Heterogeneous Quality Discernment and Competitive Pricing*

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September 14, 2023

Abstract

This paper weakens the idea of a vertically differentiated products market that competes in prices, with heterogeneous consumers who differ in their ability to evaluate (rank by quality) rival products. All consumers share the same weak ranking between the products, but higher income/type consumers - with a richer consumption experience - are able to strictly rank an ascendingly increasing number of them. Doing so overturns the prominent results in the literature: a common cost across products (sellers) now allows unlimited numbers of them to survive in the market profitably; and only when cost increases in quality can a seller be ousted from the market, and this can be the high quality seller. However, equilibrium prices are still ranked by true quality; also (weakly) so is their maximum buyer surplus.

JEL Categories: D4; D8; L1.

Keywords: quality; price competition; heterogeneous quality-discernment; numbers of sellers.

*PRELIMINARY; Please do not cite.

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1 INTRODUCTION

The literature on competitive pricing of vertically differentiated products can be traced back to Gabszewicz and Thisse (1979) with two identical cost sellers/products ranked along a single dimension of 'quality' by consumers with identical preferences but varying income levels. Consumer heterogeneity is thus the exact opposite of that modeled in the Hotelling framework of horizontal differentiation (Hotelling, 1929): of varying preferences but identical income.

A simplification of their consumer utility function that retains its main properties results in the finding of the 'finiteness property', that is, an upper bound on the number of sellers who can profitably sell positive quantities: Gabszewicz and Thisse (1980) themselves find this in a follow-up paper even though entry beyond this number continues to lower prices in their model[1]; Shaked and Sutton (1982) find that only two sellers can profitably enter or survive, beyond which entry leads all sellers to choose a common ('top') quality, lowering price to cost and erasing profits for all of them; and Shaked and Sutton (1983) characterize the bound, and find that it exists as long as all products are ranked identically by all consumers and unit variable costs do not rise too rapidly with quality.

Cremer and Thisse (1991) point out that every horizontally differentiated model with differentiable transportation costs is equivalent to a vertical product differentiation model. However, Wauthy (1996) illustrates that whether the market is covered (all buyers purchase from some seller) or not is an endogenous property; and Wauthy (2010) identifies the assumption of a covered market in vertical differentiation as the source of its similarity with horizontal product differentiation in which the finiteness property disappears. And in a restatement of the condition found by Shaked and Sutton (1983), Wauthy (2010) identifies finiteness as holding whenever all consumers have the same preferred product (when each is sold at unit cost); in contrast, each consumer in horizontal differentiation is typified by her preferred product whose location coincides with her own (Hotelling, 1929).

The findings in the literature make the case for exploring a weaker definition of vertical differentiation to examine its similarity with horizontal differentiation, its impact on price equilibrium and on the finiteness property. This is the aim of this paper. Unlike the vertically differentiated models cited above, wherein (i) higher income consumers have strictly higher reservation prices for all quality-variants, and (ii) all consumers share the same strict ranking between all rival products; I use weaker versions of both (i) and (ii) by adapting a commonly used consumer utility function.

Referring to the consumer-type variable as her quality-discernment (henceforth q-d) ability, I model this as her private bound on the discernment of true quality in the market. A consumer's q-d bound modeled thus also symbolizes her maximum valuation of (or willingness to pay for) quality, correlated with her income/purchasing power. A consumer with a higher q-d bound is able to discern the true quality of (and strictly rank) an ascendingly greater range of quality-variants.[2] Thus all consumers share only a weak ranking between all rival products, giving a weaker definition of identical preferences across consumers. A weaker monotonicity also now holds between value/reservation price and income/type such that the value of a quality-variant is non-decreasing in higher consumer income/type; that is, higher type consumers have higher reservation prices for higher (but not all) quality-variants.

An intuitive reason for why consumers have different q-d abilities (bounds) and why these are correlated with income is as follows. If evaluating quality is learned through past consumption experience, and (for certain product categories) low income consumers' experience

[1] This seems to follow from their assumption that new entry is always of higher quality than incumbent products.

[2] In contrast, Grilo and Wauthy (2000) model incomplete quality discernment homogeneously for all consumers as them knowing the expected quality and variance of each product, although consumers differ in their risk aversion types and therefore their preference for variance in quality.

is limited to low quality variants whereas higher income bestows a richer experience of a wider range of (including higher) quality, then not all consumers are able to discern the true quality of (or strictly rank between) all product variants and higher income consumers can do so for a greater range of variants. As an example, consider the difference between a college student, who desires to purchase a bottle of wine but cannot discern (or rank between) higher quality variants, and a connoisseur of wine who can rank higher quality wines because she has tasted many kinds (unconstrained by her purchasing power).

