MEDIA BIAS: A DEMAND-SIDED APPROACH

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ABSTRACT. *When a Bayesian consumer seeking information does not alter its behaviour by reading a media report, the media cannot charge a positive price for its information. For profitability, media outlets must persuade consumers to take actions they wouldn't typically consider, even when the media itself gains no direct benefit from it. Amid uncertainty about information reliability, individuals are hesitant to follow advice conflicting with their stance, especially from sources perceived as of low quality. Our paper extends this concept to illustrate how uncertainty about media quality, coupled with consumers' biased preferences and beliefs, can drive bias in a low-quality media firm. We delve deeper to characterize the precise consumer preferences and priors that induce distorting bias in a profit-maximizing, low-quality media firm under quality uncertainty. Furthermore, we establish a necessary and sufficient condition for such bias to exist. Exploring different consumer types and population sizes, we analyze their impact on the direction of this bias. Additionally, we examine how the existing polarization within the economy influences biased media, confirming the anticipated relationship, albeit in an inverse manner.*

1 Introduction

This paper aims to study the role of demand-side factors in determining a media firm's reporting behaviour in a world of uncertain quality of media. The goal here is to characterize the preferences and priors of the consumers that can make a media biased in its reporting. Information plays a very important role in any decision-making process by humans. Although it cannot be directly consumed, it helps to shape the behaviour of economic agents. The set of information that goes into any decision-making process can be very large, but not all of it is necessary. The extent of information needed or held differs from individual to individual. Most of the time people just want a summary of a vast set of raw information to make sense of the world around them. This gives rise to the existence of media firms that supply reports to meet the consumers' demand for 'processed' information in exchange for money. Moreover, media reports are classified as a form of "cheap talk" - they can report whatever they want. It is up to the consumers whether they want to purchase it or not. The interaction with consumers bounds the behaviour or the reporting strategy (i.e., roughly speaking, the extent to which it can lie in its report) of the media firms. Our focus will be on demand-side factors that can affect media firm's reporting behaviour.

Although there is no denying the role played by the supply side factors. Most commonly, media firms' ideological stances, preferences over outcomes, political pressure etc. can eventually affect their reporting strategy. Stanley and Niemi (2013), Ansolabehere et al. (2006), Baron (2006), Ellman and Germano (2009), Anderson and

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McLaren (2012) have already studied these factors. Larcinese et al. (2011), Gentzkow and Shapiro (2010), Puglisi and Snyder (2011), Mullainathan and Shleifer (2005), Mullainathan and Shleifer (2002), Gentzkow and Shapiro (2006), Suen (2004) have studied the effects of demand-side factors on the media firm.

More specifically, in this paper, our discussion will revolve around how a media becomes 'biased' in its reporting due to demand-side factors such as preferences and prior beliefs. In the literature of Media Economics, there are two ways to define bias in media reports (Gentzkow et al. (2014)): *Filtering bias* and *Distortionary bias.*

To understand distortionary bias, let us consider the following example. News channel A reports: *There has been a major road accident on the Z highway. Although many people have been injured, no death has been recorded.* Whereas, news channel B reports: *The road accident in Z highway was one of the deadliest accidents of this year and it has caused the death of 5 people.* Notice, that channel A reports no death but channel B reports the death of 5 people. This is the idea of distortionary bias where the media firm makes an objective statement of the state of the world.

To explain the filtering bias consider the following example. News channel A reports: *There has been a major road accident on the Z highway. It has caused the death of as many as 5 people. Afterwards, the crowd burnt a bus and also attacked the police van.* But the news channel B reports: *The road accident in Z highway was one of the deadliest accidents of the year and it has caused the death of at least 5 people.* Channel A chose to report about the bus and police van burning whereas Channel B did not. This phenomenon is called *selection.* Also, Channel A uses the word "as many as 5 people" but Channel B uses the word "at most 5 people". This phenomenon is termed as *information coarsening.* Filtering bias can arise due to both selection and information coarsening.

Here we construct a simple but general model with four elements - Distortionary bias, Monopoly media firm, uncertainty about the quality of a media firm, general preferences and priors of the consumers. Mullainathan and Shleifer (2005), Gentzkow and Shapiro (2006), Baron (2006), Bernhardt et al. (2008) have studied the distortionary bias in media but along different channels. Mullainathan and Shleifer (2005) studies the influence of confirmation bias of consumers on media bias. Baron (2006) shows how the career concerns of newly hired journalists make the media biased. Bernhardt et al. (2008) have shown that consumers' liking/disliking for certain types of news reports can make the media biased. Suen (2004) has already shown that the biased preferences[1] and prior beliefs of the consumers can make the media biased in its reporting but through a model of filtering bias. Gentzkow et al. (2014) shows in a model of distortionary bias that the biased preferences or prior beliefs of the consumers can not make the media biased through the simple profit-maximising pricing motive of the media firm. However, the seminal paper Gentzkow and Shapiro (2006) (From here on GS) showed that if we allow for uncertainty regarding the media firm's quality, the media firm's concern for the consumer's posterior belief about their quality can make it distortionary biased.

In this paper, using a model of distortionary bias, we study the profit-maximising pricing decision of the media allowing for uncertainty regarding the quality of the media firm as done in GS. Having said this, our paper differs from GS in three ways. Firstly, unlike them, we consider the biased preferences of the consumers. Secondly, although we include the uncertainty regarding the quality of the media firm in our model, we don't allow for the updation of it through the media reports. Thirdly, unlike GS, we incorporate the profit maximisation motive of the media firm through the price. Moreover, we have characterised in our model all the preferences and the prior beliefs of the consumers that can make a media biased in a Perfect Bayesian Equilibrium.

Our results suggest that if we allow for biased preferences or priors of the consumers without considering uncertainty regarding the quality of the media firm, the media will not be biased. This is a reaffirmation of the findings of Gentzkow et al. (2014) and Blackwell (1951). We also show the converse i.e., if we don't have

[1]By biased preferences, we mean the preferential asymmetry over the action and state of the world. For example, some actions may provide better payoffs when they are correct than others. This allows for identity-based preferences of Akerlof and Kranton (2000). Our biased preferences are different from the one defined in Bernhardt et al. (2008) as we consider the preferences over the actions and not over the media reports itself as defined in their paper.

unbiased preferences and priors of the consumers, then an uncertainty regarding the media firm's quality itself can not make the media biased. However, if we include both i.e., allow for biased preferences and priors of the consumers along with uncertainty regarding the quality of the media firm, we can show the possibility where the media will indeed be biased.

The intuition for the biased media reports is the following. Due to preferential asymmetry/biased preferences and prior beliefs of the consumer, the consumers hold a stance and are inclined to take a specific action dictated by their stance. Now as we show in the paper, the media can not charge a positive price to the consumers unless it *persuades*them to take actions that they would not have taken otherwise. Suppose that we have a monopoly with uncertain quality where quality of the media is associated with the strength of the signal that it observes from the nature. Consumers will not like to hear any suggestion to go against their stance from a media firm that is of very low quality. Their aversion stems from a higher perceived risk of deviating from their stance than in being mistaken by following it. Consequently, lower-quality media tends to display biased reporting.

To check the relationship between the preexisting level of polarization in the economy and the bias in the media, we extend the model allowing for two types of consumers who drive the direction of the bias into two different directions. We use the characterization of polarization provided in Esteban and Ray (1994) to define the polarization level in the economy. The conclusion that we can draw from this is simply that the bias in media reports is negatively related to the preexisting level of polarization in the economy.

The paper hereon proceeds: Section 2 introduces the benchmark model. The section 3 and 4 discuss the consumer and media firm's behaviour in the model. Section 5 summarizes the main result of the paper and discusses the intuition behind the result. Section 6 extends the model to include two types of consumers. Section 7 discusses an example of preferences that can cause the media to be biased. Section 8 establishes the relationship between the media's biasing decision with the preexisting polarization level in the economy. Most of the proofs are provided in the Appendix attached at the end of the paper.

2 The Benchmark Model

☐ *State of the world*: The state of the world, S is binary with $S \in \{L, R\}$, where L is the notation for *left* and R for *right*. There is a private signal $s \in \{l, r\}$ (only observed by the media firms) about the true state, whose distribution we will discuss later on. We assume that everybody in the economy has the prior belief that with probability θ, the true state is R.

☐ *Agents*: There is a unit mass of homogeneous consumers, called the *Z-type* consumers who must choose an action, $A \in \{L, R\}$. Think it as a guessing game where the agents want to guess the true state. Depending on their action and true state of the world, they get some utility given by:

$$U_Z(A, S) = \begin{cases} U_Z(R, R) & \text{if } A = S = R, \\ U_Z(L, L) & \text{if } A = S = L, \\ U_Z(R, L) & \text{if } A = R \neq S, \\ U_Z(L, R) & \text{if } A = L \neq S. \end{cases}$$

As our aim in this section is to characterize the preference structures that can make the media biased, we don't put any structure to this utility function.

☐ **Example.** Here is one example of possible preferences.

$$U_R(A,S) = \begin{cases} 1+x & \text{if } A=S=R, \\ 1 & \text{if } A=S=L, \\ x & \text{if } A=R\neq S, \\ 0 & \text{if } A=L\neq S. \end{cases}$$

where $x \in (0,1)$. Noticeably here the consumer has some bias towards the action R. It captures the ideological part of the consumer's preference. i.e., the gain from taking an action not only depends on the fact whether the action was correct but also on the ideological stance of the consumer. The given preference represents a *rightist* consumer. Similarly, we can define a leftist consumer also.

