A Theory of International Alliances *

Abstract

Keywords: information asymmetry, coordination game, information transmission, commitment, alliance formation
JEL codes: C72, D20, D74, D82, F53

*

1 Introduction

Information sharing is an integral part of policymaking in international alliances. Countries in an alliance typically have common objectives (e.g., collective defense, security cooperation, immigration, etc.), and share information in order to coordinate their policies towards achieving these shared objectives. The European Union, for example, has established multiple forums for information exchange between the member countries on a wide gamut of issues.[1] An important consideration is that information sharing is affected by policy constraints that individual countries face. Further, in order to aggregate information and make policy decisions efficiently, committing costly resources (e.g., fiscal or military) to the alliance plays an important role. To see this difference, consider migration policy as opposed to security policy. In the former, policymaking is constrained by the fact that countries have bounds imposed by their state capacity and the political preferences of their electorate. In the case of the latter, the ex-ante commitment of countries to physical resources becomes their policy set in the future.

In other words, the policy constraints faced by countries are endogenously determined as a consequence of their initial commitment decisions. For example, the NATO, which is a security and defense focused alliance, has laid an important focus on information sharing and commitment to resources.[2] Specifically, NATO proscribes an implicit "*rule of thumb*" on defense spending of 2 percent of GDP for member countries. The Permanent Structured Cooperation (PESCO) agreement, on the other hand, explicitly mandates participating countries to commit about 2.5-3 percent of GDP.[3]

However, some pertinent problems remain with introducing such commitment clauses. A majority of the commitments to NATO, for example, have been made by U.S. in comparison to European countries. In fact, this *commitment gap* has been glaringly evident in NATO since the early 2000's. In 2007, the U.S. contributed roughly 68% of the total defense spending of NATO, and this increased to 72% by 2012. As of 2012, U.S. spent over four per cent of its GDP on defense compared to European counterparts that averaged around 1.6%, and some of whom spent less than one per

[1]Some of the prominent ones are Europol for internal law enforcement within the EU, and Interparliamentary EU Information Exchange (IPEX) which is a platform for information exchange between EU national parliaments.

[2]NATO's *2010 Strategic Concept* document captures this idea succinctly: "The alliance will engage actively to enhance international security, through partnership with relevant countries and other international organizations; by contributing actively to arms control, non-proliferation and disarmament."; further, it adds "Any security issue of interest to any ally can be brought to the NATO table, to share information, exchange views and, where appropriate, forge common approaches."

[3]The PESCO agreement from 2018 reads, and we quote, "The aim is to jointly develop defence capabilities and make them available for EU military operations....The difference between PESCO and other forms of cooperation is the legally binding nature of the commitments undertaken by the participating Member States. The decision to participate was made voluntarily by each participating Member State, and decision-making will remain in the hands of the participating Member States in the Council."

cent towards defense spending. This growing *transatlantic gap* in defense spending has remained an important issue of contention in the relationship between U.S. and EU. Further, with the EU trying to form its own defense alliances through PESCO and European Defence Agency (EDA), questions pertaining to information sharing and commitment in alliances have become ever more salient.

The theoretical literature on informational incentives and commitment in international organizations, and more specifically, in alliances, is surprisingly limited.[4] The aim of the paper is to fill this gap in the literature. In this context, we are interested in the following questions. How is information aggregation within an alliance affected by the presence of policy constraints among the players? If players could instead commit to resources by investing in them ex-ante, what is the relationship between the individual preferences of players and their investments to the alliance? What factors determine the divergence in ex-ante commitments to the alliance?

In this paper, we develop a stylized model to study incentives for information aggregation and commitment in international alliances. In the baseline setup without commitment, we consider an alliance with four key features: information asymmetry about an unobservable state of nature, positive spillovers in policies, preference heterogeneity (*biases*) over final outcomes, and exogenous constraints on the policy set. The setup induces a multi-player version of a coordination game (see e.g. Venkatesh, 2023) in which each player has a coordination function that depends on the actions of all other players. Specifically, a player's policy decision has a positive spillover on every other player's coordination function. Further, players face a quadratic loss when this coordination function deviates from the players' expectation of the (bias adjusted) state of nature. Information about an underlying state of nature is *soft* in nature and *disaggregated* among the players. That is, conditional on the draw of the (unobserved) state of the world, each player receives a private signal—*low* or *high*—that imperfectly informs them about the true underlying state (Morgan and Stocken, 2008; Galeotti et al., 2013). In the communication stage, each player, publicly and simultaneously, sends a cheap talk message about their private information. After the communication stage, conditional on the private information and the messages exchanged, each player takes a policy decision from the policy set.

First, we characterize the equilibrium condition for full information revelation in the presence of exogenous policy constraints. The characterization has a surprisingly intuitive feature: information aggregation depends *only* on the biases of the two extreme players and *not* on the composition of biases in the alliance. This is in stark contrast to earlier results in the literature on multi-player cheap talk games in which

[4]Some papers have delved into the role of intellectual property rights on investment decisions by countries in the context of climate agreements (Harstad, 2012; Harstad, 2016), and the impact of domestic political incentives on international agreements (Buisseret and Bernhardt, 2018; Battaglini and Harstad, 2020) between countries.

truthful communication depends on the preference composition of the players.

The intuition for why the composition of biases do not matter in our model is as follows. In order to reveal information truthfully, the subsequent policy decision of the players must be within the feasible policy set. If this is violated, players have an incentive to misreport (exaggerate or understate) their private information in order to change the other players' actions. The policy set therefore acts as an incentive compatibility constraint for truth-telling.[5] Given this feature, players can be grouped into two types: those who always reveal the low signal but may lie about the high signal, and vice-versa. For the former, the only relevant incentive compatibility constraint is when the action exceeds the lower bound for the lowest posterior expectation of the state conditional on the player holding a high signal. If this holds, then the action is above the bound for all other higher expectations of the state. Similarly, for the latter set of players, the only constraint of interest is for the action to be below the upper bound for the highest possible posterior expectation of the state, conditional on them holding a low signal. The set of conditions for full information revelation therefore boils down to just *two* constraints, one each for the least biased player with a high signal, and the most biased player with a low signal.

Next, we use the cohesiveness of an alliance (distance between the two extreme biases) to derive the maximal size of an alliance. We show that the maximal size of an alliance depends crucially on the interdependency in policies. With a higher interdependency, there is greater spillovers in actions of players which tightens the incentive compatibility constraints in equilibrium. This reduces the size of the alliance. Further, fixing the cohesiveness of an alliance, we find that smaller alliances support more policy interdependence. The implication of these results is that alliance formation must take into account the interdependency across policy issues.

Our results on information revelation in alliances illustrates how information sharing is linked to the extent of cohesiveness and interdependency within the alliance. Consider the issue of migration control that concerns the European Union. Migration related policies have spillovers since countries migrants usually enter the "*frontline states*" and proceed to emigrate to more prosperous countries.[6] In the case of the Russia-Ukraine war that began in 2022, Hungary became the primary frontline state since most migrants fled to its borders first before trying to emigrate to other countries within the EU. When the EU formulated a policy of quotas such that every member would take in a percentage of the reported numbers, it incentivized Hungary to inflate its migration numbers. In March 2022, an article in *The Guardian* reported that the

[5]This feature is similar to the one studied by Venkatesh (2023) with two players and one-sided information asymmetry.

[6]See `https://www.unav.edu/web/global-affairs/detalle/-/blogs/refugee-crisis-the-divergence-between-the-european-union-and-the-visegrad-group` for more on this.

migration numbers were grossly exaggerated by the Hungarian government.[7] This is precisely captured by the truthful information aggregation conditions identified in the model.

Given that the incentives for truthful information revelation is closely linked to the size of the policy set, we next endogenize this constraint. Specifically, we allow for players to choose ex-ante commitments to a set of actions that act as a constraint on their subsequent decisions. The continuation game is as before in that players observe their private information after the commitments are sunk, communicate them to others, and then take an action from their available set. The commitment is costly and players pay a marginal cost upfront for every unit committed to. This could be interpreted as an "*opportunity cost*" of investment in otherwise costly resources like military infrastructure (troops, weapons, etc.), or fiscal expenditures that are diverted towards the alliance. Importantly, we assume that the commitment is implicit: it is a strategic decision and cannot be imposed on the players in the alliance.

Since greater first period investments reduce the constraints players face in the decision-making phase later on, there is a trade off between the expected benefits of committing to greater resources in the beginning and the cost of doing so. Once the investments are sunk, players aggregate information under two possible scenarios: *(a)* the investment levels are high enough such that they take the appropriate first-best actions (i.e., fully efficient); or, *(b)* the investment levels are such that the constraints do not bind up to some information level, but binds for all higher expectations of the state (i.e., partially efficient). In the latter case, there is no incentive to misrepresent private information for the players since their actions are anyway constrained beyond a threshold of information, irrespective of whether they hold a high or low signal.

We characterize the unique symmetric commitment equilibrium in which a player's ex-ante investment is proportional to their individual bias, and is a function of the composition of biases in the alliance. However, interestingly, the marginal benefits from investing an additional unit in the alliance is independent of player's bias and is uniform across all the players. This implies that, in equilibrium, each player chooses the same threshold of information up to which their actions are not constrained. Beyond this threshold there is no change in actions of players and therefore there is full revelation of information.

The equilibrium characterizes the commitment to an *action set* for each player. This set has an intuitive ordering: The commitments are increasing in the biases such that the most biased player in the alliance has the largest set of actions while the least biased player's action set is the smallest. That is, the initial preferences of players over the policy issue determines the size of the commitment (*preference effect*). Crucially,

[7] See https://www.theguardian.com/world/2022/mar/30/hungary- accused-of-inflating-number-of-ukrainian-arrivals-to-seek-eu-funds.

these differences are also exacerbated by the degree of interdependence in actions (*interdependence effect*). That is, when the actions of players become more interdependent, the action set of players shrink since the marginal benefit from a unit of investment is decreasing. From a player's perspective, higher interdependencies in actions implies an opportunity to free-ride on other players' investments. This drags down the aggregate investment in the alliance and partial efficiency threshold falls. That is, players are constrained for smaller thresholds of information.

The preference effect is driven by heterogeneity in the preferences over the policy in question. The analysis argues that what matters for greater commitment to resources is the biases of countries. If some countries cared *less* about, say, military interventions, then their willingness to contribute to defense initiatives would diminish. This could be interpreted as lower investment level that is proportionate with their individual preference. The second effect, the interdependency effect, is due to the nature of interventions that countries undertake. To be more precise, any joint action, like military strikes or peacekeeping, allows for a form of *piggybacking* in which countries that care less about the issue tend to piggyback on the efforts of those countries for whom the intervention is more salient.

In the context of alliances, the need for commitment to costly resources cannot be understated. For example, US Presidents have long advocated for greater partnership and contributions from EU countries. Bush in 2006, Obama in 2014, and more recently Trump in 2018 have all pushed for a greater share of resource contributions from NATO's European allies.[8] The NATO, since 2006, has agreed to and adopted a resolution for implementing the "*two per cent guideline*" for member nations in which each ally would contribute 2 percent of GDP (proportional contributions) for NATO's collective defense initiatives. The fact that European members of NATO have not contributed sufficiently to the joint defense budget has been well documented.[9]

The transatlantic commitment gaps have been an issue in NATO since the early 2000's. For example, at the end of the cold war in 1989/90, the U.S. contributed 6% of its GDP and European members of NATO around 3% of their GDP towards joint-defense expenditures. However, from 1990 onwards there was a steady decline in these expenditures - in 2000 the U.S. contributed 3.2% of GDP and NATO Europe around 2% of GDP. The September 2001 attack reversed the trend for the United States, which began committing more resources to fighting the war on terror. As a result, by 2009, the United States was spending about 5.3% of GDP while the expenditure by

[8]See https://www.cnbc.com/2018/07/11/obama-and-bush-also-pressed-\nato-allies-to-spend-more-on-defense.html for more on this point.

[9]Petersson (2015) writes, quoting from Obama's "West Point speech" on May 28, 2014, "We can't have a situation in which the United States is consistently spending over 3 percent of our GDP on defense," and Europe is spending 1 percent. "The gap becomes too large," he continued, and the alliance needed to make sure that everybody was doing their fair share."

European members gradually decreased to approximately 1.8% of GDP.[10]

Our results clearly provide a strategic rationale for why an alliance like NATO struggle with enforcing the "*two per cent of GDP* " rule. We argue that these rules are indexed to the GDP of the countries while in reality what matters is how much each country cares about the underlying policies that the alliance engages in.

These trends are in line with the theoretical predictions of this paper. In particular, the near universal drop in contributions between 1990-2000 could be almost entirely attributed to the decreased Soviet threat (post the cold war breakup of USSR). The ending of the Cold war is equivalent to a decline in the b_i's of the countries in NATO. However, when the terror attacks of September 2001 happened, the salience of defense related spending increased again. Correspondingly, it disproportionately affected some NATO members more than others. This explains why there was a continued decline in defense contributions by European partner countries compared to US and the UK. The nature of defense policy also changed with the '*war on terror*' in that it required coordination of allies and troops on the ground (in Afghanisthan and Iraq), which was not entirely the case during Cold war. This is akin to an increase in interdependency which diminished the incentives of doves and exacerbated those of hawks to contribute towards the war efforts and the subsequent peacekeeping interventions that followed.

Related Literature

The paper is related to the cheap talk literature with multiple senders and receivers. The two closest papers are Hagenbach and Koessler (2010) and Galeotti et al. (2013). Though the information and communication structure we adopt is identical to Galeotti et al. (2013), a fundamental difference is that they assume the actions of players are independent of each other. We allow for interdependent actions, so that a player's message affects her own action by shifting beliefs of other players. Hagenbach and Koessler (2010) study a model of strategic complementarities in actions with multiple players. In their framework, private signals of players are independent and communication between players is private. In contrast, in this paper, actions are imperfectly substitutable, private signals of players are conditionally independent but correlated, and the communication protocol is public.[11]

The paper is also related to collective decision-making literature. Gerardi and Yariv (2007) study role of communication in a collective choice (voting) problem; Jack-

[10] As of 2014, only four countries in NATO contributed over 2% of GDP to defense (U.S., Greece, United Kingdom, and Estonia). See https://csis-prod.s3.amazonaws.com/s3fs-public/publication/180816_NATO_Burden_Sharing_0.pdf for more on the infeasibility of GDP based burden sharing rules.

[11] In addition, we focus on conditions for full information aggregation that is equivalent to a complete network formulation in Corollary 4 of Hagenbach and Koessler (2010).

son and Tan (2013) analyze voting rules with informational asymmetry and (hard) information disclosure; and, both Austen-Smith and Feddersen (2005) and Austen-Smith and Feddersen (2006) look into optimal voting rules with deliberative decision-making. While this literature studies the role communication in a collective choice (voting) problem, in contrast, we study a problem of information aggregation in an interdependent decision-making environment with capacity constraints.

Finally, many papers have analyzed information aggregation in the context of elections (Bhattacharya, 2013; Feddersen and Pesendorfer, 1997), polling (Morgan and Stocken, 2008), financial markets (Werning and Angeletos, 2006; Dasgupta and Prat, 2008), and organizations (Dewan et al., 2015; Jehiel, 1999). None of the papers look at interdependent action environments, and the relationship between information aggregation and constraints.

2 Model

Consider a group of players in an alliance, indexed by $N = \{1, 2, ..., n\}$. Each player chooses a policy $x_i \in [0, 1] \equiv V$ on an issue of common interest. The payoff to each player depends on an unknown state of the world θ that is distributed uniformly on $[0, 1]$. The policies of players are interdependent in that the final policy outcome depends on the policies chosen by all the players. Further, from the perspective of each player, this final outcome is possibly heterogeneous. This is captured by a coordination function for each player, $\phi_i : V^N \to [0, 1]$, that depends on the policy vector $\mathbf{x} = (x_1, ..., x_n)$. The coordination function takes the following form:

$$\phi_i(\mathbf{x}) = \frac{x_i + \eta \sum\limits_{j \neq i} x_j}{1 + (n-1)\eta}$$

The ϕ_i function exhibits two key features. First, each player's policy, x_i, has a greater marginal effect compared to other players' policies. Second, the parameter $\eta \in (0, 1)$ in the ϕ_i function captures the degree of spillover in the policies of players. That is, a higher η indicates greater degree of substitutability in the policies. Finally, each player has a bias b_i that captures the heterogeneity of preferences within the alliance. Players want to match their respective coordination function to the bias-adjusted state, $\theta + b_i$. Specifically, player i's utility function is given by:

$$u_i(\mathbf{x}; \theta, b_i) = -\left[\phi_i(\mathbf{x}) - \theta - b_i\right]^2$$

If the biases were homogeneous and equal to b, then there would be no strategic problem since every player would want to perfectly reveal their information and take an action equal to $\theta + b$. The bias parameter b_i therefore captures the (mis)alignment of interests within the alliance. We assume without loss of generality that biases are ordered: $0 \leqslant b_1 \leqslant b_2 \leqslant .... \leqslant b_n$, with at least one strict inequality. For the sake of exposition, we define $\mathbf{b} = \{b_1, b_2, ..., b_n\}$, $\underline{b} = b_1$, $\bar{b} = b_n$, and $b_n > b_1$. We refer to a player whose bias exceeds the average bias of the group, $Avg(\mathbf{b})$, as a hawk, and one whose bias is below the average as a dove. The feasible policy set, $V = [0, 1]$, places constraints on the outcomes that players can achieve despite their heterogeneity in preferences over the final outcomes. We focus on information transmission incentives in the presence of such policy constraints.

Information Structure, Communication, and Actions

Information structure. The problem of information aggregation is critical since information about the underlying state θ is imperfect and distributed among the players. The information asymmetry among players is modeled along the lines of Morgan and Stocken (2008) and Galeotti et al. (2013). Specifically, the underlying state $\theta \in \mathbb{U}[0, 1]$ is not directly observable. Each player i receives an imperfect private signal $s_i \in S_i \equiv \{0, 1\}$ about the state of the world such that: $s_i = 1$ with probability θ, and $s_i = 0$ with probability $1 - \theta$. The conditional density $f(\theta|\{s_i\}_{i \in N})$ belongs to a standard Beta-binomial distribution. The sufficient statistic of the distribution is denoted by k, which is the number of signals $s_i = 1$. The posterior distribution of θ with uniform prior on $[0, 1]$, given k, is a Beta distribution with parameters $k + 1$ and $n - k + 1$. Consequently, $f(\theta|\{s_i\}_{i \in N}) = \frac{(n+1)!}{k!(n-k)!}\theta^k (1 - \theta)^{n-k}$ and $E[\theta|\{s_i\}_{i \in N}] = \frac{k+1}{n+2}$. The Beta distribution implies players' signals are conditionally independent but correlated.[12]

Communication. After each player receives their signal s_i, they publicly and simultaneously communicate their information through a cheap talk message to the alliance. We focus on pure messaging strategies in which each player simultaneously sends a public message $m_i(s_i)$ to every other player in the alliance. Player i's messaging strategy is given by $m_i : \{0, 1\} \longrightarrow \{0, 1\}$. A truthful message is one where $m_i(s_i) = s_i$ for $s_i = \{0, 1\}$, and a babbling message is characterized by $m_i(s_i) = m_i(1 - s_i)$. Let $\mathbf{m} = (m_1, m_2, ..., m_n)$ be the joint communication strategy of the players.

