Competing for Dominance in Technology Markets

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Abstract

Technology markets are inherently winner-take-all, and competition for dominance can be modeled as a contest. Uncertainty is a common feature of technology contests. A competitor is often uncertain about the set of rivals that it faces, and the entering firms can attempt to influence their winning chances by undertaking a pre-contest action. This may be an investment in a technology or technological improvement, the outcome of which is uncertain. We model this competition as an all-pay auction in which the entry decision is endogenous, and where – upon entry – players may invest in acquiring a better technology; a player's investment cost is private knowledge, and the outcome of the investment is stochastic. We characterize equilibrium in terms of two thresholds of the cost parameter that determine entry and then investment, and investigate how the equilibrium is affected by uncertainty related to the investment (both the likelihood of success and its return). Our model finds applications in many technology-based markets such as virtual currency mining, mass entertainment, internet technology and wealth management.

1 Introduction

Competition to become a technological standard or a leading technology exhibits winner-take-all properties. Marc Andreessen, a Silicone Valley veteran and co-founder of Netscape puts it like this: "The big companies, though, in technology tend to have 90 percent market share. So we think that generally these are winner-take-all markets. Generally, number one is going to get like 90 percent of the profits. Number two is going to get like 10 percent of the profits, and numbers three through 10 are going to get nothing".[1] This is not a new

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[1]Interview with Alexia Tsotsis inTechCrunch, 2013: Marc Andreessen On The Future Of Enterprise (retrieved 02.11.2023).

phenomenon. Barwise and Watkins (2018) note a 60-year pattern, starting with IBM and mainframe computing in the 1960's, in which technology markets are hotly contested before becoming dominated, often by a U.S. company. The sources of such enduring dominance (economies of scale and scope, direct and indirect network effects, switching costs and lock-in, and extensive use of big data) has been discussed by among others Shapiro and Varian (1999b). Economists model winner-take-all situations as a contest in which competitors make irreversible expenditures to win the main prize. In this paper, we set up a contest model that exhibits several features of the competition for technological dominance.

Several features of technological competition are notable. First, the set of participants is often not known at the beginning of the competition. A novel technology often opens new market opportunities, taking time before the set of participants is established. In the fledgling market for home video entertainment at the beginning of the 1970's several companies entered, each with their own technology. Sony launched its Betamax videotape recording system, the Victor Company of Japan (JVC) pioneered the Video Home System (VHS) format, while RCA and Philips developed needle-based record-style discs.[2] Although it is now well known that the main war was between Betamax and VHS, with the latter claiming the market standard, the companies sunk large resources into making hardware improvements and convincing film studios and distributors to choose their format. The outcome of these investments was uncertain; Betamax was widely regarded as a superior system in terms of picture and sound quality, but the availability of longer recording times on a VHS cassette seems to have tipped the balance, deciding the outcome of the war in favor of that format.[3] Uncertainty over the set of competitors and the outcome of an investment in deciding the contest is found in other applications that we expound upon below.

We develop a tractable three-period model, involving entry, pre-contest investment and then the actual contest. At the beginning, a fixed set of players are privately informed of their marginal cost of making the pre-contest investment, and must decide whether or not to enter the competition. Next, upon entry, they decide whether to invest in acquiring an advantage which can enable them to realize a larger prize value which would create a favorable imbalance in the future contest. The return to investment in advantage acquisition is uncertain − an agent acquires the advantage with some probability. Finally, the set of entrants, some of which have potentially acquired an advantage over their rivals, compete in an all-pay auction to win the prize. The nature of the advantage that may be acquired will differ between applications, and we discuss this further in Section 2. Our model is quite

[2]See Videotape format war (retrieved 01.11.23).

[3]A standard Betamax tape lasted 60 minutes, whereas a 3-hour VHS cassette was capable of recording a whole movie: see The Betamax vs VHS Format War.

generic but captures key features of the technology contest: endogenous entry, uncertainty over the set of rivals, and potential investment with an uncertain return upon entry.

By analyzing a model that combines entry and pre-contest investment decisions, we unite disparate strands of the contest literature. The interaction of these decisions appears to have been little studied previously. Investment may enhance the size of a prize that can be appropriated as in Konrad (2002)[4], or reduce the cost of competing for the prize (Fu and Lu, 2009 and Münster, 2007). Entry can occur exogenously as the result of a pre-determined stochastic process (see among others Myerson and Wärneryd, 2006, Münster, 2006, Lim and Matros, 2009 and Fu et al., 2011), or can be set endogenously as part of the equilibrium strategy (for example Fu and Lu, 2010, Fu et al., 2015, Liu and Lu, 2019, Jiao et al., 2022 and Kaplan and Sela, 2010). Some work has been done on the disclosure or concealment of the set of entrants (Jiao et al., 2022), or whether the investment decision is observable (taken simultaneously or sequentially as in Münster, 2007). We combine the approaches by considering endogenous, observable entry and simultaneous investment decisions, the outcome of which is stochastic and observable. Furthermore we explore how entry is affected by its cost. Kaplan and Sela (2010) and Liu and Lu (2019) study an all-pay auction with an entry cost. Ability is common knowledge and higher ability gives a reduced entry cost and effort cost in the former, while it is private information in the latter but does not affect the cost of entry. In common with our approach, these papers derive a threshold strategy for entry into the contest. Kaplan and Sela (2010) demonstrate that the contest may not be effective in the sense that the probability of entry is not increasing in ability. To rectify this, they show that the winner of the contest can be charged a fee, although this reduces the attractiveness of entry for all competitors. Liu and Lu (2019) focus attention on the division of the prize mass into different sizes, showing that a single winner-take-all prize is optimal if the cost of effort is linear or concave. The first stage of the model is similar to the contest entry game in Hammond et al. (2019), who have an all-pay auction between players who have different costs of effort that are private information. Assuming that the entry fee augments the prize, they derive an elegant solution for the entry threshold and total effort. In addition to deriving a threshold for entry, our analysis introduces further uncertainty by considering which of the entrants that will make an ability-enhancing investment before the actual contest is played. We show how a contest designer can set an entry fee to influence the level of entry, and how this in turn affects the amount of contest effort, the total entry fees collected and the expected number of contestants that make the investment in ability.[5]

[4]Konrad (2002) only considers pre-contest investment by the incumbent.

[5]In a similar vein, Fu et al. (2015) show that an effort-maximizing contest designer may wish to limit participation in a Tullock contest with homogeneous participants with endogenous (and concealed), costly entry.

Our findings demonstrate the complex interplay between entry and investment. In making the entry decision, an agent is enticed by the potential value of the contest prize, and upon entry weighs up the private cost of investment with its return. Even if an investment is successful, actions at the contest stage may dissipate its return. The exception is if an agent is the only one to succeed in its investment. Intuitively, contests are hard fought between a sufficiently homogeneous group of competitors, but a single strong player can dissuade rivals from making effort. This forms the incentive to enter and to invest, and we show how this is critically determined by the relationship between the number of agents and the probability of successful investment. Entry and investment are least attractive when the probability of investment success is very high or low, since this imparts a low expectation of being the single strong player (with a successful investment) at the contest stage. Agents are more likely to enter and invest if the success probability takes an intermediate value. In the context of a technology market, if an innovation or improvement is easy to achieve then many firms will have this "advantage" in the final contest for the market; if the innovation is difficult to achieve, then firms may not invest since this will most likely just represent a cost. An intermediate chance of success will entice many firms to enter the market, and to invest in order to become the dominant agent in the competition for the market.

We show also that the entry cost acts as a mechanism to exclude less efficient agents from entering the contest. Some technology contests are designed in order to only admit better competitors, and a designer can effectively use the entry fee to exclude high-cost agents, even though cost information is not freely available. Furthermore, we show how the entry cost may be set in order to achieve any vested interest that the designer may have such us maximizing entry revenue or the expected number of investing agents.[6]

The rest of the paper is organized as follows. Section 2 gives some background for our choice of model by discussing applications. The basic model framework is presented in Section 3, and Section 4 sets up the contest. Pre-contest investment decisions are analyzed in Section 5, while Section 6 considers entry strategy. The significance of the model parameters for the analysis is expounded in Section 7. Section 8 considers the achievement of different goals by a contest designer, and Section 9 concludes.

[6]There are many historical examples of technology-based contests that are arranged by a sponsor, such as the Longitude prize (Burton and Nicholas, 2017), and the Netflix prize (Bell et al., 2010). Many such competitions are ongoing, such as those run by the United States Agency for International Development (see USAID).

2 Applications

The applications outlined in this section share certain common features that we model. Firms that engage in an inter-technology competition can undertake pre-contest actions (investments) in order to gain a competitive advantage. At the same time, they are uncertain about the nature of the future contest and how the investments will pay off. Sinking resources into projects that do not give them the desired competitiveness in the future, will yield a low return on investment, implying a lower chance of becoming the dominant technology holder. Additionally, there will be a heterogeneous set of contestants, some of whom may be more competent than others. The uncertainty about the profile of contestants can impact both investment and entry strategies. In inter-technology competition, Besen and Farrell (1994) note common use of the following strategies: investment to build on an early lead, attracting/acquiring suppliers of complements, and making product pre-announcements. Below we show how rivals in technology competitions have used these instruments as investments to position themselves in the ultimate competition.

Technology markets are often built on a direct and/or indirect network externality. A direct network effect arises since users get more utility from using a service that connects many others;[7] an indirect network effect occurs when customers are matched with complementary needs.[8] Besen and Farrell (1994) characterize network markets as "tippy", noting that small differences in products, platforms or technologies - whether real or merely perceived - "can be magnified in a process in which some firms make extremely large gains and in which dominant market positions are difficult to change" (p. 119). This is historically well documented in the cases of Google, Apple, Facebook, Amazon (GAFA).[9] Adding to this Netflix and Disney, The Economist notes that the business of mass entertainment is dominated by an oligarchy of giants where technology is making the rich richer.[10] The same article notes that technology has atomized entertainment, making supply almost unlimited, while at the same time making it easier than ever to aggregate audiences through self-reinforcing recommendation algorithms. One strategy that these technological giants have adopted is to acquire other companies and make them subsidiaries; this is one type of strategy that "investment" in our stylized model may capture. At the time of the acquisition, its future return is uncertain but can be used in the upcoming contest for dominance in one dimension of the

[7]This seems to have been first suggested by Rohlfs (1974) in an analysis of the pricing of a video communication service.

[8]Barwise and Watkins (2018) mention for example that Uber connects drivers with passengers, Apple's App Store connects software developers with users, and Google/Facebook match advertisers with customers.

[9]Microsoft is often added, making GAFAM.

