Beliefs, Aggregate Productivity and Wages: Role of expectation in labor market dynamics

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Work in progress. Comments and feedback welcome

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Abstract

What happens to labor market dynamics when a share of workers are uninformed about aggregate productivity? In this paper, I use heterogeneity in workers' perception of aggregate productivity process to replicate features of US data. Firm incentives to create jobs is driven by the interactions of firm with workers of two types: full information workers and workers who do not observe aggregate productivity. Differences in worker types drive differences in wage contracts based on bargaining protocol of Binmore-Rubinstein-Wolinsky with uninformed workers negotiating a fixed wage while full information workers asking for aggregate productivity-linked flexible wages. The framework is able to replicate features of the data for reasonable calibration of the parameters.

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1. Introduction

What happens to labor market dynamics when a share of workers are uninformed? Recent empirical evidence shows that workers' perception about labor market outcomes does not match with realized outcomes(See Adams-Prassl et al. [2023], Mueller et al. [2021]). In a standard economic model, firms create jobs when output net of wages is greater than the expected cost of creating a vacancy. However, the underlying productivity process for generating that output may not be known to workers. In such a scenario wage bargaining would respond to workers' beliefs about productivity and not its actual level. In this paper, I assume that some workers are uninformed in that these workers do not observe the true productivity of the firm. These workers believe that productivity is always constant and consequently demand for a fixed wage contract. This has implications for firm responses in job creation which is the focus of the paper.

This paper builds on the Diamond-Mortensen-Pissarides (henceforth DMP) search and matching framework with the innovation that there are two types of workers: i) full information workers whose belief about productivity coincides with the true value in every period, and ii) uninformed workers who believe that productivity is constant. Period by period the uninformed worker's belief does not coincide with actual values of labor market. Through a business cycle, these workers are relatively pessimistic about their labor market outcomes when productivity is greater than their belief and relatively optimistic on the reverse. The uninformed worker is modelled on the lines of 'stubborn' worker in the innovative research of Menzio [2022] in formulating 'Stubborn Beliefs Equilibrium'.

The central argument of the paper revolves around the aspect that firms face a heterogeneous workforce whose wage demands are different due to the structure of beliefs. Full information workers demand variable wage contract that accrues a stable net surplus to firms thereby sharing the risk of volatility of productivity. Uninformed worker demand fixed wage contract that results in low surplus during recessions and high surplus during booms. When creating a vacancy, the firm takes into account that it can meet either type of worker depending upon the proportion of each type in the pool of unemployed workers. Firms' interactions with uninformed workers results in firms' incentives to co-move with productivity as wage remains fixed. Variability in productivity is therefore reflected in firm responses instead of being absorbed in variable wages. Here it is assumed that the firm knows the worker type it is matched with and pursues a best response strategy that can accommodate the wage demand.[1] This is achieved under two conditions: a) firm makes non-negative surplus, b) firm does not find it profitable to reject any worker type to hire the other worker type in next period.

Condition (a) is less strict than the requirement that surplus from each worker type be greater than fixed cost of vacancy. After vacancy is created, the firm only takes into account the continuation value of surplus when matched with a worker type as the fixed cost of vacancy creation is sunk. Hence, if full information workers are able to generate sufficient surplus at low productivity levels such that the expected weighted benefit of vacancy creation is greater than the cost, firms will create vacancies. At high productivity levels, the high surplus from uninformed workers promotes additional vacancy creation. This influence across types is an important feature of the model. An economy with only uninformed workers may not generate sufficient surplus for firms to create vacancies at low productivity levels; while an economy with only full information workers will lead to the unemployment-volatility puzzle.

Condition (b) restricts firms from selecting worker type that maximises surplus for a given productivity level. Given the wage demand, a firm would want to hire uninformed workers during periods of high productivity and full information workers during periods of low productivity. In the model, a firm meets only one worker within a period. Therefore, the firm has to compare the benefit of hiring the worker type that it is matched with against the benefit of paying fixed cost of creating a new vacancy in search of the other worker type. Condition (b) ensures that the former is greater than the latter.

1.1. Related Literature

The paper is related to three broad strands of the literature: Search and Matching, Bounded Rationality and Contract Theory. The theoretical framework in this model

[1] Adding asymmetric information in this model is an exciting direction of research, however out of scope for this paper.

builds on the standard search and matching framework. A major criticism of the DMP search and matching model with Nash Bargaining is that it fails to match the volatility of unemployment rate in the US data (Shimer [2005]). Most of the fluctuations in aggregate productivity are absorbed by flexible wages, thereby reducing volatility in firms' incentives to create jobs (Ljungqvist and Sargent [2017, 2021]). Vacancy creation rate and the unemployment rate are an order of magnitude larger than the prediction of the DMP search and matching model giving rise to the 'Unemployment-Volatility Puzzle' (colloquially the 'Shimer Puzzle'). Subsequently, various distinguished research scholars have brought innovations to the DMP model. Notable contributions include Pissarides [2011], Pissarides [2009], Shimer [2012], and Hagedorn and Manovskii [2008] among others. Conceptually, this paper is similar to innovations in wage determination protocols (Hall [2005] and Gertler et al. [2020]) and bargaining mechanisms (Hall and Milgrom [2008] and Gertler and Trigari [2009]). The distinction for this paper is that it contributes by discussing the implications of a heterogeneous workforce whereas prior work has focused on a homogeneous setup.

A recurring theme of debate in this literature centers on the elasticity of real wages with respect to productivity, quantifying which has been a challenge (see Abraham and Haltiwanger [1995], Bils [1985], Solon et al. [1994]). Hagedorn and Manovskii [2008] measure the elasticity of real wages at 0.45 using data released by the Bureau of Labor Statistics(BLS). This measure forms an important data moment for the simulation of the theoretical framework of this paper to establish the proportion of uninformed workers. In recent literature, Gertler et al. [2020] analyse the elasticity of wages by decomposing new hires into workers that join from unemployment and those that join from employment. They show that workers that move from unemployment to employment have a lower elasticity of wages compared to those who switch jobs. The current paper focuses on movement from unemployment to employment and hence workers are not allowed on-the-job search, which is a hugely important field of research on its own. (See e.g. Moscarini and Postel-Vinay [2018], Fujita et al. [2020])

The underlying theoretical framework in this paper is closely related to the growing literature on departures from full information rational expectations equilibrium; Gabaix [2019] provides an excellent survey of the same. Evidence of behavioral agents or non-standard rational agents has been well established in the last two decades in the literature. In recent work, Adams-Prassl et al. [2023] collect survey evidence on search behavior for workers searching on the job and for unemployed workers in the United Kingdom and find that workers are over-optimistic about receiving job offers conditional on any search for jobs. Caplin et al. [2023] link administrative data of survey respondents in Denmark to evaluate subjective earnings risk. They further calibrate a model of directed search over the life cycle in the vein of Menzio et al. [2016] and find that the model produces higher estimates of individual earnings risk than the subjective expectations survey data. Mueller et al. [2021] provide evidence from surveys administered in the United States of America that there is significant heterogeneity in perceived job finding rate for workers which remains unchanged over an unemployment spell. The uninformed worker in this paper can be interpreted in two ways: (i) the worker is unable to observe productivity, or (ii) the worker chooses a bounded rational behavior that ignores fluctuations in productivity. The paper does not take a stance on why these workers exhibit this behavior or the source of their stark information friction.

The economic literature has focused on non-rational expectations and its effect on worker search behavior and search outcomes. Santos-Pinto and de la Rosa [2020] provide an extensive survey of overconfidence in labor markets. Responses of firms in an economy with workers of non-rational beliefs, however, has not been discussed extensively. This paper takes the behavior of uninformed workers as given and explores macroeconomic implications arising from firm behavior in the presence of a heterogeneous workforce. In particular, this paper uses the 'Stubborn Beliefs Equilibrium' setup of Menzio [2022] who shows that an economy with stubborn agents would have larger elasticities of market tightness to productivity shock compared to a rational expectations framework. A share of uninformed workers provides a natural way to disentangle productivity from wage bargaining protocol thereby providing more variability in job creation. The main contribution of this paper is the application of Menzio [2022] to US data with a more general productivity process.

The presence of separate wage contracts for different types of worker of under informational assumptions is not new. Contracts designed under information asymmetries have a substantial literature starting from Calvo and Phelps [1977], Hall and Lilien [1979] and followed by seminal works of Azariadis [1983], Chari [1983], Cooper [1983], Grossman and Hart [1983] and many others. Cooper [2001] provides an extensive review of patterns in labor contracts. This paper takes a stance that some workers are unable to observe productivity at all while the remaining workers observe productivity in every period. This necessitates two different contracts for the two different worker types. Informed workers have a variable wage contract where the wages are indexed to the productivity, while the uniformed workers have a fixed wage contract that is linked to their belief about productivity, in this case a constant value.

1.2. Model Features, Estimation and Results

The theoretical framework in this paper has the following standard features: a stochastic process for productivity; a measure of homogeneous firms that utilize the productivity to produce a single output through a measure of employed workers; output net of wages establishes surplus that drives incentives to create jobs; a measure of unemployed workers that search and get matched with these jobs subject to search frictions. The main mechanism of the model is the difference in wage demand and contract type due to difference in worker types. The economic trade-off for the firm is two-fold: (i) when productivity is low, if the firm opens a vacancy, it can be matched with an uninformed worker which results in low surplus for firm compared to being matched with an informed worker, therefore there is moderation of vacancy creation; (ii) when productivity is high, the firm can be matched with an uninformed worker which results in higher surplus for firm compared to being matched with an informed worker, therefore there is increased impetus to vacancy creation. The combined effect produces increased volatility in the economy.