Actions. After the messages have been exchanged publicly, each player's policy is chosen simultaneously. The strategy for a player can be defined as $\tau_i : S_i \times \{0, 1\}^N \rightarrow V$. That is, $\tau_i(s_i, (m_i, m_{-i}))$ is the action of player i with private signal s_i, who sent the message m_i, and observed the messages $m_{-i} = (m_j)_{j \neq i}$ from the other players in the alliance. Let $\tau(\mathbf{s}, \mathbf{m}) = \left\{\tau_i(s_i, (m_i, m_{-i}))\right\}_{i \in N}$ be the strategy profile of the players.

[12]Specifically, $\Pr(s_j = 1 \mid s_i = 1) = \frac{2}{3}$, $\Pr(s_j = 0 \mid s_i = 1) = \frac{1}{3}$ and $\Pr(s_j = 0 \mid s_i = 0) = \frac{2}{3}$, $\Pr(s_j = 1 \mid s_i = 0) = \frac{1}{3}$.

The parameter η, bias vector $\mathbf{b}$, and the policy set V are all assumed to be common knowledge. The timing of the policymaking game is as follows:

1. The state of nature θ is drawn from a uniform distribution on $[0,1]$. Conditional on θ each player observes a private signal $s_i \in \{0,1\}$.

2. The players simultaneously send a public message $m_i(s_i)$ to the alliance. The vector $\mathbf{m}$ is the publicly available information at the end of the communication stage.

3. After observing the private signal s_i and set of messages $\mathbf{m}$ each player decides on the policy $x_i(s_i, \mathbf{m}) \in V$ simultaneously. Payoffs are realized.

Equilibrium

We focus on linear best reply functions. The equilibrium concept is perfect Bayesian equilibrium in pure strategies (henceforth equilibrium). An equilibrium is defined as a strategy profile $\left\{\mathbf{m}, \tau(\mathbf{s}, \mathbf{m})\right\} = \left\{(m_i)_{i\in N}, (\tau_i)_{i\in N}\right\}$. Further, given the messaging strategies, the players can be grouped into the "*truthful*" set and "*babbling*" set in equilibrium. We define them as follows:

Definition 1. *Truthful set,* $T = \{i : m_i(0) = 0, m_i(1) = 1\}$

Definition 2. *Babbling set,* $B = \{j : m_j(0) = m_j(1)\}$

The truthful set consists of players whose messages are believed in equilibrium as informative, while messages from the babbling set are ignored as uninformative. After the communication stage, the information available to the players consists of $|T|$ truthful messages, $m_T = \{m_i : i \in T\}$, and $|B|$ babbling messages, $m_B = \{m_j : j \in B\}$.

3 Equilibrium Policies without constraints

We first characterize the equilibrium policies in the final stage of the game in absence of policy constraints. That is, we assume that $x_i \in \mathbb{R}$ for all $i \in N$. Let $t = |T|$ and $(n - t) = |B|$ be the number of truthful players and babbling players respectively. We fully characterize the closed form solution of the policymaking stage, for any messaging equilibrium of the public communication protocol. An intuitive way to think about the policymaking stage is to abstract away from communication, and assume the following. Suppose all agents were exogenously given the information m_T, a set of t truthful signals, and a sub-group of $(n - t)$ agents were additionally provided with a private signal 0 or 1. Subject to this exogenous information structure what are the policies of each player? The solution to this problem is a Bayesian Nash equilibrium (BNE) that is equivalent to solving the case where there are t truthful players and $(n-t)$ babbling players. We define the unconstrained policies of players as simply functions of the summary statistic of the Beta-binomial, k, and the number of truthful signals, t.

Definition 3. *Let $x_i^*(k,t)$ be the policy choice of a truthful player $i \in T$, given a sufficient statistic k.*

$$\forall i \in T : x_i^*(k,t) \equiv \underset{x_i \in \mathbb{R}}{\operatorname{argmax}} \, \mathbb{E}_{\theta, s_B}\left[u_i\left(\phi_i\left(x_i, x^*_{T\setminus\{i\}}(k,t), x^*_{j\in B}(s_j,k,t)\right), \theta, b_i\right) \Big| k, m_T\right] \tag{1}$$

$x_T^*(k,t)$ represents the vector of equilibrium policies of the t truthful players.

Definition 4. *Let $x_j^*(s_j,k,t)$ be the equilibrium policy of a babbling player $j \in B$ with private signal s_j and sufficient statistic k.*

$$\begin{aligned}&\forall j \in B, s_j \in \{0,1\} : \\ &x_j^*(s_j, m_T) \equiv \underset{x_j \in \mathbb{R}}{\operatorname{argmax}} \, \mathbb{E}_{\theta, s_B}\left[u_j\left(\phi_j\left(x_j, x_T^*(m_T), x^*_{j'\in B\setminus\{j\}}(s_{j'},t,k)\right), \theta, b_j\right) \Big| s_j, k, m_T\right]\end{aligned} \tag{2}$$

$x^*_{j\in B}(s_j,k,t)$ is the vector of equilibrium policies of the $(n-t)$ babbling players.

For exposition sake, the actions can be represented without the truthful message set m_T that is publicly observable. The equilibrium profile of actions is given by,

$$\left(\{x_i^*(k,t)\}_{i\in T}, \left\{x_j^*(0,k,t), x_j^*(1,k,t)\right\}_{j\in B}\right)$$

Players choose an action that solves a system of $(|T| + 2|B|)$ equations. Since communication is public, every player knows precisely the set of truthful signals in equilibrium. Moreover, all truthful players have the same information given by m_T, while the babbling players have an additional private signal s_j. Finally, every babbling player is one of two types—0 or 1—and players in the alliance have the same posterior expectation about their type.

Definition 5. *Let $\tilde{b}_i = \frac{\eta}{1-\eta} \sum_{j\in N}(b_i - b_j)$ measure the misalignment of interests for player i.*[13]

$\tilde{b}_i$ captures the weighted distance of each player's individual bias from that of all the players in the alliance. This term measures the extent of misalignments within the alliance. Since the $\tilde{b}_i$'s are ordered according to the biases, we can deduce that in more closely aligned alliances, for example, the dispersion in $\tilde{b}_i$'s is lesser. Using this formulation, we completely characterize the policies chosen by the players when there are no constraints.

[13] We omit the exogenous parameters $\tilde{b}_i$ is dependent on, for simplicity of exposition.

Theorem 1. *Under unconstrained domain of actions ($x_i \in \mathbb{R}$) the players' sequentially rational action after receiving t truthful messages and $(n-t)$ babbling messages is given by:*

Truthful player:

$$\bar{x}_i(k,t) = \mathbb{E}[\theta \mid k, m_T] + b_i + \tilde{b}_i$$

Babbling player with low signal:

$$\bar{x}_j(0,k,t) = \mathbb{E}[\theta \mid k, m_T] + b_i + \tilde{b}_i - \frac{1}{1+h(t)}\mathbb{E}[\theta \mid k, m_T]$$

Babbling player with high signal:

$$\bar{x}_j(1,k,t) = \mathbb{E}[\theta \mid k, m_T] + b_i + \tilde{b}_i + \frac{1}{1+h(t)}\Big(1 - \mathbb{E}[\theta \mid k, m_T]\Big)$$

where $h(t) = \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}$

Proof. See Appendix A.2. □

When the policy set is unconstrained, players can either reveal their private information and choose the optimal policy dictated by the first equation of Theorem 1, or not reveal their private information and choose the policies according to the other two equations in Theorem 1.[14] Therefore, whether a player reveals information or not, their coordination function $\phi_i(\mathbf{x})$ is equal to the bias adjusted expected state. Further, it holds for any set of truthful messages. This implies that without policy constraints the coordination game has a common interest feature, in that all players can achieve their first-best outcomes irrespective of their signals and messaging strategies.

Corollary 1. *When the policy set is unconstrained, there always exists a fully revealing equilibrium in which every player's messaging strategy is truthful ($t = n$), and players' policies are given by:*

$$\bar{x}_{i\in N}(k,n) = \mathbb{E}[\theta \mid k, n] + b_i + \tilde{b}_i \tag{3}$$

4 Full Information Revelation with Constraints

The equilibrium policies of players in any fully revealing equilibrium depends on the constraint imposed on the policy set, $x_i \in [0,1]$. Whether the policies are within the bounds is driven by the sign of $b_i + \tilde{b}_i$. Intuitively, from Equation 3, if $b_i + \tilde{b}_i < 0$, the policies can never exceed the upper bound ($x_i < 1$). Correspondingly, $b_i + \tilde{b}_i > 0$, the policies are always above the lower bound ($x_i > 0$). The former never face

[14] This follows directly from Theorem 1 in Venkatesh (2023).

incentives to exaggerate their low signal but may choose to misreport their high signal. Analogously, the latter may exaggerate their low signals but truthfully report their high signals. We therefore separate the players into two types based on their incentives to reveal the low and high signal respectively. Specifically, we define $0 - type$ and $1 - type$ set of players in the following way:[15]

Definition 6. $0 - type = \{i \in N : b_i + \tilde{b}_i \leq 0\}$

Definition 7. $1 - type = \{i \in N : b_i + \tilde{b}_i > 0\}$

A player in the set $0 - type$ has incentives to reveal their low signal but may face incentives to misrepresent their high signal $s_i = 1$. This is driven by the observation that for some truthful message realizations, m_N, their optimal policy choice may be bounded at zero.[16] In contrast, players in the set $1 - type$ always reveal their high signal, but may misrepresent their low signal for analogous reasons. Since we focus on full revelation equilibrium, the policies are henceforth represented as a function of only the sufficient statistic k. Going forward, we refer to $x_i^*(k) \in [0,1]$ as the equilibrium policy of player i under full information revelation.

In the case of N player alliance, full information revelation implies that every player in the group reveals their private information $s_i = \{0,1\}$ truthfully for every possible (truthful) signal realization of the other $(n-1)$ players. In other words, player i's policy choice, conditional on message m_i being truthful, must be within the constraint set for all possible $(n-1)$ truthful message realizations of the other players. The following result characterizes the sufficient condition for full information revelation.

Theorem 2. *Under public communication protocol, given the set of policy constraints* $[0,1]$*, there is full information revelation if and only if:*

$$\forall i \in N : \qquad |b_i + \tilde{b}_i| \leqslant \frac{2}{n+2}$$

Proof. See Appendix A.3. □

Let a player, say i', from the set $0 - type$ hold a signal $s_{i'} = 1$. In any equilibrium where all $(n-1)$ other players reveal truthfully, it must hold that the equilibrium policy of player i' is greater than zero, for every possible signal realization of the remaining players. If this is not so, there are positive deviations for the $0 - type$ player. The *pivotal* constraint for the $0 - type$ player is the one where the IC is tightest, i.e., when

[15]Henceforth, I refer to a player belonging the set $j - type$ as '$j - type$ player'.

[16]Clearly, if this player held the low signal, her action would still be bounded but the player cannot report anything lower.

$k=1$, $\sum_{j\in N\setminus\{i'\}} s_j = 0$ and $s_{i'}=1$. In this case, the expectation of state is $\mathbb{E}[\theta \mid 1,n] = \frac{2}{n+2}$ and the IC constraint can be expressed as,

$$x^*_{i'}(1) \geq 0 \implies -(b_{i'} + \tilde{b}_{i'}) \leq \frac{2}{n+2}$$

For a player $i'' \in 1-type$ to reveal information truthfully in equilibrium, the policies have to be weakly below one for every possible realization of the remaining $(n-1)$ signals. As before, the pivotal *IC* constraint is one where the sufficient statistic $k=n-1$, i.e., $\sum_{j\in N\setminus\{i''\}} s_j = (n-1)$ and $s_{i''}=0$. The corresponding expectation of the state is $\mathbb{E}[\theta \mid n-1,n] = \frac{n}{n+2}$. The policy constraint can therefore be written as,

$$x^*_{i''}(n-1) \leq 1 \implies b_{i''} + \tilde{b}_{i''} \leq 1 - \frac{n}{n+2} = \frac{2}{n+2}$$

The necessary and sufficient condition requires policies to be within the bound for *almost* all the set of signal realizations. That is, each player's (expected) coordination function $\phi_i(\mathbf{x})$ be exactly equal to $\bar{\phi}^k_i = \mathbb{E}[\theta|k,n] + b_i$ for every possible set of truthful messages m_N, except when $k=0$ and $k=n$.[17]

The equilibrium condition implies that no player can do better by misreporting their private signal. In other words, if any player's policy goes above (below) the upper (lower) bound, in equilibrium, the other players can readjust their policies. This would violate the player's interim incentive compatibility constraint as their ϕ_i differs from the ideal policy given by $\mathbb{E}[\theta|m_N] + b_i$. Since the biases are ordered in ascending order and the policy constraint is homogeneous for all players, the equilibrium condition has to be satisfied only for the extreme players — player 1 in $0-type$ and n in $1-type$. Once the condition holds for these two players in the alliance, it must hold for all other players in both the $0-type$ and $1-type$ sets. That is,

$$\begin{aligned} -(\underline{b} + \tilde{b}_1) &\leqslant \frac{2}{n+2} \qquad &\implies& \qquad -\underline{b} \leqslant \frac{2}{n+2} + \tilde{b}_1 \\ \bar{b} + \tilde{b}_n &\leqslant \frac{2}{n+2} \qquad &\implies& \qquad \bar{b} \leqslant \frac{2}{n+2} - \tilde{b}_n \end{aligned} \tag{4}$$

Combining these equations, we can write down the condition for full information revelation as just a function of the two extreme biases.

$$\bar{b} - \underline{b} \leqslant \frac{1-\eta}{1+(n-1)\eta} \cdot \frac{4}{n+2} \tag{5}$$

[17] When a $0-type$ player i holds signal 0 *and* $s_{j\neq i} = 0$, such that $k=0$, the player cannot do better than revealing this information. Similarly, when a $1-type$ player holds signal 1 and $k=n$, then even if the policies are constrained the player cannot do any better than revealing this signal.

Optimal Size of an Alliance

In order to efficiently aggregate information the two extreme players must be 'closely' aligned. Going forward, we refer to this difference in biases of the extreme players as the measure of *cohesiveness* in the alliance. Clearly, expanding the alliance by including a new player changes incentives for information aggregation. Whether the additional player in the alliance has a bias in the interior of $[\underline{b}, \bar{b}]$, or outside this interval, the incentives for information aggregation with $n+1$ players is characterized by Equation 5. Crucially, if the additional player's bias is in the set $[\underline{b}, \bar{b}]$, then as we add more players to the alliance, the size of the alliance is simply determined by the largest n such that Equation 5 is satisfied.

Proposition 1. *Suppose the biases of players in an alliance is in the interval $[\underline{b}, \bar{b}]$. Then, the maximal size of the alliance, $\bar{n}$, is given by,*

$$\bar{n} \quad \equiv \quad \underset{n \in \mathbb{N}}{\operatorname{argmax}} (n+2)(1+(n-1)\eta) \leq \frac{4(1-\eta)}{\bar{b}-\underline{b}}$$

The expected state under the most pivotal constraint of player n, i.e., $\mathbb{E}[\theta \mid n-1, n] = \frac{n}{n+2}$, is increasing in n. Analogously, it is decreasing in the case of player 1, since $\mathbb{E}[\theta \mid n-1, n] = \frac{2}{n+2}$. This implies that both the extreme players face tighter truth-telling constraints to satisfy (Equation 4) as more players are added into the alliance. Moreover, as additional players enter the alliance, their policy has a spillover effect on the coordination functions of the two extreme players. For player 1, since $\frac{2}{n+2}$ decreases with n while the total policy spillover increases, player 1 has incentives to readjust her policy downwards. This continues until her policy choice hits the lower bound. An analogous argument ensues in the case of player n. Intuitively, when the alliance is more cohesive (i.e., $\bar{b}-\underline{b}$ is smaller), it can accommodate (weakly) more players without affecting the truth-telling constraints.

Lemma 1. *If $\frac{2}{n+2} > -(\underline{b}+\tilde{b}_1) > \frac{2}{n+3}$ or $\frac{2}{n+2} > \bar{b}+\tilde{b}_n > \frac{2}{n+3}$, there exists no bias b' such that adding a player maintains full information revelation.*

The observation follows from Proposition 1. In order for an alliance to include more members, it must continue to satisfy the constraints in Equation 4. Intuitively, if the constraints fail to hold for $n+1$ players, for either of the two extreme members, then new members cannot be inducted into the alliance. This is because adding a new member with any bias b' worsens the truth-telling constraints of the extreme players. To see the intuition, suppose player 1's constraint fails to hold in the case of $n+1$ players, i.e., $-(\underline{b}+\tilde{b}_1) > \frac{2}{n+3}$. Then adding a player with bias $b' > \underline{b}$ worsens truth-telling constraint, precluding information revelation. If instead $b' < \underline{b}$, then the new member becomes an extreme player in the alliance with $n+1$ members. However, clearly, for this new player the truth-telling constraint is violated since $\tilde{b_{b'}} < \tilde{b}_1$. Similarly, when $\bar{b} + \tilde{b}_n > \frac{2}{n+3}$, then if a new member's bias is to the left of $\bar{b}$, the truth-telling constraint only worsens since $\tilde{b}_n$ increases with the addition of a player. On the other hand, if the new player's bias is to the right of $\bar{b}$, then this player becomes the alliance's right extreme member. But, for this new player, $\tilde{b}_{b'} > \tilde{b}_n$ since her bias is further away from the n members compared to $\bar{b}$.

Two observations are in order. First, these results provide a natural upper bound on the maximal size of a coalition. This is similar to the "*size principle*" proposed by Riker (1962) that argues for minimal coalitions to occur in equilibrium. In contrast, when there is an informational rationale, we characterize the maximum size of an alliance for any level of cohesiveness. Second, information aggregation becomes harder as more players join the alliance. However, when players' actions are independent of each other, and there are no constraints, Penn (2015) shows that there is a trade-off between the informational benefit and cost (imposed by increased preference diversity) when a new member joins an alliance. Consequently, there is an incentive to include extremists to the alliance whenever the informational benefits outweigh the costs. In contrast, in the presence of interdependency between policies and constraints on the policy set, adding extremists only worsens the constraints for truth-telling.

Lemma 2. *The maximum permissible interdependency in the alliance, $\bar{\eta}$, is given by,*

$$\frac{1 + (n-1)\bar{\eta}}{1 - \bar{\eta}} = \frac{4}{(n+2)(\bar{b} - \underline{b})}$$

As the degree of interdependence in policies increases, players readjust their policies downwards for any fixed value of the sufficient statistic k. This in turn tightens the pivotal constraint for truth-telling since policies of player 1 could hit the lower bound for large values of η and $k = 1$. For player n, an increase in η has two effects. At low levels, an increase entails a lower policy from other members and a smaller spillover as a consequence. However, at sufficiently high values of η, the interdependency effect is greater even though actions of other players decreases. This implies for high values of η the incentive constraints for truth-telling tightens for both the extreme players.

