Is it better to be first? Sequential search with endogenous information acquisition*
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Abstract

We consider a model of sequential search in which a decision-maker (DM) has to choose one alternative from a fixed set. All available alternatives are iid random variables and ex-ante unknown to the DM. Before making a choice, contrary to the standard search literature, we allow the DM to decide how much and what kind of information to acquire about each alternative, e.g., design different job market interviews for candidates with different arrival ranks. We find that optimal interviews have an intuitive property – the first arriving candidates are treated harshly, and their interviews are harder to pass, while later candidates' interviews are easier to pass. We compare the unconditional probabilities of choice and study the discrimination the order of inspection can cause. We argue that discrimination is sensitive to the functional form of the cost of learning. For different cost functions, the discrimination may also depend on the DM's prior belief.

JEL-codes: D81, D83, D91, J71.

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1 Introduction

Job market interviews are an integral part of hiring decisions. Candidates are considered sequentially, and employers can decide what questions to ask and what tests to perform.

Economic literature lacks papers in which there is a sequential search with flexible and unconstrained learning. This paper closes the gap – we build a model of sequential search in which a decision maker can decide how much and what information to acquire at each stage of the search process. Other papers, for example, Ursu et al. (2020), model learning as flexible but restrict their analysis to a specific learning technology of acquiring signals of fixed structure, and the decision maker chooses only the amount of these signals.

We show that under general conditions, the optimal learning and hiring strategy are extremely tractable and intuitive. First, learning at each stage of the search is characterized by a set of posterior beliefs about the candidate that is considered at this stage, and this set is binary – high and low posteriors. Second, if the interview ends by approaching the high posterior, the candidate is hired, and the search stops; if, however, the low posterior is approached, the candidate is dismissed, and the search goes to the next stage. Third, both high and low posteriors are lower at each new stage than they were in the previous stage.

Our results are important for several strands of literature. First, to the best of our knowledge, our paper is the first one that considers a problem of sequential search with full flexibility of acquired information; that is, we generalize the optimal stopping problem (see, e.g., Gilbert and Mosteller (1966)). Secondly, we contribute to the literature on Bayesian learning. One of the central notions in the literature is the notion of Blackwell's order on signal structures, or, what is equivalent, experiments (see Blackwell (1951)). The issue is that, in practice, many signals are Blackwell- incomparable. We introduce a new order on information structures, and this new order is related to the third result of our paper. The third result has a very intuitive interpretation and is also important for literature in information acquisition. It can be interpreted in the following way: the interviews of each following candidate become easier and easier to pass. The importance of information acquisition literature is as follows. The signal structures designed by the decision maker are not comparable in a Blackwell sense but still can be ordered; that is, we introduce a new order on interviews – their difficulty to pass. Fourth, our paper is important for labor economics, namely, for literature on labor market discrimination.

2 Setup

We study a decision maker (DM, she) who considers T a priori identical candidates. Each candidate i (he) has an associated random variable which we call the productivity level, $\theta_i \in \Theta = \{0, 1\}$.[1] The productivity levels are iid across all candidates with $\mathbb{P}(\theta_i = 1) = \mu \in (0, 1)$.

The manager must choose one candidate; there is no outside option. Before making a decision, she may learn about the candidates' types. Learning is costly and sequential. In stage i, the manager inspects candidate i. We assume no discounting between periods, and therefore, the total costs in the problem for the manager is simply the sum of costs in each period i. The manager's payoff in the problem equals the difference between the expected value of the chosen candidate and total costs.

Learning in stage i proceeds as follows. The manager chooses a Blackwell experiment, which generates information about θ_i. This experiment is a mapping from the state space $\{0, 1\}$ to a probability measure over some compact set of signal realizations. Each signal realization is associated with posterior belief distribution on $\{0, 1\}$, and an experiment induces a distribution over posterior beliefs. Since we consider a binary random variable, we identify a posterior belief distribution with a belief about $\theta_i = 1$. It is commonly known from the literature on rational inattention (e.g., Caplin and Dean (2013)) that instead of considering the set of Blackwell experiments, one can consider the set of distribution of posterior beliefs, in which the mean equals prior. In our analysis, we apply the posterior approach. Formally, we define a feasible set of distributions as

[1] The results presented are robust with respect to the cardinality of the productivity levels.

follows

$$\Pi = \{p_i \in \Delta(\Delta(\Theta)) : \mathbb{E}_{p_i}[x] = \mu\}, \tag{1}$$

where x is a generic posterior belief about state $\theta_i = 1$. Feasible sets of posterior distributions are the same for all candidates since the productivity levels of the candidates are iid random variables. If DM does not acquire informative signals during the interview, we simply denote the resultant information structure by δ, a completely uninformative signal, that is, p puts unit weight on the prior, μ.