Borrowing the definition of preferred products from Schmidt (2009) and Wauthy (2010), unlike a single preferred product for each consumer in either product differentiation model (which, when identical across consumers results in the finiteness property) in the papers cited above, each consumer in the common cost market here has a preferred *set of* products such that she is indifferent between all products in this set. The smallest such set is non-empty and is nested within all others; this contains at least the highest quality product in the market. The condition (for finiteness) identified by Wauthy (2010) is thus satisfied weakly as there is at least one product that is preferred by all consumers, although each consumer's set of preferred products is not the same; nor are they disjointed as would be in horizontally differentiated markets.

I find that if sellers have a common marginal cost, prices are ranked by true quality in any pure strategy price equilibrium, despite the weak definition of identical consumer preferences. Moreover, a higher quality product delivers weakly larger maximum consumer surplus. And there is no upper bound for the number of distinct qualities (sellers) that can exist profitably in the market, illustrating that the finiteness ('natural oligopoly' in the words of Shaked and Sutton, 1983) property is not robust to weak vertical differentiation.

With marginal cost increasing in quality, however, selling at cost naturally creates price differences. This causes a leftward (lower quality) shift in the preferred product(s) of consumers, such that the highest quality product in the market is no longer included in every consumer's preferred set. I illustrate with a duopoly example that not all sellers (qualities) need be profitably accommodated in market equilibrium, or that finiteness can now exist. The ousted seller can be the higher quality one, unlike what Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) find.

Summarizing, weakening the strict preference ranking of consumers and the monotonicity of reservation prices in consumer income/type gives a weaker version of vertical differentiation in the common cost market where finiteness fails, but opens up the possibility of finiteness in the increasing (in quality) cost market. This is the opposite of the consensus result in the erstwhile literature that the finiteness property holds in vertical differentiation only when marginal costs do not rise (too steeply) with quality (Shaked and Sutton, 1983; Cremer and Thisse, 1991; Motta, 1993; Schmidt, 2009).

Intuitively, allowing consumers to have less than perfect/strict preference ordering and tying quality discernment monotonically with their purchasing power, makes low(er) type consumers more price-concerned because they are unable to discern differences in high quality. This in turn makes it harder for high quality products/sellers to be attractive to low type/income consumers. These factors become especially salient when unit cost increases (steeply) in quality, resulting possibly in high(er) quality rival(s) being forced out of the market. It is not surprising that this result reminds us of the classic Gresham law (Bad money drives out good), a feature of products wherein quality is confounded or cannot be perfectly discerned.

2 CONSUMER UTILITY & CHOICE

If there are n sellers selling quality-variants in the market, let the true quality[3] of the ith seller be q_i. Without loss of generality, let $q_1 < q_2 < ... < q_{n-1} < q_n$, or that true quality increases in the subscript, unless all sellers sell a homogeneous good in which case $q_i = q, \forall i$. Each seller (i) sells only one quality, and sellers have no capacity constraints.

A consumer's private type is her given q-d bound, b; this is correlated with her income level and also represents her maximum valuation/reservation price in the market. Consumer j's net utility (surplus) from purchasing good of quality q_i at price p_i is

$$u_{ij} = \min\{b_j, \theta q_i\} - p_i; \tag{1}$$

where $\theta > 0$ is a common (to all consumers) positive valuation parameter. This is an adaptation of the consumer utility in Tirole (1988) and Motta (1993) to accommodate q-d bounds; a consumer's value/utility from a particular quality-variant is its true value, θq_i, if this is within her q-d bound, and is her q-d bound itself if the true value of quality is higher and therefore indiscernible by the consumer. In other words, consumer j can discern and evaluate quality q_i as long as $\theta q_i \leq b_j$, otherwise she values it at her maximum value, b_j, unable to discern its true quality.

All consumers now share the same weak ranking of all goods: $q_n \geq q_{n-1} \geq ... \geq q_2 \geq q_1$; and a higher income/q-d type consumer is able to discern and rank an ascendingly increasing number of quality variants. That is, despite quality-ranked sellers, consumer bounds to quality discernment result in weakly identical preference orderings between consumers. According to Definition 2 in Wauthy (2010) (and also Schmidt, 2009) then, this model market is vertically differentiated in a weak sense.

