New Venture Financing and Venture Capital ‘Funding Hole’

September 2023

Amiya K Basu[a]

Siddhartha Basu[b]

Sankar De[c]

Abstract

When deciding whether to seek seed financing for a new venture from an angel investor or a venture capitalist (VC), an entrepreneur must consider the challenge of financing the next stage of the venture which is typically much larger than the seed round. Compounding the challenge, the original investors, including the angels in most cases, and the VCs in many cases, do not extend follow-on financing, forcing the entrepreneur to approach a new investor, triggering adverse valuation risk due to asymmetric information. We show, analytically using a model, as well as empirically using new venture financing data that, except when the VC retains a sufficiently high, or a sufficiently low, proportion of seed-stage ventures for continued association including funding, the adverse valuation risk is higher if seed investments are VC-financed and lower if angel-financed. In other words, VC financing and angel financing of seed investments are complimentary choices, not substitutes.¿

Keywords: angel investor, venture capital, seed funding, follow-on financing

JEL Classifications: G23, D82, D86

a Corresponding author. Whitman School of Management, Syracuse University. Email abasu@syr.edu

b Airbnb, San Francisco, California. Email sidbasu@alumni.stanford.edu

c Independent researcher. Email drsankarde@gmail.com

New Venture Financing and Venture Capital 'Funding Hole'

1. Introduction

An entrepreneur's choice of seed financing for a new venture is dominated by consideration for financing required at the next stage of the venture. A couple of salient features of the venture financing market in the USA illustrate the overwhelming importance of second stage financing. For the typical venture, financing needed at the second stage is much larger than seed financing. During our sample period 2010-2019, the median seed financing round in the USA was of the order of 750,000 dollars, while the median second stage financing round, at about 8.7 million dollars, was more than 11 times as large (www.crunchbase.com). Second, a great majority of the seed stage financiers do not fund the second round. Of the two most common sources of new venture financing, namely angel investors and venture capitalists (VCs), angel investors typically do not finance beyond the seed stage and VCs do not fund second rounds for the majority of ventures that they support with seed financing. This situation forces the majority of venture firms to seek second stage financing from new financiers (usually VCs since angels typically do not fund second rounds) who, not having been involved with the ventures since their inception, may not be in a position to evaluate correctly their potential going forward or even their true performance in the first period.

Traditionally, angel investors have been individual investors investing their own money for financial reasons as well as other objectives (for example, breeding the next generation of entrepreneurs). While the size of an average seed investment round is within the reach of many high net-worth investors, typically much larger later rounds are beyond the scope of most individual investors. In recent years angel groups such as Tech Coast Angels or Houston Angel Network that are, as their names indicate, associations of angel investors with similar investment goals and considerable investment funds at their disposal, have entered the field. However, angel firms by and large appear to have remained primarily seed stage investors. Our analysis of the financing activities of the 200 largest US-based angel investors, including angel groups, during 2010 - 2019 indicates that in their case the median rate for second stage financing for the startup firms they support with seed funds is of the order of 7% only.

By contrast, VC firms are financial intermediaries typically organized as partnerships and formed with the express purpose of maximizing returns on their investments. A VC firm generally raises multiple funds. For example Accel Partners, one of the biggest US VC firms, raised \$12.9 billion dollars across 29 funds during 2000 - 2019. However, though VC firms may have the financial resources to fund later rounds following seed investments, they often do not do so. A VC, usually an experienced investor, can effectively determine a venture's potential going forward, specifically if is NPV-positive or not. Thus, the VC may release all bad (NPV-negative) ventures, and offers second stage funding to some of the good ventures in her stable, but not all, depending on her other investment opportunities. In section 6 of this paper we provide empirical support for the view that VCs indeed decline second stage financing to some good projects. It has also been widely noted by practitioners that VCs follow a *"spray and pray approach"* with their seed stage investments, taking on many more early stage investments than they plan to support later (https://www.quora.com/Whats-the-spray-and-pray-pattern-in-the-venture-capital-industry). Our analysis of financing data from Crunchbase for a large sample of VC firms in the USA during 2010-2019 indicates that the median rate for second stage financing for the venture firms they support with seed funds is about 38%.

In section 3 of this paper, we present a formal model of the entrepreneur's choice between angel and VC financing of seed funds while keeping in view requirements of later stage financing. The model incorporates the characteristics of the venture financing market noted in academic as well as professional literatures, including that VCs are more experienced investors than angels and are, therefore, better at determining a venture's potential going forward but also providing appropriate business advice and guidance at the seed stage (Alvarez-Garrido, 2022). We model the former feature by having the VC observe more information than the angel investor and other investors, who can only see a noisy signal of the project quality. We model the VC's beneficial involvement in a venture through an increase in the success probability for VC-funded ventures. The rate at which the original VC retains the seed stage ventures for continued involvement plays a key role in our analysis of the entrepreneurial choice of seed stage financing. We note that the retention rate for a particular VC is visible in practice.

Our model identifies the conditions that make VC financing preferable for seed investments as well as those that make angel financing preferable. If the retention rate is either sufficiently low, or sufficiently high, VC seed financing is a better option than angel seed financing. In the first case, the average quality of the pool of ventures released by the original VC approaches the average quality of the original pool. While the pool of ventures released by the angel is also similar to the original pool since the angel releases nearly all seed stage ventures, the ventures released by the VC have the additional gain of value addition by the original VC at the seed stage. There is no comparable value addition by the angel during the first stage (Hellman and Puri, 2002). In the second case, where the retention rate is very high, a high probability of being retained by the original VC and avoiding asymmetric information costs in dealing with a new investor, in addition to the prospect of continued value-addition through mentorship, may outweigh the expected costs associated with the small probability of being rejected by the original VC and undergoing a high valuation loss. At this rate, VC financing at the seed stage again becomes preferable.

By contrast, when the retention rate is neither very low nor very high, angel financing of seed funds may be a more appealing option than VC financing. We formalize this intuition in two propositions

(Propositions 1 and 2) in Section 3, but sketch the intuition here. As the retention rate by the original VC increases from a very low level (equivalently, if the rejection rate decreases from a very high level), the pool of ventures rejected for further financing by the original VC is likely to have a higher proportion of bad ventures compared to good ventures. In this situation the prospect of value addition in the first period may not sufficiently make up for the loss in valuation for a good venture in the rejected pool due to dilution of quality of the pool in the second period. In fact, the dilution may be so significant at a high enough retention rate by the original VC that the new VC may not be adequately compensated for her risky investment in the venture. Then, even a good venture rejected for second period funding by the original VC may not be able to secure funding from any new investor at all and disappear in a 'funding hole. [1]

The implications of the model discussed above lead to the following two testable predictions:

- The range of values of retention rates that make angel financing preferable is bounded both below and above. Proposition 1 in section 4 defines the lower and upper bounds of retention rate where angel financing is preferable. If this prediction holds up, we should expect to observe empirically a double-peaked distribution of retention rates with the lower and the upper bounds corresponding to the two peaks.

- For retention rates between the two bounds, the firms released by VCs should perform worse in the second period than firms released by angels in terms of recognized performance indicators, such as average number of funding rounds, average amount of funds raised, and the ability to raise funds after release by the original investor.

The predictions hold up in empirical tests. In section 5 of this paper, we present our analysis of financing data for a large sample of US VC firms during 2010 - 2019 from Crunchbase. The sample includes top 200 VC firms by number of investments among those that offered at least one seed financing in the sample period. The data indicate a double-peaked distribution of retention rates by the VCs supporting our model prediction.

From Crunchbase, we also obtain performance data for a large sample of firms released by the top 200 VC firms and the top 200 angel investors in the USA by number of investments. Our analysis of the data shows that firms released by VCs with high retention rates perform worse on several metrics than firms released by angel investors, including funds raisd after release and progression beyond the seed stage. By contrast, firms released by VC firms with low retention rates perform almost as well

[1] The label funding hole is a take on the black hole in astronomy where cosmic constellations supposedly disappear for ever. A "funding hole" is different from a "funding gap" which is used to capture the difference between funds needed for a project and funds available (https://www.smartcapitalmind.com/what-is-a-funding-gap.htm)

as firms released by angels on these metrics. This is consistent with our model prediction that firms released by higher retention rate VCs enter a diluted pool with more bad ventures in relation to good ventures and thus fare worse when raising funds after release. In this analysis, we include the year of seed funding, the size of the seed funding, and prominence of the entrepreneur as control variables to account for selection bias.

Our results, including the possibility of a funding hole, are new to the existing literature, as we discuss in the next section of this paper. The results have interesting implications, including one that appears counter-intuitive. It is in the interest of entrepreneurs seeking VC funding at the seed stage that the VC rejects either a sufficiently high proportion of seed stage ventures for follow-on funding (corresponding to a low retention rate), or a sufficiently small proportion (corresponding to a high retention rate), but not in a proportion in the medium range which will make them undergo valuation loss due to quality dilution in the pool in the second period, even leaving them in danger of slipping into a funding hole. The first implication is certainly counter-intuitive. In a competitive VC market, VCs pay heed to the entrepreneurs' preference. Therefore, empirically we should observe that some VCs offer follow-on financing to a small proportion of their early-stage companies, and some other VCs to a high proportion, but not many VCs in a proportion in the medium range, resulting in a double-peaked distribution of retention rates in practice.

Another important implication is that angel investors and VCs are actually complementary sources of capital in a range of situations. In other words, unlike the conventional academic as well as professional wisdom, they are not always substitutes for each other. When VC funding of seed funds leads to high valuation loss, or even the possibility of a funding hole, in the second period, angel funding may come to the rescue of the concerned ventures.

Finally, our analysis contributes to the understanding of a phenomenon widely noted by finance professionals and labelled a 'Series A crunch,' where a profusion of early stage financing to new ventures has led to a sharp 'cliff' with new financing at the second stage very hard to obtain[2], by showing that success in raising funds at later stages of a project depends on the type of financier that provided seed funding to the project.

The remainder of the paper is organized as follows. Section 2 presents a review of relevant literature. Section 3 presents our model, and Section 4 the implications of the model. Section 5 presents the findings from our data analysis. Section 6 provides robustness checks and addresses alternative explanations of the fndings. Section 7 presents some concluding observations. An appendix at the end presents the proofs of our results and other technical details.

[2]See http://www.cbinsights.com/blog/seed-investing-report for an analysis of this phenomenon.

2. Relevant Literature

The present paper focuses on the choice between angel financing and VC financing of seed investments required for a new project. We examine in particular what role, if any, considerations of follow-on financing play in this choice, given that angels typically do not provide follow-on funding and VCs decline further funding to many ventures they support with seed funding. Our work includes both theoretical and empirical analysis of the decision-making framework of the entrepreneur faced with this choice. The existing literature, empirical as well as theoretical, is relatively sparse in our particular area of interest. In fact we find only one empirical paper (Hellman, Schure and Vo; 2013) and one purely theoretical paper (Kim and Wagman; 2016), and no paper with both empirical and theoretical analysis like ours that examines the implications of follow-on financing for the choice of seed investments.

Hellman, Schure and Vo (2013) analyze whether angel investors and VCs are complements or substitutes. By using time variation in the government of British Columbia's tax credits for entrepreneurial investments as an instrument, the authors find that taking initial funding from a VC makes a venture firm less likely to have angel funding in future, but not the other way around. While this makes sense, both effects can be explained by the fact that angels typically do not provide follow-on financing, leaving VC financing as the only possibility for flow-on funding. Kim and Wagman (2016) develop a two-round theoretical model similar in some respects to ours. In their model an entrepreneur chooses the first round (seed) funding from either an angel who does not provide follow-on funding or a VC who can determine the exact probability of success before the second round and only funds a project if its probability of success exceeds a threshold. Any project not funded by the first round investor is funded by a new VC who only sees the distribution of the success probability of the venture. A project released by a VC gets a lower valuation as the new VC knows that the probability of success is below the cut-off imposed by the original VC. There is a separating equilibrium in their model where the entrepreneur chooses seed funding from an angel or a VC depending on whether he himself knows the probability of success beforehand, and accessibility of VC funding. While this paper does consider implications of follow-on funding, it differs from our paper in two key areas. First, in Kim and Wagman (2016) an incumbent VC never rejects a good project, that is, one that meets its probability threshold. Second, all projects are funded in equilibrium, and there is no funding hole. Our data analysis supports our model's predictions that even good projects may be released and funding holes likely exist.

In a broader context, some existing papers compare different aspects of angel financing and VC financing than we do. Casamatta (2003) presents a model that compares the financing and advisory roles of VCs and other external investors. The paper considers the choice of appropriate financial instruments for compensating angel investors and VCs rather than the choice of angel and VC financing in the first place as we do. The paper also does not consider the implications of follow-on financing.

Chemmanur and Chen (2014) analyze the evolution of financing contracts between VC's, angels and entrepreneurs. The primary frictions that they consider are the fact that VCs can exert efforts to increase a venture's probability of success, and that VC financing is scarce relative to angel financing at the seed stage. Their main finding is that, while the optimal contract in both early and late stage VC financing appears to be similar to convertible preferred equity (with downside protection and upside gains), the relative magnitudes of the upside and downside components change over time. Further, they find that contracts with angel investors are less likely to incorporate such features. While our model also incorporates the VC's value additive role, a fundamental difference between angels and VCs in that angels typically do not have the liquidity to fund follow on rounds for a seed project plays an important role in our model. Chemmanur and Chen in fact predict that in some circumstances, previously VC-backed projects will be entirely financed by angels in a later round. As we have noted above, venture financing data do not support this prediction.

Some existing papers compare VC financing with other financing sources, including traditional sources of financing such as bank financing. Ueda (2004) and Farboodi (2013) consider value addition by the VC due to better industry knowledge. However, they consider different frictions that may make bank financing preferable at the margin, such as risk of appropriation of the project idea by the VC (Ueda) or private benefits of control to the entrepreneur (Faboodi). Bettignies and Brander (2007) present a two-sided moral hazard problem where the VC and the entrepreneur provide unverified effort and conclude that VC financing is preferred when VC productivity is high. Winton and Yerrimilli (2008) examine the effect of the venture capitalist's cost of capital on the choice between bank and VC. The papers do not consider implications of follow on financing and have a different scope from ours.

3. Model

3.1 Setting

We model a penniless entrepreneur who seeks financing for a new project. There are two periods (1 and 2) and three dates in our model: 0, 1, and 2. The entrepreneur seeks seed funding I_0 at date 0 and follow-up funding I_1, usually much larger than I_0, at date 1. If not funded at either date, the project is terminated with zero payoff. The project's payout, either V (success) or 0 (failure), is realized at date 2. Even though V is assumed same, the probability of success and hence expected payoff varies across projects.

We assume that all projects are identical at date 0 to both the entrepreneur and investors. The project's type, defined by probability of success, is revealed at date 1, when the project can have either positive or zero expected value. Some information about the value of the project can be public (hard information such as revenue growth), but other information necessary for valuation is available only to an experienced incumbent investor (soft information) at date 1. The project can be at one of four states at date 1: superior (S), good (G), mediocre (M), or bad (B), with date 1 probabilities of success $p_s > p_g > p_m > p_b = 0$. Any investor, incumbent or external, can correctly identify the S and B states without additional information. B projects, with zero probability of success, are never funded.

As all projects are identical at date 0, either all projects are funded, or no project is funded, at date 0. As discussed later, in our model the presence of S projects ensures that all projects receive seed funding at date 0. G projects are NPV positive and M projects are NPV negative at date 1. An external investor cannot distinguish between G and M projects, but can determine the probabilities of the two states.

Under angel financing, we denote the probabilities at date 0 of reaching the states S, G, M, and B by q_s, q_g, q_m, and q_b, respectively, where $q_s + q_g + q_m + q_b = 1$. In our framework as well as in practice, there are a few key differences between angel financing and VC financing. First, a VC adds value to a project financed by her with suitable mentorship. We model the difference in value addition to the project between the VC and the angel with a multiplier μ, where $\mu \geq 1$, on the date 0 probabilities of states G or M when initially funded by a VC and letting q_b decline appropriately. If there is no value addition, we can just set the multiplier equal to one.

Second, though the entrepreneur can obtain I_0 from an angel investor or a VC, the angel investor is liquidity constrained and can only provide I_0 to the project, but not I_1. Thus, under angel financing, the entrepreneur must obtain financing from a different source (a new VC) at date 1 if he wishes to continue the project.[3]

However, if the entrepreneur obtains I_0 from a VC, then he may or may not receive I_1 from the same VC. The VC, an experienced investor, precisely knows the state of the project at date 1. She releases all M and B projects as they have negative NPV, and funds all S projects since they have a high probability of success. She provides follow-on funding to a G project with probability R $(0 < R < 1)$ and releases one with probability $(1 - R)$.

Projects released by the original VC attempt to raise financing from a new VC. B projects are never financed since their state is visible. However G and M projects are pooled since the new VC cannot distinguish between them. Therefore, the released G projects get worse financing terms, and might not even be financed due to informational asymmetry.

Summarizing, the entrepreneur has three possible financing paths:

Financing Path 1. Receive seed funding from angel, and seek funding at date 1 from a new VC.

[3] We break any indifference by assuming that the entrepreneur continues with the project. This can be modeled explicitly with a private benefit of control (as in Farboodi (2013)).

Financing Path 2. Receive seed funding from VC, and get funding from the original VC at date 1.

Financing Path 3. Receive seed funding from VC, be released by the original VC, and seek funding from a new VC at date 1.

