The Changing Polarization of Party Ideologies: The Role of Sorting

Satyajit Chatterjee and Burcu Eyigungor

Federal Reserve Bank of Philadelphia

September 2023

Abstract

Ideology scores derived from U.S. congressional roll-call voting patterns show that the ideological distance between the two parties along the primary dimension changes inversely with the ideological distance along the secondary dimension. To explain this inverse association, a model of party competition with endogenous party membership and a two-dimensional ideology space is developed. If the distribution of voter preferences is uniform on a disk, equilibrium ideological distances along the two dimensions are inversely related. The model can quantitatively account for the historical movements in ideological distances as a function of changes in the ideological orientation of the two parties.

1 Introduction

Much of the attention on Poole and Rosenthal's (1997) widely cited and widely used roll-call ideology scores has been on the polarization of political parties along the first ("liberal versus conservative" or "left versus right") dimension. Figure 1 plots the absolute

Figure 1:
Polarization Along the Liberal-Conservative Dimension
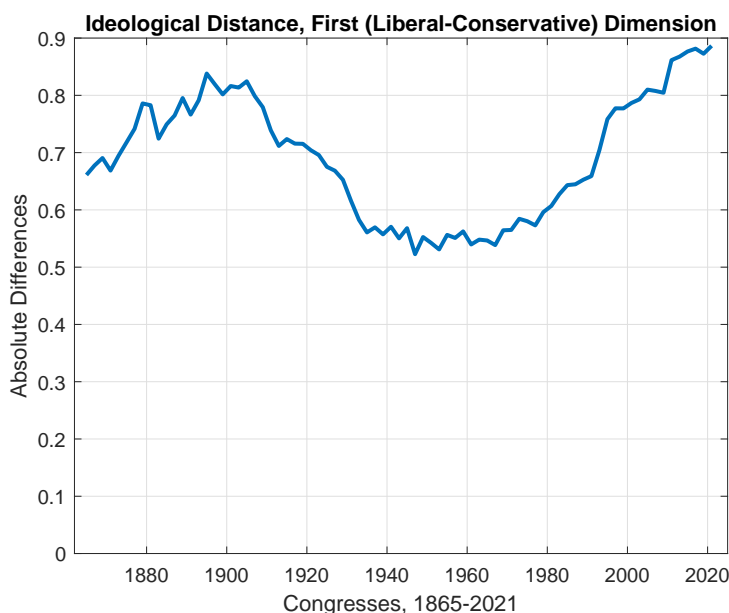
Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022) and authors' calculation. The ideological distance for each Congress is the absolute value of the difference in average ideological scores of Democrats and Republicans along the primary dimension for that Congress.

difference in the mean ideological position of Democrats and Republicans along the first (liberal-conservative) dimension, from the 39th to the 117th Congresses. The ideological distance between the two parties has waxed and waned: The distance shrank during the first eight decades of the 20th century but has been rising since 1980. Currently, the distance is the widest it has been since 1865, corroborating the general perception that political polarization has increased substantially in recent decades.

In contrast, the ideological distance along the second dimension has received less attention. Figure 2 plots the absolute difference in mean ideological positions along the second dimension. Historically, the second dimension has picked up differences within the two parties on issues such as slavery and currency — which show up as the peaks in

1865 and 1890 — and the long-running differences regarding civil rights which peaked in the 1940s and 1950s and then declined following the 1964 Civil Rights Act.

Figure 2:
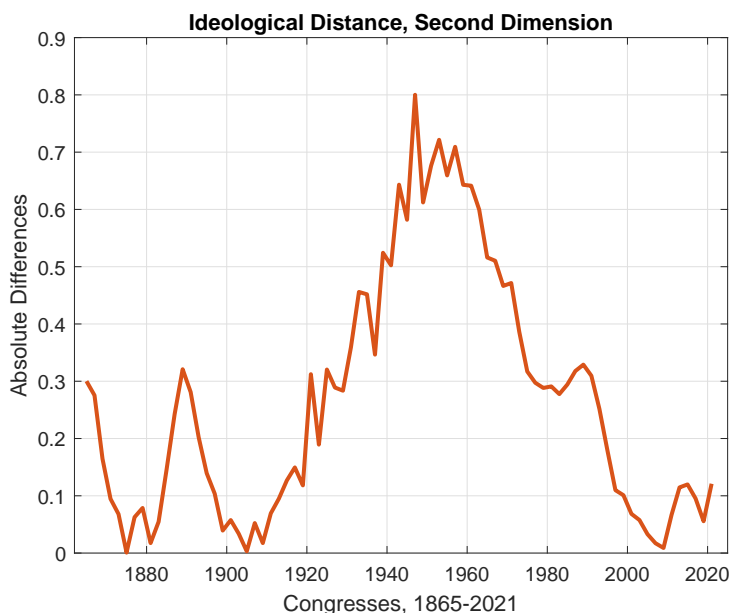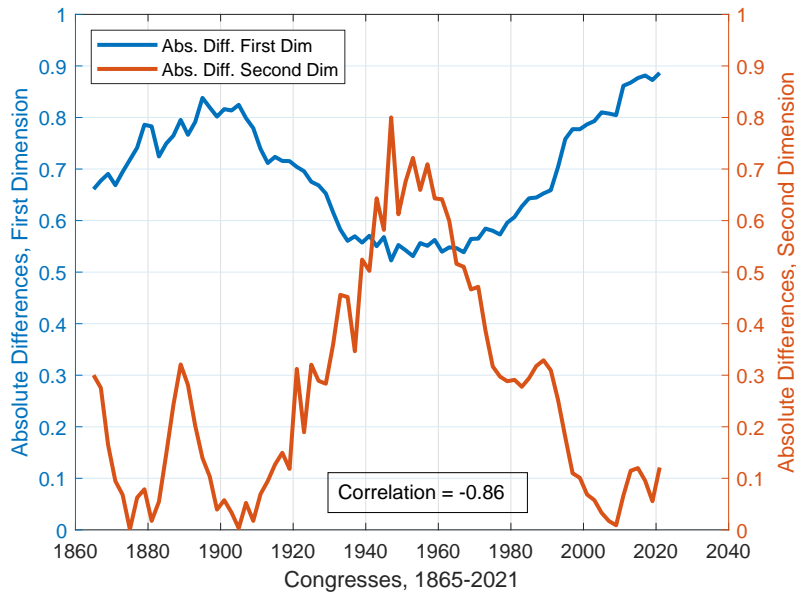Polarization Along the Second Dimension

Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022) and authors' calculation. The ideological distance for each Congress is the absolute value of the difference in average ideological scores of Democrats and Republicans along the second dimension for that Congress.

For the purposes of this paper, the key takeaway from these plots is the manner in which the ideological distances move relative to each other: When ideological distance increases along one dimension, it tends to *decrease* along the other dimension. This pattern of inverse association is readily seen in Figure 3: Periods when the ideological distance along the second dimension rose tend to be periods when the ideological distance along the primary dimension fell. Indeed, the correlation between the two series is −0.86.