Assume that each consumer desires to buy one unit/good as long as purchasing it leaves her with a positive surplus (assuming zero surplus if nothing is purchased), and desires to maximize net utility across rival products/sellers in the market. Also assume that sellers cannot identify buyer types, and thus cannot price discriminate between them, but are aware of the cumulative distribution of b given by $F(b)$, which is atomless on the interval, $[b_L, b_H]$.[4] And the support of sellers' qualities is such that their valuation by consumers also lies in the same interval: $\theta q_i \in [b_L, b_H], \forall i$.

Given this setup, the maximum surplus across consumer types offered by a seller/quality is the difference between the true value of its quality and its price; I define this below.

Definition 1. *Define $\bar{u}_i = \theta q_i - p_i$ as the maximum net utility/surplus of quality q_i to any buyer; i.e. $\bar{u}_i = \max_j\{u_{ij}\}$.*

In a market of n sellers, the support of consumer/buyer types, $[b_L, b_H]$, can be segmented into $n+1$ categories where the smallest types fall in category 1 defined as $b \leq \theta q_1$; the largest fall in category $n+1$ defined as $b > \theta q_n$; and any intermediary category h, is defined as $b \in (\theta q_{h-1}, \theta q_h]$.

Using (1), which every consumer wants to maximize by choosing one quality-variant to purchase, it is thus straightforward that each category 1 buyer compares only prices and either buys the cheapest product or does not purchase anything if the cheapest price exceeds her bound. While category $n+1$ buyers compare value net of price ($\theta q_i - p_i$) for all products and purchase that quality with the highest such measure, or do not purchase anything if this is negative. More generally, the following lemma details how buyers in different categories compare between the rival qualities, given their prices, to choose which to purchase to optimize surplus.

[3] Quality is treated as one-dimensional but can easily extend to a weighted average of multiple dimensions as long as all consumers have the same weights for all dimensions.

[4] $F(b)$ is thus the fraction/probability of consumers of q-d types less than or equal to b.

Lemma 1. *Buyer of type b in category h compares $\theta q_i - p_i, \forall i = \{1, 2, 3, ..., h-1\}$ with $b - p_i, \forall i = \{h, h+1, ..., n\}$ and chooses the seller that gives the largest of these measures, or does not purchase anything; where by definition of her category, $b \in (\theta q_{h-1}, \theta q_h]$.*

Because a lower income/type consumer - in a lower category - can discern the true quality for a smaller number of quality-variants in the market, between all other (a larger number of higher quality) products, she simply compares prices. That is, the lower a consumer's type, the more price concerned she is in comparing sellers. Such consumer behavior is intuitively appealing as low income buyers often base their purchase decisions more heavily on price comparisons versus high income buyers who are often epicures/gourmet purchasers, are more aware and discerning of quality (and thus surplus) differences.

3 HOMOGENEOUS SELLERS

As a preliminary illustration of the model, suppose each of n sellers sells the same product at the same constant marginal cost, that is $q_i = q, c_i = c, \forall i$. What is the symmetric equilibrium price then?

Segmenting buyers gives only two categories now: category 1 buyers are those with $b \leq \theta q$, and category 2 those with $b > \theta q$. From lemma 1, category 1 buyers only compare prices across sellers in choosing who to purchase from. And category 2 buyers compare $\theta q - p_i$ across sellers; this also thus reduces to a comparison of only prices because all qualities are the same. Therefore, all buyers choose to purchase from the seller with the lowest price, if at all.

The only symmetric pure strategy price equilibrium therefore is $p_i = c, \forall i$. The market for homogeneous goods is therefore uninteresting from a modeling point of view as it reduces to the standard Bertrand pricing game with buyer values distributed in $[b_L, b_H]$ and lacks novel insights. It is also undesirable by rival sellers in the market as it erodes their profits.

4 QUALITY-RANKED SELLERS

We return to n sellers, each (i) with pre-determined and distinct quality, q_i, increasing in i, such that q_1 is the lowest quality and q_n the highest.

4.1 COMMON COST

As in Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) assume that all sellers have a common marginal cost of production/sale, $c \in [0, b_L)$. Referring to consumer j's preferred product(s) as those that maximize her surplus when all products are offered at unit cost (as defined in Wauthy, 2010 and Schmidt, 2009), and using $\tilde{q}_j$ to denote this set, from (1), the assumption of common cost gives $\tilde{q}_j = \{q_n\} \bigcup \{q_i : \theta q_i \geq b_j\}$. Notice that $q_n \in \tilde{q}_j, \forall j$, or that the highest quality is in every consumer's preferred products set. Using lemma 1, it is now possible to characterize price equilibria between sellers, as in the following results.