The maximal interdependency parameter that supports information revelation is given in Lemma 2. Intuitively, we can observe that in more cohesive alliances, this parameter is higher. That is, more cohesive alliances allow for greater interdependencies in policies. This is because, in more cohesive alliances the actions of the two extreme players are closer to each other. In order to satisfy truth-telling, therefore, a greater range of η can be supported. Further, larger alliances admit lesser interdependencies. This is driven by the fact that fixing cohesiveness, as we add more members to the alliance the net policy spillover in every player's coordination function increases. This puts a pressure on the left extreme player to readjust her policy downwards. In the same vein, the right extreme player readjusts her policy upwards since the expected state under the tightest constraint is greater.

An implication of Lemma 1 is that more interdependent the policy is, the more cohesive an alliance must be in order to achieve full information aggregation. Lemma 1 argues why having a one-size-fits-all type of policy-making might be infeasible, especially so when the alliance size and cohesiveness are fixed. Our analysis indicates that in the case of migration policy, an alliance member can exaggerate their numbers in order to extract greater resources from other members.

4.1 Example

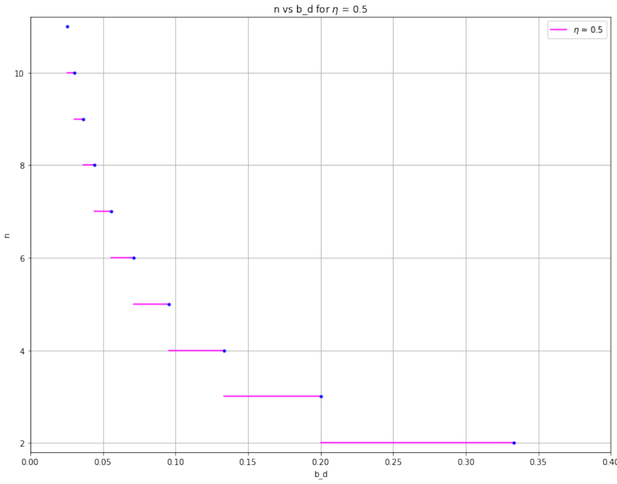
Consider an alliance with $\bar{b} = \frac{1}{10}$ and $\underline{b} = 0$. In this case we immediately compute the maximum size of permissible alliance for different values of η using the equation in Proposition 1.

Specifically, if $\eta = \frac{2}{5}$ then the RHS of equation is 24. Simple computation yields $\bar{n}\left(\frac{2}{5}\right) = 6$. Similarly, when $\eta = \frac{1}{10}$, the optimal size of the alliance is $\bar{n}\left(\frac{1}{10}\right) = 13$. Figure 1 captures this trade-off between cohesiveness and alliance size for different values of η. Crucially, as the interdependency in policies decreases, the alliance can support a greater size for a fixed value of the cohesiveness parameter. This is because greater interdependency implies greater spillover in the policies of other players, i.e., a higher aggregate spillover from all other players' policies. This reduces the incentives for the lowest bias $\underline{b}$ player to reveal the highest signal. In order to counteract this, the alliance size must be smaller in order for the overall spillover to be within the limit for truthful communication.

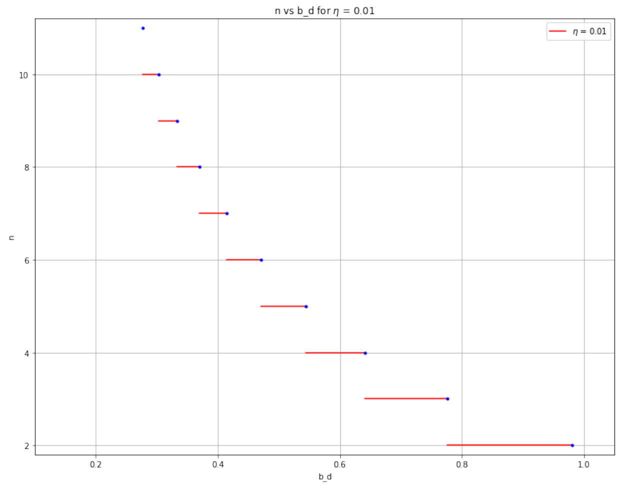
We can also compute the cutoff η such that there is truthful communication for a fixed alliance size. Suppose $n = 3$. Using Lemma 2, we can calculate $\bar{\eta}(3) = \frac{7}{10}$. Similarly, $\bar{\eta}(5) = \frac{33}{68}$ and $\bar{\eta}(10) = \frac{7}{37}$. That is, as the size of an alliance increases, the cutoff interdependency is smaller (see Figure 2). This is because for the extreme players' truth-telling constraint to be satisfied with a greater n, the spillover term must readjust by decreasing the extent of interdependency in policies. This way the overall spillover is maintained for truth-telling.

5 Commitment and Investment in Alliances

One of the main insights of the analysis so far is that the policy (action) set available to the players directly affects information aggregation. However, two restrictions were imposed on this set. First, it was assumed to be homogeneous across players. Second,



(a) $\eta = 0.5$



(b) $\eta = 0.01$

Figure 1: The cohesiveness parameter is b_d on the x-axis. When $b_d = 0.1$ and $\eta = 0.5$, the maximum size of alliance is only 4; instead, if $\eta = 0.01$ the size increases to 31.

the set was exogenous in that players do not choose how much to invest in the alliance. However, as is often the case in international alliances, an important component of decision-making is to do with allocation of otherwise costly resources (e.g., military personnel and weaponry, defense budgets). The need for commitment by members to such resources arises from two possible sources: *i)* an explicit contract via treaties and agreements (see e.g. Harstad, 2012; Harstad, 2016), and *ii)* an implicit commitment-by-interest (see e.g. Harstad et al., 2019; Snyder, 2007).[18] With commitment, the action set of the players are determined endogenously, and depending on their alignment of interest, it could vary across players. Specifically, each player chooses ex-ante the level

[18] Snyder (2007), for example, argues that both explicit agreements-driven and implicit interests-driven commitment are sustained by a combination of moral, legal, and reputational considerations.

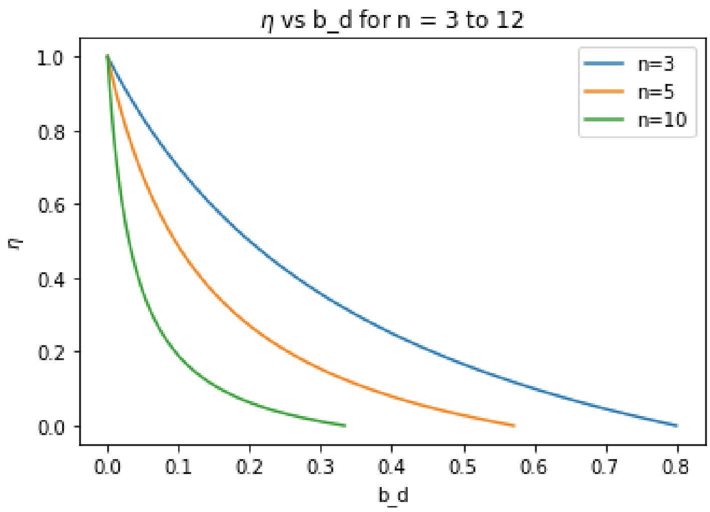


Figure 2: Fixing $b_d = 0.1$ on the x-axis, we can notice that the as n increases, the cutoff $\bar{\eta}$ for full information revelation falls drastically.

of commitment $\bar{R}_i$ at marginal cost c. The ex-ante utility of a player i is given by,

$$u_i(\mathbf{x}, \bar{R}_i, \bar{R}_{-i}) = -\left[\phi_i(\mathbf{x}) - \theta - b_i\right]^2 - c\bar{R}_i$$

The sequence of the alliance commitment game is as follows.

1. Players simultaneously choose an investment $V_i = [0, \bar{R}_i]$.
2. The state of nature θ is drawn from a uniform distribution on $[0, 1]$. Conditional on θ each player observes a private signal $s_i \in \{0, 1\}$.
3. The players simultaneously send a public message $m_i(s_i)$ to the group. The vector $\mathbf{m} = (m_1, m_2, ..., m_n)$ is the publicly available information at the end of the communication stage.
4. After observing the set of messages $\mathbf{m}$ and k, each player decides on the action $x_i^*(k) \in [0, \bar{R}_i]$ simultaneously. Payoffs are realized.

The modified commitment game has an additional first "investment" stage in which members simultaneously commit to resources. This could be interpreted as an investment that each member must explicitly comply with in order to be part of the alliance. The commitment clauses are typically proscribed in alliance agreements that specify "*rules of thumb*" for members. Such clauses impose both an implicit (e.g. NATO) and explicit (e.g. PESCO) commitment requirement on countries to ex-ante invest resources into the alliance. At the same time they also specify an upper bound on the levels of spending that members are obligated to undertake. Typically, being part of an alliance entails strategic benefits in monetary terms (Konrad, 2014). Here, we abstract away from the pecuniary advantages of joining an alliance and instead focus on the informational incentives associated with ex-ante commitments.

When $\bar{R}_i$'s are endogenously chosen and therefore possibly heterogeneous across the players, information aggregation incentives are also affected. This is because players now choose different levels of initial investments that in turn affect their ability to allocate resources in the continuation game, which involves strategic communication and decision-making. Full information revelation can ensue due to *two* factors. *First*, each player invests enough resources in the beginning such that $\bar{R}_i$ is never binding irrespective of the sufficient statistic k that is realized from truthful communication. *Second*, the constraints are binding for a particular level of $\bar{q} < n$, and for any $k > \bar{q}$ the actions of all players are bounded at $\bar{R}_i$. This implies that players can truthfully reveal all information since they have already exhausted their resources and there is no gain from misrepresenting their private information.

For example, suppose players commit to investments in the first stage such that $\bar{R}_i = \bar{x}_i(0)$ (defined according to Equation 3). In the continuation game their action set is binding for all $k = \{1, 2, ..., n\}$. This holds for every player in the alliance, and no one player has an incentive to lie and exaggerate their signal $s_i = 0$. This is because the actions of other players are *invariant* and bounded at $\bar{x}_i(0)$, irrespective of the value of the sufficient statistic. Therefore, full information revelation ensues eventhough all players have *under-invested* in the alliance.

Definition 8. *Let q_i be the highest sufficient statistic up to which actions are not binding for player i, given an investment $\bar{R}_i$. That is,*

$$q_i \equiv \underset{q}{\operatorname{argmax}}\, \bar{x}_i(q) \leq \bar{R}_i \quad \textit{such that} \quad \phi_i\Big(\bar{x}_i(q), \bar{x}_{-i}(q)\Big) = \mathbb{E}[\theta \mid q, n] + b_i \equiv \bar{\phi}_i^q$$

From the property of the Beta-Binomial function and the nature of equilibrium actions (Equation 3), it follows that,

$$\bar{x}_i(k+1) = \bar{x}_i(k) + \frac{1}{n+2}$$

We characterize the equilibrium levels of investment in an alliance in terms of $q_i \leq n$, the highest value of the sufficient statistic such that player i's coordination function matches their expected bias-adjusted state (first-best). Therefore, the ex-ante commitment problem can be viewed as the optimal choice of q_i, which is in turn determined by the marginal cost of investment, c. From Definition 8, if $\bar{R}_i = \bar{x}_i(n)$, then $q_i = n$ for all players. This implies they can truthfully reveal their information and take actions such that the coordination functions are equal to the first-best levels, resulting in full efficiency. The following definition formalizes this concept.

Definition 9. *Full Efficiency: Coordination function of each player exactly matches the bias-adjusted expected state for all possible values of k.*

$$\forall i, k : \phi_i\Big(\bar{x}_i(k), \bar{x}_{-i}(k)\Big) = \mathbb{E}[\theta \mid k, n] + b_i \equiv \bar{\phi}_i^k$$

Intuitively, when the marginal cost of investment goes to zero, players can achieve full efficiency, and as c becomes sufficiently large investment falls to zero and $q_i \to 0$ for all the players. Since the marginal costs are the same across players, we focus on symmetric investment equilibrium of the alliance commitment game, such that $q_1 = q_2 = ... = q_n = \bar{q}$. If players under-invest such that $\bar{q} < n$, the actions in the continuation game are bounded for $k > \bar{q}$. That is, there is some inefficiency induced by the actions since the coordination function is below the first-best.

Definition 10. *Partial Efficiency: There exists $\bar{q} \leq n-1$ such that,*

$$\forall k \in \{0, 1, .., \bar{q}\}, \forall i \in N : \qquad \phi_i\Big(\bar{x}_i(k), \bar{x}_{-i}(k)\Big) = \bar{\phi}_i^k$$

$$\forall k \in \{\bar{q}+1, ..., n\}, \exists i \in N : \qquad \phi_i\Big(\bar{R}_i, \bar{R}_{-i}\Big) < \bar{\phi}_i^k$$

That is, $\bar{q}$ is the upper bound on the sufficient statistic for which all players achieve first-best coordination. Beyond this, there is miscoordination losses due to lack of commitment to sufficient resources. Using the notion of Partial Efficiency, we characterize the *unique* symmetric equilibrium under which there is full information revelation.

Proposition 2. *The alliance commitment game has a unique full information revelation equilibrium in symmetric investment strategies given by $\bar{R}_i = \bar{x}_i(\bar{q}) + y$, where $\bar{q} < n$ and y solves,*

$$\frac{(n-\bar{q})(n-\bar{q}+1)}{(n+1)(n+2)(1+(n-1)\eta)} - \frac{2(n-\bar{q})}{(n+1)(1+(n-1)\eta)}y = c \tag{6}$$

$$\textit{where} \qquad y \in \left[0, \frac{1}{n+2}\right)$$

Proof. See Appendix A.4. □

The reason why this constitutes an equilibrium with full revelation can be gleaned by looking at the best response functions of players. Specifically, consider player i's best response strategy given that all other players $j \neq i$ have invested $\bar{x}_i(\bar{q}) + y$. We consider three intervals broadly: *(i)* $\bar{R}_i < \bar{x}_j(\bar{q}-1)$; *(ii)* $\bar{R}_i \in [\bar{x}_j(\bar{q}-1), \bar{x}_j(\bar{q}) + \tilde{y}]$, where $\tilde{y} = \frac{1+[1-(n+2)y](n-1)\eta}{n+2}$; and *(iii)* $\bar{R}_i > \bar{x}_j(\bar{q}) + \tilde{y}$.

In the first, there is *under-investment* by i which results in incentives for lying in the continuation game. That is, by committing to substantially lesser resources in the first stage, player i can exaggerate the low signal $s_i = 0$ and report $m_i = 1$. In this case, when $k = \{\bar{q}-1, \bar{q}, ..., n\}$, player i can free-ride on the investments made by the other players. Analogously, in the third case, player i is *over-investing* in the alliance. Any player j can exaggerate her signal in the messaging stage and benefit from the over-investment by i when $k = \{\bar{q}+1, ..., n\}$. Both these case therefore preclude truthful communication by the players.

Suppose the marginal benefit at any generic $\bar{q}$ is $MB_{\bar{q}}$. In the intermediate interval given in (ii), the marginal benefits to player i is decreasing while the marginal costs of investment is fixed at c (see Equation 6). Since the marginal costs are the same across players, it follows that the residual investment y is also equal. Consequently, uniqueness follows from noting that, when $y = 0$ is substituted in equation 6, the marginal benefit function is clearly decreasing in $\bar{q}$. That is, we can start with the case when $\bar{q} = 0$. The marginal benefit in this case is $MB_0 = \frac{n}{(n+2)(1+(n-1)\eta)}$. When $\bar{q} = 1$ instead, the marginal benefit is $MB_1 = \frac{n(n-1)}{(n+1)(n+2)(1+(n-1)\eta)} < MB_0$. Since marginal benefit function decreases discontinuously between $\bar{q}$ and $\bar{q}+1$ while the marginal cost is constant at c, a unique residual investment y ensures that both marginal benefits and costs are equal in the investment stage.

Proposition 3. *The ex-ante expected welfare for a player under the symmetric investment equilibrium is:*

$$W_i(\bar{q}, y) = -Var(\theta \mid n) - \frac{(n-\bar{q})(n-\bar{q}+1)(2(n-\bar{q})+1)}{6(n+1)(n+2)^2} + \left(\frac{n-\bar{q}}{n+1}\right)\left[\frac{(n-\bar{q}+1)}{(n+2)} - y\right] \cdot y - c \cdot \bar{R}_i \tag{7}$$

$$\textit{where} \qquad Var(\theta \mid n) = \frac{1}{6(n+2)}$$

Proof. See Appendix A.5. □

The first term of Equation 7 captures the variance of the underlying state variable given an alliance size (e.g. Penn, 2015). The second term is the additional variance generated due to partial efficiency ($\bar{q} < n$); the third term quantifies the losses induced by residual investment y. The final term captures the cost of ex-ante commitment to the alliance. Clearly, investing more resources into the alliance (weakly) increases $\bar{q}$ and decreases miscoordination losses resulting from inefficient actions. Similarly, welfare of members is strictly increasing in $y \in [0, \frac{1}{n+2})$, fixing a level of $\bar{q}$. The optimal commitment in an alliance therefore involves a fundamental trade off between the efficiency benefits of a higher investment, $(\bar{q}, y)$, and the opportunity costs of contributing, c.

Corollary 2. *$\bar{q}$ is (weakly) increasing in n and (weakly) decreasing in c.*

When the alliance size increases, the possible realizations of the sufficient statistic q also increases. This implies that if members fix $\bar{q}$, the miscoordination losses —the second and third terms in Equation 7—increase with n. Therefore the marginal partial efficiency gains from increasing the cutoff $\bar{q}$ is greater. As a consequence when the alliance size increases, members choose weakly higher levels of $\bar{q}$. Similarly, as the marginal costs of investment increases, fixing n, members would choose a weakly lower $\bar{q}$ in order to equate the increased marginal costs with greater miscoordination losses.

Corollary 3. *The following statements hold true about commitments in alliances.*

1. *Preference effect: Bias differences exacerbates the investment differences in the alliance.*
2. *Interdependence effect: Higher interdependence reduces investments made by players.*

Though the choice of $\bar{q}$ and y are independent of b_i's, the final equilibrium investments are proportional to individual biases of players:

$$\bar{R}_i \propto b_i + \tilde{b}_i \tag{8}$$

The action (policy) sets $V_i \equiv [0, \bar{R}_i]$ are therefore ordered according to the bias parameter b_i. Henceforth, we refer to a player whose bias exceeds the average bias, $Avg(\mathbf{b}) = \frac{\sum_{i \in N} b_i}{n}$, as a hawk, and one whose bias is below the average as a dove. Clearly, we can rewrite $\tilde{b}_i$ as,

$$\tilde{b}_i = \frac{\eta}{1-\eta} \sum_{j \in N} (b_i - b_j) \quad = \quad \frac{n\eta}{1-\eta}(b_i - Avg(\mathbf{b}))$$

Using the above and taking the differences in $\bar{R}_i$'s, we get,

$$\forall i, j \in N, j > i : \qquad \bar{R}_{ji} \quad \equiv \quad \bar{R}_j - \bar{R}_i = \frac{(1-\eta)(b_j - b_i)}{1 + (n-1)\eta} \tag{9}$$

The differences in investments exhibit an intuitive ordering: Greater the difference in biases, the greater are the differences in the investments made by the players. Further, when η goes up the marginal benefits from investing decreases for any level of efficiency $\bar{q}$. Intuitively, this is because of the greater spillover as a result of a higher η. As a consequence, when η increases, either y must decrease keeping $\bar{q}$ fixed in order to balance the marginal benefits with c, or the extent of efficiency achieved by the alliance also decreases. The following example illustrates these observations.