Our contribution is to show that this inverse association might not be a historical accident and is, in fact, implied by a spatial model of voting in which party membership is endogenous and party ideologies are formed in primary elections that precede the national election.

In our model, voters are heterogeneous regarding their preferences (equivalently, ideologies) and sort into the two parties depending on what they believe each party stands

Figure 3:
Inverse Association of Ideological Distances

for. The primary elections then turn the preferences of primary attendees into the party platforms on which the parties compete in the national election. We show that when voter preferences are uniformly distributed on a disk there is a circle in the two-dimensional ideology space such that any two diametrically opposite points on the circle constitute *equilibrium* beliefs: If voters associate the two parties with these diametrically opposing ideologies, the resulting sorting of voters into the two parties generates party ideologies that confirm those beliefs. Comparing across equilibria, a larger ideological distance along one dimension is always associated with a smaller ideological distance along the other dimension.

Less formally, the main idea is this: When some issue causes parties to polarize along a social dimension — such as the issue of desegregation prior to the passage of the Civil Rights Act — parties will attract voters with varying economic circumstances who are similarly disposed toward segregation (either for or against it). The greater mixing of economically disparate voters within each party means that party members will not necessarily see eye-to-eye on many bread-and-butter issues. This fact will force the parties to

offer platforms that are moderate with respect to economic issues and thus closer along the primary left-right dimension.

In our model, what matters for the sorting of voters into the two parties (and thus for the policy outcomes) are simply their beliefs regarding the *direction* of party ideologies. In our quantitative work, we exploit this feature to obtain model-implied ideological distances between the two parties, given the observed direction of party ideology for each Congress. We show that our model gives a good quantitative account of the movements in ideological distances seen in Figures 1 and 2. Specifically, it can account for the downward and upward drifts in the first- and second-dimension distances from the mid-1920s to the mid-1960s and their subsequent reversals up to the mid-1990s. However it cannot account for the rise in the first-dimension distances since the mid-1990s as the observed direction of party ideologies has not changed much in the last 30 years.

Related Literature: Poole and Rosenthal (1984) and Poole and Rosenthal (1997) drew attention to the rising polarization along the primary dimension since the early 1980s.[1] In follow-up work, McCarty, Poole, and Rosenthal (2006) focused on the fact that the two parties were more stratified by income (with a higher share of higher income individuals identifying with the Republican party and a higher share of lower income individuals with the Democratic party) than 50 years ago. They viewed rising income inequality as the likely cause of increasing polarization of the two parties.[2]

A second force that has attracted attention are campaign contributions. Herrera, Levine, and Martinelli (2008) present a model in which increasing uncertainty about

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[1]Rising political polarization has been found in other data. Using the text of *Congressional Record* and published political discourse more generally, Jensen, Naidu, Kaplan, Wilse-Samson, Gergen, Zuckerman, and Spirling (2012) found evidence of rising polarization since the mid 1990s (Figure 3, bottom panel, p. 23). Gentzkow, Shapiro, and Taddy (2019) substantially sharpened this message by correcting for finite-sample bias (Figure 2, p. 1321). Survey-based measures that examine affective polarization also find an increase in political polarization since the mid 1990s (see Iyengar, Lelkes, Levendusky, Malhotra, and West-wood (2019), Figure 1, p.132). The fact that alternative measures of political polarization show no change until the mid 1990s seem inconsistent with Figures 1 and 2. However, the average Euclidean distance between the ideology points of the two parties for the 1865-1945 congresses and the 1947-1993 congresses are roughly the same, being 0.77 and 0.76, respectively, but jumps to 0.83 for the 1995-2021 congresses. In this sense, Figures 1 and 2 may not be inconsistent with the findings from text-based and survey-based measures.

[2]See Barber and McCarty (2013) for a survey of the more empirical research on the causes and consequences of polarization.

election outcomes increases the size of campaign contributions as well as polarization of party platforms. Drautzburg, Livshits, and Wright (2021) show how increases in legal contribution limits to political campaigns can lead to greater polarization in candidate positions, provided campaign contributions generate private benefits for contestants.

A third force is imperfect understanding of the true state of affairs. Dixit and Weibull (2007) present a model in which policy failures lead some individuals to want to reverse course and others to advocate an even stronger form of the failed policy. Nimark and Sunderasan (2019) present a model in which information is freely available but costly to process and endogenously divides people into camps with opposite beliefs. Also using a model of costly information processing, Matějka and Tabellini (2021) show how increased granularity of information can cause policy divergence. Azzimonti and Fernandes (2023) present a model in which the advent of social media as a news source causes polarization of beliefs.

Unlike the studies mentioned thus far, two recent studies stress the interaction between polarization along two different dimensions, as we do. Krasa and Polborn (2014) examine how polarization along the economic dimension is affected by polarization along the social/cultural dimension. They present a model in which contestants have opposing views on cultural issues but choose positions along the economic dimension. Under certain conditions, increased divergence in candidate views along the social dimension can cause increased polarization of positions chosen on the economic dimension. Our approach differs from theirs in that party platforms are determined by the composition of voters in each party and our goal is to explain the inverse association in ideological distances documented earlier.

Konishi and Pan (2020) study a model in which policy choices also have social and economic dimensions. In their model, third parties lobby *both* contestants to ensure the economic dimension is not politically salient. This leaves contestants to differentiate themselves by espousing different social policies, and divergence along this dimension can increase if lobbying on the economic dimension becomes more intense.

Beyond papers that focus on polarization, there is a closely related literature on party positioning and policy divergence.[3] In an early paper, Coleman (1971) showed how differences in party preferences can arise if candidates must win a primary election before contesting the national election. Relatedly, Wittman (1973) and Calvert (1985) sought to locate policy divergence in exogenous differences in party preferences. In all these studies, party membership is exogenous. Roemer (2001) and Poutvaara (2003) proposed models in which party preferences are derived from voter preferences and party membership is endogenous. Our modeling approach is in the spirit of the latter two papers, the main difference being that we model two-dimensional as opposed to single-issue polarization.

Finally, there is a macroeconomic literature on the consequences of political polarization. This literature assumes two types of agents, each represented by a political party, where the probability of a party becoming the ruling party is either given or determined in a general election.[4] This framework has been used to study the impact of polarized preferences on fiscal policy in Persson and Svensson (1989) and Alesina and Tabellini (1990); on aggregate investment in Azzimonti (2011); on sovereign borrowing and default in Cuadra and Sapriza (2008); on allocative inefficiencies in Acemoglu, Golosov, and Tsyvinski (2011); on policy extremism and reelection probabilities in Chatterjee and Eyigungor (2020); and on status-quo effect and efficiency in Bowen, Chen, and Eraslan (2014) and Piguillem and Riboni (2021). All these studies are explicitly dynamic and most are quantitative. Our paper shares the quantitative focus of this literature but abstracts from explicit dynamics in order to delve deeper into the formation of party preferences.

The rest of the paper is organized as follows. Section 2 lays out the model environment and Section 3 explains how we can get an inverse association between the two ideological

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[3]The impetus for this literature came from Downsian models of party competition (Downs (1957)) in which both parties declared the same platform. As pointed out by many, this prediction is at odds with the fact that parties offer different platforms and it is quite inconsistent with Figures 1 and 2, which show that the two U.S. parties have consistently differed in their ideologies.

