Automation and Human Capital: Modeling the Impact on Labor Share*

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Abstract

As the automation revolution reshapes industries, the imperative to understand its implications on labor share and economic dynamics intensifies. This paper delves into the nexus of human capital growth and the pressing challenge of declining labor share engendered by automation. Theoretically employing a triple-model approach, we examine human capital's exogenous influence within a neoclassical framework. Our findings reveal that automation accounts for 18.6 per cent of the global wage share decline, yet augmenting human capital by just 1 per cent can mitigate this effect by 3.23 per cent. The ratio of robot share to labor share increases with a decline in workers' skill sets and an increase in the number of robots adopted in the production process. In the later sections, we model human capital endogenously following Mankiw et. al. (1992) and Lucas (1988) and find a positive correlation between economic growth and investment in physical and human capital. Given the assumptions of the balanced growth path, output growth exceeds the growth of physical and automation capital due to the exogenous accumulation of human capital. The labor share will become constant in the long run, whereas the automation share will become zero.

Keywords: Human Capital, Automation, Labor Share, Economic Growth, Neoclassical Growth Model.

JEL codes: J24, E25, O41

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*This is the first and preliminary draft of the paper that is part of the first author's PhD research work at IIT Delhi. Any errors and omissions are our sole responsibility.

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1 Introduction

The demand for industrial robots is rising inexorably. According to estimations from the International Federation of Robotics (IFR)[1], there is a 14 per cent surge in the worldwide operating stock of industrial robots from 2016 to 2021. Moreover, the projected demand for industrial robots anticipates an annual escalation of 33 per cent from 2021 to 2025. Countries such as China, the United States, Japan and many others portray an increased demand for industrial robots in 2021 (IFR Report, 2022). Robotic models built by American firms like Boston Dynamics exhibit the capacity to supplant human labor from the production process who are engaged in routine activities. These developments in recent years have threatened the global labor force involved in low-skilled or routine tasks.

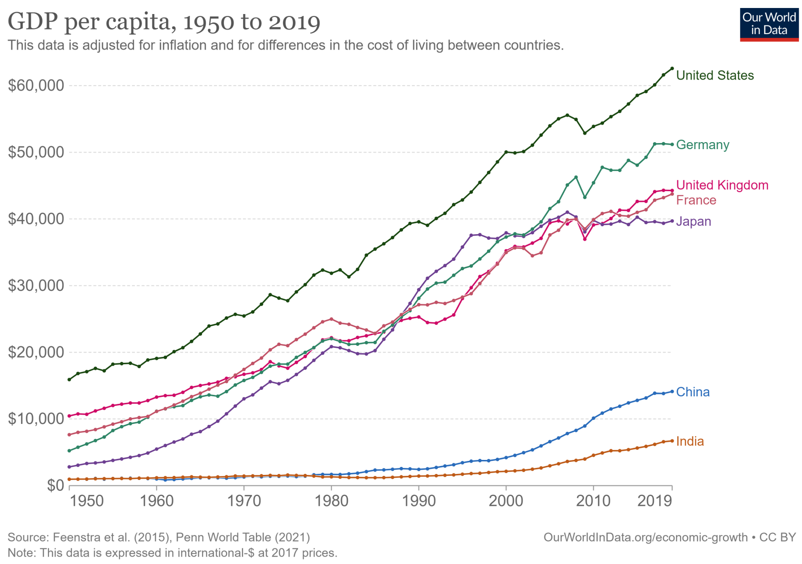


Figure 1: GDP per capita, 1950-2019 (Source: The World Bank)

Figure 1 illustrates the trajectory of GDP per capita from 1950 to 2019 for a few selected nations. The United States has the highest GDP per capita compared to other nations. European countries like Germany, France and the UK have a higher GDP per capita than Asian countries like India and China. Japan, which is not known for having abundant natural resources, has fared admirably since 1950 and is developing at a rate comparable to European nations. Even though China and India have lower per capita GDPs than other affluent countries, their growth rates remain optimistic.

Figure 2 portrays a positive association between increased GDP per capita and improved

[1] World Robotics Report 2022: Click here to see the Report

life expectancy at birth. Countries such as China, Australia, Norway, and Denmark have more than 82 years of life expectancy. In stark contrast, parts of Africa grapple with 50 to 60 years of life expectancies.

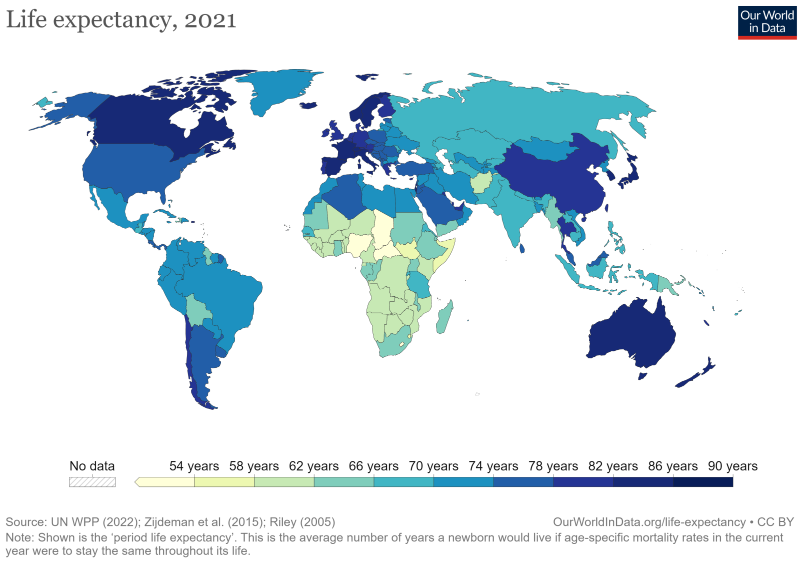


Figure 2: Life Expectancy, 2021 (Source: The World Bank)

With an increase in per capita GDP or economic growth, the life expectancy of a nation increases, which changes the population composition, i.e., the dependence of the old population increases if the rate of population ageing is greater than the population growth rate, as shown in the figure below, which is a significant setback for a country's progress. In such a situation, we witness a shrink in the working-age population, which compels a nation to look for alternative measures to produce goods and services.

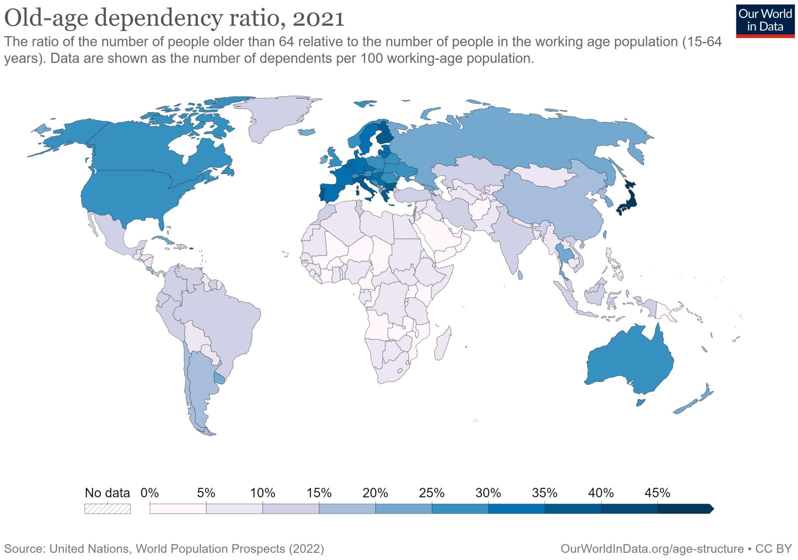


Figure 3: Old-age dependency ratio, 2021 (Source: The World Bank)

In summary, countries with higher economic growth or per capita GDP guarantee a higher living standard in the face of declining population growth, a shrinking the working-age population, and an increase in the old-age dependency ratio, which forces them to seek alternative measures to replace declining labor from the production process.