5.1 Example

Consider an alliance with $N = 4$ and individual biases $(b_1, b_2, b_3, b_4) = (\frac{1}{20}, \frac{1}{15}, \frac{1}{12}, \frac{1}{10})$. For different values of η we can compute the marginal benefit at $\bar{q} = \{0, 1, 2, 3\}$, given by $MB_{\bar{q}}$, using the equation specified in Equation 6. In particular, by substituting $y = 0$, we can compute precisely the jumps in marginal benefit function as $\bar{q}$ increases.[19] We carry out this precise exercise when the marginal cost is $c = \{0.25, 0.1\}$. The values in red marks the highest $\bar{q}$ such that the marginal benefit is above c. Given this, it is straightforward to calculate y by substituting for $\bar{q}$ in Equation 6. Once $(\bar{q}, y)$ is known, we can directly compute the total investments of each player. This is carried out in Table 1 and Table 2. We compute the partial efficiency parameter $\bar{q}$, and then use this to compute the optimal residual investment y, and finally the action $\bar{x}_i(\bar{q})$ which together with y gives the equilibrium investments.

η	MB_0	MB_1	MB_2	MB_3
$\frac{1}{2}$	0.266	0.16	0.08	0.027
$\frac{1}{4}$	0.381	0.228	0.114	0.038
$\frac{1}{10}$	0.513	0.308	0.154	0.051

$\bar{q}$	y	$\bar{R}_1$	$\bar{R}_2$	$\bar{R}_3$	$\bar{R}_4$
0	0.025	0.142	0.225	0.308	0.392
0	0.143	0.326	0.365	0.404	0.476
1	0.062	0.435	0.459	0.483	0.507

Table 1: Case of $c = 0.25$

η	MB_0	MB_1	MB_2	MB_3
$\frac{1}{2}$	0.266	0.16	0.08	0.027
$\frac{1}{4}$	0.381	0.228	0.114	0.038
$\frac{1}{10}$	0.513	0.308	0.154	0.051

$\bar{q}$	y	$\bar{R}_1$	$\bar{R}_2$	$\bar{R}_3$	$\bar{R}_4$
1	0.125	0.408	0.492	0.575	0.658
2	0.016	0.549	0.587	0.626	0.665
2	0.087	0.626	0.651	0.675	0.698

Table 2: Case of $c = 0.1$

Notice that the investments are ordered depending on the size of the players' biases, i.e. $\bar{R}_1 < \bar{R}_2 < \bar{R}_3 < \bar{R}_4$ irrespective of the degree interdependence. In a similar vein, $\bar{R}_i$ is decreasing in η across all the players, which captures the interdependency effect.

Corollary 4. *The total investment in an alliance is increasing in the size of the alliance.*

From the symmetric equilibrium investments we can compute the aggregate investment as,

$$\bar{R} = \sum_{i \in N} \bar{R}_i = n\Big[\mathbb{E}[\theta|\bar{q}] + y\Big] + \sum_{i \in N} b_i$$

[19]The fully efficient case is ignored since $MB_n = 0$ by definition.

Clearly, as more players join the alliance it has an equilibrium investment effect in that either $\bar{q}$, or y increases. Further there is an additional bias effect since investments are proportional to the individual players' biases. This implies that bigger alliances tend to increase both individual and aggregate commitments.

6 Conclusion

References

AUSTEN-SMITH, D. AND T. FEDDERSEN (2005): *Deliberation and voting rules*, Springer.

AUSTEN-SMITH, D. AND T. J. FEDDERSEN (2006): "Deliberation, preference uncertainty, and voting rules," *American Political Science Review*, 100, 209–217.

BATTAGLINI, M. AND B. HARSTAD (2020): "The political economy of weak treaties," *Journal of Political Economy*, 128, 544–590.

BHATTACHARYA, S. (2013): "Preference monotonicity and information aggregation in elections," *Econometrica*, 81, 1229–1247.

BUISSERET, P. AND D. BERNHARDT (2018): "Reelection and renegotiation: International agreements in the shadow of the polls," *American Political Science Review*, 112, 1016–1035.

DASGUPTA, A. AND A. PRAT (2008): "Information aggregation in financial markets with career concerns," *Journal of Economic Theory*, 143, 83–113.

DEWAN, T., A. GALEOTTI, C. GHIGLINO, AND F. SQUINTANI (2015): "Information aggregation and optimal structure of the executive," *American Journal of Political Science*, 59, 475–494.

FEDDERSEN, T. AND W. PESENDORFER (1997): "Voting behavior and information aggregation in elections with private information," *Econometrica: Journal of the Econometric Society*, 1029–1058.

GALEOTTI, A., C. GHIGLINO, AND F. SQUINTANI (2013): "Strategic information transmission networks," *Journal of Economic Theory*, 148, 1751–1769.

GERARDI, D. AND L. YARIV (2007): "Deliberative voting," *Journal of Economic Theory*, 134, 317–338.

HAGENBACH, J. AND F. KOESSLER (2010): "Strategic communication networks," *The Review of Economic Studies*, 77, 1072–1099.

HARSTAD, B. (2012): "Climate contracts: A game of emissions, investments, negotiations, and renegotiations," *Review of Economic Studies*, 79, 1527–1557.

——— (2016): "The dynamics of climate agreements," *Journal of the European Economic Association*, 14, 719–752.

HARSTAD, B., F. LANCIA, AND A. RUSSO (2019): "Compliance technology and self-enforcing agreements," *Journal of the European Economic Association*, 17, 1–29.

JACKSON, M. O. AND X. TAN (2013): "Deliberation, disclosure of information, and voting," *Journal of Economic Theory*, 148, 2–30.

JEHIEL, P. (1999): "Information aggregation and communication in organizations," *Management Science*, 45, 659–669.

KONRAD, K. A. (2014): "Strategic aspects of fighting in alliances," *The Economics of Conflict: Theory and Empirical Evidence. MIT Press, Cambridge, MA*.

MORGAN, J. AND P. C. STOCKEN (2008): "Information aggregation in polls," *The American Economic Review*, 98, 864–896.

PENN, E. M. (2015): "Engagement, Disengagement, or Exit: A Theory of Equilibrium Associations," *American Journal of Political Science*.

PETERSSON, M. (2015): *The US NATO Debate: From Libya to Ukraine*, Bloomsbury Publishing USA.

RIKER, W. H. (1962): *The theory of political coalitions*, Yale University Press.

SNYDER, G. H. (2007): *Alliance politics*, Cornell University Press.

VENKATESH, R. (2023): "Communication and Coordination with Constraints," *Available at SSRN 4350506*.

WERNING, I. AND G.-M. ANGELETOS (2006): "Crises and Prices: Information Aggregation, Multiplicity, and Volatility," *American Economic Review*, 96, 1720–1736.

A Proofs

A.1 Equilibrium Definition

The equilibrium concept is perfect Bayesian equilibrium in pure strategies (henceforth equilibrium). An equilibrium is defined as a strategy profile $(m, \tau) = ((m_i)_{i\in N}, (\tau_i)_{i\in N})$ such that,

1. Actions are sequentially rational, given messages and beliefs:

   $\forall i \in N, m_{-i} \in M_{-i}$:

$$\tau_i(s_i, (m_i, m_{-i})) \in \arg\max_{x_i \in V_i} \int_0^1 \sum_{s_{-i}\in\{0,1\}^{n-1}} u_i(x_i,(\tau_j(s_j,(m_j,m_{-j})))_{j\neq i};\theta, b_i) \Pr(s_{-i} \mid \theta) f(\theta|m_{-i}, s_i) d\theta$$

2. Messages are truthful if and only if they satisfy the IC for truth-telling:

   $\forall i \in N, s_i \in \{0,1\}$:

$$-\int_0^1 \sum_{s_{T-1}\in\{0,1\}^{t-1}} \sum_{s_B\in\{0,1\}^{n-t}} u_i(\tau_i(s_i,(s_i,m_{-i})),(\tau_j(s_j,(s_i,m_{-i})))_{j\in T-1},$$
$$(\tau_k(s_B(k),(s_i,m_{-i})))_{k\in B};\theta,b_i) f(\theta,s_{T-1},s_B|s)d\theta$$
$$\geq$$
$$-\int_0^1 \sum_{s_{T-1}\in\{0,1\}^{t-1}} \sum_{s_B\in\{0,1\}^{n-t}} u_i(\tau_i(s_i,(1-s_i,m_{-i})),(\tau_j(s_j,(1-s_i,m_{-i})))_{j\in T-1},$$
$$(\tau_k(s_B(k),(1-s_i,m_{-i})))_{k\in B};\theta,b_i) f(\theta,s_{T-1},s_B|s)d\theta$$

   where s_{T-1} is the set of $(T-1)$ truthful signals, apart from player i and s_B is the set of babbling signals.

A.2 Proof of Theorem 1

Before proceeding to prove Theorem 1, we begin by providing some basic insights into the nature of the maximization problem that each type of player faces, and in general, lay out some important properties of the Beta-Binomial distribution. We start by reformulating the maximization problem faced by a truthful player, given in Equation 1, as follows:

$$\max_{x_i} \int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} u_i((x_i, x_{T\setminus\{i\}}, x_B(s_B));\theta,b_i) \Pr(s_B|\theta) f(\theta|m_T)d\theta$$

The conditional density $f(\theta|m_T)$ belongs to a standard Beta-binomial distribution. Letting $k = \sum_{i\in T} s_i$, the number of signals s_i with $i \in T$ that are equal to one, the posterior distribution of θ with uniform prior on $[0,1]$, given k successes in t trials, is a Beta distribution with parameters $k+1$ and $t-k+1$. As a consequence, $f(\theta|m_T) = \frac{(t+1)!}{k!(t-k)!}\theta^k (1-\theta)^{t-k}$ and $E[\theta|m_T] = [k+1]/[t+2]$. Further, for any s_B, letting $\ell(s_B) = \sum_{q\in B} s_q$, it is the case that $\Pr(s_B|\theta) = \theta^{\ell(s_B)} (1-\theta)^{n-t-\ell(s_B)}$.

In a similar way, the problem of every babbling player $j \in B$ with a private signal s_j, stated in Equation 2, can be expanded as the following:

$$\max_{x_j(s_j)} \int_0^1 \sum_{s_{B\setminus\{j\}}\in\{0,1\}^{n-t-1}} u_j((x_j(s_j), x_T, x_{B\setminus\{j\}}(s_{B\setminus\{j\}}));\theta,b_j) \Pr(s_{B\setminus\{j\}}|\theta)$$
$$f(\theta|m_T,s_j)d\theta$$

The posterior density $f(\theta|m_T, s_j)$ with $k+s_j$ successes in $t+1$ signals is a Beta distribution with parameters $k+s_j+1$ and $(t-k-s_j+2)$. Consequently, $f(\theta|m_T, s_j) = \frac{(t+2)!}{(k+s_j)!(t+1-k-s_j)!}\theta^{k+s_j}(1-\theta)^{t+1-k-s_j}$ and $E[\theta|m_T, s_j] = [k+s_j+1]/[t+3]$. As before, for any $s_{B\setminus\{j\}}$, $\Pr(s_{B\setminus\{j\}}|\theta) = \theta^{\ell(s_{B\setminus\{j\}})}(1-\theta)^{n-t-\ell(s_{B\setminus\{j\}})}$.

The characterization involves solving the best responses of each of the three types of players from Equation 1 and Equation 2.

Case 1. *Truthful player's problem:*

$$\mathbb{E}_{\theta,s_B}[u_i(\mathbf{x},\mathbf{m})] = \\ -\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \left(\frac{x_i + \eta\sum_{j\in T\setminus\{i\}} x_j + \eta\sum_{j\in B} x_j(s_j)}{1+(n-1)\eta} - \theta - b_i\right)^2 \Pr(s_B|\theta) f(\theta|m_T) d\theta$$

where $f(\theta|m_T) = \frac{(t+1)!}{k!(t-k)!}\theta^k(1-\theta)^{t-k}$, iff $0 \leq \theta \leq 1$. Differentiating the above with respect to x_i, we get the following FOC:

$$\int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \left(\frac{x_i + \eta\sum_{j\in T\setminus\{i\}} x_j + \eta\sum_{j\in B} x_j(s_j)}{1+(n-1)\eta} - \theta - b_i\right) \\ \Pr(s_B|\theta) f(\theta|m_T) d\theta = 0$$

Simplifying, we obtain:

$$x_i + \eta\left[\sum_{j\in T\setminus\{i\}} x_j + \int_0^1 \sum_{s_B\in\{0,1\}^{n-t}} \sum_{j\in B} x_j(s_j)\Pr(s_B|\theta) f(\theta|m_T) d\theta\right] = \\ (1+(n-1)\eta)(b_i + \mathbb{E}[\theta|m_T]) \tag{10}$$

Case 2. *Babbling player's problem:*

With analogous procedures, the expected utility of a babbling player i with signal s_i is:

$$\mathbb{E}_{\theta,s_B}[u_i(\mathbf{x},\mathbf{m})] = -\mathbb{E}_{\theta,s_{B\setminus\{i\}}}\left[\left(\frac{x_i(s_i) + \eta\sum_{j\in T} x_j + \eta\sum_{j\in B\setminus\{i\}} x_j(s_j)}{1+(n-1)\eta} - \theta - b_i\right)^2 \mid m_T, s_i\right]$$

$$= -\int_0^1 \sum_{s_{B\setminus\{i\}} \in \{0,1\}^{n-t-1}} \left(\frac{x_i(s_i) + \eta \sum_{j\in T} x_j + \eta \sum_{j \in B\setminus\{i\}} x_j(s_j)}{1+(n-1)\eta} - \theta - b_i \right)^2 \Pr(s_{B\setminus\{i\}}|\theta)$$
$$f(\theta|m_T, s_i)d\theta \tag{11}$$

Again, the density $f(\theta|m_T, s_i)$ belongs to the Beta-binomial family such that,

$$f(\theta|m_T, s_i) = \frac{(t+2)!}{(k+s_i)!\,(t+1-k-s_i)!}\theta^{k+s_i}\,(1-\theta)^{t+1-k-s_i}$$

Differentiating Equation 11 with respect to $x_i(s_i)$,

$$\int_0^1 \sum_{s_{B\setminus\{i\}} \in \{0,1\}^{n-t-1}} \left(\frac{x_i(s_i) + \eta \sum_{j\in T} x_j + \eta \sum_{j \in B\setminus\{i\}} x_j(s_j)}{1+(n-1)\eta} - \theta - b_i \right) \Pr(s_{B\setminus\{i\}}|\theta)$$
$$f(\theta|m_T, s_i)d\theta = 0$$

Simplifying yields,

$$x_i(s_i) + \eta \left[\sum_{j\in T} x_j + \int_0^1 \sum_{s_{B\setminus\{i\}} \in \{0,1\}^{n-t-1}} \sum_{j \in B\setminus\{i\}} x_j(s_j) \Pr(s_{B\setminus\{i\}}|\theta) f(\theta|m_T, s_i)d\theta \right] = \tag{12}$$
$$(b_i + E[\theta|m_T, s_i])\,[1+(n-1)\eta]$$

We focus on linear equilibrium strategies of the form where x_i and $x_i(s_i)$ are both only functions of the individual bias b_i, the vector of group biases $\mathbf{b}$, and the expectation of the state given the information – m_T for truthful players and (m_T, s_i) for the babbling players.[20] Since the signals are conditionally independent, the information contained in m_T captures everything that the players know about each babbling players' privately held signal. As a result we can rewrite Equation 10 as the following:

$$x_i + \eta \sum_{j \in T\setminus\{i\}} x_j + \eta \int_0^1 \sum_{j\in B} \sum_{s_j \in \{0,1\}} x_j(s_j) \Pr(s_j|\theta) f(\theta|m_T)d\theta =$$
$$(1+(n-1)\eta)\,(b_i + \mathbb{E}[\theta|m_T])$$

[20]For example, a linear functional form where $x_i = [P(b_i + E[\theta|m_T]) + Q]$ and $x_i(s_i) = [P_{s_i}(b_i + \mathbb{E}[\theta|m_T, s_i]) + Q_{s_i}]$ could be applied to the best response equations.