[4]A closely related literature models fiscal policy as the outcome of legislative bargaining game; see Battaglini and Coate (2008).

distances. Section 4 takes this insight further and shows that it has some quantitative bite. Section 5 concludes.

2 The Model

Our model consists of a continuum of voters and two political parties. The parties contest a general election to determine which party's policies are adopted. Leading up to the general election, the parties determine their respective policy platforms in separate primary elections.

The timeline of events is as follows. At the start of the period, voters sort into primaries depending on their beliefs about the ideology of each party. Then, there are elections for the primaries of each party that determine each party's platform for the national election. Finally, the national election takes place and the governing party is determined. We solve the model by backward induction.

2.1 Voters

There is a set of *voters* who care about two issues. In the space of issues, a voter's *type* is defined by $(x,y) \in \mathbb{R}^2$. If policies chosen by the government are (w,z), the utility of a type (x,y) voter is:

$$U(x,y;w,z) = -[(w-x)^2 + (z-y)^2]. \tag{1}$$

Thus (x,y) denotes the voter's ideal policies. The density of the distribution of voter types is $q(x,y) \geq 0$. For the moment, we assume only that this distribution is symmetric around $(0,0)$, i.e.,

$$q(x,y) = q(-x,-y) \quad \forall (x,y) \in \mathbb{R}^2. \tag{2}$$

Later on, $q(x,y)$ will be specialized to a uniform density on a disk centered at $(0,0)$.

2.2 Political Parties and the National Election

There are two political parties referred to as the D party and the R party. We denote their respective platforms by (w_k, z_k), $k \in \{D, R\}$. In this (sub)section we take these policies as given and discuss how they affect the outcome of the national elections. The determination of these policies will be discussed in the next section.

Let $\mathcal{P}$ denote the 4-tuple (w_D, z_D, w_R, z_R). The party that gets to govern is determined in a national election via majority vote. A type (x, y) voter votes for party D if

$$U(x, y; w_D, z_D) + A \geq U(x, y; w_R, w_D). \tag{3}$$

Otherwise, she votes for party R. Here, $A \in \mathbb{R}$ is a net preference for the D party *per se* that is realized at the time of the election and is common to all voters. It is a random variable symmetrically distributed around 0 with a CDF $F(A)$ continuous in A.

Given (3), D party will win the national election if A exceeds some $\mathcal{P}$-dependent threshold $\bar{A}(\mathcal{P})$.[5] We assert that $\bar{A}(\mathcal{P})$ is the value of A for which the mean voter, i.e., the voter of type $(0, 0)$, is indifferent between the two parties. To see why, suppose, without loss, that $w_D \neq w_R$. For each y, let $x(y, A, \mathcal{P})$ be such that the voter of type $(x(y, A, \mathcal{P}), y)$ is indifferent between the two parties. From (3) we get

$$x(y, A, \mathcal{P}) = \frac{\left[(z_D - y)^2 - (z_R - y)^2\right] - A + \left[w_D^2 - w_R^2\right]}{2\left[w_D - w_R\right]}. \tag{4}$$

The value of A for which the voter of type $(0, 0)$ is indifferent between the two parties solves $x(0, A, \mathcal{P}) = 0$. This yields:

$$\bar{A}(\mathcal{P}) = [w_D^2 + z_D^2] - [w_R^2 + z_R^2]. \tag{5}$$

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[5]If $w_D > w_R$, we may verify that $U(x, y; w_D, z_D) + A - U(x, y; w_R, z_R)$ is increasing in x. In this case a voter of type (x, y) votes for the D party if $x \geq x(y, A, \mathcal{P})$ and votes for the R party if $x < x(y, A, \mathcal{P})$. On the other hand, if $w_D < w_R$ then $U(x, y; w_D, z_D) + A - U(x, y; w_R, z_R)$ is decreasing in x and a voter of type (x, y) votes for the D party if $x \leq x(y, A, \mathcal{P})$ and votes for the R party if $x > x(y, A, \mathcal{P})$. Regardless, an increase in A expands the set of voters who vote for the D party.

Next, observe that (4) and (5) together imply

$$x(y, \bar{A}(\mathcal{P}), \mathcal{P}) = y \left[\frac{z_R - z_D}{w_D - w_R} \right]. \tag{6}$$

Equation (6) implies that if $\tilde{x} > x(\tilde{y}, \bar{A}(\mathcal{P}), \mathcal{P})$, then $-\tilde{x} < x(-\tilde{y}, \bar{A}(\mathcal{P}), \mathcal{P})$. Therefore, given $A = \bar{A}(\mathcal{P})$, every pair of voters with symmetrically opposite preferences will vote for different parties. From the symmetry of $q(x, y)$, it follows that the measure of voters voting for the two parties must be exactly one-half. Since the l.h.s. of (3) is increasing in A, the D party will achieve a majority if $A > \bar{A}(\mathcal{P}) = [w_D^2 + z_D^2] - [w_R^2 + z_R^2]$.[6]

2.3 Primaries and the Formation of Party Platforms

We turn now to the formation of party platforms. We imagine that voters have common expectations about the policy platform each party will ultimately declare. Based on these expectations, each voter participates in the primary of his or her preferred party.

Figure 4:
Sorting of Voters Based on Beliefs
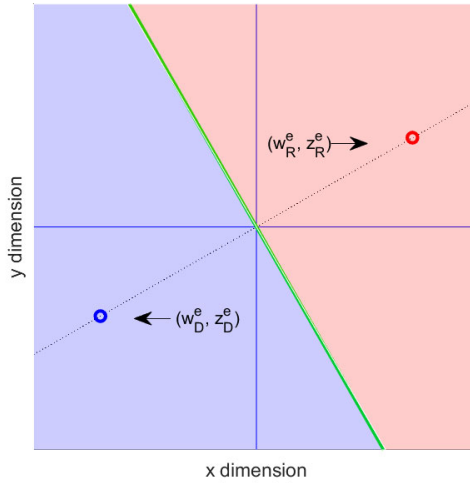
The (common) beliefs of voters determine which primary election they participate in. Let $\mathcal{P}^e = (w_D^e, z_D^e, w_R^e, z_R^e)$ denote the voters' (common) expectations of party platforms.

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[6]If $w_D = w_R$ but $z_D \neq z_R$, an analogous argument establishes (5). If $w_D = w_R$ and $z_D = z_R$, then all voters (not just the ones with mean preferences) are indifferent between the two parties and the D party will win if $A > 0$. Thus $\bar{A}(\mathcal{P}) = 0$ and (5) remains true.