Choice of p_i is costly. We define the cost as a mapping $C : \Pi \to \mathbb{R}_+$ such that

$$C(p) = \lambda \mathbb{E}_p[c(x)], \tag{2}$$

where $\lambda \in \mathbb{R}_{++}$ is the marginal cost of information and $c(x)$ is convex and belongs to C^∞ class. This cost specification falls into the class of the posterior-separable cost function, the most commonly used function in the rational inattention literature (Caplin et al. (2022)).

After observing the realized posterior belief x_i about candidate i, the DM has to make a decision about this candidate. We assume no recall property, that is, if a candidate is not hired right after an interview, he is no longer available to the DM.[2] Thus, DM has only two available actions: whether or not hire i, denoted by $a_i \in \{0, 1\}$. If DM chooses $a_i = 1$ and hires a candidate i, she stops the search process.[3]

To formulate the DM's dynamic problem formally, we introduce a history up to time i. We define a set H_i that consists of all realized posteriors such that all candidates before i have been rejected.[4] Formally,

$$\begin{aligned} &H_i = \left\{[0,1]^{i-1} \times \{0,1\}^{i-1}\right\} \text{ if } i \in \{2, \dots, T\}, \\ &H_1 = \emptyset. \end{aligned}$$

The DM's strategy at i is a pair of functions denoted by p_i and a_i where[5]

$$\begin{aligned} &p_i : H_i \to \Pi, \\ &a_i : H_i \times [0,1] \to \{0,1\}. \end{aligned}$$

The first function gives the choice of signal for i for each $h_i \in H_i$. The second function dictates the hiring decision for i for each posterior x_i in support of optimal signal.[6] Collecting these two functions for each candidate, the DM's strategy is given by $(\boldsymbol{p}, \boldsymbol{a}) = ((p_1, a_1)), \dots, (p_T, a_T))$.

Given any posterior belief, x_i DM has two actions. If she hires a candidate, she gets utility x_i and if she does not she reaches the history $h_{i+1}|h_i = (x_i, 0, h_i)$. We denote a continuation value, the maximal expected value that DM achieves at this history as $V(h_{i+1}|h_i)$. Given a strategy (p_i, a_i) we can write the DM's payoff from interviewing candidate i as

$$\mathbb{E}_{p_i}\Big[a_i x_i + (1 - a_i)V(h_{i+1}|h_i) - \lambda c(x_i)\Big]. \tag{3}$$

We formulate DM's dynamic problem using dynamic consistency and normalization requirements.

Definition 1. *The full dynamic problem of the manager is given by*

[2] There can be several rationales for such assumption: no-recall can be caused by the psychological factors of the rejected agent (pride, etc.), or of the DM (extreme case of limited memory), or by conditions on the labor market (other firms immediately hire rejected candidate).

[3] Usually, interviews with other candidates are prescheduled in advance. In this case, DM simply does not obtain informative signals about candidates from $i + 1$ to T.

[4] If the DM has already been hired, the history is terminal. Consequently, there are no actions to take.

[5] Throughout we restrict our attention to pure actions, both for signals and hiring decisions. It is shown that this is without loss.

[6] Note that the hiring decision is invariant across signals if their supports coincide.

$$(\boldsymbol{p}, \boldsymbol{a}) \in \left\{ \arg\max_{(p_i, a_i)} \Big[\mathbb{E}_{p_i} \Big[a_i x_i + (1 - a_i) V(h_{i+1}|h_i) - \lambda c(x_i) \Big] \Big] \right\}_{(i,h_i) \in \{1,\dots,T\} \times H_i} \tag{4}$$

s.t.

$$V(h_{i+1}|h_i) = \left\{ \max_{(p_{i+1}, a_{i+1})} \Big[\mathbb{E}_{p_{i+1}} \Big[a_{i+1} x_{i+1} + (1 - a_{i+1}) V(h_{i+2}|h_{i+1}) - \lambda c(x_{i+1}) \Big] \Big] \right\} \forall\ i \neq T, h_i \in H_i \tag{DC}$$

$$V(h_T) = 0 \ \forall\ h_T \in H_T \tag{FC}$$

Constraint (DC) ensures dynamic consistency, that is DM behaves optimally in every history. Constraint (FC) captures the intuition that if the final candidate is indeed reached, it implies that the DM has rejected all candidates before. In that case DM rejects all candidates and ends up with zero payoff.