Substituting $\Pr(s_j|\theta) = \theta$,

$$(1+(n-1)\eta)\left(b_i + \mathbb{E}[\theta|m_T]\right) = x_i + \eta \sum_{j\in T\setminus\{i\}} x_j + \quad \eta \int_0^1 \sum_{j\in B} x_j(0)(1-\theta)f(\theta|m_T)d\theta$$
$$+\eta \int_0^1 \sum_{j\in B} x_j(1)\theta f(\theta|m_T)d\theta$$

Since $\int_0^1 \theta f(\theta|m_T)d\theta = \mathbb{E}[\theta|m_T]$ and $\int_0^1 (1-\theta)f(\theta|m_T)d\theta = 1 - \mathbb{E}\left[\theta|m_T\right]$, the above equation can be further simplified as:

$$(1+(n-1)\eta)\left(b_i + \mathbb{E}[\theta|m_T]\right) = x_i + \eta \sum_{j\in T\setminus\{i\}} x_j + \eta\left(1 - \mathbb{E}\left[\theta|m_T\right]\right) \sum_{j\in B} x_j(0)$$
$$+\eta\mathbb{E}[\theta|m_T] \sum_{j\in B} x_j(1)$$

Similarly, applying the same principles to Equation 12,

$$(1+(n-1)\eta)\left(b_i + \mathbb{E}[\theta|m_T, s_i]\right) = x_i(s_i) + \eta \sum_{j\in T} x_j$$
$$+\eta \int_0^1 \sum_{j\in B\setminus\{i\}} \sum_{s_j\in\{0,1\}} x_j(s_j)\Pr(s_j|\theta)f(\theta|m_T, s_i)d\theta$$

Performing the substitutions $\int_0^1 \theta f(\theta|m_T, s_i)d\theta = \mathbb{E}[\theta|m_T, s_i]$ and $\int_0^1 (1-\theta)f(\theta|m_T, s_i)d\theta = 1 - \mathbb{E}\left[\theta|m_T, s_i\right]$,

$$(1+(n-1)\eta)\left(b_i + \mathbb{E}[\theta|m_T, s_i]\right) = x_i(s_i) + \eta \sum_{j\in T} x_j + \eta\left(1 - \mathbb{E}\left[\theta|m_T, s_i\right]\right) \sum_{j\in B\setminus\{i\}} x_j(0)$$
$$+\eta\mathbb{E}[\theta|m_T, s_i] \sum_{j\in B\setminus\{i\}} x_j(1)$$

Together, we can sum up the best responses for the three types of players as the following:

1. Truthful player $i \in T$:

$$x_i = (b_i + \mathbb{E}[\theta|m_T])\,[1 + (n-1)\eta] - \eta \sum_{j \in T\setminus\{i\}} x_j - \eta\,(1 - \mathbb{E}\,[\theta|m_T]) \sum_{j \in B} x_j(0) \\ - \eta \mathbb{E}[\theta|m_T] \sum_{j \in B} x_j(1) \tag{13}$$

2. Babbling player with low signal $i \in B$, $s_i = 0$:

$$x_i(0) = (b_i + E[\theta|m_T, 0])\,[1 + (n-1)\eta] - \eta \sum_{j \in T} x_j - \eta\,(1 - E\,[\theta|m_T, 0]) \sum_{j \in B\setminus\{i\}} x_j(0) \\ - \eta E[\theta|m_T, 0] \sum_{j \in B\setminus\{i\}} x_j(1) \tag{14}$$

3. Babbling player with high signal $i \in B$, $s_i = 1$:

$$x_i(1) = (b_i + \mathbb{E}[\theta|m_T, 1])\,[1 + (n-1)\eta] - \eta \sum_{j \in T} x_j - \eta\,(1 - \mathbb{E}\,[\theta|m_T, 1]) \sum_{j \in B\setminus\{i\}} x_j(0) \\ - \eta \mathbb{E}[\theta|m_T, 1] \sum_{j \in B\setminus\{i\}} x_j(1) \tag{15}$$

There are essentially three types post the communication round – the truthful type, the babbling type with low private signal, and one with high private signal. Let $\mathbb{E}\,[\theta|m_T] = c$, $\mathbb{E}\,[\theta|m_T, 0] = c_0$ and $\mathbb{E}\,[\theta|m_T, 1] = c_1$. We apply the following linear guessing strategies for the players respectively:

$$x_i = Pb_i + Q \sum_{j \neq i} b_j + K$$

$$x_i(0) = Pb_i + Q \sum_{j \neq i} b_j + K_0$$

$$x_i(1) = Pb_i + Q \sum_{j \neq i} b_j + K_1$$

Plugging the above functional forms into equations 13, 14 and 15, we get the following:[21]

Case. *Best response of truthful players:*

[21] The algebra is omitted and available on request.

$$x_i = (1+(n-1)(1-Q)\eta)b_i - \eta(P+(n-2)Q).\sum_{j\neq i} b_j - \eta(t-1)K$$
$$+(1+(n-1)\eta)c - \eta b.(cK_1 + (1-c)K_0) \tag{16}$$

Case. *Best response of babbling players with low signal*

$$x_i(0) = (1+(n-1)(1-Q)\eta).b_i - \eta(P+(n-2)Q).\sum_{j\neq i} b_j - \eta t K$$
$$-\eta(n-t-1).(c_0K_1 + (1-c_0)K_0) + (1+(n-1)\eta).c_0 \tag{17}$$

Case. *Best response of babbling players with high signal*

This is very similar to the low signal case, except for one expression. Following the same steps as in the case with the low signal,

$$x_i(1) = (1+(n-1)(1-Q)\eta).b_i - \eta(P+(n-2)Q).\sum_{j\neq i} b_j - \eta t K$$
$$-\eta(n-t-1).(c_1K_1 + (1-c_1)K_0) + (1+(n-1)\eta).c_1 \tag{18}$$

COMPARING COEFFICIENTS:

Using Equation 16, Equation 17, and Equation 18 to compare coefficients:

$$P = (1+(n-1)(1-Q)\eta) \qquad Q = -\eta(P+(n-2)Q)$$
$$\implies Q = -\eta(1+(n-1)(1-Q)\eta + (n-2)Q)$$

Solving the above equations:

$$P = \frac{(1+(n-2)\eta)}{1-\eta} \qquad Q = -\frac{\eta}{1-\eta}$$

Proceeding similarly, we solve for three equations in three unknowns (K, K_0, K_1).[22]

$$K = \frac{(1+\eta(n-t-1))(1-\eta(c_1-c_0))c - (n-t)\eta c_0}{(1-\eta)\,(1+\eta(n-t-1)(c_1-c_0))}$$

$$K_0 = \frac{(1+\eta(t-1))c_0 - \eta t(1-\eta(c_1-c_0))c}{(1-\eta)((1+\eta(n-t-1)(c_1-c_0))}$$

[22]For exposition sake, we omit the part where we solve for the coefficients. It is available to interested readers on request

$$K_1 = \frac{((n-1)\eta - (n-t-1))\eta c_0 + (1+(n-1)\eta)(1-\eta)c_1 - \eta t(1-\eta(c_1-c_0))c}{(1-\eta)\,(1+\eta(n-t-1)(c_1-c_0))}$$

Suppose that out of the T truthful messages, k signals are 1, then $E[\theta \mid k, m_T] = \frac{k+1}{t+2}$. Similarly, the babbling player with low signal then has an expectation given by $E[\theta \mid k, m_T, 0] = \frac{k+1}{t+3}$, and the babbling player with a high signal has $E[\theta \mid k, m_T, 1] = \frac{k+2}{t+3}$.

$$c = \frac{k+1}{t+2} \qquad c_0 = \frac{k+1}{t+3} \qquad c_1 = \frac{k+2}{t+3} \qquad c_1 - c_0 = \frac{1}{t+3}$$

Substituting for $h(t) = \frac{(2+t(1-\eta))}{(1+(n-1)\eta)}$ and the above values in the expressions for K, K_0 and K_1,

$$K = \frac{k+1}{t+2}$$

$$K_0 = \frac{k+1}{t+2}\frac{1}{(1+\frac{1+(n-1)\eta}{2+t(1-\eta)})} = \frac{k+1}{t+2}\cdot\frac{h(t)}{1+h(t)}$$

$$K_1 = \frac{\left(1+\frac{(k+1)}{(t+2)}\frac{(2+t(1-\eta))}{(1+(n-1)\eta)}\right)}{\left(1+\frac{2+t(1-\eta)}{1+(n-1)\eta}\right)} = \frac{k+1}{t+2}\cdot\frac{h(t)}{1+h(t)} + \frac{1}{1+h(t)}$$

Truthful players' equilibrium action:

$$\bar{x}_i(k,t) = \frac{(1+(n-2)\eta)}{1-\eta}b_i - \frac{\eta}{1-\eta}\sum_{j\neq i} b_j + \mathbb{E}[\theta \mid k, m_T]$$

$$\bar{x}_i(k,t) = \mathbb{E}[\theta \mid k, m_T] + b_i + \tilde{b}_i \tag{19}$$

Low signal babbling players' equilibrium action:

$$\bar{x}_j(0,k,t) = \frac{(1+(n-2)\eta)}{1-\eta}b_i - \frac{\eta}{1-\eta}\sum_{j\neq i} b_j + \frac{h(t)}{1+h(t)}\cdot\mathbb{E}[\theta \mid k, m_T]$$

$$\bar{x}_j(0,k,t) = \mathbb{E}[\theta \mid k, m_T] + b_i + \tilde{b}_i - \frac{1}{1+h(t)}\mathbb{E}[\theta \mid k, m_T] \tag{20}$$

High signal babbling players' equilibrium action:

$$\bar{x}_j(1,k,t) = \frac{(1+(n-2)\eta)}{1-\eta}b_i - \frac{\eta}{1-\eta}\sum_{j\neq i} b_j + \frac{h(t)}{1+h(t)}\cdot\mathbb{E}[\theta \mid k, m_T] + \frac{1}{1+h(t)}$$

$$\bar{x}_j(1,k,t) = \mathbb{E}[\theta \mid k, m_T] + b_i + \tilde{b}_i + \frac{1}{1+h(t)}\Big(1 - \mathbb{E}[\theta \mid k, m_T]\Big) \tag{21}$$

This completes the proof. **QED**

A.3 Proof of Theorem 2

Sufficiency:

From arguments made in Section 3 and 4, a $0 - type$ player always reveals the low signal and the $1 - type$ player never misreports a high signal. The only cases of relevance then is one where $0 - type$ $(1 - type)$ gets a high (low) signal.

Take the case of a $0 - type$ player. For i to reveal a high signal $s_i = 1$, it must be that, for any possible realization of the other $(n-1)$ players' signals, sending a truthful message $m_i = s_i = 1$ must be optimal. This means that the equilibrium action of i, $x_i^*(1 + \sum m_{-i}) \geqslant 0$ for any set of (truthful) messages from the other players, m_{-i}. Since the posterior on the state θ is a beta-binomial distribution, what matters is the sufficient statistic k, the number of 1's in the set of messages (m_i, m_{-i}).

Therefore, for i to reveal $s_i = 1$, a set of n constraints (corresponding to $k = 1$ to n). However, the tightest constraint that would ensure this is when every other player reveals 0, meaning that $\sum m_{-i} = 0$. In this case, if $m_i = 1$, then $k = \sum\limits_{j \in N} m_j = 1$ and therefore the expected value of θ, $E[\theta \mid k] = \frac{2}{n+2}$. Once this constraint is satisfied, every other IC for player i must be satisfied. From Equation 3, it must be that,

$$b_i + \tilde{b}_i + \frac{2}{(n+2)} \geqslant 0$$

$$-(b_i + \tilde{b}_i) \leq \frac{2}{n+2} \tag{22}$$

A similar argument ensues for a player $i \in 1 - type$. For i to reveal a low signal truthfully, it must be that for any other order of $(n-1)$ truthful signals from the other players, player i's optimal action upon sending the message $m_i = s_i = i0$ must be within the upper bound of the action set. As before, we only need to concentrate on the tightest IC that satisfies this condition. This is the constraint when $\sum m_{-i} = (n-1)$, the case in which every other player reveals a high signal.

In this case, if $m_i = 0$, then $k = \sum\limits_{N} m = (n-1)$ and therefore the expected value of θ is $E[\theta \mid k] = \frac{n}{n+2}$. Once this constraint is satisfied, every other IC for player i must be satisfied. Again applying the upper bound condition for Equation 3,

$$b_i + \tilde{b}_i + \frac{n}{(n+2)} \leqslant 1$$

$$(b_i + \tilde{b}_i) \leq \frac{2}{n+2} \tag{23}$$

Equation 22 and Equation 23 together imply that for every possible signal realization, every player's action is within the action set $[0, 1]$. This means that $\phi_i(\mathbf{x}) = \bar{\phi}_i^k = \mathbb{E}[\theta \mid k, m_N] + b_i$ for all $i \in N$ and $m_N \in \{0, 1\}^N$. That is, there is no additional residual variance (in expectation) and players cannot do better by misrepresenting their signals. By combining Equation 22 and Equation 23, we conclude that there is full information aggregation if:

$$\forall i \in N : \qquad |b_i + \tilde{b}_i| \leqslant \frac{2}{n+2} \tag{24}$$

Necessity:

We prove by contradiction. Suppose there is a full revelation equilibrium in which for $(n-1)$ players Equation 24 is satisfied and for some player $i \in N$, this condition is violated. It is then enough to show a profitable deviation for this player i conditional on truthful messaging strategy of the other players in the group. Without loss of generality, let the condition be violated for player n, with conflict of interest b_n.[23] Then, given that each of remaining $(n-1)$ players are being truthful and the sufficient condition holding for them, it requires to be checked if n has an incentive to misreport her signal. Since $b_n = \sup\{b_i : i \in N\}$, n is a $1 - type$ player. Further, as before, $s_n = 0$ and n reports truthfully. Then, if each of the other signals are such that $\sum m_{-n} = (n-1)$, then the equilibrium action of n is $x_n^*(n-1) = \min\{1, b_n + \tilde{b} + \frac{n}{(n+2)}\} = 1$, since Equation 24 is violated by construction. This implies that the other $(n-1)$ players readjust their action by compensating for player n's lower action as opposed to one dictated by Equation 3. Henceforth, $\bar{x}_i(k)$ is the first-best action of player i written in terms of only the sufficient statistic.

We proceed in two steps. First, we characterize exactly how the rest of the players readjust their actions when $\bar{x}_n(n-1) > 1$, in which case the action is binding ($x_n^*(n-1) = 1$). Second, using this we show that the readjustment process results in miscoordination losses for player n. In other words, $\phi_n(x_n^*(n-1), x_{-n}^*(n-1)) < \bar{\phi}_n^{n-1} = \mathbb{E}[\theta \mid n-1, m_N] + b_n$, where $(x_n^*(n-1), x_{-n}^*(n-1))$ is the vector of actions after the readjustment. This results in miscoordination since the readjusted actions result in a lower value of coordination function than what the first best actions $\bar{\mathbf{x}}(n-1)$ entails. We let $x_j(k)$ to be the readjusted actions of players $j \in \{1, 2, ...(n-1)\}$ when the sufficient statistic is k, and let $x_{-n}(k)$ be the joint vector of (readjusted) actions of the $(n-1)$ players.

STEP 1:

[23]For example, the same set of arguments are valid for players in the set $0 - type$.

Clearly, the readjusted action $x_j(n-1) \geq \bar{x}_j(n-1)$. For example, the readjusted action binds for player $n-1$ if $\bar{x}_{n-1}(n-1) \geq 1$, according to Equation 3. We drop the sufficient statistic k as an argument in the player's action in order to simplify notation. Let $\epsilon^*_{n-1} = \bar{x}_n - 1$ be the residual action that player n could not take when $k = (n-1)$. Then the extra action that remaining $(n-1)$ players have to compensate is given by $\frac{\eta}{1+(n-1)\eta}\epsilon^*_{n-1}$. Since the marginal spillover (η) of each player's action on every other players' coordination function ϕ_i is homogeneous, the $(n-1)$ players must share this extra $\frac{\eta}{1+(n-1)\eta}\epsilon^*_{n-1}$ equally. Let X^*_{n-1} be the extra action of each of the remaining players. Then, the following condition solves for X^*_{n-1}:

$$\frac{1}{1+(n-1)\eta}X^*_{n-1} + \frac{(n-2)\eta}{1+(n-1)\eta}X^*_{n-1} = \frac{\eta}{1+(n-1)\eta}\epsilon^*_{n-1}$$

The above expression simply implies that the total sum of the actions must equal to the residual that is required to be compensated. Solving gives,

$$X^*_{n-1} = \frac{\eta}{1+(n-2)\eta}\epsilon^*_{n-1}$$

There are two cases to be considered. If $x_{n-1} = \bar{x}_{n-1} + X^*_{n-1} \leq 1$ then each of the other $(n-1)$ players take an action that is given by $x_i = \bar{x}_i + X^*_{n-1}$ and $\phi_n\Big(x_n(n-1), x_{-n}(n-1)\Big) \equiv \phi^{n-1}_n$ is,

$$\phi^{n-1}_n = \frac{\Big(\bar{x}_n(n-1) - \epsilon^*_{n-1}\Big) + \eta\left(\sum_{j\in N\setminus n}\bar{x}_j(n-1) + (n-1)X^*_{n-1}\right)}{(1+(n-1)\eta)}$$

$$\phi^{n-1}_n = \bar{\phi}^{n-1}_n - \underbrace{\left[\frac{\epsilon^*_{n-1}}{1+(n-1)\eta} - \frac{(n-1)\eta}{1+(n-1)\eta}X^*_{n-1}\right]}_{\Delta^1_n(n-1)}$$

The residual variance term $\Delta^q_n(n-1)$ is where the superscript q denotes the number of players for whom the constraint binds. In the case where $x_{n-1}(n-1) \leq 1$, we write $\Delta^1_n(n-1)$ as the residual variance when the constraint binds only for one player, n. Since by construction $\bar{\phi}^{n-1}_n = \mathbb{E}[\theta \mid k = (n-1)] + b_n$, the final expressions for $\Delta^1_n(n-1)$ and ϕ^{n-1}_n is simply given by,

$$\Delta^1_n(n-1) = \frac{1-\eta}{1+(n-2)\eta}\epsilon^*_{n-1}$$

$$\phi^{n-1}_n = \bar{\phi}^{n-1}_n - \Delta^1_n(n-1)$$

If on the other hand $x_{n-1} = \bar{x}_{n-1} + X^*_{n-1} > 1$, but $\bar{x}_j + X^*_{n-1} \leq 1$ for all $j \in \{1, 2, ...(n-$

$2)\}$, we can define ϵ^*_{n-2} in a similar manner, i.e.,

$$\epsilon^*_{n-2} = \bar{x}_{n-1} + X^*_{n-1} - 1$$

As before X^*_{n-2} solves:

$$\frac{1}{1+(n-1)\eta} X^*_{n-2} + \frac{(n-3)\eta}{1+(n-1)\eta} X^*_{n-2} = \frac{\eta}{1+(n-1)\eta}\epsilon^*_{n-2}$$

$$X^*_{n-2} = \frac{\eta}{1+(n-3)\eta}\epsilon^*_{n-2}$$

The actions of players $j \in \{1, 2, ..., (n-2)\} : x_j = \bar{x}_j + X^*_{n-1} + X^*_{n-2}$, that of $(n-1)^{th}$ player is $x_{n-1} = \left(\bar{x}_{n-1} + X^*_{n-1}\right) - \epsilon^*_{n-2}$, and as before, $x_n = \bar{x}_n - \epsilon^*_{n-1}$.
As before, if $x_j = \bar{x}_j + X^*_{n-1} + X^*_{n-2} \leq 1$ for all $j \in \{1, 2, ..., (n-2)\}$,

$$\phi_n^{n-1} = \bar{\phi}_n^{n-1} - \underbrace{\left[\frac{\epsilon^*_{n-1} - (n-1)\eta X^*_{n-1}}{1+(n-1)\eta}\right]}_{\Delta_n^1(n-1)} - \underbrace{\left[\frac{\eta\epsilon^*_{n-2} - (n-2)\eta X^*_{n-2}}{1+(n-1)\eta}\right]}_{\Delta_n^2(n-1)}$$

$$\Delta_n^2(n-1) = \frac{\eta}{1+(n-1)\eta} \cdot \frac{1-\eta}{1+(n-3)\eta}\epsilon^*_{n-2}$$

Rewriting $\Delta_n^1(n-1)$ in a similar manner, we get,

$$\Delta_n^1(n-1) = \frac{1-\eta}{1+(n-1)\eta}\epsilon^*_{n-1} + \frac{\eta}{1+(n-1)\eta} \cdot \frac{1-\eta}{1+(n-2)\eta}\epsilon^*_{n-1}$$

Using these expressions, the readjusted coordination function is,

$$\begin{aligned}\phi_n^{n-1} = \bar{\phi}_n^{n-1} &- \frac{1-\eta}{1+(n-1)\eta}\epsilon^*_{n-1} \\ &- \frac{\eta}{1+(n-1)\eta}\left[\frac{1-\eta}{1+(n-2)\eta}\epsilon^*_{n-1} + \frac{1-\eta}{1+(n-3)\eta}\epsilon^*_{n-2}\right]\end{aligned}$$

Recursively writing the above equation we get,

$$\phi_n^{n-1} = \bar{\phi}_n^{n-1} - \frac{1-\eta}{1+(n-1)\eta}\epsilon^*_{n-1} - \frac{\eta(1-\eta)}{1+(n-1)\eta}\sum_{j=1}^{n-1}\frac{\epsilon^*_{n-j}}{1+(n-j-1)\eta}$$

The total miscoordination losses when q players' action constraint binds, is given by,

$$\Lambda_n^q(n-1) = \sum_{\ell'=1}^{q}\Delta_n^{\ell'}(n-1) = \frac{1-\eta}{1+(n-1)\eta}\left[\epsilon^*_{n-1} + \eta\sum_{\ell'=1}^{q}\frac{\epsilon^*_{n-\ell'}}{1+(n-\ell'-1)\eta}\right]$$

The action of the players in this case is,

$$\forall j \in \{1, 2, ..., (n-q)\} : x_j(n-1) = \bar{x}_j(n-1) + \sum_{h=1}^{q} X^*_{n-h}$$

$$\forall j \in \{(n-q+1), (n-q+2)...., n\} : x_j(n-1) = 1$$

Clearly, irrespective of how large or small the q is, there is miscoordination losses for player n. Further, this total miscoordination loss is increasing in q, the number of players facing a binding constraint as a result of the readjustment process. This implies there is always under-provision from n's point of view.