The model is estimated using method of simulated moments. I follow the search and matching literature by choosing aggregate labor market flows reported in Shimer [2005] as the moments to target for the US economy. Proportion of each worker type in the pool of unemployed workers is an important parameter in the model as it drives the vacancy creation incentives. To discipline this parameter, I target the elasticity of wage with respect to productivity. A model period corresponds to 1 month. Each simulation is run for 1639 months of which first 1000 are discarded which gives monthly series equivalent to 213 quarters as in Shimer [2005]. I run 5000 simulations and report the parameters as mean across all simulations at quarterly level.

The benchmark estimation uses parameters calibrated from external sources while allowing 3 parameters: the bargaining power, the curvature of the match function and the proportion of uninformed worker to be estimated within the simulations. The latter parameter is the variable of interest for this paper. The data targets for the simulation are the average monthly job finding rate of 0.45 (Shimer [2005]), the elasticity of job finding relative to market tightness of 0.5 (Petrongolo and Pissarides [2001]) and the elasticity of wage relative to aggregate productivity of 0.449 (Hagedorn and Manovskii [2008]). The model is able to replicate the necessary volatility of the vacancy creation and the unemployment rate thereby providing an alternate resolution to 'Unemployment-volatility puzzle'. The proportion of uninformed workers is about 0.54 which represents more than half the workers in the unemployment pool. The bargaining power for workers estimated from the simulations is 0.64 which is slightly higher than the standard value of 0.5 but less than the 0.72 utilized by Shimer [2005].

The elasticity of wage with respect to productivity forms an important parameter that determines the proportion of uninformed workers across the simulation specifications. Fixed wage contracts for uninformed workers forms an alternate channel to achieve wage rigidity central to matching the data. In alternate calibrations, when I increase the value from non-market activity the elasticity of wage with respect to productivity for full information worker decreases, therefore a lower proportion of uninformed workers fits the data better. The contribution of this paper is in this regard that across different specifications of reasonable parameter calibration, the presence of uninformed workers is necessary to match the data.

Roadmap: The paper is organized as follows: I introduce the theoretical framework in section 2. In section 3, I elaborate the calibration exercise to match the data moments from Shimer [2005]. The simulation results are are discussed in section 4. Section 5 concludes.

2. Model

The framework is based upon a standard DMP search-and-matching model and the theoretical advancement of Menzio [2022] through the 'Stubborn Agent'. The framework has (i) homogeneous firms that operate a constant returns to scale technology, (ii) aggregate fluctuations in productivity, and (iii) unemployed workers whose beliefs are that aggregate productivity is fixed at the long run average. The framework does not account for endogenous separation, and there is no on the job search. There are only two states of employment status: Unemployed and Employed.

2.1. Population and Technology

Time is discrete and denoted by t. The economy is populated by a continuum of measure 1 of equally productive, infinitely-lived workers and an endogenous measure of firms with free entry. Firms and workers are risk neutral and share the same discount rate β. Firms all produce an identical homogeneous good. Aggregate productivity is described by a variable y that takes a finite number of values in $\mathcal{Y}$ and follows a Markov process with transition distribution $\mathcal{F}(y' \mid y)$ defined over $\mathcal{Y}$.

There is a single market for all vacancies where all unemployed workers search for open vacancies. Existing matches are destroyed at a constant exogenous rate $\delta \in (0,1)$. Workers are identical in all regard except their beliefs about the process of aggregate productivity. There are two types of workers: $\{R, N\}$; type R worker is the full information rational expectations agent as in the standard DMP model, while type N worker is the uninformed worker or stubborn agent as in Menzio [2022].

2.2. Workers' Problem

When unemployed, workers engage in home production that gives them per period value b while they search for vacancies which gives them wages when matched with firm. I describe the worker types in detail below.

Type N:

Type N are workers who believe that the aggregate productivity process is constant at some $y_t = y^*$ where $y^* \in \mathcal{Y}$. Consequently, they conjecture that their probability of finding a vacancy when unemployed is constant at $p(\theta_N)$ evaluated at the market tightness level corresponding to the solution of a static DMP search framework with productivity at y^*. An implication of these beliefs is that when $y_t < y^*(y_t > y^*)$, type N workers would over-estimate (under-estimate) their probability of finding a job and thereby would be optimistic (pessimistic) relative to actual job finding prospects.

Value from employment $\hat{V}$ and unemployment $\hat{U}$ for type N are defined as follows:

$$\textit{Employment} \qquad \hat{V}(y^*, w_N) = w_N + \beta[\delta \hat{U}(y^*) + (1-\delta)\hat{V}(y^*, w_N)] \tag{1}$$

$$\textit{Unemployment} \qquad \hat{U}(y^*) = b + \beta[\, p(\theta_N)V(w_N) + \{1 - p(\theta_N)\}\hat{U}(y^*)\,] \tag{2}$$

Type R:

Type R workers are as in the standard full information rational expectations model. Their belief about the aggregate productivity process coincides with the actual aggregate productivity process. Type R workers observe y_t and conjecture that market tightness is a function of aggregate productivity, i.e., $\theta_R(y_t)$. Value of employment V and unemployment U for type R are same as in the full information rational expectations model and defined as follows:

$$\textit{Employment} \qquad V(y_t, w_R) = w_R + \beta \mathbf{E}_{|y_t}[\delta U(y_{t+1}) + (1-\delta)V(y_{t+1}, w'_R))] \tag{3}$$

$$\textit{Unemployment} \qquad U(y_t) = b + \beta \mathbf{E}_{|y_t}[p(\theta_R)V(y_{t+1}, w'_R)) + \{1 - p(\theta_R)\}U(y_{t+1})] \tag{4}$$

2.3. Firms' Problem

Firms create vacancies at cost c per vacancy. These vacancies determine the market tightness, θ, defined as the ratio of total vacancies v to total unemployed workers u in the market. The number of matches is determined through a matching function $m(u, v)$ with standard properties. Firms have full information about the type of worker they meet. Firms when not matched with a worker do not produce any output. The probability of a firm getting matched with a worker is given by $q(\theta) = m(u, v)/v = m(1/\theta, 1)$ where θ denotes the market tightness or v-u ratio. Let the negotiated wage for worker of type $i \in \{R, N\}$ be $w_{i,t}$. When matched with a worker of type i, firm's profit function is

$$J_i(y_t, w_{i,t}) = y_t - w_{i,t} + \beta \mathbf{E}_{|y_t}[(1 - \delta)J(y_{t+1}, w_{i,t+1})] \tag{5}$$

Firms can not screen the type of worker they get matched to as there is a single market for all vacancies with random search. Consequently, a firm's expected surplus from creating a vacancy is a weighted average of surplus from type N and type R scaled by the probability of filling the vacancy, $q(\theta(y))$. Let ζ be the proportion of type N workers in the economy. Using (5), expected surplus from a vacancy is

$$q(\theta(y_t))\mathcal{J}(y_t, w_R, w_N, \zeta) = q(\theta(y_t))(\zeta J_N(y_t, w_N) + (1 - \zeta)J_R(y_t, w_R)) \tag{6}$$

The above equation is applicable only under the assumption that firms do not reject workers based on worker type. This is delivered by two conditions that should be fulfilled in equilibrium:

- $J_i(y_t, w_{i,t}) > 0$ for $i \in \{R, N\}$ and $\forall y \in \mathcal{Y}$, i.e., once matched with a worker, firm do not make non-negative surplus.

- $J_i(y_t, w_{i,t}) > \beta E_y[q(\theta(y_{t+1}))\mathcal{P}_{-i} J_{-i}(y_{t+1}, w_{-i,t+1})] - c$ for $i \in \{R, N\}$ and $\forall y \in \mathcal{Y}$. Here, $\mathcal{P}_{-i}$ is the proportion of worker of type $-i$, i.e., ζ or $1 - \zeta$, in the pool of unemployed workers. This condition states that firms do not find it profitable to reject a worker of either type to hire the other worker type in next period.

The two conditions ensure that the firm if matched with a worker hires at every productivity level irrespective of the worker type.

2.4. Wages and Bargaining

On meeting a firm, there is bargaining for wages in the spirit of Binmore et al. [1986]. Type N's perceived surplus is different from actual surplus as they are either overestimating current aggregate productivity (if $y < y^*$) or underestimating it (if $y > y^*$). Wage bargaining as elaborated in Menzio [2022] leads to the following closed form solution.[2]

$$w_N(y^*) = \Big[\gamma y^* + (1-\gamma)(1-\beta)\hat{U}(y^*)\Big] \tag{7}$$

Here, γ can be treated as the bargaining power of the worker in the BRW setup. The equation (7) shows that type N worker's wage is a weighted average of the static belief about aggregate productivity and perceived unemployment value.

For worker of Type R, the solution to the bargaining problem is as in the standard DMP framework with Nash Bargaining.

$$w_R(y) = \gamma y + (1-\gamma)(1-\beta)U_R(y) \tag{8}$$

2.5. Market Interaction

Each type of worker produces the same output when matched with a firm but their wages are different, denoted w_N and w_R for type N and R respectively. By free entry assumption, per vacancy cost should be equal to expected surplus from the vacancy:

$$c = \beta E_{y_t}[q(\theta(y_{t+1}))\mathcal{J}(y_{t+1}, w_R, w_N, \zeta)] \tag{9}$$

Each type of worker interprets (9) based on their beliefs, which provides the departure from market dynamics of the full information rational expectations model. Type N evaluates the market tightness at $y_t = y^*$ using their perception of firm surplus. Both

[2] See Appendix A for more details

perceived firm surplus and perceived market tightness are fixed as per the belief, and therefore wages are fixed. As a result, firm surplus and thereby market tightness are pro-cyclical with aggregate productivity. Type R evaluates the market tightness at y_t. The difference of beliefs drives the differences in wage contracts resulting in increased volatility in the incentives to create vacancies.