Figure 1 presents a visual representation of the angel financing and VC financing systems.

Figure 1 about here

In financing paths 1 and 3, we assume that the new VC observes a noisy signal of the project's state before deciding whether to invest or not. The signal takes two values, high (H) and low (L), where the 'correct' signal is observed with probability λ. Since the S and B projects are always visible to an outside investor, the new VC does not observe a signal in their case. Thus we have:

$P(H|G) = \lambda$, $P(H|M) = 1 - \lambda$

$P(L|M) = \lambda$, $P(L|G) = 1 - \lambda$

The precision of the signal, λ, determines whether the new VC funds a project at date 1. If $\lambda = 0.5$, the posterior probabilities of G and M are same as the prior probabilities. If λ is 1, then the new VC has perfect ability to distinguish between G and M projects. Thus we assume that $.5 < \lambda < 1$. As λ increases, the new VC can identify the true state of the project more precisely.

Funding Costs and Viable Paths: The financing paths outlined above can be subdivided into alternative paths the project may proceed along. For example, within financing path 1, there are four possible paths depending on which of the four states the project reaches at date 1. In each path, there is a funding cost at each date, denoted F_0 at date 0 and F_1 and date 1, that the entrepreneur must pay the investor if successful at date 2. (As V is same for all projects, this is equivalent to offering the investor(s) equity shares α_0 and α_1 as repayment for investment.)

We call a path "viable" if the project is funded at both date 0 and date 1 along the path. We now make three assumptions to define the funding costs and the viability of a project.

Assumption 1: At both date 0 and date 1, financial markets are competitive, with zero expected profit for the financier.

Assumption 1 implies that

(1) $-I_0 + P(\text{success at date 0})F_0 = 0$, that is, $F_0 = \dfrac{I_0}{P(\text{success at date 0})}$

(2) $-I_1 + P(\text{success at date 1})F_1 = 0$, that is, $F_1 = \dfrac{I_1}{P(\text{success at date 1})}$

Under Assumption 1, an incumbent VC charges a G project it retains at date 1 a funding cost of $F_1 = \dfrac{I_1}{p_g}$, which is lower than what the entrepreneur can receive from an external investor who sees a mixed pool of G and M projects. Thus, we do not allow the possibility that the incumbent investor exploits the entrepreneur by only matching the funding cost offered by the external market. There are two reasons for this assumption. First, exploitation can damage the reputation of the VC and make her a less attractive funding source in the future. Second, in a competitive VC market with zero expected profit, a positive expected profit at date 1 means the VC must have a negative expected profit at date 0 by offering a discounted funding cost. It seems counterintuitive to entice entrepreneurs with discounts only to exploit them later. We analyzed the case of exploitation for both competitive and non-competitive VC markets. Results regarding the entrepreneur's choice of seed funding are similar to what we present here and are available upon request.

The funding costs F_0 and F_1 depend on the path the project proceeds along. Clearly, the entrepreneur can repay the investors fully upon success if and only if $F_0 + F_1 \leq V$.

Assumption 2: A path is viable, that is, the project is funded at both date 0 and date 1 in the path, if and only if $F_0 + F_1 \leq V$, that is, if successful at date 2, the entrepreneur can fully repay the date 0 and date 1 investors.

For example, no path through state B at date 1, with infinitely large F_1, is viable. To complete the model, we make the following assumption:

Assumption 3: $p_m I_1 > V$, and $V - \dfrac{I_0}{p_s q_s} - \dfrac{I_1}{p_g} > 0$.

The first part of Assumption 3 implies that if the investor knows that a project is in state M at date 1, she will not fund the project because $F_1 = \dfrac{I_1}{p_m} > V$, and hence $F_0 + F_1 > V$ for the path. Regarding the second part of assumption 3, note that at date 0, the probability of success at date 2 is the sum of probabilities of success through all the viable paths. If an S project is funded at date 1, the date 0 probability of success along the path through state S is $p_s q_s$, that is, the probability of success at date 0 is at least $p_s q_s$. Hence, $F_0 \leq \dfrac{I_0}{p_s q_s}$. Since $p_s > p_g$, for an S project $F_1 = \dfrac{I_1}{p_s} < \dfrac{I_1}{p_g}$. Combining, assumption 3 implies that $F_0 + F_1 < V$ for the path through state S at date 1, that is, this path is viable. Since there is at least one viable path to success, all projects are funded at date 0. Intuitively, we assume that there is a sufficiently large probability that a project will be a superior project to attract investors to the market.

Consider now the case where the date 1 investor correctly identifies a G project, that is, $F_1 = \dfrac{I_1}{p_g}$. Since $F_0 \leq \dfrac{I_0}{p_s q_s}$, it follows from Assumption 3 that $F_0 + F_1 < V$ here, that is the G project is also funded if it is correctly identified.

3.2 Funding Costs and Entrepreneur Outcomes

We now describe the scenarios that may arise in the different financing paths and provide the funding costs and the expected outcomes of the entrepreneur. In any scenario, we can show (see Appendix 1.1) that the ex ante expected outcome of the entrepreneur is

(3) $\Pi = P(\text{success at date } 0) * V - I_0 - (\text{Probability at date 0 that project is funded at date 1}) * I_1$

The three financing paths are examined next. We only sketch the derivations here, details are provided in Appendix 1.1.

Financing path 1

In this path, the project receives seed funding from an angel investor at date 0 and seeks funding from a VC at date 1, since angels do not fund follow-on rounds. At date 1, the new VC always funds S projects with $F_1 = \dfrac{I_1}{p_s}$, and never funds B projects. For G and M projects, she observes a signal H or L. Using Bayes rule, the conditional probabilities of success given low and high signals are:

(4) $P(success|H) = \dfrac{\lambda p_g q_g + (1-\lambda)p_m q_m}{\lambda q_g + (1-\lambda)q_m}$, and $P(success|L) = \dfrac{(1-\lambda)p_g q_g + \lambda p_m q_m}{(1-\lambda)q_g + \lambda q_m}$

This makes the investor's break even repayments:

(5) $F_{1H}^A = \dfrac{I_1[\lambda q_g + (1-\lambda)q_m]}{\lambda p_g q_q + (1-\lambda)p_m q_m}$, and $F_{1L}^A = \dfrac{I_1[(1-\lambda)q_g + \lambda q_m]}{(1-\lambda)p_g q_q + \lambda p_m q_m}$

The superscript A means angel financing and the subscripts refer to signals H and L. If $\lambda = .5$, the signal is not informative and the posterior probabilities of states G and M are same as the prior probabilities. Then, $F_{1H}^A = F_{1L}^A$. As signal precision (λ) increases from 0.5, the probability of success given the H signal increases while the probability of success given the L signal decreases, that is, $F_{1H}^A < F_{1L}^A$ if $\lambda > .5$. Thus, if the L signal is funded, the H signal is also funded. Three cases are possible depending on the behavior of the new VC.

Case 1A: The VC only funds S projects for which the state is visible. She does not consider B projects which have negative NPV for certain. She also does not invest even if H signal is observed for the other projects which are either G or M. In this case, the probability of success at date 0 is $p_s q_s$, that is, the angel requires a break even payment of

(6) $F_0^{A,S}(I_0) = \dfrac{I_0}{p_s q_s}$

Hence the expected outcome of the entrepreneur at date 0 is

(7) $\Pi^{A,S} = p_s q_s(V - F_0 - F_1) = q_s(p_s V - I_1) - I_0$

Case 1B: The new VC uses the signal as separation, and chooses to fund the H signals, but not the L ones. If a project is in state S at date 1, its probability of success is p_s. On the other hand, if the project is in state G or M, then the probability of H signal is $\lambda q_g + (1-\lambda)q_m$. Therefore the probability of success given the date 0 information set is:

(8) $p_s q_s + P(success|H)P(H) \quad = \quad p_s q_s + \lambda p_g q_g + (1-\lambda)p_m q_m$

Hence,

(9) $F_0^{A,H}(I_0) = \dfrac{I_0}{p_s q_s + \lambda p_g q_g + (1-\lambda) p_m q_m}$

where superscript A means angel funding and H means H but not L is funded. The expected outcome of the entrepreneur Π in this case is given by

(10) $\Pi^{A,H} = (p_s q_s + \lambda p_g q_g + (1-\lambda) p_m q_m)V - I_0 - (q_s + \lambda q_g + (1-\lambda) q_m) I_1,$

which can be restated as

(11) $\Pi^{A,H} = (p_s V - I_1) q_s + \lambda (p_g V - I_1) q_g - (1-\lambda)(I_1 - p_m V) q_m - I_0$

The expression for the expected outcome now depends on the precision of the signal λ. Only a fraction λ of the G projects will receive funding by obtaining a high signal. On the other hand, a fraction $1-\lambda$ of the M projects can trick the new VC into investing by getting the high signal. However, if the precision of the signal is not too low, in other words if the signal accomplishes reasonable though not complete separation of types, prevention of loss from too many M projects receiving funding overweighs the probable loss from not funding G projects that receive L signal.

Case 1C: The new VC funds S, H and L projects, making appropriate changes in F_1. In this case, all projects that reach states S, G and M are funded at date 1. Hence, the probability of success at date 0 is $(p_s q_s + p_g q_g + p_m q_m)$, that is,

(11) $F_0^{A,HL} = \dfrac{I_0}{p_s q_s + p_g q_g + p_m q_m}$

where HL means H and L are both funded. Then, the entrepreneur's expected outcome at date 0 is

(12) $\Pi^{A,HL} = q_s(p_s V - I_1) + q_g(p_g V - I_1) - q_m(I_1 - p_m V) - I_0$

Case 1A (only S funded) arises if $V - F_0^{A,H} - F_{1H}^{A} < 0$, and Case 1C ($S$, H and L all funded) arises if $V - F_0^{A,HL} - F_{1L}^{A} \geq 0$. In all other cases, we have Case 1B where S and H are funded but L is not funded. In Case 1C, the signal λ is not precise enough for the VC to eliminate the L signal. In Appendix 1.2, we identify a threshold such that the new VC does not fund the L signal under both angel and VC financing once λ exceeds that threshold.

Financing paths 2 and 3

When the entrepreneur takes seed financing from a VC, the VC perfectly observes the state of the project at date 1. If the project is in M state, the original VC always releases the project because the expected NPV is negative. If the project is in G state, the VC continues funding the project with probability R. In this situation, an entrepreneur with a good project has two possible paths to success. If the original VC retains the project at date 1 (financing path 2), she offers the entrepreneur a fair contract, that is, requires a repayment $F_1(I_1) = \dfrac{I_1}{p_g}$.

The other path to success is funding by a new VC if released by the original VC (financing path 3). The new VC observes a signal of the project's state, and then either offers the entrepreneur a break-even funding contract based on this signal, or declines to fund the project. The new VC computes the probability of success with the knowledge that a fraction $(1 - R)$ of the G projects and all M projects that are initially VC financed are now in her pool. Using Bayes rule and proceeding as with angel financing, at date 1 the new VC requires repayments

(13) $F_{1H}^{V} = \dfrac{I_1[\lambda(1-R)q_g + (1-\lambda)q_m]}{\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m}$ and $F_{1L}^{V} = \dfrac{I_1[(1-\lambda)(1-R)q_g + \lambda q_m]}{(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m}$

As with angel financing, $F_{1H}^{V} = F_{1L}^{V}$ if $\lambda = .5$ and $F_{1H}^{V} < F_{1L}^{V}$ if $\lambda > .5$. Thus, once again, the H signal is funded if the L signal is funded.

As in financing path 1, the probability of success and funding requirement at date 0 depends on the choice of the new VC at date 1 and three cases are possible, parallel to the three cases for angel seed financing.

Case 3A. The new VC does not fund either the H or the L signal. Hence,

(14) $F_0^{V,S} = \dfrac{I_0}{p_s q_s + \mu R p_g q_g}$,

and the entrepreneur's ex ante expected outcome at date 0 is:

(15) $\Pi^{V,S} = q_s(p_s V - I_1) + \mu R q_g(p_g V - I_1) - I_0$

Case 3B. The new VC funds H but not L projects. Here,

(16) $F_0^{V,H} = \dfrac{I_0}{p_s q_s + \mu[\{R + \lambda(1-R)\}p_g q_g + (1-\lambda)p_m q_m]}$

where V means VC financing and H means H but not L is funded. The entrepreneur's ex-ante expected outcome is:

(18) $\Pi^{V,H} = q_s(p_s V - I_1) + \mu q_g\{R + \lambda(1-R)\}(p_g V - I_1) - \mu q_m(1-\lambda)(I_1 - p_m V) - I_0$

Case 3C. The new VC funds both H and L signals. Thus, all projects that reach states S, G and M at date 1 are funded. Here,

(19) $F_0^{V,HL} = \dfrac{I_0}{p_s q_s + \mu(p_g q_g + p_m q_m)}$,

where V means VC seed financing at HL means both H and L signals are funded. The entrepreneur's ex-ante expected outcome is

(20) $\Pi^{V,HL} = q_s(p_s V - I_1) + \mu q_g(p_g V - I_1) - \mu q_m(I_1 - p_m V) - I_0$

4. Implications and Propositions

4.1 VC retention rate and funding hole

We now examine how the parameter R, the fraction of good projects retained by the original VC for second stage funding, affects the expected outcome of the entrepreneur and informs the decision to seek seed funding from an angel or a VC. If $R = 0$, a comparison of (5) and (13) shows that the date 1 funding cost for either signal (F_{1H} or F_{1L}) is same for angel and VC seed financing. Also, if $R = 0$, a comparison of F_0 under parallel conditions for angel seed financing and VC seed financing shows that F_0 is same in both cases if $\mu = 1$, and F_0 is lower for VC seed financing if $\mu > 1$, as the project is more likely to succeed ex ante given the boost from the original VC. Thus, if the new VC funds either signal under angel seed financing, she will also fund the same signal under VC seed financing if $R = 0$. Intuitively, if the original VC does not follow on any of her seed investments, all G projects are available to the new VC, and there is no dilution.

As R increases from 0, the pool of projects left for the new VC becomes worse, and G projects in the pool face tougher financing conditions. When R approaches 1, almost every project in the pool available to the new VC is an M project. Thus, when R is very high, the new VC does not fund either signal. We present the results from our model and the intuition underlying them in the rest of this section. The formal proofs of the results are provided in Appendices 1.2 and 1.3.

The effect of R on funding costs and the expected outcome of the entrepreneur is moderated by λ, the precision of the signal observed by the new VC. If λ is higher, the new VC is better able to discern the true state of the project, somewhat mitigating the effect of pool dilution from a larger R. In the extreme case of $\lambda = 1$, there is no information asymmetry and all good projects are funded by the new VC. At the other extreme, if the signal is not informative, the new VC either funds all G and M projects (no funding hole), or no G or M projects at all, leaving all good projects released by the original VC in the funding hole. While these extreme cases are theoretically possible, they should not occur too often. In order to identify the level of signal precision to examine, we are guided by two results. First, we show that for both angel and VC seed financing, the new VC does not fund the L signal if λ exceeds threshold levels that are less than or equal to

$$(22)\ \lambda_L = \frac{(p_g V - I_1)q_g}{(p_g V - I_1)q_g + (I_1 - p_m V)q_m}$$

Thus, the new VC finds it easier to deny funding to the L signal if M projects are more likely to occur or are more NPV-negative. We also show that under general conditions, easily satisfied if $I_1 \gg I_0$, the H signal is funded under angel seed financing and under VC seed financing when R is low. We treat this case, where the L signal is never funded and the H signal is funded at least when R is low, as our baseline scenario.

We start with the case of VC seed financing where the new VC has a signal precision high enough that she never funds the L signal and funds the H signal when $R = 0$. As R increases, the pool available to the new VC becomes progressively worse with fewer G projects until the new VC stops funding H projects also. We formalize this intuition in Proposition 1.

Proposition 1: Consider VC seed financing where $\lambda < 1$ is large enough that the new VC never funds the L signal, and funds the H signal when $R = 0$. Then, there is a threshold $\overline{R}$, $0 \leq \overline{R} < 1$, such that the new VC funds the H signal if $R \leq \overline{R}$ but does not fund the H signal if $R > \overline{R}$.

The proof of Proposition 1 is presented in Appendix 1.3. By assumption, for the H signal, $(F_0 + F_1) \leq V$ if $R = 0$. We show that $(F_0 + F_1)$ is a strictly convex function of R and that $F_0 + F_1 > V$ when R is large. Strict convexity implies that the graph of $(F_0 + F_1)$ crosses the level V upwards at a single R, which is $\overline{R}$.

Proposition 1 establishes that if R exceeds a threshold, any G project released by the original VC is not funded again, that is, falls into a funding hole. Thus, external VC financing is not a viable option for the entrepreneurs who opt for VC financing at date 0 for any R greater than $\overline{R}$. However, while the projects that are dropped by their original VCs are in a worse shape when R increases, the projects that do receive second stage funding from the original VC has the advantage over a project with angel seed funding that the original VC knows the true state of the project at date 1. So, there is no signalling risk, which reduces funding cost at date 1. This raises the question, can angel seed financing ever be a better option than VC seed financing, given that the project will receive funding from the original VC at date 1 with probability R ? If yes, for what range of R does angel financing dominate VC financing? Proposition 2 answers the question.