STEP 2:
Now instead if n misreports her signal and sends a message $m_n = 1 - s_n = 1$, and as before the rest of the players all have a signal $s_i = 1$, then the actions of every other player apart from n is increased in equilibrium. The additional increase is just $\mathbb{E}[\theta \mid k = n] - \mathbb{E}[\theta \mid k = (n-1)] = \frac{1}{(n+2)}$. As in *STEP 1*, the players $\forall j \in \{(n - q+1), (n-q+2)...., n\} : x_j = 1$ and the remaining players have to compensate for an additional $\frac{1}{n+2}$ increase in the expectation of θ. For all players $j \in \{1, 2, ..., (n-q)\}$, the action is $x_j(n) = x_j(n-1) + \gamma(n)$ where $\gamma(n)$ solves,

$$\left[\frac{1}{1+(n-1)\eta} + \frac{(n-q-1)\eta}{1+(n-1)\eta}\right]\gamma(n) = \frac{1}{n+2}$$

$$\gamma(n) = \frac{1+(n-1)\eta}{(n+2)(1+(n-q-1)\eta)} < 1$$

It is straightforward to see that when each of the non-binding player's action increases by an additional $\gamma(n)$, the marginal increase in ϕ_n^{n-1} is just $\chi_n^d(q) = \frac{(n-q)\eta}{(1+(n-1)\eta)}\gamma(n)$. The overall value of the coordination function for player n from playing the deviation strategy is given by,

$$\phi_n^{n-1} = \bar{\phi}_n^{n-1} - \Lambda_n^q(n-1) + \chi_n^d(q)$$

If $\Lambda_n^q(n-1) > \chi_n^d(q)$, the total miscoordination losses from deviating and sending the higher message is lower and therefore there is a gain from deviation. If on the other hand, $\chi_n^d(q) > \Lambda_n^q(n-1)$, then since $\chi_n^d(q) < \frac{1}{1+(n-1)\eta}$, it immediately implies that $\chi_n^d(q) - \Lambda_n^q(n-1) < \frac{1}{1+(n-1)\eta}$. Player n can readjust her actions and choose a deviation action $x_n^d \in (0,1)$ such that $\phi_n^{n-1} = \bar{\phi}_n^{n-1}$.

This concludes both the requisite steps. A similar argument holds true for every other signal realization of the remaining $(n-1)$ players. This can be observed by noting that q — the number of players for whom the constraint is binding — is weakly increasing in k, the sufficient statistic of the Beta-binomial distribution. It is straightforward to note that as before, either $\Lambda_n^q(k) > \chi_n^d(q)$ in which case the miscoordination is smaller from the deviation strategy, or $\Lambda_n^q(k) < \chi_n^d(q)$, in which case the miscoordination loss is reduced or eliminated irrespective of the value of q. That is, player n can always misrepresent her private signal and subsequently readjust her action according to the realization of k. This way n can reduce miscoordination, or eliminate it, for all possible realizations of the other signals from the remaining players. It concludes to observe that irrespective of whether $x_n(s_n + \sum m_{-n}) \leqslant 1$ or not, n is better off deviating to the higher message when $s_n = 0$, since the actions of other players have unequivocally risen and decreases miscoordination as a consequence. Thus, n benefits from deviating to $m_n = 1$ when $s_n = 0$. But if this is true, then a $n-$player equilibrium ceases to exist, contradicting the starting presumption.

An analogous argument holds for players $j \in 0 - type$. This concludes the proof. **QED**

A.4 Proof of Proposition 2

The characterization of a fully revealing equilibrium in the alliance commitment game relies on an intuitive property of the payoff function. Specifically, if players are truthful in the continuation game after the commitment stage, it must be that their actions always binds for some values of the sufficient statistic k. Suppose not, and let $\bar{R}_i = \bar{x}_i(n)$. In this case, there is full efficiency and the coordination function of all the players exactly match the bias-adjusted state, for all $k = \{0, 1, ..., n\}$. Therefore, the ex-ante welfare is simply the variance of θ.[24] This implies that the marginal benefit is zero while the marginal cost is positive. Therefore $\bar{R}_i < \bar{x}_i(n)$ and the investment can never be such that players achieve full efficiency in the action stage.

To establish that a symmetric strategy described in Proposition 2 is indeed an equilibrium, we fix the strategies of all players but one. Suppose all players $j \neq i$ choose an investment level $\bar{R}_j = \bar{x}_j(\bar{q}) + y$. We check if player i can deviate from this symmetric strategy and invest $\bar{R}_i \neq \bar{x}_i(\bar{q}) + y$. Since the investment decisions are taken simultaneously, we look at the best response function of player i given players $-i$ are investing according to the symmetric strategies. For $\bar{q} \geq 1$, we separate the best response function of i into five intervals:

(a) $\bar{R}_i \in [0, \bar{x}_i(0))$

(b) $\bar{R}_i \in [\bar{x}_i(0), \bar{x}_i(\bar{q} - 1))$

(c) $\bar{R}_i \in [\bar{x}_i(\bar{q} - 1), \bar{x}_i(\bar{q}))$

[24]See Galeotti et al. (2013) and Penn (2015) for the precise derivation of welfare given a Beta-Binomial information structure.

(d) $\bar{R}_i \in [\bar{x}_i(\bar{q}), \bar{x}_i(q) + \tilde{y}]$ where $\tilde{y} = \frac{1+(1-(n+2)y)(n-1)\eta}{n+2}$

(e) $\bar{R}_i > \bar{x}_i(\bar{q}) + \tilde{y}$

The reason for splitting the best response function of i into these intervals is the following. Given the structure of the distribution, all values of $k = \{0, 1, ..., n\}$ have an ex-ante probability of $\frac{1}{n+1}$ at the commitment stage. Therefore if player i, fixing the investment in one of the intervals, is able to deviate from truthful communication in the subsequent period and readjust her actions accordingly for every realization of k, then from an ex-ante perspective this precludes possibility of truthful communication.

It follows that when the best response is according to (a) and player i is truthful, then i's action is always bounded at $\bar{R}_i$ irrespective of the realization of k. Since $\eta < 1$, this implies that i suffers from miscoordination losses if she reports the truth. To see this, suppose $\bar{R}_i = \bar{x}_i(0) - \vartheta$ and i is truthful in the ensuing game. If $k_T = 0$, then the action of i in equilibrium is $x_i^*(0) = \bar{x}_i(0) - \vartheta$. Player $j \neq i$ takes an action that compensates for this under-provision by player i. The action of j, $x_j^*(0) = \bar{x}_j(0) + \varepsilon_+^0$, where ε_+^k is the additional action required to ensure that j's coordination function is equal to the first best. That is,

$$\frac{[\bar{x}_j(0) + \varepsilon_+^0] + \eta \sum_{j' \neq j,i} [\bar{x}_{j'}(0) + \varepsilon_+^0] + \eta[\bar{x}_i(0) - \vartheta]}{1 + (n-1)\eta} = \bar{\phi}_j^0$$

$$\implies \varepsilon_+^0 = \frac{\eta\vartheta}{1 + (n-2)\eta}$$

However, if we substitute this into the coordination function of i, it gives,

$$\frac{[\bar{x}_i(0) - \vartheta] + \eta \sum_{j \neq i} \bar{x}_j(0) + (n-1)\eta\varepsilon_+^0}{1 + (n-1)\eta} = \bar{\phi}_i^0 - \underbrace{\frac{\vartheta - \frac{(n-1)\eta^2\vartheta}{1+(n-2)\eta}}{1 + (n-1)\eta}}_{>0}$$

There is miscoordination in that from i's perspective, the readjusted actions are not sufficient to achieve her first-best, $\bar{\phi}_i^0$. Instead, if i exaggerates her signal in the communication stage, then if the other players take an action corresponding with $k_D = 1$, i reduces the miscoordination loss. A similar logic applies to all $k_T > 0$. Since each of the k_T's occur with probability $\frac{1}{n+2}$, there is positive benefit from deviating to an inflated messaging strategy. Therefore best responses in this interval are ruled out as an equilibrium strategy under full revelation.

Next, we proceed as follows. First, we show that if the best response of i is in (b) or (c), then full information revelation breaks down since i would have incentives to exaggerate her low signal. Second, if the best response is according to (e), then player i has over-invested in the alliance and player j would have incentives to lie. The implication of (b), (c) and (e) is that in any equilibrium where information is fully revealed, the investments are such that actions are bounded for all $k > \bar{q}$. That is there is partial efficiency and miscoordination for some values of k. Finally, using this finding, we characterize the ex-ante welfare of players and then verify that the best response of i indeed coincides with the symmetric strategy which is in the interval (d).

Claim 1. *If $\bar{R}_i \in [\bar{x}_i(0), \bar{x}_i(\bar{q}-1))$ then player i has incentives to misrepresent her signal $s_i = 0$ and report $m_i(s_i) = 1$ for both signal types.*

Proof. We proceed by first assuming that $\bar{R}_i \in [\bar{x}_i(0), \bar{x}_i(\bar{q}-1))$, and show that conditional on all other players $j \neq i$ investing $\bar{R}_j = \bar{x}_j(\bar{q}) + y$ and revealing information truthfully in the continuation game, there is an incentive for player i to deviate from truth-telling. Intuitively, given the investments in the first stage, $\bar{R} \equiv \{\bar{x}_1(\bar{q}) + y, ..., \bar{x}_{i-1}(\bar{q}) + y, \bar{R}_i, \bar{x}_{i+1}(\bar{q}) + y, ..., \bar{x}_n(\bar{q}) + y\}$, there exists a profitable deviation action for player i that makes her better off under the messaging strategy $m_i(0) = 1$ instead of the truthful strategy.

We fix the investment at some $\bar{R}_i < \bar{x}_i(\bar{q}-1)$ and compute the actions of i for the different values of the sufficient statistic, k_T, under truthful messaging in the subsequent communication round. Once this is done, we show that for "every" possible realization of the sufficient statistic k_T (realizing with probability $\frac{1}{n+2}$), a deviation strategy exists where the message by i is inflated ($m_i(0) = 1$) and an associated deviation action under the inflated sufficient statistic $k_D = k_T + 1$ gives i a weakly higher expected payoff. Crucially, the equilibrium beliefs of other players is fixed implying that they believe i's deviation message to be truthful. Their equilibrium actions $x^*_{j\neq i}(\cdot)$ are therefore conditioned on k_D. Meanwhile, i's deviation action is conditioned on k_T and is a best response to $x^*_{j\neq i}(k_D)$.

We begin by letting $\bar{R}_i = \bar{x}_i(\bar{q}-1) - \vartheta$, where $\vartheta \in (0, \frac{1}{n+2}]$. Note that $\bar{R}_i = \bar{x}_i(\bar{q}-1) - \frac{1}{n+2} = \bar{x}_i(\bar{q}-2)$. The same set of arguments would carry through for $\bar{R}_i = \bar{x}_i(\bar{q}-2) - \vartheta$, and so on until $\bar{R}_i = \bar{x}_i(1) - \vartheta$.

Given $\bar{R}_i = \bar{x}_i(\bar{q}-1) - \vartheta$, we consider three cases, where under truthful revelation, $k_T = \{\bar{q}-2, \bar{q}-1, \bar{q}\}$. We present the incentives for deviation and the associated deviation strategies for each of the cases below.

Case 1. $k_T = \bar{q} - 2$

In this case, under truthful revelation, the investments of players are not binding. Therefore they all take the action dictated by Equation 3. That is, $x^*_j(\bar{q}-2) = \bar{x}_j(\bar{q}-2)$, for all $j \in N$. Given this, player i achieves first-best coordination levels, $\bar{\phi}_i^{\bar{q}-2} = \mathbb{E}[\theta|\bar{q}-2] + b_i$. However, we argue that even by deviating to an inflated messaging strategy, player i can achieve the same, if we fix equilibrium beliefs. That is fixing the fact that other players believe i's message to be truthful, if i chooses to inflate her signal $s_i = 0$, then the realized $k_D = \bar{q}-1$ instead of $k_T = \bar{q}-2$. In this case, players $j \neq i$ anticipate that i's actions are bounded at $\bar{x}_i(\bar{q}-1) - \vartheta$ instead of the optimal $\bar{x}_i(\bar{q}-1)$. They would choose their actions accordingly. Remember that in the coordination function of any player $j \neq i$, the reduced action of i, $\bar{x}_i(\bar{q}-1) - \vartheta$ implies a net reduction of $\eta\vartheta$. This reduction can be compensated for by the $(n-1)$ players $j \neq i$ if they choose an additional $\varepsilon_{k_D}^+$ such that,

$$[1+(n-2)\eta]\varepsilon_{\bar{q}-1}^+ = \eta\vartheta \quad \implies \quad \varepsilon_{\bar{q}-1}^+ = \frac{\eta\vartheta}{1+(n-2)\eta}$$

That is, the player i knows that in the case where $k_D = \bar{q}-1$, all other players would play an action $x_j^*(\bar{q}-1) = \bar{x}_j(\bar{q}-2) + \frac{1}{n+2} + \varepsilon_{\bar{q}-1}^+$. This means that according to the true k_T, player i could instead choose an action $x_i^*(\bar{q}-2) = \bar{x}_i(\bar{q}-2) - \vartheta_{\bar{q}-2}^-$ such that the coordination function of i is given by,

$$\phi_i^{\bar{q}-2} = \frac{\left[\bar{x}_i(\bar{q}-2) - \vartheta_{\bar{q}-2}^-\right] + \eta\sum_{j\neq i}\left[\bar{x}_j(\bar{q}-2) + \frac{1}{n+2} + \varepsilon_{\bar{q}-1}^+\right]}{1+(n-1)\eta}$$

$$\implies \phi_i^{\bar{q}-2} = \bar{\phi}_i^{\bar{q}-2} + \left[(n-1)\eta\left(\frac{1}{n+2} + \frac{\eta\vartheta}{1+(n-2)\eta}\right) - \vartheta_{\bar{q}-2}^-\right]$$

By setting $\vartheta_{\bar{q}-2}^- = (n-1)\eta\left(\frac{1}{n+2} + \frac{\eta\vartheta}{1+(n-2)\eta}\right)$, the player can readjust her actions such that there is no miscoordination. Therefore, in the subgame where $m_i(0) = 1$ and $k_D = \bar{q}-1$, player i gets the same expected payoff as in the case of truth-telling. Indeed, for any lower realization of $k \leq \bar{q}-2$, the same incentives hold and player i's expected payoff under lying is the same as truth-telling.

Case 2. $k_T = \bar{q}-1$

In this case, under truthful revelation, the investments of players $j \neq i$ are not binding, while player i's action is binding at $\bar{x}_i(\bar{q}-1) - \vartheta$. In order to compensate for this, player $j \neq i$ chooses an additional $\varepsilon_{\bar{q}-1}^+ = \frac{\eta\vartheta}{1+(n-2)\eta}$ (similar to the previous case). From the perspective of player i this results in miscoordination given by,

$$\phi_i^{\bar{q}-1} = \frac{\left[\bar{x}_i(\bar{q}-1) - \vartheta\right] + \eta\sum_{j\neq i}\left[\bar{x}_j(\bar{q}-1) + \varepsilon_{\bar{q}-1}^+\right]}{1+(n-1)\eta} = \bar{\phi}_i^{\bar{q}-1} + \frac{(n-1)\eta\varepsilon_{\bar{q}-1}^+ - \vartheta}{1+(n-1)\eta}$$

$$\phi_i^{\bar{q}-1} = \bar{\phi}_i^{\bar{q}-1} - \frac{(1-\eta)\vartheta}{1+(n-2)\eta}$$

If i instead deviated to an inflated messaging strategy, player i can reduce the miscoordination losses. Again, we fix equilibrium beliefs. If i chooses to inflate her signal, then the realized $k_D = \bar{q}$ in which case players $j \neq i$ choose an additional $\varepsilon_{\bar{q}}^+$ that takes into account the fact that $x_i^*(\bar{q}) = \bar{x}_i(\bar{q}) - \frac{1}{n+2} - \vartheta$. The action of player $j \neq i$ is therefore $x_j^*(\bar{q}) = \bar{x}_j(\bar{q}) + \varepsilon_{\bar{q}}^+$, where the residual additional action is given by,

$$\varepsilon_{\bar{q}}^+ = \frac{\eta\left(\vartheta + \frac{1}{n+2}\right)}{1+(n-2)\eta}$$

We consider two sub-cases. First one is where $\varepsilon_{\bar{q}}^+ < y$. In this sub-case, from earlier arguments, player i can anticipate it in equilibrium with $k_D = \bar{q}$, where all other players take the action $x_j^*(\bar{q}) = \bar{x}_j(\bar{q}-1) + \frac{1}{n+2} + \varepsilon_{\bar{q}}^+$. As before, i can readjust her action according to the true k_T, and instead take an action $x_i^*(\bar{q}-1) = \bar{x}_i(\bar{q}-1) - \vartheta - \vartheta_{\bar{q}-1}^-$ such that the coordination function of i becomes,

$$\phi_i^{\bar{q}-1} = \frac{\left[\bar{x}_i(\bar{q}-1) - \vartheta - \vartheta_{\bar{q}-1}^-\right] + \eta\sum_{j\neq i}\left[\bar{x}_j(\bar{q}-1) + \frac{1}{n+2} + \varepsilon_{\bar{q}}^+\right]}{1+(n-1)\eta}$$

$$\implies \phi_i^{\bar{q}-1} = \bar{\phi}_i^{\bar{q}-1} + \left[\frac{(n-1)\eta\left(\frac{1}{n+2} + \frac{\eta\left(\vartheta+\frac{1}{n+2}\right)}{1+(n-2)\eta}\right) - \vartheta - \vartheta_{\bar{q}-1}^-}{1+(n-1)\eta}\right]$$

$$\implies \phi_i^{\bar{q}-1} = \bar{\phi}_i^{\bar{q}-1} + \left[\frac{\frac{(n-1)\eta}{n+2}\frac{1+(n-1)\eta}{1+(n-2)\eta} + \frac{(1-\eta)(1+(n-1)\eta)\vartheta}{1+(n-2)\eta} - \vartheta_{\bar{q}-1}^-}{1+(n-1)\eta}\right]$$

By setting $\vartheta_{\bar{q}-1}^- = \left(\frac{1+(n-1)\eta}{1+(n-2)\eta}\right)\left[\eta \cdot \frac{n-1}{n+2} + (1-\eta)\vartheta\right]$, the player can readjust her actions such that there is no miscoordination.