3. Calibration

I follow the search-and-matching literature in calibrating the model which is estimated using a method of simulated moments. Primarily, the simulation exercise is aimed to match the aggregate labor market flows as in Shimer [2005]. Parameters that define the model are: those related to the productivity process, the match function, the bargaining power, the discount rate, the home production value, the cost of creating vacancy, the separation rate and the proportion of uninformed workers. The simulation exercise is focused on the proportion of uninformed workers that can resolve the unemployment volatility puzzle.

Time is discrete and a period of the model corresponds to one month. I set the discount rate β to 0.9966 which is approximately a 4% annual interest rate. The separation rate δ is set to 0.033 which corresponds to a worker staying in the same job for approximately 30 months or 2.5 years. In the baseline estimation, I fix the holding cost of a vacancy $c = 0.213$ as in Shimer [2005]. In alternate estimations, I allow the cost c to be estimated within the simulations.

The process for productivity is defined by $y_t = b + e^{z_t}(y^* - b)$. Here, y^* is the long run average of productivity which is normalized to 1, and b is the home production value which is set to 0.4. The latent variable z is taken as an AR-1 process $z_t = \rho z_{t-1} + \epsilon$ with $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon)$. This ensures that productivity y is mean reverting to y^*. I use Tauchen [1986] method to create a grid of 2001 points with +/- 2 standard deviations around 0 for the latent variable which is used to create a grid for productivity and its corresponding transition matrix. The productivity parameters are set to replicate the quarterly standard deviation of 0.020 and the quarterly autocorrelation of 0.878 of the

Table 1—Details of Calibrated Parameters

Parameter	Value	Description
Externally Calibrated		
Monthly discount rate β	0.9967	Annual discount rate of 4%
Monthly separation rate δ	0.033	Shimer (2005)
Value of home-production b	0.4	Shimer (2005)
Cost of vacancy c	0.213	Shimer (2005)
Estimated Parameters:		
Productivity process AR-1	$\rho = 0.953$ $\sigma_y = 0.0457$	Shimer (2005)
Parameters of focus:		
Match function curvature l	0.799	Estimated in simulations
Bargaining Power γ	0.64	
Prop. of uninformed ζ	0.539	

average labor productivity (Shimer [2005]). I also run experiments with stochastic job destruction rate where $\delta_t = e^{-z_t}\delta$. This allows the job destruction to be high during periods of low productivity and vice-versa.

The match function is taken to as a CES function with job finding probability given by as $p(\theta) = \dfrac{\theta}{(1+\theta^l)^l}$; corresponding vacancy filling probability $q(\theta) = \dfrac{1}{(1+\theta^l)^l}$. This function keeps the probability bounded in $[0,1]$. The parameter l is determined in the estimation to match the elasticity of probability of job finding with respect to market tightness of 0.5 as reported in Petrongolo and Pissarides [2001].[3] I target an average monthly job finding rate of 0.45 as reported in Shimer [2005] for the simulations.

Each simulation is run for 1639 months of which the first 1000 months are discarded. I run 5000 simulations and report the parameters as mean across all simulations at quarterly level. In all specifications the estimated parameters are: the bargaining power γ and the proportion of uninformed workers ζ and these are established by the method of simulated moments. The main data target of the simulation exercise is the elasticity of wages with respect to aggregate productivity 0.449. In alternate specification, I estimate the cost c for vacancy creation while adding the quarterly standard deviation of vacancy creation as a data target.

[3] The curvature of match function has multiple estimates in the literature. For example, Shimer [2005] uses a value of 0.72, Cooper et al. [2007] estimate a value 0.36, while Hall [2005] report a value of 0.24.

4. Results

The results of simulations to estimate $\Theta = \{\zeta, \gamma, l\}$, i.e., the proportion of uniformed workers, the bargaining power and the curvature of match function are reported in table 2. The model performs very well on the targeted moments. The elasticity of wages with respect to productivity is 0.448 which is driven by the composition of uninformed workers who get fixed wage contracts while full information workers have an elasticity of 0.9746. The latter value is the major criticism of DMP search and matching framework with Nash Bargaining in that it leads to wages which absorb all volatility of the productivity, i.e., the 'Shimer Puzzle' The estimated proportion of uninformed workers in the economy is 0.539, which suggests that more than half of the workforce is uninformed. The estimated bargaining power of 0.64 for workers is lower than the 0.72 value taken by Shimer [2005].

Table 2—Data vs Model: Estimating $\Theta = \{\zeta, \gamma, l\}$

Moment	Data	Model
TARGETED		
Average monthly job finding rate	0.45	0.45
Elasticity of job finding w.r.t to θ	0.5	0.5
Elasticity of wages w.r.t agg. productivity	0.449	0.448
UN-TARGETED		
Qtr. std. dev. of vacancy rate	0.202	0.256
Qtr. std. dev. of job finding rate	0.118	0.199
Qtr. std. dev. of unemployment rate	0.190	0.171
Qtr. std. dev. of market tightness	0.382	0.399

The model does a good job in terms of the non-targeted moments of the data. The volatility of vacancy rate is higher than the data and correspondingly, there is increased volatility in job finding rate. The model is able to replicate volatility in the unemployment rate and the market tightness thereby resolving the 'Shimer Puzzle'.

The excess volatility in vacancy rate is due to the cost of vacancy creation of 0.213 which is lower than that argued by Hagedorn and Manovskii [2008] and Pissarides [2009] among others. To fit the data better on the volatility of vacancy rate, I use it as a targeted data moment and re-estimate the model by allowing the cost of vacancy creation to be estimated within the simulations. The results are as below.

Table 3—Data vs Model: Estimating $\Theta = \{\zeta, \gamma, l, c\}$ with home-production $b = 0.4$

Moment	Data	Model
TARGETED		
Average monthly job finding rate	0.45	0.45
Elasticity of job finding w.r.t to θ	0.5	0.5
Elasticity of wages w.r.t agg. productivity	0.449	0.449
Qtr. std. dev. of vacancy rate	0.202	0.202
UN-TARGETED		
Qtr. std. dev. of job finding rate	0.118	0.161
Qtr. std. dev. of unemployment rate	0.190	0.141
Qtr. std. dev. of market tightness	0.382	0.324

The targeted moments are matched perfectly. The corresponding estimated parameter values are: proportion of uninformed workers $\zeta = 0.536$, the bargaining power $\gamma = 0.596$, the curvature of match function $l = 0.805$, and the cost of creating vacancies $c = 0.283$. Notice that the proportion of uniformed workers does not change much between first set of simulations and the current one. The parameter for proportion of uninformed workers is driven primarily by the elasticity of wage with respect to aggregate productivity which is determined by the curvature of match function l and the home production value b.

4.1. Effect of home-production value b

The value of home-production has been contentious in the literature. Shimer set b to equal 0.4 to match a 40% replacement rate of unemployment benefits. Rudanko [2011] argues that this value should be calibrated to match the observed drop in consumption upon job loss for models where workers live hand-to-mouth.[4] Hagedorn and Manovskii [2008] take the non-market activity to be 0.955 but bargaining power of the workers in their model is 0.052 to match the wage elasticity relative to productivity. Hall and Milgrom [2008] estimate a value of 0.71 to match unemployment volatility induced by productivity fluctuations.

As the focus of the paper is the unemployment volatility, I run simulations with non-market activity level of 0.71 estimating the parameters $\Theta = \{\zeta, \gamma, l, c\}$, i.e., the proportion

[4]See Aguiar and Hurst [2005], Browning and Crossley [2001], Gruber [1994], Stephens Jr [2001] for estimates of reduction in consumption on earnings shock.

of uniformed workers, the bargaining power, the curvature of match function and the cost of creating vacancies[5]

Table 4—Data vs Model: Estimating $\Theta = \{\zeta, \gamma, l, c\}$ with home-production $b = 0.71$

Moment	Data	Model
TARGETED		
Average monthly job finding rate	0.45	0.45
Elasticity of job finding w.r.t to θ	0.5	0.5
Elasticity of wages w.r.t agg. productivity	0.449	0.449
Qtr. std. dev. of vacancy rate	0.202	0.201
UN-TARGETED		
Qtr. std. dev. of job finding rate	0.118	0.162
Qtr. std. dev. of unemployment rate	0.190	0.141
Qtr. std. dev. of market tightness	0.382	0.325

The corresponding estimated parameter values are: proportion of uninformed workers $\zeta = 0.503$, the bargaining power $\gamma = 0.403$, the curvature of match function $l = 0.805$, and the cost of creating vacancies $c = 0.283$. Here, as the non-market activity or home production value increases, the bargaining power of workers has to decline to ensure that firms still create vacancies. The proportion of uninformed workers also declines as the elasticity of wages relative to aggregate productivity for full information workers declines from about 0.97 in baseline specification to approximately 0.90 in this estimation.