Proposition 1. *If sellers have a common marginal cost, c, then in equilibrium, $c < p_i \leq \theta q_i, \forall i$; and each seller makes positive expected sales and profit.*

Proof. See Appendix A. □

This result is in direct contrast to the finding of a bound on the number of profitable sellers in the market with common marginal cost of quality (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982; Shaked and Sutton (1983)). The bound existed in their models because with a common strict preference ordering between quality variants, increased competition led prices for high quality-variants to plunge low, eliminating positive profits for low quality products/sellers and forcing them out of the market. In this model, however, the weak ranking between all products (shared by consumers) leads lower type consumers - constrained by their q-d bounds or reservation prices - to value higher quality products at less than their true value and compare them simply by price. Lower quality sellers thus find it easier (than in the earlier literature) to beat the surpluses offered by their higher quality rivals to lower type consumers. Imperfect quality discernment thus creates market niches of consecutively lower type consumers for lower qualities/sellers to profitably exist.

The proof uses the argument that as long as prices of higher qualities exceed the common cost, lower quality sellers can price marginally above cost and remain the preferred seller for some small subset of consumer types; there is thus no limit or upper bound to the number of sellers such a market can accommodate. The following proposition further characterizes the properties of such a price equilibrium.

Proposition 2. *If sellers have a common marginal cost, c, then in equilibrium:*
(i) prices are ranked by true quality;
(ii) a higher quality delivers a maximum consumer surplus no smaller than that of all lower quality products, i.e. $\bar{u}_h \geq \bar{u}_i, \forall i < h,\ \forall h \in \{2, 3, ..., n\}$.

Proof. See Appendix A. □

Part (i) of the above proposition is both an expected and common result in vertically differentiated markets (Wauthy, 1996); but given the weak nature of the common preference ordering in this model, this is surprising and affirms the model's closeness with vertical differentiation. Part (ii) reveals that high(er) quality goods must offer weakly larger maximum surpluses because given that prices are ranked by quality, consumers need to be enticed away from purchasing lower quality cheaper goods.

4.2 COST INCREASING IN QUALITY

When higher quality is expensive to produce/source, then consumers' preferred products sets depend on how cost increases in quality. From (1), if $\theta q_i \geq b_j$, then $u_{ij} = b_j - c_i$. This implies that from amongst sellers/qualities such that $\theta q_i \geq b_j$, only the lowest cost seller *can be* a preferred product of consumer j (when all products are sold at unit cost). More importantly, unlike in the common cost market in the last subsection, q_n need no longer be an element of the preferred products set, $\tilde{q}_j$, for consumers, $j : b_j < \theta q_n$. That is, there is no longer a quality/product that is necessarily preferred by all consumers.

However, this makes it possible for the numbers of active sellers in market equilibrium to be bounded; in other words it makes it possible for a seller with particular (high) quality-cost combination to be unable to sell in market equilibrium; the following illustrates.

Consider qualities q_2, q_1 such that $q_2 > q_1$ and $c_2 > c_1$; where $c_1 < b_L \leq \theta q_1 < \theta q_2 \leq b_H$ as in the last section. Seller 2 pricing below cost is dominated, and therefore $p_2 \geq c_2 > c_1$ holds in any equilibrium. With $n = 2$, buyers can be divided into three categories. Using lemma 1, $p_1 = p_2 > c_1$ results in category 1 consumers ($b \leq \theta q_1$) indifferent between both sellers, and categories 2 ($b \in (\theta q_1, \theta q_2]$) and 3 ($b > \theta q_2$) preferring seller 2. But this cannot be an equilibrium because seller 1 has a profitable price cut to win over at least all of category 1 buyers and (or if category 1 is empty) some of category 2. Similarly, $p_1 > p_2 > c_1$ results in consumers of all three categories preferring seller 2, but being the lower cost seller, seller 1 has a profitable price cut to win over at least all category 1 consumers. Therefore, the following generalizes the result (i) of proposition 2 for an increasing cost quality duopoly.

Claim 1. *In a duopoly market, as long as marginal cost is non-decreasing in quality, equilibrium prices are ranked by true quality.*

The following proposition uses the above structure to draw an example in which a seller with given quality and cost (the high quality/cost seller) is unable to sell profitably in any pure strategy price equilibrium.