[10]The Economist: Mass entertainment in the digital age is still about blockbusters, not endless choice, retrieved 02.11.2023.

entertainment industry. Besen and Farrell (1994) note that attracting suppliers of complements is a usual strategy to employ in inter-technology competition; the tech giants can also attract by acquisition. Google (Alphabet) has for example acquired You Tube, Fitbit and Motorola, Facebook has acquired Instagram and WhatsApp, Amazon has strengthened its grip on consumer robotics by purchasing iRobot and its movie offerings have been bolstered by acquiring the film studio MGM in 2021. In 2018 Microsoft branched out into the world of open-source software in acquiring GitHub, and its has further strengthened its position in communication through its purchase of Skype and LinkedIn.[11] The fight for dominance in the entertainment industry is ongoing. As The Economist noted in 2017: "The best time to gain (or lose) audience—and to challenge the dominance of an established platform—is when technology makes a leap. That is why media, gaming and tech companies are investing billions in virtual reality and augmented reality".[12] The next winner-take-all contest in the entertainment industry is predicted to have a value of nearly 1 trillion USD by 2030, to the firm that conquers the metaverse, the next big platform incorporating "extended reality" and Artificial Intelligence (AI).[13]

Indeed, the big tech firms are pursuing different strategies in their approach to AI. Microsoft has currently a 49% stake in OpenAI, the developer of ChatGPT, allowing the capabilities of this this large language learning model to be integrated into several Microsoft products such as the GitHub Copilot that assists software developers in writing code, and Microsoft 365 Copilot to enhance office productivity. Meta, the owner of Facebook, has developed its own AI model, Llama, as has Google (Bard). Apple appeared to come out of the starting block quite late, choosing to invest in developing its Apple Vision Pro headset.[14] Apple is now reportedly investing millions of dollars a day on several AI models across several teams, some members of which have previously been employed at Google.[15] Building on an early lead is one strategy that Besen and Farrell (1994) mention that firms use in inter-technology competition. Another is product pre-announcements. On March 16th 2023 Microsoft announced the integration of AI into virtually all of its productivity software, a few days after Alphabet (Google's parent company) revealed an AI-based upgrade for its Gmail and Sheets. Google revealed Bard several days after Microsoft's announcement. According to The Economist, all of the tech giants have revealed AI-related product pre-announcements.[16]

As mentioned, a leap in technology can give rise to a new contest; several of these

[11] More details can be found here: The Big Five Tech Companies & Thier Big Five Acquisitions
[12] The Economist: Mass entertainment in the digital age is still about blockbusters, not endless choice
[13] Bloomberg: Metaverse Market to Surpass US$ 993.86 Billion by 2030
[14] Big Tech Giants Invest Heavily in AI: An Analysis of Microsoft, Meta, and Apple
[15] Apple is reportedly spending millions of dollars a day training AI
[16] Big tech and the pursuit of AI dominance

involve fighting for the adoption of a standard in a two-sided market with network effects (Shapiro and Varian, 1999a). The VHS platform for home entertainment was eventually challenged and replaced by digital video/versatile discs (DVD) at the end of the 1990's, and although there were different types of disc, most were compatible with standard DVD-players. A new format or standardization war erupted over the next generation of this technology, namely high-definition optical discs, with Blu-ray and HD DVD optical disc as the competing formats. Sony and its allies in the Blu-ray Disc Association and the DVD Forum (chaired by rival Toshiba) tried to negotiate a unified standard in 2005 in order to avoid a costly war that would erode future profits, but the talks stalled.[17] As part of the war, each side tried to attract complementors in the form of movie studios, with Universal, Paramount and Warner Bros. initially favoring HD DVD, whilst Sony Pictures, Disney and 20th Century Fox supported Blu-ray. In 2007, Blockbuster announced that it would only supply the Blu-ray format to the movie rental market, and the following year Warner Bros. withdrew its support for HD DVD. Sony incorporated a Blu-ray disc player into the PlayStation 3, and this eventually became a feature of Microsoft's Xbox One, even though the competing format had originally used Microsoft's technology. In early 2008, Toshiba announced the end of production for HD DVD players and recorders. It is estimated that Sony used at least 3bn USD in fighting the contest against Toshiba.[18]

Uncertainty over the number and quality of rivals is present in new markets such as cryptocurrency mining. Since the introduction of the first cryptocurrency *Bitcoin* by Nakamoto (2008), the total market capitalization of cryptocurrencies has steadily increased over time and numerous cryptocurrencies, including Litecoin, Dogecoin and the original version of Ethereum, have emerged (see Pessa et al., 2023). Cryptocurrency mining involves introducing new blocks of a currency to its existing circulation using a consensus mechanism; mining is often related to Proof of Work in which complex mathematical puzzles must be solved to validate transactions. This process is underpinned by a protocol that allows cryptocurrencies to function as a peer-to-peer (P2P) decentralized network (see Narayanan et al., 2016, for a detailed description of the mechanism). Miners verify transactions between participants over the blockchain network and add them to the distributed public ledger. In exchange, miners receive a block reward in the form of new currency plus a transaction fee. Bitcoin miners, for instance, currently receive 6.25 bitcoin as a block reward.[19]

Mining can be a profitable venture, but its profitability is subject to various factors. Min-

[17]Den Hartigh et al. (2009) document developments in this case study.

[18]The Times: How the Blu ray war was won Sony outspent, outsold Toshiba

[19]Not all cryptocurrencies use Proof of Work for validation. Cardano and the upgraded Ethereum 2.0 use Proof of Stake in which block validators are chosen based on the number of coins they are willing to stake as collateral. See Sriman et al. (2021) for more on these consensus mechanisms.

ers compete against each other to solve a cryptographic puzzle, with the winner receiving the block reward. Solving a puzzle requires expensive resources, such as specialized hardware equipment and a considerable amount of electricity. The cost and efficiency of mining equipment can vary significantly. For example, a general-purpose central processing unit (CPU) may not be expensive, but it is inefficient in solving the puzzle. Given the current level of difficulty, utilizing a high-end CPU to find a new block would require hundreds of years (Narayanan et al. 2016). A network of CPUs has potential to reduce the search time, but the associated electricity costs make it an impractical pursuit. Professional miners therefore invest in more efficient but expensive networks of application-specific integrated circuits (ASICs), which can only mine one type of cryptocurrency. Despite the potential for profit, miners are confronted with uncertainty regarding the return on their investment in hardware equipment and the selection of mining center locations (Halaburda et al., 2022). At the time of investment, they are uncertain about the future market value of the specific cryptocurrency that the ASIC can mine, as well as the future price of electricity at the mining location. Those factors determine the expected prize value and the cost of effort at the time the contest takes place. The uncertainty can discourage investment and entry in the mining industry; it is not surprising that a small number of large entities control cryptocurrency mining, despite the currency being conceptualized as a decentralized mean for facilitating transactions.

Application of investment for competitive advantage with uncertain returns can also be found in the context of developing technology-driven products such as alternative fuel vehicles. For instance, various potential technologies like hydrogen fuel cells, Sodium-ion batteries, Lithium-sulphur batteries, can not only replace the traditional fuel cars but also bring significant improvements to the currently dominant lithium-ion battery technology used in the electric cars (Das et al., 2023). Investing early in one of these technologies could provide an investor with a substantial competitive advantage and the potential for a monopoly rent. However, it's uncertain which technology will become dominant in the market.

The growing market for managing wealth is characterized as a winner-take-all contest for a prize of 100trn USD.[20] While wealth management is currently described as fragmented, several large actors are positioning themselves by acquiring subsidiaries that will ensure technological supremacy, economies of scale and global reach. Morgan Stanley for example purchased a brokerage platform in 2020 in order to increase service to a large mass of customers, and a stock plan administration firm with a huge customer base in 2019. Mean-

[20]This example draws heavily on The $100trn battle for the world's wealthiest people, The Economist 5th September 2023.

while UBS, another large actor, has acquired Credit Suisse, gaining access to large customer markets in Brazil and South-East Asia. The winner-take-all nature of this market is being accelerated by AI-based tools that will allow advisers to build wealth for their customers, possibly through an automatic transaction technology. Alluding to the uncertain nature of returns from pre-contest investment ahead of the ultimate battle, The Economist (05.09.23) says this of the main competitors Morgan Stanley and UBS: "Either firm could falter. Although the two are chasing different strategies, it is surely only a matter of time before they clash".

3 Model

Consider $N \geq 2$ agents who can enter an all-pay auction by incurring an entry cost $c > 0$. Denote the set of entrants by E, which has $n \leq N$ members. After entering, agents can invest in acquiring an advantage that affects their valuation of the contest prize. In this model, there are two equivalent ways of modeling how a successful investment affects payoffs; one can either assume that successful agents have a lower marginal cost of exerting effort in the contest, or that such an agent has a larger prize value.[21] We shall follow the latter interpretation, so that a successful agent has a value of winning the contest of αv where $\alpha > 1$ is common knowledge; the valuation of an unsuccessful or non-investing agent is v. Agent $i \in E$ has an investment cost of θ_i, where $\theta_i, i \in E$ are independent random variables uniformly distributed over $[0, 1]$. The return to investment is stochastic. The likelihood of success is $q \in (0, 1)$, which is the same for every investor and is common knowledge. Those who do not invest do not acquire the advantage. We denote by $m \leq n$ the number of agents that realize a successful return.

The agents that enter the contest exert efforts in order to win the contest prize, given by $\mathbf{x} = (x_1, x_2, ..., x_n)$. In an all-pay auction, the winner is the contestant with the highest effort; if several agents have the same maximal effort, they each have an equal probability of winning. Let $W(\mathbf{x}) = \{j \in E \mid x_j \geq x_z \text{ for every } z \in E\}$ represent the set of agents that have maximal effort. The probability of agent $i \in E$ winning the contest is given by

$$p_i(\mathbf{x}) = \begin{cases} \frac{1}{|W(\mathbf{x})|} & \text{if } i \in W(\mathbf{x}) \\ 0 & \text{otherwise.} \end{cases}$$

[21]See Vojnović (2015). In an early paper, Konrad (2002) considered an investment made by a single agent - the incumbent - that increased the size of the prize that this agent and the single rival could fight over. Our model permits investment by all agents, and its return accrues as a private benefit.

The expected payoff of agent $i \in E$ with prize $V \in \{v, \alpha v\}$ is

$$\pi_i = p_i(\mathbf{x})V - x_i. \tag{1}$$

The game proceeds follows:

- Stage 0: Nature chooses the type θ_i of agent $i = 1, 2, \ldots, N$. The type of an agent is private information.

- Stage 1 (Entry): Agent $i \in \{1, 2, \ldots, N\}$ decides whether to enter, after paying the entry fee. The subset of entering agents is E with $|E| = n$ as the number of entrants; n is public information. If $n = 0$, the game ends.

- Stage 2 (Investment): Agent $i \in E$ decides whether to invest by incurring an investment cost of θ_i. Then, nature decides whether an agent realizes a successful investment. The number of agents that realize a successful return, $m \leq n$, is public information.

- Stage 3 (Contest): If $n = 1$, the sole entrant wins the prize. If $n > 1$, agents participate in an all-pay auction to win the prize.

We study the perfect Bayesian equilibrium of the game.

This model captures many of the salient features of the battle for dominance in technology. There is initially a set of possible firms that can compete to win the market; we can imagine that these firms have a technology that they may improve upon, and that the cost of doing this is known only to the firm itself. If a competitor succeeds in improving its technology, it gains an advantage in the coming contest. In the fight for supremacy in the early market for home entertainment, Sony, JVC and Philips entered with the prototypes of their technologies (Betamax, VHS and Video Disc), and subsequently invested in making them more attractive for customers. In practice, the form of the investment involved making the technology more suitable for home use, and attracting complement products such as films in the appropriate format. Sony and JVC succeeded in doing this, and Philips was at a distinct disadvantage when trying to attract a new mass of customers. Although stylized, our model captures the uncertainties that firms face in such a winner-take-all market, namely about the number and type of competitors that will be faced in the final competition.