Proposition 2. Suppose $\lambda < 1$ is large enough that new VC never funds the L signal and funds the H signal for angel seed financing. Let $\overline{R}$ be as defined in Proposition 1. Then,

(1) Under VC seed financing, the new VC funds the H signal when $R = 0$.

(2) The ex ante expected outcome of the entrepreneur is higher under angel financing than under VC financing if

(23) $\overline{R} < R < \tilde{R} = \dfrac{\lambda q_g(p_g V - I_1) - (1-\lambda) q_m (I_1 - p_m V)}{\mu q_g (p_g V - I_1)}$

(3) The ex ante expected outcome of the entrepreneur is higher under VC financing than under angel financing if $R \leq \overline{R}$ or $R > \tilde{R}$.

The proof of Proposition 2 is provided in Appendix 1.3. The numerator in the expression for $\tilde{R}$ indicates the marginal value from angel financing if the project gets an H signal, while the denominator indicates the marginal value if the project is retained by the original VC. The proposition shows that if R is sufficiently high, then the prospect of value addition and absence of signalling risk if retained by the original VC outweighs the risk of being dropped by the original VC and passing into the funding hole with no follow-on financing from any source. In other words, the zone where angel seed financing is preferable to VC seed financing is bounded below by $\overline{R}$ and above by $\tilde{R}$, in a range where R is neither sufficiently low nor sufficiently high.

From Proposition 2, the entrepreneur can have higher expected payoff under angel financing if $\overline{R} < \tilde{R}$. If signal precision (λ) is too high, $\overline{R}$ is also high as the new VC can separate good from mediocre projects more easily, and we may have $\overline{R} \geq \tilde{R}$. Then, there is no zone of R where angel financing brings higher expected payoff. Thus, Proposition 2 implies that the expected ex ante payoff of the entrepreneur can be higher under angel financing only when λ is high enough that H signals are funded under angel financing, but not too high. In Section 4.2, we explore this issue further using a numerical example.

4.2 Signal Precision and Funding Hole

Consider VC seed financing where signal precision is high enough that the L signal is never funded and the H signal is funded if R is low. We established that if R exceeds a threshold $\overline{R}$, a good project released by the original VC falls in the funding hole and is not funded again. We now present results of comparative statics and a numerical example to show how $\overline{R}$ depends on some model parameters.

Result: Suppose, for VC financing, λ is large enough that the new VC never funds the L signal, and funds the H signal if $R = 0$. If $\overline{R} > 0$, then $\dfrac{d\overline{R}}{d\mu} > 0$. If, in addition,

(24) $p_m q_m I_0 \leq (1 - \overline{R})[\mu(p_g - p_m) q_g q_m I_1 + p_g q_g I_0]$,

then $\dfrac{d\overline{R}}{d\lambda} > 0$.

Intuitively, when the original VC boosts probability of success by mentoring, the ex ante probability of success at date 0 increases, which reduces the date 0 funding cost F_0. Hence, when μ is higher, the new VC is willing to invest over a larger range of R, raising $\overline{R}$.

If signal precision (λ) increases, the date 1 funding cost F_1 decreases as the H signal is more likely to imply a good project. However, a larger λ may increase F_0 by eliminating a path to success. Condition (24), which is easily satisfied if $I_1 \gg I_0$, is a sufficient condition that ensures that the total funding cost $(F_0 + F_1)$ strictly decreases with λ. Thus, if λ increases, the threshold $\overline{R}$ also increases. Therefore, an increase in λ or μ reduces funding cost and compensates for the effect of dilution from a larger R.

From (23), $\tilde{R}$ also increases with λ. This happens because the new VC can identify G projects more accurately, reducing funding cost and thus increasing the ex ante expected outcome of the entrepreneur under angel financing. Thus, the retention rate of the original VC under VC financing must be higher to match the increase in expected outcome under angel financing. Note that the zone between $\overline{R}$ and

$\tilde{R}$, where angel seed financing is preferable, only exists if $\overline{R} < \tilde{R}$.

Numerical Example: We now present a numerical example to show how the parameters λ and μ affect the funding hole and the ex ante expected outcomes with angel financing (path 1) and VC financing (paths 2 and 3) of seed funds. We use the following baseline parameter values:

p_s	q_s	p_g	q_g	p_m	q_m	V	I_0	I_1	μ
1	.2	.4	.3	.1	.3	35	1	10	1

With the baseline parameters, the new VC does not fund the H or the L signal in path 1 if $\lambda < .721$, and only the H signal if $\lambda \geq .721$. We denote this value of λ by λ_1. $\tilde{R} = .2433$ if $\lambda = .721$ and it increases linearly as λ increases from .721. Consider VC financing when λ is just above .721. Since $\mu = 1$, F_0 and F_1 under VC financing when $R = 0$ are same as F_0 and F_1 under angel financing, that is, the H signal is funded here. However, if R increases from zero, $(F_0 + F_1)$ soon exceeds V and H signals are no longer funded, that is, $\overline{R}$ is close to zero here (for example, $\overline{R} = .0043$ if $\lambda = 0.722$). Since $\overline{R} < \tilde{R}$, we have a zone of R where angel financing is preferable. Numerically, the expected outcome in path 1 is 7.44% higher when R just exceeds $\overline{R}$.

As λ increases from .721, the gap between $\overline{R}$ and $\tilde{R}$ decreases, and the fractional difference in expected outcomes also decreases until both gaps become zero at $\lambda = .945$, which we call λ_2. For higher levels of λ, path 2 provides higher expected outcome. Figure 2 plots $\overline{R}$ and $\tilde{R}$ against λ for the baseline parameter values. Note that a zone where expected payoff under angel financing is higher than expected payoff under VC financing only exists if λ is high enough that H signals are funded under angel financing but not so high that $\overline{R} \geq \tilde{R}$.

Figure 2 about here

Starting from the baseline parameters, we use five additional levels of μ: 1.02, 1.04, 1.06, 1.08 and 1.1. Table 1 summarizes the results of this example. From Table 1, it can be seen that if μ increases from the baseline level, the range of R over which angel financing (path 1) is preferable to VC financing (path 2) shrinks, and the level of λ where $\overline{R}$ becomes equal to $\tilde{R}$ decreases. For each case, Table 1 also provides $\overline{R}$, $\tilde{R}$, the expected outcomes for angel and VC financing when R is just above $\overline{R}$, and the % difference in expected outcome for angel and VC financing when λ is the average of λ_1 and λ_2.

Table 1 about here

Figure 3 provides $\bar{R}$ and $\tilde{R}$ for the parameter values given above when $\lambda = 0.8$. As R increases, the entrepreneur's ex ante expected outcome under VC seed funding increases initially. However, when $\bar{R}$ is hit, there is a sharp drop in expected outcome since now there are many G projects that are not funded at date 1. Above $\bar{R}$, the expected outcome increases more rapidly than before. This happens because the G projects can only be funded by the original VC and are no longer pooled with mediocre projects. $\tilde{R}$ is the point where the red line and the blue line intersect for a second time. From there, the expected outcome keeps on increasing in R until it hits the maximal level of $R = 1$.

Figure 3 about here

5. Data and Evidence

In this section we verify our theoretical analysis in the preceding sections with the help of new venture financing data from Crunchbase (www.crunchbase.com). We look for evidence in data for a double-peaked pattern in retention rates as predicted by our model. In particular, we wish to verify with data the basic premise of our model that as a VC provides follow-on funding to only a fraction of good prospects for further financing, a good prospect released by a VC enters a diluted pool of projects with a greater proportion of mediocre prospects, compared to a good prospect released by an angel investor who releases all good projects. Consequently, a firm released by a VC should find it more difficult to raise funds after release than a firm released by an angel investor.

5.1 Sample

1/1/2010 is the starting point of our sample period for seed funding in order to avoid the recession of 2008-2009, and the end point is 12/31/2015 to allow sufficient time for follow-on funding. Any follow-on funding occurring between 1/1/2010 and 12/31/2019 is included. The data indicate that during our sample period (1/1/2010 - 12/31/2015), there were a total of 16487 cases of seed funding by investors located in the USA, with a median investment of \$500,000. In the same period there were 9953 cases of early stage venture funding (series A and series B) by US investors, with a median investment of \$7,000,000. For the purpose of the present exercise, we consider seed financing data in Crunchbase for the largest 200 VC firms and the largest 200 angels or angel groups based in the USA by the total number of investments. Out of the 200 VC firms, we included 169 firms in our final sample that made at least one seed investment in our sample period. The maximum number of seed investments that a firm in the sample made was 790, while the minimum was 1 which is the cut-off number for the sample. In aggregate, the 169 sample firms made 6640 seed investments and followed up with series A or higher series funding in 1804 cases, for a 27.17% aggregate retention rate. Considering the firms individually, the median number of seed fundings was 19 and the median retention rate was 38.10%. Figure 4 presents a histogram of the distribution of seed fundings, and Figure 5 presents a histogram of retention rates of the VC firms. Note that there is evidence of a double-peaked pattern in retention rates as predicted by our model with a first peak is at 0.333 and a second peak is at 0.50.

Figures 4 and 5 about here

194 out of the 200 angel or angel groups that made at least one seed funding in our sample period are included in our analysis. The maximum number of seed investments for a firm in the sample was 118, while the minimum of course was 1. In aggregate, they made 3455 seed investments and followed up with series A or higher funding in 358 cases, for a 10.36% aggregate retention rate. Considered individually, the median retention rate for firms in this sample was only 7.11%. Figure 6 presents a histogram of the distribution of the number of seed investments by this group. Figure 7 presents the histogram of the retention rates of these 194 investors and indicates that there was no peak in the distribution. Table 2 presents the summary statistics for the samples discussed above.

Table 2 and Figures 6 and 7 about here

5.2 Testing the model's premise

According to our model, a firm released by a VC should find it more difficult to raise funds after release than a similar firm released by an angel investor, and this difficulty should increase with an increase in R, the fraction of good projects retained by the VC. To test this premise, we examine samples of firms released by the original investors. We start by dividing the investors that provided at least one seed funding between 2010 and 2015 into five categories: angel investors, and four quartiles of VC investors grouped by retention rates:

Category	Number of Investors	Retention Rate	Average Retention Rate	Number of Unique Firms Released
Angel	194		11.94%	2048
VC 1st quartile (Q1)	42	$\leq$ 23.57%	11.31%	2652
VC 2nd quartile (Q2)	42	24% to 37.84%	30.88%	891
VC 3rd quartile (Q3)	42	38% to 54.55%	46.15%	523
VC 4th quartile (Q4)	43	$\geq$ 54.92%	74.72%	236

While we cannot directly measure R, we posit that venture capitalists with larger retention rates should have larger values of R also, and we later present evidence to support this position. The average retention rate for Q1 investors is similar to that of angel investors. Thus, according to our theory, firms released by Q1 should perform similarly to firms released by angels while firms released by Q2, Q3 and Q4 should perform progressively worse. We use the following sample.

- A random sample of 800 firms released by angel investors.

- A random sample of 800 firms released by Q1 investors.

- Of the 891 firms released by Q2 investors, Crunchbase provides data for 884 firms. All are included.
- Of the 523 firms released by Q3 investors, Crunchbase provides data for 517 firms. All are included.
- Of the 236 firms released by Q4 investors, Crunchbase provides data for 234 firms. All are included.

There were some overlaps as some firms were released by different investor categories. Removing these firms, we obtain a sample of 2123 firms (566 angel, 638 Q1, 521 Q2, 272 Q3, and 126 Q4). For this sample, the following information was obtained from the Crunchbase database in September 2022.

- Number of funding rounds (NFRound). (A larger number of rounds should indicate greater success in securing financing.)
- Seed funding from original investor (Seed), available for 1448 out of the 2123 firms in the sample (68.21%).
- Total amount of funds raised (Fund), available for 1796 out of the 2123 firms (84.60%) in our sample. Of the 327 firms with Fund missing, Seed was also missing in 326 cases.
- Crunchbase Rank (CB). This rank is "determined by an algorithm that takes into account the number of connections of a profile within the platform, the amount of community engagement, funding events, news articles, acquisitions, and more." (https://about.crunchbase.com/blog/influential-companies/, October, 2022). A smaller value of CB implies greater prominence. For our sample, the correlation of CB and Fund is -0.089 ($P < .0001$), suggesting that CB is largely determined by factors other than funds raised.
- Closed: Crunchbase lists each firm as "active" or "closed." We code the variable "Closed" as 1 if the firm is closed and 0 if it is active. (In our sample, Closed is not related to whether a firm underwent merger and acquisition as the cross tabulation of the two variables gives $\chi^2 = 0.110$ at $df = 1$ ($P = .740$).)
- Raisedzero and Raisedafter: For the firms for which seed funding from original investor and total funds raised were both available (1447 cases, 68.16% of the sample), we defined Raisedzero as 1 where Fund = Seed, that is, no funds were raised after release, and Raisedzero = 0 otherwise. For the cases where the firm raised funds after release, Raisedafter is computed as Fund $-$ Seed.
- Lastseed: Crunchbase provides the last funding type for each firm. We code Lastseed as 1 if the last funding type is seed, and 0 if not. Note that a firm may have multiple seed funding rounds or can raise funds from multiple investors in the same round. A value of 1 indicates lack of progress beyond the seed stage.

There were no missing data for NFRound, CB, Closed or Lastseed. Table 3 provides summary statistics for the metrics for the aggregate sample as well as for the investor categories separately.

Table 3 about here

Results show that for the aggregate sample, firms released by angel investors have larger average means of NFRound, Fund and Raisedafter, and lower proportions of Lastseed and Raisedzero than firms released by all other four categories of investors. Also, firms released by angels have a lower proportion of Closed than three of the four other categories of investors. However, results also show considerable variation in types of ventures funded by the different categories of investors. In particular, the mean seed funding of firms released by Q1 investors is much lower than that of all other investor groups. And, for the firms for which Seed is available, the median seed funding for firms released by Q1 is only \$125,000 (compared to \$1000,000 or more for angel, Q2, Q3 or Q4). Also Seed is available for only 330 of the 638 released by Q1 investors. As firms for which Seed is not available have much higher CB ranks than firms for which Seed is available, the actual mean and median for firms released by Q1 investors are likely to be even lower. Thus, Q1 investors appear to be "spray and pray" investors that fund many smaller projects and retain only a small fraction of the projects. In contrast, Q2, Q3 and Q4 investors fund projects of greater scope and retain more of these projects. A proper comparison of firms released by different categories of investors must control for such differences.

We next compare the different groups of released organizations after controlling for the following factors available from the data:

- Seed: A higher Seed is likely to imply a project with greater scope.
- CB: A lower CB should denote greater prominence and networking ability.
- Dummy variables for the year seed funding was received to capture variations in economic conditions. These are 1/0 variables for 2011, 2012, 2013, 2014 and 2015, with 2010 as the baseline.

Results of Table 3 also show that total funds raised has very high coefficients of variation (aggregate sample: 5.82, angel: 5.26, Q1: 8.91, Q2: 3.61, Q3: 31.19, Q4: 3.12). To eliminate the extremes, we only include the firms for which Fund is available, and winsorize the data by removing the top 2.5% and bottom 2.5% of Fund from the aggregate sample. By eliminating the extremes, winsorizing should also remove "superior" and "bad" prospects and retain more "good" and "mediocre" prospects. As Seed is a control variable, we next remove the cases where Seed is missing. This gives a final sample of 1378 cases (angel: 358, Q1: 303, Q2: 408, Q3: 217, Q4: 92). We now proceed to compare the performance of the five categories of the released firms.

Analysis with Aggregate Data: With the winsorized aggregate data, we estimate regression models with dependent variables NFRound, Fund, and logit models with dependent variables Closed, Raisedzero and Lastseed. Also, using the subset of the aggregate data where the released firm succeeded in raising funds after release (Raisedzero = 0), we estimate a regression model with dependent variable Raisedafter. NFRound, Fund, and Lastseed indicate the performance of the firm through the funding rounds. Raisedzero and Raisedafter capture performance after release. Closed indicates whether the firm could survive at all after release by the original investor.

In all the models, the predictors were the same: Q_1, Q_2, Q_3 and Q_4, which are 1/0 dummy variables for the four quartiles of VC investors, respectively, and the control variables Seed, CB and dummy variables for the years of seed funding 2011-2015. Results are presented in Table 4.

Table 4 about here

From Table 4, we find that once control variables are included, there is no significant difference between angel and Q1 in NFRound, Fund, Raisedafter, Lastseed, and Closed. Raisedzero is higher for Q1, but the difference is small compared to firms released by VC's with higher retention rates. Thus, the performance of firms released by angel and Q1 investors are similar, which is expected because both groups had similar retention rates. Comparing angel with the other three categories of VC investors, we note the following:

- Firms released by the three categories of VC investors had lower NFRound, higher Lastsed and higher Raisedzero than firms released by angel investors with all differences significant at confidence levels higher than 99%.

- Compared to angel, Q2, Q3 and Q4 all have lower Fund and Raisedafter. The differences are significant for Q3, but not significant at a 90% level of confidence for Q2 and Q4. (For Q4, the difference in Fund is significant at a 90% level of confidence for a one-sided test.)

- Firms released by Q2, Q3 and Q4 all were more likely to be Closed than firms released by than angel investors, but the difference is not statistically significant for Q2.

Overall, results show that compared to firms released by angel investors, firms released by investors with higher retention rates perform worse after release. However, results on Raisedafter indicate that if a released firm manages to secure funding after release, it performs almost as well as firms released by angel investors subsequently. Generally, the results support our theoretical framework.