Table 3 and table 4 produce the exact same results with two different parameter estimations for proportion of uninformed workers and the bargaining power. With cost of creating vacancies estimated at $c = 0.283$, the increase in home production value from $b = 0.4$ to $b = 0.71$ decreases the proportion of uninformed workers from $\zeta = 0.539$ to $\zeta = 0.503$ while the bargaining power reduces from $\gamma = 0.596$ to $\gamma = 0.403$.

4.2. Effect of exogenous separation δ

An increase in the separation or job destruction rate δ makes job duration to decrease and therefore both firms and workers are worse off. For uninformed workers, the perceived match surplus declines and consequently the perceived market tightness and the

[5]Estimation while keeping cost of creating vacancies at $c = 0.213$ as in baseline specification also produce similar results and are available on request.

perceived job finding rate declines. Uniformed workers' perceived outside option falls and therefore the wage demand is lower. From firm's perspective, the reduced duration of job match lowers the continuation value of hiring an uninformed worker, but the reduced wage demand increases the continuation value. The overall effect depends upon the relative size of these two opposing forces and driven by the bargaining power.

Stochastic Process for separation rate: An important consideration in the data is that job destruction rate is not constant over the business cycle. A comprehensive theory of endogenous separation would be appropriate to match the data. However, an approximation in this regard is to allow job destruction to be variable through the business cycle in the simulations. I generate a stochastic separation rate $\delta_t = e^{-z_t}\delta$ where $\delta = 0.033$. The firm decisions are same as in the benchmark case with constant job destruction but while simulating the model, I impose the stochastic job destruction rate that is high during periods of aggregate productivity and vice versa.

Table 5—Data vs Model: Estimating $\Theta = \{\zeta, \gamma, l, c\}$ with stochastic job destruction

Moment	Data	Model
TARGETED		
Average monthly job finding rate	0.45	0.45
Elasticity of job finding w.r.t to θ	0.5	0.5
Qtr. std. dev. of vacancy rate	0.202	0.202
Elasticity of wages w.r.t agg. productivity	0.449	0.449
UN-TARGETED		
Qtr. std. dev. of job finding rate	0.118	0.178
Qtr. std. dev. of unemployment rate	0.190	0.182
Qtr. std. dev. of market tightness	0.382	0.357

The corresponding estimated parameter values are: proportion of uninformed workers $\zeta = 0.537$, the bargaining power $\gamma = 0.62$, and the curvature of match function $l = 0.792$. This specification matches the data best, with only the volatility of job finding rate higher than in data.

5. Conclusion

Empirical findings of Shimer [2005] brought to light shortcomings of the standard Diamond Mortensen Pissarides search model in matching data from U.S. unemployment time series. The seminal work opened up multiple avenues to address the puzzle of large volatility in unemployment and vacancy creation present in the data in response to relatively small changes in aggregate productivity. However, much of the literature has focused on homogeneous workers that are fully informed. Recent empirical evidence suggests that workers' beliefs are heterogeneous and do not change over an unemployment spell. In this paper, I move away from the assumption of rational belief with complete information and model workers as having beliefs similar to Menzio [2022]; which shows that introducing workers with stubborn beliefs provides increased elasticity of response of market tightness to aggregate shock.

This project argues that existence of uninformed workers can prompt firms to respond accordingly. This allows disentanglement of perception from actual labor market conditions which provides for responses to aggregate productivity that are larger than the full information rational expectations equilibrium. Using reasonable calibration of the model, I find that an economy with about half of the pool of unemployed workers being uninformed is able to resolve the 'unemployment-volatility puzzle'. The question on source of such information frictions is fascinating and an area of future research.

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Appendix A: Binmore et al. [1986] offer-counter-offer bargaining

In this appendix, I elaborate the wage determination as illustrated in Menzio [2022]. In a Binmore et al. [1986] offer-counter-offer setup, parties to a negotiation propose their offer to the other which can be rejected or accepted. In case the offer is accepted, the negotiation is complete and allocation is decided based on the offer. In the offer is rejected, the negotiation breaks down with some probability. In case the negotiation continues, the other party gets to propose their offer and the game continues.

In this context, let $1 - e^{-\lambda\Delta}$ be the probability that negotiation breaks down after worker's offer is rejected by the firm. Similarly, let $1 - e^{-\mu\Delta}$ be the probability that negotiation breaks down after firm's offer is rejected by the worker. The complementary probability defines the probability of negotiations continuing to next round. Here, $\lambda > 0; \mu > 0;$ are parameters and $\Delta > 0$ is the time step between two successive negotiations. An important assumption in this regard is that there is no learning or updating of belief after an offer is rejected, i.e., when worker's offer is rejected, the worker will make the same offer on their next turn.

Let value from unemployment for type N be $\hat{U}$ and that for type R by $U_R(y)$. Similarly, let value for employment for type N be $\hat{V}$ and that for type R be $V_R(y)$. Perceived probability of finding a job in next period for the type N is constant $p(\theta_N)$ as the perceived labor market tightness θ_N is constant. The perceived probability of finding a job next period for the type R is $p(\theta(y'))$ which is a function of next period's productivity. The value from unemployment is written as:

$$U_R(y) = b + \beta \mathbf{E}_{|y}[p(\theta(y'))V_R(y', w(y')) + \{1 - p(\theta(y'))\}U_R(y')] \tag{10}$$

$$\hat{U} = b + \beta[p(\theta_N)\hat{V}(w(y^*)) + \{1 - p(\theta_N)\}\hat{U}] \tag{11}$$

The value from employment is written as:

$$V_R(y, w(y)) = w(y) + \beta \mathbf{E}_{|y}[\delta U_R(y') + (1 - \delta)V_R(y', w(y'))] \tag{12}$$

$$\hat{V}(w(y^*)) = w(y^*) + \beta[\delta\hat{U} + (1 - \delta)\hat{V}(w(y^*))] \tag{13}$$

Interaction with type N worker : From type N worker's perspective, firm's surplus for any wage $w(y^*)$ is

$$\hat{J}(y^*, w(y^*)) = y^* - w(y^*) + \beta[(1-\delta)\hat{J}(y^*, w(y^*))] \quad (14)$$

$$\hat{J}(y^*, w(y^*)) = \frac{y^* - w(y^*)}{1-\beta(1-\delta)} = \frac{y^* - w(y^*)}{A} \quad (15)$$

Here, $A \equiv 1 - \beta(1-\delta)$.

Let the wage offered by firm be $w_{N,o}$ and $w_{N,d}$ be the wage demanded by worker. When worker demands wage $w_{N,d}$, they are trying to make firm indifferent between accepting the wage demand $w_{N,d}$ and rejecting the offer, and putting wage offer as $w_{N,o}$, i.e., workers solve

$$\hat{J}(y^*, w_{N,d}) = e^{-\lambda\Delta}\hat{J}(y^*, w_{N,o}) \quad (16)$$

$$y^* - w_{N,d} = e^{-\lambda\Delta}(y^* - w_{N,o}) \quad (17)$$

$$w_{N,d} = (1 - e^{-\lambda\Delta})y^* + e^{-\lambda\Delta}w_{N,o} \quad (18)$$

Here, equation (16) has the perceived value to firm if it accepts the wage demand $w_{N,d}$ on the left hand side and the perceived value to firm if it rejects and subsequently makes an offer $w_{N,o}$ which gets accepted. The firm's strategy is a best response to worker's belief. They offer $w_{N,o}$ to make worker indifferent between accepting in this round of negotiation and rejecting $w_{N,o}$ in which case the worker will get $\hat{U}$ if negotiation breaks down or demand $w_{N,d}$ in next round which the firm accepts and get $\hat{V}(w_{N,d})$.Therefore, the firm solves:

$$\hat{V}(w_{N,o}) = (1 - e^{-\mu\Delta})\hat{U} + e^{-\mu\Delta}\hat{V}(w_{N,d}) \quad (19)$$

$$\frac{w_{N,o}}{A} + \frac{\beta\delta\hat{U}}{A} = (1 - e^{-\mu\Delta})\hat{U} + e^{-\mu\Delta}[\frac{w_{N,d}}{A} + \frac{\beta\delta\hat{U}}{A}] \quad (20)$$

$$w_{N,o} = (1 - e^{-\mu\Delta})(A\hat{U} - \beta\delta\hat{U}) + e^{-\mu\Delta}w_{N,d} \quad (21)$$

$$w_{N,o} = (1 - e^{-\mu\Delta})(1-\beta)\hat{U} + e^{-\mu\Delta}w_{N,d} \quad (22)$$

Here, left had side of eq (19) has the perceived value of accepting the wage $w_{N,o}$ for worker. The right had side of eq (19) has the combined value from rejecting $w_{N,o}$ that includes value from negotiation breakdown and value from getting wage demand $w_{N,d}$ accepted in next round. Solving (18) and (22) together, we get

$$w_{N,o} = \frac{(1-e^{-\mu\Delta})}{(1-e^{-(\mu+\lambda)\Delta}}(1-\beta)\hat{U} + \frac{(1-e^{-\lambda\Delta})}{(1-e^{-(\mu+\lambda)\Delta}}e^{-\mu\Delta}y^* \tag{23}$$

$$w_{N,d} = \frac{(1-e^{-\mu\Delta})}{(1-e^{-(\mu+\lambda)\Delta}}e^{-\lambda\Delta}(1-\beta)\hat{U} + \frac{(1-e^{-\lambda\Delta})}{(1-e^{-(\mu+\lambda)\Delta}}y^* \tag{24}$$