4 Contest stage

We begin our analysis at stage 3, where the number of entrants n and the number of successful agents m are common knowledge. Denote the expected contest-stage payoff of a successful agent by $\pi_s(n,m)$, and an unsuccessful one by $\pi_u(n,m)$.[22] Let $T(n,m)$ be the total expected effort exerted in the contest.

If $n=1$, there is no contest and the sole entrant wins the prize. The entrant's payoff is αv if its investment has paid off, and v if it did not invest, or if the investment failed. We can therefore set $\pi_s(1,1)=\alpha v$, $\pi_u(1,0)=v$, and $T(1,1)=T(1,0)=0$.

Consider $n\geq 2$ and $m\in\{0,1,\ldots,n\}$. Each agent chooses effort to maximize (1). This is then a standard all-pay auction under complete information, which has been extensively studied by Baye et al. (1996). We can use their results directly. There are three cases to consider:[23]

Lemma 1. *(Baye et al., 1996) Suppose that $n\geq 2$.*

(i) $m=0$. Then $\pi_u(n,0)=0$, $T(n,0)=v$.

(ii) $n\geq m\geq 2$. Then $\pi_s(n,m)=\pi_u(n,m)=0$, $T(n,m)=\alpha v$.

(iii) $m=1$. Then $\pi_s(n,1)=(\alpha-1)\,v$, $\pi_u(n,1)=0$, and $T(n,1)\in[T^{min}(n,1),T^{max}(n,1)]$ where

$$T^{min}(n,1)=\frac{v}{n}\left[(n-1)^2\alpha^2-(2n-1)(n-1)\alpha+n^2-(n-1)^2(\alpha-1)^{\frac{2n-1}{n-1}}\alpha^{\frac{-1}{n-1}}\right], \quad (2)$$

$$T^{max}(n,1)=\frac{\alpha+1}{2\alpha}v. \quad (3)$$

In case (i), no agent has realized a successful investment, and all expect a payoff of zero since they exert an expected amount of effort in aggregate that equals the value of the prize. In case (ii), there are at least two successful agents and these exert efforts that are expected to equal their prize value αv. Agents that have not acquired the investment advantage do not exert effort, and all participants expect a payoff of zero. In both of these cases, there are a continuum of equilibria, but Baye et al. (1996) show that they all lead to the same expected total effort. A single successful agent - as in case (iii) - will have a positive expected payoff equal to the difference in the prize between it and a rival with that has not acquired the investment advantage. All unsuccessful agents expect a payoff of zero. Again, there are a continuum of equilibria also in this case, but they do not lead to the same amount of aggregate expected effort. $T(n,1)$ is minimized when the unsuccessful agents all compete in the contest, using a symmetric strategy; this leads to expected effort $T^{min}(n,1)$. On the other hand, Baye et al. (1996) show that $T(n,1)$ is maximized when all but one of the unsuccessful agents have an effort of zero, yielding an expected effort of $T^{max}(n,1)$. When

[22] At this stage, an agent may be "unsuccessful" because it did not invest or because it did not invest. Whatever the source of this lack of success, any investment cost is sunk and this does not affect payoffs at the contest stage.

[23] Cases (i) and (ii) use Theorem 1 in Baye et al. (1996), and case (iii) uses Theorem 2.

$n = 2$, these equilibria coincide since both imply that one successful agent competes with one unsuccessful rival, and $T(2,1) = T^{min}(2,1) = T^{max}(n,1)$.

Because $\alpha > 1 > \frac{\alpha+1}{2\alpha}$, the principal always gets the lowest effort when there is exactly one successful agent, and highest when there is more than one.

5 Investment

Consider stage 2, where the number of entrants n is common knowledge. The investment strategy affects the number of successful players. An agent's investment decision is contingent on his type θ. If there are $m-1 \in \{0, 1, \ldots, n-1\}$ successful agents among the other $n-1$ rivals, the return to investment for an agent of type θ is $q\triangle(n,m) - \theta$, where

$$\triangle(n,m) = \pi_s(n,m) - \pi_u(n,m-1).$$

An agent's expected return to investment is $q\mathbb{E}_{m-1}(\triangle(n,m)) - \theta$, where the expectation is taken over the probability distribution of $(m-1)$, the number of successful players among $(n-1)$ competitors.

As investment success is a binary event in our model, the number of successful players follows a Binomial distribution. Specifically, $(m-1) \sim \mathit{Binomial}(n-1, \kappa)$ where κ is the probability of finding a successful agent conditional upon entry. This probability κ depends on the investment and entry strategies of the players.

Proposition 1 below shows that every agent's optimal investment strategy is a threshold strategy whenever all agents follow a threshold entry strategy. The intuition is straightforward $-$ conditional upon entry, every agent's expected payoff is decreasing in its own investment cost, and therefore investment pays off only if the cost is sufficiently low. In this case, $\kappa = q\theta_I/\theta_E$, where $\theta_I \in [0, \theta_E]$ and $\theta_E \in [0,1]$ denote the investment and entry thresholds respectively.

Given θ_E, we can derive the investment threshold from the investment-indifference condition:

$$q\left[\sum_{m-1=0}^{n-1}\binom{n-1}{m-1}\left(\frac{q\theta_I}{\theta_E}\right)^{m-1}\left(1-\frac{q\theta_I}{\theta_E}\right)^{n-m}\triangle(n,m)\right] - \theta_I = 0. \tag{4}$$

Among the $(n-1)$ rivals that the indifferent agent competes against, $(m-1)$ successful agents can be drawn in $\binom{n-1}{m-1}$ ways with the probability of each draw being $\left({}^{q\theta_I}/_{\theta_E}\right)^{m-1}\left(1 - {}^{q\theta_I}/_{\theta_E}\right)^{n-m}$ and the indifferent agent becomes the m-th successful agent with probability q after incurring the investment cost θ_I.

Using Lemma 1, we find that

$$\triangle(n,m) = \begin{cases} (\alpha-1)\,v & \text{if } m = 1 \\ 0 & \text{if } m \geq 2 \end{cases},$$

which reduces (4) to

$$(\alpha-1)\,vq\left(1-\frac{q\theta_I}{\theta_E}\right)^{n-1} - \theta_I = 0. \tag{5}$$

The first term of (5) is the probability of a single agent being successful in a pool of n entrants multiplied by the increment to the agent's payoff in this case; it is easily verified to be strictly decreasing and strictly convex in θ_I. The second term is the threshold investment cost. There will be two possibilities. The left-hand-side of (5) is positive for all $\theta \leq \theta_E$; in this case, every agent with $\theta \leq \theta_E$ invests, and so we can set the investment threshold $\theta_I = \theta_E$. Otherwise, the indifference condition (5) will have a unique solution $\theta_I \leq \theta_E$, which determines the investment threshold. Thus, the following possibilities can arise in equilibrium.

- **Full investment**: For given θ_E and n, all entrants invest: $\theta_I = \theta_E$.
- **Limited investment**: For given θ_E and n, a subset of entrants invest: $\theta_I < \theta_E$.

The condition for full investment will be determined by the marginal entrant's expected return to investment. Denote the gross expected return to investment (without subtracting the investment cost) of an agent under full investment in an $n-$player contest by $\xi(n)$ where

$$\xi(n) := (\alpha-1)\,vq\,(1-q)^{n-1}. \tag{6}$$

It is straightforward to verify that $\xi(n)$ is strictly decreasing in n. The following proposition documents the equilibrium investment strategy for given θ_E and n.

Proposition 1. *Fix θ_E, n, and suppose that all agents with type $\theta \leq \theta_E \leq 1$ enter. There exists $0 < \theta_I \leq \theta_E$ such that all agents with type $\theta \leq \theta_I$ invest in equilibrium.*

1. If $\theta_E \leq \xi(n)$, then $\theta_I = \theta_E$ so that there is full investment.

2. If $\theta_E > \xi(n)$, then there is limited investment and the investment threshold θ_I uniquely solves (5). The investment threshold θ_I weakly increases in θ_E, v and α. Further, (θ_I/θ_E) strictly decreases in θ_E.

Proof. In Appendix. □

Whether all entrants invest or not depends critically on the value of $\xi(n)$. For $n \geq 2$ entrants, it follows from (6) that $\xi(n)$ is the gross expected return to investment under full investment. The regime of full investment is most likely when $\xi(n)$ is high, i.e. when α (the return to investment) or v (the contest prize) are high, and the number of entrants (n) is low. The effect of the probability of successful investment (q) is ambiguous, since $\xi(n)$ is concave in this parameter, increasing for $q \in (0, 1/n)$, and decreasing thereafter. Relatively low or high values of the success probability depends also on the number of entrants since $\xi(n)$ is maximized at $q = 1/n$. When q is relatively low, it is unlikely that an investing agent will succeed, but at the same time if it does succeed, it is likely to be alone; in this case this agent expects $\pi_s(n, 1) = (\alpha - 1)\,v$ at the contest stage by Proposition 1. A relatively high value of q makes it more likely than an investing agent will succeed, but decreases the chances of being the sole successful agent at the contest stage. This decreases the gross expected return of the investment. This nicely demonstrates the interplay between the entry decision and the success probability in determining whether all entrants invest or not.

Limited investment is most likely to occur if the expected return to investment at the contest stage are low, i.e. low α and/or v, a high number of entrants and a probability of investment success that deviates greatly from $q = 1/n$. The gross expected return to investment under limited investment is θ_I, and this is weakly increasing in the prize parameters associated with the contest stage (α, v).

For given θ_E and n, let $\pi(\theta, n)$ denote the expected payoff of an agent of type θ at the investment stage, given by:

(a) In case of limited investment, i.e., when $\theta_E > \xi(n)$:

$$\pi(\theta, n) = \begin{cases} \theta_I - \theta - c & \text{if } \theta \leq \theta_I < \theta_E \\ -c & \text{if } \theta_I < \theta \leq \theta_E \\ 0 & \text{if } \theta > \theta_E \end{cases}. \tag{7}$$

(b) In case of full investment, i.e., when $\theta_E \leq \xi(n)$:

$$\pi(\theta, n) = \begin{cases} \xi(n) - \theta - c & \text{if } \theta \leq \theta_I = \theta_E \\ 0 & \text{if } \theta > \theta_E \end{cases}. \tag{8}$$

For $n = 1$, the sole entrant invests if $\theta \leq qv(\alpha - 1) = \xi(1)$, and its expected payoff is $v + \xi(1) - \theta - c$ if it invests, and is $v - c$ if it doesn't invest. Therefore,

$$\pi(\theta, 1) = \begin{cases} v + \xi(1) - \theta - c & \text{if } \theta \leq \min\{\theta_E, \xi(1)\} \\ v - c & \text{if } \min\{\theta_E, \xi(1)\} < \theta \leq \theta_E \\ 0 & \text{if } \theta > \theta_E \end{cases}. \tag{9}$$

For $n = 0$, $\pi(\theta, 0)$ is set to zero.