☐ *Media firm*: There is a monopolist media firm with some uncertainty about it's quality. With probability $\lambda \in (0,1)$, the firm is "good" quality and has a signal that perfectly reveals the true state. With probability $1-\lambda$, the firm is "normal" and has an imperfect but informative signal s distributed according to

$$\pi = Pr(s=r|S=R) = Pr(s=l|S=L).$$

The media firm, after observing the signal, creates a report $\hat{s} \in \{\hat{l}, \hat{r}\}$. Report $\hat{s}$ can be thought as a suggestion to the reader about the true state being S. The reporting strategy of the media firm is to chooses $\sigma_s(\hat{s}) := Pr(\hat{s}|s)$. i.e., the probability with which a firm will report $\hat{s}$ after observing a signal s. Media firms are not restricted to be truthful reporting irrespective of their quality. We assume that good-quality media firm chooses an unbiased and truthful reporting strategy, later in the paper we will argue that it has no incentive to deviate from it in a babbling pooling Perfect Bayesian Equilibrium. We assume that both types of media firms must charge a strictly positive otherwise they will not be able to sell the report. For simplicity, we assume that the cost of production is zero irrespective of the firm type.

Definition 1. *We say that a firm is 'distortionary' biased towards right if $\sigma_r(\hat{r}) > \sigma_l(\hat{l})$ which is also the same thing as saying $\sigma_r(\hat{l}) > \sigma_l(\hat{r})$ and similarly we define a left biased media firm.*

☐ *Timeline*: We can summarize the game timeline as follows:

- Stage 1: Nature chooses the firm type and the state of the world.
- Stage 2: The firm commits to it's reporting strategy and the price for the report, which becomes common knowledge.
- Stage 3: The firm receives a signal and makes a report.
- Stage 4: Individuals decide whether to buy or not.
- Stage 5: Individuals choose action $A \in \{L, R\}$.

☐ *Strategy spaces*: The strategy space of the normal or good-quality media firm consists of the decision to sell or not, price (p), reporting strategy $(\sigma_r(\hat{r}), \sigma_l(\hat{l}))$. The strategy space of the consumer consists of purchasing decisions depending on the price and reporting strategies of the media firms and the actions they take after reading each type of report.

☐ *Solution concept*: The solution concept used for solving the above game is Perfect Bayesian Equilibrium (PBE).

- *Separating PBE*: In the appendix B, we show that there can never be a separating Perfect Bayesian Equilibrium in this model.
- *Pooling PBE*: In the appendix B, we show that there always exists Pooling PBE, and we characterize all the possible PBE in this model. One noticeable characteristic of these equilibria is that they don't specifically pinpoint the reporting strategy of the media firm and there are infinite sets of prices and reporting strategies that can be sustained as a pooling PBE. With the same price, the firm can pick a different reporting strategy at a pooling PBE. As long as their reporting strategy makes the rational Bayesian consumer choose to buy the report, the reporting strategy can be sustained at a pooling PBE.

  Let $\mathcal{P}$ be the set of all possible prices that can be sustained as a pooling PBE. The supremum of $\mathcal{P}$, or simply, $\sup\{\mathcal{P}\}$ can also be sustained as a babbling pooling PBE. This (babbling) equilibrium, gives an unequivocal set of price and reporting strategy. Here we can unequivocally characterise the bias in the reporting. *So, this paper focuses on the babbling PBE.*

3 Consumer's behaviour

A consumer's motive to read a media report is to create a better posterior to achieve a higher expected utility. The expected utility depends on the posterior and the actions taken after observing each report. The consumer needs to decide what action she should take after observing each report even before reading the report. This is also a part of the strategy of the consumer. For the ease of exposition of the strategies, we give the following definition.

Definition 2. *We say that a consumer "follows" a report $\hat{r}$ (or $\hat{l}$) if she takes action R (or L) whenever the report says $\hat{r}$ (or $\hat{l}$). Similarly, we define "not follow".*

Straightforward calculations with Bayesian updation lead to the following observation.

Observation 1. *The Z-type consumer follows*

- *$\hat{r}$ report iff,*

$$\begin{aligned}\theta[\lambda + (1-\lambda)\{\sigma_r(\hat{r})\pi + (1-\sigma_l(\hat{l}))(1-\pi)\}](U_Z(R,R) - U_Z(L,R))\\ \geq (1-\theta)(1-\lambda)(\sigma_r(\hat{r})(1-\pi) + (1-\sigma_l(\hat{l}))\pi)(U_Z(L,L) - U_Z(R,L))\end{aligned} \tag{1}$$

  This is just saying that given that the report says $\hat{r}$, the expected utility from taking the action R is greater than the expected utility of taking the action L.

- *$\hat{l}$ report iff,*

$$\begin{aligned}(1-\theta)[\lambda + (1-\lambda)((1-\sigma_r(\hat{r}))(1-\pi) + \sigma_l(\hat{l})\pi)](U_Z(L,L) - U_Z(R,L))\\ \geq \theta(1-\lambda)\{(1-\sigma_r(\hat{r}))\pi + \sigma_l(\hat{l})(1-\pi)\}(U_Z(R,R) - U_Z(L,R))\end{aligned} \tag{2}$$

A consumer who chooses to buy the report (but has not read the report yet) has four strategies: follow both the reports (we call it FF), follow $\hat{r}$ reports but not follow $\hat{l}$ reports (FN), not follow $\hat{r}$ reports but follow $\hat{l}$ reports (NF), not follow either of the reports (NN). Which of these strategies will be chosen by the consumer will depend on the above observation.

The purchasing decision of the consumer requires the comparison of the expected utility that can be achieved after reading the report (i.e., the case when the consumer takes the action based on the posterior), and the expected utility that can be achieved without reading the report (i.e., the case when the consumer takes the action based on her priors).

The expected utility of the consumer, without the report, $E(U_Z(A,S))|_{without\ report}$, will be

$$\max\{\theta U_Z(R,R) + (1-\theta)U_Z(R,L), (1-\theta)U_Z(L,L) + \theta U_Z(L,R)\}$$

But the expected utility with the report depends on what the consumer chooses after reading a certain report. So, the expected utility of the consumer is determined by the reporting strategy of the media firm through the conditions 1 and 2. For example, if the condition 1 is satisfied but not the 2, then the consumer will follow the rightist report but will not follow the leftist report. So, we introduce the following notations for the expected utility of the consumer with the media report. If 1 and 2 both hold, then the consumer follows both the types of reports (i.e., FF) and the expected utility will be denoted as $E(U_Z(A,S))|^{FF}_{with\ report}$. Whereas if 1 holds but 2 does not hold, the consumer follows the rightist reports but does not follow the leftist reports (i.e., FN) and the expected utility will be denoted as $E(U_Z(A,S))|^{FN}_{with\ report}$. Similarly we define $E(U_Z(A,S))|^{NF}_{with\ report}$ and $E(U_Z(A,S))|^{NN}_{with\ report}$.

Proposition 1. *Without persuasion, the media can never ask for a positive price. i.e., A reporting strategy that makes the consumers follow only one type of media report, can never make the media earn a positive profit.*

The proof for the above proposition is given in the A.1. The conclusion that we can draw from here is that at any equilibrium where the firm makes a positive profit, the firm will never choose a reporting strategy such that only one of 1 and 2 holds as it indeed gives a non-positive profit to the firm. This is similar to the straightforward signalling defined in Kamenica and Gentzkow (2011) and says that every positive profit-earning media is a persuader of the Bayesian consumer. The intuition here is that, after reading the report, if the consumers still choose the same action that they would have taken without reading the report, the expected utility of the consumers with and without the report stays the same. This is just an implication of the fact that the expected posterior belief is the prior belief.

Using the observation 1 and proposition 1, we have something even more substantial.

Claim 1. *For the survival of media, the preference of the consumers must satisfy*[2]

$$\text{sgn}(U_Z(R,R) - U_Z(L,R)) = \text{sgn}(U_Z(L,L) - U_Z(R,L))$$

Proof. The proof of the above claim is very simple. Suppose to the contrary that the $\text{sgn}(U_Z(R,R) - U_Z(L,R)) \neq \text{sgn}(U_Z(L,L) - U_Z(R,L))$. Then we can never have the situation where either both of 1 and 2 are satisfied or both of them are unsatisfied. Now from proposition 1, we see that the media will not be able to sell the report at a positive price. So, for a media to earn a positive profit we need $\text{sgn}(U_Z(R,R) - U_Z(L,R)) = \text{sgn}(U_Z(L,L) - U_Z(R,L))$. □

In words, the above proposition is actually a necessary condition for the consumers to pay a positive price for the media report. All it requires is that they should gain by being correct in each true state or lose by being correct in every true state. If consumers gain from being correct in one true state and looses from being correct in another true state then in that case the media can never ask for a positive price.

Notice that the scenario with $\text{sgn}(U_Z(R,R) - U_Z(L,R)) = \text{sgn}(U_Z(L,L) - U_Z(R,L)) = +1$ is exactly same as $\text{sgn}(U_Z(R,R) - U_Z(L,R)) = \text{sgn}(U_Z(L,L) - U_Z(R,L)) = -1$ if we just switch the true states and the actions. So, without loss of generality, we make the following simplifying assumption.

[2]sgn denotes the signum function which basically captures the sign.

□ *Assumption:* People don't loose by taking the correct action. i.e.,

$$\text{sgn}(U_Z(R,R) - U_Z(L,R)) = \text{sgn}(U_Z(L,L) - U_Z(R,L)) \in \{+1, 0\}$$

or

$$\min\{U_Z(R,R) - U_Z(L,R), U_Z(L,L) - U_Z(R,L)\} \geq 0$$

4 Media's behaviour

The profit-maximizing price in this model will always be set at the expected net gain caused by the media report. i.e., $E(U_Z(A,S))|_{with\ report} - E(U_Z(A,S))|_{without\ report}$. But the value of this difference will depend on what sort of behaviour of consumer gets imposed by the media firm's reporting strategy. So, the media firm faces three optimization problems here. One is where the media chooses a reporting strategy such that the consumers choose the FF strategy and the other one is with the NN strategy of the consumers. And then chooses the reporting strategy that gives the maximum between these two.