A slew of studies on automation, factor share, and economic development are underway. Lankisch and Prettner (2019)[7] considered two types of labor in their model, namely high-skilled and low-skilled laborers, and found that automation displaces the low-skilled labor engaged in routine activities. If the degree of substitution between high-skilled and low-skilled workers is high, then the accumulation of robots will decrease the share of low-skilled workers.

Acemoglu and Restrepo (2019)[3] tried to solve the puzzle of the effect of automation, i.e., will automation replace human labor or increase the demand for human labor by creating more productive opportunities? But, they remain inconclusive on the effect of automation on labor.

Strulik and Prettner (2020) [11] modelled a research and development sector where automation and skilled laborers work together to generate output used as intermediaries in another sector. However, these intermediaries pose a threat to unskilled workers working in the manufacturing sector. R&D-based innovations in machine technology lead to automation, increasing the skill premium, thus increasing inequality in income and wealth. Their model suggested workers would lose the ”race with machines”.

Ray and Mookherjee (2022)[12] used a sectoral task-based production function where they showed that given a set of laborers, automation will completely substitute human labor, but the process will be prolonged. Therefore, the share of human labor will be zero in the long run.

Guerreiro et al. (2022) [5] modelled an OLG setup with two types of workers and endogenous skill formation to solve the optimal tax system using the Mirrleesian tax system (a non-linear tax system that is designed to maximize social welfare). As robots become cheaper, the share of routine workers declines, so a planner tries to redistribute income towards routine workers. In their framework, a social planner provides the young generation incentives to invest in skills and become non-routine workers, so it is optimal to tax robots when the initial old generation of routine workers are in the labor-force (short-run), and once the non-routine workers retire, the optimal robot tax is zero (long-run).

The main aim of our study is to determine how the introduction of automation would affect human labor, considering whether or not workers will have the chance to expand their skill sets. In the first part of this article, we explain the dynamics of wage share in total income with exogenous human capital formation when automation is a factor input. In the second part of this article, we endogenize the process of human capital formation. It enables us to examine how the labor share shifts over time and determine whether or not our model allows for any room for policy intervention.

In the initial part, We employ the framework of neoclassical growth model with a single sector (Solow 1956[13]) with exogenous human capital formation, similar to the exposition of Prettner (2019)[10], where he showed the implication of automation is very distressing. Automation could explain 14 per cent of the total decline in the global share of income reported by Karabarbounis and Neiman (2014)[6] for the United States. We show that when workers get training and improve their skill sets, the decline in the wage share is slower than compared to a situation where workers do not update their skill sets. This is one of the significant policy implications of our research.

In the second part, we endogenize the model on the lines of Mankiw et. al. (1992)[9] and showed that long-run growth can be increased if a higher proportion of aggregate savings is diverted to accumulate human capital. Higher skill accumulation enables workers to handle high-tech machines, increasing their compatibility with physical capital and reducing the substitutability with automation capital.

In the final part, we endogenize human capital formation using Lucas's (1988) model, showing the effect of different parameters on output growth and other variables. Factor shares of robots and workers are also discussed broadly in this section, where we find a negative relationship between the wage share and time allocated by workers for production. In the long run, with a decrease in the time allocated for production by workers, the robot share declines at a slower rate.

Our paper is divided into four sections. The first section focuses on a theoretical model that incorporates automation and human capital into the Solow framework. We also include a numerical assessment section where we measure the impact of exogenous human capital on explaining the declining global wage share due to automation. In the second section, we discuss the effect of investment in human capital accumulation on growth by endogenizing human capital formation using Mankiw et.al. (1992). Here, we show the transitional dynamics of human capital growth. In the third section, we endogenize human capital following Lucas (1988), which converts our baseline model to a Ramsey-Solow framework. We discuss in detail the determinants of growth rates of our main variables here. Moreover, we show the dynamics of the long-run factor shares of robots and humans. Finally, we conclude in the last section.

2 Baseline Model

We consider the production function of a closed economy consisting of representative households to be Cobb-Douglas in form. The final output is a function of physical capital $(K(t))$, automation capital $(P(t))$, amount of time devoted to the production process $(u \in (0,1])$, and human capital $(h(t))$. To eliminate any effect of technological advancement on our main variables, the technical parameter $A(t)$ is normalised to unity. The average level of human capital in the economy is represented by h_a, while γ is an external parameter. In this section, we will consider per capita human capital as given and treat it as an exogenous variable, so we assume workers spend their entire time in the production process $(u = 1)$, while in the later section, we will endogenize the human capital growth following Mankiw et. al. (1992)[8].

$$Y(t) = A(t)\ K(t)^{\beta}\ [u(t)\ h(t)\ N(t) + P(t)]^{1-\beta}\ h_a^{\gamma} \tag{1}$$

The stream of physical capital accumulation and automation capital accumulation are formalised as below.

$$\dot{K}(t) = s_k \, s \, Y(t) - \delta \, K(t) \tag{2}$$

$$\dot{P}(t) = s_p \, s \, Y(t) - \delta \, P(t) \tag{3}$$

The additional amount of physical and automation capital is produced according to the Solow model (1956). The stock of physical capital available for the next period is the difference between a fraction of total savings (s_k amount of total savings) invested in physical capital production and the amount of physical capital depreciated in the current period. Automation capital for the period $(t+1)$ is generated similarly, with a savings of $(1 - s_k) \equiv s_p$ invested for the formulation of automation capital minus the depreciated amount in the current period. To keep things simple, we keep the physical and automation capital depreciation rates uniform; changing them will not improve our results but will complicate the model.

In a competitive market, an investor would like to invest in automation capital if the returns are at least as good as those from physical capital ($r_p = r_k$). From the no-arbitrage condition, we establish a relationship between physical, automation and human capital. Its per capita counterpart has been written below:

$$k(t) = \frac{\beta}{1-\beta} \, [h(t) + p(t)] \tag{4}$$

By utilizing the definition of $k(t)$ and inserting it into the per capita production function derived from equation (1), we can derive a simplified output function as shown below:

$$y(t) = \left(\frac{\beta}{1-\beta}\right)^{\beta} [h(t) + p(t)] \, {h_a}^{\gamma} \tag{5}$$

Differentiating equation (4) with respect to time we get,

$$\dot{k} = \frac{\beta}{1-\beta} \, (\dot{h} + \dot{p}) \tag{6}$$

Solving the neoclassical growth model:

The fundamental equation of motion of the Solow model, where θ denotes the population growth rate. We use equations (2) and (3) in per capita form.

$$\dot{k} + \dot{p} = s_k \; s \; y - (\theta + \delta) \; k + s_p \; s \; y - (\theta + \delta) \; p \tag{7}$$

Using equations (4-6) we obtain the growth rate of automation capital (g_p) as follows:

$$g_p \equiv \frac{\dot{p}}{p} = s \; \beta^{\beta} \; 1 - \beta^{(1-\beta)} \; \left(\frac{h}{p} + 1\right) \; {h_a}^{\gamma} - (\theta + \delta) \left(1 + \frac{\beta h}{p}\right) \tag{8}$$

In the balanced growth path, we follow the assumption that all the variables grow at a constant rate. Thus, imposing this condition, in the long run, all the variables grow at a constant rate g.

$$g = s \; \beta^{\beta} \; (1 - \beta)^{1-\beta} \; {h_a}^{\gamma} - (\theta + \delta) \tag{9}$$

In general, the growth rate is positive, implying that the above equation's left-hand side will be strictly greater than the right-hand side of the equation. Because an economy's aggregate savings rate (s) is positive and h_a is a positive variable, the left side will always be greater than the right side. This is the sustainability condition in our model.