6 Entry

Consider stage 1. An agent's entry strategy is contingent on its type, which is private information. From (7), (8), and (9), it follows that when there is at least one entrant ($n \geq 1$), the expected payoff of the marginal entrant of type θ_E is

$$\pi(\theta_E, n) = \begin{cases} \begin{cases} \xi(n) - \theta_E - c & \text{if } \theta_E \leq \xi(n) \\ -c & \text{if } \theta_E > \xi(n) \end{cases} & \text{if } n \geq 2 \\ \\ \begin{cases} v + \xi(1) - \theta_E - c & \text{if } \theta_E \leq \xi(1) \\ v - c & \text{if } \theta_E > \xi(1) \end{cases} & \text{if } n = 1 \end{cases}. \tag{10}$$

For the marginal entrant who is indifferent between entry and no entry, the following must hold:

$$\mathbb{E}_{n-1}[\pi(\theta_E, n)] \; = \sum_{n-1=0}^{N-1} \binom{N-1}{n-1} (\theta_E)^{n-1} (1-\theta_E)^{N-n} \pi(\theta_E, n) \; = 0, \tag{11}$$

where $n - 1 \backsim \text{Binomial}(N-1, \theta_E)$. The expression in the entry-indifference condition (11) is derived as follows. Among the $(N-1)$ rivals that the entry-indifferent agent with type θ_E competes against, $n - 1 \in \{0, 1, \ldots, N-1\}$ other entrants can be drawn in $\binom{N-1}{n-1}$ ways and the probability of each draw is $(\theta_E)^{n-1}(1-\theta_E)^{N-n}$. In each of these draws, the entry-indifferent agent receives an expected payoff of $\pi(\theta_E, n)$ after entering.

The following proposition formally proves that $\mathbb{E}_{n-1}[\pi(\theta_E, n)]$ is decreasing in θ_E. This observation implies that two possibilities may arise. First, $\mathbb{E}_{n-1}[\pi(\theta_E, n)]$ is always positive for all $\theta \leq 1$; in this case, all types enter, and we can set the entry threshold $\theta_E = 1$. Second, the entry-indifference condition (11) has a unique solution at $\theta_E < 1$, which determines the entry threshold. Thus, we observe the following two possible regimes in equilibrium:

- **Full entry**: All types of agents enter: $\theta_E = 1$.
- **Limited entry**: A subset of agents enters: $\theta_E < 1$.

The expected payoff of the agent of type $\theta = 1$ determines the condition for full entry.

Observe that

$$\mathbb{E}_{n-1}\left[\pi\left(1,n\right)\right]=\pi\left(1,N\right)=\begin{cases}\xi\left(N\right)-1-c & \text{if } 1\leq\xi\left(N\right)\\ -c & \text{if } 1>\xi\left(N\right)\end{cases}$$
$$=\underline{c}\left(N\right)-c, \tag{12}$$

where $\underline{c}(N) := \max\{\xi(N)-1,0\}$. The following proposition formally characterizes the equilibrium entry strategy.

Proposition 2. *Fix N. There exists* $0<\theta_E\leq 1$ such that all agents with type $\theta\leq\theta_E$ enter.

1. *If $c\leq\underline{c}(N)$, then $\theta_E=1$ so that there is full entry.*
2. *If $c>\underline{c}(N)$, then there is limited entry and the entry threshold θ_E uniquely solves (11). Further, θ_E increases in v and α.*

Proof. In Appendix. □

From the entry and investment thresholds, we can fully characterize the distribution of the number of entrants and the number of agents who succeed with an investment. Specifically, n follows *Binomial* (N,θ_E) and m follows *Binomial* $(n,q\theta_I/\theta_E)$.

Full entry is most likely to occur when $\xi(N)$ is large and/or the entry cost (c) is small; the former occurs when the rewards from the contest stage (α,v) are large and the probability of investment success is at an intermediate value (i.e. close to $q={}^1\!/\!_N$). A straightforward implication of Proposition 2 is that for a contest with an entry fee, full investment occurs whenever there is full entry. This follows from the following observations. First, if $0<c\leq\underline{c}(N)$, then $\underline{c}(N)>0$ and hence $n=N$. Further, $\underline{c}(N)>0$ implies that $\theta_E\leq 1<\xi(N)$ and by Proposition 1, we get $\theta_I=\theta_E=1$, summed up in Corollary 1:

Corollary 1. *If $0<c\leq\underline{c}(N)$, then $\theta_I=\theta_E=1$.*

The intuition is as follows. If the full entry condition is satisfied, the marginal entrant faces no uncertainty about the number of entrants. She will always compete against the remaining $N-1$ players in an all-pay auction. Since a player that does not realize a successful investment receives zero payoff in the all-pay auction, the marginal entrant can never recover its entry cost by not investing. Therefore, in the full-entry regime, the marginal entrant, and consequently every entrant, is committed to invest.

Limited entry implies $n\leq N$, occurring when $c>\underline{c}(N)$, and the investment threshold depends on the realized value of n. The marginal entrant faces uncertainty regarding the number of entrants at the entry stage. If it faces no competition upon entry, which happens when it is the sole entrant, it expects to receive a positive payoff even if it has not realized a successful investment. After entering, the entry cost becomes a sunk cost, and the agent will only invest if the potential gain from investing is sufficient to recover the investment cost. Limited investment can be observed alongside limited entry if $\xi(n)$ falls below the entry threshold θ_E for any given n. Because $\xi(n)$ is decreasing in n, an agent's incentive to invest is reduced with the number of entrants.

The interplay between the uncertainty that arises from entry and that from investment is complex but present in the real world applications that we want to capture with our model. For a contest with an entry fee, full entry implies full investment, and that is a clean result. Limited entry can give rise to both full investment and limited investment in equilibrium. We explore this further in the next section.

7 Comparative statics

In this section, we discuss the comparative statics effects of some key parameters of our model on entry and investment incentives.[24]

7.1 Likelihood of successful investment

In Section 2, we outlined possible strategies that tech companies have adopted in trying to get an edge in the upcoming contest situation, such as purchasing competitors or suppliers of complements. Cryptocurrency miners try to get an edge by acquiring up to date hardware, and wealth management firms position themselves by acquiring subsidiaries that are supposed to provide a technological edge and worldwide reach. The returns to these pre-contest investments are uncertain. To capture this, we have assumed in our model that the return on investment is stochastic. The likelihood of success q has contrasting effects on incentives for investment and entry. When q is high, an agent is more likely to succeed in investment, which favorably affects the payoff from entry. However, this agent also anticipates competing against more agents that have made a successful investment, which has a dampening effect. Typically, both entry and investment incentives are high at an intermediate range of q, and this range is concentrated around $q = {}^{1}\!/_{N}$.

To see why, let us examine the full-entry condition: $c \leq \underline{c}(N)$. Observe that $\xi(N)$ is concave for $q \in [0, 1]$, is equal to zero at $q = \{0, 1\}$, and it increases with q for $q < 1/N$,

[24] It is customary in contest models to focus attention on expected total effort. From Lemma (1) effort is αv, in all cases except $m = 0$ (giving effort v) and $m = 1$ in which case it is $\left\{T^{min}, T^{max}\right\}$. Computing the probabilities of the latter events requires closed-form solutions for the entry and investment thresholds, and these we cannot calculate. Hence we focus on entry and investment incentives.

but decreases thereafter. Therefore, the full-entry condition can only be satisfied at an intermediate level of q. We can hence find an interval $\left[\underline{q}, \overline{q}\right], 0 < \underline{q} \leq \overline{q} < 1$ such that $\left[\underline{q}, \overline{q}\right] = \{q \in [0,1] : c \leq \underline{c}(N)\}$. Further, this interval can be vacuous if $\max_{q\in[0,1]} \xi(N) < 1 + c$, which holds if $(N-1)^{N-1}/N^N < (1+c)/v(\alpha-1)$. We state this formally:

Proposition 3. *Fix $N \geq 2$.*

1. If $(N-1)^{N-1}/N^N < (1+c)/v(\alpha-1)$, there is limited entry in equilibrium for every $q \in [0,1]$.

2. If $(N-1)^{N-1}/N^N \geq (1+c)/v(\alpha-1)$, there exist $0 < \underline{q} \leq 1/N \leq \overline{q} < 1$ such that for $q \in \left[\underline{q}, \overline{q}\right]$, there is full entry in equilibrium.

Part 2 indicates that when the contest is sufficiently favorable (high potential prize value, a low entry cost and few potential competitors), all agents will enter if the success probability balances the positive effect of achieving the advantage with the negative one of meeting potentially strong rivals. When there is full entry, there is also full investment. When the full-entry condition does not hold, the full-investment condition is given by $\theta_E \leq \xi(n)$, where n is the realized number of entrants. For given θ_E and n, it easily follows from the shape of $\xi(n)$ that the full-investment condition, if satisfied, only occurs at an intermediate level of q.

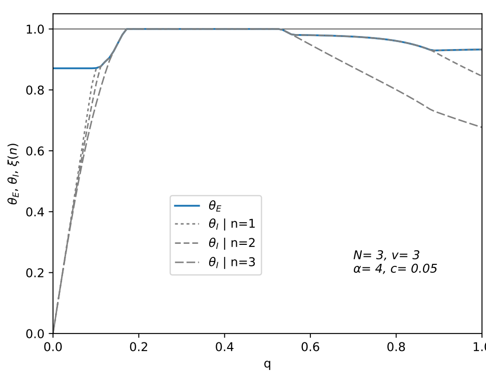

Figure 1: θ_E and θ_I against q

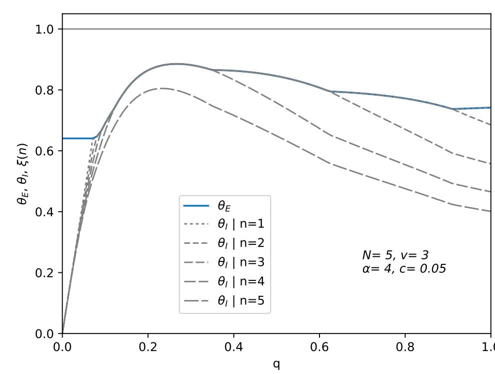

Figure 2: θ_E and θ_I against q

Notes. We consider the following parameter specifications: $v = 3$, $\alpha = 4$, and $c = 0.05$. Figure 1 plots θ_E (the continuous graph) and θ_I (the dashed graph) against q for $N = 3$. The investment threshold θ_I depends on the realized number of entrants. For $q \in [0.169, 0.531]$, $\xi(3) > 1+c$, and we have full entry in equilibrium. In Figure 2, we consider $N = 5$; in this case, $\xi(N) < 1 + c$ for every q, and so limited entry in equilibrium.

Part 1 shows that when the contest is expected to be less favorable, then no value of the investment success parameter will entice all agents to enter. This is intuitively straightforward. However, it is less obvious how the thresholds θ_E and θ_I change in relation to q in regimes with limited entry and limited investment. The following figures illustrate how the two thresholds move against q. In Figure 1, which plots θ_E and θ_I against q for $N = 3$, there is full entry in equilibrium when $q \in [0.169, 0.531]$. Figure 2 plots the threshold for $N = 5$ and there is limited entry for every q. These figures also plot θ_I, contingent on $n \in \{1, 2, \ldots, N\}$.