Taking the limit $\Delta \to 0$ and denoting $\gamma = \frac{\lambda}{\lambda+\mu}$ as worker's bargaining power, we get

$$w_{N,d} = w_{N,o} = \gamma y^* + (1-\gamma)(1-\beta)\hat{U} \tag{25}$$

Using a similar analysis we can find the wage offer and wage demand for type R. The difference arises in aggregate productivity and the continuation value of unemployment.

$$w_{R,o} = \frac{(1-e^{-\mu\Delta})}{(1-e^{-(\mu+\lambda)\Delta})}(1-\beta)U_R(y) + \frac{(1-e^{-\lambda\Delta})}{(1-e^{-(\mu+\lambda)\Delta})}e^{-\mu\Delta}y \tag{26}$$

$$w_{R,d} = \frac{(1-e^{-\mu\Delta})}{(1-e^{-(\mu+\lambda)\Delta})}e^{-\lambda\Delta}(1-\beta)U_R(y) + \frac{(1-e^{-\lambda\Delta})}{(1-e^{-(\mu+\lambda)\Delta})}y \tag{27}$$

Taking the limit $\Delta \to 0$ and denoting $\gamma = \frac{\lambda}{\lambda+\mu}$ as worker's bargaining power, we get

$$w_{R,d} = w_{R,o} = \gamma y + (1-\gamma)(1-\beta)U_R(y) \tag{28}$$

Appendix B: Perceived unemployment, match surplus and market tightness

Define perceived probability of job finding as $p(\theta_N)$. Using the wage from equation (25), the perceived value of unemployment $\hat{U}$ becomes:

$$
\begin{aligned}
\hat{U} &= b + \beta[p(\theta_N)V(w(y^*)) + \{1 - p(\theta_N)\}\hat{U}] \\
\hat{U} &= b + \beta[p(\theta_N)\{\frac{w(y^*)}{A} + \frac{\beta\delta\hat{U}}{A}\} + \{1 - p(\theta_N)\}\hat{U}] \\
A\hat{U} &= Ab + \beta p(\theta_N)w(y^*) + \beta[\beta\delta\hat{U}p(\theta_N) - p(\theta_N)(1 - \beta + \beta\delta))\hat{U} + A\hat{U}] \\
A\hat{U} &= Ab + \beta p(\theta_N)w(y^*) + \beta[A\hat{U} - p(\theta_N)(1 - \beta)\hat{U}] \\
A\hat{U} &= Ab + \beta p(\theta_N)[\gamma y^* + (1 - \gamma)(1 - \beta)U] + \beta[A\hat{U} - p(\theta_N)(1 - \beta)\hat{U}] \\
A\hat{U} &= Ab + \beta p(\theta_N)\gamma y^* + \beta[p(\theta_N)(1 - \gamma)(1 - \beta)\hat{U} + A\hat{U} - p(\theta_N)(1 - \beta)\hat{U}] \\
A\hat{U} &= Ab + \beta p(\theta_N)\gamma y^* + \beta[p(\theta_N)(-\gamma)(1 - \beta)\hat{U} + A\hat{U}] \\
\hat{U} &= \frac{Ab + \beta p(\theta_N)\gamma y^*}{(A + p(\theta_N)\gamma\beta)(1 - \beta)} \qquad (29)
\end{aligned}
$$

Similarly, solving for perceived value of firm's profit, $\hat{J}(y^*, w(y^*))$

$$
\begin{aligned}
\hat{J}(y^*, w(y^*)) &= \frac{y^* - w(y^*)}{A} \\
&= \frac{y^* - \gamma y^* - (1 - \gamma)(1 - \beta)\hat{U}}{A} \\
&= \frac{1 - \gamma}{A}\{y^* - (1 - \beta)\hat{U}\} \\
&= \frac{1 - \gamma}{A}\{y^* - (1 - \beta)\frac{Ab + \beta p(\theta_N)\gamma y^*}{(A + p(\theta_N)\gamma\beta)(1 - \beta)}\} \\
&= \frac{1 - \gamma}{A}\{y^* - \frac{Ab + \beta p(\theta_N)\gamma y^*}{(A + p(\theta_N)\gamma\beta)}\} \\
&= \frac{1 - \gamma}{A(A + p(\theta_N)\gamma\beta)}\{Ay^* - Ab\} \\
\hat{J}(y^*, w(y^*)) &= \frac{1 - \gamma}{(A + p(\theta_N)\gamma\beta)}\{y^* - b\} \qquad (30)
\end{aligned}
$$

From the equation (??) of vacancy creation, the perceived market tightness would be:

$$
\begin{aligned}
c &= q(\theta_N)\hat{J}(y^*, w(y^*)) \\
c &= q(\theta_N)\frac{1-\gamma}{(A + p(\theta_N)\gamma\beta)}\{y^* - b\} \\
c(1 - \beta(1-\delta)) &= q(\theta_N)(1-\gamma)\{y^* - b\} - cp(\theta_N)\gamma\beta
\end{aligned} \tag{31}
$$

Effect of discount rate β

$\frac{\partial \theta_N}{\partial \beta}$: Differentiating equation (31) with respect to β, we have :

$$
\begin{aligned}
c(-(1-\delta)) &= q'(\theta_N)(1-\gamma)\{y^* - b\}\frac{\partial \theta_N}{\partial \beta} - cp(\theta_N)\gamma - cp'(\theta_N)\gamma\beta\frac{\partial \theta_N}{\partial \beta} \\
c\left[(1-\delta) - p(\theta_N)\gamma\right] &= \frac{\partial \theta_N}{\partial \beta}\left[cp'(\theta_N)\gamma\beta - q'(\theta_N)(1-\gamma)\{y^* - b\}\right] \\
\frac{\partial \theta_N}{\partial \beta} &= \frac{c\left[(1-\delta) - p(\theta_N)\gamma\right]}{\left[cp'(\theta_N)\gamma\beta - q'(\theta_N)(1-\gamma)\{y^* - b\}\right]}
\end{aligned} \tag{32}
$$

For, $p'(\theta_N) > 0$ and $q'(\theta_N) < 0$, the denominator on RHS of equation (32) is positive. Therefore, $\frac{\partial \theta_N}{\partial \beta} > 0$ if $(1-\delta) - p(\theta_N)\gamma > 0$.

$\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \beta}$: Define $\kappa \equiv [cp'(\theta_N)\gamma\beta - q'(\theta_N)(1-\gamma)\{y^* - b\}]$. Differentiating equation (30) with respect to β and using (32), we have :

$$
\begin{aligned}
\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \beta} &= -\frac{(1-\gamma)(y^* - b)}{(A + p(\theta_N)\gamma\beta)^2}\left(-(1-\delta) + p(\theta_N)\gamma + \frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \beta}\gamma\beta\right) \\
\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \beta} &= \frac{(1-\gamma)(y^* - b)\left[\kappa(1-\delta) - \kappa p(\theta_N)\gamma + \frac{\partial p(\theta_N)}{\partial \theta_N}c\left[(1-\delta) - p(\theta_N)\gamma\right]\gamma\beta\right]}{(A + p(\theta_N)\gamma\beta)^2\kappa} \\
\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \beta} &= \frac{(1-\gamma)(y^* - b)\left[(1-\delta)cp'(\theta_N)\gamma\beta - (1-\delta)q'(\theta_N)(1-\gamma)\{y^* - b\} - p(\theta_N)\gamma cp'(\theta_N)\gamma\beta\right]}{(A + p(\theta_N)\gamma\beta)^2\kappa} \\
&\quad - \frac{(1-\gamma)(y^* - b)\left[-p(\theta_N)\gamma q'(\theta_N)(1-\gamma)\{y^* - b\} + p'(\theta_N)c\left[(1-\delta) - p(\theta_N)\gamma\right]\gamma\beta\right]}{(A + p(\theta_N)\gamma\beta)^2\kappa} \\
\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \beta} &= \frac{(1-\gamma)(y^* - b)\left[-q'(\theta_N)(1-\gamma)\{y^* - b\}\left[(1-\delta) - p(\theta_N)\gamma\right]\right]}{(A + p(\theta_N)\gamma\beta)^2\kappa}
\end{aligned} \tag{33}
$$

As $-q'(\theta_N) > 0$, $\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \beta} > 0$ if $(1-\delta) - p(\theta_N)\gamma > 0$

Effect of discount rate β on perceived value of unemployment $\hat{U}$:

Differentiating equation (29) with respect to β, we have :

$$\frac{\partial \hat{U}}{\partial \beta} = \frac{[(A+p(\theta_N)\gamma\beta)(1-\beta)]\frac{\partial[Ab+\beta p(\theta_N)\gamma y^*]}{\partial\beta} - [Ab+\beta p(\theta_N)\gamma y^*]\frac{\partial[(A+p(\theta_N)\gamma\beta)(1-\beta)]}{\partial\beta}}{[(A+p(\theta_N)\gamma\beta)(1-\beta)]^2} \tag{34}$$