Proposition 3. *If marginal cost increases in quality, it is possible that a seller with given* q_i, c_i*, is unable to sell profitably in any pure strategyy price equilibrium.*

Proof. See Appendix A □

In the example in the proof, not only are there buyers in the market who value the high quality good above its cost (i.e. $\exists b \in (c_2, b_H])$, in (some) equilibrium the seller is unable to sell despite its $\bar{u}_i$ being positive because the low quality seller is priced more attractively.

But if a given quality-cost seller is unable to sell profitably, it implies that the market is limited in its accommodation of quality-variants and finiteness becomes possible. Cost increasing in quality can therefore enable an upper bound to the number of sellers profitably accommodated in market equilibria by limiting the competitive ability (of slashing prices) of high quality sellers. This is the exact opposite of what is found in Shaked and Sutton (1983) where cost increasing in quality can erase the finiteness property that exists in common cost markets. Notice that unlike in Gabszewicz and Thisse (1979) and Shaked and Sutton (1982), here it is the high(est) quality product that is unable to sell profitably.

5 CONCLUSION & DISCUSSION

In this paper I explore weakening the vertical differentiated-ness of a market where sellers/qualities compete in prices. I do this by modeling heterogeneous quality-discernment (correlated with income) by consumers, that leads the lowest income/type consumers to compare only prices, those with the highest income/type to compare only net surpluses, and in general, the lower the consumer's income/type the greater the number of products between which she compares only prices.

The assumption of a complete and perfectly informed (identical) consumer ranking of all products by quality in a classical vertically differentiated products market is a demanding and unrealistic theoretical abstraction. Relaxing it in this model using the ideas of quality discernment being learned through past consumption and therefore being correlated with purchasing power, results in a failure of the finiteness property in the common cost market, undermining the tendency of such markets to be natural oligopolies. Prices, however, necessarily rank by true quality in any equilibrium. Finiteness can however result when marginal cost increases steeply in quality. Both results together present an antithesis of the common result that finiteness holds in vertically differentiated markets as long as cost does not rise (steeply) in quality (Shaked and Sutton, 1983; Motta, 1993; Schmidt, 2009).

The results in this model are driven by how the relationship of unit cost and quality affects the (sets of) preferred products of consumer types. But unlike in strictly vertically differentiated models where finiteness exists whenever all consumers have the same preferred product (Wauthy, 2010), here finiteness fails whenever all consumers' preferred products sets necessarily include a common product (the highest quality). This is because here (unlike in the older models) the inability to profitably survive in the market is a possibility for the highest quality product(s), which becomes real when they are not preferred products for all consumer types.

APPENDIX

A Proofs

Proposition 1.

Proof.
Step 1: In any pure strategy price equilibrium, $p_i \in [c, \theta q_i], \forall i$.

Proof. Seller i will never set $p_i > \theta q_i$ because at such a high price consumers with $b < \theta q_i$ earn a negative surplus from its product and so do consumers with $b \geq \theta q_i$; no consumer would want to purchase from seller i. And it is trivial that $p_i < c$ is dominated $\forall i$. Therefore, in equilibrium, $p_i \in [c, \theta q_i], \forall i$. □

Step 2: In any pure strategy price equilibrium, $p_n > c$.

Proof. At $p_i = c$, seller i earns 0 profit. For the highest quality, seller n, this is dominated as follows. Because $\forall i \neq n, \ \max\{\theta q_i - p_i\} \leq \theta q_{n-1} - c$, there exists $p_n > c$ such that $\theta q_n - p_n > \max\{\theta q_i - p_i\}, \forall i \neq n$; that is, from lemma 1 it is possible for seller n to price above cost and yet be the preferred seller (quality) for category $n+1$ consumers. Even if there is zero mass of buyers in category $n+1$, which occurs if $\theta q_n = b_H$, because $\forall i \neq n, \ \max\{\theta q_i - p_i\} \leq \theta q_{n-1} - c$, there exists $p_n > c$ and b'' in category n such that $b'' - p_n > \theta q_{n-1} - c \geq \max\{\theta q_i - p_i\}, \forall i \neq n$; that is, from lemma 1 seller n can price above cost and be preferred by some buyers in category n. Therefore in any equilibrium, $p_n > c$; this also implies that $p_i = p = c, \forall i$ cannot be an equilibrium. □

Step 3: If $p_i > c, \forall i \geq h$, then in equilibrium $p_{h-1} > c$.