7.2 Number of agents

The entry incentive weakly diminishes as the total number of agents N increases; this follows from two observations. Firstly, the full-entry condition is satisfied for sufficiently small values of N. Additionally, in cases of limited entry, the entry threshold decreases as N increases. The following proposition documents formally how the entry threshold changes with respect to N.

Proposition 4. *Define* $\bar{N} := \max\{0, 1 + \lfloor(\ln(1+c) - \ln((\alpha-1)vq))/\ln(1-q)\rfloor\}$ *where* $\lfloor x \rfloor$ *is the largest integer less than or equal to* x*.*

1. *For* $N \leq \bar{N}$*,* $\theta_E(N) = 1$*.*
2. *For* $N > \bar{N}$*,* $\theta_E(N)$ *weakly decreases in* N*.*

Part 1 is the familiar full-entry condition. Since $N \geq 2$, full entry cannot occur if $\bar{N} < 2$, which happens if then entry cost is very large: $1 + c > \xi(2)$; in this case, two agents will not both find it profitable to enter, and full entry will certainly not occur for additional agents. Entry is less attractive the more agents there are. To get the intuition behind the result, consider from the perspective of the marginal entrant when there are N players. The marginal entrant expects a positive payoff in two scenarios. First, it might be the only entrant, and the expected post-entry payoff in this event is $v + \max\{0, \xi(1) - \theta_E\}$. Second, there could be $(n-1)$ other entrants for various values of n, and the marginal entrant's expected post-entry payoffs in these events are $\max\{0, \xi(n) - \theta_E\}$. When the number of players increases by 1, the likelihood of the first scenario decreases, and the payoffs associated with the second scenario decrease for every n. Consequently, the marginal entrant's expected post-entry payoff declines as N increases.

Describing the impact of N on the investment threshold is more complex because the threshold depends on the realized number of entrants, and the distribution of the number of entrants changes as N moves. If we fix the number of entrants at a given n and examine how changing N affects the investment threshold, we can infer from Proposition 1 that θ_I will also decrease. This is because the two thresholds are positively related.

Figure 3 numerically illustrates the relationship between the entry threshold and N. We also plot the investment threshold when all agents have entered ($n = N$); in this case the investment threshold solves:

$$(\alpha - 1)\, vq \left(1 - \frac{q\theta_I}{\theta_E(N)}\right)^{N-1} - \theta_I = 0. \tag{13}$$

As N increases, it affects θ_I in two ways: first, by directly influencing the threshold that solves (13), and second, by decreasing the entry threshold. Proposition 1 implies that the second effect leads to a decline of the investment threshold. Furthermore, it can be shown that the direct effect of N decreases θ_I.[25]

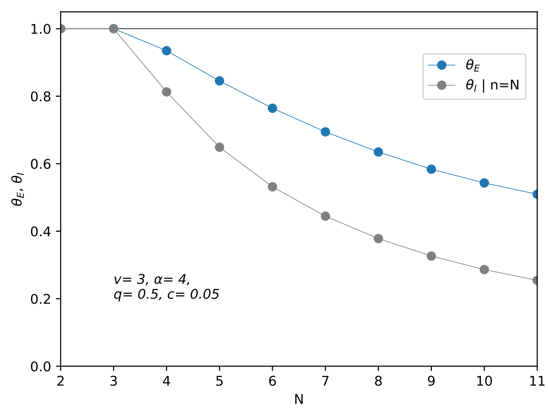

Figure 3: θ_E and $\theta_I\ |_{n=N}$ against N

Notes. We consider the following parametric specifications: $v = 3$, $\alpha = 4$, $c = 0.05$, and $q = 0.5$, and plots θ_E (blue) against N. The grey-colored curve plots the investment threshold in the event of full entry (when $n = N$) against N.

7.3 Entry fee

An entry fee adversely affects the entry incentive. As with our analysis of the effects of N, we can illustrate the dampening effect of c with two observations.

Firstly, the full-entry condition is satisfied only for sufficiently small values of c, specifically, for $c \leq \underline{c}(N)$. In addition, when there is limited entry, we can examine how θ_E moves with respect to c by analyzing (11). Proposition 5 documents the effect of c on θ_E.

Proposition 5. *For $c \leq \underline{c}(N)$, $\theta_E = 1$. For $c > \underline{c}(N)$, θ_E is strictly decreasing in c.*

The mechanism behind this result is straightforward: entry fees directly reduce the marginal entrant's payoff in all possible scenarios, thereby dampening the incentive to enter. Figure 4 plots the relationship between $\theta_E(c)$ and c for two parameter combinations, one illustrating the case $\underline{c}(N) > 0$,in which there is full entry for sufficiently low entry cost, and the other illustrating limited entry (case $\underline{c}(N) = 0$).

[25] The proof of this follows the technique used to show part 2 of Proposition 4, and is omitted here.

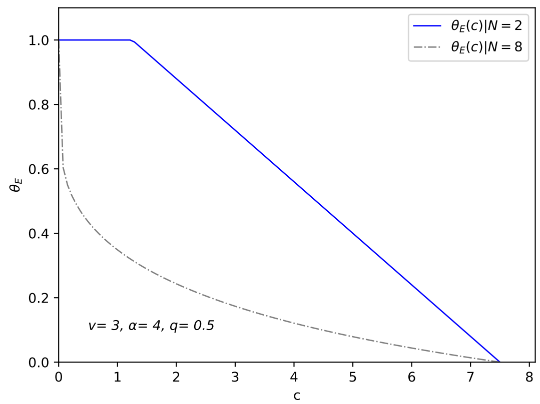

Figure 4: θ_E against c

Notes. We consider the following parametric specifications: $v = 3$, $\alpha = 4$, and $q = 0.5$, and plots θ_E against c. The continuous (blue-colored) curve plots the threshold when $N = 2$, in which case, $\underline{c}(N) = \xi(2) - 1 = 1.25 > 0$. The dashed (grey-coloured) curve plots the threshold when $N = 8$, in which case $\underline{c}(N) = 0$.

The monotone relationship between the entry fee and the entry threshold has important implications for design problems. A contest designer can achieve her desired entry threshold by adjusting the entry fee. For a given θ_E, let $\widehat{c}(\theta_E)$ be the maximum level of entry cost that results in an entry threshold equal to θ_E. Then we can prove the following proposition.

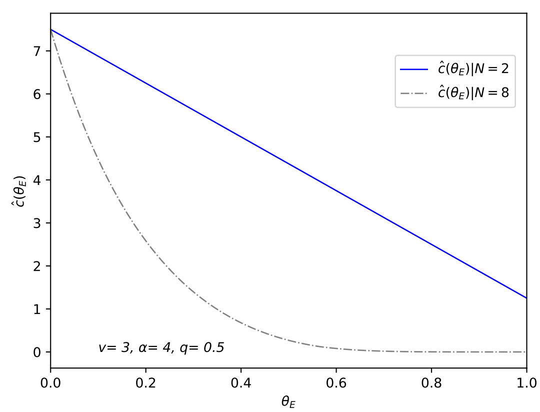

Figure 5: $\widehat{c}(\theta_E)$ against θ_E

Notes. We consider the following parametric specifications: $v = 3$, $\alpha = 4$, and $q = 0.5$, and plots $\widehat{c}(\theta_E)$ against θ_E. The continuous (blue-colored) curve plots $\widehat{c}(\theta_E)$ when $N = 2$, in which case, $\underline{c}(N) = \xi(2) - 1 = 1.25 > 0$. The dashed (grey-colored) curve plots $\widehat{c}(\theta_E)$ when $N = 8$, in which case $\underline{c}(N) = 0$.

Proposition 6. *Any entry threshold $\theta_E \in [0, 1]$ can be implemented by choosing an entry fee $c = \widehat{c}(\theta_E)$. Furthermore, $\widehat{c}(\theta_E)$ is continuous and differentiable.*

Figure 5 plots the relationship between $\widehat{c}(\theta_E)$ and θ_E for two parameter combinations, one illustrating the case $\underline{c}(N) > 0$ and the other illustrating the case $\underline{c}(N) = 0$.

8 Endogenous preference over entry

Contest models often purport the existence of a designer that sets various instruments in the competition in order to achieve some objective. In the context of technological competition, a designer can be a national government that may be interested in attracting or discouraging large multinationals from competing in the national market. Given Proposition 6, an entry fee may be one such instrument. While national governments do not specifically charge an entry fee to allow rivals to compete in a national market, several policies may be regarded as fulfilling this role. In 2023, the Office of the United States Trade Representative (USTR) has documented world trade barriers facing US firms, and some of these may have the function of an entry fee.[26] One example is forcing online information service providers (such as Google or Meta) to pay national news publishers for the right to host their content.[27] Another is a network usage fee that has been implemented in South Korea and is being considered in the European Union.[28] Furthermore, strictly regulated markets may require the payment of compliance-related services. To comply with the General Data Protection Regulation (GDPR) in the European Union involves costs for legal advice, data security and certification costs; tech companies operating in China must comply with Chinese Cybersecurity Law, and this can involve significant costs. Social media companies often incur costs in order to adhere to national privacy laws such as the Online Safety Act 2023 in the United Kingdom that regulates online content.[29]

At a microeconomic level, technology and innovation contests date at least as far back as the British government's Longitude Prize of 1714 (MacCormack et al. 2013). Entry fees are sometimes charged in order to recoup the expenses from running the contest, or to limit participation, especially by low-quality agents. Taylor (1995) notes that the US Federal Communications Commission opened a contest to design the technology standard for HD-TV, charging an entry fee of 200,000 USD. Competitions in music, writing, sports and architecture often charge an entry fee. In 2021 the participation fees for the Eurovision song contest totaled 6.2 million Euros.[30]

[26] 2023 National Trade Estimate Report on Foreign Trade Barriers, USTR

[27] Under the News Media Bargaining Code of 2019 in Australia, or the Online News Act of 2023 in Canada.

[28] Network cost contribution debate, European Parliamentary Research Service, April 2023

[29] A set of rules are currently under formation for the European Union, see The Digital Services Act package, European Commission, September 2023.

[30] See: https://web.archive.org/web/20200623153206/https://eurovision.tv/about/faq/.

We examine two distinct objectives pursued by the designer. In the first, the designer maximizes the expected total entry fees received. In the second, the designer maximizes the expected number of investors. The analyses below primarily focus on finding conditions in which the designer prefers limited entry.

8.1 Total fees collected

The contest designer can extract surplus from the participants by charging an entry fee, c, to maximize the expected value of the total fees received. As the number of entrants n follows $Binomial\left(N,\theta_E\right)$, the expected value of fees received is $c\mathbb{E}\left(n\right)=cN\theta_E\left(c\right)$ for a given c. The optimal choice of c, therefore, maximizes $Nc\theta_E\left(c\right)$.