6. Robustness Checks

6.1. Binary Comparisons using Propensity Score Matching: As an additional check, we used subsets of the winsorized data to compare firms released by angel investors with firms released by Q1, Q2, Q3 and Q4 separately. For each comparison (such as angel with Q1), propensity score matching is performed using logit propensity score with procedure PSMatch in SAS for the dependent variable for the VC category (such as Q1) and independent variables Seed, CB, and dummy variables for years 2011-2015. The matched samples are compared on NFRound, Fund, Lastseed, Raisedzero, and Closed. Next, matched samples are created using cases where the released firms were successful in raising funds after release (Raisedzero = 0), and Raisedafter was compared for the matched samples. Results are presented in Table 5 and are consistent with results for the aggregate sample.

Table 5 about here

6.2 Addressing Alternative Explanation by Kim and Wagman: Kim and Wagman (2016), henceforth KW, use a continuous formulation where an incumbent VC can observe the probability of success p at date 1, and funds the project if and only if p exceeds a threshold p^*. Therefore, all "good" projects are retained unlike our model where only a fraction R of G projects are retained. To compare how well the KW model explains the data, we compare the Q2 and Q3 venture capitalists. We do not include Q1 and Q4 as Q1 venture capitalists tend to fund projects of much smaller scope, and the sample size from Q4 is small. Q2 and Q3 both provide large samples of released firms with a low retention rate for Q2 and a high retention rate for Q3. The KW framework offers two possibilities.

Possibility 1: Threshold p^* is similar for Q2 and Q3: If so, the external market should find the pools of firms released by Q2 and Q3 venture capitalists similar and treat the two pools of applicants similarly. To test this premise, we examine the subset of the winsorized data from Q2 and Q3. Propensity score matching is performed to create matched samples from Q2 and Q3 using logit propensity score with procedure PSMatch in SAS using independent variables Seed, CB, and dummy variables for years 2011-2015. The matched samples are compared on the metrics NFRound, Fund, Lastseed, Raisedzero, Closed, and Raisedafter. Results, presented in Table 6, show that on all the metrics, firms released by Q2 investors perform better than firms released by Q3 investors, and all differences except for Raisedafter are significant at $P < .05$. For Raisedafter, the difference is significant at a 90% level of confidence in a one-sided test. Clearly, the financial market finds the pool of firms released by Q2 superior to firms released by Q3.

Table 6 about here

Possibility 2. Threshold p^* is different for Q2 and Q3: The difference in performance between firms released by Q2 and Q3 may also arise if Q2 investors are more selective than Q3 investors and apply a higher threshold p^* when deciding which projects to retain. If so, firms retained by Q2 investors should perform better than firms retained by Q3 investors in the financial market. In our sample, the Q2 investors retained 420 unique firms and the Q3 investors retained 413 unique firms, with 90 firms retained by both Q2 and Q3. Removing the overlaps, we have a sample of 653 firms (330 Q2, 323 Q3). Winsorizing to remove the highest 2.5% and lowest 2.5% of Fund, we have a sample of 615 retained firms (307 Q2, 308 Q3). Propensity score matching is performed to create matched samples from Q2 and Q3 as before, and the matched samples are compared on the metrics NFRound, Fund, Raisedafter (defined as Fund − Seed), and Closed. (As these firms obtained later round funding, Raisedzero and Lastseed are not relevant here.) Results are presented in Table 6 and show that there is no statistically significant difference between firms retained by Q2 and Q3 on Fund, Raisedafter and Closed. While firms retained by Q2 had more funding rounds than firms retained by Q3, the difference is smaller than the difference for released firms. Thus, the financial market did not see a major difference between firms retained by Q2 and Q3 investors.

Combining the results for released and retained organizations, we find that the KW model, where no good project is released, cannot explain the difference between Q2 and Q3, which our model can.

6.3 Note on Missing Data: Our analysis was performed with the cases where Seed and Fund were both available. Table 3 also provides summary statistics on NFRound, CB, Lastseed and Closed separately for (1) the cases where Seed and Fund are both available, and (2) the cases where Seed, or Fund, or both are missing. Comparing, we find that for each investor category, cases with missing data have lower mean NFRound, higher mean CB (that is, lower prominence), and higher % of Lastseed. However, the % of Closed is similar. So, it is likely that the missing data come from organizations with more limited scope, competing in a different ecosystem from the organizations included in the analysis.

7. Concluding Observations

In this paper we used a model to examine how financing contracts between an entrepreneur seeking financing for his new venture and the two common sources of seed financing, namely an angel investor and a VC, are determined in a competitive venture capital market. Our analysis shows that the contracts are influenced by the implications of follow-on financing required at the next stage of the venture, given that the median second stage financing round is typically several times the size of a seed financing round and that the two first stage investors often do not extend second stage financing, forcing the entrepreneur to approach a new external financier at that stage. We have observed that angel investors typically do not finance beyond the seed stage and VCs do not fund second rounds for the majority of ventures that they support with seed financing.

Our analysis indicates that the entrepreneur faces three possible funding zones, where each zone is demarcated from the others by the retention rate for the VC. In the first zone, where retention rate is sufficiently low, the entrepreneur prefers VC financing to angel financing for seed funds. In case the project is funded again in the second period by the original VC, then of course it is the best financing outcome for the project, but it has a low probability. On the other hand, the pool of projects that are abandoned by the original VC is not overwhelmed with bad projects as it includes most of the good projects along with all bad projects in the original population of projects. While there is some risk of a good project being accorded less than fair valuation in this situation, value addition by the original VC in the first period serves as a compensating factor either fully or partly. In the case of angel seed financing, the pool seeking second stage funding reflects the average quality in the original population of projects since the angel does not provide follow-on financing to any one of them, and there is no comparable value addition by the angel. Therefore, an entrepreneur prefers seed financing from the VC.

In the second zone where retention rate is sufficiently high (but not too high), the pool of projects rejected for further financing by the original VC is likely to have a high proportion of bad projects compared to good projects. The valuation loss may even be so significant that the new VC may not be adequately compensated for her risky investment in the project and consequently decline to fund it. In this situation even a good project rejected for second period funding by the original VC may not get follow-on funding from any external source at all, and disappear for ever in a funding hole. Angel financing is a better option now. We formalized this intuition in Section 4 above.

In the third zone where retention rate is very high, a high probability of being funded again by the original VC with attendant benefit of zero asymmetric information costs may outweigh the costs associated with the small probability of being rejected by the original VC and passing into the funding hole. Thus, the range of values of retention rate where seed financing by angel is preferred is bounded both below and above. Outside this zone, seed financing by VC is preferable.

We have noted that our results imply, interestingly, that it is in the interest of entrepreneurs seeking VC funding at the seed stage that the VC rejects either a sufficiently high proportion of them for follow-on funding (corresponding to a low R), or a sufficiently small proportion (corresponding to a high R), but not in a proportion in the medium range which will make them confront a funding hole. We examined financing data for a large sample of US VC firms during 2010 - 2019 from www.crunchbase.com and showed that retention rates for VC firms had a double-peaked distribution, consistent with the prediction of our model. From the same data, we also showed that compared to firms released by angel investors, firms released by VC firms, particularly VC firms with high retention rates, were less likely to raise any funds after release or progress beyond the seed stage. This also supports our model framework.

We have also noted another important implication of our findings. Angel investors and VCs are actually complementary sources of capital in a range of situations. In other words, unlike in most academic research as well as professional thinking, they are not always substitutes for each other. When VC funding of seed funds leads to a funding hole in the second period, angel financing may still be available for seed funds.

References

Alvarez-Garrido, E. (2022), “Move-in ready or Fixer-Upper? VC Specialization and Start-up Innovation,” Strategy Science, forthcoming.

Bettignie J. and J. Brander (2007), “Financing entrepreneurship: Bank finance versus venture capital,” Journal of Business Venturing, 22, pages 808-832.

Casamatta, C. (2003), “Financing and Advising: Optimal Financial Contracts with Venture Capitalists,” The Journal of Finance, 58(5), 2059-2085.

Chemmanur, T.J. and Z. Chen (2014), “Venture Capitalists Versus Angels: The Dynamics of Private Firm Financing Contracts,” The Review of Corporate Finance Studies, 3 (1-2), pages 39-86.

Farboodie, M. (2013), “Financing and Control Rights: Entrepreneurial Choice of Funding Source,” working paper, University of Chicago.

Hellmann, T. and M.Puri (2002), “Venture Capital and the Professionalization of Start-Up Firms: Empirical Evidence,” The Journal of Finance, 57(1), pp 169-197

Hellmann, T., P. Schure, and D.Vo (2013), “ Angels and Venture Capitalists: Complements or Substitutes?”, working paper.

Hochberg, Y., A. Ljungqvist, A. Vissing-Jorgensen (2014), “Informational Holdup and Performance Persistence in Venture Capital,” The Review of Financial Studies, 27(1), pages 102-152.

Kim, J. and L. Wagman (2016), “Early-stage entrepreneurial financing: A signaling perspective,” Journal of Banking and Finance, 67, pages 12-22.

Ueda, M. (2004), “Banks versus Venture Capital,” The Journal of Finance, 59(2), pages 601-621.

Winton, A. and V. Yerramilli (2008), “Entrepreneurial finance: Baks versus venture capital,” Journal of Financial Economics 88, pages 51-79.

Table 1: Numerical Example

μ	λ_1	λ_2	λ_m	$\overline{R}_m$	$\tilde{R}_m$	Π_m^A	Π_m^V	$(\frac{\Pi_m^A - \Pi_m^V}{\Pi_m^A}) \times 100\%$
1.00	0.721	0.945	0.833	0.4994	0.5616	4.6740	4.5993	1.5976
1.02	0.721	0.895	0.808	0.4062	0.4864	4.5954	4.4972	2.1363
1.04	0.721	0.873	0.797	0.3634	0.4488	4.5601	4.4535	2.3367
1.06	0.721	0.857	0.789	0.3334	0.4209	4.5354	4.4241	2.4533
1.08	0.721	0.845	0.783	0.3103	0.3984	4.5163	4.4021	2.5277
1.10	0.721	0.835	0.778	0.2915	0.3793	4.5007	4.3848	2.5747

Values of other parameters:

$p_s = 1,\ q_s = .2,\ p_g = .4,\ q_g = .3,\ p_m = .1,\ q_m = .3,\ V = 35,\ I_0 = 1,\ I_1 = 10$

Legends:

λ_1: Value of λ at which the H signal start getting funded by new VC under angel financing.

λ_2: Value of λ above which $\tilde{R}$ is less than $\overline{R}$.

λ_m: Average of λ_1 and λ_2

$\overline{R}_m$, $\tilde{R}_m$: $\overline{R}$ and $\tilde{R}$ at $\lambda = \lambda_m$

Π_m^A, Π_m^V: Expected ex ante outcomes under angel financing and under VC financing just above $R = \overline{R}_m$ when $\lambda = \lambda_m$.

The expected outcome under angel financing is higher than the expected outcome under VC financing if $\lambda_1 \leq \lambda < \lambda_2$, and $\overline{R} < R < \tilde{R}$.

Table 2. Seed Fundings and Retention Rates

	Seed Fundings	Retention Rates
All VC Firms		
Number of firms	169	169
Minimum	1	0
Maximum	790	1
Mean	39.29	0.4097
Standard Deviation	90.25	0.2525
Median	19	0.381
VC Firms with above median seed fundings		
Number of firms	84	84
Minimum	20	0.0130
Maximum	790	0.7600
Mean	71.5	0.3322
Standard Deviation	119.9	0.1710
Median	40	0.3181
VC Firms with median or below median seed fundings		
Number of firms	85	85
Minimum	1	0
Maximum	19	1
Mean	7.471	.4862
Standard Deviation	5.349	0.2945
Median	7	0.5000
Angel Investors		
Number of firms	194	194
Minimum	1	0
Maximum	118	1
Mean	17.81	0.1194
Standard Deviation	16.18	0.1556
Median	13.0	0.0711
Aggregate Results		
All venture capital firms (169 firms)	6640	0.2717
Three VC firms with most seed fundings (292, 789, 790)	1871	0.1074
VC firms excluding firms with three most seed fundings	4769	0.3361
Venture capital firms with above median number of seed fundings	6005	0.2506
Venture capital firms with median or below number of seed fundings	635	0.4706
Angel investors	3455	0.1036

Table 3: Summary Statistics on Released Organizations

	Aggregate	Released by				
		by Angel	Q1	Q2	Q3	Q4
Number of Funding Rounds						
Mean (standard deviation)	3.465 (2.716)	4.099 (3.000)	3.172 (2.620)	3.547 (2.714)	2.945 (2.223)	2.881 (2.226)
Median	3.000	3.000	2.000	3.000	2.000	2.000
Fund ($ million)						
Mean (standard deviation)	37.780 (217.070)	47.300 (248.600)	32.800 (292.200)	40.400 (145.670)	26.560 (84.600)	30.000 (93.670)
Median	2.750	3.900	1.000	4.150	2.500	2.750
CB Rank						
Mean (standard deviation)	188,168 (310,484)	170,800 (288,186)	303,056 (419,944)	92,401 (137,524)	110,255 (150,444)	248,627 (332,882)
Median	78,158	71,954	123,697	49,402	65,359	101,286
Seed ($ million)						
Mean (standard deviation)	1.279 (1.296)	1.173 (1.039)	0.561 (0.799)	1.713 (1.583)	1.577 (1.212)	1.606 (1.301)
Median	1.000	1.000	0.125	1.500	1.293	1.400
% Closed	26.75%	23.85%	29.15%	23.42%	29.78%	34.92%
% Raisedzero	30.27%	21.35%	34.85%	29.09%	37.67%	38.30%
Raisedafter ($ million)						
Mean	54.780	64.400	55.800	54.700	40.840	35.200
(standard deviation)	(280.890)	(314.500)	(425.100)	(175.000)	(107.600)	(84.300)
Median	5.000	5.300	2.700	7.200	3.520	4.400
% Lastseed	52.36%	43.46%	59.25%	47.02%	61.99%	58.73%
Seed and Fund both available						
Number of Cases	1447	384	330	416	223	94
Mean number of funding rounds	3.664	4.224	3.564	3.692	3.085	2.979
Mean CB Rank	117,762	123,025	180,690	64,038	80,629	201,200
% Closed	27.71%	24.22%	31.82%	24.04%	30.49%	37.23%
% Lastseed	48.86%	40.10%	5.15%	44.47%	59.19%	57.45%
Seed or Fund missing						
Number of Cases	675	182	308	105	48	32
Mean number of funding rounds	3.039	3.835	2.753	2.971	2.292	2.594
Mean CB Rank	338,872	271,599	434,162	204,772	245,085	387,945
% Closed	24.70%	23.08%	26.30%	20.95%	26.53%	28.13%
% Lastseed	59.85%	50.55%	63.64%	57.14%	75.00%	62.50%

Legend

Q1-Q4: VC firms with retention rates in the bottom 25% (retention rate < 23.79%), next 25% (retention rate between 24% and 37.84%), next 25% (retention rate between 38% and 54.55%), and highest 25% (retention rate 54.92% or greater)

Closed: 1 if listed as "closed" (not active) by Crunchbase in September 2022, 0 if active.

Raisedzero: 1 if firm raised no funds after release, 0 otherwise.

Raisedafter: Total Funding Amount − Seed Money (original investor), calculated if > 0.

Lastseed: 1 if last funding type is "seed," zero if not.

Table 4: Results on released Organizations using Aggregate Data

Regression Models			
Predictor	Dependent variable		
	NFRound	Fund	Raisedafter
	Parameter Estimate (t Statistic, P value)	Parameter Estimate (t Statistic, P value)	Parameter Estimate (t Statistic, P value)
Constant	5.3039** (16.40, < .0001)	27.500** (4.68, < .0001)	34.880** (4.25, < .0001)
Seed	0.1499** (2.93, .0034)	10.180** (10.96, < .0001)	10.147** (8.62, < .0001)
CB	−.0044** (−12.55, < .0001)	−0.042** (−6.57, < .0001)	−0.0713** (−5.91, < .0001)
Q1	-0.0769^{ns} (−0.43, 0.6689)	2.861^{ns} (0.87, .3818)	4.023^{ns} (0.93, .3529)
Q2	−0.7931** (−4.77, < .0001)	-3.450^{ns} (−1.14, .2536)	-0.951^{ns} (−0.24, .8126)
Q3	−1.3306** (−6.76, < .0001)	−11.192** (−3.13, .0018)	−9.159 (−1.83, .0673)
Q4	−0.8282** (−3.11, .0019)	-6.788^{ns} (−1.40, .1608)	-3.575^{ns} (−0.52, .6033)
Logit Models			
Predictor	Dependent variable		
	Lastseed	Raisedzero	Closed
	Parameter Estimate (Wald Chi-sq, P value)	Parameter Estimate (Wald Chi-sq, P value)	Parameter Estimate (Wald Chi-sq, P value)
Constant	−2.722** (39.852, < .0001)	−2.875** (48.688, < .0001)	−1.588** (20.694, < .0001)
Seed	−0.386** (33.461, < .0001)	−0.125* (3.883, .0488)	−0.134* (4.405, .0358)
CB	0.012** (109.289, < .0001)	.006** (79.320, < .0001)	0.003** (34.278, < .0001)
Q1	0.125^{ns} (0.456, .4995)	0.358 (3.048, .0808)	0.160^{ns} (0.725, .3947)
Q2	0.876** (26.725, < .0001)	1.003** (28.017, < .0001)	0.216^{ns} (1.489, .2224)
Q3	1.502** (54.195, < .0001)	1.389** (43.391, < .0001)	0.505* (6.364, .0116)
Q4	0.818** (8.386, .0038)	0.854** (8.951, .0028)	0.496 (3.439, .0637)

Note: Constant corresponds to seed funding in 2010. Coefficients for dummy variables for 2011, 2012, 2013, 2014 and 2015 are omitted from table.