Proposition 1: *We show that, without any technological progress, there is the possibility of perpetual growth along the balanced growth path using a different variant of the Solow model (1956) with automation and traditional capitals and exogenous human capital.*

- *(i) With the increase in the aggregate savings rate, g increases.*
- *(ii) The rise in the average rate of human capital (h_a) increases g.*
- *(iii) An increase in population growth and depreciation of both types of capital will hinder g.*

Proof: Inspecting equation (9), which explains the dynamics of growth along the balanced growth path, shows that increasing the aggregate savings rate will enhance growth in the long-run as physical and automation capital will accumulate faster.
The average level of human capital in an economy, which is an exogenous variable, also positively impacts economic growth. Intuitively, an increase in the average human capital in the economy will enhance labor productivity. Enhancing the productivity of laborers will enable them to be complementary to traditional capital, which will positively impact long-run growth.

Whereas an increase in population growth (θ) and an increase in the depreciation of both types of capitals (δ) will reduce the long-run economic growth rate, as an increase in population growth will create a shortage of per capita output, which will significantly lower the aggregate savings rate and further reduce the process of capital accumulation. Similarly, if the rate of depreciation of both types of capital increases, then the replacement rates of both types of capital will increase, which will hamper economic growth in the long-run.

Proposition 2: *With our version of the neoclassical growth model, where automation capital and traditional capital accumulate endogenously as a function of output, savings rate, population growth, and depreciation rate, and exogenous human capital formation, we can show the factor share of laborers is an increasing function of human capital (h) and a decreasing function of automation capital (p).*

Proof: The aggregate share of laborers in the total output is described as the ratio of their returns times the total laborers in an economy and the aggregate output. Using equation (1) to derive the labor share, we get:

$$\frac{wh}{Y} = (1-\beta)\,\frac{hN}{hN+P} \tag{10}$$

We can deduce from the above equation that the wage share to national income will decline as automation capital accumulation increases in an economy. Human capital should grow faster than automation capital to maintain a higher wage share in the total income. This analogy will be taken forward to the later section, where we endogenize the human capital formation. On the other hand, with an increase in human capital, the labor share increases. The intuition behind this proposition is that when laborers upgrade their skill sets, they will be complementary to traditional capital, enabling them to earn more wages than before. Due to automation, workers will benefit if they upgrade their skills and become complements to traditional capital, which decreases their substitutability and increases wages.

Proposition 3: *Using our theoretical baseline model, we can highlight a few key findings:*

- *(i) The ratio of aggregate factor shares of automation capital and labor increases with an increase in automation capital (P) and decreases with an increase in labor supply (N) and an increase in labor skill-set (h).*

- *(ii) The ratio of the aggregate share of physical capital and automation capital increases as there is a decrease in automation capital (P), an increase in the supply of labor (N), and upgradation of skill sets by laborers (h).*

Proof: We use equation (1) to calculate the aggregate share of automation capital to total output for proposition 3. (i):

$$\frac{r_p P}{Y} = (1-\beta)\frac{P}{hN+P} \tag{11}$$

Similarly, obtain the ratio of automation capital share to labor share:

$$\frac{r_p P}{wh} = \frac{P}{hN} \tag{12}$$

Inspecting the above equation, we can find a one-to-one relationship between the share of automation capital in total income and the automation units in the economy. On the other hand, with the increase in the supply of laborers (N) and the increase in human capital accumulation of the existing laborers (h), the ratio of automation capital share and labor share to total income decreases in the short-run.

Intuitively, a rise in automation capital will remove workers from the production process as the substitutability factor kicks in. And with an increase in the stock of human capital accumulated by the laborers, they become more compatible with machines.

Similarly, we prove proposition 3. (ii) by calculating the factor share of physical capital using equation (1) and dividing by the share of automation capital given in equation (11) to derive the ratio of the physical capital share to the automation capital share.

$$\frac{r_k K}{r_p P} = \frac{\beta}{1-\beta}\frac{hN+P}{P} \tag{13}$$

Inspecting the above equation, the ratio rises as the level of automation capital falls and the number of laborers rises. This has a similar intuition as earlier, where machines and humans work in complementary to each other.

Our results improve on the findings of Prettner (2019) [10] and C.Lankisch et al. (2019) [7], where we showed the possibility of perpetual growth if there exists an exogenous human capital formation that enhances the skill sets of laborers without bifurcating the types of laborers. Our theoretical framework also shows the decline in wages will be gradual despite robots being welcomed in the production process because upgrading skill sets will empower laborers and will be hard to replace. Our paper pays attention to the vast literature on research and development, and learning by doing models that control human capital formation. Our model is lucid and straightforward, which helps in explaining the objectives effectively.

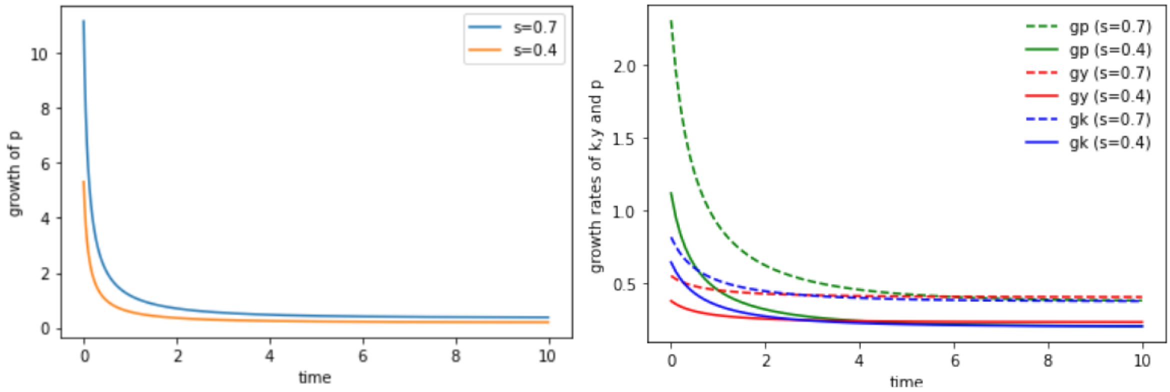
2.1 Model Calibrations and Numerical Illustrations

In this section, we have calibrated our model to visualise the off-steady-state paths of our main variables, which constitute the transitional dynamics of the study. The following are the parametric values borrowed from various literatures.

Parameter Source	Description	Estimate
β Karabarbounis and Neiman (2014) [6]	Output elasticity of capital	0.33
θ Prettner (2019) [10]	Population growth	0.009
δ Grossmann et.al. (2013) [4]	Depreciation rate	0.04
s To show the dynamics	Savings rate	0.4 and 0.7
h To show the dynamics	Human capital	50

Table 1: Parametric values

Arbitrary values have been assigned for the savings rate and stock of human capital. Altering these values will remain the main results of our analysis.



(a) Plot of g_p with s=0.4 and s=0.7

(b) Plot of growth rates with s=0.4 and s=0.7

Figure 4: Transitional dynamics

The above figure captures the transitional dynamics of our stylized economy. Equation (8) has been plotted with the above parametric values, whereas Figure 4. (a) portrays the importance of aggregate savings rate. With an increase in the aggregate savings rate from 0.4 to 0.7, the growth rate of automation shifts rightward, which signifies positive robot accumulation. The effect of an increase in the aggregate savings rate is uniform at other growth rates in the model. Figure 4. (b) depicts the transition of all of our main variables in the model. The growth rates of physical capital, output, and automation capital converge to the long-run steady-state growth rate (g), depicted by equation (9). The dotted lines represent the growth rate of our main variables when $s = 0.7$, and the solid lines represent the same with $s = 0.4$.