3 Solution

3.1 Reduction to Static Problems

Using the notation from the previous Section, we introduce a building block for our analysis and consider a special class of static problems. We later show how the problem (4) can be reduced to a collection of the introduced static problems.

We consider a decision problem with two states $\theta = \{0, 1\}$ such that $\mathbb{P}(\theta = 1) = \mu$ and two available actions. First action $a = 1$ is state-dependent and gives utility $u(a = 1, \theta) = \theta$. Another action $a = 0$ is state-independent and gives fixed utility $u(a = 0, \theta) = \bar{u} \in (0, 1)$. Prior to making a decision, DM may reduce the uncertainty regarding the state. DM's learning technology and related cost are summarized by set Π and function $C(p)$ from (1) and (2) correspondingly.[7]

Given a realized posterior x, the DM chooses an action that delivers higher utility; therefore, her ex-post reward from x equals $\max\{x, \bar{u}\}$. Formally, DM solves the following problem

Definition 2. *The static problem (or the problem with exogenous outside option) is*

$$\max_{p(x) \in \Pi} \mathbb{E}_p \Big[\max\{x, \bar{u}\} - \lambda c(x) \Big]. \tag{5}$$

We show how the full DM's problem (4) can be reduced to the collection of the problems (5). The key observation is that continuation values are history-independent. Suppose that DM reaches a history $h_{i+1}|h_i$. Continuation value $V(h_{i+1}|h_i)$ equals the maximum achieved payoff for that history. Because of the no-recall property, DM faces the same problem for any history h_{i+1} given that all $a_{i'} = 0$ for $i' \leq i$. Therefore, given that DM did not hire a candidate up to time i, a sufficient statistic for her problem at any history h_{i+1} is the number of candidates left. Thus, we can write $V(h_{i+1}|h_i) = V_{T-i}$, because DM discarded first i candidates and $T - i$ is left.

In stage i given a posterior realization x_i in order to make a choice a_i DM simply compares value x_i and a fixed number V_{T-i}. Therefore, a problem (4) can be reduced to a collection of problems (2) with different outside options $\bar{u}$. We can formulate the simplified DM's problem as follows.

Definition 3. *The (reduced) dynamic DM's problem is*

$$\begin{gathered} \max_{p_i(x) \in \Pi} \mathbb{E}_{p_i} \Big[\max\{x_i, V_{T-i}\} - \lambda c(x_i) \Big] \quad \forall\ i \neq T, \\ s.t. \\ V_{T-i} = \left\{ \max_{p_{i+1}(x) \in \Pi} \mathbb{E}_{p_{i+1}} \Big[\max\{x_{i+1}, V_{T-i-1}\} - \lambda c(x_{i+1}) \Big] \right\}, \\ V_T = 0. \end{gathered} \tag{6}$$

[7]Such a problem is a problem of rationally inattentive agent with an exogenous outside option, see, e.g. Matějka and McKay (2015), Wei (2021) for the previous reference.

In the analysis of the DM's problem below, we use the formulation (6) and refer to it as dynamic DM's problem. To solve the problem (6), we start with the analysis of the static problem (5).

3.2 Analysis of the Static Problem

We start with a characterization lemma that exploits the convexity of the cost function $c(x)$.[8]

Lemma 1. *1. The optimal posterior distribution $p(x)$ in the problem (5) consists of at most two posterior beliefs.*

2. There exist two thresholds x_{lnl}, x_{hnl} such that if $\bar{u} \leq x_{lnl}$ or $\bar{u} \geq x_{hnl}$ DM chooses degenerate posterior distribution with unit mass on μ and chooses $a = 1$ if $\bar{u} \leq x_{lnl}$ and $a = 0$ if $\bar{u} \geq x_{hnl}$. If $\bar{u} \in (x_{lnl}, x_{hnl})$ DM chooses a posterior distribution with two posterior beliefs x^L, x^H such that $x^L < \mu < x^H$.