In the second sub-case where $\varepsilon_{\bar{q}}^+ \geq y$, the additional action by the players $j \neq i$ is simply $(n-1)\eta\left[\frac{1}{n+2} + y\right]$. In this sub-case, $\vartheta_{\bar{q}-1}^- = (n-1)\eta\left[\frac{1}{n+2} + y\right] - \vartheta$.

Case 3. $k_T = \bar{q}$

In this case, under truthful revelation, player i's action is binding at $\bar{x}_i(\bar{q}-1) - \vartheta$. In order to compensate for this, player $j \neq i$ chooses an additional $\varepsilon_{\bar{q}}^+ = \frac{\eta\left[\vartheta+\frac{1}{n+2}\right]}{1+(n-2)\eta}$ (similar to the previous case).

If $\varepsilon_{\bar{q}}^{+} < y$, from the perspective of player i this results in miscoordination given by,

$$\phi_i^{\bar{q}} = \frac{\left[\bar{x}_i(\bar{q}) - \frac{1}{n+2} - \vartheta\right] + \eta \sum_{j \neq i} \left[\bar{x}_j(\bar{q}) + \varepsilon_{\bar{q}}^{+}\right]}{1 + (n-1)\eta} = \bar{\phi}_i^{\bar{q}} + \frac{(n-1)\eta \varepsilon_{\bar{q}}^{+} - \frac{1}{n+2} - \vartheta}{1 + (n-1)\eta}$$

$$\phi_i^{\bar{q}} = \bar{\phi}_i^{\bar{q}} - \frac{(1-\eta)\left[\vartheta + \frac{1}{n+2}\right]}{1 + (n-2)\eta}$$

If i inflates her signal, then the realized $k_D = \bar{q} + 1$ in which case players $j \neq i$ choose an additional $\varepsilon_{\bar{q}+1}^{+}$ that takes into account the fact that $x_i^*(\bar{q}) = \bar{x}_i(\bar{q}) - \frac{1}{n+2} - \vartheta$. The action of player $j \neq i$ is therefore $x_j^*(\bar{q}) = \bar{x}_j(\bar{q}) + y$. The additional action by the players $j \neq i$ is simply $(n-1)\eta y$. Clearly, since $\varepsilon_{\bar{q}}^{+} < y$, making the other players take a higher action reduces the miscoordination losses experienced by player i under truthful communication.

If $\varepsilon_{\bar{q}}^{+} \geq y$, then all players' actions are bounded irrespective of whether communication is truthful or not. Indeed, for all $k_T \geq \bar{q} + 1$ the same intuition applies since players' actions are always binding.

So far, we have established that the best response by i to symmetric strategies cannot be such that $\bar{R}_i = \bar{x}_i(\bar{q} - 1) - \vartheta$. For every $\bar{R}_i = \{\bar{x}_i(\bar{q} - q') - \vartheta$ where $q' = \{2, 3, ..., \bar{q} - 2\}$ and $\vartheta = (0, \frac{1}{n+2}]$, we can similarly consider the three cases: *(i)* $k_T = \bar{q} - q' - 1$, emph(ii) $k_T = \bar{q} - q'$, emph(iii) $k_T = \bar{q} - q' + 1$. These coincide with precisely the same incentive problem as in Cases 1-3 presented above. Therefore, under truthful revelation, there are incentives for deviation and the associated deviation strategies for i. Consequently, the equilibrium best-response to symmetric strategies is such that $\bar{R}_i \notin [\bar{x}_i(0), \bar{x}_i(\bar{q} - 1))$.

□

Claim 2. *If $R_i \in [\bar{x}_i(\bar{q} - 1), \bar{x}_i(\bar{q}))$, then there exists a cutoff $\tilde{z}(y) \geq 0$ such that i has incentives to misrepresent signal $s_i = 0$ and report $m(s_i) = 1$ if $R_i \in (\bar{x}_i(\bar{q} - 1) + \tilde{z}(y), \bar{x}_i(\bar{q}))$.*

Proof. The intuition follows along the same lines as the previous case. If $k_T = \{0, 1, ..., \bar{q} - 1\}$, players' investments are not binding and they take the first-best action such that $x_j^*(k_T) = \bar{x}_j(k_T)$ for all $j \in N$. In the case where $k_T = \{\bar{q} + 1, ..., n\}$, all player's actions are binding and therefore they cannot do better than reporting truthfully since misreporting does not change the actions of the players in the subsequent stage.

Consider the only salient case, $k_T = \bar{q}$. In this case, under truthful communication, suppose $x_{j \neq i}^*(\bar{q}) = \bar{R}_j$. The cutoff $\tilde{z}$ can be computed by looking at the coordination function of $j \neq i$ under these actions:

$$\phi_j^{\bar{q}} = \frac{\Big[\bar{x}_j(\bar{q}) + y\Big] + \eta \sum_{j' \neq j,i} \Big[\bar{x}_{j'}(\bar{q}) + y\Big] + \eta \Big[\bar{x}_i(\bar{q}) - \frac{1}{n+2} + \tilde{z}\Big]}{1 + (n-1)\eta}$$

$$\implies \phi_j^{\bar{q}} = \bar{\phi}_j^{\bar{q}} + \left[\frac{[1 + (n-2)\eta]y - \eta\left(\frac{1}{n+2} - \tilde{z}\right)}{1 + (n-1)\eta}\right]$$

Clearly,

$$[1 + (n-2)\eta]y - \eta\left(\frac{1}{n+2} - \tilde{z}\right) \leq 0$$

$$\implies \quad \tilde{z}(y) \leq \max\left\{0, \frac{1}{n+2} - \frac{[1 + (n-2)\eta]y}{\eta}\right\} \tag{25}$$

Case. $y \geq \frac{1}{n+2} \cdot \frac{\eta}{1+(n-2)\eta}$

That is, if $y > \frac{1}{n+2} \cdot \frac{\eta}{1+(n-2)\eta}$, then the actions of players $j \neq i$ are not binding for any $\bar{R}_i \in [\bar{x}_i(\bar{q}-1), \bar{x}_i(\bar{q}))$. This results in miscoordination losses for i when $k_T = \bar{q}$. To see this, suppose player i has invested $\bar{R}_i = \bar{x}_i(\bar{q}) - \vartheta$ while the players $j \neq i$ have invested $\bar{R}_j = \bar{x}_j(\bar{q}) + y$ where $y > \frac{1}{n+2} \cdot \frac{\eta}{1+(n-2)\eta}$. In this case, clearly, the additional action taken by j is such that,

$$[1 + (n-2)\eta]\varepsilon_{\bar{q}}^+ = \eta\vartheta \quad \implies \quad \varepsilon_{\bar{q}}^+ = \frac{\eta\vartheta}{1 + (n-2)\eta}$$

Since $\varepsilon_{\bar{q}}^+ < y$, the miscoordination for i given the set of actions $x_i^*(\bar{q}) = \bar{x}_i(\bar{q}) - \vartheta$ and $x_{j\neq i}^*(\bar{q}) == \bar{x}_j(\bar{q}) + \varepsilon_{\bar{q}}^+$ is computed as before,

$$\phi_i^{\bar{q}} = \frac{\Big[\bar{x}_i(\bar{q}) - \vartheta\Big] + \eta \sum_{j \neq i} \Big[\bar{x}_j(\bar{q}) + \varepsilon_{\bar{q}}^+\Big]}{1 + (n-1)\eta}$$

$$\implies \phi_i^{\bar{q}} = \bar{\phi}_i^{\bar{q}} - \frac{(1-\eta)\vartheta}{1 + (n-2)\eta}$$

This miscoordination can be corrected by i by sending an inflated message which extracts the highest possible action by the other players, i.e., $x_{j\neq i}^*(\bar{q}+1) = \bar{x}_j(\bar{q}) + y$. This is clearly better for the player i since,

$$\frac{\Big[\bar{x}_i(\bar{q}) - \vartheta\Big] + \eta \sum_{j \neq i} \Big[\bar{x}_j(\bar{q}) + \varepsilon_{\bar{q}}^+\Big]}{1 + (n-1)\eta} < \frac{\Big[\bar{x}_i(\bar{q}) - \vartheta\Big] + \eta \sum_{j \neq i} \Big[\bar{x}_j(\bar{q}) + y\Big]}{1 + (n-1)\eta} \leq \bar{\phi}_i^{\bar{q}}$$

Or,

$$\frac{\left[\bar{x}_i(\bar{q}) - \vartheta\right] + \eta \sum_{j \neq i} \left[\bar{x}_j(\bar{q}) + y\right]}{1 + (n-1)\eta} > \bar{\phi}_i^{\bar{q}}$$

In the latter case, clearly, player i can choose a downward readjustment in actions, $\vartheta_{\bar{q}}^{-} = (n-1)\eta y - \vartheta$. That is the readjusted action to eliminate miscoordination is therefore, $x_i^*(\bar{q}) = \bar{x}_i(\bar{q}) - \vartheta - \vartheta_{\bar{q}}^{-} = \bar{x}_i(\bar{q}) - (n-1)\eta y$.

Case. $y < \frac{1}{n+2} \cdot \frac{\eta}{1+(n-2)\eta}$

In this case, there exists a $\tilde{z}(y) > 0$ such that whenever $\bar{R}_i \in (\bar{x}_i(\bar{q}-1), \bar{x}_i(\bar{q}-1) + \tilde{z}(y)]$, the actions of all players $j \neq i$ are binding at $\bar{R}_j$ when $k_T = \bar{q}$. This implies player i cannot do better by exaggerating her private signal and therefore truthful communication can be an equilibrium messaging strategy.

Analogously, if instead $\bar{R}_i \in (\bar{x}_i(\bar{q}-1) + \tilde{z}(y), \bar{x}_i(\bar{q}))$, then there is miscoordination concerns for i. To see this, suppose $\bar{R}_i = \bar{x}_i(\bar{q}) - \tilde{\vartheta}$ where $\tilde{\vartheta} \in \left(0, \frac{1}{n+2} - \tilde{z}(y)\right)$. In this case, when $k_T = \bar{q}$, the actions of players $j \neq i$ are not binding. That is, $x_j^*(\bar{q}) < \bar{R}_j$. Specifically, $x_j^*(\bar{q}) = \bar{x}_j(\bar{q}) + \varepsilon_{\bar{q}}^{+}$ where, as computed in earlier cases, $\varepsilon_{\bar{q}}^{+} = \frac{\eta\tilde{\vartheta}}{1+(n-2)\eta}$. This results in miscoordination losses for i thereby precluding full revelation of information in equilibrium. Finally, the action of player i cannot be in the interval $(\bar{x}_i(\bar{q}-1), \bar{x}_i(\bar{q}-1) + \tilde{z}(y)]$ either since the marginal costs are the same across all the players and if i's best response is in the above interval, the marginal benefits will be below the marginal costs (this is shown formally in Claim 4). □

Claim 3. *If $R_i > \bar{x}_i(q) + \tilde{y}$ then analogously player $j \neq i$ has incentives to misrepresent her signal $s_j = 0$ and report $m(s_j) = 1$ for both signal types. The intuition follows along the same lines as the previous case.*

Proof. Notice that $\tilde{y}$ is the action at which if $k_T = \bar{q}+1$, then $\phi_i^{\bar{q}+1} = \bar{\phi}_i^{\bar{q}+1}$. This is because when $k_T = \bar{q}+1$, all other players $j \neq i$ take the action $x_j^*(\bar{q}+1) = \bar{x}_j(\bar{q}) + y$. If $x_i^*(\bar{q}+1) = \bar{x}_i(\bar{q}) + \tilde{y}$, then

$$\phi_i^{\bar{q}+1} = \frac{\left[\bar{x}_i(\bar{q}) + \tilde{y}\right] + \eta \sum_{j \neq i} \left[\bar{x}_j(\bar{q}) + y\right]}{1 + (n-1)\eta} \tag{26}$$

$$= \frac{\left[\bar{x}_i(\bar{q}+1) + \frac{(n-1)\eta}{n+2} - (n-1)\eta y\right] + \eta \sum_{j \neq i} \left[\bar{x}_j(\bar{q}+1) - \frac{1}{n+2} + y\right]}{1 + (n-1)\eta} = \bar{\phi}_i^{\bar{q}+1}$$

However, this results in a miscoordination loss for player $j \neq i$. To see this, we can substitute the actions into the coordination function of j:

$$\phi_j^{\bar{q}+1} = \frac{\left[\bar{x}_j(\bar{q}+1) - \frac{1}{n+2} + y\right] + \eta \sum_{j' \neq j,i} \left[\bar{x}_{j'}(\bar{q}+1) - \frac{1}{n+2} + y\right] + \eta \left[\bar{x}_i(\bar{q}+1) + \frac{(1-(n+2)y)(n-1)\eta}{n+2}\right]}{1+(n-1)\eta}$$

$$\implies \phi_j^{\bar{q}+1} = \bar{\phi}_j^{\bar{q}+1} - \frac{(1-\eta)\left(\frac{1}{n+2} - y\right)}{1+(n-1)\eta}$$

That is, if player i's initial investment is such that $R_i > \bar{x}_i(q) + \tilde{y}$, the actions of i are not binding when $k_T = \bar{q}$. Since this results in miscoordination, player j can use inflated messaging strategy and derive a higher action from i. This would either reduce or eliminate, depending on how high the investments made by i are, the miscoordination losses faced by j. This precludes truthful communication by j.

□

Claim 4. *For all $\bar{R}_i \in [\bar{x}_i(q), \bar{x}_i(q) + \tilde{y}]$, the marginal benefit from investing $\bar{x}_i(q) + y_i$ is strictly decreasing in y_i. Since the marginal cost of investing resources is constant at c, the symmetric equilibrium exists and is unique.*

We write the ex-ante welfare in terms of the investment $\bar{R}_i = \bar{x}_i(\bar{q}) + y_i$, where $y_i \leq \tilde{y}$. We allow heterogeneity via the term y_i and then check if there is an equilibrium in which $y_i = y \in [0, \frac{1}{n+2})$ for all $i \in N$.

$$W_i(\bar{q}, y_i, y_{-i}) \equiv -\mathbb{E}_\theta\left[\mathbb{E}_k\left[\phi_i\Big(\min\{\bar{x}_i(k), \bar{R}_i\}, \min\{\bar{x}_{-i}(k), \bar{R}_{-i}\}\Big) - \theta - b_i\right]^2\right] - c\bar{R}_i$$

Define $y_{-i} = \sum_{j \neq i} y_j$ and $Y_i = \frac{y_i + \eta y_{-i}}{1+(n-1)\eta}$. Let the expected miscoordination loss (i.e., ignoring the cost term) be:

$$L_i = -\mathbb{E}_\theta\left[\mathbb{E}_k\left[\phi_i\Big(\min\{\bar{x}_i(k), \bar{R}_i\}, \min\{\bar{x}_{-i}(k), \bar{R}_{-i}\}\Big) - \theta - b_i\right]^2\right]$$

$$L_i = -\int_0^1 \sum_{k=0}^{n} \left[\phi_i\Big(\min\{\bar{x}_i(k), \bar{R}_i\}, \min\{\bar{x}_{-i}(k), \bar{R}_{-i}\}\Big) - \theta - b_i\right]^2 f(k \mid n, \theta) d\theta$$

$$L_i = -\frac{1}{n+1} \sum_{k=0}^{n} \int_0^1 \left[\phi_i\Big(\min\{\bar{x}_i(k), \bar{R}_i\}, \min\{\bar{x}_{-i}(k), \bar{R}_{-i}\}\Big) - \theta - b_i\right]^2 f(\theta \mid k, n) d\theta$$

where, $f(k \mid n, \theta) = \frac{f(\theta|k,n)}{n+1}$ from the property of Beta-Binomial distribution.

$$L_i = -\frac{1}{n+1}\sum_{k=0}^{\bar{q}}\int_0^1 \left[\bar{\phi}_i^k - \theta - b_i\right]^2 f(\theta \mid k,n)d\theta - \frac{1}{n+1}\sum_{k=\bar{q}+1}^{n}\int_0^1 \left[\bar{\phi}_i^{\bar{q}} + Y_i - \theta - b_i\right]^2 f(\theta \mid k,n)d\theta$$

The above expansion of the equation follows from noting that $\bar{q}$ is the highest sufficient statistic up to which players' actions can achieve first best coordination, i.e., $\bar{\phi}_i^k$ for all $i \in N$ as long as $k \in \{0, 1, ..., q\}$. Beyond the cutoff $\bar{q}$, there is inefficiency in decision-making in that players to do not achieve first-best coordination. Focusing on the integral inside the second term:

$$\left[\bar{\phi}_i^{\bar{q}} + Y_i - \theta - b_i\right]^2 = \left[\Big(\mathbb{E}[\theta \mid k,n] - \theta\Big) - \Big(\mathbb{E}[\theta \mid k,n] - \mathbb{E}[\theta \mid \bar{q},n] - Y_i\Big)\right]^2$$

This follows from making the substitution $\bar{\phi}_i^{\bar{q}} = \mathbb{E}[\theta \mid \bar{q}, n] + b_i$, then adding and subtracting $\mathbb{E}[\theta \mid k, n]$, and rearranging the terms.

$$\left[\bar{\phi}_i^{\bar{q}} + Y_i - \theta - b_i\right]^2 = \Big(\mathbb{E}[\theta \mid k,n] - \theta\Big)^2 + \Big(\mathbb{E}[\theta \mid k,n] - \mathbb{E}[\theta \mid \bar{q},n] - Y_i\Big)^2 - 2\Big(\mathbb{E}[\theta \mid k,n] - \theta\Big) \cdot \Big(\mathbb{E}[\theta \mid k,n] - \mathbb{E}[\theta \mid \bar{q},n] - Y_i\Big)$$

When taken into the integral, the last term cancels out. We therefore omit it and continue rewriting the welfare.

$$\begin{aligned} L_i = & -\frac{1}{n+1}\sum_{k=0}^{\bar{q}}\int_0^1 \left[\mathbb{E}[\theta \mid k,n] - \theta\right]^2 f(\theta \mid k,n)d\theta \\ & -\frac{1}{n+1}\sum_{k=\bar{q}+1}^{n}\int_0^1 \left[\mathbb{E}[\theta \mid k,n] - \theta\right]^2 f(\theta \mid k,n)d\theta \\ & -\frac{1}{n+1}\sum_{k=\bar{q}+1}^{n}\int_0^1 \left[\mathbb{E}[\theta \mid k,n] - \mathbb{E}[\theta \mid \bar{q},n] - Y_i\right]^2 f(\theta \mid k,n)d\theta \end{aligned}$$

$$\begin{aligned} L_i = & -\frac{1}{n+1}\sum_{k=0}^{n}\int_0^1 \left[\mathbb{E}[\theta \mid k,n] - \theta\right]^2 f(\theta \mid k,n)d\theta \\ & -\frac{1}{n+1}\sum_{k=q+1}^{n}\int_0^1 \left[\mathbb{E}[\theta \mid k,n] - \mathbb{E}[\theta \mid \bar{q},n] - Y_i\right]^2 f(\theta \mid k,n)d\theta \end{aligned}$$

$$where \qquad -\frac{1}{n+1}\sum_{k=0}^{n}\int_0^1 \Big[\mathbb{E}[\theta \mid k,n]-\theta\Big]^2 f(\theta \mid k,n)d\theta = -\frac{1}{n+1}\sum_{k=0}^{n}Var(\theta|k,n)$$