As the denominator in (34) is positive, we can focus on the sign of only the numerator. Let $DNR \equiv [(A+p(\theta_N)\gamma\beta)(1-\beta)]^2$, and solving further:

$$\begin{aligned}
DNR\frac{\partial \hat{U}}{\partial \beta} &= [(A+p(\theta_N)\gamma\beta)(1-\beta)]\frac{\partial[Ab+\beta p(\theta_N)\gamma y^*]}{\partial\beta} - [Ab+\beta p(\theta_N)\gamma y^*]\frac{\partial[(A+p(\theta_N)\gamma\beta)(1-\beta)]}{\partial\beta} \\
DNR\frac{\partial \hat{U}}{\partial \beta} &= [(A+p(\theta_N)\gamma\beta)(1-\beta)][-(1-\delta)b + p(\theta_N)\gamma y^* + \beta\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma y^*] \\
&\quad - [Ab+\beta p(\theta_N)\gamma y^*][\{-(1-\delta) + p(\theta_N)\gamma + \frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma\beta\}(1-\beta) - (A+p(\theta_N)\gamma\beta)] \\
DNR\frac{\partial \hat{U}}{\partial \beta} &= -(A+p(\theta_N)\gamma\beta)(1-\beta)(1-\delta)b + (A+p(\theta_N)\gamma\beta)(1-\beta)p(\theta_N)\gamma y^* \\
&\quad + (A+p(\theta_N)\gamma\beta)(1-\beta)\beta\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma y^* + [Ab+\beta p(\theta_N)\gamma y^*](A+p(\theta_N)\gamma\beta) \\
&\quad + [Ab+\beta p(\theta_N)\gamma y^*](1-\delta)(1-\beta) - [Ab+\beta p(\theta_N)\gamma y^*]p(\theta_N)\gamma(1-\beta) \\
&\quad - (Ab+\beta p(\theta_N)\gamma y^*)\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma\beta(1-\beta) \\
DNR\frac{\partial \hat{U}}{\partial \beta} &= -(A+p(\theta_N)\gamma\beta)(1-\beta)(1-\delta)b + (A+p(\theta_N)\gamma\beta)(1-\beta)p(\theta_N)\gamma y^* \\
&\quad + (A+\cancel{p(\theta_N)\gamma\beta})(1-\beta)\beta\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma y^* + [Ab+\beta p(\theta_N)\gamma y^*](A+p(\theta_N)\gamma\beta) \\
&\quad + [Ab+\beta p(\theta_N)\gamma y^*](1-\delta)(1-\beta) - [Ab+\beta p(\theta_N)\gamma y^*]p(\theta_N)\gamma(1-\beta) \\
&\quad - (Ab+\cancel{\beta p(\theta_N)\gamma y^*})\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma\beta(1-\beta) \\
DNR\frac{\partial \hat{U}}{\partial \beta} &= -(A+p(\theta_N)\gamma\beta)(1-\beta)(1-\delta)b + (A+p(\theta_N)\gamma\beta)(1-\beta)p(\theta_N)\gamma y^* \\
&\quad + A(1-\beta)\beta\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma y^* + [Ab+\beta p(\theta_N)\gamma y^*](A+p(\theta_N)\gamma\beta) \\
&\quad + [Ab+\beta p(\theta_N)\gamma y^*](1-\delta)(1-\beta) - [Ab+\beta p(\theta_N)\gamma y^*]p(\theta_N)\gamma(1-\beta) \\
&\quad - Ab\frac{\partial p(\theta_N)}{\partial\theta_N}\frac{\partial\theta_N}{\partial\beta}\gamma\beta(1-\beta)
\end{aligned}$$

Collecting similar terms and solving further,

$$
\begin{aligned}
DNR\frac{\partial \hat{U}}{\partial \beta} &= (A + p(\theta_N)\gamma\beta)[(1-\beta)p(\theta_N)\gamma y^* + \beta(1-\delta)b - (1-\delta)b + Ab + \beta p(\theta_N)\gamma y^*] \\
&\quad + A(1-\beta)\beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \beta}\gamma[y^* - b] + [Ab + \beta p(\theta_N)\gamma y^*][(1-\delta) - p(\theta_N)\gamma] \\
DNR\frac{\partial \hat{U}}{\partial \beta} &= (A + p(\theta_N)\gamma\beta)[p(\theta_N)\gamma y^* + \delta b] + A(1-\beta)\beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \beta}\gamma[y^* - b] \\
&\quad + [Ab + \beta p(\theta_N)\gamma y^*][(1-\delta) - p(\theta_N)\gamma] \qquad (35)
\end{aligned}
$$

As all terms of equation (35) are positive if $(1-\delta) - p(\theta_N)\gamma > 0$, we have $\frac{\partial \hat{U}}{\partial \beta} > 0$.

Effect of discount rate β on wage w_N :

Wage w_N can be written using (25) and (29) as,

$$
\begin{aligned}
w_N &= \gamma y^* + (1-\gamma)(1-\beta)\hat{U} \\
&= \gamma y^* + (1-\gamma)\frac{Ab + \beta p(\theta_N)\gamma y^*}{(A + p(\theta_N)\gamma\beta)} \\
&= \gamma y^* + (1-\gamma)\frac{Ab + Ay^* - Ay^* + \beta p(\theta_N)\gamma y^*}{(A + p(\theta_N)\gamma\beta)} \\
&= \gamma y^* + (1-\gamma)\frac{Ab - Ay^* + y^*(A + \beta p(\theta_N)\gamma)}{(A + p(\theta_N)\gamma\beta)} \\
&= \gamma y^* + (1-\gamma)y^* + (1-\gamma)\frac{Ab - Ay^*}{(A + p(\theta_N)\gamma\beta)} \\
&= y^* + (1-\gamma)\frac{A(b - y^*)}{(A + p(\theta_N)\gamma\beta)} \qquad (36)
\end{aligned}
$$

Differentiating equation (36) with respect to β, we have :

$$
\begin{aligned}
\frac{\partial w_N}{\partial \beta} &= (1-\gamma)\frac{(A + p(\theta_N)\gamma\beta)\left[-(b - y^*)(1-\delta)\right] - A(b - y^*)\left[-(1-\delta) + \beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \beta}\gamma + p(\theta_N)\gamma\right]}{(A + p(\theta_N)\gamma\beta)^2} \\
\frac{\partial w_N}{\partial \beta} &= \frac{(A + p(\theta_N)\gamma\beta)\left[(y^* - b)(1-\delta)\right] + A(y^* - b)\left[-(1-\delta) + \beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \beta}\gamma + p(\theta_N)\gamma\right]}{(1-\gamma)^{-1}(A + p(\theta_N)\gamma\beta)^2} \\
\frac{\partial w_N}{\partial \beta} &= \frac{p(\theta_N)\gamma(y^* - b) + A(y^* - b)\left[\beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \beta}\gamma\right]}{(1-\gamma)^{-1}(A + p(\theta_N)\gamma\beta)^2} \qquad (37)
\end{aligned}
$$

As $\frac{\partial p(\theta_N)}{\partial \theta_N} > 0$ and $\frac{\partial \theta_N}{\partial \beta} > 0$ if $(1-\delta) - p(\theta_N)\gamma > 0$, we have $\frac{\partial w_N}{\partial \beta} > 0$.

Effect of separation rate δ

On perceived market tightness $\frac{\partial \theta_N}{\partial \delta}$:

Differentiating equation (31) with respect to δ, we have :

$$\begin{aligned} c\beta &= q'(\theta_N)(1-\gamma)\{y^* - b\}\frac{\partial \theta_N}{\partial \delta} - cp'(\theta_N)\gamma\beta\frac{\partial \theta_N}{\partial \delta} \\ c\beta &= \frac{\partial \theta_N}{\partial \delta}\left[q'(\theta_N)(1-\gamma)\{y^* - b\} - cp'(\theta_N)\gamma\beta\right] \\ \frac{\partial \theta_N}{\partial \delta} &= \frac{c\beta}{\left[q'(\theta_N)(1-\gamma)\{y^* - b\} - cp'(\theta_N)\gamma\beta\right]} \end{aligned} \tag{38}$$

For, $p'(\theta_N) > 0$ and $q'(\theta_N) < 0$, the denominator on RHS of equation (38) is negative. Therefore, $\frac{\partial \theta_N}{\partial \delta} < 0$.

On perceived value of firm's profit $\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \delta}$:

Define $\kappa \equiv [cp'(\theta_N)\gamma\beta - q'(\theta_N)(1-\gamma)\{y^* - b\}]$. Differentiating equation (30) with respect to β and using (32), we have :

$$\begin{aligned} \frac{\partial \hat{J}(y^*, w(y^*))}{\partial \delta} &= -\frac{(1-\gamma)(y^* - b)}{(A + p(\theta_N)\gamma\beta)^2}\left[\beta + p'(\theta_N)\frac{\partial \theta_N}{\partial \delta}\gamma\beta\right] \\ \frac{\partial \hat{J}(y^*, w(y^*))}{\partial \delta} &= \frac{(1-\gamma)(y^* - b)\left[\beta cp'(\theta_N)\gamma\beta - \beta q'(\theta_N)(1-\gamma)\{y^* - b\} - p'(\theta_N)c\beta\gamma\beta\right]}{-(A + p(\theta_N)\gamma\beta)^2\kappa} \\ \frac{\partial \hat{J}(y^*, w(y^*))}{\partial \delta} &= \frac{(1-\gamma)(y^* - b)\left[-\beta q'(\theta_N)(1-\gamma)\{y^* - b\}\right]}{-(A + p(\theta_N)\gamma\beta)^2\kappa} \end{aligned} \tag{39}$$

As $-q'(\theta_N) > 0$ and $\kappa > 0$, $\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \delta} < 0$.

On perceived value of unemployment $\frac{\partial \hat{U}}{\partial \delta}$: Differentiating equation (29) with respect to δ, we have :

$$(1-\beta)\frac{\partial \hat{U}}{\partial \delta} = \frac{[(A + p(\theta_N)\gamma\beta)]\frac{\partial [Ab + \beta p(\theta_N)\gamma y^*]}{\partial \delta} - [Ab + \beta p(\theta_N)\gamma y^*]\frac{\partial [(A + p(\theta_N)\gamma\beta)]}{\partial \delta}}{[(A + p(\theta_N)\gamma\beta)]^2} \tag{40}$$

As the denominator in (40) is positive, we can focus on the sign of only the numerator.