The first statement is a consequence of two available actions and convexity of $c(x)$. If two different posterior beliefs lead to the same action, a merged posterior, which equals their linear combination, is cheaper and leads to the same expected reward. The second statement determines the so-called learning region. If the value of the outside option is too low or too high compared to the expected value of the random option, DM does not obtain any informative signal and chooses the best ex-ante action. If $\bar{u} \in (x_{lnl}, x_{hnl})$ then DM chooses a posterior distribution p with two posterior realizations x^L, x^H. Given x^L she chooses $a = 0$ and given x^H chooses $a = 1$. Inequality $x^L < \mu < x^H$ follows from the feasibility condition $p \in \Pi$.

Exact values of x^L and x^H depend on the parameters of the model and function $c(x)$. We use the concavification technique as a solution method to the problem.[9] Using the language from Caplin et al. (2022) we introduce net utility $u(x) = \max\{x, \bar{u}\} - \lambda c(x)$. Optimal beliefs x^L, x^H belong to the concave closure $\hat{u}(x)$ of the net utility. Moreover, $\hat{u}(x) = u(x)$ if $x \leq x^L$ or $x \geq x^H$ and $\hat{u}(x)$ equals to the straight line, connecting points $(x^L, u(x^L)), (x^H, u(x^H))$.

Using tangency conditions, we can characterize necessary and sufficient conditions for the optimal posteriors x^L, x^H. We formulate those in the following technical lemma.

Lemma 2. *Suppose the optimal solution to the problem (5) is not degenerate. Optimal posterior beliefs solve the system*

$$\begin{cases} -\lambda c'(x^L) = 1 - \lambda c'(x^H) \\ x^H - \lambda c(x^H) - (\bar{u} - \lambda c(x^L)) = -\lambda c'(x^L)(x^H - x^L). \end{cases}$$

Concave closure $\hat{u}(x)$ may not depend on the prior belief μ. For example, it happens if the cost function $C(p)$ falls into the class of uniformly posterior-separable cost functions with $C(p) = \lambda \mathbb{E}_p[\tilde{c}(x)] - \tilde{c}(\mu)$, where function $\tilde{c}(x)$ does not depend on μ. In this case posterior beliefs x^L, x^H that satisfy conditions from Lemma 2 are a solution of the problem (6) if $x^L < \mu < x^H$. To analyze how the solution to the static problems in (6) changes for different i, we investigate the dynamics of continuation values V_i.

3.3 Continuation Value Dynamics

We observe that a continuation value equals the maximal attained value in a static problem (5) for a particular value of outside option $\bar{u}$. We define a function $g : [0, 1] \to [0, 1]$ such that

$$g(y) = \max_{p(x) \in \Pi} \mathbb{E}_p \Big[\max\{x, y\} - \lambda c(x) \Big],$$

that equals value of the problem (5) for an outside option y. Clearly, if $y \leq x_{lnl}$ then $g(y) = \mu$ and if $y \geq x_{hnl}$ then $g(y) = y$. We characterize function $g(y)$ on the interval (x_{lnl}, x_{hnl}) using the Envelope theorem.

[8] A variant of this lemma appears in the literature, and we state it without proof. For the reference, see, e.g., Matysková and Montes (2023), Wei (2021).

[9] See, e.g., for recent use of concavification to the related rationally inattentive problems Jain and Whitmeyer (2021), Kim et al. (2022).

Lemma 3. *If $y \in (x_{lnl}, x_{hnl})$ function $g(y)$ is strictly increasing and strictly convex. Moreover,* $\lim\limits_{y \to x_{lnl}+0} g'(y) = 0$ *and* $\lim\limits_{y \to x_{hnl}-0} g'(y) = 1$.

Proof. We consider optimal posterior beliefs x^L, x^H and denote $q(x^L), q(x^H)$ as marginal probabilities to receive such beliefs. Using this notation we can can express $g(y)$ as expected value in the optimum: $g(y) = q(x^L)(y - \lambda c(x^L)) + q(x^H)(x^H - \lambda c(x^H))$.

By the Envelope theorem equality $g'(y) = q(x^L)$ holds. Because $q(x^L) \in (0, 1)$ inequality $g'(y) > 0$ holds and function $g(y)$ is strictly increasing.