For the Beta-Binomial distribution,

$$\frac{1}{n+1}\sum_{k=0}^{n}Var(\theta|k,n) = Var(\theta|n) = \frac{1}{6(n+2)}$$

$$L_i \quad = \quad -\frac{1}{6(n+2)} - \frac{1}{n+1}\sum_{k=q+1}^{n}\int_0^1 \Big[\mathbb{E}[\theta \mid k,n]-\mathbb{E}[\theta \mid \bar{q},n]-Y_i\Big]^2 f(\theta \mid k,n)d\theta$$

Since none of the terms in the bracket of the integral depend on the realization of θ, we can rewrite them omitting the integral,

$$L_i \quad = \quad -\frac{1}{6(n+2)} - \frac{1}{n+1}\sum_{k=q+1}^{n}\underbrace{\Big[\mathbb{E}[\theta \mid k,n]-\mathbb{E}[\theta \mid \bar{q},n]-Y_i\Big]^2}_{\equiv \mathcal{Z}}$$

Expanding the term in the brackets further,

$$\mathcal{Z} \equiv \left[\Big(\mathbb{E}[\theta \mid k,n]-\mathbb{E}[\theta \mid \bar{q},n]\Big)^2 - 2Y_i\cdot\Big(\mathbb{E}[\theta \mid k,n]-\mathbb{E}[\theta \mid \bar{q},n]\Big)+Y_i^2\right]$$

Where, $\mathbb{E}[\theta \mid k,n]-\mathbb{E}[\theta \mid \bar{q},n] = \frac{k+1}{n+2}-\frac{\bar{q}+1}{n+2} = \frac{k-\bar{q}}{n+2}$. Substituting this back into the ex-ante welfare function,

$$W_i(\bar{q},y_i,y_{-i}) \quad = \quad -\frac{1}{6(n+2)} - \frac{1}{n+1}\sum_{k=\bar{q}+1}^{n}\left[\left(\frac{k-\bar{q}}{n+2}\right)^2 - 2Y_i\cdot\left(\frac{k-\bar{q}}{n+2}\right)+Y_i^2\right] - c\bar{R}_i$$

Using the identities $1^2+2^2+...+z^2 = \frac{z(z+1)(2z+1)}{6}$ and $1+2+..+z = \frac{z(z+1)}{2}$, we get,

$$W_i(\bar{q},y_i,y_{-i}) = -\frac{1}{6(n+2)} - \frac{(n-\bar{q})(n-\bar{q}+1)(2(n-\bar{q})+1)}{6(n+1)(n+2)^2} + \frac{(n-\bar{q})(n-\bar{q}+1)}{(n+1)(n+2)}Y_i - \frac{(n-\bar{q})}{(n+1)}Y_i^2 - c\bar{R}_i$$

To see that the above expressions are consistent, we check for the limit case where $Y_i = 0$ and $Y_i = \frac{1}{n+2}$. In the former, the terms containing Y_i disappear and the expression is trivially equal to one where $\bar{R}_i = \bar{x}_i(\bar{q})$. In the latter case, the 2nd, 3rd, and 4th terms in the welfare function simplifies to,

$$-\frac{(n-\bar{q})(n-\bar{q}+1)(2(n-\bar{q})+1)}{6(n+1)(n+2)^2}+\frac{(n-\bar{q})^2}{(n+1)(n+2)^2} = \\ -\frac{(n-\bar{q}-1)(n-\bar{q})(2(n-\bar{q})-1)}{6(n+1)(n+2)^2}$$

However we know that when $\bar{R}_i = \bar{x}_i(\bar{q}+1)$ and $Y_i = 0$,

$$W_i(\bar{q}+1, y_i, y_{-i})\Big|_{Y_i=0} = -\frac{1}{6(n+2)} - \frac{(n-\bar{q}-1)(n-\bar{q})(2(n-\bar{q})-1)}{6(n+1)(n+2)^2} - c\bar{R}_i$$

This implies the following:

$$W_i(\bar{q}+1, y_i, y_{-i})\Big|_{Y_i=0} = W_i(\bar{q}, y_i, y_{-i})\Big|_{Y_i=\frac{1}{n+2}}$$

The final expression for the welfare function is therefore,

$$\begin{aligned} W_i(\bar{q}, y_i, y_{-i}) = &-\frac{1}{6(n+2)} - \frac{(n-\bar{q})(n-\bar{q}+1)(2(n-\bar{q})+1)}{6(n+1)(n+2)^2} \\ &+ \frac{(n-\bar{q})}{(n+1)}\left[\frac{(n-\bar{q}+1)}{(n+2)} - Y_i\right] Y_i - c\bar{R}_i \end{aligned} \tag{27}$$

In the symmetric equilibrium, when every other player invests $y_i = y$ in the first stage such that $Y_i = \frac{y_i+(n-1)\eta y}{1+(n-1)\eta}$, the equilibrium welfare is,

$$\begin{aligned} W_i(\bar{q}, y_i, y) = &-\frac{1}{6(n+2)} - \frac{(n-\bar{q})(n-\bar{q}+1)(2(n-\bar{q})+1)}{6(n+1)(n+2)^2} \\ &+ \frac{(n-\bar{q})}{(n+1)}\left[\frac{(n-\bar{q}+1)}{(n+2)} - Y_i\right] Y_i - c[\bar{x}_i(\bar{q}) + y_i] \end{aligned}$$

The first order condition with respect to y_i is therefore,

$$\frac{(n-\bar{q})(n-\bar{q}+1)}{(n+1)(n+2)(1+(n-1)\eta)} - \frac{2(n-\bar{q})}{(n+1)(1+(n-1)\eta)} y_i = c$$

This is the same expression for all players due to symmetry. Therefore $y_i = y$ and,

$$\frac{(n-\bar{q})(n-\bar{q}+1)}{(n+1)(n+2)(1+(n-1)\eta)} - \frac{2(n-\bar{q})}{(n+1)(1+(n-1)\eta)} y = c$$

A.5 Proof of Proposition 3

The expression for welfare directly follows from plugging $Y_i = y$ under the symmetric equilibrium strategies into Equation 27.

B Proof of Theorem 2: Two Player Case

Since there are only two players and second player is more hawkish than the first, it directly follows that the latter is a $0 - type$ and the former is a $1 - type$. This implies the first player always has incentives to reveal the low signal while the second, the high one. Further, the signals s_i are conditionally independent but correlated in that $Pr(s_2 = 1|s_1) = \frac{2}{3}$ and $Pr(s_2 = 0|s_1) = \frac{1}{3}$, and vice versa for s_2. Suppose signals $s = (s_1, s_2)$ are *publicly* observed such that,

$$\mathbb{E}[\theta|s] = \frac{s_1 + s_2 + 1}{4}$$

The actions of players in this case is given by,

$$x_1(s) = \mathbb{E}[\theta|s] - \frac{\eta}{1-\eta}b < 1 \tag{28}$$

$$x_2(s) = \mathbb{E}[\theta|s] + \frac{1}{1-\eta}b > 0 \tag{29}$$

The actions depend crucially on whether the bounds on actions $x_i \in [0,1]$ are binding or not. It is very clear from the above equations that $x_1(s)$ is always less than one while $x_2(s)$ is always greater than zero. As long as actions remain within the bound, it is straightforward to observe that $\phi_1(x_1(s), x_2(s)) = \mathbb{E}[\theta|s]$ and $\phi_2(x_1(s), x_2(s)) = \mathbb{E}[\theta|s] + b$.

However, suppose the constraints are binding for certain signal realizations. In particular $x_1(s) < 0$ and/or $x_2(s) > 1$ for some realization of signals s. Suppose $x_1(s) < 0$, then $x_1 = 0$ and $x_2(s) = (1+\eta)\,(\mathbb{E}[\theta|s] + b)$. Further, $x_1(s) < 0$ implies $\mathbb{E}[\theta|s] < \eta.\,(\mathbb{E}[\theta|s] + b)$. It can be verified trivially that $\phi_1(0, x_2(s)) = \eta.\,(\mathbb{E}[\theta|s] + b) > \mathbb{E}[\theta|s]$. Similarly, if $x_2(s) > 1$, then $x_2 = 1$ and $x_1(s) = (1+\eta)\mathbb{E}[\theta|s] - \eta$. As before, $\phi_2(x_1(s), 1) = \eta.\mathbb{E}[\theta|s] + (1-\eta) < \mathbb{E}[\theta|s] + b$ since $x_2(s) > 1$ implies $\mathbb{E}[\theta|s] + b > \eta.\mathbb{E}[\theta|s] + (1-\eta)$.

Whenever $x_1(s) < 0$ there is *over-provision* concern for player 1 and whenever $x_2(s) > 1$ there is *under-provision* concern for player 2. To see how this can lead to babbling in equilibrium, let us consider the case where the signals are privately observed and the players communication with each other via simultaneous cheap talk messages. Say truthful messages are such that m_i^T: $m(s_i) = s_i$ for $s_i = 0$ or 1. Consider the case where $s_1 = 1$.[25] Let $\mathbb{E}U_1(1, m_1^T)$ be the ex ante EU of player 1 with information $s_1 = 1$ under truthful messaging, conditional on the other player reporting truthfully.

[25] An analogous argument follows for the player 2.

$$\mathbb{E}U_1(1, m_1^T) = -\sum_{s_2 \in \{0,1\}} Pr(s_2|s_1 = 1) \int_0^1 \left(\frac{x_1(1,s_2) + \eta x_2(1,s_2)}{1+\eta} - \theta \right)^2 f(\theta \mid 1, s_2) d\theta$$

Given the interim expectation of s_2 conditional on s_1, I can expand the above equation as follows:

$$\begin{aligned}\mathbb{E}U_1(1, m_1^T) = &-\frac{1}{3}\int_0^1 \left(\frac{x_1(1,0) + \eta x_2(1,0)}{1+\eta} - \theta \right)^2 f(\theta \mid 1, 0) d\theta \\ &-\frac{2}{3}\int_0^1 \left(\frac{x_1(1,1) + \eta x_2(1,1)}{1+\eta} - \theta \right)^2 f(\theta \mid 1, 1) d\theta\end{aligned}$$

Suppose $x_1(1,1) > 0$ but $x_1(1,0) < 0$ according to Equation 28 and Equation 29. Then it follows from previous arguments that,

$$\frac{x_1(1,1) + \eta x_2(1,1)}{1+\eta} = \mathbb{E}[\theta|(1,1)]$$

$$\frac{x_1(1,0) + \eta x_2(1,0)}{1+\eta} = \mathbb{E}[\theta|(1,0)] + (\eta b - (1-\eta)\mathbb{E}[\theta|(1,0)])$$

Where,

$$\Delta_1(1,0) = (\eta b - (1-\eta)\mathbb{E}[\theta|(1,0)]) > 0$$

$$\mathbb{E}U_1(1, m_1^T) = -\frac{1}{3}\int_0^1 (\mathbb{E}[\theta|1,0] - \theta + \Delta_1(1,0))^2 f(\theta \mid 1,0) d\theta - \frac{2}{3}\int_0^1 (\mathbb{E}[\theta|1,1] - \theta)^2 f(\theta \mid 1,1) d\theta \quad (30)$$

The additional term $\Delta_1(1,0)$ increases the expected losses over and above the standard variance term $(\mathbb{E}[\theta|1,0] - \theta)^2$. This provides incentives for under-reporting the high signal. To see this clearly, suppose player 1 misrepresents her signal and sends a deviation message m_1^D: $m(1) = m(0) = 0$. Player 2 treats the deviation message as if it were *on equilibrium path*. This implies that her action is simply given by $x_2(0,0)$ and $x_2(0,1)$ when the signal $s_2 = 0$ or 1 respectively. Of course, if $x_2(1,0)$ and $x_2(1,1)$ were both above the upper bound implying $x_2(1,0) = x_2(1,1) = 1$, then it is also possible $x_2(0,0) = x_2(0,1) = 1$. If so, then the deviation strategy does not alter the expected utility and player 1 is indifferent. However, if $x_2(0,0) < x_2(0,1) < 1$, then player 1's deviation actions $x_1^D(1,0)$ and $x_1^D(1,1)$ anticipate player 2's actions and correspondingly is readjusted. That is,

$$x_2(0,0) = \frac{1}{4} + \frac{1}{1-\eta}b \implies x_1^D(1,0) = (1+\eta)\frac{1}{2} - \eta.x_2(0,0)$$

$$x_2(0,1) = \frac{1}{2} + \frac{1}{1-\eta}b \implies x_1^D(1,1) = (1+\eta)\frac{3}{4} - \eta.x_2(0,1)$$

This implies that $\phi_1\left(x_1^D(1,0), x_2(0,0)\right) = \mathbb{E}[\theta|1,0]$ or,

$$\phi_1\left(x_1(1,0), x_2(1,0)\right) \geq \phi_1\left(x_1^D(1,0), x_2(0,0)\right) \geq \mathbb{E}[\theta|1,0]$$

In the former case, it is obvious $\Delta_1^D(1,0) = 0$ while in the latter $\Delta_1^D(1,0) \leq \Delta_1(1,0)$. That is,

$$\begin{aligned}\mathbb{E}U_1^D(1,m_1^D) = &-\frac{1}{3}\int_0^1 \left(\mathbb{E}[\theta|1,0] - \theta + \Delta_1^D(1,0)\right)^2 f(\theta \mid 1,0)d\theta \\ &-\frac{2}{3}\int_0^1 \left(\mathbb{E}[\theta|1,1] - \theta\right)^2 f(\theta \mid 1,1)d\theta \qquad \geq \qquad \mathbb{E}U_1(1,m_1^T)\end{aligned}$$

As a result player 1 will always prefer the deviation message and truthful revelation breaks down. In the case where $x_1(1,0) < x_1(1,1) < 0$ the deviation message weakly benefits player 1 when $s_2 = 0$ and $s_2 = 1$. That is with truthful messaging the expected utility is,

$$\begin{aligned}\mathbb{E}U_1(1,m_1^T) = &-\frac{1}{3}\int_0^1 \left(\mathbb{E}[\theta|1,0] - \theta + \Delta_1(1,0)\right)^2 f(\theta \mid 1,0)d\theta \\ &-\frac{2}{3}\int_0^1 \left(\mathbb{E}[\theta|1,1] - \theta + \Delta_1(1,1)\right)^2 f(\theta \mid 1,1)d\theta\end{aligned}$$

Under the deviation message, the expected utility is instead,

$$\mathbb{E}U_1^D(1, m_1^D) = -\frac{1}{3}\int_0^1 \left(\mathbb{E}[\theta|1,0] - \theta + \Delta_1^D(1,0)\right)^2 f(\theta \mid 1,0)d\theta$$
$$-\frac{2}{3}\int_0^1 \left(\mathbb{E}[\theta|1,1] - \theta + \Delta_1^D(1,1)\right)^2 f(\theta \mid 1,1)d\theta$$

For analogous arguments made above, $\Delta_1(1,0) \geq \Delta_1^D(1,0) \geq 0$ and $\Delta_1(1,1) \geq \Delta_1^D(1,1) \geq 0$. Therefore it follows that $\mathbb{E}U_1^D(1, m_1^D) \geq \mathbb{E}U_1(1, m_1^T)$. Finally,it can be inferred from the arguments that the relevant IC constraints for truth-telling are $s_1 = 1$ for player 1 and $s_2 = 0$ for player 2.

That is to check if full revelation is possible by both players, it is necessary and sufficient to check for IC constraint of player 1 with the high signal and player 2 with low signal. Given the incentives for player 1 to deviate from revealing the high signal, as described above, it is also clear that the *pivotal* IC is the one where $s = (1,0)$ and $x_1(s) < 0$, i.e the one in which the other player holds a low signal and the resulting action of player 1 after truthfully revealing the high signal is below the lower bound of actions. The vice-versa holds for player 2. The pivotal constraint to check is one in which $s = (1,0)$ and $x_2(1,0) > 1$. (Remember that for player 2 the following inequalities hold when the actions are above the upper bound: $1 \leq x_2(0,0) \leq x_2(1,0)$.) The reason is intuitive. Though $x_2(0,0) < 1$ (or $x_1(1,1) > 0$) is a necessary condition for truthful revelation, it is not sufficient. Simply put, as in the case analyzed earlier, it could be that $x_1(1,1) > 0$ but $x_1(1,0) < 0$ ($x_1(0,0) < 1$ and $x_2(1,0) > 1$ in the case of player 2). In this case, sufficiency breaks down since player's have an incentive to deviate since $x_1(1,0) < 0$ ($x_2(1,0) > 1$). Therefore the pivotal case is one in which the signal $s = (1,0)$ and the actions corresponding to the truthful revelation of s is such that,

$$x_1(1,0) = \frac{1}{2} - \frac{\eta}{1-\eta}b \geq 0 \qquad x_2(1,0) = \frac{1}{2} + \frac{1}{1-\eta}b \leq 1$$

Rearranging gives,

$$b \leq \frac{1-\eta}{2\eta} \qquad \text{and} \qquad b \leq \frac{1-\eta}{2}$$

Since $\eta < 1$ the condition for x_2 is the one that is binding.

In the case of two players, it is straightforward to see that player 1 is a $0 - type$ and player 2 is a $1 - type$. As before, $q = s_1 + s_2$ and $q \in \{0, 1, 2\}$. From **??**, it follows that $x_1(2) \geq 0$ and $x_2(0) \leq 1$. Player 1 has an incentive to under-report the high signal whenever for some realization of player 2's signal, player 1's action under truth-telling is below the lower bound. In this case, $x_1(q) = 0$ and player 2 readjusts her actions accordingly. Crucially, this readjustment results in $\phi_1(\mathbf{x}) > \mathbb{E}[\theta \mid s]$, increasing the overall ex ante variance. This is sufficient to induce player 1 to deviate and under-report.

Since the utility functions satisfy single crossing, if player 1 is truthful and her action is below the lower bound for $s_2 = 1$, then it *must* be below this bound for $s_2 = 0$. The vice versa need not be true. That is, it *may* be that when $s = (1,0)$ the action $x_1(1) < 0$, and when $s = (1,1)$ the action $x_1(2) > 0$. As a result, the pivotal IC constraint for player 1 is the case when $s_2 = 0$ and $x_1(1) < 0$. A similar argument for player 2 implies that her pivotal IC is when $s_2 = 0$ and $s_1 = 1$ such that $x_2(1) > 1$. Together, these conditions yield the characterization for the two player case.

QED