Let $DNR \equiv [A + p(\theta_N)\gamma\beta]^2$, and solving further:

$$(1-\beta)DNR\frac{\partial \hat{U}}{\partial \delta} = [A + p(\theta_N)\gamma\beta](\beta b + \beta p'(\theta_N)\gamma y^* \frac{\partial \theta_N}{\partial \delta}) - [Ab + \beta p(\theta_N)\gamma y^*](\beta + p'(\theta_N)\gamma\beta\frac{\partial \theta_N}{\partial \delta})$$

$$(1-\beta)DNR\frac{\partial \hat{U}}{\partial \delta} = [A\beta b + \beta b p(\theta_N)\gamma\beta + A\beta p'(\theta_N)\gamma y^* \frac{\partial \theta_N}{\partial \delta} + p(\theta_N)\gamma\beta\beta p'(\theta_N)\gamma y^* \frac{\partial \theta_N}{\partial \delta}]$$
$$- [Ab\beta + \beta p(\theta_N)\gamma y^* \beta + Abp'(\theta_N)\gamma\beta\frac{\partial \theta_N}{\partial \delta} + \beta p(\theta_N)\gamma y^* p'(\theta_N)\gamma\beta\frac{\partial \theta_N}{\partial \delta}]$$

$$(1-\beta)DNR\frac{\partial \hat{U}}{\partial \delta} = A\beta p'(\theta_N)\gamma\frac{\partial \theta_N}{\partial \delta}(y^* - b) - \beta p(\theta_N)\beta\gamma(y^* - b) \quad (41)$$

As $\frac{\partial \theta_N}{\partial \delta} < 0$ and $p'(\theta_N) > 0$, right hand side of equation (41) are negative, we have $\frac{\partial \hat{U}}{\partial \delta} < 0$.

Effect of discount rate δ on wage $\frac{\partial w_N}{\partial \delta}$: Differentiating equation (25) with respect to δ, we have :

$$\frac{\partial w_N}{\partial \delta} = (1-\gamma)(1-\beta)\frac{\partial \hat{U}}{\partial \delta} \quad (42)$$

As $\frac{\partial \hat{U}}{\partial \delta} < 0$ we have $\frac{\partial w_N}{\partial \delta} < 0$.

Effect of bargaining power γ

$\dfrac{\partial\theta_N}{\partial\gamma}$: Rewriting equation (31) as :

$$cp(\theta_N)\gamma\beta = q(\theta_N)(1-\gamma)\{y^* - b\} - c(1-\beta(1-\delta)) \tag{43}$$

Differentiating equation (43) with respect to γ, we have :

$$\begin{aligned}
cp(\theta_N)\beta + cp'(\theta_N)\gamma\beta\frac{\partial\theta_N}{\partial\gamma} &= q'(\theta_N)(1-\gamma)\{y^*-b\}\frac{\partial\theta_N}{\partial\gamma} - q(\theta_N)\{y^*-b\} \\
cp(\theta_N)\beta + q(\theta_N)\{y^*-b\} &= \frac{\partial\theta_N}{\partial\gamma}\left[-cp'(\theta_N)\gamma\beta + q'(\theta_N)(1-\gamma)\{y^*-b\}\right] \\
\frac{\partial\theta_N}{\partial\gamma} &= \frac{cp(\theta_N)\beta + q(\theta_N)\{y^*-b\}}{[q'(\theta_N)(1-\gamma)\{y^*-b\} - cp'(\theta_N)\gamma\beta]} \qquad (44)
\end{aligned}$$

For, $p'(\theta_N) > 0$ and $q'(\theta_N) < 0$, the denominator on RHS of equation (44) is negative. Therefore, $\frac{\partial\theta_N}{\partial\gamma} < 0$.

Effect of bargaining power γ on perceived value of unemployment $\hat{U}$:

Differentiating equation (29) with respect to γ, we have :

$$(1-\beta)\frac{\partial\hat{U}}{\partial\gamma} = \frac{[(A+p(\theta_N)\gamma\beta)]\frac{\partial[Ab+\beta p(\theta_N)\gamma y^*]}{\partial\gamma} - [Ab+\beta p(\theta_N)\gamma y^*]\frac{\partial[(A+p(\theta_N)\gamma\beta)]}{\partial\gamma}}{[(A+p(\theta_N)\gamma\beta)]^2} \tag{45}$$

As the denominator in (45) is positive, we can focus on the sign of only the numerator. Let $DNR \equiv [(A+p(\theta_N)\gamma\beta)]^2(1-\beta)$, and solving further:

$$\begin{aligned}
DNR\frac{\partial\hat{U}}{\partial\gamma} &= [A+p(\theta_N)\gamma\beta][\beta p(\theta_N)y^* + \beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma y^*] - [Ab+\beta p(\theta_N)\gamma y^*][p(\theta_N)\beta + \beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma] \\
DNR\frac{\partial\hat{U}}{\partial\gamma} &= \left[A\beta p(\theta_N)y^* + \cancel{p(\theta_N)\gamma\beta\beta p(\theta_N)y^*} + A\beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma y^* + \cancel{p(\theta_N)\gamma\beta\beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma y^*}\right] \\
&\quad - \left[Abp(\theta_N)\beta + \cancel{\beta p(\theta_N)\gamma y^* p(\theta_N)\beta} + Ab\beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma + \cancel{\beta p(\theta_N)\gamma y^*\beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma}\right] \\
DNR\frac{\partial\hat{U}}{\partial\gamma} &= \left[A\beta p(\theta_N)(y^*-b) + A\beta p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma(y^*-b)\right] \\
DNR\frac{\partial\hat{U}}{\partial\gamma} &= A\beta(y^*-b)\left[p(\theta_N) + p'(\theta_N)\frac{\partial\theta_N}{\partial\gamma}\gamma\right] \qquad (46)
\end{aligned}$$

From equation (46), $\frac{\partial \hat{U}}{\partial \gamma} > 0$ if $p(\theta_N) + p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}\gamma > 0$. For the matching function, note that $p(\theta) = \theta q(\theta)$ and $p'(\theta) = q(\theta) + \theta q'(\theta)$. Solving further, $\frac{\partial \hat{U}}{\partial \gamma} > 0$ if

$$\begin{aligned}
p(\theta_N) + p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}\gamma >& 0 \\
p(\theta_N) >& -p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}\gamma \\
p(\theta_N) >& -p'(\theta_N)\frac{cp(\theta_N)\beta + q(\theta_N)\{y^* - b\}}{[q'(\theta_N)(1-\gamma)\{y^* - b\} - cp'(\theta_N)\gamma\beta]}\gamma \\
p(\theta_N) >& p'(\theta_N)\frac{cp(\theta_N)\beta + q(\theta_N)\{y^* - b\}}{[-q'(\theta_N)(1-\gamma)\{y^* - b\} + cp'(\theta_N)\gamma\beta]}\gamma
\end{aligned}$$

As $[-q'(\theta_N)(1-\gamma)\{y^* - b\} + cp'(\theta_N)\gamma\beta] > 0$, multiplying on both sides by it, we get

$$\begin{aligned}
-q'(\theta_N)(1-\gamma)\{y^* - b\}p(\theta_N) + cp'(\theta_N)\gamma\beta p(\theta_N) >& p'(\theta_N)cp(\theta_N)\beta\gamma + p'(\theta_N)q(\theta_N)\{y^* - b\}\gamma \\
-q'(\theta_N)(1-\gamma)\{y^* - b\}p(\theta_N) >& p'(\theta_N)q(\theta_N)\{y^* - b\}\gamma \\
-q'(\theta_N)(1-\gamma)p(\theta_N) >& p'(\theta_N)q(\theta_N)\gamma \\
-q'(\theta_N)(1-\gamma)\theta_N q(\theta_N) >& [\theta_N q'(\theta_N) + q(\theta_N)]q(\theta_N)\gamma \\
-q'(\theta_N)(1-\gamma)\theta_N q(\theta_N) >& \theta_N q'(\theta_N)q(\theta_N)\gamma + q(\theta_N)q(\theta_N)\gamma \\
-q'(\theta_N)\theta_N q(\theta_N) >& q(\theta_N)q(\theta_N)\gamma \\
-\frac{q'(\theta_N)\theta_N}{q(\theta_N)} >& \gamma
\end{aligned} \tag{47}$$

Equation (47) gives a relation between the elasticity of the matching function and the bargaining power. For the case of a Cobb-Douglas matching function $m(u, v) = \mu u^{\lambda} v^{1-\lambda}$, if λ is greater than bargaining power γ, then $\frac{\partial \hat{U}}{\partial \gamma} > 0$. On increasing bargaining power, the workers get more of the match surplus but there is increased risk of not finding a job when unemployed which is governed by the elasticity parameter of matching function λ. For matching functions, we have, $p(\theta) = \theta q(\theta)$ and $p'(\theta) = \theta q'(\theta) + q(\theta)$.
Define, $\frac{q'(\theta_N)\theta_N}{q(\theta_N)} = \eta_q$ and $\frac{p'(\theta_N)\theta_N}{p(\theta_N)} = \eta_p$ as elasticities of vacancy filling and job finding respectively. We have, $\eta_p = 1 + \eta_q$