Legend

Seed, Fund, Raisedafter: Unit = $1,000,000

CB: CB rank of company (unit = 1000)

Q1, Q2, Q3, Q4: 1/0 dummy variables for organizations released by Q1, Q2, Q3 and Q4 VC-s.

$** \rightarrow$ P value $< .01$, $* \rightarrow$ P value $< .05$, ns $\rightarrow$ P value ≥ 0.10

Table 5: Comparisons of Released Organizations using Propensity Score Matching

Comparison of Angel and Q1	
Matched Sample Sizes	Angel 209, Q1 209
Propensity Score (PS)	$\overline{PS}(angel)$.5447, $\overline{PS}(Q1)$.5259, P (equal means) .241, P (equal variances) .704
NFRound	Mean Angel = 3.880, Mean Q1 = 3.890, t Statistic = −0.04, P value = .969
Fund ($1,000,000)	Mean Angel = 15.212, Mean Q1 = 17.220, t Statistic = −0.53, P value = .599
Lastseed	Angel 46.89%, Q1 47.37%, χ^2 = 0.0096, P value = .9219
Raisedzero	Angel 20.57%, Q1 24.88%, χ^2 = 1.1034, P value = .2935
Closed	Angel 28.23%, Q1 29.19%, χ^2 = 0.0468, P value = .8288
Raisedafter*	Mean Angel = 18.489, Mean Q1 = 20.708, t Statistic = −0.49, P value = .6247
Comparison of Angel and Q2	
Matched Sample Sizes	Angel 295 , Q2 295
Propensity Score (PS)	$\overline{PS}(angel)$.4786, $\overline{PS}(Q2)$.4669, P (equal means) .216, P (equal variances) .261
NFRound	Mean Angel = 4.305, Mean Q2 = 3.244, t Statistic = 5.45, P value < .0001
Fund ($1,000,000)	Mean Angel = 23.121, Mean Q2 = 16.884, t Statistic = 1.69, P value = .092
Lastseed	Angel 35.25%, Q2 53.56%, χ^2 = 20.020, P value < .0001
Raisedzero	Angel 14.58%, Q2 35.25%, χ^2 = 33.713, P value < .0001
Closed	Angel 22.37%, Q2 26.10%, χ^2 = 1.117, P value = .291
Raisedafter*	Mean Angel = 29.963, Mean Q2 = 25.035, t Statistic = 0.98, P value = .3284
Comparison of Angel and Q3	
Matched Sample Sizes	Angel 202, Q3 202
Propensity Score (PS)	$\overline{PS}(angel)$.6059, $\overline{PS}(Q3)$.5902, P (equal means) .218, P (equal variances) .680
NFRound	Mean Angel 4.233, Mean Q3 2.965, t Statistic = 5.58, P value < .0001
Fund ($1,000,000)	Mean Angel = 25.901, Mean Q3 = 13.993, t Statistic = 2.74, P value = .0065
Lastseed	Angel 36.14%, Q3 61.39%, χ^2 = 25.768, P value < .0001
Raisedzero	Angel 17.33%, Q3 40.10%, χ^2 = 25.589, P value < .0001
Closed	Angel 23.27%, Q3 32.67%, χ^2 = 4.435, P value = .0352
Raisedafter*	Mean Angel = 32.460, Mean Q3 = 17.275, t Statistic = 2.70, P value = .0075
Comparison of Angel and Q4	
Matched Sample Sizes	Angel 88, Q4 88
Propensity Score (PS)	$\overline{PS}(angel)$.7720, $\overline{PS}(Q4)$.7602, P (equal means) .493, P (equal variances) .994
NFRound	Angel 3.796, Q4 3.000, t Statistic = 2.46, P value = .0149
Fund ($1,000,000)	Mean Angel = 23.204, Mean Q4 = 15.121, t Statistic = 1.25, P value = .2133
Lastseed	Angel 39.77%, Q4 57.95%, χ^2 = 5.821, P value = .0158
Raisedzero	Angel 22.73%, Q4 38.64%, χ^2 = 5.236, P value = .0221
Closed	Angel 31.82%, Q4 36.36%, χ^2 = 0.405, P value = .5247
Raisedafter*	Mean Angel = 24.730, Mean Q4 = 22.717, t Statistic = 0.23, P value = .8223

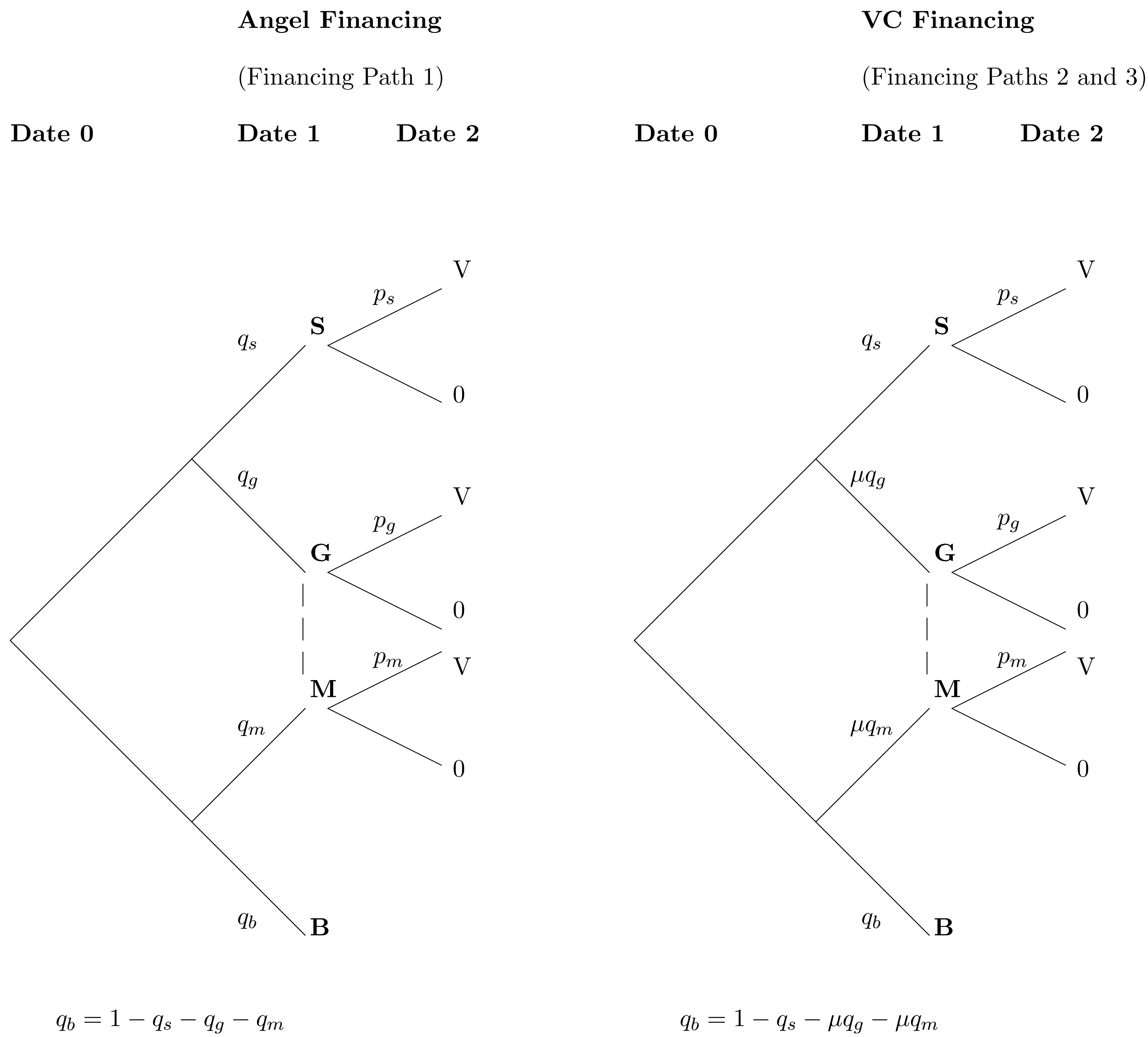
∗ For the comparison of Raisedafter, propensity score matching was used to create matched samples only for cases where Raisedafter > 0.

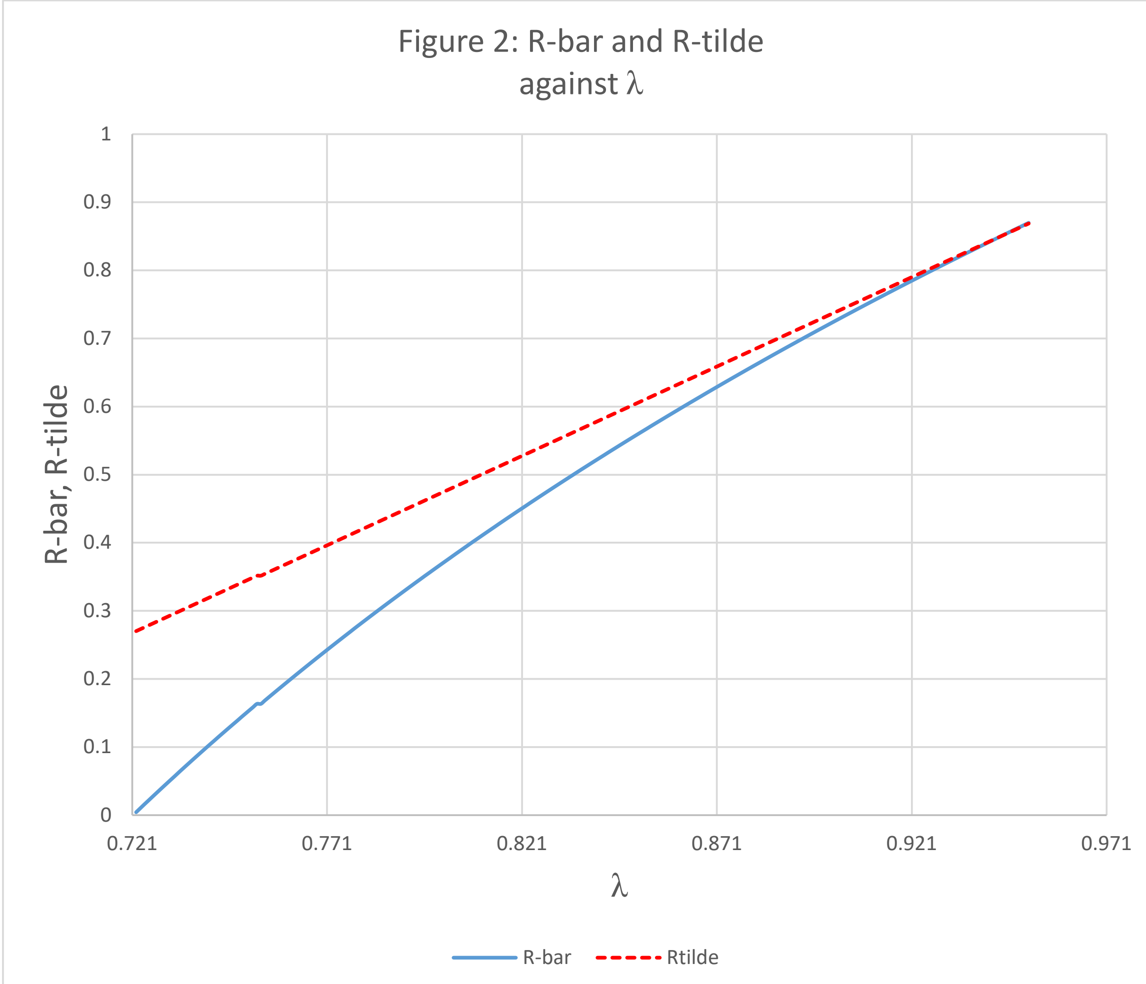
Table 6: Comparison of Q2 and Q3 Venture Capitalists

Comparison of Released Organizations	
Full Sample	
Matched Sample Sizes	210 Q2, 210 Q3
Propensity Score (PS)	$\overline{PS}(Q2) = .6530$, $\overline{PS}(Q3) = .6438$, P (equal means) .152, P (equal variances) .979
NFRound	Mean Q2 = 3.619, Mean Q3 = 2.952, t Statistic = 2.94, P value = .0035
Fund ($1,000,000)	Mean Q2 = 24.961, Mean Q3 = 13.912, t Statistic = 2.42, P value = .0158
Lastseed	Q2 43.33%, Q3 60.48%, $\chi^2 = 12.361$, P value = .0004
Raisedzero	Q2 26.19%, Q3 37.62%, $\chi^2 = 6.313$, P value = .0120
Closed	Q2 21.43%, Q3 31.90%, $\chi^2 = 5.893$, P value = .0152
Cases with Raisedzero = 0	
Matched Sample Sizes	130 Q2, 130 Q3
Propensity Score (PS)	$\overline{PS}(Q2) = .6779$, $\overline{PS}(Q3) = .6665$, P (equal means) .283, P (equal variances) .922
Raisedafter	Mean Q2 = 32.486, Mean Q3 = 21.887, t Statistic = 1.49, P value = .1383 P value = .069 for a one-sided test
Comparison of Retained Organizations	
Matched Sample Sizes	249 Q2, 249 Q3
Propensity Score (PS)	$\overline{PS}(Q2) = .5056$, $\overline{PS}(Q3) = .5084$, P (equal means) .565, P (equal variances) .693
NFRound	Mean Q2 = 5.329, Mean Q3 = 4.831, t Statistic = −2.25, P value = .0246
Fund	Mean Q2 = 78.954, Mean Q3 = 76.653, t Statistic = 0.21, P value = .8325
Raisedfafter*	Mean Q2 = 76.81, Mean Q3 = 74.48, t Statistic = 0.21, P value = .8302
Closed	Q2 22.89%, Q3 25.30%, $\chi^2 = 0.3952$, P value = .5296

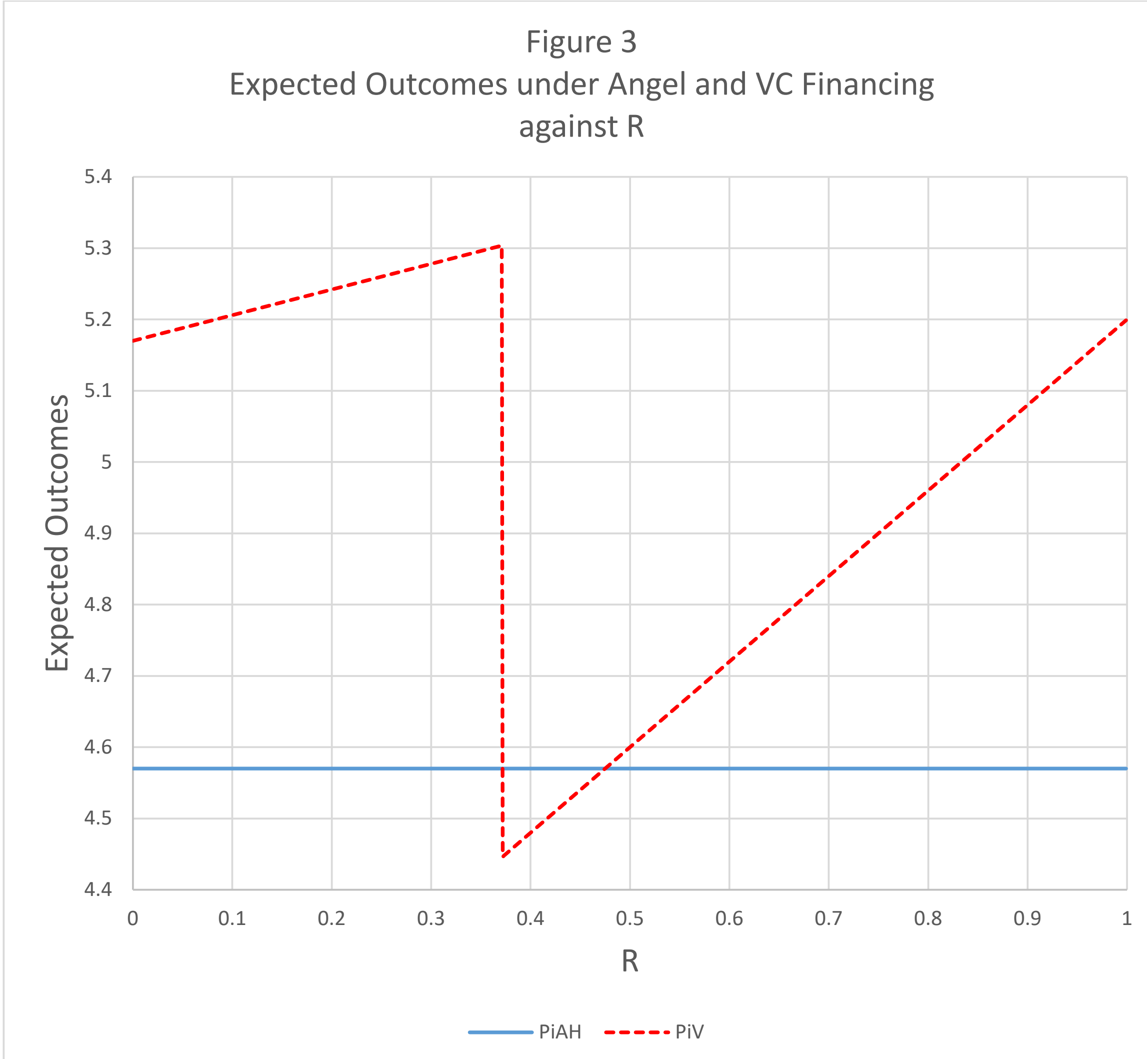
* Since all retained firms received follow-on funding, Raisedafter is computed for the full sample as Fund − Seed.