The final task of this section is to compare the impact of human capital on wage share in our model using equation (10). We follow a similar methodology as Prettner (2019) demonstrated, but we will look at the global share, unlike the previous literature, which focused on the USA. The average labor force of the world between 1990-2021 is 2937 million people (World Bank 2022)[1]. According to the report provided by the International Federation of Robotics (2022)[2], the world estimate of industrial robots installed is 141 robots per 10,000 workers $(P(t))$. We have taken the value of β from Karabarbounis and Neiman (2014) as 0.33, which is also common in this genre of literature. Our calculations are very conservative as we do not account for heterogeneity in our estimation. Our results are a mere tool to support our theoretical results and should be perceived as a lower bound to any conclusion drawn. To benefit our analysis, we develop two cases for comparison:

Case 1: In this case, we assume the human capital $(h(t))$ to be kept at unity. This case is closely related to the earlier studies that do not account for human capital in their models. We also put $\gamma = 0$ to shut any exogenous channel in our analysis.

(i) We assume that the number of robots in 1990 is zero. We get the wage share as follows:

$$\begin{aligned}\left[\frac{w(t)h(t)}{Y(t)}\right]_{1990} =& (1-\beta)\frac{hN}{hN+P} \\ =& 66.67\%\end{aligned}$$

(ii) In 2021 we use the measure of $P(t)$ from IFR (2022) as discussed above.

$$\begin{aligned}\left[\frac{w(t)h(t)}{Y(t)}\right]_{2021} =& (1-\beta)\frac{hN}{hN+P} \\ =& 65.74\%\end{aligned}$$

Comparing the two scenarios, with the inclusion of automation in the production process, the global wage share has declined by 0.93 percentage points. Karabarbounis and Neiman (2014) highlighted a decline of 5 percentage point of the global wage share, and our model explains that around 18.6 per cent of the global wage share decline is due to automation.

Case 2: Keeping the above ingredients, we increase the human capital by 1 per cent and analyse two similar sub-cases.

(i) When automation is zero in 1990, the wage share corresponds to the earlier case, i.e., 66.67 per cent.

(ii) With an increase in human capital by 1 per cent the wage share boils down to:

$$\begin{aligned}\left[\frac{w(t)h(t)}{Y(t)}\right]_{2021} =& (1-\beta)\frac{hN}{hN+P} \\ =& 65.75\%\end{aligned}$$

Comparing the two sub-sections, the decrease in the wage share due to the inclusion of automation is by 0.9 percentage points. Following the estimates of Karabarbounis and Neiman (2014), we can say that 18 per cent of the reduction in the global wage share can be explained by automation when there is a 1 per cent exogenous rise in human capital accumulation.

Comparing the two cases, keeping everything unaltered, if there is a policy tool that increases the average human capital by 1 per cent, then the reduction in global wage share due to automation will reduce by 3.23 per cent. Therefore, training and education will slow the decline of the wage share rate when the threat of losing jobs to automation is high.

3 Endogenous Human Capital Formation: Mankiw, Romer and Weil Framework

In the previous section we have used exogenous human capital formation to show the impact of a marginal increase in the education level on growth rate and wage ratios. In this section we will use the Mankiw et. al. (1992)[8] specification to determine the path of human capital formation.

In this section, we will use the production function from the theoretical model and exclude the external accumulation of human capital ($\gamma = 0$). We assume that workers devote their full time to the production process ($u = 1$) because in the Mankiw et. al. (1992) system, human capital is accumulated by investing a certain fraction of household savings into education and training. This investment results in individuals having better skill sets than those who invest less or nothing in accumulating human capital. We also exclude external technical changes ($A = 1$) in this section. Therefore, the production function is written as follows:

$$Y = K^{\beta} \, [h \, N + P]^{1-\beta} \tag{14}$$

The per capita accumulation of physical and automation capitals are identical as in the baseline model, but the per capita human capital formation is defined as:

$$\dot{h}(t) = s_h \, s \, y(t) - (\theta + \delta) \, h(t) \tag{15}$$

Here s_h denotes a fraction of total savings channelised to accumulate human capital. Therefore $s_h + s_p + s_k = 1$. For simplicity, we assume that the depreciation rate of human capital is similar to the other forms of capital in the model.

Using the no-arbitrage condition similar to the baseline model, we derive a reduced form output-per capita equation as shown below:

$$y = \left(\frac{\beta}{1-\beta}\right)^{\beta} (h+p) \tag{16}$$

Now solving for the aggregate capital accumulation of the economy:

$$\dot{k} + \dot{p} + \dot{h} = s_k\, s\, y - (\theta+\delta)\, k + s_p\, s\, y - (\theta+\delta)\, p + s_h\, s\, y - (\theta+\delta)\, h \tag{17}$$

By substituting the values of k, $\dot{k}$, and y from equations (4), (6), and (16) into the fundamental equation of the Solow model, which includes the no-arbitrage condition and reduced form per capita output, we obtain:

$$\dot{h} = s\, \beta^{\beta}\, (1-\beta)^{1-\beta}(h+p) - (\theta+\delta)(h+p) - \dot{p} \tag{18}$$

We can substitute the definition of $\dot{p}$ from equation (3) into the per capita equation above to obtain:

$$\frac{\dot{h}}{h} = s\, \beta^{\beta}\, (1-\beta)^{-\beta}\, (s_k + s_h - \beta)\, (1 + \frac{p}{h}) - (\theta+\delta) \tag{19}$$

To obtain the long-run growth rates in our model, making certain assumptions about the balanced growth path is pivotal.

Assumption 1: In balanced growth, the growth of human capital is strictly greater than the growth of automation capital $(g_h > g_p)$. Therefore in the long-run $\frac{h}{p} \to \infty$.

Using the stated assumption and re-writing equation (19) to obtain the long-run growth as:

$$g_h \equiv \frac{\dot{h}}{h} = s\, \beta^{\beta}\, (1-\beta)^{-\beta}\, (s_k + s_h - \beta) - (\theta+\delta) \tag{20}$$

If $g_h < g_p$, the growth will explode, which doesn't make sense in this setup. On the other hand, if $g_h = g_p$, then the growth rate of g_h will be higher than the earlier equation.

Proposition 4: *If we consider the accumulation of human capital following Mankiw et.al. (1992), then:*

- *(i) There exists a possibility of perpetual long-run growth without technological progress in a neoclassical framework.*
- *(ii) The population growth rate can hinder the perpetual long-run growth in our model.*
- *(iii) If there is a long run growth, then it increases with an increase in the proportion of savings devoted to accumulating physical and human capital.*
- *(iv) The share of human capital in total income increases with an increase in human capital and a decrease in automation capital.*

Proof: Propositions 4. (i) and 4. (ii) can be proven by inspecting equation (20). Proposition 4. (iii) can be proven as follows:

$$\frac{\partial g_h}{\partial s_h} = \frac{\partial g_h}{\partial s_k} = s\ \beta^{\beta}\ (1-\beta)^{-\beta} > 0$$

With an increase in the proportion of savings devoted to the accumulation of physical and human capital, there is a possibility of an increase in growth. Proposition 4. (iv) can be proven by deriving the share of human capital in total income. We use equation (14) to get:

$$\frac{\frac{\partial Y}{\partial h}\,h}{Y} = (1-\beta)\frac{hN}{hN+P} \tag{21}$$

We can see that the wage share increases when h increases and P decreases. Therefore, an increase in human capital may secure workers from the rise of automation in the short-run.