Define set of voters who weakly prefer the platform (w_k^e, z_k^e) as $H_k(\mathcal{P}^e) = \{(x,y) \in \mathbb{R}^2 :$ $U(x,y;w_k^e,z_k^e) \geq U(x,y;w_{\sim k}^e,z_{\sim k}^e)\}$, where $\sim k = R(D)$ if $k = D(R)$. As shown in Figure 4, these sets are separated by the straight line passing through the midpoint of the line joining (w_D^e, z_D^e) and (w_R^e, z_R^e) and perpendicular to it:

$$\bar{x} = \frac{w_D^{e\,2} + z_D^{e\,2} - [w_R^{e\,2} + z_R^{e\,2}]}{2(w_D^e - w_R^e)} - \left[\frac{z_D^e - z_R^e}{w_D^e - w_R^e}\right]y. \tag{7}$$

Utility is the negative of the square of the Euclidean distance from (w_k^e, z_k^e), so all voters on this line are indifferent between the two (expected) platforms. Voters on either side have a strict preference for one of the parties and, so, attend the primary of that party. We assume that voters on the line attend both primaries.

In each primary, two candidates vie to represent their party in the national election. Candidates are office motivated, i.e, care only about winning the primary election. By the Downsian logic, the candidates offer the same platform. If policies do not fully pin down votes for the two candidates because of random variation in how much voters like a candidate independent of his or her proposed policy, then, as shown in Lindbeck and Weibull (1987) and Persson and Tabellini (2000), the equilibrium platform coincides with the solution of the utilitarian social welfare maximization problem.

With this equivalence in mind, we assume that the D-party platform is determined by the following programming problem:

$$\max_{(w,z)} \left\{ \begin{array}{l} \pi(w,z,w_R,z_R)\,\mathbb{E}_{(x,y)}\left(\left[-(w-x)^2 - (z-y)^2\right]\mathbb{1}_{\{(x,y)\in\mathbb{H}_D(\mathcal{P}^e)\}}\right) \\ + [1 - \pi(w,z,w_R,z_R)]\,\mathbb{E}_{(x,y)}\left(\left[-(w_R-x)^2 - (z_R-y)^2\right]\mathbb{1}_{\{(x,y)\in\mathbb{H}_D(\mathcal{P}^e)\}}\right) \end{array} \right\}. \tag{8}$$

Here $\pi(w,z,w_R,z_R)$ is the probability of the D party winning the national election given these policies. Thus, we assume that a party's social welfare takes account of the likelihood of the party winning the national election on the basis of any given platform.[7] Note

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[7]At this choice stage, the parties contemplate platforms that are different from what voters expect. In equilibrium, the chosen platforms will coincide with expected ones.

that the social welfare of the D party depends only on the voters who weakly prefer the D party given beliefs $\mathcal{P}^e$, i.e., those who belong to $H_D(\mathcal{P}^e)$.

Using the fact that the probability of a D party win is $(1 - F([w^2 + z^2] - [w_r^2 + z_R^2]))$, the choice problem can be expressed as

$$\max_{(w,z)} \left\{ \begin{array}{c} -L(w_R, z_R) + \\ \left[1 - F\left(w^2 + z^2 - [w_R{}^2 + z_R{}^2]\right)\right] \times \\ m(H_D(\mathcal{P}^e)) \mathbb{E}_{(x,y)|H_D(\mathcal{P}^e)} \left\{ -\left[(w - x)^2 + (z - y)^2\right] + \left[(w_R - x)^2 + (z_R - y)^2\right] \right\} \end{array} \right\}, \quad (9)$$

where $m(H_D(\mathcal{P}^e))$ is the measure of the set of people who attend the D-party primary, $\mathbb{E}$ is expectation taken with respect to the distribution of (x, y) conditional on $(x, y) \in H_D(\mathcal{P}^e)$, and $L(w_R, z_R)$ is shorthand for $\mathbb{E}_{(x,y)|H_D(\mathcal{P}^e)}[(w_R - x)^2 + (z_R - y)^2]$. Ignoring the constant term L and the multiplicative term $m(\cdot)$, the objective function is thus the product of the probability of D party winning the national election and the total net gain from doing so.

Analogously, but recognizing that R party's probability of winning the election is $F([w_D^2 + z_D^2] - [w^2 + z^2])$, R party's choice problem reduces to

$$\max_{(w,z)} \left\{ \begin{array}{c} -L(w_D, z_D) + \\ F\left(w_D^2 + z_D^2 - [w^2 + z^2]\right) \times \\ m(H_R(\mathcal{P}^e)) \mathbb{E}_{(x,y)|H_R(\mathcal{P}^e)} \left\{ -\left[(w - x)^2 + (z - y)^2\right] + \left[(w_D - x)^2 + (z_D - y)^2\right] \right\} \end{array} \right\}. \quad (10)$$

If the k party cared *only* about winning the national election, its optimal policy would be to set $(w_k, z_k) = (0, 0)$ because this choice maximizes the probability of a k-party win. On the other hand, if it cared *only* about maximizing the total net gain from winning, its optimal policy would be to set $(w_k, z_k) = (\mathbb{E}(x|H_k(\mathcal{P}^e)), \mathbb{E}(y|H_k(\mathcal{P}^e))$ as $\mathbb{E}_{(x,y)|H^k(\mathcal{P}^e)}\left[(w - x)^2 + (z - y)^2\right]$ is minimized if w_k and z_k are set to the conditional means of x and y, respectively. Generally speaking, these two goals are mutually incompatible and the party's best response strikes a balance between them.[8]

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[8]In Roemer (2001), these two extremes figure as motivations ascribed to "opportunists" (who care only about winning) and "militants" (who care only about the gain, conditional on winning). What we are referring to as the social planner is called "reformists."

For more intuition, first-order conditions are helpful. If at the optimum $1 - F(\cdot) > 0$ and $F'(\cdot)$ exists, k party's optimal policy will satisfy the following marginal conditions:

$$(1 + \phi_k)w = \mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}x \quad \text{and} \quad (1 + \phi_k)z = \mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}y, \tag{11}$$

where

$$\phi_D = \left[\frac{F'\left(w^2 + z^2 - w_R^2 - z_R^2\right)}{1 - F\left(w^2 + z^2 - w_R^2 - z_R^2\right)} \right] \times$$

$$\mathbb{E}_{(x,y)|H_D(\mathcal{P}^e)}\{-(w - x)^2 - (z - y)^2 + [(w_R - x)^2 + (z_R - y)^2]\}, \tag{12}$$

and

$$\phi_R = \left[\frac{F'\left(w_D^2 + z_D^2 - w^2 - z^2\right)}{1 - F\left(w_D^2 + z_D^2 - w^2 - z^2\right)} \right] \times$$

$$\mathbb{E}_{(x,y)|H_R(\mathcal{P}^e)}\{-(w - x)^2 - (z - y)^2 + [(w_D - x)^2 + (z_D - y)^2]\}. \tag{13}$$

The terms ϕ_D and ϕ_R can be negative only if the expectation terms in (12) and (13), respectively, are negative. But these terms are the net gain from winning the election for party D and R, respectively. At an optimum, these net gains can never be negative because a party always has the option of choosing the other party's policies and ensuring a zero net gain. Therefore, at an optimum, $(1 + \phi_k) \geq 1$. It follows that the optimal (w, z) for each party is on the line segment connecting $(0, 0)$ to $(\mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}x, \mathbb{E}_{(x,y)|H_k(\mathcal{P}^e)}y)$. This clearly shows that parties balance the two polar objectives mentioned above, namely, maximizing the net gain from winning and maximizing the probability of a win. Furthermore, the closeness of (w_k, z_k) to the origin depends in an intuitive way on the distribution of A at the optimum. The more concentrated the distribution, i.e., the larger F', the larger is ϕ_k and closer is k party's optimal choice to the origin: The party is willing to put up with a platform closer to the origin if moving toward its mean preferences lowers the probability of a win substantially.