To show that function $g(y)$ is strictly convex, we compute derivative $q(x^L)$ with respect to y. We express $q(x^L)$ using Bayes rule as $q(x^L) = \frac{x^H - \mu}{x^H - x^L}$. Simple algebra shows that condition $q'(x^L) > 0$ is equivalent to the conditions $\frac{dx^L}{dy} > 0$, $\frac{dx^H}{dy} > 0$. Rearranging the system from Lemma (2) we get that

$$\begin{cases} \frac{dx^L}{dy} = \frac{1}{\lambda c''(y^L)(y^H - y^L)} \\ \frac{dx^H}{dy} = \frac{1}{\lambda c''(y^H)(y^H - y^L)}, \end{cases}$$

therefore function $g(y)$ is convex.

By the Envelope theorem and the fact that if $y = x_{lnl}$ DM chooses $a = 1$ and if $y = x_{lnl}$ DM chooses $a = 0$, conditions $\lim\limits_{y \to x_{lnl}+0} g'(y) = 0$ and $\lim\limits_{y \to x_{hnl}-0} g'(y) = 1$ hold. □

Our first characterization relates to the learning strategy. We claim that for any value of T, DM learns during the first interview and continues learning until she gets a high posterior realization.

We first show that if DM in the solution to problem (5) learns with outside option y, then she learns in the solution to the problem with outside option $g(y)$. For such a result we need to show that if $y \in (x_{lnl}, x_{hnl})$ then $g(y) \in (x_{lnl}, x_{hnl})$. Function $g(y)$ is increasing, convex and $g'(x_{hnl}) = 1$. Therefore, line $y = x$ is a tangent line to $g(y)$ at point $y = x_{hnl}$. Thus for any $y \in (x_{lnl}, x_{hnl})$ value $g(y) < x_{hnl}$. Inequality $g(y) > x_{lnl}$ is trivial.

Therefore, continuation values satisfy $V_{T-i} = g^i(\mu)$. The sequence $g^i(\mu)$ is increasing and clearly bounded. Sequence $g^i(\mu)$ converges if the number of candidates approaches infinity. Because equation $g(y) = y$ has a single root $y = x_{hnl}$ limit equals $\lim\limits_{i \to \infty} g^i(\mu) = x_{hnl}$.

We draw a dynamics of the continuation values for the entropy cost function and a particular set of parameters in Figure 1 and summarize the analysis above in the Proposition below.

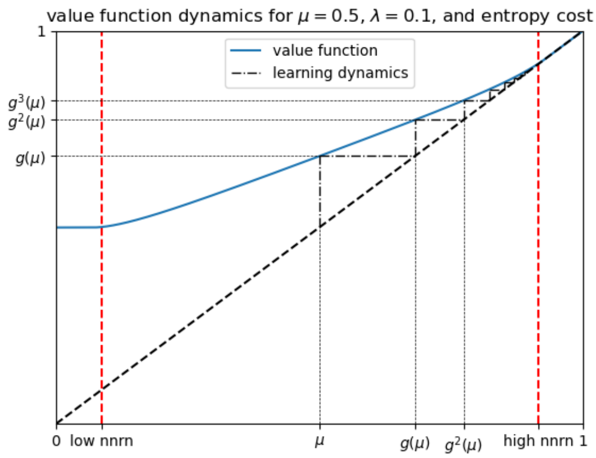

Figure 1: Dynamics of the continuation values

Proposition 1. *For any T, the manager always acquires information during the first interview and chooses distribution with two posterior beliefs x_1^L, x_1^H.*

If during interview i for all $i < T-1$ low posterior belief x_L^i has been realized, manager acquires information during the interview $i+1$ and chooses distribution with two posterior beliefs x_{i+1}^L, x_{i+1}^H.

We characterize and discuss the optimal learning strategy in the next Section.

4 Optimal Interviews

Recall that the manager acquires information during the interview i, and she chooses two posterior beliefs. We apply the binary test interpretation of the interview. Suppose a candidate i fails the test. In that case, the manager gets the expected estimate of the candidate's productivity x_i^L, and if the candidate succeeds, the manager gets an estimate of x_i^H with $x_i^H > x_i^L$.