Proposition 2. *At optimal the normal quality media firm will choose a reporting strategy such that the consumers follow both types of reports.*

The proof is given in appendix A.2. As an immediate implication of the above proposition, we can say that the profit-maximising media at optimal will never choose a reporting strategy such that the consumers choose not to follow both types of reports. So, from now on we will consider only the reporting strategies where the media chooses a reporting strategy such that the consumers choose to follow both types of reports.

5 Characterization

Proposition 3. *With any prior belief $\theta \in (0,1)$ and $\lambda > 0$, the normal media will be biased (in a babbling pooling PBE) iff*

$$0 < \frac{\min\{\theta a, (1-\theta)b\}}{\theta a + (1-\theta)b} \leq \pi \leq \frac{\max\{\theta a, (1-\theta)b\}}{\theta a + (1-\theta)b} < 1$$

where $a := U_Z(R,R) - U_Z(L,R)$ and $b := U_Z(L,L) - U_Z(R,L)$.

More specifically, the normal media firm will be right biased with a reporting strategy of $\sigma_r(\hat{r}) = 1; \sigma_l(l) = 0$, good quality media will be unbiased and truthful with a reporting strategy of $\sigma_r(\hat{r}) = 1; \sigma_l(l) = 1$ and both will be charging a price of $\lambda b(1-\theta)$ if and only if

$$0 < \frac{(1-\theta)b}{\theta a + (1-\theta)b} \leq \pi \leq \frac{\theta a}{\theta a + (1-\theta)b} < 1$$

The normal media firm will be left biased with a reporting strategy of $\sigma_r(\hat{r}) = 0; \sigma_l(l) = 1$, good quality media will be unbiased and truthful with a reporting strategy of $\sigma_r(\hat{r}) = 1; \sigma_l(l) = 1$ and both will be charging a price of $\lambda a\theta$ if and only if

$$0 < \frac{\theta a}{\theta a + (1-\theta)b} \leq \pi \leq \frac{(1-\theta)b}{\theta a + (1-\theta)b} < 1$$

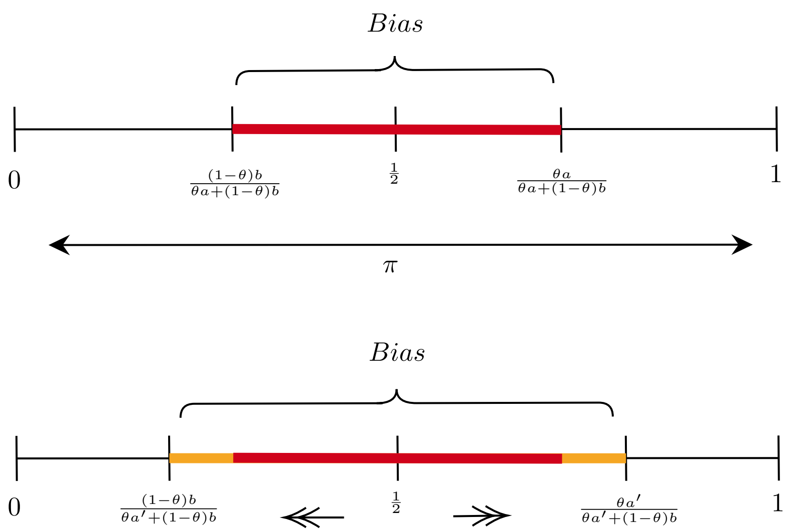
The proof is given in appendix A.3.

5.1 Discussion on the result

Without loss of generality let us focus on the right bias in normal media reports. The condition for the right bias in this model is:

$$0 < \frac{(1-\theta)b}{\theta a + (1-\theta)b} \leq \pi \leq \frac{\theta a}{\theta a + (1-\theta)b} < 1$$

Notice that the bounds here are always equidistant from 0.5, and they always include 0.5. The worst signal generator for the media is with $\pi = 0.5$, and the signal generator becomes weaker as π comes closer to 0.5 it becomes stronger as it comes closer 0 or 1. So, we can interpret the condition for the bias by saying that the bias exists when the signal generator is *weak enough*. Also, the bounds are increasing in θa and decreasing in $(1-\theta)b$. So, if people in the economy are more right-biased, in prior or preference, the set of values of π, that dictates the media to be biased, expands. The below figure illustrates this, where $a < a'$.



The seminal work of Kamenica and Gentzkow (2011) characterises the Bayesian consumers that can be persuaded. In order to do so, they focused on the sender-preferred Perfect Bayesian Equilibrium. What we have focused on is the receiver-preferred equilibria as the monopolist, in order to maximise profit, chooses a reporting strategy that maximises the expected utility of the consumer. Our Proposition 1 highlights that the media also persuades the Bayesian reader. Rather than solely exploring when persuasion is feasible, our paper delves into a broader inquiry: *When does persuasion necessitate a biased report or signal?*

Our investigation reveals that for normal media to exhibit bias, the signal strength must be exceptionally weak. The rationale behind biased media reports is rooted in the asymmetry of preferences and preexisting beliefs among consumers, guiding them toward a particular course of action. As demonstrated in our paper, the media cannot charge consumers a positive price unless it effectively persuades them to act differently than their natural inclinations. Consider a monopoly situation where the media's quality, tied to the accuracy of the signals it receives from nature, is uncertain. Consumers are wary of receiving suggestions conflicting with their beliefs from a low-quality media entity. This wariness stems from their biased preferences and prior beliefs, as they perceive greater loss from deviating from their stance than from being mistaken by adhering to it. Consequently, media of significantly low quality tends to exhibit biased reporting.

6 Extension: Two types of consumers

So far we have only one type (namely, *Z-type*) consumers in the economy. Now with the characterization proposed in the last section, we extend the benchmark model by allowing for two different types of consumers defined as:

L-type: A consumer, with utility function of $U_L(A,S)$, is said to be *L-type* if: $0 < a_l\theta_l < (1-\theta_l)b_l$ where, θ_l is the prior belief of the consumer about the true state being R and $a_l := U_L(R,R) - U_L(L,R)$, $b_l := U_L(L,L) - U_L(R,L)$.

R-type: A consumer, with utility function of $U_R(A,S)$, is said to be *R-type* if: $a_r\theta_r > (1-\theta_r)b_r > 0$ where θ_r is the prior belief of the consumer about the true state being R and $a_r := U_R(R,R) - U_R(L,R)$, $b_r := U_R(L,L) - U_R(R,L)$.

Basically, from our characterization, the L-type consumers drive the media to be left biased and the R-type consumers drive it to be right biased. Let the size of the R-type in the population be $\phi_r \in [0,1]$ and the L-type be $\phi_l := 1 - \phi_r$.

Symmetry: We make the following simplifying assumptions about the economic environment

$$\theta_r U_R(R,R) = (1-\theta_l)U_L(L,L)$$

$$\theta_r U_R(L,R) = (1-\theta_l)U_L(R,L)$$

$$(1-\theta_r)U_R(R,L) = \theta_l U_L(L,R)$$

$$(1-\theta_r)U_R(L,L) = \theta_l U_L(R,R)$$

As a consequence of our *symmetry* assumption, we have $\theta_r a_r = (1-\theta_l)b_l$ and $(1-\theta_r)b_r = \theta_l a_l$. Moreover we have $a_r\theta_r + (1-\theta_r)b_r = \theta_l a_l + (1-\theta_l)b_l$ and

$$\frac{\theta_r a_r}{\theta_r a_r + (1-\theta_r)b_r} = \frac{(1-\theta_l)b_l}{\theta_l a_l + (1-\theta_l)b_l}$$

$$\frac{\theta_l a_l}{\theta_l a_l + (1-\theta_l)b_l} = \frac{(1-\theta_r)b_r}{\theta_r a_r + (1-\theta_r)b_r}$$

Proposition 4. *With $\theta_r \in (0,1)$ and $\lambda \in (0,1)$, the profit-maximizing media firm will be biased iff*

$$0 < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r, 1-\phi_r\})}{(\theta_r a_r + (1-\theta_r)b_r)(1-\lambda)} \le \pi \le 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r, 1-\phi_r\})}{(\theta_r a_r + (1-\theta_r)b_r)(1-\lambda)} < 1$$

More specifically, it will be right biased iff in the above condition we have $\phi_r > 0.5$ and left biased iff we have $\phi_r < 0.5$ and any reporting goes through if $\phi_r = 0.5$.

The proof of the above proposition is shown in A.4

7 Ideologically driven consumers: an example

So far we have characterized all the preferences that can make the media biased. In this section we discuss one such type of preferences. It is natural to believe that almost everybody holds some sort of ideology and perception about the real world. This infects their choice through affecting their preferences. Here we study about the consumers whose preferences are determined by their ideologies and so they are called ideologically driven consumers.

☐ *Ideological agents*: There is a unit mass of heterogeneous consumers, with two different world views and who each must choose a binary action, $A \in \{L, R\}$ which gives them utility depending on their type. The utility function of the consumer consists of two parts, valence and identity. Valence arises if the action taken by the agent matches the true state. Identity arises if the action taken by the agent matches her ideology/identity. It is the ideological stance of the individuals that defines their type. Here individuals are either *rightist* or *leftist*.