Proof. Suppose $p_i > c, \forall i \geq h$, implying $\min_{i \geq h}\{p_i\} > c$; and from Step 1 above we have that $p_i \geq c, \forall i < h$. For seller $h-1$ then, pricing at c and earning zero profit is dominated if $\exists p_{h-1} > c$ and $\exists b' > \theta q_{h-1}$ in category h such that (i) $\max_{i \leq h-2}\{\theta q_i - p_i\} < \theta q_{h-1} - p_{h-1}$, and (ii)$\theta q_{h-1} - p_{h-1} = b' - \min_{i \geq h}\{p_i\}$ hold; where (i) and (ii) together result in buyers of types $[\theta q_{h-1}, b')$ preferring to purchase from seller $h-1$ at price p_{h-1}, earning it positive profit[5].

To find b' and $p_{h-1} > c$, from Step 1 we need $c < p_{h-1} \leq \theta q_{h-1}$. From (ii), $\theta q_{h-1} - b' + \min_{i \geq h}\{p_i\} = p_{h-1}$. For $p_{h-1} > c$, the required condition is $\theta q_{h-1} - c + \min_{i \geq h}\{p_i\} > b'$; and for $p_{h-1} \leq \theta q_{h-1}$ it is $\min_{i \geq h}\{p_i\} \leq b'$. Together therefore, we require $\theta q_{h-1} - c > b' - \min_{i \geq h}\{p_i\} \geq 0$. Moreover, for (i) also to hold, it must be that $\max_{i \leq h-2}\{\theta q_i - p_i\} < b' - \min_{i \geq h}\{p_i\}$, the LHS of which is bounded above by $\theta q_{h-2} - c$. Define $b' = \theta q_{h-1} + \min_{i \geq h}\{p_i\} - c - \epsilon$, for some $\epsilon < \theta[q_{h-1} - q_{h-2}]$; b' thus satisfies all required conditions, and gives $p_{h-1} = c + \epsilon$, using (ii) from above. Therefore, $p_i > c, \forall i \geq h$ implies $\exists p_{h-1} > c$, at which seller $h-1$ makes positive sales. □

From Step 2 we know $p_n > c$ in any equilibrium; using Step 3, this implies $p_{n-i} > c$, and both together imply $p_{n-2} > c$, and so on. Therefore $p_i > c, \forall i$ in equilibrium. □

Proposition 2.

[5]Notice that this implies that seller 1 (the lowest quality seller) will have positive sales at a price above cost even if $b_L = \theta q_1$.

Proof. (i) Suppose for some $k \in \{1, n-1\}$, $p_{n-k} > p_{n-k+1}$ holds in equilibrium. From lemma 1, buyer categories 1, 2, 3, ..., $n-k$ find seller $n-k+1$ more attractive than seller $n-k$ because for all of them $b - p_{n-k} < b - p_{n-k+1}$ holds and is the deciding criterion between these two sellers.

For category $n-k+1$ buyers, by definition of their category, $\theta q_{n-k} - p_{n-k} < b - p_{n-k}$ holds, and by the above ranking between the two relevant prices, $b - p_{n-k} < b - p_{n-k+1}$ holds; therefore $\theta q_{n-k} - p_{n-k} < b - p_{n-k+1}$ and using lemma 1, these buyers also prefer seller $n-k+1$. The same strict preference between these two sellers also holds for buyer categories $n-k+2, n-k+3, ..., n, n+1$ because $\theta q_{n-k} - p_{n-k} < \theta q_{n-k+1} - p_{n-k+1}$ holds and is the deciding criterion from lemma 1.

But then seller $n-k$ is unable to sell to any buyers; this cannot be an equilibrium as it contradicts proposition 1. This rules out $p_{n-k} > p_{n-k+1}$ in equilibrium.

Next suppose, $p_{n-k} = p_{n-k+1}$ in equilibrium. Seller $n-k$ can now sell only to buyer categories 2, 3, 4, ..., $n-k$, who are indifferent between these two sellers because $b - p_{n-k} = b - p_{n-k+1}$ (using lemma 1); all higher buyer categories prefer seller $n-k+1$ because $\theta q_{n-k} - p_{n-k} < b - p_{n-k+1}, \forall b > \theta q_{n-k}$.