$\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma}$: Define $\kappa_\gamma \equiv [q'(\theta_N)(1-\gamma)\{y^*-b\} - cp'(\theta_N)\gamma\beta]$. Differentiating equation (30) with respect to γ and using (44), we have :

$$(y^*-b)^{-1}\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} = \frac{(A+p(\theta_N)\gamma\beta)(-1) - (1-\gamma)\left[p(\theta_N)\beta + p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}\gamma\beta\right]}{(A+p(\theta_N)\gamma\beta)^2}$$
$$(y^*-b)^{-1}\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} = \frac{-A - p(\theta_N)\gamma\beta - p(\theta_N)\beta + \gamma p(\theta_N)\beta - (1-\gamma)p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}\gamma\beta}{(A+p(\theta_N)\gamma\beta)^2}$$
$$(y^*-b)^{-1}\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} = -\left[\frac{A + p(\theta_N)\beta + (1-\gamma)p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}\gamma\beta}{(A+p(\theta_N)\gamma\beta)^2}\right]$$
$$(y^*-b)^{-1}\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} = -\left[\frac{A\kappa_\gamma + p(\theta_N)\beta\kappa_\gamma + (1-\gamma)p'(\theta_N)\gamma\beta[cp(\theta_N)\beta + q(\theta_N)\{y^*-b\}]}{(A+p(\theta_N)\gamma\beta)^2\kappa_\gamma}\right]$$

Solving further by using κ_γ,

$$\begin{aligned}(y^*-b)^{-1}\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} &= -\left[\frac{A\kappa_\gamma + p(\theta_N)\beta q'(\theta_N)(1-\gamma)\{y^*-b\} - p(\theta_N)\beta cp'(\theta_N)\gamma\beta}{(A+p(\theta_N)\gamma\beta)^2\kappa_\gamma}\right] \\ &= -\left[\frac{+(1-\gamma)p'(\theta_N)\gamma\beta(cp(\theta_N)\beta + q(\theta_N)\{y^*-b\})}{(A+p(\theta_N)\gamma\beta)^2\kappa_\gamma}\right] \\ (y^*-b)^{-1}\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} &= -\left[\frac{A\kappa_\gamma + p(\theta_N)\beta q'(\theta_N)(1-\gamma)\{y^*-b\} - p'(\theta_N)\beta^2\gamma^2 p(\theta_N)}{(A+p(\theta_N)\gamma\beta)^2\kappa_\gamma}\right] \\ &= -\left[\frac{+(1-\gamma)p'(\theta_N)\gamma\beta q(\theta_N)\{y^*-b\}}{(A+p(\theta_N)\gamma\beta)^2\kappa_\gamma}\right] \qquad (48)\end{aligned}$$

As $\kappa_\gamma < 0$ terms on the denominator of RHS of equation (48) are negative. Therefore,

$$-[(1-\gamma)p'(\theta_N)\gamma\beta q(\theta_N)\{y^*-b\} + p(\theta_N)\beta q'(\theta_N)(1-\gamma)\{y^*-b\} + A\kappa_\gamma - p'(\theta_N)\beta^2\gamma^2 p(\theta_N)] > 0$$
$$(1-\gamma)p'(\theta_N)\gamma\beta q(\theta_N)\{y^*-b\} + p(\theta_N)\beta q'(\theta_N)(1-\gamma)\{y^*-b\} + A\kappa_\gamma < p'(\theta_N)\beta^2\gamma^2 p(\theta_N)$$
$$(1-\gamma)\{y^*-b\}[A + \beta p(\theta_N)q'(\theta_N) + \beta\gamma p'(\theta_N)q(\theta_N)] - Acp'(\theta_N)\gamma\beta < p'(\theta_N)\beta^2\gamma^2 p(\theta_N)$$
$$(1-\gamma)\{y^*-b\}[A/q(\theta_N)^2 + \beta\eta_q + \beta\gamma(1+\eta_q)] - Ac(1+\eta_q)/q(\theta_N)\gamma\beta < (1+\eta_q)\theta\beta^2\gamma^2$$

Solving further, $\frac{\partial \hat{J}(y^*, w(y^*))}{\partial \gamma} < 0$ if the elasticity of job finding holds for below

$$\eta_q < \frac{\theta\beta^2\gamma^2 - (1-\gamma)\{y^*-b\}[A/q(\theta_N)^2 + \beta\gamma] + Ac\gamma\beta/q(\theta_N)}{(1-\gamma)\{y^*-b\}[\beta + \beta\gamma] - Ac\gamma\beta/q(\theta_N) - \theta\beta^2\gamma^2} \qquad (49)$$

Effect of discount rate γ on wage w_N :

$$w_N = y^* + (1-\gamma)\frac{A(b-y^*)}{(A+p(\theta_N)\gamma\beta)}$$

Differentiating wage with respect to γ , we get

$$\frac{\partial w_N}{\partial \gamma} = \frac{(A+p(\theta_N)\gamma\beta)\left[-A(b-y^*)\right] - (1-\gamma)A(b-y^*)\left[\beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \gamma}\gamma + p(\theta_N)\beta\right]}{(A+p(\theta_N)\gamma\beta)^2}$$

$$\left[\frac{(A+p(\theta_N)\gamma\beta)^2}{A(y^*-b)}\right]\frac{\partial w_N}{\partial \gamma} = (A+p(\theta_N)\gamma\beta) + (1-\gamma)\left[\beta\frac{\partial p(\theta_N)}{\partial \theta_N}\frac{\partial \theta_N}{\partial \gamma}\gamma + p(\theta_N)\beta\right]$$

$$\left[\frac{(A+p(\theta_N)\gamma\beta)^2}{A(y^*-b)}\right]\frac{\partial w_N}{\partial \gamma} = A + p(\theta_N)\beta + (1-\gamma)\gamma\beta p'(\theta_N)\frac{\partial \theta_N}{\partial \gamma}$$

$$\left[\frac{(A+p(\theta_N)\gamma\beta)^2}{A(y^*-b)}\right]\frac{\partial w_N}{\partial \gamma} = A + p(\theta_N)\beta + (1-\gamma)\gamma\beta p'(\theta_N)\frac{cp(\theta_N)\beta + q(\theta_N)\{y^*-b\}}{[q'(\theta_N)(1-\gamma)\{y^*-b\} - cp'(\theta_N)\gamma\beta]}$$

Define $\mathcal{K} = \left[\frac{(A+p(\theta_N)\gamma\beta)^2}{A(y^*-b)}\right]$. Solving further,

$$\mathcal{K}\frac{\partial w_N}{\partial \gamma} = Aq'(\theta_N)(1-\gamma)\{y^*-b\} - Acp'(\theta_N)\gamma\beta + p(\theta_N)\beta q'(\theta_N)(1-\gamma)\{y^*-b\} - p(\theta_N)\beta cp'(\theta_N)\gamma\beta$$
$$+ (1-\gamma)\gamma\beta p'(\theta_N)cp(\theta_N)\beta + (1-\gamma)\gamma\beta p'(\theta_N)q(\theta_N)\{y^*-b\}$$

$$\mathcal{K}\frac{\partial w_N}{\partial \gamma} = Aq'(\theta_N)(1-\gamma)\{y^*-b\} - Acp'(\theta_N)\gamma\beta + p(\theta_N)\beta q'(\theta_N)(1-\gamma)\{y^*-b\}$$
$$- \gamma^2\beta p'(\theta_N)cp(\theta_N)\beta + (1-\gamma)\gamma\beta p'(\theta_N)q(\theta_N)\{y^*-b\}$$

$$\mathcal{K}\frac{\partial w_N}{\partial \gamma} = \frac{1}{q(\theta_N)^2}\left[\frac{A\eta_q}{\theta_N q(\theta_N)}(1-\gamma)\{y^*-b\} - \frac{Ac\eta_p}{q(\theta_N)}\gamma\beta + \beta\eta_q(1-\gamma)\{y^*-b\}\right.$$
$$\left. - \frac{\gamma^2\beta\eta_p c}{\theta_N}\beta + (1-\gamma)\gamma\beta\{y^*-b\}\eta_p\right] \tag{50}$$

Define: $\mathcal{D} = (1-\gamma)\{y^*-b\}$. As, $q(\theta_N)^2\mathcal{K} > 0$, $\frac{\partial w_N}{\partial \gamma} > 0$ if

$$\frac{A\eta_q}{\theta_N q(\theta_N)}\mathcal{D} - \frac{Ac\eta_p}{q(\theta_N)}\gamma\beta + \beta\eta_q\mathcal{D} - \frac{\gamma^2\beta\eta_p c}{\theta_N}\beta + (1-\gamma)\gamma\beta\{y^*-b\}\eta_p > 0$$

$$\eta_q\left[\mathcal{D}\{\frac{A}{\theta_N q(\theta_N)} + \beta + \gamma\beta\} - \frac{Ac}{q(\theta_N)}\gamma\beta - \frac{\gamma^2\beta^2 c}{\theta_N}\right] + \mathcal{D}\gamma\beta - \frac{Ac}{q(\theta_N)}\gamma\beta - \frac{\gamma^2\beta^2 c}{\theta_N} > 0$$

$$\eta_q\left[\mathcal{D}\{\frac{A}{\theta_N q(\theta_N)} + \beta + \gamma\beta\} - \frac{Ac}{q(\theta_N)}\gamma\beta - \frac{\gamma^2\beta^2 c}{\theta_N}\right]^{-1} > \frac{Ac}{q(\theta_N)}\gamma\beta + \frac{\gamma^2\beta^2 c}{\theta_N} - \mathcal{D}\gamma\beta$$