If the person is a leftist (or type L) then her utility is,

$$U_L(A,S) = \begin{cases} 1+x & \text{if } A = S = L, \\ 1 & \text{if } A = S = R, \\ x & \text{if } A = L \neq S, \\ 0 & \text{if } A = R \neq S. \end{cases} \tag{3}$$

If the person is a rightist (or type R) then her utility is,

$$U_R(A,S) = \begin{cases} 1+x & \text{if } A = S = R, \\ 1 & \text{if } A = S = L, \\ x & \text{if } A = R \neq S, \\ 0 & \text{if } A = L \neq S. \end{cases} \tag{4}$$

where $x > 0$, the ideological gain made by the consumers from taking an ideologically aligned action. This resembles the identity based ultity structure proposed by Akerlof and Kranton (2000). Noticeably, every individual is identical in the valence part, they only differ in the identity part. We denote the size of rightists in population by ϕ_r, and of the leftists by ϕ_l (where $\phi_r + \phi_l = 1$). Whenever $x > 0$ in the utility function of the consumer, we will call such preferences *biased* and if that is zero we will call such preferences *unbiased*. We assume that the prior belief of the *every consumer* that the true state of the world is R, is $\frac{1}{2}$.

All the other stuffs of the extended model are kept as they were earlier.

Now from the proposition 4, we say that the media will be biased in this economy iff:

$$0 < \frac{(1-x)(1-\lambda \max\{\phi_r, 1-\phi_r\})}{2(1-\lambda)} \leq \pi \leq 1 - \frac{(1-x)(1-\lambda \max\{\phi_r, 1-\phi_r\})}{2(1-\lambda)} < 1$$

In other words, the optimal reporting strategy of the media firm can be characterized as:

$$\{\sigma_r^*(\hat{r}), \sigma_l^*(\hat{l})\} = \begin{cases} \{1,1\} & \text{if } x \in (0, 2\pi - 1] \\ \begin{cases} \{1,1\} & \text{if } \frac{1}{2} \leq \phi_r \leq \frac{\lambda + (1-\lambda)\pi - \frac{1}{2} - \frac{x}{2}}{\frac{\lambda}{2}(1-x)} \\ \{1,0\} & \text{if } \phi_r > \max\left\{\frac{\lambda + (1-\lambda)\pi - \frac{1}{2} - \frac{x}{2}}{\frac{\lambda}{2}(1-x)}, \frac{1}{2}\right\} \\ \{1,1\} & \text{if } \frac{1}{2} \geq \phi_r \geq 1 - \left(\frac{\lambda + (1-\lambda)\pi - \frac{1}{2} - \frac{x}{2}}{\frac{\lambda}{2}(1-x)}\right) \\ \{0,1\} & \text{if } \phi_r < \min\left\{1 - \left(\frac{\lambda + (1-\lambda)\pi - \frac{1}{2} - \frac{x}{2}}{\frac{\lambda}{2}(1-x)}\right), \frac{1}{2}\right\} \end{cases} & \text{if } x \in (2\pi - 1, 1) \end{cases}$$

This tells us that the stronger ideological stance of the consumers is a necessary condition for the media to be biased but not sufficient. With high enough ideological stance, the group size has to be large enough to make the media biased.

8 Bias and polarization

In this section, we study the extended model with two types of consumers and study the effects of preexisting polarization on the media and the effect of media on the polarization level in the economy. Using the characterization of the polarization index in Esteban and Ray (1994), we define the polarization here as.

Definition 3. *Let $\mathcal{R}$ and $\mathcal{L}$ be the expected number of people who choose to take rightist and leftist action respectively. The polarization level in the economy, denoted as* **Pol**, *is defined as*

$$\mathbf{Pol} = \mathcal{R}.\mathcal{L} = \mathcal{R}(1-\mathcal{R}) \tag{5}$$

In our model, $\mathbf{Pol} \in [0, 0.25]$. It takes value zero when there is no polarization and 0.25 when there is complete polarization. The complete polarization is the scenario where the half of the economy takes rightist and the other half leftist action (in expectation). The zero polarization is the scenario where everybody takes rightist or leftist action.

8.1 Effect of polarization on media bias

Without the media report a rightist (leftist) always takes a rightist (leftist) action. So, the polarization level without the media report or just the preexisting level of polarization in the economy is,

$$\mathbf{Pol}_0 = \phi_r \phi_l = \phi_r(1-\phi_r) \tag{6}$$

So, the preexisting polarization in the economy is affected solely by the size of the rightists (or leftists) in the economy. In this economy, higher polarization is associated with a ϕ_r closer to 0.5 and as the ϕ_r comes closer to 0 or 1, the polarization goes down.

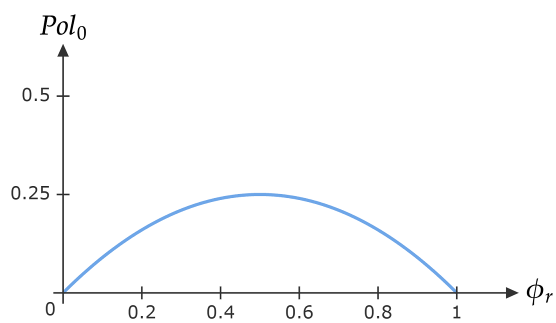


Figure 1: Preexisting level of polarization

Now let us recall from proposition 4 that the media chooses to be biased iff

$$0 < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r, 1-\phi_r\})}{(\theta_r a_r + (1-\theta_r)b_r)(1-\lambda)} \leq \pi \leq 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r, 1-\phi_r\})}{(\theta_r a_r + (1-\theta_r)b_r)(1-\lambda)} < 1$$

Notice that the bounds on π (for a given preference and θ) expand as ϕ_r comes closer to 1 or 0 and shrink as ϕ_r comes closer to 0.5. So, we have the following proposition:

Proposition 5. *The set of values of π, which makes the media bias, expands as the economy's preexisting polarisation level decreases (or ϕ_r becomes closer to 0 or 1) and vice-versa.*

8.2 Effect of media reports on polarization in the economy

We make the following assumption to simplify the calculation.

Assumption: The prior beliefs of both the types of consumers are assumed to be equal to $\frac{1}{2}$. i.e., $\theta_r = \theta_l = 0.5$

In future versions of the model, we aim to get away from this assumption. Now, we begin to study how the media reports affect the level of polarization in the economy. Also, recall that whenever the media chooses to be unbiased, it serves the whole market but whenever it chooses to be biased, it serves only one type of customer (the consumers whose type is aligned with the direction of bias). Also, we showed that whoever chooses to buy the media reports at equilibrium chooses to *follow* all the reports. So, the probability that a consumer chooses rightist action is nothing but $Pr(\hat{r})$. Similarly, the probability that a consumer chooses leftist action is nothing but $Pr(\hat{l})$. Using these ideas we can derive the level of polarization in the economy with the media reports as:

$$\mathbf{Pol}_1 = \begin{cases} \frac{1}{4} & \text{if } \pi \notin \mathcal{G} \\ \begin{cases} \phi_r\left(1-\frac{\lambda}{2}\right)\left\{1-\phi_r\left(1-\frac{\lambda}{2}\right)\right\} & \text{if } \phi_r \geq \frac{1}{2} \\ (1-\phi_r)\left(1-\frac{\lambda}{2}\right)\left\{1-(1-\phi_r)\left(1-\frac{\lambda}{2}\right)\right\} & \text{if } \phi_r \leq \frac{1}{2} \end{cases} & \text{if } \pi \in \mathcal{G} \end{cases}$$

or

$$\mathbf{Pol}_1 = \begin{cases} 1 & \text{if } \pi \notin \mathcal{G} \\ \max\{\phi_r, 1-\phi_r\}\left(1-\frac{\lambda}{2}\right)\left\{1-\max\{\phi_r, 1-\phi_r\}\left(1-\frac{\lambda}{2}\right)\right\} & \text{if } \pi \in \mathcal{G} \end{cases}$$

where $\mathcal{G} := \left[\frac{b_r(1-\lambda\max\{\phi_r,1-\phi_r\})}{(a_r+b_r)(1-\lambda)}, 1-\frac{b_r(1-\lambda\max\{\phi_r,1-\phi_r\})}{(a_r+b_r)(1-\lambda)}\right] \cap (0,1)$ which is nothing but the possible values of π that makes the media biased (We will refer $\mathcal{G}$ as the *"zone of bias"*). One immediate observation that we can make from the above equation is that the polarization level with the media reports will be the highest whenever the media chooses to be biased. This is simply a result of the fact that people follow the media report's recommendation as everybody chooses to buy the unbiased reports at equilibrium, in expectation the economy gets polarized to the maximum level. To get a better sense about the effect of media on the polarization level we can calculate the change in the polarization $\mathbf{\Delta Pol} = \mathbf{Pol}_1 - \mathbf{Pol}_0$ as:

$$\mathbf{\Delta Pol} = \begin{cases} \frac{1}{4} - \phi_r(1-\phi_r) & \text{if } \pi \notin \mathcal{G} \\ \max\{\phi_r, 1-\phi_r\}\left(1-\frac{\lambda}{2}\right)\left\{1-\max\{\phi_r, 1-\phi_r\}\left(1-\frac{\lambda}{2}\right)\right\} - \phi_r(1-\phi_r) & \text{if } \pi \in \mathcal{G} \end{cases}$$

In the above expression, the change in polarization due to unbiased media $\frac{1}{4} - \phi_r(1-\phi_r) > 0$ iff $\phi \neq \frac{1}{2}$. But there is ambiguity about the direction of this change for a biased media as $\mathbf{\Delta Pol}$ may be positive or negative. This means that the level of polarization with biased media may increase or decrease. Similar to the left-biased media. so, we can summarize it as,

$$\mathbf{\Delta Pol} = \begin{cases} \geq 0 & \text{if } \pi \notin \mathcal{G} \\ \lesseqqgtr 0 & \text{if } \pi \in \mathcal{G} \end{cases}$$

Proposition 6. *Media report decreases the polarization in the economy* ***iff*** $\phi_r \in \left[\frac{2-\lambda}{4-\lambda}, \frac{2}{4-\lambda}\right]$ *and* $\pi \in \mathcal{G}$. *This is possible only through biased reporting. Unbiased reporting never decreases polarization.*

In words, the above proposition says that for the media to decrease the polarization, it necessarily has to be biased in it's reporting. A sufficiency condition for the media to decrease the polarization is that the economy is highly polarized enough to begin with and the media is biased. Noticeably as the economy is highly polarized (i.e., high enough ϕ_r), the zone of bias shrinks.