The usual way to compare such tests is by their informativeness using the Blackwell order. A test i is more informative than j if distribution p_i is the mean-preserved spread of distribution p_j. In the case of binary tests, the condition simplifies into two inequalities $x_i^L \leq x_j^L, x_i^H \geq x_j^H$. The interpretation for such a condition is simply that the test i includes test j. Although the Blackwell ordering has intuitive properties, it is incomplete, and very often, two tests are incomparable in the Blackwell sense. We introduce an alternative order of tests (interviews) that allows comparing them to the current problem.

Definition 4. *We say that an interview i is **more difficult** that an interview j if*

$$x_i^L > x_j^L, x_i^H > x_j^H.$$

The condition above has a simple interpretation. Suppose that the same agent is offered test i and j. We say that test i is more difficult than test j if the manager gets higher estimates about the candidate's productivity given both low and high realizations. Stating differently, if the agent succeeds on the test i, he has relatively high expected productivity, which means he solves a relatively hard problem. On the contrary, if he fails the test j, he has relatively low expected productivity and does not solve a relatively easy problem. It is straightforward from the definition that the probability of success is lower for the harder interview.

Our main result says that the optimal tests decrease in difficulty.

Theorem 1. *The optimal difficulty of informative interviews decreases, that is, for any fixed T*

$$x_i^L, x_i^H \text{ are decreasing in } i.$$

Moreover,

$$\lim_{T\to\infty} x_1^L = \mu, \qquad \lim_{T\to\infty} x_1^H = x',$$

where $x' \in (x_{hnl}, 1]$.

Proof. The proof mostly follows from Lemma 3. For the first part, it is sufficient to show that both optimal posterior beliefs in the static problem (5) increase with respect to the outside option. It follows from the proof of Lemma 3.

If $T \to \infty$ then continuation value V_1 converges to x_{hnl}. In the solution to the problem (5) with an outside option x_{hnl}, DM does not learn because her lower posterior beliefs equal to the prior μ. By the continuity argument, $\lim_{T\to\infty} x_1^L = \mu$ holds.

□

In the Figure below, we show an example of an optimal learning strategy for the entropy cost of learning.

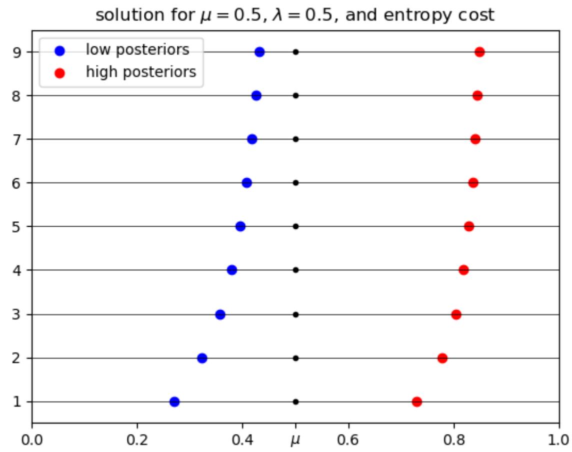

Figure 2: Optimal posterior beliefs given the number of candidates left

Decreasing the high posterior realizations x_i^H is intuitive. If a posterior x_i^H is realized on the interview i, the manager stops the search and chooses candidate i. The manager stops the search earlier if she gets a higher expected estimate about the quality of a candidate.

Decreasing the low posterior realizations x_i^L is rather surprising. By using such a strategy, the manager optimally procrastinates: instead of acquiring the most information during the first interviews, she wants to spread expected information acquisition towards all interviews. Intuitively, during the first interviews, she offered hard tests for the applicants because she had some applicants that were left. The manager wants to bear the risk and try to «catch a big fish» at the beginning. The fewer candidates are left, the safer the strategy used by the manager.

Such interview design is the result of the flexible information technology that is available to the manager. Below, we briefly show why flexible information is useful. We consider restricted a very stylized version of a problem (6). We fix uniform prior belief $\mu = 0.5$ and restrict set Π such that DM can choose only symmetric binary information structures. In particular, binary distribution p with x^L, x^H is feasible only if $\mu - x^L = x^H - \mu$. The cost of such information structure can be summarized in a number ϕ, where $\phi(p) = x^H - \mu$. We denote a cost for ϕ as $\tilde{c}(\phi)$ with standard properties. Because of the symmetry, the expected payoff of the manager in the interview i is given by $0.5(0.5 + \phi) + 0.5V_{T-i} - \tilde{c}(\phi)$. Thus, optimal ϕ is independent from i, and the manager uses the same posterior distribution on all interviews.