2.4 Equilibrium

An equilibrium is a 4-tuple $(w_D^*, z_D^*, w_R^*, z_R^*)$ such that if $\mathcal{P}^e = (w_D^*, z_D^*, w_R^*, z_R^*)$ then (w_D^*, z_D^*) solves (9) and (w_R^*, z_R^*) solves (10).

3 Polarized Equilibria and the Inverse Association

We turn now to explaining the inverse association. We focus on equilibria that are symmetric and polarized along at least one dimension, i.e., equilibria in which $(w_D^*, z_D^*) = -(w_R^*, z_R^*)$ and $(w_k^*, z_k^*) \neq (0,0)$.[9] We assume that A is uniformly distributed on the interval $[-\alpha, \alpha]$, $\alpha > 0$, and that voter ideal points are uniformly distributed on a disk with radius $\theta > 0$ centered at $(0,0)$.

The determination of symmetric equilibria is done in three steps. In the first step, the mean preferences of voters attending each primary is determined, given common (symmetric) beliefs about party platforms. In the second step, optimal policies of the two parties are solved, given mean preferences. In the final step, the first two steps are combined to isolate the set of beliefs that generate optimal policy platforms that, in turn, confirm those beliefs.

Step 1. *Beliefs to Conditional Means*

We consider all $\mathcal{P}^e$ such that $(w_D^e, z_D^e) = -(w_R^e, z_R^e) \neq (0,0)$, i.e., beliefs that are symmetrically polarized along at least one dimension. For concreteness, we will assume that $w_D^e = -w_R^e \neq 0$. The solution for the $w_k^e = 0$ (but $z_k^e \neq 0$) case is given by the limit of the solution as $w_k^e \to 0$.

Given $w_D^e \neq 0$ and symmetry of beliefs, it follows from (7) that the line separating D-party primary attendees from R-party primary attendees is given by

$$\bar{x}(y; \mathcal{P}^e) = -\left[\frac{z_D^e - z_R^e}{w_D^e - w_R^e}\right] y = -\left[\frac{z_D^e}{w_D^e}\right] y.$$

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[9]A symmetric equilibrium in which both parties offer $(0,0)$ is possible, but we ignore this equilibrium as it's not relevant for addressing the facts.

This is a straight line that goes through the origin and divides the circular support of the voters ideal point distribution into two semicircles.

Figure 5:
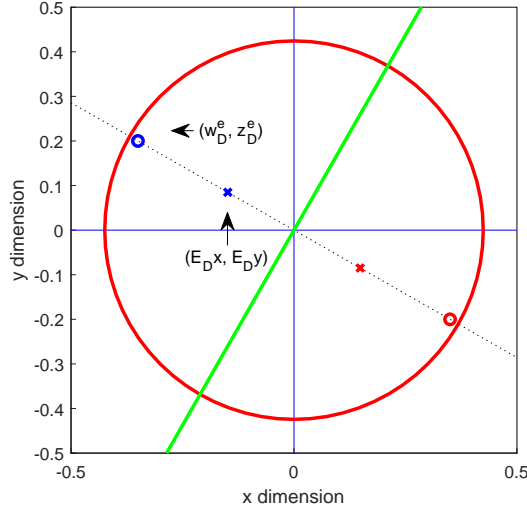Sorting When Voters are Distributed on a Disk

Figure 5 shows an example where points marked by circles represent the common symmetric beliefs about the platforms of the D and R parties, respectively. In this figure, $w_D^e < 0$ and $z_D^e > 0$ and, so, the line separating voters is the positively sloped straight line going through the origin and perpendicular to the dotted line joining the circle points. All voters with ideal points that lie in the semicircle to the left of the straight line attend the D-party primary, and all voters with ideal points in the semicircle to the right of the straight line attend the R-party primary.

Since the distribution of the voter ideal points is uniform over each semicircle, the points $(\mathbb{E}_D x, \mathbb{E}_D y)$ and $(\mathbb{E}_R x, \mathbb{E}_R y)$ coincide with the center of gravity, or centroid, of the D-party and R-party semicircles, respectively. This allows for an easy computation of conditional means $(\mathbb{E}_k x, \mathbb{E}_k y)$ using the fact that the centroid of a semicircle of radius θ lies on the radius that is perpendicular to the base of the semicircle and a distance $4\theta/3\pi$ from the center. In Figure 5, the conditional means of the D- and R-party semicircles are denoted by points marked by 'x.'

To determine the coordinates of 'x,' note that since the distance of 'x' from the origin is $\left(\frac{4\theta}{3\pi}\right)$, we must have that $(\mathbb{E}_k x)^2 + (\mathbb{E}_k y)^2 = \left(\frac{4\theta}{3\pi}\right)^2$. And, since the conditional mean is

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located on the radius perpendicular to the base, we must have $\mathbb{E}_k y / \mathbb{E}_k x = z_k^e / w_k^e$. Then, denoting z_k^e / w_k^e by β, we have:

$$(\mathbb{E}_k x)^2 = \left[\frac{4\theta}{3\pi}\right]^2 \frac{1}{1+\beta^2} \text{ and } (\mathbb{E}_k y)^2 = \beta^2 (\mathbb{E}_k x)^2. \tag{14}$$

These equations show that β^2 determines the magnitudes of $\mathbb{E}_k x$ and $\mathbb{E}_k y$ but not their signs. For their signs, note that $(\mathbb{E}_k x, \mathbb{E}_k y)$ is on the line connecting $(0,0)$ to (w_k^e, z_k^e) and, so, the signs of $\mathbb{E}_k x$ and $\mathbb{E}_k y$ must be the signs of w_k^e and z_k^e, respectively. Combining,

$$\begin{bmatrix} \mathbb{E}_k x \\ \mathbb{E}_k y \end{bmatrix} = \begin{bmatrix} \text{sgn}(w_k^e)\frac{4\theta}{3\pi}\frac{1}{\sqrt{1+\beta^2}} \\ \text{sgn}(z_k^e)\frac{4\theta}{3\pi}\sqrt{\frac{\beta^2}{1+\beta^2}} \end{bmatrix}. \tag{15}$$

These formulae also work for the case where $w_k^e = 0$ and $z_k^e \neq 0$. To see this note that as $\beta^2 \to \infty$, we get

$$\begin{bmatrix} \mathbb{E}_k x \\ \mathbb{E}_k y \end{bmatrix} \to \begin{bmatrix} 0 \\ \text{sgn}(z_k^e)\frac{4\theta}{3\pi} \end{bmatrix}, \tag{16}$$

which is exactly the outcome we get from geometry.