9 Conclusion

This paper studied how a media becomes distortionary biased due to consumer side factors such as preferences and prior beliefs. It offered a fresh angle on understanding biased media, emphasizing that biased outcomes can result from weaker signal strengths in media channels. Notably, it suggested that enhancing the information sources of media firms could potentially mitigate biases. While the study primarily focused on monopolistic media entities, the subsequent phase might explore the impacts of competition, particularly within an oligopoly framework. Expanding the model to view bias as a continuous filtering mechanism could further refine our understanding by detailing preferences and prior beliefs. Interestingly, Suen (2004) demonstrated that media bias can exist independently of uncertainties about media quality, sparking an intriguing inquiry into the distinctions between discrete and continuous models of media bias. This sets the stage for an open research question: the divergence between discrete and continuous models in understanding media bias, marking a potential direction for future exploration.

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10 Appendix

A A1

A.1 Proof of proposition 1

Proof. Let us define $E(U_Z(A,S))|^j = E(U_Z(A,S))|^j_{with\ report} - E(U_Z(A,S))|_{without\ report}$, where $j \in \{FF, NF, FN, NN\}$ denotes the strategy of the consumers, as the net gain made by the consumer after reading the report. This must be greater than or equal to the price to make the consumers purchase the report.

Noticeably if 1 holds but 2 does not hold, then we have

$$\begin{aligned} E(U_Z(A,S))|^{FN} =& \theta U_Z(R,R) + (1-\theta)U_Z(R,L) \\ &- \max\left\{\theta U_Z(R,R) + (1-\theta)U_Z(R,L), (1-\theta)U_Z(L,L) + \theta U_Z(L,R)\right\} \leq 0 \end{aligned}$$

and if 2 holds but 1 does not hold, then we have

$$\begin{aligned} E(U_Z(A,S))|^{NF} =& (1-\theta)U_Z(L,L) + \theta U_Z(L,R) \\ &- \max\left\{\theta U_Z(R,R) + (1-\theta)U_Z(R,L), (1-\theta)U_Z(L,L) + \theta U_Z(L,R)\right\} \leq 0 \end{aligned}$$

As these net expected gain (which is also the maximum price that the media can charge) is always non-positive, we have the proposition. □

A.2 Proof of proposition 3

Proof. Media firm faces the following optimization problem if it chooses a reporting strategy such that the consumer follows both types of reports.

$$\max_{\{\sigma_r(\hat{r}), \sigma_l(\hat{l})\}} E(U_Z(A,S))|^{FF} \tag{7}$$

such that, 1 and 2 both are satisfied and $\sigma_r(\hat{r}), \sigma_l(\hat{l}) \in [0,1]$.

Let us define $\max E(U_Z(A,S))|^{FF}_{con}$ and $\arg\max E(U_Z(A,S))|^{FF}_{con}$ be the maximum value and the maximizer of $E(U_Z(A,S))|^{FF}$ given the above stated constraints. Whereas $\max E(U_Z(A,S))|^{FF}_{uncon}$ and $\arg\max E(U_Z(A,S))|^{FF}_{uncon}$ be the maximum value and the maximizer of $E(U_Z(A,S))|^{FF}$ when we don't consider any constraints.

Whereas, the media firm faces the following optimisation problem if it chooses a reporting strategy such that the consumer does not follow both types of reports.

$$\max_{\{\sigma_r(\hat{r}), \sigma_l(\hat{l})\}} E(U_Z(A,S))|^{NN} \tag{8}$$

such that, such that, 1 and 2 both are unsatisfied and $\sigma_r(\hat{r}), \sigma_l(\hat{l}) \in [0,1]$

Let us define $\max E(U_Z(A,S))|^{NN}_{con}$ and $\arg\max E(U_Z(A,S))|^{NN}_{con}$ be the maximum value and the maximizer of $E(U_Z(A,S))|^{NN}$ given the above stated constraints. Whereas $\max E(U_Z(A,S))|^{NN}_{uncon}$ and $\arg\max E(U_Z(A,S))|^{NN}_{uncon}$ be the maximum value and the maximizer of $E(U_Z(A,S))|^{NN}$ when we don't consider any constraints.

To prove: $\max_{\{\sigma_r(\hat{r}), \sigma_l(\hat{l})\}} E(U_Z(A,S))|^{FF}_{con} > \max_{\{\sigma_r(\hat{r}), \sigma_l(\hat{l})\}} E(U_Z(A,S))|^{NN}_{con}$ We define $a := U_Z(R,R) -$

$U_Z(L,R)$ and $b := U_Z(L,L) - U_Z(R,L)$. Now let us consider few cases:

- If $\pi \in \left[0, \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}\right)$
  $\max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_R(A,S))|^{FF}_{con} = \min\{\theta a,(1-\theta)b\} - A(1-\lambda)\pi > A(1-\lambda)(1-\pi) - \max\{\theta a,(1-\theta)b\} = \max E(U_R(A,S))|^{NN}_{uncon} \geq \max E(U_R(A,S))|^{NN}_{con}$ where $A := a\theta + (1-\theta)b$.
  $\arg\max E(U_R(A,S))|^{FF}_{uncon} = \arg\max E(U_R(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (0,0)$
- $\pi \in \left(\frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}, 1\right]$
  $\max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_R(A,S))|^{FF}_{con} = A(1-\lambda)(\pi-1) + \min\{\theta a,(1-\theta)b\} > A(1-\lambda)\pi - \max\{\theta a,(1-\theta)b\} = \max E(U_R(A,S))|^{NN}_{uncon} \geq \max E(U_R(A,S))|^{NN}_{con}$
  And the $\arg\max E(U_R(A,S))|^{FF}_{uncon} = \arg\max E(U_R(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,1)$
- $\pi \in \left[\frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}, \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}\right]$:
  Here $\max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_R(A,S))|^{FF}_{con} = \lambda \min\{\theta a,(1-\theta)b\} > -\lambda \max\{\theta a,(1-\theta)b\} = \max E(U_R(A,S))|^{NN}_{uncon} \geq \max E(U_R(A,S))|^{NN}_{con}$
  And the $\arg\max E(U_R(A,S))|^{FF}_{uncon} = \arg\max E(U_R(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,0)$

$\square$

A.3 Proof of proposition 4

Proof. If direction. i.e.,
If $0 < \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} \leq \pi \leq \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} < 1$*, the media chooses to be biased.*

Then it is necessarily true that neither θa nor $(1-\theta)b$ is equal to 0 otherwise the lower and upper bound would become equal to 0 and 1 respectively. So, we have $\min\{\theta a,(1-\theta)b\} > 0$

For the time being let us assume that $\theta a \geq (1-\theta)b > 0$. So, we have to show that $0 < \frac{(1-\theta)b}{\theta a+(1-\theta b)} \leq \pi \leq \frac{\theta a}{\theta a+(1-\theta)b} < 1 \Rightarrow$ Media bias.

Suppose $0 < \frac{(1-\theta)b}{\theta a+(1-\theta b)} \leq \pi \leq \frac{\theta a}{\theta a+(1-\theta)b} < 1$.

Now this implies $\arg\max E(U_Z(A,S))|^{FF}_{uncon} = E(U_Z(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,0)$ and $\max E(U_Z(A,S))|^{FF}_{uncon} = E(U_Z(A,S))|^{FF}_{con} = \lambda(1-\theta)b$

This means the profit from choosing the reporting strategy $(\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,0)$ is highest. i.e., the media chooses to be right biased. Similarly, we can show that for $(1-\theta)b \geq \theta a > 0$ or $\frac{(1-\theta)b}{\theta a+(1-\theta b)} \leq \pi \leq \frac{\theta a}{\theta a+(1-\theta)b}$ we will have media bias. Moreover for this case the media will become left biased with a reporting strategy as $(\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (0,1)$. So, this proves that if $\frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} \leq \pi \leq \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} \Rightarrow$ media bias. It will be right biased when we have $\theta a > (1-\theta)b$ and left biased if $\theta a > (1-\theta)b$ in the given condition.

Only if direction:
First of all, let us talk about the bounds on π. We can make a stronger claim here.

Lemma 1. *If the media firm is able to sale the report at a positive price then it is necessarily the case that*

$$\frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} < 1;$$

$$\frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} > 0.$$

Proof. Notice that if the upper bound of π in this condition coincides with 1 that is the same as saying that the lower bound on π coincides with 0. i.e., the bounds on π are always equidistant from 0.5. It is also the case that the above terms will necessarily lie in between 0 and 1. So, all that we have to prove is that these are not equal to 1 and 0 respectively. Now let us suppose to the contrary that the media firm is selling the report at a positive price and $\frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} = 1$ which also means that $\frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} = 0$. Also notice that by assumption we have here $\min\{\theta a,(1-\theta)b\} > 0$. That means here we have either (1) $\theta a > 0$ & $(1-\theta)b = 0$ or (2) $\theta a = 0$ & $(1-\theta)b > 0$. For now let us just focus in the first case where we have $\theta a > 0$ & $(1-\theta)b = 0$. Then we have

$$\max E(U_Z(A,S))|^{FF}_{uncon} = \lambda(1-\theta)b = 0 \geq \max E(U_Z(A,S))|^{FF}_{con}$$

So, the media will not be able to charge a positive price at optimal. This contradicts the fact that the media firm is able to charge a positive price in this setting. Similarly, we can reach to contradictions if we had $\theta a = 0$ & $(1-\theta)b > 0$. So, this proves our claim. $\square$

Now we show that the media is biased only if

$$0 < \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} \leq \pi \leq \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} < 1$$

.

