W^{LL} < W^{HH}$ in zone X of Figure 4.

7.4.3 Shift from HL to LL

Let us first consider zone $Y1$ of Figure 4. In this zone instead of producing both the high quality product and the low quality product independently, the firms collude and produce only the low quality product (LL) as discussed in Section 5.3. On the other hand, if the firms do not collude, then they both produce the high quality product and the low quality product (HL) independently for different markets as discussed in Section 4.2. First, after comparing the outputs we observe that if the firms collude, then the production of the high quality product stops and the production of the low quality product occurs (which is higher than in situation HL).

After comparing the consumer surplus of the two scenarios (HL and LL), using equation (46) and equation (41), we observe that after collusion the consumer surplus will increase or $CS^{LL} > CS^{HL}$ in zone $Y1$ of Figure 4 if

$$(4-3\gamma^2) + (4-3\gamma^2)\theta^2 + 2\gamma^3\theta < 2(1+\gamma)(2-\gamma)^2\theta^2. \tag{52}$$

However, it is never possible, thus consumer surplus falls in zone $Y1$ of Figure 4 if the firms collude as the total production of the high quality product falls and the production of the low quality product increases.

Moreover, as $\Pi^A_{1LL} + \Pi^B_{1LL} = \Pi^A_{1LL} + \Pi^A_{2LL} > \Pi^A_{1HL} + \Pi^B_{1LH} = \Pi^A_{1HL} + \Pi^A_{2HL}$, the industry profit will increase.

Now after comparing the welfare of the two scenarios, using equation (43) and equation (48), we observe that after collusion the welfare will increase or $W^{LL} > W^{HL}$ in zone $Y1$ of Figure 4 if

$$(12-\gamma^2) + (12-\gamma^2)\theta^2 - 2\gamma\theta(8-\gamma^2) - 2F(4-\gamma^2)^2 < 2(3+\gamma)(2-\gamma)^2\theta^2. \tag{53}$$

However, this is not possible. Thus, after collusion the welfare will fall as $W^{LL} < W^{HL}$ in zone $Y1$ of Figure 4.

7.4.4 Results

The main finding of this section is summarized in the following proposition.

Proposition 5 *i) If the firms collude, then the total production of the high quality product falls and the production of the low quality product increases.*

ii) For $\underline{f} > f$ so that HH is the unique Nash equilibrium, if γ is very high or the horizontal product differentiation is very low, then welfare will increase if the firms form the cartel and

choose HL or LH (i.e. one firm produces the high-quality product and the other firm produces the low-quality product) as the collusive strategy. Otherwise, after collusion welfare always reduces.

8 Conclusion

There exists a large literature on multimarket contact facilitating collusion and sustaining higher market prices. Though there is also evidence of under-provision of the quality of products in multimarket contact settings, there does not exist any theory analysing this aspect of multi-market contact. Ours is the first theoretical model to analyse the effect of multimarket contact on product qualities chosen by firms. In this paper, we have developed a model of multi-market contact and collusion with both horizontally and vertically differentiated products.

We have considered two identical markets which are geographically separated having identical demands. Two firms are serving both markets. A two-stage game is considered, where firms choose the product qualities in the first stage for both markets and in the second stage, they compete in quantities in both markets. Firms have the option of choosing either a low or a high-quality product and the production of higher quality product requires a fixed cost. In this simple setting, we have demonstrated that multi-market contact may play a crucial role in the quality choice stage. Thus, firms might collude in the quality choice game even though they compete in quantities in the second stage. As a result of this collusion in quality choice, the firms will under-invest in product quality improvement (and thereby save the fixed cost). This will lead to the availability of more low-quality products and the availability of less high-quality products in the market. Thus, our paper provides a theoretical foundation for the empirically observed phenomena.

In the paper, we have first characterized the outcomes when the firms don't collude but select the quality of the product strategically in a non-cooperative setting. First, both firms will produce the higher quality product in both markets if the fixed cost is relatively low or the quality difference (net of cost) between the products is very high. In the opposite case, both firms will produce the lower quality product in both markets if the fixed cost is relatively high or the quality difference (net of cost) is very low. The third possibility is that in both markets, the firms will produce different types of products in each market if the fixed cost is neither very high nor very low or the quality difference (net of cost) is neither very high nor very low.

When we allow the possibility of collusion between firms regarding the choice of product quality in both markets, we find that when the fixed cost is low so that producing only the high-quality product is the unique Nash equilibrium along with low horizontal product differentiation, then irrespective of the quality difference (net of cost), the firms will form the cartel and either of the firms would switch to the lower quality product (instead of producing, the higher quality product). On the other hand, if the fixed cost is neither very high nor very low so that one firm produces the high-quality product and the other firm produces the low-quality product non-cooperatively (vice-verse in the other market), if the quality difference (net of cost) is low, then the firms will form the cartel and produce only the low-quality product. Interest-

ingly, it is shown that there also exist cases where the firms will collude and produce only the low-quality product in both markets instead of producing only the high-quality product, which is the outcome of a non-cooperative equilibrium. Thus, in essence, due to multi-market contact, if the firms collude and choose the quality of the product jointly, in general the production of the lower-quality product will increase, and the production of the higher-quality product will decrease. Thus, we have shown that the product quality supplied by firms may be affected due to multi-market contact as it opens up the possibility of coordination and collusion between firms.

We also show that when the fixed cost is low, so that producing only the high-quality product is the unique Nash equilibrium, if the horizontal product differentiation is very low then welfare will increase if the firms form the cartel and either of the firms switches to the lower quality product (instead of producing, the higher quality product). Otherwise, after collusion which dampens the supply of high quality product, welfare always decreases.