Step 2. *Conditional Means to Party Platforms*

In a symmetric equilibrium, $(w_k^*, z_k^*) = (-w_{\sim k}^*, -z_{\sim k}^*)$. Making this substitution in the k party's first-order condition (11), using the fact that $F'(0) = 1/[2\alpha]$ and $F(0) = 1/2$, and substituting $h^* \mathbb{E}_k x$ and $h^* \mathbb{E}_k y$ for w_k^* and z_k^*, respectively, leads to the following pair of first-order conditions:

$$[\alpha + 4h^*(\mathbb{E}_k x)^2 + 4h^*(\mathbb{E}_k y)^2]h^* \mathbb{E}_k x = \alpha \mathbb{E}_k x \tag{17}$$

$$[\alpha + 4h^*(\mathbb{E}_k x)^2 + 4h^*(\mathbb{E}_k y)^2]h^* \mathbb{E}_k y = \alpha \mathbb{E}_k y. \tag{18}$$

From (14), the sum of the squared conditional means is $[4\theta/3\pi]^2$ (for both parties). Furthermore, at least one of the conditional means is non-zero. Eliminating the non-zero

conditional mean from the appropriate first-order condition leads to a quadratic in h^*:

$$4\left[\frac{4\theta}{3\pi}\right]^2 h^{*2} + \alpha h^* - \alpha = 0. \tag{19}$$

The quadratic has positive and negative roots. Since h^* is just $(1 + \phi)^{-1}$ and $\phi \geq 0$ at an optimum, the positive root is the relevant one. Thus, the solution is:

$$h^* = \frac{-\alpha + \sqrt{\alpha^2 + 16\alpha[4\theta/3\pi]^2}}{8[4\theta/3\pi]^2} < 1 \tag{20}$$

and the optimal party platforms are given by

$$\begin{bmatrix} w_k^* \\ z_k^* \end{bmatrix} = \begin{bmatrix} \text{sgn}(w_k^e)\frac{4\theta}{3\pi}\frac{1}{\sqrt{1+\beta^2}}h^* \\ \text{sgn}(z_k^e)\frac{4\theta}{3\pi}\sqrt{\frac{\beta^2}{1+\beta^2}}h^* \end{bmatrix} \quad k \in \{D, R\}. \tag{21}$$

Step 3. *Equilibrium*:

Equation (21) says that optimal platforms depend on beliefs only through the value of β and the signs of w_k^e and z_k^e, i.e., they depend on the *direction* of beliefs only and not on the distance of the belief points from the origin. Therefore, to construct an equilibrium, we may choose any direction for beliefs, determine the optimal policy platforms, and then scale the position of the belief points up or down (i.e., set the distance of the belief points from the origin) to match the optimal platforms.

To see this more clearly, consider Figure 5 again. As shown, the distance of the belief vector from the origin (the circle points) is greater than the distance of the two conditional mean vectors (the 'x' points). Since the implied policy platforms are on the line segment connecting the respective conditional means to the origin, the situation depicted is not an equilibrium: The belief vectors are further away from the origin than the implied policy platforms. However, it is easy to ensure an equilibrium by scaling the belief points down to the optimal policy platforms. Since the scaling down does not change the direction of the beliefs, it leaves the optimal policy platforms unchanged and ensures the equality of beliefs and outcomes.

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The implication of this situation is that there is a continuum of equilibria indexed by the direction of the belief vector. In other words, the equilibrium set is a circle of radius $[4\theta/3\pi]h^*$ centered at $(0,0)$. This fact brings us to the key equilibrium implication of the model: Across equilibria, there is an inverse association between the ideological distances of the two parties along x and y dimensions.

Proposition (Inverse Association). *Let $\mathcal{P}^*$ and $\mathcal{P}^{*\prime}$ be any two symmetric equilibria, with w_D^* and $w_D^{*\prime}$ not equal to zero. For a given equilibrium, denote the ideological distance along the x dimension by $\delta_x = |w_D^* - w_R^*|$ and along the y dimension by $\delta_y = |z_D^* - z_R^*|$. Then, $\delta_x > \delta_x'$ if and only if $\delta_y < \delta_y'$.*

Proof. Let $\beta = z_D^*/w_D^*$ and $\beta' = z_D'^*/w_D'^*$. By Proposition 5 and 6, we have that

$$|w_D^e| = h^* \left[\frac{4\theta}{3\pi}\right] \frac{1}{\sqrt{1+\beta^2}} \text{ and } |w_D'^e| = h'^* \left[\frac{4\theta}{3\pi}\right] \frac{1}{\sqrt{1+\beta'^2}}, \qquad (22)$$

$$|z_D^e| = h^* \left[\frac{4\theta}{3\pi}\right] \sqrt{\frac{\beta^2}{1+\beta^2}} \text{ and } |z_D'^e| = h'^* \left[\frac{4\theta}{3\pi}\right] \sqrt{\frac{\beta'^2}{1+\beta'^2}}. \qquad (23)$$

By equations (20) and (14), we have that

$$h^* = \frac{-\alpha + \sqrt{16\alpha\left[\frac{4\theta}{3\pi}\right]^2 + \alpha^2}}{8\left[\frac{4\theta}{3\pi}\right]^2} = h'^*. \qquad (24)$$

In a symmetric equilibrium $\delta_x = 2|w_D^*|$ and $\delta_y = 2|z_D^*|$. Therefore, $\delta_x > \delta_x'$ if and only if $\beta^2 < \beta'^2$ and, so, if and only if $\delta_y < \delta_y'$. $\qquad \square$

Less formally, the reason for the inverse association can be readily seen in Figure 6. The figure plots the circular equilibrium set. In this figure, any two diametrically opposed points is a symmetric polarized equilibrium. Moving along the circle, we trace out different equilibria. Comparing across equilibria, it is evident that the ideological distance along the x dimension can increase only at the expense of the ideological distance along the y dimension and vice versa.

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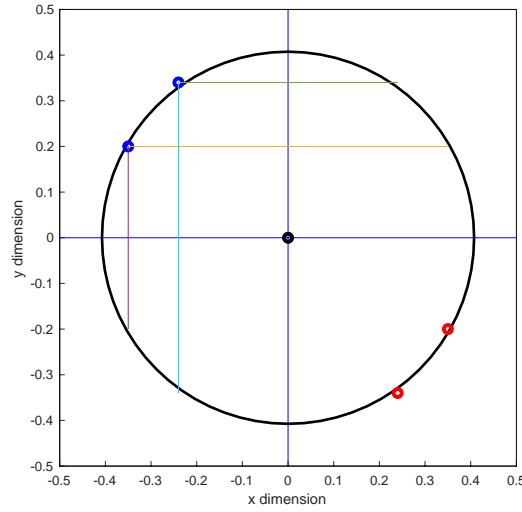
4 Secular Changes in Polarization of Political Parties in the US

Up to this point, we have presented a model in which the ideological distance along one dimension moves inversely with the ideological distance along the other dimension. In this section, we ask if this implication of the model has quantitative bite. That is, if the model is supplied with an empirical analog of β^2 for each Congress, can it reproduce the ideological distances for each Congress we see in Figures 1 and 2?