If the media is biased and selling at a positive price then from the above lemma, we can argue that the above bounds on π are strictly in between 0 and 1. So neither θa nor $(1-\theta)b$ is equal to zero.

- *Media is biased and* $\pi > \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$.

Now suppose to the contrary that the media is biased and $\pi > \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$. Here, as the media is biased then it must have chosen a reporting strategy such that $\sigma_r(\hat{r}) \neq \sigma_l(\hat{l})$.

Notice that here we have $\min\{\theta a,(1-\theta)n\} > 0$ and more specifically let us just assume that $\theta a \geq (1-\theta)b > 0$. So, this implies that $\pi > \frac{\theta a}{\theta a+(1-\theta)b} \geq \frac{(1-\theta)b}{\theta a+(1-\theta)b}$. Now we have $\arg\max E(U_Z(A,S))|^{FF}_{uncon} = \arg\max E(U_Z(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}),\sigma_l(\hat{l})) = (1,1)$ and
$\max E(U_Z(A,S))|^{FF}_{con} = E(U_Z(A,S))|^{FF}_{uncon}$. So, the reporting strategy $(\sigma_r(\hat{r}),\sigma_l(\hat{l})) = (1,1)$ leads to the highest profit which contradicts our assumption that the firm's profit maximizing reporting strategy is $\sigma_r(\hat{r}) \neq \sigma_l(\hat{l})$. Similarly we can show that if $(1-\theta)b \geq \theta a > 0$ then also if media is biased and $\pi > \frac{(1-\theta)b}{\theta a+(1-\theta)b}$, we will have a contradiction. So, we claim that if the media is biased then we can not have $\pi > \frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$.

- *Media is biased and* $\pi < \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$.

Now again suppose to the contrary that the media is biased and $\pi < \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$.

Notice that we have $\theta a,(1-\theta)b\}>0$. Moreover let us be more specific and just consider the case with $\theta a \geq (1-\theta)b > 0$ i.e., $\pi < \frac{(1-\theta)b}{\theta a+(1-\theta)b} \leq \frac{\theta a}{\theta a+(1-\theta)b}$. Now again we have $\arg\max E(U_Z(A,S))|^{FF}_{uncon} = \arg\max E(U_Z(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}),\sigma_l(\hat{l})) = (0,0)$, and
$\max E(U_Z(A,S))|^{FF}_{con} = \max E(U_Z(A,S))|^{FF}_{uncon}$. So, the profit maximizing media should choose the reporting strategy $(0,0)$ and this contradicts our assumption that the media is biased. Similarly, with $(1-\theta)b \geq \theta a > 0$ we reach to contradiction if we assume that the media is biased.

This completes our proof that if the media is biased then we can not have $\pi < \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$.

So, we have proved that if the media is biased then we must have
$\pi \not< \frac{\min\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b}$ and $\frac{\max\{\theta a,(1-\theta)b\}}{\theta a+(1-\theta)b} \not< \pi$. This ends our proof for the only if part.

So, we have proved that the media will be biased in this economy iff

$$0 < \frac{\min\{\theta a, (1-\theta)b\}}{\theta a + (1-\theta)b} \leq \pi \leq \frac{\max\{\theta a, (1-\theta)b\}}{\theta a + (1-\theta)b} < 1$$

$\square$

A.4 Proof of proposition 6

Before beginning the proof formally, we first summarize some observations and lemmas which will be required for the proof to come.

Observation 2. $\forall j \in \{FF, NN\}$

- $E(U_R(A,S))|^j_{(\sigma,\sigma)} = E(U_L(A,S))|^j_{(\sigma,\sigma)} \quad \forall\, \sigma_r(\hat{r}) = \sigma_l(\hat{l}) = \sigma \in [0,1]$
  i.e., People get same expected utility from the unbiased reports irrespective of their type.
- $E(U_R(A,S))|^j_{(\sigma_r(\hat{r}),\sigma_l(\hat{l}))} > E(U_L(A,S))|^j_{(\sigma_r(\hat{r}),\sigma_l(\hat{l}))} \quad \forall\, \sigma_r(\hat{r}) > \sigma_l(\hat{l})$
  i.e., Right biased report gives higher expected utility to the R-type consumers.
- $E(U_R(A,S))|^j_{(\sigma_r(\hat{r}),\sigma_l(\hat{l}))} < E(U_L(A,S))|^j_{(\sigma_r(\hat{r}),\sigma_l(\hat{l}))} \quad \forall\, \sigma_r(\hat{r}) < \sigma_l(\hat{l})$
  i.e., Left biased report gives higher expected utility to the L-type consumers.

Lemma 2. *No reporting strategy of the media firm will make one type of consumers to choose FF and the other to choose NN strategy.*

Proof. Let the reporting strategy chosen by the media be $\sigma_r(\hat{r}) = \sigma_r$, $\sigma_l(\hat{l}) = \sigma_l$. Now suppose to the contrary and without loss of generality that the R-type is choosing FF whereas the L-type is choosing NN then that requires following four inequalities:

$$\theta_r a_r[\lambda + (1-\lambda)\{\sigma_r\pi + (1-\sigma_l)(1-\pi)\}] \geq (1-\theta_r)b_r(1-\lambda)\{\sigma_r(1-\pi) + (1-\sigma_l)\pi\}$$

$$(1-\theta_r)b_r[\lambda + (1-\lambda)\{(1-\sigma_r)(1-\pi) + \sigma_l\pi\}] \geq \theta_r a_r(1-\lambda)\{(1-\sigma_r)\pi + \sigma_l(1-\pi)\}$$

$$\theta_r a_r(1-\lambda)\{\sigma_r(1-\pi) + (1-\sigma_l)\pi\} > (1-\theta_r)b_r[\lambda + (1-\lambda)\{\sigma_r\pi + (1-\sigma_l)(1-\pi)\}]$$

$$(1-\theta_r)b_r(1-\lambda)\{(1-\sigma_r)\pi + \sigma_l(1-\pi)\} > \theta_r a_r[\lambda + (1-\lambda)\{(1-\sigma_r)(1-\pi) + \sigma_l\pi\}]$$

First two inequalities, make the R-type follow $\hat{r}$ and $\hat{l}$ reports and the other two inequalities, make the L-type not follow $\hat{r}$ and $\hat{l}$ reports.

The first two inequalities give us:

$$\frac{[\lambda + (1-\lambda)\{(1-\sigma_r)(1-\pi) + \sigma_l\pi\}]}{(1-\lambda)\{(1-\sigma_r)\pi + \sigma_l(1-\pi)\}} \geq \frac{\theta_r a_r}{(1-\theta_r)b_r} \geq \frac{(1-\lambda)\{\sigma_r(1-\pi) + (1-\sigma_l)\pi\}}{[\lambda + (1-\lambda)\{\sigma_r\pi + (1-\sigma_l)(1-\pi)\}]}$$

The other two inequalities give us:

$$\frac{(1-\lambda)\{(1-\sigma_r)\pi + \sigma_l(1-\pi)\}}{[\lambda + (1-\lambda)\{(1-\sigma_r)(1-\pi) + \sigma_l\pi\}]} > \frac{\theta_r a_r}{(1-\theta_r)b_r} > \frac{[\lambda + (1-\lambda)\{\sigma_r\pi + (1-\sigma_l)(1-\pi)\}]}{(1-\lambda)\{\sigma_r(1-\pi) + (1-\sigma_l)\pi\}}$$

These two inequalities can not hold simultaneously given that we know $\theta_r a_r > (1-\theta_r)b_r > 0$

$\square$

Observation 3. *We have the following observations for different values of* π*:*

- $\pi \in \left[0, \frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}\right)$*: Here* $E(U_R(A,S))|^{FF}$ *and* $E(U_L(A,S))|^{FF}$ *both are decreasing in* $\sigma_r(\hat{r})$ *and* $\sigma_l(\hat{l})$*. Whereas* $E(U_R(A,S))|^{NN}$ *and* $E(U_L(A,S))|^{NN}$ *both are increasing in* $\sigma_r(\hat{r})$ *and* $\sigma_l(\hat{l})$
- $\pi \in \left(\frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}, 1\right]$*: Here* $E(U_R(A,S))|^{FF}$ *and* $E(U_L(A,S))|^{FF}$ *both are increasing in* $\sigma_r(\hat{r})$ *and* $\sigma_l(\hat{l})$ *whereas* $E(U_R(A,S))|^{NN}$ *and* $E(U_L(A,S))|^{NN}$ *both are decreasing in* $\sigma_r(\hat{r})$ *and* $\sigma_l(\hat{l})$
- $\pi \in \left[\frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}, \frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}\right]$*:*
  - *Here* $E(U_R(A,S))|^{FF}$ *is increasing in* $\sigma_r(\hat{r})$ *and decreasing in* $\sigma_l(\hat{l})$*, &* $E(U_R(A,S))|^{NN}$ *is decreasing in* $\sigma_r(\hat{r})$ *and increasing in* $\sigma_l(\hat{l})$*.*
  - *But* $E(U_L(A,S))|^{FF}$ *is decreasing in* $\sigma_r(\hat{r})$ *and increasing in* $\sigma_l(\hat{l})$*, &* $E(U_L(A,S))|^{NN}$ *is increasing in* $\sigma_r(\hat{r})$ *and decreasing in* $\sigma_l(\hat{l})$*.*

The above observation implies that if π takes a value close enough to 1, which is the same as saying that the signal received by the media is much better, the consumers would like the media to be truthful. Whereas, if π is close enough to 0, which is the same as saying that the signals are labelled wrong (i.e., the meanings of r and l are interchanged), the consumers want the media to not follow their signals and report the opposite of their observed signal. When π approaches 0.5, indicating a low-quality signal generator for the media, biased reports will provide higher expected utility for consumers based on their type. With these in mind, we can say that *if the media has a weak signal generator, consumers will prefer biased reporting and the direction of bias will depend on the type of consumer.*

Lemma 3. $\max E(U_i(A,S))|^{FF} > \max E(U_i(A,S))|^{NN} \ \forall\ i \in \{R, L\}$

Proof. Let's consider the case with $i = R$. A similar proof can be written for the L-type consumers too.