References

[1] Akinbosoye, O., Bond, E.W., & Syropoulos, C. (2012). "On the stability of multimarket collusion in price-setting supergames", *International Journal of Industrial Organization*, 30, 253-264.

[2] Ashournia, D., Hansen, P. S. & Hansen, J. W. (2013). "Trade liberalization and the degree of competition in international duopoly", *Review of International Economics*, 21, 1048–1059.

[3] Belleflamme, P., & Peitz, M. (2015), *Industrial Organization: Markets and Strategies*, Cambridge University Press, New York.

[4] Bernheim, D., & Whinston, M. (1990). "Multimarket contact and collusive behavior", *RAND Journal of Economics*, 21, 1-26.

[5] Bhattacharjea, A., & Sinha, U. B. (2015). "Multi-market collusion with territorial allocation", *International Journal of Industrial Organization*, 41, 42-50.

[6] Bond, E.W., & Syropoulos, C. (2008). "Trade costs and multimarket collusion", *RAND Journal of Economics*, 39, 1080-1104.

[7] Busse, M. (2000). "Multimarket Contact and Price Coordination in the Cellular Telephone Industry", *Journal of Economics & Management Strategy*, 9, 287–320.

[8] Byford, M.C., & Gans, J. S. (2014). "Collusion at the extensive margin", *International Journal of Industrial Organization*, 37, 75–83.

[9] Choi, J.P., & Gerlach, H. (2012). "International antitrust enforcement and multimarket contact", *International Economic Review*, 53, 635-657.

[10] Ciliberto, F. & Williams, J.W. (2014). "Does Multimarket Contact Facilitate Tacit Collusion? Inference on Conduct Parameters in the Airline Industry", *RAND Journal of Economics*, 45, 764-791.

[11] Colombo, L., & Labrecciosa, P. (2007). "Sustaining collusion under economic integration", *Review of International Economics*, 15, 905–915.

[12] Edwards, C. D., (1955). "Conglomerate Bigness as a Source of Power", *Business Concentration and Price Policy* (NBER Conference Report) (Princeton, NJ: Princeton University Press), pp. 331-352.

[13] Evans, W. N. & Kessides, I.N. (1994). "Living by the "Golden Rule": Multimarket Contact in the U. S. Airline Industry", *Quarterly Journal of Economics*, 109, 341-366.

[14] Feinberg, R. M. (2014). "Price effects of multimarket contact among movie chains in small US metropolitan areas", *Economics Letters*, 123, 6–8.

[15] Fernandez, N. & Marin, P. (1998). "Market Power and Multimarket Contact: Some Evidence from the Spanish Hotel Industry", *Journal of Industrial Economics*, 46, 301–315.

[16] Hackner, J. (2000). "A Note on Price and Quantity Competition in Differentiated Oligopolies", *Journal of Economic Theory*, 93, 233-239.

[17] Hylton, K.N. & Deng, F. (2007). "Antitrust Around the World: An Empirical Analysis of the Scope of Competition Laws and Their Effects, in 74 Antitrust Law Journal 271 (2007). Available at: $https://scholarship.law.bu.edu/faculty\_scholarship/852$

[18] Jans, I. & Rosenbaum, D.I. (1997). "Multimarket Contact and Pricing: Evidence from the U.S. Cement Industry", *International Journal of Industrial Organization*, 15, 391-412.

[19] Leheyda, N., 2008. "Market Power, Multimarket Contact and Pricing: Some Evidence from the US Automobile Market", *ZEW Discussion Papers 08-118, ZEW - Leibniz Centre for European Economic Research.*

[20] Lin, H. & McCarthy, I.M. (2023). "Multimarket Contact in Health Insurance: Evidence from Medicare Advantage", *Journal of Industrial Economics*, 71, 212-255.

[21] Lommerud, K.E., & Sørgard, L. (2001). "Trade liberalization and cartel stability", *Review of International Economics*, 9, 343-55.

[22] Mishra, A. K., & Rao, G. (2015). "Analysing Anti-competitive Behaviour: The Case for Indian Telecom Industry", *Science, Technology and Society*, 20, 21–43.

[23] OECD (Organisation for Economic Cooperation and Development). (2013). "The Role and Measurement of Quality in Competition Analysis", Policy Roundtables. Paris: OECD. http://www.oecd.org/competition/Quality-in-competition-analysis-2013.pdf.

[24] Parker, P. M. & Röller, L.H. (1997). "Collusive Conduct in Duopolies: Multimarket Contact and Cross-Ownership in the Mobile Telephone Industry", *RAND Journal of Economics*, 28, 304-322.

[25] Pinto, B. (1986). "Repeated games and the 'reciprocal dumping' model of trade", *Journal of International Economics*, 20, 357-366.

[26] Prince, J. T. & Simon, D. H. (2009). "Multimarket Contact and Service Quality: Evidence from On-Time Performance in the US Airline Industry", *Academy of Management Journal*, 52, 336–354.

[27] Sen, N., Kaul, S. & Biswas, R. (2021). "Technology licensing under product differentiation", *Journal of Economics* 134, 219–260.

[28] Sinha, U.B. (2018). "FDI and International Collusion", CDE Working Paper No. 295 http://www.cdedse.org/pdf/work295.pdf

[29] Symeonidis, G. (1999). "Cartel stability in advertising-intensive and R&D-intensive industries", *Economics Letters*, 62, 121–129.

[30] Waldfogel, J. & Wulf, J. (2006). "Measuring the Effect of Multimarket Contact on Competition: Evidence from Mergers Following Radio Broadcast Ownership Deregulation", *The B.E. Journal of Economic Analysis & Policy*, vol. 5, no. 1, 2006, pp. 000010151515380645142O. https://doi.org/10.1515/1538-0645.1420