For this, we treat the average ideology of the two parties, as revealed in roll-call voting, as the ideology of two candidates who face each other in the national election.[10] For a given Congress, let $D1$ and $R1$ be the mean ideological positions of Democrats and Republicans along the primary dimension and let $D2$ and $R2$ be the mean ideological positions along the secondary dimension. As an example, Figure 7 shows these points for the 79th Congress. The blue circle corresponds to $(D1,D2)$ and the red circle to $(R1,R2)$.

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[10]For this mapping to work, we are assuming that voters in local primary elections take signals from the two parties' national agendas and sort accordingly. For instance, if the party opposes desegregation, then sorting at the local level reflects this and leads to a candidate in some district who leans more toward the center on economic matters than the average party representative but has the desired leaning with regard to segregation. If this candidate is elected to Congress, she will contribute an ideology point that leans conservative on the second dimension and toward the center along the economic dimension.

Figure 7:
The Predicted Line Separating Republicans and Democrats
79th Congress (1945-47)
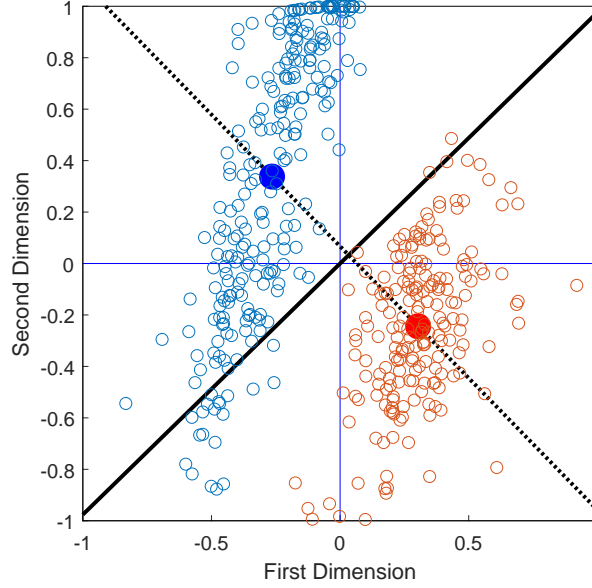
Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022) and authors' calculation.

We take the empirical analog of β^2 to be the slope of the dotted line, i.e.,

$$\hat{\beta}^2 = \left(\frac{D2 - R2}{D1 - R1}\right)^2.$$

Note that the theory counterpart of $\hat{\beta}^2$ is indeed β^2 since $w_D = -w_R$ and $z_D = -z_R$ and $z_D/w_D = \beta$.

In our theory, as a consequence of symmetry, the dotted line is predicted to pass through the origin. In this instance, it does not quite do so but it comes close. The solid line passing through the origin and perpendicular to the dotted line is the line that, in theory, should pass through the midpoint of the dotted line and fully separate Democrats from Republicans. Again, it does not quite do so but comes close. These discrepancies reflect the fact that the average ideology of Democrats and Republicans are not exactly symmetrically opposed.

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In order to connect $\hat{\beta}^2$ to the theoretically predicted ideological distances, we need values of θ and α. For this, note that in any equilibrium the (common) Euclidean distance of a party's platform from the origin is a constant given by

$$
\begin{aligned}
\sqrt{(w_k^*)^2 + (z_k^*)^2} &= \sqrt{h^{*2}[(\mathbb{E}_k x)^2 + (\mathbb{E}_k y)^2]} \\
&= h^*\left[\frac{4\theta}{3\pi}\right] \\
&= \frac{-\alpha + \sqrt{16\alpha\left[\frac{4\theta}{3\pi}\right]^2 + \alpha^2}}{8\left[\frac{4\theta}{3\pi}\right]}.
\end{aligned}
\tag{25}
$$

In actuality, the Euclidean distance (from the origin) of a party's ideology has not been constant over time and neither have the Euclidean distances of the two parties been exactly equal to each other at all times. However, it is the case that the distances of the D-party platform averaged over time are quite close to the distances of the R-party platform averaged over time, these being 0.39 and 0.40, respectively.[11] We set $\theta = 1$ and pick α so that the expression in (25) is equal to $(0.39 + 0.40)/2$.[12] With these settings of θ and α, we use time series on $\hat{\beta}^2$ to get the theoretically predicted time series for δ_x and δ_y using the expressions in (22) and (23).

Figures 8 and 9 plots δ_x along with $|D1-R1|$ and δ_y along with $|D2-R2|$, respectively. In these figures, the solid line is history and the dotted line is the prediction of the calibrated model. In both figures, the predicted ideological distance tracks the actual ideological distances quite closely. These plots show that knowledge of $\hat{\beta}^2$, which is the *ratio* $[(D2 - R2)/(D1 - R1)]^2$, allows prediction of the *levels* of $|D2 - R2|$ and $|D1 - R1|$. Thus, during the '40s and '50s, when $\hat{\beta}^2$ rose the model predicts that $|D1 - R1|$ should fall as observed,

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[11]The near-equality of the time averages is a reflection of the rough symmetry of ideological positions of the two parties.

[12]If we had picked a different value for θ but altered the value of α so that the expression in (25) evaluated to 0.395, the theoretically predicted equilibrium outcomes would remain unchanged. In this sense, the choice of θ is a normalization. We note, however, that the method employed by the DW-Nominate program to infer the ideological positions of members of Congress assumes that the position is always contained in $[-1,1] \times [-1,1]$. Thus, by construction, the mean ideological position of a party along any dimension cannot exceed 1 in absolute value. For our model to be consistent with this restriction, θ cannot exceed $3\pi/4 = 2.37$; if it did, there will exist values of α and β^2 for which the equilibrium ideological position of parties along at least one dimension will exceed 1.

Figure 8:
Ideological Distance, First Dimension
Data and Model Predictions
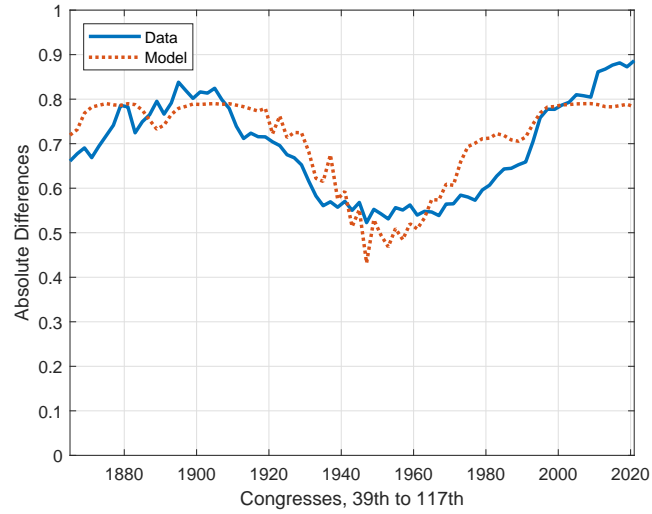
Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022) and authors' calculation. For the data, the ideological distance for each Congress is the absolute value of the difference in average ideological scores of Democrats and Republicans along the first dimension for that Congress. In the model, the ideological distance is δ_x.