By replacing c by $\widehat{c}\left(\theta_E\right)$, we can rewrite the optimization problem as a choice problem over the possible entry threshold values. The designer's preferred choice of θ_E maximizes the expected value of fees collected, denoted by V_f:

$$V_f\left(\theta_E\right):=N\widehat{c}\left(\theta_E\right)\theta_E. \tag{14}$$

Because of continuity and differentiability of $\widehat{c}\left(\theta_E\right)$, V_f is continuous and differentiable in θ_E. Therefore, if the optimization problem has an interior solution, this must satisfy the first-order necessary condition:

$$\widehat{c}\left(\theta_E\right)+\theta_E\frac{d\widehat{c}\left(\theta_E\right)}{d\theta_E}=0.$$

In general, the objective function (14) can exhibit both concave and convex properties. Proposition 7 outlines the sufficient condition under which limited entry is preferred.

Proposition 7. *Consider a contest designer who maximizes the total fees received. If* $\underline{c}\left(N\right)=0$*, then the designer prefers limited entry. If* $\underline{c}\left(N\right)>0$*, then a sufficient condition for the designer to prefer limited entry is given by*

$$\xi\left(N-1\right)\left(1-Nq\right)-2-v\cdot\mathbf{1}_{\{N=2\}}<0, \tag{15}$$

where $\mathbf{1}_{\{N=2\}}$ *is an indicator function that takes the value* 1 *if* $N=2$*, and* 0 *otherwise.*

Limited entry is preferred for large N values. This is because $\underline{c}\left(N\right)=0$ for sufficiently high N values. Further, when $\underline{c}\left(N\right)>0$, the sufficient condition (15) is more likely to hold for high values of N: $\xi\left(N-1\right)\left(1-Nq\right)\leq 0$ for $N\geq 1/q$ and is positive but decreasing in N for $N<1/q$. Similarly, for sufficiently large q values, limited entry is preferred. Although

$\underline{c}(N)$ moves non-monotonically with respect to q, it is decreasing in q for $q \geq 1/N$, and the sufficient condition is always negative for $q \geq 1/N$. It is important to note that if $dV_f/d\theta_E > 0$ as θ_E approaches 1, we cannot definitely conclude that full entry is preferred, as there could be a local interior maximum even if V_f is increasing at $\theta_E = 1$. This is illustrated in Figure 6.

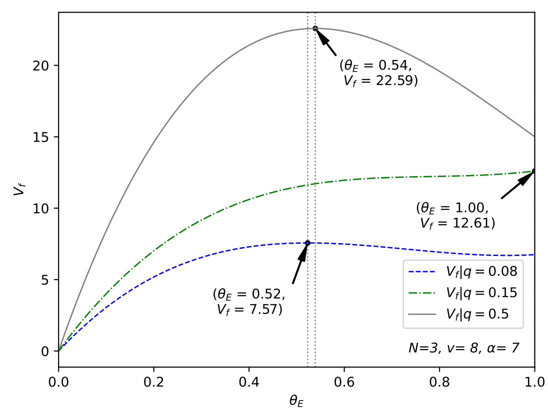

Figure 6: V_f against θ_E for different q values

Notes. We consider the following parametric specifications: $N = 3$, $v = 8$, and $\alpha = 7$, and plot V_f against θ_E for q values of 0.08, 0.15, and 0.5.

Figure 6 depicts how V_f changes with respect to θ_E under various scenarios. We set $N = 3$, $v = 8$, $\alpha = 7$, and vary q within the set $\{0.08, 0.15, 0.5\}$. In all these scenarios, $\underline{c}(N) > 0$. The sufficiency condition in (15) is met when $q = 0.5$, but it is not satisfied for $q = 0.08$ and $q = 0.15$. For $q = 0.5$, V_f (represented by the continuous curve) reaches its maximum value of 22.59 at $\theta_E = 0.54$. For $q = 0.15$, V_f (represented by the green dot-dashed curve) achieves its maximum at the boundary $\theta_E = 1$. For $q = 0.08$,V_f (shown as the blue dashed curve) attains its interior maximum at $\theta_E = 0.52$.

8.2 The expected number of investors

Consider that the contest designer's objective is to maximize the expected number of investors. The likelihood of investment by a player, conditional upon entry, is θ_I/θ_E. As the number of entrants follows Binomial(N, θ_E), the expected number of investors, denoted by V_{inv}, is given by

$$V_{inv}(\theta_E) = \sum_{n=0}^{N} \binom{N}{n} (\theta_E)^n (1-\theta_E)^{N-n} \frac{n\theta_I(n)}{\theta_E}, \tag{16}$$

where $\theta_I(n)$ and θ_E satisfy the conditions described in Proposition 1 and Proposition 2. After rearranging terms, (16) can be simplified as

$$V_{inv}(\theta_E) = N\mathbb{E}_{n-1}[\theta_I(n)], \tag{17}$$

where $(n-1)$ follows Binomial$(N-1, \theta_E)$.

Consider first the case $\underline{c}(N) > 0$, which occurs when $\xi(N) > 1$. Note that if $\theta_E \leq \xi(N)$, then $\theta_E \leq \xi(n)$ for every $n \leq N$, and consequently, $\theta_I(n) = \theta_E$. From (17), $V_{inv}(\theta_E) = N\theta_E$, which is maximized at $\theta_E = 1$. Therefore, the designer prefers full entry.

Next, consider the case when $\underline{c}(N) = 0$, which occurs when $\xi(N) \leq 1$. As we have argued in the previous case, replacing c by $\widehat{c}(\theta_E)$ in (17), we can express the designer's problem as a choice problem over the possible values of θ_E. Thus, we can study the derivatives of V_{inv} with respect to θ_E at the boundary values to derive a sufficient condition for the existence of a preferred entry threshold strictly below 1. Proposition 8 documents the sufficient condition under which limited entry is preferred.

Proposition 8. *Consider a contest designer who maximizes the expected number of investors. If $\underline{c}(N) > 0$, then the designer prefers full entry. If $\underline{c}(N) = 0$, then a sufficient condition for the designer to prefer limited entry is given by*

$$\frac{(N-1)\,q\hat{\theta}^2}{(N-2)\,q\hat{\theta}+1} + (N-1)\left(\hat{\theta} - \hat{\hat{\theta}}\right) < 0, \tag{18}$$

where $\hat{\theta} := \lim_{\theta_E \to 1} \theta_I(N)$ and $\hat{\hat{\theta}} := \lim_{\theta_E \to 1} \theta_I(N-1)$.

Why might a designer choose to limit entry? In scenarios where full investment occurs (i.e., when $\theta_I = \theta_E$), the designer generally benefits from raising the entry threshold. However, she might consider limiting entry specifically when the investment threshold is significantly lower than the entry threshold for certain values of n. In such cases, the designer's motivation for increasing θ_E is influenced not just by the investment thresholds across different events with varying n values, but also by the rate at which the probabilities of these events shift. Notably, as θ_E approaches 1, the rate of change in probabilities of all events, except when $n = N$ and $n = N-1$, asymptotically approaches zero. In contrast, the probabilities of the events of $n = N-1$ and $n = N$ decrease and increase, respectively, as θ_E nears 1. Additionally, given that $\theta_I(N-1)$ is strictly larger than $\theta_I(N)$ (when both are below θ_E), the reduction in probability of $n = N-1$ can sometimes weaken the designer's motivation to raise the entry threshold. The sufficient condition outlined in (18) precisely characterizes such scenarios.

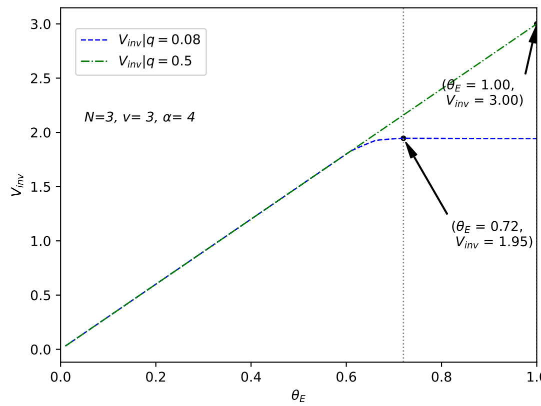

Figure 7: V_{inv} against θ_E for different q values

Notes. We consider the following parametric specifications: $N = 3$, $v = 3$, and $\alpha = 4$, and plot V_{inv} against θ_E for q values of 0.08 and 0.5.

Figure 7 illustrates how V_{inv} changes with θ_E. We set $N = 3$, $v = 3$, $\alpha = 4$, and consider cases where q is 0.08 and 0.5. For $q = 0.5$, $\underline{c}(N) = 0.12$, and V_{inv} (depicted by the green dot-dashed curve) achieves its maximum at the boundary $\theta_E = 1$. For $q = 0.08$, $\underline{c}(N) = 0$ and the sufficiency condition in (18) is satisfied; V_{inv} (represented by the blue dashed curve) reaches its maximum at $\theta_E = 0.72$, with the maximum value being 1.95. Notably, for $q = 0.08$, V_{inv} at $\theta_E = 1$ is marginally lower, measured at 1.94.

9 Conclusion

This paper has investigated a generic contest model with uncertain entry and investment. As such it is widely applicable, but we have concentrated on applications related to technology. In these markets, firms are often unsure about which and how many rivals they will face, as well as the efficacy of actions taken to gain a competitive edge in the upcoming contest for the market. Our analysis shows a complex interplay between the two types of uncertainty, captured by the relationship between the probability of success and the total potential number of participants. While the potential size of the prize and the cost of entering the contest have predictable effects on entry and investment, the probability of a successful investment has a non-monotonic effect. Furthermore, the region of non-monotonicity is inextricably linked to the total number of competitors. The threshold values of marginal cost that determine entry and investment in equilibrium resolve the complex decision making process. Agents know that if they meet homogeneous rivals in the final contest (whether they are all successful or unsuccessful in their investment), they will compete away a large proportion of the contested prize, lowering the incentive to enter and make a pre-contest investment. Meeting one or more stronger rivals in the contest also leads to a low expected payoff and weak incentives. The driving force behind entry/investment is the promise of being the lone strong agent in the contest, who is guaranteed a large prize for low effort. If the probability of achieving a successful investment is very low, then an agent is likely to meet equally weak rivals in the upcoming contest. If the success probability is high, an agent will expect to meet several equally strong rivals. An intermediate probability of success balances these two scenarios, making entry and investment more attractive.

We have shown that any entry threshold can be implemented by appropriate setting of the cost of entering the contest. A sufficiently low (but positive) fee can entice full entry into the contest, and this in turns guarantees that all entrants invest. A higher entry cost discourages entry by those who have the highest marginal cost of investment. It may well still be the case that all entrants invest also in this scenario. All other things equal, a large initial number of competitors weakens the incentive for both entry and investment.

Our model captures several salient features of technology contests, but also has some drawbacks. A common focus of contest models is how total expected effort is affected by the model parameters. We have not been able to calculate total effort, since we cannot derive explicit solutions for the entry and investment thresholds. We have also assumed that the result of entry and investment decisions is immediately disclosed to the participants, and this may not suit all applications. We plan to return to this in later work.

Appendix

The Appendix contains the proofs that are omitted in the main text. We will begin with documentation of two additional results, Lemma A.1 and Lemma A.2, that will be useful in proving our main findings.