- If $\pi \in \left[0, \frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}\right)$:

  $\max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_R(A,S))|^{FF}_{con} = (1-\theta_r)b_r - A_r(1-\lambda)\pi > A_r(1-\lambda)(1-\pi) - \theta_r a_r = \max E(U_R(A,S))|^{NN}_{uncon} \geq \max E(U_R(A,S))|^{NN}_{con}$ where $A_r := a_r\theta_r + (1-\theta_r)b_r$.

  $\arg\max E(U_R(A,S))|^{FF}_{uncon} = \arg\max E(U_R(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (0,0)$
- $\pi \in \left(\frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}, 1\right]$:

  $\max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_R(A,S))|^{FF}_{con} = A_r(1-\lambda)(1-\pi) + (1-\theta_r)b_r > A_r(1-\lambda)\pi - \theta_r a_r = \max E(U_R(A,S))|^{NN}_{uncon} \geq \max E(U_R(A,S))|^{NN}_{con}$

  And the $\arg\max E(U_R(A,S))|^{FF}_{uncon} = \arg\max E(U_R(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,1)$
- $\pi \in \left[\frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}, \frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}\right]$:

  Here $\max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_R(A,S))|^{FF}_{con} = \lambda(1-\theta_r)b_r$
  $> -\lambda\theta_r a_r = \max E(U_R(A,S))|^{NN}_{uncon} \geq \max E(U_R(A,S))|^{NN}_{con}$

  And the $\arg\max E(U_R(A,S))|^{FF}_{uncon} = \arg\max E(U_R(A,S))|^{FF}_{con} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,0)$

□

Only FF: Using the above lemma and noting the fact that the price charged by the media firm is nothing but the maximum expected gain made by the consumers from the report, we can stop considering the reporting strategy and prices so that the consumers chooses not to follow any type of reports. Also we have already argued that the FN or NF strategies of the consumers give them zero or non-positive expected gain and so the maximum profit the media can make for such reporting is non-positive. So, from now on we will only consider FF strategy of the consumers.

Notice that the media firm here need to decide whether it wants to choose a reporting strategy and price such that it serves both the types of consumers or only one type of consumers.

If the media wants to sell to both the types of the consumers then the optimization problem faced will be

$$maximize_{\{\sigma_r(\hat{r}),\sigma_l(\hat{l})\}} \min\{E(U_R(A,S))|_{con}^{FF}, E(U_L(A,S))|_{con}^{FF}\}$$

Noticeably here

$\arg\max(\min\{E(U_R(A,S))|_{uncon}^{FF}, E(U_L(A,S))|_{uncon}^{FF}\})$

$= \arg\max(\min\{E(U_R(A,S))|_{con}^{FF}, E(U_L(A,S))|_{con}^{FF}\})$

$= (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,1)$ if $\pi \geq \max\left\{\frac{1}{2}, 1 - \frac{(1-\theta_r)b_r}{A_r(1-\lambda)}\right\}$

and

$\arg\max(\min\{E(U_R(A,S))|_{uncon}^{FF}, E(U_L(A,S))|_{uncon}^{FF}\})$

$= \arg\max(\min\{E(U_R(A,S))|_{con}^{FF}, E(U_L(A,S))|_{con}^{FF}\})$

$= (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (0,0)$ if $\pi \leq \min\left\{\frac{1}{2}, \frac{(1-\theta_r)b_r}{A_r(1-\lambda)}\right\}$

Also,

$\max(\min\{E(U_R(A,S))|_{uncon}^{FF}, E(U_L(A,S))|_{uncon}^{FF}\}) = (1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}$

So, as long as $\max(\min\{E(U_R(A,S))|_{uncon}^{FF}, E(U_L(A,S))|_{uncon}^{FF}\}) \geq 0$, we have

$\max(\min\{E(U_R(A,S))|_{uncon}^{FF}, E(U_L(A,S))|_{uncon}^{FF}\})$

$= \max(\min\{E(U_R(A,S))|_{con}^{FF}, E(U_L(A,S))|_{con}^{FF}\})$

From here we claim that the maximum non-negative profit that the media can earn from serving both types of consumers is

$$(1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}$$

and that is only through unbiased reporting (Either complete truthful or complete truthless depending on π.). From observation 3, we can claim that this is also going to be the maximum profit that unbiased media can make.

If the media wants to serve only one type consumers then it faces the following optimization problem:

$$maximize_{\{\sigma_r(\hat{r}),\sigma_l(\hat{l})\}} \max\{\phi_r E(U_R(A,S))|_{con}^{FF}, (1-\phi_r)E(U_L(A,S))|_{con}^{FF}\}$$

such that $E(U_R(A,S))|_{con}^{FF} \neq E(U_L(A,S))|_{con}^{FF}$. This optimization will have a solution if and only if $\pi \in \left[\frac{b_r(1-\theta_r)}{a_r\theta_r + b_r(1-\theta_r)}, \frac{a_r\theta_r}{a_r\theta_r + b_r(1-\theta_r)}\right]$. This is evident from lemma 3, that is for these values of π only, the expected gains from the report are behaving differently for the two types of consumers in the reporting strategy of the media firm. In other words, for all $\pi \notin \left[\frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}, \frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}\right]$ a media will not choose a reporting strategy and price such that it serves only one type of consumers. And as we showed that such media will only choose unbiased reporting.

So, we consider $\pi \in \left[\frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}, \frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}\right]$.

Notice that here

$\arg\max E(U_R(A,S))|^{FF}_{con} = \arg\max E(U_R(A,S))|^{FF}_{uncon} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (1,0)$
$\arg\max E(U_L(A,S))|^{FF}_{con} = \arg\max E(U_L(A,S))|^{FF}_{uncon} = (\sigma_r(\hat{r}), \sigma_l(\hat{l})) = (0,1)$.

In fact, we have $\max E(U_R(A,S))|^{FF}_{con} = \max E(U_R(A,S))|^{FF}_{uncon} = \max E(U_L(A,S))|^{FF}_{con} = \max E(U_L(A,S))|^{FF}_{uncon} = \lambda(1-\theta_r)b_r = \lambda\theta_l a_l$.

So, the maximum profit that the media can make by optimally choosing the reporting strategy such that it serves only one type of consumers, is:

$$\lambda \max\{\phi_r, 1-\phi_r\}(1-\theta_r)b_r$$

If the R-types are larger in population, the reporting strategy will be completely right biased and vice-versa. So, in other words, it is the maximum profit that a biased media can make.

Lemma 4. *Maximum profit that a biased media can make is $\lambda\max\{\phi_r, 1-\phi_r\}(1-\theta_r)b_r$ whereas the maximum profit that an unbiased media can make is $(1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}$.*

Now we begin the formal proof with all the knowledge built through the above claims.

Proof. If direction:
If $0 < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} \leq \pi \leq 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} < 1$, the profit maximising media firm will choose to be biased.

Let's assume that $0 < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} \leq \pi \leq 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} < 1$. That means necessarily we have $\frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} \in (0, 0.5)$. Noticeably here

$\pi \in \left[\frac{(1-\theta_r)b_r(1-\lambda\max\{\phi_r,1-\phi_r\})}{A_r(1-\lambda)}, 1 - \frac{(1-\theta_r)b_r(1-\lambda\max\{\phi_r,1-\phi_r\})}{A_r(1-\lambda)}\right] \subset \left[\frac{b_r(1-\theta_r)}{a_r\theta_r+b_r(1-\theta_r)}, \frac{a_r\theta_r}{a_r\theta_r+b_r(1-\theta_r)}\right]$

Now we can rewrite $\frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} \leq \pi \leq 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}$ as

$$\underbrace{\lambda\max\{\phi_r, 1-\phi_r\}(1-\theta_r)b_r}_{\text{Maximum profit from biased reporting}} \geq \underbrace{(1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}}_{\text{Maximum profit from unbiased reporting}}$$

And so the media chooses to be biased here. The direction of bias will depend on the value ϕ_r. If $\phi_r > 0.5$ the media chooses to be right biased with the reporting strategy $\sigma_r(\hat{r}) = 1$ and $\sigma_l(\hat{l}) = 0$ whereas if $\phi_r < 0.5$ the media chooses to be left biased with the reporting strategy $\sigma_r(\hat{r}) = 0$ and $\sigma_l(\hat{l}) = 1$.

So, *if* $0 < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} \leq \pi \leq 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} < 1$ *then the profit maximizing media firm choose to be biased.*

Only if direction:
If the media is biased then we must have
$0 < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} \leq \pi \leq 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)} < 1$. First of all note that the bounds on π are equidistant from $\frac{1}{2}$ and neither of them can be zero if the media firm is choosing to sell the report at a positive price.

Now to prove the only if part let us suppose to the contrary that the media is biased and

$$\pi \in \left[0, \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r, 1-\phi_r\})}{(\theta_r a_r + (1-\theta_r)b_r)(1-\lambda)}\right) \cup \left(1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r, 1-\phi_r\})}{(\theta_r a_r + (1-\theta_r)b_r)(1-\lambda)}, 1\right]$$

As we previously argued that as long as $(1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\} \geq 0$ the profit of the unbiased media is $(1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}$. This is ensured here as we have,

$$\min\{\pi, 1-\pi\} < \frac{(1-\theta_r)b_r(1-\lambda\max\{\phi_r, 1-\phi-r\})}{A_r(1-\lambda)} < \frac{(1-\theta_r)b_r}{A_r(1-\lambda)}$$

We will have immediate contradiction for $\pi \in \left[0, \frac{(1-\theta_r)b_r}{a_r\theta_r+(1-\theta_r)b_r}\right) \cup \left(\frac{a_r\theta_r}{a_r\theta_r+(1-\theta_r)b_r}, 1\right]$ because using lemma 3 we know that the firm can make higher profit by being unbiased.