Figure 1. Angel Financing and VC Financing Paths

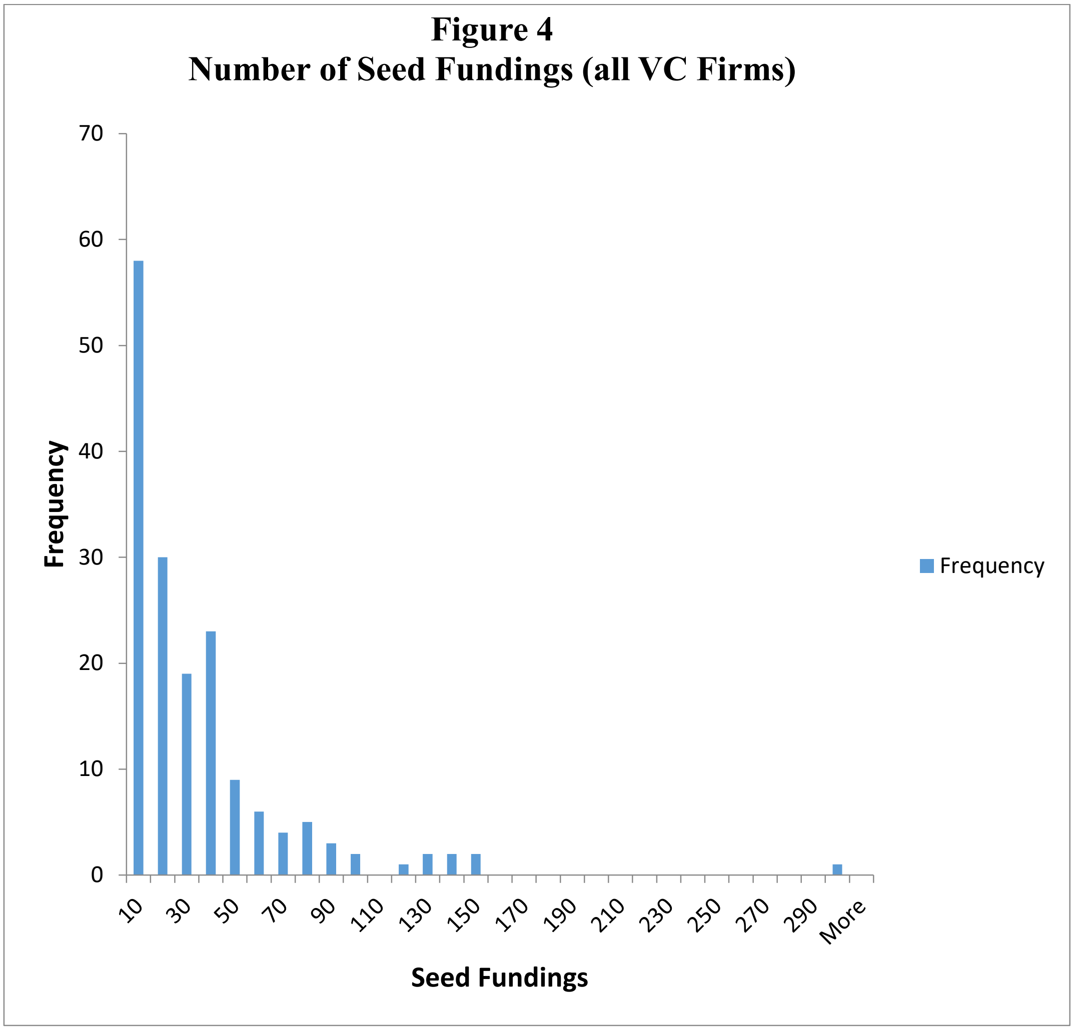




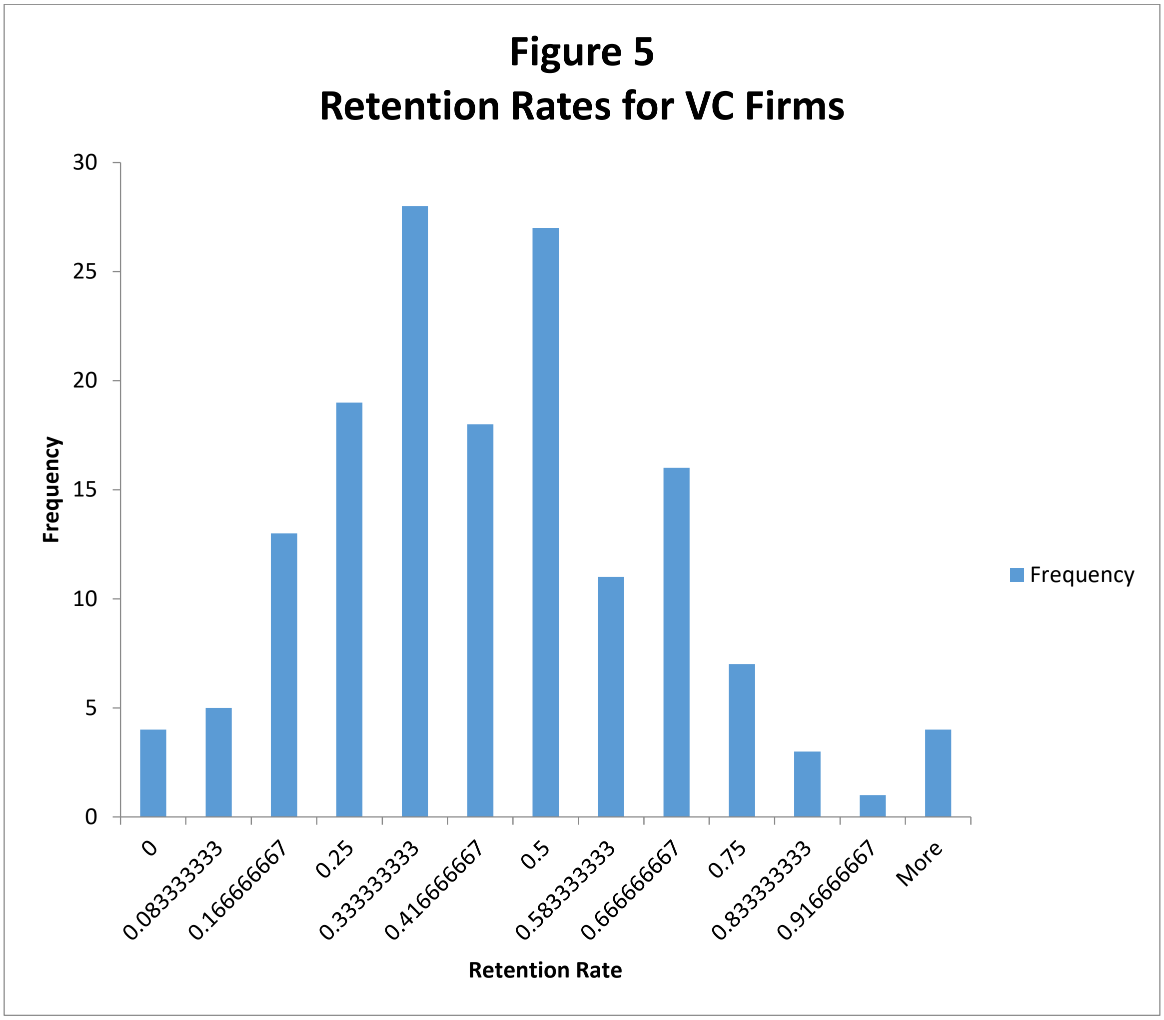
$p_s = 1.0$, $q_s = 0.2$, $p_g = 0.4$, $q_g = 0.3$, $p_m = 0.1$, $q_m = 0.3$, $V = 35$, $I_0 = 1$, $I_1 = 10$, $\mu = 1.0$, $K = 1.0$



$p_s = 1.0$, $q_s = 0.2$, $p_g = 0.4$, $q_g = 0.3$, $p_m = 0.1$, $q_m = 0.3$, $V = 35$, $I_0 = 1$, $I_1 = 10$, $\mu = 1.0$, $K = 1.0$



Two VC firms with more than 300 seed fundings (789 and 790) excluded from histogram.



VC firms with one seed funding excluded from the histogram.

Figure 6
Number of Seed Fundings of Angel Investors

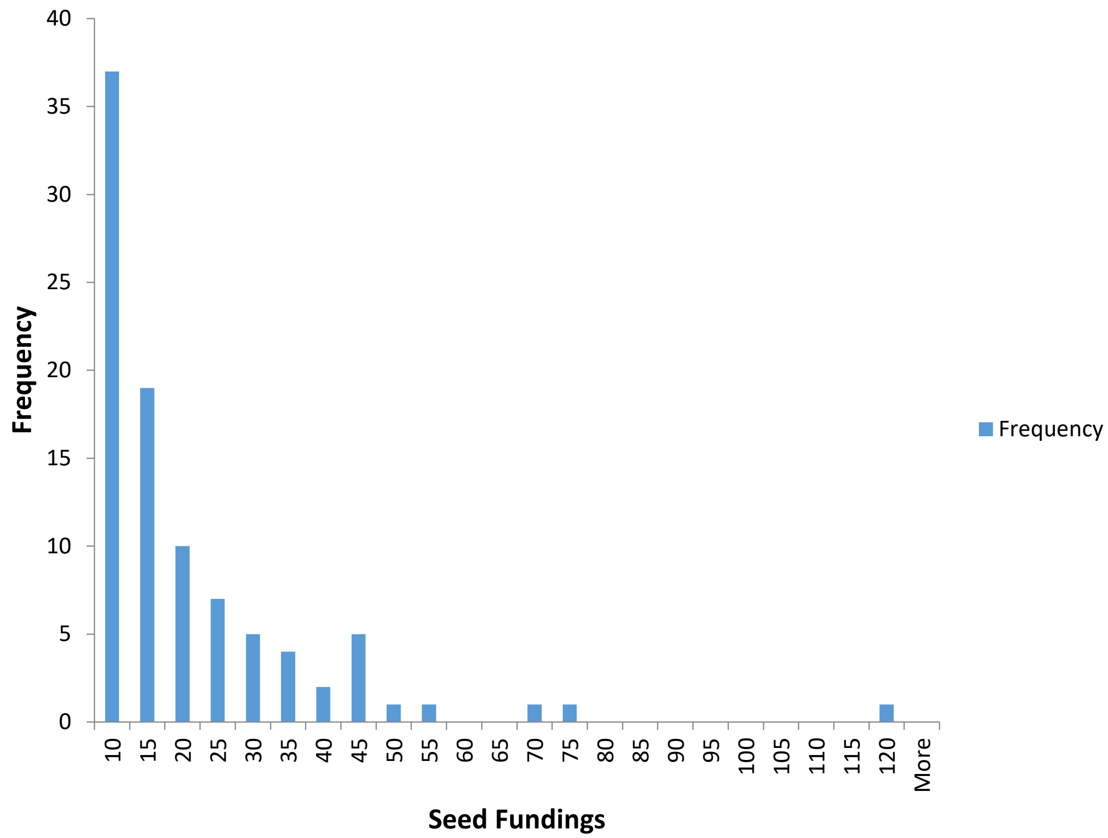
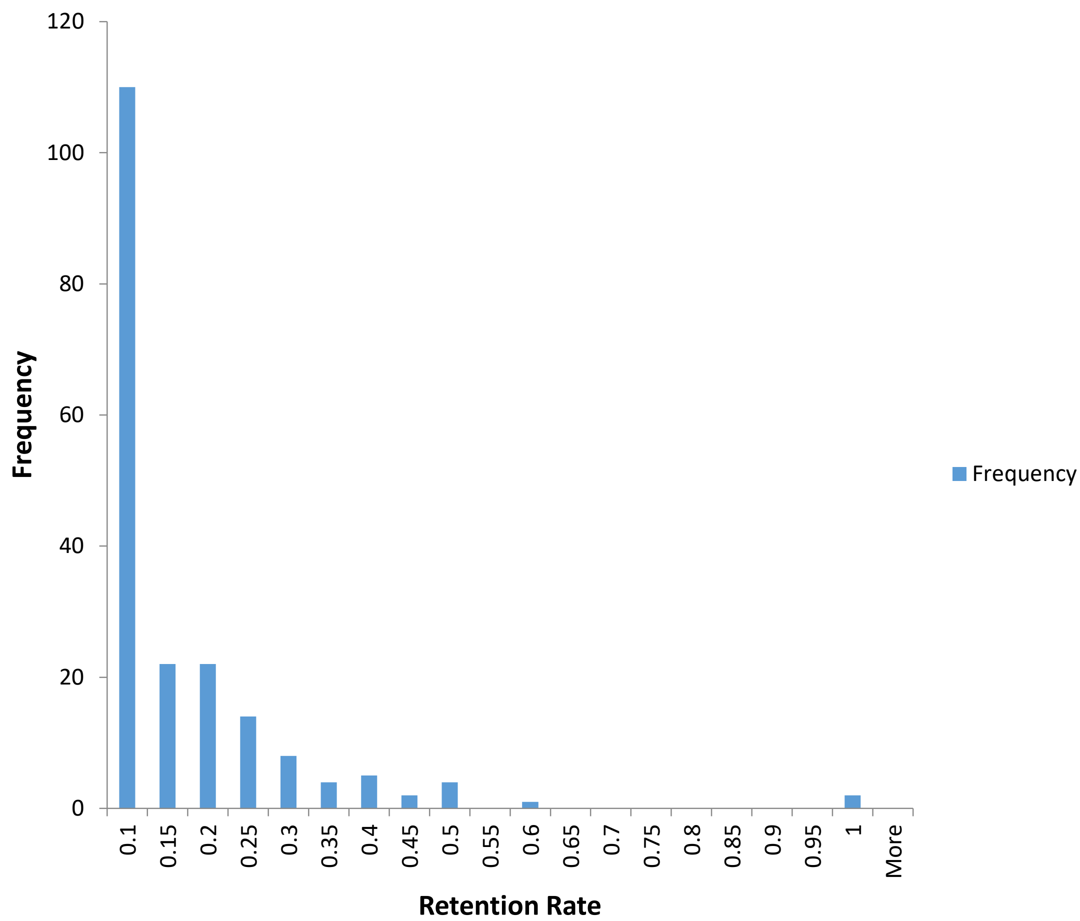


Figure 7
Retention Rates of Angel Investors



Appendix 1. Expected Outcomes, Funding Costs, and Proofs of Propositions

Terms and Notations: Throughout the appendix, we use superscripts A and V to represent angel financing and VC financing at date 0 (seed stage), respectively. Superscripts S, H and HL represent the cases where only the S projects are funded, the H signals and not the L signals are funded, and both H and L signals are funded at date 1, respectively. Subscripts H and L refer to the H signal and the L signal, respectively.

$F_0^{A,H}$	Date 0 funding cost under angel financing when external VC funds H but not L at date 1.
$F_0^{A,HL}$	Date 0 funding cost under angel financing when external VC funds both H and L at date 1.
$F_0^{A,S}$	Date 0 funding cost under angel financing when external VC only funds S (neither H nor L) at date 1.
F_{1H}^{A}	Date 1 funding cost with external VC for signal H under angel financing.
F_{1L}^{A}	Date 1 funding cost with external VC for signal L under angel financing.
$\Pi^{A,H}$	Entrepreneur's ex-ante expected outcome under angel financing when external VC funds H but not L at date 1.
$\Pi^{A,HL}$	Entrepreneur's ex-ante expected outcome under angel financing when external VC funds both H and L at date 1.
$\Pi^{A,S}$	Entrepreneur's ex-ante expected outcome under angel financing when external VC only funds S (neither H nor L) at date 1.
$F_0^{V,H}$	Date 0 funding cost under VC financing when external VC funds H but not L at date 1.
$F_0^{V,HL}$	Date 0 funding cost under VC financing when external VC funds both H and L at date 1.
$F_0^{V,S}$	Date 0 funding cost under VC financing when external VC only funds S (neither H nor L) at date 1.
F_{1H}^{V}	Date 1 funding cost with external VC for signal H under VC financing.
F_{1L}^{V}	Date 1 funding cost with external VC for signal L under VC financing.
$\Pi^{V,H}$	Entrepreneur's ex-ante expected outcome under VC financing when external VC funds H but not L at date 1.
$\Pi^{V,HL}$	Entrepreneur's ex-ante expected outcome under VC financing when external VC funds both H and L at date 1.
$\Pi^{V,S}$	Entrepreneur's ex-ante expected outcome under VC financing when external VC only funds S (neither H nor L) at date 1.