To show the role of flexibility, we consider a marginal change for both symmetric posterior beliefs x^L, x^H by $\delta > 0$. The first order effect on the expected rewards equals $p_t^H \Delta x_t^H + \Delta p_t^H V_{T-i} - \Delta p_t^H x_t^H$. Using symmetry and $m = 0.5$ assumptions, the first-order effect can be simplified into $\Delta p_t^H (V_{T-i} - 0.5)$. The last expression is always positive. Therefore, the manager has an incentive to use the flexibility of information, increase both posterior beliefs, and decrease the difficulty of the interview.

5 Discrimination

To study discrimination, we compare the unconditional probabilities of the choice of the manager. The optimal design of an interview generates two opposite effects on probabilities. Because the first interviews are more difficult, they are harder to pass. However, if a candidate passes the interview, he is immediately hired.

We present our results mainly for the case $T = 2$. We assume that function $c(x)$ has an axis of symmetry $x = z$ such that for any x' and x'' such that $x' < z < x''$ and $z - x' = x'' - z$ equality $c(x') = c(x'')$ holds. For example, for the quadratic cost $c(x) = (x - \mu)^2$ the axis of symmetry is $x = \mu$, for the entropy cost $c(x) = -x\log(x) - (1 - x)\log(1 - x)$ the axis of symmetry equals $x = \frac{1}{2}$.

We denote P_2^1 and P_2^2 as unconditional probabilities of choosing the first and the second agent correspondingly. We formulate the discrimination result in terms of sufficient conditions for the cost function $c(x)$.

Proposition 2. *Let $\mu \in (z-\varepsilon, z+\varepsilon)$ for small enough ε. If $\mu = z$ then $P_2^1 = P_2^2$. If $c'(x)$ is linear on $(y', y'+\varepsilon)$ then $P_2^1 = P_2^2$, if $c'(x)$ is concave on $(y', y'+\varepsilon)$ then $P_2^1 < P_2^2$, if $c'(x)$ is convex on $(y', y'+\varepsilon)$ then $P_2^1 > P_2^2$.*

Proof. We first consider a problem with two available candidates that may differ in their expected qualities. We denote μ as the expected quality of the first candidate and y as the expected quality of the second candidate (outside option).

We consider a value $x = z$ such that posterior beliefs x^L, x^H are symmetric around z: $x^H - z = z - x^L$. Therefore, equalities $c'(x^L) + c'(x^H) = 0$ and $c(x^L) = c(x^H)$ hold. From the system of first-order equations, we get that

$$\frac{x^H - z}{x^H - x^L} = \frac{1}{2}.$$

holds. Therefore $x^H - y = y - x^L$ holds and $z = y$ also holds. If $y = \mu$ then $P_2^1 = P_2^2 = \frac{1}{2}$.

Below, we analyze the case with the candidates with the same expected qualities $y = \mu$. We compute the sign of the derivative of the P_2^2:

$$\frac{dP_2^2}{dy} = \left(\frac{x^H - y}{x^H - x^L}\right)' = \frac{((x^H)' - 1)(x^H - x^L) - ((x^H)' - (x^L)')(x^H - y)}{(x^H - x^L)^2}.$$

The sign of the numerator determines the sign of the expression. We rearrange the numerator in the following form:

$$((x^H)' - 1)(x^H - x^L) - ((x^H)' - (x^L)')(x^H - y) = ((x^H)' - 1)(y - x^L) - ((x^L)' - 1)(x^H - y).$$

Because points x^L and x^H are symmetric around z equality $c''(x^L) = c''(x^H)$ holds and, therefore, $(x^H)' = (x^L)'$. Thus, the sign of the expression above is determined from the sign of the expression $(x^H)' - 1$. Using the expression for $(x^H)'$ and for $c'(x^H)$ we write

$$(x^H)' - 1 = \frac{1}{\lambda c''(x^H)(x^H - x^L)} - 1 = 2\frac{c'(x^H)}{c''(x^H)(x^H - x^L)} - 1.$$

The sign of the expression above is determined from the sign of the expression $\frac{c'(x^H)}{c''(x^H)} - \frac{x^H - x^L}{2}$.