Lemma A.1. *Consider a function* $f(p, X) : [0, 1] \times \mathbb{N} \to \mathbb{R}$ *that is decreasing in both arguments,* p *and* X*. Let* $m \geq 2$ *be an integer and let* X *follow Binomial*(m, p)*. Then,* $d\mathbb{E}_X [f(p, X)] / dp \leq 0$*.*
Furthermore, if f *is strictly decreasing in* X *for some* $X \in \{0, 1, ..., m\}$*, or if* f *is strictly decreasing in* p *at some* $X \in \{0, 1, ..., m\}$*, then* $d\mathbb{E}_X [f(p, X)] / dp < 0$*.*

Proof of Lemma A.1. Assume $X \backsim \text{Binomial}(m, p)$. Then,

$$\mathbb{E}_X [f(p, X)] = \sum_{j=0}^{m} f(p, j) \binom{m}{j} (p)^j (1-p)^{m-j}. \tag{A.1}$$

Claim 1: $d\mathbb{E}_X [f(p, X)] / dp = m\mathbb{E}_Y [f(p, Y+1) - f(p, Y) + (pf_p(p, Y+1)) / Y + 1] + (1-p)^m f_p(p, 0)$, where $Y \backsim \text{Binomial}(m-1, p)$ and $f_p(p, X) = \partial f(p, X) / \partial p$, which is the partial derivative of f with respect p.

Proof of Claim 1: Differentiating (A.1) with respect to p, we get

$$\begin{aligned}
\frac{d}{dp}\mathbb{E}_X [f(p, X)] = & \sum_{j=0}^{m} f(p, j) \binom{m}{j} \left[jp^{j-1} (1-p)^{m-j} - (m-j) p^j (1-p)^{m-j-1} \right] \\
& + \sum_{j=0}^{m} f_p(p, j) \binom{m}{j} p^j (1-p)^{m-j} \\
= & \sum_{j=1}^{m} f(p, j) j \binom{m}{j} p^{j-1} (1-p)^{m-j} \\
& - \sum_{j=0}^{m-1} f(p, j) (m-j) \binom{m}{j} p^j (1-p)^{m-j-1} \\
& + \sum_{j=1}^{m} f_p(p, j) \binom{m}{j} p^j (1-p)^{m-j} + (1-p)^m f_p(p, 0)
\end{aligned}$$

Replacing $j\binom{m}{j}$, $(m-j)\binom{m}{j}$, and $\binom{m}{j}$ by $m\binom{m-1}{j-1}$, $m\binom{m-1}{j}$, and $\frac{m}{j}\binom{m-1}{j-1}$, respectively, we get

$$\begin{aligned}\frac{d}{dp}\mathbb{E}_X\left[f\left(p,X\right)\right]=&\sum_{j=1}^{m}f\left(p,j\right)m\binom{m-1}{j-1}p^{j-1}\left(1-p\right)^{(m-1)-(j-1)}\\&-\sum_{j=0}^{m-1}f\left(p,j\right)m\binom{m-1}{j}p^{j}\left(1-p\right)^{m-j-1}\\&+\sum_{j=1}^{m}pf_p\left(p,j\right)\frac{m}{j}\binom{m-1}{j-1}p^{j-1}\left(1-p\right)^{(m-1)-(j-1)}+\left(1-p\right)^{m}f_p\left(p,0\right)\end{aligned}$$

Replacing $j-1$ by j in the first and the third terms,

$$\begin{aligned}\frac{d}{dp}\mathbb{E}_X\left[f\left(p,X\right)\right]=&\sum_{j=0}^{m-1}f\left(p,j+1\right)m\binom{m-1}{j}p^{j}\left(1-p\right)^{(m-1)-j}\\&-\sum_{j=0}^{m-1}f\left(p,j\right)m\binom{m-1}{j}p^{j}\left(1-p\right)^{m-j-1}\\&+\sum_{j=0}^{m-1}pf_p\left(p,j+1\right)\frac{m}{j+1}\binom{m-1}{j}p^{j}\left(1-p\right)^{(m-1)-j}+\left(1-p\right)^{m}f_p\left(p,0\right)\\=&m\mathbb{E}_Y\left[f\left(p,Y+1\right)-f\left(p,Y\right)+\frac{pf_p\left(p,Y+1\right)}{Y+1}\right]+\left(1-p\right)^{m}f_p\left(p,0\right),\end{aligned}\tag{A.2}$$

where $Y\backsim\text{Binomial}\left(m-1,p\right)$. This proves claim 1.

Observe that f is decreasing in both arguments, we have $f\left(p,Y+1\right)\leq f\left(p,Y\right)$ and $f_p\left(p,Y\right)\leq 0$ for any Y. Therefore, $d\mathbb{E}_X\left[f\left(p,X\right)\right]/dp\leq 0$.

Further, it follows from (A.2) that if f is strictly decreasing in Y for some $Y\in\{0,1,\ldots,m-1\}$ or if $f_p<0$ at some $Y\in\{0,1,\ldots,m-1\}$, then $d\mathbb{E}_X\left[f\left(p,X\right)\right]/dp$ is strictly negative, which proves the final part of the Lemma. □

Lemma A.2. *Consider a function $f(X):\mathbb{N}\to\mathbb{R}$ that is decreaing in X. Fix $p\in[0,1]$ and define a function $F:\mathbb{N}\to\mathbb{R}$ by $F\left(m\right)=\mathbb{E}_X\left[f\left(X\right)\right]$ where $X\backsim\text{Binomial}\left(m,p\right)$. Then, $F\left(m+1\right)\leq F\left(m\right)$. Furthermore, the inequality holds strictly if f is strictly decreasing for some $X\in\{0,1,...,m\}$.*

Proof of Lemma A.2. Since $X\backsim\text{Binomial}\left(m,p\right)$, it can be expressed as the sum of m Bernoulli variables: $X=X_1+\ldots+X_m$ where $X_i\backsim\text{Bernoulli}\left(p\right)$. Then, $F\left(m\right)=$

$\mathbb{E}_X [f(X)] = \mathbb{E}_{X_1} \cdots \mathbb{E}_{X_m} [f(X_1 + \ldots + X_m)]$ for any m. Further, given that X_{m+1} follows Bernoulli, we can write

$$\begin{aligned} F(m+1) =& \mathbb{E}_{X_1} \cdots \mathbb{E}_{X_{m+1}} [f(X_1 + \ldots + X_{m+1})] \\ =& \mathbb{E}_{X_1} \cdots \mathbb{E}_{X_m} [pf(X_1 + \ldots + X_m + 1) + (1-p) f(X_1 + \ldots + X_m)] \\ =& \mathbb{E}_{X_1} \cdots \mathbb{E}_{X_m} [p(f(X_1 + \ldots + X_m + 1) - f(X_1 + \ldots + X_m))] \\ &+ \mathbb{E}_{X_1} \cdots \mathbb{E}_{X_m} [f(X_1 + \ldots + X_m)] \\ =& p\mathbb{E}_X [(f(X+1) - f(X))] + F(m), \end{aligned} \tag{A.3}$$

where $X \backsim \text{Binomial}(m, p)$. Because $f(X)$ is decreasing in X, it follows that $F(m+1) \leq F(m)$.

Finally, if $f(X+1) < f(X)$ for some $X \in \{0, 1, \ldots, m\}$, then it follows from (A.3) that $F(m+1) < F(m)$. □

Proof of Lemma 1. Parts (i) and (ii) follow directly from Baye et al. (1996, Theorem 1). Part (iii) uses their Theorem 2. Denoting the expected effort of the skilled agent by e_s, we can use Baye et al. (1996, Theorem 2C) to write the expected sum of efforts as

$$T(n, 1) = \sum_{i=1}^{n} \mathbb{E}x_i = \frac{v}{\alpha} + \left(1 - \frac{1}{\alpha}\right) \mathbb{E}x_s, \tag{A.4}$$

where $\mathbb{E}x_s$ is the expected effort of the single skilled agent, and this varies across the continuum of equilibria. Denoting the mixed strategy of the skilled agent by $G_s(x_s), x_s \in \left[\underline{x_s}, \overline{x_s}\right]$, we have

$$\mathbb{E}x_s = \int_{\underline{x_s}}^{\overline{x_s}} (1 - G_s(x_s) dx_s. \tag{A.5}$$

In the equilibrium leading to the least effort, we use Baye et al. (1996, eq. 4) to find the mixed strategy of the skilled agent as

$$G_s(x_s) = \frac{x_s}{v} \left(1 - \frac{1}{\alpha} + \frac{x_s}{\alpha v}\right)^{\frac{2-n}{n-1}}, x_s \in [0, v]. \tag{A.6}$$

Inserting (A.6) into (A.5) and then into (A.4) gives $T^{min}(n, 1)$ after some rearrangement.

Further, when only one unskilled agent is active, Baye et al. (1996, eq. 4) implies

$$G_s(x_s) = \frac{x_s}{v}, x_s \in [0, v]. \tag{A.7}$$

Inserting (A.7) into (A.5) and into (A.4) gives $\mathrm{T}^{max}(n,1)$. It is straightforward to verify by substitution that $T^{min}(2,1) = T^{max}(n,1)$. □

Proof of Proposition 1. Consider that agents are following a threshold entry strategy: all types less than θ_E enter. An agent's return to investment $q\triangle(n,m) - \theta$ is decreasing in its investment cost θ, implying that its investment strategy follows a cutoff rule as well. Further, because all agents have the same entry cost c, the investment cutoff will also be the same across all agents who enter. Denoting the investment threshold by θ_I, the probability that a randomly picked agent would have a successful investment conditional on entry is $q\Pr[\theta \le \theta_I] / \Pr[\theta \le \theta_E] = q\theta_I/\theta_E$. Since agents' success are independent events, the probability that an agent faces exactly $m-1$ successful agents out of $n-1$ entrants is given by $\binom{n-1}{m-1}(q\theta_I/\theta_E)^{m-1}(1-(q\theta_I/\theta_E))^{n-m}, m-1 \in \{0,\dots,n-1\}$. Therefore, the expected return to investment is

$$q\left[\sum_{m-1=0}^{n-1}\binom{n-1}{m-1}\left(\frac{q\theta_I}{\theta_E}\right)^{m-1}\left(1-\frac{q\theta_I}{\theta_E}\right)^{n-m}\triangle(n,m)\right]-\theta,$$

which further reduces to $(\alpha-1)vq(1-(q\theta_I/\theta_E))^{n-1} - \theta$ because $\triangle(n,m) = 0$ for all $m \ge 2$. If the expected return for the marginal entrant is positive, which happens if $(\alpha-1)vq(1-q)^{n-1} - \theta_E \ge 0$, or equivalently, $\xi(n) \ge \theta_E$, then all agents who enter must invest. In this case, $\theta_I = \theta_E$. If the expected return is negative for the agent with type θ_E, which happens if $\xi(n) < \theta_E$, then only a subset of agents must invest, and θ_I uniquely satisfies (5). The uniqueness follows from the fact that the marginal investor's expected return is also decreasing in θ_I.