1.1 Expected Outcomes and Funding Costs

Result: In any given scenario, the expected ex ante outcome of the entrepreneur is

(A1) $\Pi = P(\text{success at date 0}) * V - I_0 - (\text{Probability at date 0 that project is funded at date 1}) * I_1$

Proof: The S project is always funded at date 1. Thus, under any scenario, at date 0 the entrepreneur has one or more viable paths to success. Suppose, in a given scenario, the entrepreneur has n viable paths to success: $VP_1, \ldots, VP_n$. Then,

(A2) $P(\text{success at date 0}) = \sum_{i=1}^{n} P(\text{success at date 0 through } VP_i)$

Denoting the probability at date 0 that the project will proceed through VP_i by $P(VP_i)$,

(A3) $P(\text{success at date 0 through } VP_i) = \quad P(\text{success at date 1}|VP_i) * P(VP_i)$,

For VP_i, date 1 funding cost is

(A4) $F_{1i} = \dfrac{I_1}{P(\text{success at date 1}|VP_i)}$

Combining, the expected outcome of the entrepreneur at date 0 is

$\Pi = \sum_{i=1}^{n}(V - F_0 - F_{1i})P(\text{success at date 0 through } VP_i)$

$= (V - F_0) * P(\text{success at date 0}) - \sum_{i=1}^{n}(\frac{I_1}{P(\text{success at date 1}|VP_i)}) * P(\text{success at date 0 through } VP_i)$

$= V * P(\text{success at date 0}) - I_0 - (\sum P(VP_i)) * I_1$

$= P(\text{success at date 0}) * V - I_0 - P(\text{Probability at date 0 project is funded at date 1}) * I_1$

Angel Financing

The following cases may receive funding at date 1.

1. State S: Probability q_s, probability of success at stage 1 is p_s, $F_1 = \frac{I_1}{p_s}$

At date 0, the probability of success through this path is $p_s q_s$.

2. Signal H at date 1. These consist of:

2(a) G projects and signal H: Probability at date $0 = \lambda q_g$

The probability of success through this path at date $0 = \lambda p_g q_g$

2(b) M projects and signal H: Probability at date $0 = (1-\lambda)q_m$

Probability of success through this path at date $0 = (1-\lambda)p_m q_m$

Combining, the probability of H signal at date 0 is

(A5) $P(H) = \lambda q_g + (1-\lambda)q_m$,

and the probability of success at date 0 through this path is

(A6) $P(success \cap H) = \lambda p_g q_g + (1-\lambda)p_m q_m$

Hence,

(A7) $P(success|H) = \frac{P(success \cap H)}{P(H)} = \frac{\lambda p_g q_g + (1-\lambda)p_m q_m}{\lambda q_g + (1-\lambda)q_m}$

Therefore, for the H signal, the funding cost at date 1 is

(A8) $F_{1H}^{A} = \frac{I_1}{P(success|H)} = \frac{I_1\{\lambda q_g + (1-\lambda)q_m\}}{\lambda p_g q_g + (1-\lambda)p_m q_m}$

3. Signal L at date 1. These projects consist of:

3(a) G projects and signal L. Probability at date 0 is $(1-\lambda)q_g$. Probability of success through this path at date 0 is $(1-\lambda)p_g q_g$.

3(b) M projects and signal L. Probability at date 0 is λq_m. Probability of success through this path at date 0 is $\lambda p_m q_m$.

Combining, probability of signal L at date 0 is

(A9) $P(L) = (1-\lambda)q_g + \lambda q_m$,

and probability of success through this path is

(A10) $P(success \cap L) = (1-\lambda)p_q q_g + \lambda p_m q_m$

Hence,

(A11) $P(success|L) = \frac{P(success \cap L)}{P(L)} = \frac{(1-\lambda)p_g q_q + \lambda p_m q_m}{(1-\lambda)q_g + \lambda q_m}$

The funding cost at date 1 for the L signal is

(A12) $F_{1L}^{A} = \dfrac{I_1}{P(success|L)} = \dfrac{I_1\{(1-\lambda)q_g + \lambda q_m\}}{(1-\lambda)p_g q_g + \lambda p_m q_m}$

$F_{1H}^{A} = F_{1L}^{A}$ at $\lambda = .5$ and, as discussed later (Appendix 1.2, Result IR1) , $F_{1H} < F_{1L}$ if $\lambda > .5$. Since $\lambda \geq .5$, $F_{1H} \leq F_{1L}$.

Case 1A. Only S (neither H nor L) projects are funded by new VC at date 1: At date 0 the probability of success is $p_s q_s$, and $F_0^{A,S} = \dfrac{I_0}{p_s q_s}$

Expected outcome of the entrepreneur at date 0

(A13) $\Pi^{A,S} = p_s q_s V - I_0 - q_s I_1 = q_s(p_s V - I_1) - I_0$

Case 1B. At date 1, H projects funded by new VC and L projects are not: At date 0, the probability of success is

$p_s q_s + \lambda p_g q_g + (1-\lambda) p_m q_m$

Hence,

(A14) $F_0^{A,H} = \dfrac{I_0}{p_s q_s + \lambda p_g q_g + (1-\lambda) p_m q_m}$

Expected outcome at date 0 is

(A15) $\Pi^{A,H} = [p_s q_s + \lambda p_g q_g + (1-\lambda) p_m q_m]V - I_0 - [q_s + \lambda q_g + (1-\lambda) q_m] I_1$

$= q_s(p_s V - I_1) + \lambda q_g(p_g V - I_1) - (1-\lambda) q_m (I_1 - p_m V)$

Case 1C. Both H and L projects are funded by new VC at date 1: In this case, any project that reaches state S, G or M is funded. Hence, at date 0, the probability of success is

$p_s q_s + p_g q_g + p_m q_m$, and the date 0 funding cost is

(A16) $F_0^{A,HL} = \dfrac{I_0}{p_s q_s + p_g q_g + p_m q_m}$

Expected outcome at date 0

(A17) $\Pi^{A,HL} = [p_s q_s + p_g q_g + p_m q_m]V - I_0 - (q_s + q_g + q_m) I_1 = q_s(p_s V - I_1) + q_g(p_g V - I_1) - q_m(I_1 - p_m V) - I_0$

Conditions for the cases: Given a path, a project is funded by the new VC at stage 1 if and only if $(V - F_0 - F_1) \geq 0$ for the path. Since $F_{1H}^{A} \leq F_{1L}^{A}$, the H project is funded any time the L project is funded. Thus, we have the following cases.

- Only the S projects are funded, that is, neither H nor L projects are funded, if and only if the H project is not funded, that is, $V - F_0^{A,H} - F_{1H}^{A} \quad < \quad 0$
- Both H and L are funded if and only if the L project is funded, that is,

  $V - F_0^{A,HL} - F_{1L}^{A} \geq 0$
- In all other cases the H project is funded and the L project is not funded.

VC financing

The following states may receive funding at date 1.

1. S state: Probability q_s, probability of success at date 1 is p_s, $F_1 = \dfrac{I_1}{p_s}$

At date 0, the probability of success through this path is $p_s q_s$.

2. G and retained by original VC: Probability $\mu R q_g$, probability of success at date 1 is p_g, $F_1 = \dfrac{I_1}{p_g}$

At date 0, the probability of success through this path is $\mu R p_g q_g$.

3. Released by original VC and signal H at date 1. These consist of:

3(a) G projects released by original VC and signal H:

Probability at date $0 = \mu q_g(1-R)\lambda = \mu\lambda(1-R)q_g$

The probability of success through this path at date 0 is $\mu\lambda(1-R)p_g q_g$

3(b) M projects with signal H: Probability at date $0 = \mu(1-\lambda)q_m$

Probability of success through this path at date 0 is $\mu(1-\lambda)p_m q_m$

Combining, the probability of H signal at date 0 is

(A18) $P(H) = \mu[\lambda(1-R)q_g + (1-\lambda)q_m]$,

and the probability of success at date 0 through this path is

(A19) $P(success \cap H) = \mu[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]$

Hence,

(A20) $P(success|H) = \dfrac{\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m}{\lambda(1-R)q_g + (1-\lambda)q_m}$, and $F_{1H}^V = \dfrac{I_1\{\lambda(1-R)q_g + (1-\lambda)q_m\}}{\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m}$

4. Released by original VC and signal L at date 1. These consist of:

4(a) G projects released by original VC and signal L:

Probability at date $0 = \mu q_g(1-R)(1-\lambda) = \mu(1-\lambda)(1-R)q_g$

The probability of success through this path at date 0 is $\mu(1-\lambda)(1-R)p_g q_g$

4(b) M projects with signal L: Probability at date $0 = \lambda\mu q_m$

Probability of success through this path at date 0 is $\lambda\mu p_m q_m$

Combining, the probability of L signal at date 0 is

(A21) $P(L) = \mu[(1-\lambda)(1-R)q_g + \lambda q_m]$,

and the probability of success at date 0 through this path is

(A22) $P(success \cap L) = \mu[(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m]$

Hence,

(A23) $P(success|L) = \dfrac{(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m}{(1-\lambda)(1-R)q_g + \lambda q_m}$, and $F_{1L}^V = \dfrac{I_1\{(1-\lambda)(1-R)q_g + \lambda q_m\}}{(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m}$

Case 3A. The new VC funds only S signals (neither H, nor L): Only the S projects and the G projects retained by the incumbent VC receive funding at date 1. Hence, the probability of success at stage 0 is $(p_s q_s + \mu R p_g q_g)$ and

$$F_0^{V,S} = \frac{I_0}{p_s q_s + \mu R p_g q_g}$$

The expected outcome at date 0 is,

(A24) $\Pi^{V,S} = q_s(p_s V - I_1) + \mu R q_g(p_g V - I_1) - I_0$

Case 3B. H signals and L signals are not funded by new VC: Probability of success at date 0 is

$p_s q_s + \mu[R p_g q_g + \lambda(1-R)p_g q_g + (1-\lambda)p_m q_m] = p_s q_s + \mu[\{\lambda(1-R) + R\}p_g q_g + (1-\lambda)p_m q_m]$

(A25) $F_0^{V,H} = \dfrac{I_0}{p_s q_s + \mu[\{\lambda(1-R)+R\}p_g q_g + (1-\lambda)p_m q_m]}$

Proceeding as for angel financing, the expected outcome at date 0 is

(A26) $\Pi^{V,H} = q_s(p_s V - I_1) + \mu q_g\{R + \lambda(1-R)\}(p_g V - I_1) - \mu(1-\lambda)(I_1 - p_m V) - I_0$

Case 3C. Both H and L signals are funded by the new VC: Probability of success at date 0 is

$p_s q_s + \mu[R p_g q_g + (1-R)p_g q_g + p_m q_m] = p_s q_s + \mu(p_g q_g + p_m q_m)$

(A27) $F_0^{V,HL} = \dfrac{I_0}{p_s q_s + \mu(p_g q_g + p_m q_m)}$

Proceeding as in the angel seed funding case, the expected outcome at date 0 is

(A28) $\Pi^{V,HL} = q_s(p_s V - I_1) + \mu q_g(p_g V - I_1) - \mu q_m(I_1 - p_m V) - I_0$

Conditions for the cases: Given a path, a project is funded at date 1 if and only if $(V - F_0 - F_1) \geq 0$ for the path. Since $F_{1H}^V \leq F_{1L}^V$ (Result IR1, Appendix 1.2), the H project is funded any time the L project is funded. Thus, we have the following cases.

- Only the S projects are funded, that is, neither H nor L projects are funded, if and only if the H project is not funded, that is,

  $$V - F_0^{V,H} - F_{1H}^V \quad < \quad 0$$

- Both H and L are funded if and only if the L project is funded, that is,

  $$V - F_0^{V,HL} - F_{1L}^A \quad \geq \quad 0$$

- In all other cases the H signal is funded and the L signal is not funded.

1.2 Intermediate results (IR) used to prove propositions

Effect of λ and R on Funding Costs

IR1. Under both angel financing and VC financing, F_{1H} is strictly decreasing in λ, F_{1L} is strictly increasing in λ, $F_{1H} = F_{1L}$ if $\lambda = .5$, and $F_{1H} < F_{1L}$ if $\lambda > .5$

Proof: We provide the proof for VC financing. The proof is similar for angel financing and is omitted.

From (A20) and (A23), $F_{1H}^V = F_{1L}^V$ if $\lambda = .5$,

(A29) $\dfrac{\partial}{\partial\lambda}F_{1H}^V = -\dfrac{I_1(1-R)(p_g - p_m)q_g q_m}{[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]^2} < 0$, and

(A30) $\dfrac{\partial}{\partial\lambda}F_{1L}^V = \dfrac{I_1(1-R)(p_g - p_m)q_g q_m}{[(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m]^2} > 0$

Hence, if $\lambda > .5$, $F_{1H}^V < F_{1L}^V$

IR2. The total funding cost under angel financing when H is funded and L is not funded is strictly decreasing in λ, that is, $\frac{\partial}{\partial\lambda}[F_0^{A,H} + F_{1H}^A] < 0$, if the following condition is satisfied:

(A31) $(\dfrac{I_1}{I_0})(1 - \dfrac{p_m}{p_g})q_m \geq (\dfrac{p_m q_m}{p_g q_g}) - 1$

Proof: From (A8) and (A14),

$$\frac{\partial}{\partial\lambda}[F_0^{A,H} + F_{1H}^A] = \frac{(p_m q_m - p_g q_g)I_0}{[p_s q_s + \lambda p_g q_g + (1-\lambda)p_m q_m]^2} - \frac{(p_g - p_m)q_g q_m I_1}{[\lambda p_g q_g + (1-\lambda)p_m q_m]^2}$$

Since $[p_s q_s + \lambda p_g q_g + (1-\lambda)p_m q_m]^2 > [\lambda p_g q_g + (1-\lambda)p_m q_m]^2$,

$\dfrac{\partial}{\partial\lambda}(F_0^{A,H} + F_{1H}^A) < 0$ if $(p_g - p_m)q_g q_m I_1 \geq (p_m q_m - p_g q_g)I_0$.

IR3. The total funding cost under VC financing where H is funded and L is not funded, is strictly decreasing in λ, that is, $\frac{\partial}{\partial\lambda}[F_0^{V,H} + F_{1H}^V] < 0$, if the following condition is satisfied:

(A32) $p_m q_m I_0 \leq (1-R)[\mu(p_g - p_m)q_g q_m I_1 + p_g q_g I_0]$

Proof: From (A20) and (A25),

$$\frac{\partial}{\partial\lambda}[F_0^{V,H} + F_{1H}^V] = \frac{\mu[p_m q_m - (1-R)p_g q_g]I_0}{\{p_s q_s + \mu[\{\lambda(1-R) + R\}p_g q_g + (1-\lambda)p_m q_m]\}^2} - \frac{(1-R)(p_g - p_m)q_g q_m I_1}{[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]^2}$$

$$< \frac{\mu[p_m q_m - (1-R)p_g q_g]I_0}{\{\mu[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]\}^2} - \frac{(1-R)(p_g - p_m)q_g q_m I_1}{[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]^2}$$

$$= \frac{\mu\{p_m q_m I_0 - (1-R)[\mu(p_g - p_m)q_g q_m I_1 + p_g q_g I_0]\}}{\{\mu[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]\}^2}$$

Hence the result.

IR4. F_{1H}^V and F_{1L}^V are strictly increasing and $F_0^{V,H}$ is strictly decreasing in R, $F_0^{V,HL}$ does not depend on R, and $(F_0^{V,HL} + F_{1H}^V)$, $(F_0^{V,HL} + F_{1L}^V)$, $(F_0^{V,H} + F_{1H}^V)$ and $(F_0^{V,H} + F_{1L}^V)$ are all strictly convex functions of R.

Proof: From (A20), (A23), (A25) and (A27),

(A33) $\dfrac{\partial F_{1H}^V}{\partial R} = \dfrac{\lambda(1-\lambda)(p_g - p_m)q_g q_m I_1}{[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]^2} > 0$,

(A34) $\dfrac{\partial F_{1L}^V}{\partial R} = \dfrac{\lambda(1-\lambda)(p_g - p_m)q_g q_m I_1}{[(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m]^2} > 0$, (A35) $\dfrac{\partial F_0^{V,H}}{\partial R} = -\dfrac{\mu(1-\lambda)p_g q_g I_0}{\{p_s q_s + \mu[\{\lambda(1-R)+R\}p_g q_g + (1-\lambda)p_m q_m]\}^2} < 0$, and

(A36) $\dfrac{\partial F_0^{V,HL}}{\partial R} = 0$

Hence,

(A37) $\dfrac{\partial^2 F_{1H}^V}{\partial R^2} = \dfrac{2\lambda^2(1-\lambda)(p_g-p_m)p_g q_g^2 q_m I_1}{[\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]^3} > 0,$

(A38) $\dfrac{\partial^2 F_{1L}^V}{\partial R^2} = \dfrac{2\lambda(1-\lambda)^2(p_g-p_m)p_g q_g^2 q_m I_1}{[(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m]^3} > 0,$

(A39) $\dfrac{\partial^2 F_0^{V,H}}{\partial R^2} = \dfrac{2[\mu(1-\lambda)p_g q_g]^2 I_0}{\{p_s q_s + \mu[\{\lambda(1-R)+R\}p_g q_g + (1-\lambda)p_m q_m]\}^3} > 0$, and

(A40) $\dfrac{\partial^2 F_0^{V,HL}}{\partial R^2} = 0$

Hence, $(F_0^{V,HL} + F_{1H}^V)$, $(F_0^{V,HL} + F_{1L}^V)$, $(F_0^{V,H} + F_{1H}^V)$ and $(F_0^{V,H} + F_{1L}^V)$ are strictly convex functions of R.