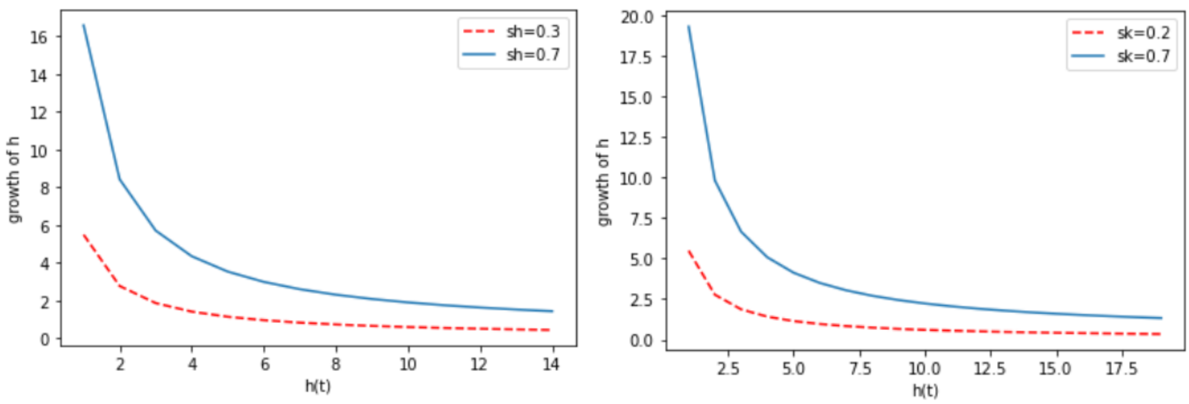
As mentioned in the previous literature by Prettner (2019), we obtained a similar finding that violates the neoclassical assumption of growth along the balanced growth path. There lies a possibility of positive long-run growth in our model without technical progress due to the inclusion of automation in the production process.

endogenizing human capital in this section enables us to establish a relationship between the savings rate diverted to human capital formation and economic growth. We find a positive relationship between the two variables, which is quite intuitive. An increase in the investment in human capital can boost skill formation, increasing labor productivity and making it immune from automation.

In the last part of our results, we find that the share of workers in total income increases if workers get proper training, which increases their human capital, thus making them hard to replace by automation capital in the production process. Intuitively, if people invest more in accumulating human capital, a.k.a attaining higher education, they are endowed with higher skill sets, which help them complement physical capital. Therefore, countries with higher working-age populations should focus on providing better educational facilities and training that will immune their workers from automation, which is also an important policy implication of our research.

3.1 Model Calibrations

In this section, the model has been adapted to accurately showcase the fluctuating patterns of our prominent variables, highlighting their transitions over time. Different parametric values have been taken from the earlier-mentioned sources. We use equation (19) for our analysis, and through calibration, we show how the growth rate of human capital behaves when we alter the proportion of savings diverted to human and physical capital accumulation.



(a) Plot of g_h with $s_h = 0.3$ and $s_h = 0.7$ (b) Plot of g_h with $s_K = 0.2$ and $s_k = 0.7$

Figure 5: Transitional dynamics

Figure 5 shows the transition of human capital growth over time. It confirms our earlier proposition that human capital growth can be increased if we increase the proportion of savings for physical and human capital accumulation. Figure 5. (a) depicts the increase in g_h when the proportion of savings diverted to human capital increases from 0.3 to 0.7. On the other hand, figure 5. (b) portrays the change in g_h when savings are channelled for accumulating more physical capital.

The portion of income attributed to wages remains the same as in the baseline model. Therefore, the results obtained in the numerical example of the baseline model using the equation for wage share will also apply to this section. However, in the previous section, we increased human capital through external means, while in this section, we increased human capital by allocating more savings towards its accumulation. When the proportion of savings devoted to human capital, denoted as s_h, increases, the growth rate also increases, and it helps workers combat automation by allowing them to complement machines as they are equipped with better skills now. Thus, by endogenizing human capital, we observe a twofold impact of raising s_h, which was not present in the baseline model.

4 Endogenous Human Capital Formation: Ramsey-Solow Model

To this point, we have shown there remains a positive growth with automation capital as one of the factor inputs even though we shut down technical progress given a level of human capital. In this section, we will endogenize human capital formation in line with Lucas (1988), making it a Ramsey-Solow framework and analysing its effect on our baseline model. This section will add to the existing automation literature as it directly extends the findings of Prettner (2019). We use the same formation as in the baseline model where equation (1) implies the total output in the economy, and its per capita counterpart can be seen as

$$y = k^{\beta} \, (uh + p)^{1-\beta} \, {h_a}^{\gamma} \tag{22}$$

As described earlier, h_a represents the average level of human capital formation; following Lucas (1988), it is defined as:

$$h_a = \frac{\int_0^{\infty} \, h \; N(h) \; dh}{\int_0^{\infty} \; N(h) \; dh}$$

where the total labor force of an economy has a certain level of human capital that ranges from zero to infinity, individuals can allocate their non-working hours $(1-u)$ to accumulate a higher level of h. It is considered an external component in the model, which implies nobody has prior information regarding the average level of human capital in the economy before deciding on their level of human capital. To keep our model simple, we exclude the ability constraint of individuals in accumulating human capital. We do not consider the learning-by-doing framework in our model, where individuals can upgrade their existing levels of human capital by working at a particular job for a more extended period of time.

We also exclude the channel of heritable learning in accumulating human capital, where individuals obtain knowledge from their previous generations. The technology related to human capital formation should be related to the non-working hours $(1-u)$ allocated by individuals to accumulate human capital. Uzawa (1965)[14] formulated a simple linear relationship between the time allocated to accumulate human capital and the growth of human capital, which Lucas has used in his seminal work. Therefore, the technology related to human capital formation is given by:

$$\frac{\dot{h}}{h} = \delta(1-u) \tag{23}$$

where δ is the efficiency parameter, which is homogeneous for all individuals in the model. Workers devote a u fraction of total time in the production process; therefore, the left-over time $(1-u)$ goes for human capital accumulation. There is no leisure in our framework, but u is a choice variable, which implies that individuals can choose between the time allocated for production, which will give them wages and for accumulating human capital, which will increase their stock of human capital.

For simplicity, we derive the no-arbitrage condition similarly to equation (4) to develop a reduced form production function where output is a function of automation capital and human capital. In the household sector, the budget constraint for individuals is given as follows:

$$c(t) + \dot{p}(t) = r_p(t)\ p(t) + w(t)\ u(t)\ h(t) \tag{24}$$

where the left-hand side of the above equation represents the expenditure part of the households in our economy, it comprises of two parts- per capita consumption ($c(t)$) and investment to accumulate automation capital ($\dot{p}(t)$). The right-hand side describes the income channel of households in our system. There are two sources of income in our model- (a) wage income, which is earned by supplying effective labor to the firm, and (b) interest income earned from automation capital. At equilibrium, total spending by households should equal the total income they earn. The utility function used in our model is a constant relative risk aversion utility, where σ denotes the coefficient of relative risk aversion. It is written as:

$$U(c) = \frac{c(t)^{1-\sigma}}{1-\sigma} \tag{25}$$

Present value Hamiltonian:

$$J = e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} + \psi_1 \, e^{-\rho t} \, [r_p(t) \; p(t) + w(t) \; u(t) \; h(t) - c(t)] + \psi_2 \, e^{-\rho t} \, [h(t) \; \delta \; (1 - u(t))] \tag{26}$$

Where $c(t)$ and $u(t)$ are the control variables, and $p(t)$ and $h(t)$ are the state variables. The shadow prices of automation capital and human capital are denoted by ψ_1 and ψ_2, respectively. For simplicity, we are working with the reduced form of output, but if we use the per capita counterpart without using the no-arbitrage condition, then our model will also include physical capital in the budget equation. Then ψ_1 will represent the shadow price of both physical and automation capitals. Later, we will use the no-arbitrage condition to establish a relationship between automation and physical capital.