Figure 9:
Ideological Distance, Second Dimension
Data and Model Predictions
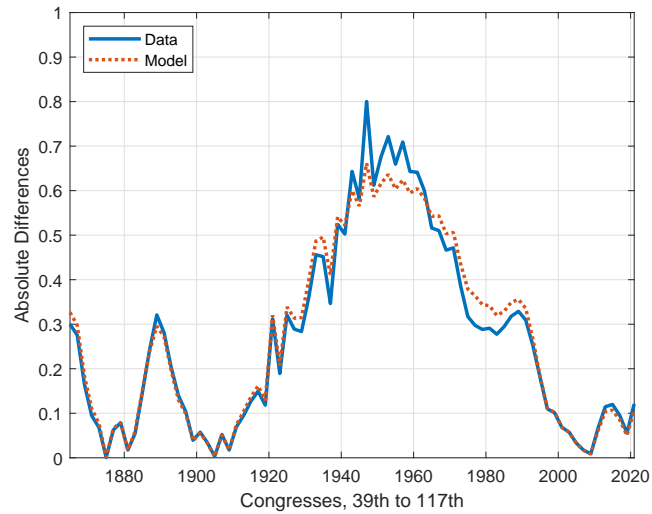
Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022) and authors' calculation. For the data, the ideological distance for each Congress is the absolute value of the difference in average ideological scores of Democrats and Republicans along the second dimension for that Congress. In the model, the ideological distance is δ_y.

and in the post-1964 era when $\hat{\beta}^2$ fell, the model predicts that $|D1 - R1|$ should rise, as observed.

The premise on which this effect is based is diversity with respect to the social/cultural ideology among both economic liberals and economic conservatives. When a polity is polarized along a social issue such as desegregation, it is possible — i.e., it can be an equilibrium outcome — that modest economic conservatives (liberals) who are socially very conservative (liberal) can align themselves with the economically *liberal* (*conservative*) party because the economically liberal (conservative) party happens also to be the more socially conservative (liberal) party. When they do, the alignment leads to *less* sorting, i.e., *more* homogeneity, along the economic dimension for both parties and, therefore, smaller ideological distance between them along the first dimension.

As an example of this type of sorting, Figure 10 displays the ideological positions of members of the House of Representatives for the 88th (1963-1965) Congress. Note the presence of (first-dimension) economic conservatives in the Democratic party and economic liberals in the Republican party. As a contrast to this type of sorting, Figure 11 displays the ideological map of the 117th (2021-2023) Congress. We no longer see any economic conservatives in the Democratic party or economic liberals in the Republican party. The result of this sorting pattern is that the two parties are more ideologically distant along the economic (first) dimension.

These findings bring a novel perspective to the decline and subsequent rise in polarization along the first dimension in the post-WWII era. In the years leading up to the Civil Rights Act of 1964, the ideological distance between the Democratic and Republican parties widened along the second dimension. According to our theory, this widening can account for the *narrowing* of ideological distance along the economic dimension during the same period. Similarly, the narrowing of ideological distance along the social/cultural dimension can account for the *widening* of ideological distance along the economic dimension since 1964.

Figure 10:
Member Ideologies, 88th Congress (1963-1965)
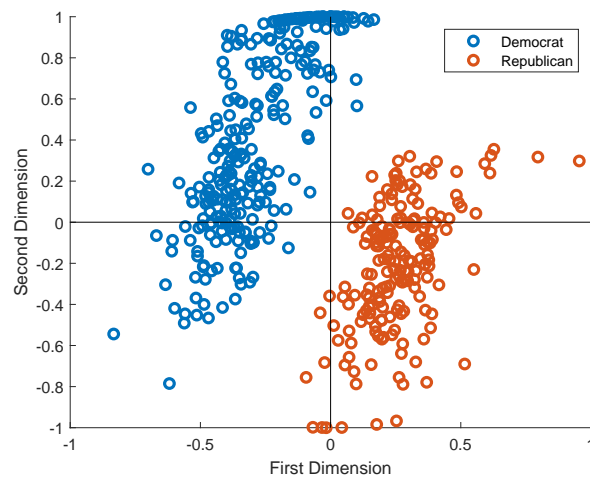
Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022)

Figure 11:
Member Ideologies, 117th Congress (2021-2023)
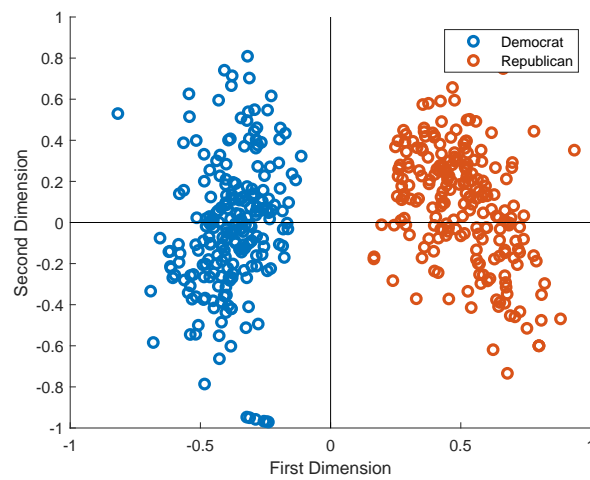
Source: Lewis, Poole, Rosenthal, Boche, Rudkin, and Sonnet (2022)

Finally, we note that the increase in polarization that has occurred along the first dimension since the mid-1990s is not explained by changes in the ideological distance along the second dimension. During the last three decades, changes in the ideological distance along the second dimension have been small and do not imply significant changes to polarization along the first dimension, which is predicted to be essentially flat. Thus, our results are not in conflict with studies (cited earlier) that have focused on reasons (rising inequality, intensifying lobbying, the rise of social media) to account for increasing polarization along the first dimension in recent decades.

5   Conclusion

Viewed over a long stretch of time, roll call voting records for Democratic and Republican party representatives in Congress show an inverse association between the mean ideological distance along first dimension and the mean ideological distance along the second dimension.

We presented a model of party competition with a two-dimensional ideology space to explain this fact. In our model, voters have (common) beliefs about the ideological stance of the two parties and join a party based on these beliefs. Each party chooses its national platform by balancing the preferences of the voters who compose the party and the preferences of the polity at large since a party can come to power only if it wins the general election.

If the distribution of voter ideologies is uniform on a disk, there is a continuum of polarized equilibria indexed by voter beliefs. Moving from an equilibrium in which the two parties are more polarized along a dimension to one in which they are less polarized along the same dimension implies an *opposite* movement for polarization along the other dimension. This occurs as a result of the differential sorting into the two parties induced by the different beliefs about what each party stands for.

Remarkably, the sorting and re-sorting of a stable distribution of voters into the two parties gives a good *quantitative* account of the variation in ideological distances along the

two dimensions over a long stretch of history. This finding brings a new perspective on the potential causes of political polarization that highlights the role of changes in beliefs about party ideologies as opposed to changes in preferences, law, or technology.

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