Results regarding when H and L signals are funded

IR5. Under both angel and VC financing, if the L signal is not funded at a given λ, it is not funded at any higher level of λ either.

Proof: We present the proof for angel financing. The proof for VC financing is similar and is omitted. Consider a level of λ where the L signal is not funded, that is, $F_0^{A,HL} + F_1^{A,L} > V$. From (A16), $F_0^{A,HL} = \dfrac{I_0}{p_s q_s + p_g q_g + p_m q_m}$ is same for all λ. Since $F_1^{A,L}$ is a strictly increasing function of λ (Result IR1), $F_0^{A,HL} + F_1^{A,L} > V$ at any higher level of λ and the L signal is not funded.

IR6. Under both angel financing and VC financing, the H signal is funded by the new VC any time the L signal is funded.

Proof: A signal is funded at stage 1 if $V - F_0 - F_1 \geq 0$. Since, for $\lambda \geq .5$, $F_{1H} \leq F_{1L}$, the result follows.

IR7. Under either angel financing or VC financing, there is a threshold λ_L such that the new VC funds L signals if $\lambda \leq \lambda_L$ and does not fund L signals if $\lambda > \lambda_L$. Under both types of financing,

(A41) $\lambda_L \leq \dfrac{q_g(p_g V - I_1)}{q_g(p_g V - I_1) + q_m(I_1 - p_m V)}$

Proof: For a given type of financing, $\lim_{\lambda \downarrow 0} F_{1L} = \dfrac{I_1}{p_g}$. Hence, if λ is sufficiently small, L projects are funded. (Such λ may be less than 0.5.) From Result IR5, if the L signal is not funded at a given level of λ, it is not funded at any higher level of λ either.

Consider first angel financing. Since $F_0 \geq I_0 > 0$, $(V - F_0 - F_{1L}) < 0$ and the L signal is not funded if

$$V \leq F_{1L}^A = \frac{I_1[(1-\lambda)q_g + \lambda q_m]}{(1-\lambda)p_g q_g + \lambda p_m q_m}$$

$$\longleftrightarrow \quad (1-\lambda)(p_g V - I_1)q_g \leq \lambda(I_1 - p_m V)q_m$$

$$\longleftrightarrow \quad \frac{\lambda}{1-\lambda} \geq \frac{(p_g V - I_1)q_g}{(I_1 - p_m V)q_g} \quad \longleftrightarrow \quad \lambda \geq \lambda_L = \frac{(p_g V - I_1)q_g}{(p_g V - I_1)q_g + (I_1 - p_m V)q_m}$$

Similarly, for VC financing, $(V - F_0 - F_{1L}) < 0$ and the L signal is not funded if

$$V \leq F_{1L}^{V} = \frac{I_1[(1-\lambda)(1-R)q_g + \lambda q_m]}{(1-\lambda)(1-R)p_g q_g + \lambda p_m q_m}$$

$$\longleftrightarrow \quad (1-\lambda)(1-R)(p_g V - I_1)q_g \leq \lambda(I_1 - p_m V)q_m \quad \longleftrightarrow \quad \lambda \geq \frac{(1-R)(p_g V - I_1)q_g}{(1-R)(p_g V - I_1)q_g + (I_1 - p_m V)q_m}$$

Since $0 \leq R \leq 1$,

$$\lambda_L = \frac{(p_g V - I_1)q_g}{(p_g V - I_1)q_g + (I_1 - p_m V)q_m} \quad \geq \quad \frac{(1-R)(p_g V - I_1)q_g}{(1-R)(p_g V - I_1)q_g + (I_1 - p_m V)q_m}$$

Hence the result.

IR8. Under both angel financing and VC financing, the H signal is funded if λ is sufficiently high.

Proof: From (A4) and (A19),

$$\lim_{\lambda \to 1} F_{1H}^{A} = \lim_{\lambda \to 1} F_{1H}^{V} = \frac{I_1}{p_g}$$

The result follows from model assumptions.

IR9. Suppose, for angel financing, the following condition holds:

(A42) $(\frac{I_1}{I_0})(1 - \frac{p_m}{p_g})q_m \geq (\frac{p_m q_m}{p_g q_g}) - 1$

Then, there is a threshold λ^A such that the H signal is funded if $\lambda \geq \lambda^A$.

Proof: Consider first angel seed financing and any $\lambda < 1$ such that an L project is not funded. Then, an L project is not funded at any higher level of λ as well (Result IR5). At this level of λ, an H project is either not funded or funded.

Case A1. H project is funded at this λ: By condition (A44), $(F_0^{A,H} + F_{1H}^{A})$ is strictly decreasing in λ (Result IR2). Hence, an H project is funded at any higher level of λ also.

Case A2. H project is not funded at this λ: Suppose λ increases from this level. Since $(F_0^{A,H} + F_{1H}^{A})$ is strictly decreasing in λ and an H project is funded if λ is sufficiently high (Result IR8), there is a threshold λ^A such that $F_0^{A,H} + F_{1H}^{A} \leq V$ if $\lambda \geq \lambda^A$, that is, H projects are funded if $\lambda \geq \lambda^A$.

IR10. Suppose, for VC financing, the following condition holds:

(A43) $p_m q_m I_0 \leq (1-R)[\mu(p_g - p_m)q_g q_m I_1 + p_g q_g I_0]$

Then, there is a threshold λ^V, such that the H signal is funded if $\lambda \geq \lambda^V$.

Proof: Start with any $\lambda < 1$ such that an L project is not funded. Then, an L project is not funded at any higher level of λ as well (Result IR5). At this level of λ, an H project is either not funded or funded.

Case V1. H project is funded at the starting λ: By condition (A45), $(F_0^{V,H} + F_{1H}^{V})$ is a strictly decreasing in λ (Result IR3). Hence, an H project is funded at any higher level of λ also.

Case V2. H project is not funded at the starting λ: Suppose λ increases from this level. Since $(F_0^{V,H} + F_{1H}^{V})$ is strictly decreasing in λ and the H project is funded if λ is sufficiently high (Results IR3 and IR8), there is a threshold λ^V such that $F_0^{V,H} + F_{1H}^{V} \leq V$ if $\lambda \geq \lambda^V$, that is, H projects are funded if $\lambda \geq \lambda^V$.

IR11. Under VC financing, the H signal is not funded if $R > R_H = \dfrac{\lambda q_g(p_g V - I_1) - (1-\lambda)q_m(I_1 - p_m V)}{\lambda q_g(p_g V - I_1)}$ and the L signal is not funded if $R > R_L = \dfrac{(1-\lambda)q_g(p_g V - I_1) - \lambda q_m(I_1 - p_m V)}{(1-\lambda)q_g(p_g V - I_1)}$.

Proof: $R > R_H = \dfrac{\lambda q_g(p_g V - I_1) - (1-\lambda)q_m(I_1 - p_m V)}{\lambda q_g(p_g V - I_1)}$

$\longrightarrow \quad [\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m]V < [\lambda(1-R)q_g + (1-\lambda)q_m]I_1 \longrightarrow V < F^V_{1H}$ (from A19).

Hence, $V < F_0 + F_1$, that is, H projects are not funded if $R > R_H$. Similarly, $V < F^V_{1L}$, and L projects are not funded if $R > R_L$.

1.3. Proofs of Propositions and Results on Funding Hole

Proposition 1: Consider VC seed financing where $\lambda < 1$ is large enough that the new VC never funds the L signal, and funds the H signal when $R = 0$. Then, there is a threshold $\overline{R}$, $0 \leq \overline{R} < 1$, such that the new VC funds the H signal if $R \leq \overline{R}$ but does not fund the H signal if $R > \overline{R}$.

Proof: From Result IR6, we can always choose λ large enough that the L signal is not funded at any any level of R, for example by choosing $\lambda \geq \lambda_L$.

Consider now the H signal and define $\phi(R) = F_0^{V,H} + F^V_{1H} - V$. Thus, the H signal is funded if and only if $\phi(R) \leq 0$. By assumption, $\phi(0) \leq 0$. By Result IR11, $\phi(R) > 0$ if $R \geq R_H$. By Result IR4, $\phi(R)$ is a strictly convex function of R. Also, from (A35) and (A36), $\phi'(R)$ is a continuous function of R. Two cases are possible.

Case 1: $\phi'(R) \geq 0$ at $R = 0$. Then, by the strict convexity of $\phi(R)$, $\phi'(R) > 0$ $\forall R > 0$, that is, $\phi(R)$ is strictly increasing in R $\forall R > 0$. Since $\phi(0) \leq 0$ and $\phi(R_H) > 0$, continuity of $\phi(R)$ implies that $\exists R_1$, $0 \leq R_1 < R_H$, such that $\phi(R_1) = 0$. Since ϕ is strictly increasing in R here, $\phi(R) < 0$ if $R < R_1$ and $\phi(R) > 0$ if $R > R_1$. Hence $\overline{R} = R_1$.

Case 2: $\phi'(R) < 0$ at $R = 0$. By continuity of $\phi'(R)$, we can have $h > 0$ such that $\phi'(R) < 0$ if $0 \leq R < h$. Since $\phi(0) \leq 0$, $\phi(R) < 0$ if $0 < R < h$. Since $\phi(R_H) > 0$, we must have R, $0 < R < R_H$, such that $\phi(R) = 0$. By strict convexity of ϕ, we can have $\phi(R) = 0$ at at most two values of R. Let R_1 be the smallest $R > 0$ where $\phi(R) = 0$. We now show that $\overline{R} = R_1$.

Zone $R > R_1$: By the mean value theorem, $\exists \hat{R}$, $0 < \hat{R} < R_1$, such that

$R_1 * \phi'(\hat{R}) = (R_1 - 0)\phi'(\hat{R}) = \phi(R_1) - \phi(0) \geq 0$, that is, $\phi'(\hat{R}) \geq 0$. Since $R_1 > \hat{R}$, it follows from the strict convexity of $\phi(R)$ that $\phi'(R) > 0$ if $R \geq R_1$. Hence, if $R > R_1$, $\phi(R) > 0$ and the H signal is not funded.

Zone $R \leq R_1$: Since $\phi(R_1) = 0$, the H signal is funded at $R = R_1$. We now show that $\phi(R) \leq 0$ for any R between 0 and R_1. Suppose otherwise, that is, we have R_2, $0 < R_2 < R_1$, such that $\phi(R_2) > 0$. Since $\phi(R) < 0$ if $0 < R < h$ and clearly $h < R_1$, we can also have R_3, $0 < R_3 < R_1$, such that $\phi(R_3) < 0$. Then, by continuity of $\phi(R)$, we must have R between R_3 and R_2 such that $\phi(R) = 0$, which is a contradiction since R_1 is the smallest $R > 0$ where $\phi(R) = 0$. Therefore, $\phi(R) \leq 0$, that is, the H signal is funded, if $R < R_1$ also.

Corollary: Under the conditions of Proposition 1, $\phi'(\overline{R}) > 0$ if $\overline{R} > 0$.

Proof: In Case 1, $\phi'(R) > 0$ at any $R > 0$, hence $\phi'(R) > 0$ at $R = \overline{R}$. From the proof in Case 2, $\phi'(\hat{R}) \geq 0$ where $\hat{R} < \overline{R}$. Strict convexity of $\phi(R)$ implies that $\phi'(\overline{R}) > 0$.

Proposition 2. Suppose $\lambda < 1$ is large enough that new VC never funds the L signal and funds the funds the H signal for angel seed financing. Let $\overline{R}$ be as defined in Proposition 1. Then,

(1) Under VC seed financing, the new VC funds the H signal when $R = 0$.

(2) The ex ante expected outcome of the entrepreneur is higher under angel financing than under VC financing if

(A44) $\overline{R} < R < \tilde{R} = \dfrac{[\lambda q_g(p_g V - I_1) - (1-\lambda)q_m(I_1 - p_m V)}{\mu q_g(p_g V - I_1)}$

(3) The ex ante expected outcome of the entrepreneur is higher under VC financing than under angel financing if $R \leq \overline{R}$ or $R > \tilde{R}$.

Proof: If $R = 0$, F_{1H} is same for both angel and VC financing. Comparing parallel cases for angel and VC seed financing, F_0 for VC financing if $R = 0$ is never greater than F_0 for angel seed financing. Therefore, since the new VC funds the H signal under angel financing, the new VC also funds the H signal under VC financing when $R = 0$. From Proposition 1, there is a threshold $\overline{R} \geq 0$ such that under VC financing, the H signal is funded if $R \leq \overline{R}$, and not funded if $R > \overline{R}$. We examine the two zones $R > \overline{R}$ and $R \leq \overline{R}$ separately.

Zone $R > \overline{R}$: Here, the H and S projects are funded by the new VC under angel financing, and only the S project is funded by the new VC under VC financing. From (A15) and (A26), the entrepreneur's ex ante expected outcomes for the two systems are:

$$\Pi^A = \Pi^{A,H} = q_s(p_sV - I_1) + \lambda q_g(p_gV - I_1) - (1-\lambda)q_m((I_1 - p_mV) - I_0$$

$$\Pi^V = \Pi^{V,S} = q_s(p_sV - I_1) + \mu R q_g(p_gV - I_1) - I_0$$

Hence,

$$\Pi^A - \Pi^V = \lambda q_g(p_gV - I_1) - (1-\lambda)q_m(I_1 - p_mV) - \mu R q_g(p_g - I_1)$$

Therefore, $\Pi^A > \Pi^V$ if

$$\overline{R} < R < \tilde{R} = \frac{\lambda q_g(p_gV - I_1) - (1-\lambda)q_m(I_1 - p_mV)}{\mu q_g(p_gV - I_1)},$$

and $\Pi^V > \Pi^A$ if $R > \tilde{R}$.

Zone $R \leq \overline{R}$: Under angel financing, only the H and S projects are funded by the new VC at date 1, and the ex ante expected outcome the entrepreneur is

$$\Pi^A = \Pi^{A,H} = [p_sq_s + \lambda p_gq_g + (1-\lambda)p_mq_m]V - I_0 - [q_s + \lambda q_g + (1-\lambda)q_m]I_1$$

In VC financing, since $R \leq \overline{R}$, H and S projects are funded, and the entrepreneur's expected outcome is: $\Pi^V = \Pi^{V,H} = q_s(p_sV - I_1) + \mu q_g\{R + \lambda(1-R)\}(p_gV - I_1) - \mu(1-\lambda)q_m(I_1 - p_mV) - I_0$,

that is,

$$\frac{\partial \Pi^V}{\partial \mu} = Rq_g(p_gV - I_1) + [\lambda(1-R)p_gq_g + (1-\lambda)p_mq_m]V - [\lambda(1-R)q_g + (1-\lambda)q_m]I_1$$

$$= Rq_g(p_gV - I_1) + [\lambda(1-R)p_gq_g + (1-\lambda)p_mq_m](V - F^V_{1H}) > 0,$$

since $p_gV > I_1$ (assumption) and $V \geq F_0 + F_1 \geq 0$ (since $R \leq \overline{R}$).

Now, if $\mu = 1$,

$$\Pi^V - \Pi^A = q_g\{R + \lambda(1-R) - \lambda\}(p_g - I_1) = q_gR(1-\lambda)(p_gV - I_1) > 0$$

since $\lambda < 1$ and $p_gV > I_1$. Hence, $\Pi^V > \Pi^A$ for any $\mu \geq 1$.

Result on Relation between Model Parameters and Funding Hole. Suppose, for VC financing, λ is large enough that the new VC never funds the L signal, and funds the H signal if $R = 0$. If $\overline{R} > 0$, then $\dfrac{d\overline{R}}{d\mu} > 0$ and if, in addition,

(A45) $p_mq_mI_0 \leq (1 - \overline{R})[\mu(p_g - p_m)q_gq_mI_1 + p_gq_gI_0]$,

then $\dfrac{d\overline{R}}{d\lambda} > 0$.

Proof: Under the condition specified, $\overline{R}$ is the unique solution of $\phi(R) = 0$, where

(A46) $\phi(R) = F_0^{V,H} + F_{1H}^{V} - V$

$$= \frac{I_0}{p_s q_s + \mu[\{\lambda(1-R) + R\}p_g q_g + (1-\lambda)p_m q_m]} + \frac{I_1[\lambda(1-R)q_g + (1-\lambda)q_m]}{\lambda(1-R)p_g q_g + (1-\lambda)p_m q_m} - V$$

From the corollary to Proposition 1, $\frac{\partial \phi}{\partial R} > 0$ (since $\overline{R} > 0$). Also, $\frac{\partial \phi}{\partial \mu} < 0$. Therefore,

$$\frac{d\overline{R}}{d\mu} = -\frac{\frac{\partial \phi}{\partial \mu}}{\frac{\partial \phi}{\partial R}} > 0.$$

If (A45) is also satisfied, then $\frac{\partial \phi}{\partial \lambda}|_{R=\overline{R}} < 0$ (Result IR3, Appendix 1.2).

Hence, $\frac{d\overline{R}}{d\lambda} = -\frac{\frac{\partial \phi}{\partial \lambda}|_{R=\overline{R}}}{\frac{\partial \phi}{\partial R}} > 0$