The first order conditions are:

$J_c = 0$:

$$c(t)^{-\sigma} = \psi_1 \tag{27}$$

$J_u = 0$:

$$\psi_1 \; w(t) = \psi_2 \; \delta \tag{28}$$

$J_p = -\frac{d}{dt} \; (\psi_1 \; e^{-\rho t})$:

$$\frac{\dot{\psi}_1}{\psi_1} = \rho - r_p(t) \tag{29}$$

$J_h = -\frac{d}{dt} \; (\psi_2 \; e^{-\rho t})$:

$$\frac{\dot{\psi}_2}{\psi_2} = \rho - \frac{\psi_1}{\psi_2} \; w(t)u(t) - \delta \; [1 - u(t)] \tag{30}$$

The two transversality conditions are:

$$\lim_{t\to\infty}[e^{-\rho t}\ \psi_1\ p(t)] = 0$$

$$\lim_{t\to\infty}[e^{-\rho t}\ \psi_2\ h(t)] = 0$$

We can re-write equation (28) as:

$$\frac{\psi_1}{\psi_2} = \frac{\delta}{w(t)} \tag{31}$$

Differentiating the above equation with respect to time, we get:

$$\frac{\dot{w}(t)}{w(t)} = \frac{\dot{\psi}_2}{\psi_2} - \frac{\dot{\psi}_1}{\psi_1} \tag{32}$$

Using equations (29) and (30) and re-write the above equation as:

$$\frac{\dot{w}(t)}{w(t)} = r_p(t) - \frac{\psi_1}{\psi_2} w(t) u - \delta(1-u)$$

Re-writing the above equation using equation (31), we get:

$$\frac{\dot{w}(t)}{w(t)} = r_p(t) - \delta \tag{33}$$

Using equations (27) and (29) to establish a relationship between the growth of consumption (g_c) and $r_p(t)$. First, by differentiating equation (27), we get:

$$\frac{\dot{\psi}_1}{\psi_1} = -\sigma\ g_c$$

Now replacing it in equation (29), we get $r_p(t)$ as a function of g_c:

$$r_p(t) = \rho + \sigma\ g_c \tag{34}$$

Turning our attention to the firms' side. Firms produce output and employ workers from the household sector. They also invest in machine and automation capital for the production of goods and services. Firms provide factor payments to all the factor inputs used in the production, and their motive of operation is to earn profits. Firms do not take into account the exogenous variables to form their decision. Therefore, the profit maximisation problem of an individual firm is as follows:

$$max \ \ \pi: \ \ k(t)^{\beta} \ [u(t) \ h(t) + p(t)]^{1-\beta} \ {h_a}^{\gamma} - [r_p(t) \ p(t) + w(t) \ u(t) \ h(t) + r_k(t) \ k(t)] \tag{35}$$

The first order conditions are as follows:

$\pi_k = 0$:

$$r_k(t) = \beta \, k(t)^{\beta-1} \ [u(t) \ h(t) + p(t)]^{1-\beta} \ {h_a}^{\gamma} \tag{36}$$

$\pi_p = 0$:

$$r_p(t) = (1-\beta) \ k(t)^{\beta} \ [u(t) \ h(t) + p(t)]^{-\beta} \ {h_a}^{\gamma} \tag{37}$$

$\pi_h = 0$:

$$w(t) = (1-\beta) \ k(t)^{\beta} \ [u(t) \ h(t) + p(t)]^{-\beta} \ {h_a}^{\gamma} \tag{38}$$

Differentiating the wages from the firm side and implementing the market clearing condition $(h_a = h)$, we get:

$$\frac{\dot{w}}{w} = -\beta \, \frac{\dot{h}}{h} \, \frac{u\,h}{u\,h+p} - \beta \, \frac{\dot{p}}{p} \, \frac{p}{u\,h+p} - \beta \, \frac{\dot{u}}{u} \, \frac{u\,h}{u\,h+p} + \beta \, \frac{\dot{k}}{k} + \gamma \, \frac{\dot{h}}{h} \tag{39}$$

Connecting the wages from both sectors. Using equations (33) and (39), we get:

$$r_p(t) - \delta = -\beta \, \frac{\dot{h}}{h} \, \frac{u}{u+\frac{p}{h}} - \beta \, \frac{\dot{p}}{p} \, \frac{1}{u\frac{h}{p}+1} - \beta \, \frac{\dot{u}}{u} \, \frac{u}{u+\frac{p}{h}} + \beta \, \frac{\dot{k}}{k} + \gamma \frac{\dot{h}}{h} \tag{40}$$

We use short-hand notations for representing the growth variables: $\frac{\dot{x}}{x} \equiv g_x$, where x is any of the main variables in the model.

To derive the long-run growth, we have used certain assumptions pertaining to the balanced growth path, which have been stated below:

Assumption 4.1: In the balanced growth path, g_p and g_c grow at the same rate as the growth of g_k. Assigning a common growth rate g to indicate a common rate of growth for consumption, physical and automation capitals.

Assumption 4.2: In the balanced growth path, $g_h > g_p$. It means in the long-run, h grows at a faster rate than p. Or, $\frac{h}{p} \to \infty$ in the long-run.

- (i) If $g_h = g_p$, all the variables will grow at a constant rate in the long-run. This case is equivalent to estimating the model putting $\gamma = 0$ [2], a.k.a. without any external effect.
- (ii) If $g_h < g_p$; then the solution will be indeterminate.

Assumption 4.3: In the balanced growth path, $\beta > \gamma$ [3]. It means the output elasticity of capital should be strictly greater than the external parameter in our model.

- (i) If $\beta = \gamma$, then the growth will be indeterminate.
- (ii) If $\beta < \gamma$, then the growth will be negative.

Assumption 4.4: In the balanced growth path, $\dot{u} = 0$. It means that in the balanced growth path, the fraction of time allocated for production and human capital accumulation remains constant.

Using the above assumptions and equation (40) to establish a relationship between physical and human capital growth rates.

$$g_k = g_h \frac{\beta - \gamma}{\beta} \tag{41}$$

When $\gamma = 0$, the above equation implies $g_k = g_h$, which means all the variables grow at a similar rate in the balanced growth path.

Using equation (40) and the assumptions stated above, we derive the common long-run growth rate g as follows:

$$g_k + \left(\frac{\gamma}{\beta} - 1\right) g_h + \frac{1}{\beta}\left[\delta - r_p(t)\right] = 0 \tag{42}$$

Using equation (34) from the household sector to replace $r_p(t)$ and equation (41) to write g_h

[2]We will show this case in the comparative statics section.

[3]By inspecting the final growth equations of g and g_h, we can prove the two sub-cases.

in terms of g_k in the above equation with the balanced growth assumptions, we get:

$$g \equiv g_c = \frac{1}{\sigma}\,(\delta - \rho) \tag{43}$$

Putting the value of g in equation (41) to derive g_h in the long-run, we get:

$$g_h = \frac{\beta}{\sigma}\,\frac{(\delta - \rho)}{(\beta - \gamma)} \tag{44}$$

Inspecting equations (43) and (44), we find that $g_h > g$, given $\beta > \gamma$.

In this part, we derive the growth rate of per capita output (g_y) by differentiating equation (22) and assuming the market clearing condition holds. We get:

$$g_y = \beta\, g_k + (1-\beta)\, g_h\, \frac{uh}{uh+p} + (1-\beta)\, g_u\, \frac{uh}{uh+p} + (1-\beta)\, g_p\, \frac{p}{uh+p} + \gamma\, g_h$$

Now, using the balanced growth path assumptions, we can rewrite the growth of output in the long-run as:

$$g_y = \beta\, g_k + (1 - \beta + \gamma)\, g_h \tag{45}$$

Using equation (41) to replace g_k in terms of g_h, we find the output growth rate is equal to the growth rate of human capital, i.e., $g_y = g_h$. Therefore, it concludes that the output growth is more than the common growth rate (g) due to the external parameter (γ) in our model. All the variables grow uniformly without any external effect $(\gamma = 0)$.