But then either seller $n-k$ shares purchases from buyer categories 2, 3, 4, ..., $n-k$ with seller $n-k+1$, or all these buyers prefer some other seller and neither seller $n-k$ nor $n-k+1$ are able to sell to them. In the first case because $p_{n-k+1} > c$ from proposition 1, seller $n-k$ can profit by marginally undercutting p_{n-k+1}. And in the second case, seller $n-k$ is unable to sell to any buyers; from proposition 1 this cannot be an equilibrium. Therefore prices must be ranked by true quality in any price equiilbrium. □

Proof. (ii) Given consumers' q-d bounds, the above implies $b - p_{i-1} > b - p_i; \forall i$ in any pure strategy price equilibrium. From lemma 1 then, category h consumer can never (in any equilibrium) prefer to purchase from seller $i > h$. Equivalently, seller h can only be preferred by consumers in categories $\{h, h+1, ..., n, n+1\}$. But from lemma 1, the necessary condition for any of these consumers to prefer seller h (or be indifferent between purchasing q_h and some other quality) is $\theta q_h - p_h \geq \max_{i<h}\{\theta q_i - p_i\}$; using definition 1, this is equivalent to $\bar{u}_h \geq \bar{u}_i, \forall i < h,\ \forall h \in \{2, 3, ..., n\}$. □

Proposition 3.

Proof. To prove this result, it is sufficient to show an example in which the high cost/quality seller is unable to sell profitably in any equilibrium. Assume b is normally distributed in $[b_L, b_H]$, such that $F(b) = \frac{b-b_L}{b_H-b_L}$. Using claim 1 along with lemma 1, in equilibrium, category 1 consumers prefer to purchase from seller 1. For higher categories, $\hat{b}$ is the marginal buyer defined as follows

$$\hat{b} = \theta q_1 + p_2 - p_1; \tag{2}$$

such that $b < \hat{b}$ prefers seller 1, and vice versa as long as and $\min\{\theta q_2, b\} \geq p_2$ and $\hat{b} < \theta q_2$ (without which no buyer strictly prefers seller 2 because this condition is equivalent to $\theta q_2 - p_2 > \theta q_1 - p_1$). And in case $\hat{b} = \theta q_2$, then consumers in categories 1 & 2 prefer seller 1, and category 3 buyers are indifferent (and thus split equally) between both sellers.

Also because prices are ranked by true quality (cost) in equilibrium (from claim 1), (2) implies $\hat{b} > \theta q_1$, and by assumption we have $\theta q_1 \geq b_L$; therefore $\hat{b} > \theta q_1 \geq b_L$, or that there is always a positive mass of consumers preferring the low quality seller 1.

Thus from above, if $\hat{b} < \theta q_2$, then $\pi_2 = [1 - F(\hat{b})](p_2 - c_2)$, and

$$\pi_1 = \begin{cases} F(\hat{b})(p_1 - c_1), & \textit{if}\, p_1 \leq b_L; \\ [F(\hat{b}) - F(p_1)](p_1 - c_1), & \textit{if}\, p_1 > b_L. \end{cases}$$

If however, $\hat{b} = \theta q_2$, then $\pi_2 = [\frac{1-F(\hat{b})}{2}](p_2 - c_2)$; and

$$\pi_1 = \begin{cases} [\frac{1+F(\hat{b})}{2}](p_1 - c_1), & if\, p_1 \leq b_L; \\ [\frac{1+F(\hat{b})}{2} - F(p_1)](p_1 - c_1), & if\, p_1 > b_L; \end{cases}$$

where all consumers below $\hat{b}$ (categories 1 & 2, or $F(\hat{b})$) prefer seller 1, and category 3 consumers are indifferent and split equally between the two sellers, each getting $\frac{1-F(\hat{b})}{2}$.

And lastly if $\hat{b} > \theta q_2$, then all consumers prefer seller 1 if they purchase at all; $\pi_2 = 0$, and

$$\pi_1 = \begin{cases} 1(p_1 - c_1), & if\, p_1 \leq b_L; \\ [1 - F(p_1)](p_1 - c_1), & if\, p_1 > b_L. \end{cases}$$

Notice that $\hat{b} = \theta q_2$ cannot be sustained in equilibrium because at least one seller can profitably reduce its price marginally and attract all $1 - F(\hat{b})$ buyers, unless both are already selling at cost (zero profit). This rules $\hat{b} = \theta q_2$ out as a possible equilibrium scenario, unless seller 2 is unable to sell profitably. And from the above it is straightforward that if $\hat{b} > \theta q_2$ in equilibrium, then no buyer prefers to purchase from seller 2; again seller 2 is unable to sell profitably.