So, we assume
$\pi \in \left[\frac{(1-\theta_r)b_r}{a_r\theta_r+(1-\theta_r)b_r}, \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}\right) \cup \left(1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}, \frac{a_r\theta_r}{a_r\theta_r+(1-\theta_r)b_r}\right]$
Here $\pi < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}$ and $\pi > 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}$ ensures the following inequality.

$$\lambda\max\{\phi_r, 1-\phi_r\}(1-\theta_r)b_r < (1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}$$

Whereas $\pi \geq \frac{(1-\theta_r)b_r}{a_r\theta_r+(1-\theta_r)b_r}$ and $\pi \leq \frac{\theta_r a_r}{a_r\theta_r+(1-\theta_r)b_r}$ ensures that the left hand side of the above inequality is the maximum profit that the media can earn by being biased. And $\pi < \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}$ and $\pi > 1 - \frac{b_r(1-\theta_r)(1-\lambda\max\{\phi_r,1-\phi_r\})}{(\theta_r a_r+(1-\theta_r)b_r)(1-\lambda)}$ ensures that the right hand side of the above inequality is the maximum profit that an unbiased media can make.

So, we have here,

$$\underbrace{\lambda\max\{\phi_r, 1-\phi_r\}(1-\theta_r)b_r}_{\text{Maximum profit from biased reporting}} < \underbrace{(1-\theta_r)b_r - A_r(1-\lambda)\min\{\pi, 1-\pi\}}_{\text{Maximum profit from unbiased reporting}}$$

This contradicts the fact that the media is biased because here the media can make a higher profit by being unbiased.

□

B Perfect Bayesian Equilibrium

B.1 Discussion on *Perfect Bayesian Equilibrium (PBE)*

Without loss of any generality let us assume that $a\theta \geq (1-\theta)b$. In this section we prove that the following strategy profile is a babbling pooling PBE: Both types of firms choose to sell the report and charge the same price $\lambda(1-\theta)b$ but the reporting of the normal quality firm is $\sigma_r(\hat{r}) = 1$, $\sigma_l(\hat{l}) = 0$ whereas the good quality media firm chooses to be unbiased and truthful. Consumers choose to *follow* both the types of reports and chooses to buy the report as long as the net expected gain from the report is non-negative. It is shown in the proof of Proposition 4 that if the good quality follows the above strategy, the consumers and normal media firm follow the above strategies too. So, all we need to show now is that the good quality media firm also has no incentive to deviate from that strategy. Suppose that consumers and the normal quality media are following the above strategy. Recall that the primary difference between good quality and normal quality media firms is that the former has an information/signal generator which perfectly reveals the true state. Notice that as it is a babbling pooling PBE that we are talking about, the posterior beliefs about the firm quality will be the prior belief λ only. Now we state the following observation:

Observation 4. *In any babbling PBE, the behavior of the good quality media firm is same as that of the normal quality media firm with just* $\pi = 1$.

With the above observation and the characterization of proposition 4, we know that the good quality media firm will choose to be unbiased and charge a price $\lambda \min\{a\theta, b(1-\theta)\}$ to sell the report. The argument for this price is given in the proof of the proposition 4.

B.2 Separating PBE:

Proposition 7. *There is no separating PBE in this economy.*

Proof. Let us suppose to the contrary that there is a separating PBE where the selling takes place (at least for the good quality media firm). Let that PBE be where the good quality media firm is charging some $\bar{p}$ and the normal quality firm is charging some $\tilde{p}$. Now consider the normal quality media firm. If it charges $\tilde{p}$ then the consumers hold a posterior of $\lambda = 0$. i.e., the firm is of normal quality with certainty. Now we have two cases to consider: one where the condition posed in proposition 3 is satisfied and the one where it is not. In the first case, at optimal, the normal quality media is biased and earns a profit (/charges a price) of $\lambda \min\{\theta a, (1-\theta)b\} = 0$ and in the later case the normal quality media firm chooses to be unbiased and earns a profit of $\min\{\theta a, (1-\theta)b\} - (a\theta + (1-\theta)b)\,(1-\lambda) \min\{\pi, 1-\pi\} = \min\{\theta a, (1-\theta)b\} - (a\theta + (1-\theta)b) \min\{\pi, 1-\pi\} < 0$ where the last inequality is from the assumption that the condition of proportion 3 is not satisfied. So, here the normal media firm can only earn a non-positive profit. Now if the normal media firm charges a price exactly equal to $\bar{p}$, it will earn a positive profit of $\bar{p}$. □

B.3 All pooling Perfect Bayesian Equilibria

□ *Assumption:* $0 < \frac{(1-\theta)b}{\theta a + (1-\theta)b} \le \pi \le \frac{\theta a}{\theta a + (1-\theta)b} < 1$

We begin with the above assumption. Suppose a pooling PBE exists where both types of media firms charge the same price, say p^*. Let that strategy be:

$$S_{nm} = (\text{sell}, p^*, (\hat{\sigma}_r(\hat{r}), \hat{\sigma}_l(\hat{l})))$$
$$S_{gm} = (\text{sell}, p^*, (\tilde{\sigma}_r(\hat{r}), \tilde{\sigma}_l(\hat{l})))$$

One thing that we can say for sure is that $p^* \le E(U)|_{with\ report} - E(U)|_{without\ report}$ where the expected utilities are calculated given the reporting strategies defined in the above strategy profile and the consumer chooses to follow both the types of report. One interesting observation that we can see here is that if the above strategy is a pooling PBE for some belief system, the below strategy should also be a pooling PBE too:

$$S_{nm} = (\text{sell}, p^*, (1, 0))$$

$$S_{gm} = (\text{sell}, p^*, (1, 1))$$

where the p^* is that same price. We must have $p^* \le \lambda(1-\theta)b$ as this is the highest price that the media can charge in any pooling PBE. Now the question is what sort of beliefs we can have so that this strategy can be sustained as a pooling PBE. First of all, notice that irrespective of the media firm's type, no firm has any incentive to charge a price that is lower than p^* as that can only lead to a lower profit which in fact is the price that they charge if they get to sell. So, the only possible deviations that can ruin such pooling PBE are those where either of these firms charge a higher price. Now let's first think of a normal quality media firm. Given the strategies of the other players, the firm will not want to change the reporting strategy as that will only reduce the expected utility of the consumers with the report and it will have no effect on the belief of the consumers regarding the quality of the media firm. So, any profitable deviation should be the one where the media charges a price, $p' > p^*$ with the same reporting strategy. Now this will be a profitable deviation for the media iff the consumers choose to buy at this price. Notice that to make the consumers buy the report at this price the required condition is just $p' \leq \lambda(p')(1-\theta)b$. So, if we can ensure that $\forall\, p' > p^*$ we have $p' > \lambda(p')(1-\theta)b$, the strategy profile will be a pooling PBE.

So, we can summarize all the polling PBE as:

$$S_{nm} = (\text{sell}, p^*, (\hat{\sigma}_r(\hat{r}), \hat{\sigma}_l(\hat{l})))$$
$$S_{gm} = (\text{sell}, p^*, (\tilde{\sigma}_r(\hat{r}), \tilde{\sigma}_l(\hat{l})))$$

The consumers choose to *follow* both types of reports and buy the report as long as the expected net gain made from the report is at least as large as the price charged by the media firm for the report. The quality belief system is as follows:

$$\lambda(p) = \begin{cases} \lambda & \text{if } p = p^* \\ \\ [0,1] & \text{if } p < p^* \\ \\ \lambda(p) < \frac{p}{(1-\theta)b} & \text{if } p > p^* \end{cases}$$

Notice that from the above pooling PBE strategy profile, we get the babbling pooling PBE, where $\lambda(p) = \lambda \;\; \forall\, p$, as

$$S_{nm} = (\text{sell}, p^*, (1,0)$$
$$S_{gm} = (\text{sell}, p^*, (1,1)$$

where $p^* = \lambda b(1-\theta)$, and the strategy of the consumers, $S_c = (\text{buy}, F, F)$.

Similarly, for $\pi \in (0,1) \setminus \left(\frac{(1-\theta)b}{\theta a+(1-\theta)b}, \frac{\theta a}{\theta a+(1-\theta)b}\right)$, the set of all pooling PBE can be summarized as:

$$S_{nm} = (\text{sell}, p^*, (\hat{\sigma}_r(\hat{r}), \hat{\sigma}_l(\hat{l})))$$
$$S_{gm} = (\text{sell}, p^*, (\tilde{\sigma}_r(\hat{r}), \tilde{\sigma}_l(\hat{l})))$$

where $p^* \leq (1-\theta)b - (a\theta + (1-b)\theta)(1-\lambda)\min\{\pi, 1-\pi\}$, the price that can be achieved only through unbiased reporting strategy. The consumers choose to *follow* both types of reports and buy the report as long as the expected net gain made from the report is at least as large as the price charged by the media firm for the report.

The quality belief system is as follows:

$$\lambda(p) = \begin{cases} \lambda & \text{if } p = p^* \\ \\ [0,1] & \text{if } p < p^* \\ \\ \lambda(p) < \frac{p-(1-\theta)b+A\min\{\pi,1-\pi\}}{A\min\{\pi,1-\pi\}} & \text{if } p > p^* \end{cases}$$