4.1 Comparative Statics

In the balanced growth path, we compare our long-run growth rates across various parametric values using equations (43) and (44).

- (i) If $\gamma = 0$, all the variables in the model grow at a constant rate g following equation (43), i.e., $g_k = g_h = g_p = \frac{1}{\sigma}\,(\delta - \rho)$.
- (ii) If $\gamma > 0$ and $\beta > \gamma$, then the growth rate of human capital is more than the common growth rate.
- (iii) If ρ increases, consumers become less patient and try to consume more in the

present, then the accumulation of automation and physical capital declines, and vice-versa.

- (iv) If σ increases, consumers become more risk averse, and the accumulation of physical and automation capitals declines, as consumers will prefer consumption smoothing over capital accumulation.

We derive the factor shares equations similarly as in the baseline model and compare them in this section using the long-run assumptions in the balanced growth path.

- (i) The long-run wage share is derived similarly as equation (10), using the balanced growth assumptions, we obtain:

$$\lim_{\frac{h}{p}\to\infty} \frac{w(t)\ h(t)}{y(t)} = (1-\beta)\ \frac{u(t)}{u(t)+\frac{p(t)}{h(t)}} = 1-\beta \tag{46}$$

The share of wages to national income will hit a constant if the growth rate of human capital exceeds the growth rate of automation capital in the balanced growth path.

- (ii) The long-run automation share is derived similarly as equation (11) and using the balanced growth assumptions, we obtain:

$$\lim_{\frac{h}{p}\to\infty} \frac{r_p(t)\ p(t)}{y(t)} = (1-\beta)\ \frac{1}{u(t)\ \frac{h(t)}{p(t)}+1} = 0 \tag{47}$$

The share of automation capital will converge to zero in the long-run if the growth rate of human capital is more than that of automation capital. Therefore, this result also signals essential policy implications of our research. By providing education and better training, humans will difficult to be substituted from the production process by robots.

- (iii) The long-run share of physical capital is constant as in the previous sections. Due to the inclusion of automation in the production process, the share of physical capital becomes sticky.

4.2 Model Calibrations

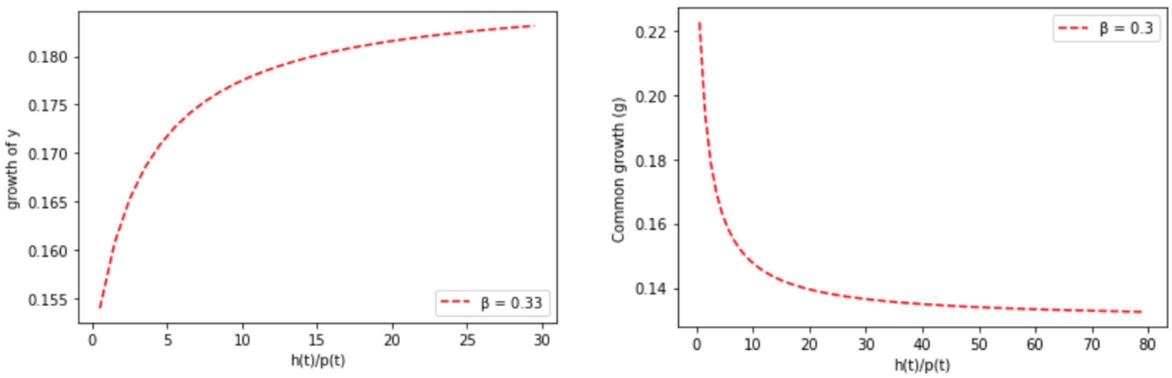
In this section, we show a detailed analysis of our results using calibrations where we visualise the transitional dynamics of our major variables' growth rates and how they approach their steady-state estimates. In the later part of this section, we have also calibrated the factor shares' transitions, which ought to be similar to the previous sections.

The parametric values assigned for this exercise have been borrowed from the previous literature, and a few have been taken at our discretion for comparative analysis of our model's performance.

Parameter Source	Description	Estimate
β Karabarbounis and Neiman (2014)	Output elasticity of capital	0.33
ρ To show the dynamics	Discount rate	0.001
δ Grossmann et.al. (2013)	Depreciation rate	0.04
σ To show the dynamics	Coefficient of relative risk aversion	0.3
γ To show the dynamics	Exogenous parameter	0.1
u International Labor Organisation	Fraction of time allocated for production	0.3

Table 2: Parametric values

We use the parametric values mentioned in Table 2 to establish the transitional paths of our variables.

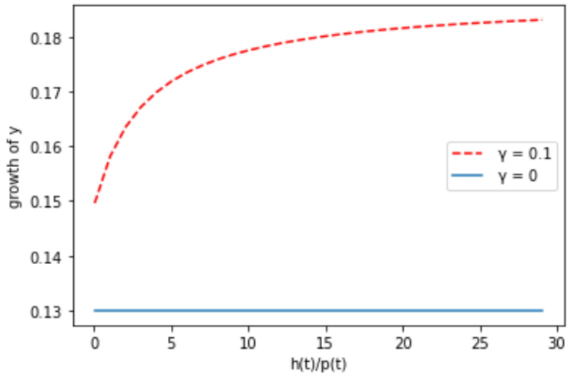


(a) Plot of g_y with $\beta = 0.33$

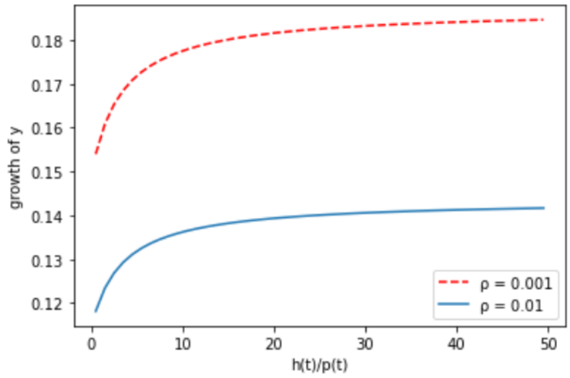
(b) Plot of g with $\beta = 0.33$

Figure 6: Transitional dynamics

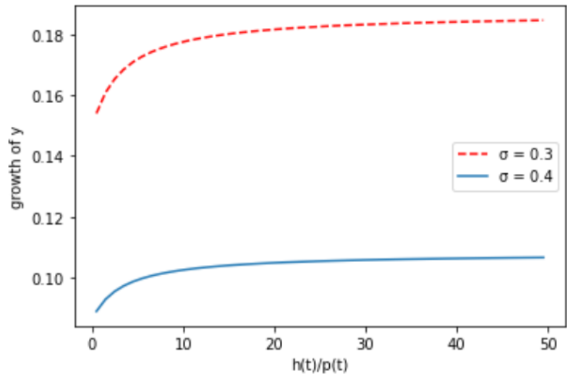
Figure 6 portrays the transitional dynamics of output and common growth rates. Figure 6. (a) shows the path of long-run growth of output as $\frac{h}{p} \to \infty$. Figure 6. (b) portrays the long-run growth of machines, robots and consumption as $\frac{h}{p} \to \infty$.