Therefore, only if $\hat{b} < \theta q_2$, can seller 2 sell profitably at all. The following parameters construct an example where this condition cannot be sustained in equilibrium: $c_1 = 0 < b_L = 2 < \theta q_1 = 5 < c_2 = \frac{11}{2} < \theta q_2 = 6 < b_H = 8$; where $\theta = 1$. Using (2), for the case $\hat{b} < \theta q_2$, the first order condition for maximizing π_1, gives

$$p_1^* = \begin{cases} \frac{\theta q_1 + p_2 - b_L + c_1}{2} = \frac{3+p_2}{2}, & if\, p_1^* \leq b_L; \\ \frac{\theta q_1 + p_2 + 2c_1}{4} = \frac{5+p_2}{4}, & if\, p_1^* > b_L. \end{cases} \tag{3}$$

Ignoring dominated values for seller 2's price gives $p_2 \geq c_2 = \frac{11}{2}$. But then $p_1^* = \frac{3+p_2}{2} \geq \frac{17}{4}$ which contradicts $p_1^* \leq b_L = 2$. Therefore, what remains feasible is $p_1^* = \frac{5+p_2}{4}$ as long as this exceeds b_L.

Seller 2's profit for the case $\hat{b} < \theta q_2$ is $\pi_2 = [1 - F(\hat{b})](p_2 - c_2)$. Let its argument maximizer be p_2^*, where because $p_1^* \leq \theta q_1$ (otherwise no one purchases at p_1^*), we have that $\theta q_1 - p_1^* \geq 0$, and therefore $\hat{b}^* \geq p_2^*$ using (2), in other words all consumers above $\hat{b}^*$ find p_2^* affordable. Using (2) and the given parameters, the FOC gives $p_2^* = \frac{b_H - \theta q_1 + p_1 + c_2}{2} = \frac{17}{4} + \frac{p_1}{2}$. Solving both feasible best response functions simultaneously, gives $p_1^* = \frac{37}{14}$ and $p_2^* = \frac{39}{7}$. Therefore $\hat{b}^* = 5 + \frac{41}{14}$; but this exceeds $\theta q_2 = 6$, which is a contradiction to the case we started out with, and which implies that no consumers purchase from seller 2 at these prices. There is therefore no equilibrium that gives $\hat{b} < \theta q_2$; in other words, seller 2 is unable to sell profitably in any equilibrium, proving the required result.

Notice that an equilibrium exists, and is of the type $\hat{b} > \theta q_2$; this is $p_1^* = 4; p_2^* = c_2 = \frac{11}{2}$.[6] Seller 2 cannot profitably deviate because in order to sell anything it needs to lower its price, which already at unit cost cannot be lowered. Seller 1 cannot profitably deviate because given that all consumers (who purchase at all) prefer it to its rival given its rival's price, its profit is $[1 - F(p_1)](p_1 - c_1)$, and this is maximum at its current price of $p_1 = 4$ that solves the respective FOC, $b_H = 2p_1$. □

[6] This gives $\hat{b}^* = \theta q_1 + p_2^* - p_1^* = 6.5 > \theta q_2$.

References

Cremer, H. and J.-F. Thisse (1991). Location models of horizontal differentiation: A special case of vertical differentiation models. *The Journal of Industrial Economics 39*(4), 383–390.

Gabszewicz, J. J. and J. Thisse (1979). Price competition, quality and income disparities. *Journal of Economic Theory 20*, 340–359.

Gabszewicz, J. J. and J. Thisse (1980). Entry (and exit) in a differentiated industry. *Journal of Economic Theory 22*, 327–338.

Grilo, I. and X. Wauthy (2000). Price competition when product quality is uncertain. *Louvain Economic Review 66*(4), 415–438.

Hotelling, H. (1929). Stability in competition. *The Economic Journal 39*(153), 41–57.

Motta, M. (1993). Endogenous quality choice: Price vs. quantity competition. *The Journal of Industrial Economics 41*(2), 113–131.

Schmidt, R. C. (2009). Welfare in differentiated oligopolies with more than two firms. *International Journal of Industrial Organization 27*(1), 501–507.

Shaked, A. and J. Sutton (1982). Relaxing price competition through product differentiation. *The Review of Economic Studies 49*(1), 3–13.

Shaked, A. and J. Sutton (1983). Natural oligopolies. *Econometrica 51*(5), 1469–1484.

Tirole, J. (1988). *The Theory of Industrial Organization*. The MIT Press.

Wauthy, X. (1996). Quality choice in models of vertical differentiation. *The Journal of Industrial Economics 44*(3), 345–353.

Wauthy, X. (2010). Market coverage and the nature of product differentiation: A note. *Economics Bulletin 30*(2), 1129–1136.