There exists a straight line, that connects points $(x^H, c'(x^H)), (z, 0), (x^L, c'(x^L))$. Therefore, if $c'(x)$ is linear, then the expression above equals 0. If function $c'(x)$ is concave on $[z, x^H]$ then the tangent line to $c'(x)$ at point x^H intersects Ox axis below z and, therefore, the expression is positive; if function $c'(x)$ is convex on $[z, x^H]$ then the tangent line to $c'(x)$ at point x^H intersects Ox axis above z and, therefore, the expression is negative.

Thus, if $c'(x)$ is linear then $P_2^1 = P_2^2$, if $c'(x)$ is concave on $(z, z+\varepsilon)$ then $P_2^1 < P_2^2$, if $c'(x)$ is convex on $(z, z+\varepsilon)$ then $P_2^1 > P_2^2$.

□

We discuss the implications of the Proposition 2 for two cost functions that are popular in the literature, quadratic cost and entropy cost.[10] In the case of the quadratic cost of learning, DM does not discriminate between candidates and chooses them uniformly.

However, with entropy cost, the manager behaves differently. Because the third derivative of an entropy is positive for $x > \frac{1}{2}$, the manager is involved in discrimination. She chooses the first candidate more often if the candidates are ex-ante relatively good, $\mu < 0.5$, and the second candidate more often if the candidates are ex-ante relatively bad, $\mu < 0.5$. Such a pattern of behavior is similar to the «cherry-picking» and «lemon-dropping» from Bartoš et al. (2016). In the case of two candidates, the manager acquires information only during the first interview. Such attention discrimination favors the first candidate if his expected productivity is above uniform productivity, and attention discrimination harms him in the other case.

[10]For the reference, see, e.g., Lipnowski et al. (2022) for quadratic cost and Caplin et al. (2019) for entropy cost.

For the case $T > 2$, the ordering of the unconditional choice probabilities can be highly non-trivial. In the Figure below, we present the unconditional probabilities of choice for the entropy cost.

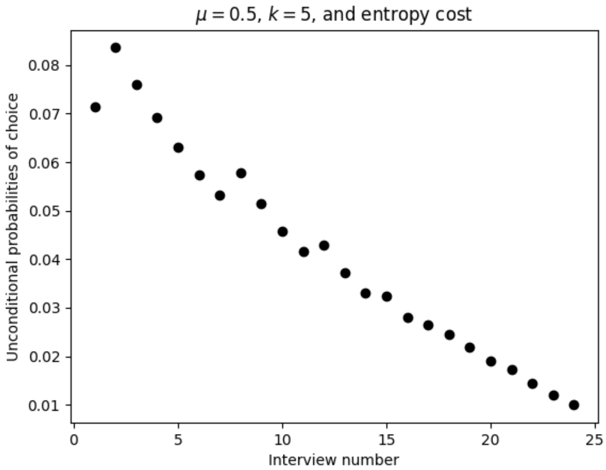

Figure 3: Non-monotone unconditional probabilities

For example, the probability of being hired for the first candidate is lower than for the third candidate. The order is determined by the combination of the curvature of the function $c(x)$ with prior belief μ and marginal cost λ. We leave the investigation of the sufficient conditions for the discrimination when $T > 2$ for future research.

6 Conclusion

As documented in the economic literature, see, e.g., Bertheau et al. (2023), hiring is difficult for firms, and one of the reasons is that the firms face time constraints while hiring candidates. This means that firms do not learn the potential workers' productivities perfectly (since it will take too long time) but instead acquire noisy information about those. In this paper, we model the process of sequential search with costly but flexible learning in each stage.

The hiring firm observes several candidates who arrive sequentially and can design interviews for each candidate individually. We show that the optimal learning strategy has a simple feature – the later the candidate appears (the higher the serial number she has), the easier questions she will be facing. That is, the optimal interviews are decreasing in their difficulty in time. However, it does not mean that the workers should try to be interviewed in the end since the probability of being hired as a function of time of arrival is not necessarily increasing.

Our paper is the first step in studying sequential search with flexible and endogenous information acquisition. Therefore, many research questions are left for the future. For instance, we study only the situation in which the candidates are ex-ante identical, and the order of their arrival is random. The problem of studying a similar problem with a priori heterogeneity in workers' productivities and choice of order of the candidates is interesting and intriguing.

Another suggestion for future research is to consider a model similar to ours but with an opportunity for recall. We suspect that the decreasing difficulty property will remain present in this class of problems.

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