(a) Plot of g_y with $\gamma = 0.1$ and 0



(b) Plot of g_y with $\rho = 0.001$ and 0.01

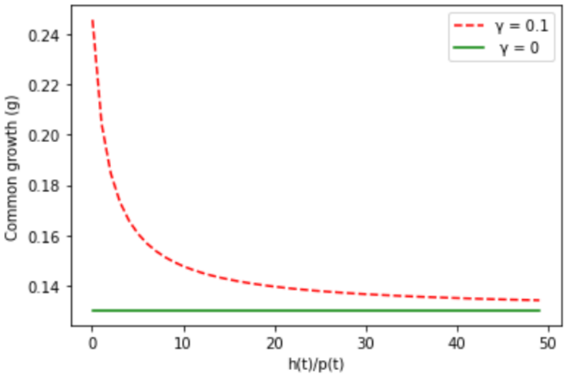


(c) Plot of g_y with $\sigma = 0.3$ and 0.4

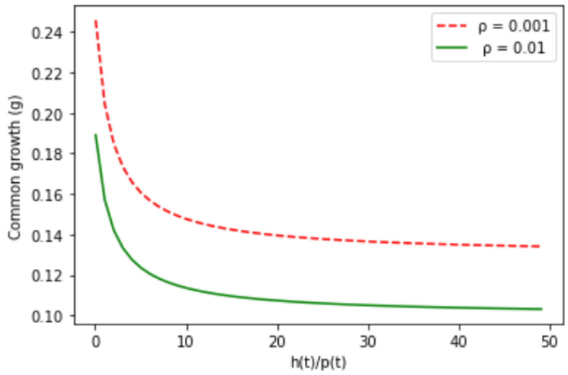
Figure 7: Comparative Statics: changes in g_y in different scenarios

Figure 7 depicts the transitional dynamics of the comparative statics section pertaining to g_y. We have explained the changes in the long-run growth of output and human capital when the estimates of the parameters alter. Figure 7. (a) shows the effect of the external parameter (γ). As described earlier, without any exogenous effect ($\gamma = 0$), all our model's variables grow uniformly. Figure 7. (b) and (c) show the transition path of g_y when there is an increase in the future discount rate (ρ) and the coefficient of relative risk aversion (σ). As discussed earlier, we find that it will distort the growth rate of output and human capital in the long-run, which is portrayed by the downward shift in the g_y curve.

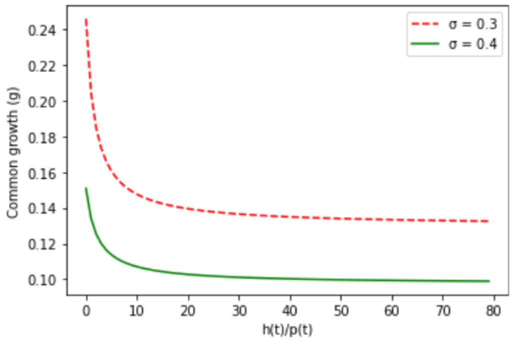
Similarly, Figure 8 below shows the transitional dynamics of the comparative statics section for the common growth rate g. Changes in the parametric values will also affect the long-run growth rates of traditional capital, automation capital and consumption. The parameter that captures the external effect in our model does not have any impact in explaining the common growth rate in our analysis. But, a positive external effect makes the human capital (or output) grow at a higher rate than the common growth rate (g). Therefore, a positive γ increases the human capital accumulation in the long-run, and the path has been calibrated as in Figure 8. (a). Figures 8. (b) and (c) depict the decrease in the common growth by a downward shift of the g curve as ρ and σ increases.



(a) Plot of g with $\gamma = 0.1$ and 0



(b) Plot of g with $\rho = 0.001$ and 0.01



(c) Plot of g with $\sigma = 0.3$ and 0.4

Figure 8: Comparative Statics: changes in g in different scenarios

In this part, we show the transitional dynamics of factor shares, as discussed in the earlier sections. We follow equations (46) and (47) to outline the factor shares of human labor and automation, and it is consistent across all the sections of the article.

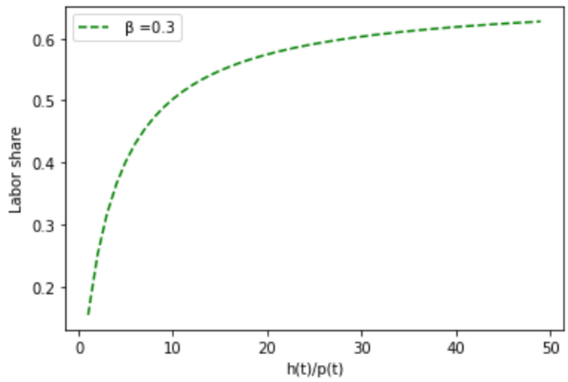


Figure 9: Dynamics of labor share

We use equation (46) to draw the transitional dynamics of labor share in Figure 9. We use $\beta = 0.33$ to show the off-steady state behaviour of labor share, as depicted in Figure 9.

Using our earlier assumptions, as the ratio of human capital to automation capital tends to infinity in the long run, the labor share converges to $(1-\beta)$.

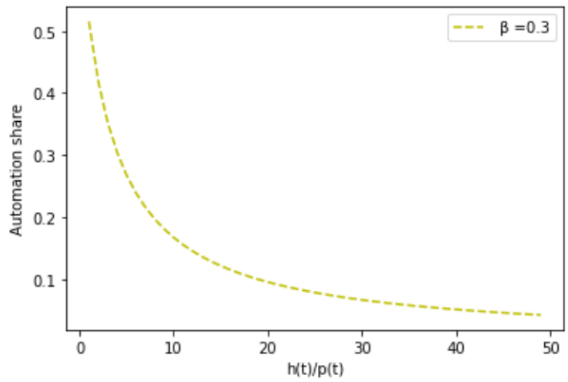


Figure 10: Dynamics of automation share

Now, we use equation (47) to outline the transitional dynamics of automation share in Figure 10. We use the exact estimate of β to show the transitional path of robot share in Figure 10. As the ratio of human capital to automation capital tends to infinity in the long run, the automation share converges to zero. Automation share behaves this way mainly due to the assumption that human capital growth exceeds automation capital growth in the long run.

5 Conclusion

In the past few years, a surge of literature has been devoted to unravel the intricate dynamics linking automation and the labor market. Our research also tries to contribute to the genre of "race against machine," an idea popularised by the earlier works of Acemoglu, Restrepo, and Prettner. The primary objective of this paper is to see the effect of human capital and whether it can save the declining labor share from the threat of automation.

To address this, our exploration commences with an exogenous human capital model, aligning closely with the work of Prettner (2019). We found that an exogenous improvement in average human capital increases the per capita growth rate given technology. The aggregate labor share decreases as more robots are taken up in the production process, but the decrease in labor share is slower when we allow workers to accumulate human capital in our model. Using aggregate data from the World Bank, we have shown without any human capital accumulation; automation explains about 18.6 per cent of the total decline in the global wage share. Whereas with an increase of 1 per cent of human capital, the reduction in global wage share due to automation can be reduced by 3.23 per cent. This finding highlights human capital as a major policy tool in our research.

In the later sections of our paper, we model human capital endogenously following the steps shown by Mankiw et. al. (1992) and Lucas (1988) to investigate the existence of any other channels to secure labor from automation. Following the former literature, we found that long-run growth is guaranteed if a larger chuck of aggregate savings is diverted to accumulate physical and human capital. A policymaker can always attract firms to invest more in physical and human capital, which will provide adequate training to the existing workers, thus making them complementary to machines and hard to substitute with robots. In countries with a higher percentage of the population in the labor-force, better educational facilities and proper training to develop the existing skill sets are pivotal for economic growth and better complementary with machines. Using certain assumptions along the balanced growth path in the latter method, we obtain the output growth equivalent to the human capital growth but greater than the common growth rates of physical and automation capital. This inequality exists due to the external parameter. In the long run, it also results in higher per capita human capital than per capita automation capital. This result also implies that the total share of workers will become constant in the long run, whereas the share of automation capital will decline to zero.

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