Let $\Omega := q(1-(q\theta_I/\theta_E))^{n-1} v(\alpha-1)$. From (5) we can find

$$\begin{aligned}\frac{d\theta_I}{d\theta_E} &= \frac{\frac{\partial\Omega}{\partial\theta_E}}{1-\frac{\partial\Omega}{\partial\theta_I}} \\ &= \frac{q\theta_I(n-1)\Omega}{\theta_E(\theta_E - q\theta_I + q(n-1)\Omega)} > 0.\end{aligned}$$

The positive marginal effects of v and α can also be derived similarly. Furthermore,

$$\begin{aligned}\frac{\partial}{\partial\theta_E}\left(\frac{\theta_I}{\theta_E}\right) &= \frac{\theta_E\frac{d\theta_I}{d\theta_E} - \theta_I}{\theta_E^2} \\ &= \frac{-\theta_I(\theta_E - q\theta_I)}{\theta_E^2(\theta_E - q\theta_I + q(n-1)\Omega)} < 0.\end{aligned}$$

□

Proof of Proposition 2. Consider the entry decisions of two agents with types θ_1 and θ_2, where $\theta_1 < \theta_2$. The θ_1-type agent can achieve a payoff as high as that of the θ_2-type agent simply by replicating the strategy followed by the θ_2-type agent, and even higher if the strategy involves investment in subsequent subgames. Therefore, the expected payoff of the θ_1-type agent from its optimal entry strategy is greater than that of the θ_2-type agent, for any given strategy profile followed by other players. This observation implies that an agent would adopt a cutoff strategy: enter if and only if θ is below a certain threshold. Furthermore, since all agents face the same entry cost, the threshold is the same for all of them. We denote this threshold as θ_E.

At the entry stage, the expected payoff of the marginal agent of type θ_E is $\mathbb{E}_{n-1}\left[\pi\left(\theta_E, n\right)\right]$ where the number of other players, $n-1$, is a random variable following a Binomial distribution with parameters N and θ_E. It directly follows from Lemma $A.1$ that $\mathbb{E}_{n-1}\left[\pi\left(\theta_E, n\right)\right]$ is decreasing in θ_E. The full-entry condition can therefore be derived from the expected payoff of the agent of type $\theta = 1$, which is given by $\mathbb{E}_{n-1}\left[\pi\left(1, n\right)\right] = \pi\left(1, N\right) = \underline{c}\left(N\right) - c$. Therefore, if $c \leq \underline{c}\left(N\right)$, every agent has an incentive to enter, and $\theta_E = 1$. On the other hand, if $c > \underline{c}\left(N\right)$, $\pi\left(1, N\right)$ is negative and (11) has a unique solution determining the entry threshold.

Further, considering $\mathbb{E}_{n-1}\left[\pi\left(\theta_E, n\right)\right]$ as a function $G\left(\theta_E, z\right)$ of θ_E and a generic parameter z, we can work with the total derivative of (11) to get

$$\frac{d\theta_E}{dz} = -\frac{\partial G/\partial z}{\partial G/\partial \theta_E}.$$

As $\partial G/\partial \theta_E \leq 0$, $d\theta_E/dz$ has the same sign as $\partial G/\partial z$, whenever both terms are well-defined. Applying this observation and the fact that $\pi\left(\theta_E, n\right)$ is increasing in v and α, we conclude that θ_E increases in v and α. □

Proof of Proposition 3. It follows from Proposition 2 that there is limited entry if $c > \underline{c}\left(N\right)$, or equivalently, if $\xi\left(N\right) < 1 + c$, which holds if

$$f_1\left(q\right) := q\left(1-q\right)^{(N-1)} < \frac{1+c}{v\left(\alpha-1\right)}.$$

Examining the first derivative, we get that f_1 is increasing in $q \leq 1/N$, and decreasing thereafter, implying

$$\max_{q\in[0,1]} f_1\left(q\right) = \frac{\left(N-1\right)^{N-1}}{N^N}.$$

If $\max_{q\in[0,1]} f_1\left(q\right) < \left(1+c\right)/v\left(\alpha-1\right)$, there is limited entry for every $q \in [0,1]$, which proves part (i) of the proposition. Further, if $\max_{q\in[0,1]} f_1(q) \geq (1+c)/v(\alpha-1)$, then there will be full entry for some q. Given that f_1 is increasing up to $1/N$ and decreasing thereafter, $f_1(q)$ must be higher than $(1+c)/v(\alpha-1)$ at an interval $[\underline{q}, \overline{q}]$, containing $1/N$. □

Proof of Proposition 4. Observe that the full-entry condition $c \leq \underline{c}(N)$ can be rewritten as

$$(1+c) \leq \xi(N) \Leftrightarrow (1-q)^{N-1} \geq \frac{(1+c)}{vq(\alpha-1)}.$$

By taking the logarithm on both sides and noting that $\ln(1-q)$ is negative, we can express the above inequality as

$$N \leq 1 + \frac{\ln((1+c)/(vq(\alpha-1)))}{\ln(1-q)}.$$

Defining $\bar{N}$ as $\max\{0, 1+\lfloor(\ln(1+c)-\ln((\alpha-1)vq))/\ln(1-q)\rfloor\}$, where $\lfloor x \rfloor$ is the largest integer less than or equal to x, part (1) of the proposition directly follows.

Next, suppose $N > \bar{N}$, in which case, there is limited entry and θ_E satisfies (11). We express $\mathbb{E}_{n-1}[\pi(\theta_E, n)]$, where $(n-1)$ follows the distribution Binomial$(N-1, \theta_E)$, as a function of θ_E and N and denoted by $G_1(\theta_E, N)$:

$$G_1(\theta_E, N) = \sum_{n-1=0}^{N-1} \binom{N-1}{n-1} (\theta_E)^{n-1} (1-\theta_E)^{N-n} \pi(\theta_E, n).$$

The entry threshold $\theta_E(N)$ implicitly solves $G_1(\theta_E, N) = 0$. Therefore,

$$\begin{aligned} & G_1(\theta_E(N+1), N+1) - G_1(\theta_E(N), N) = 0 \\ \Leftrightarrow & [G_1(\theta_E(N+1), N+1) - G_1(\theta_E(N+1), N)] \\ & + [G_1(\theta_E(N+1), N) - G_1(\theta_E(N), N)] = 0. \end{aligned}$$

Since $\pi(\theta_E, n)$ is decreasing in n, it follows from Lemma $A.2$ that $G_1(\theta_E, N)$ is decreasing in N, which implies that $G_1(\theta_E(N+1), N+1) \leq G_1(\theta_E(N+1), N)$. Consequently, $G_1(\theta_E(N+1), N) \geq G_1(\theta_E(N), N)$. However, as $\pi(\theta_E, n)$ is also decreasing in θ_E, by applying Lemma $A.1$, we find that $G_1(\theta_E, N)$ decreases in θ_E. Therefore, it must be that $\theta_E(N+1) \leq \theta_E(N)$, which completes the proof. □

Proof of Proposition 5. The first part of the proposition directly follows from Proposition 2. In order to show that θ_E is decreasing in c, we consider $\mathbb{E}_{n-1}[\pi(\theta_E, n)]$, where $(n-1)$ follows the distribution Binomial$(N-1, \theta_E)$, as a function of θ_E and N and denote it by

$G_2(\theta_E, c)$. Note that $\theta_E(c)$ implicitly solves $G_2(\theta_E, c) = 0$. Taking the total derivative of G_2 along the path of $\theta_E(c)$, we get $d\theta_E/dc = -(dG_2/dc)/(dG_2/d\theta_E)$.

Note that $\pi(\theta_E, c)$ is strictly decreasing in c, and therefore it follows from Lemma A.1 that $\partial G_2/\partial c < 0$. Further, as θ_E solves $\mathbb{E}_{n-1}[\pi(\theta_E, n)] = 0$, it must be that $\theta_E \leq \xi(n)$ at least for some n (as otherwise $\mathbb{E}_{n-1}[\pi(\theta_E, n)]$ will be independent of θ_E), which implies that $\pi(\theta_E, n)$ is strictly decreasing in θ_E for some n. Therefore, by applying Lemma A.1, we get $\partial G_2/\partial\theta_E < 0$. Hence, $d\theta_E/dc < 0$, which completes the proof. □

Proof of Proposition 6. To construct $\widehat{c}(\theta_E)$, we consider the two cases separately, $\underline{c}(N) > 0$ and $\underline{c}(N) = 0$.

Consider first $\underline{c}(N) > 0$. It follows from Proposition 2 that for all $c \leq \underline{c}(N)$, $\theta_E = 1$, and therefore, $\widehat{c}(1) = \underline{c}(N)$. For $c > \underline{c}(N)$ and $\theta_E < 1$, θ_E and c have a one-to-one relationship satisfying (11). Therefore, for all $\theta_E < 1$, $\widehat{c}(\theta_E)$ is uniquely determined by the solution of (11). Further, by Proposition 5, $\widehat{c}(\theta_E)$ is strictly decreasing for $0 \leq \theta_E \leq 1$. Because $G_2(\theta_E, c)$ is continuous and differentiable in c, $\theta_E(c)$ is also continuous and differentiable in c, ensuring the continuity and differentiability of $\widehat{c}(\theta_E)$ in $\theta_E \in [0, 1]$. Further, because of strict monotonicity of $\widehat{c}(\theta_E)$, any entry threshold θ_E in $[0, 1]$ can be implemented by choosing an entry fee $c = \widehat{c}(\theta_E)$.

Next, consider $\underline{c}(N) = 0$, which occurs when $\xi(N) \leq 1$. We claim that if $c = 0$, then $\theta_E = 1$ is a unique solution of (11). The proof follows from two observations. Firstly, at $\theta_E = 1$, $\mathbb{E}_{n-1}[\pi(1, n)] = \pi(1, N) = 0$. Secondly, with $c = 0$, we have for all $\theta_E < 1$, $\pi(\theta_E, 1) > 0$ and $\pi(\theta_E, n) \geq 0$ for $n \geq 2$. Therefore, $\mathbb{E}_{n-1}[\pi(\theta_E, n)] > 0$ for all $\theta_E < 1$, implying that any $\theta_E < 1$ cannot be a solution of (11) if $c = 0$.

For $c > 0$ and $\theta_E < 1$, θ_E and c have a one-to-one relationship satisfying (11), and therefore, $\widehat{c}(\theta_E)$ is uniquely determined by the solution of (11). Further, as we have argued in the previous case, $\widehat{c}(\theta_E)$ is differentiable and strictly decreasing in θ_E for all $\theta_E \in [0, 1]$. Therefore, any entry threshold θ_E in $[0, 1]$ can be implemented by setting $c = \widehat{c}(\theta_E)$.

[**Proof of Proposition 7**]It follows from the discussion in Section 7.3 that $\widehat{c}(\theta_E)$ is strictly positive for all $\theta_E < 1$. Therefore, the maximum value of $V_f(\theta_E)$ must be positive, and it must reach its maximum at some $\theta_E > 0$.

Let us first consider the case $\underline{c}(N) = 0$. Then, $\widehat{c}(1) = \underline{c}(N) = 0$, and therefore, $V_f(1) = 0$, which implies that V_f is maximized at some interior $\theta_E \in (0, 1)$, and so the designer prefers limited entry.

Next, consider $\underline{c}(N) > 0$. Therefore, $\widehat{c}(1) = \underline{c}(N) = \xi(N) - 1$. We derive a sufficient condition for an interior maximum by examining the derivative of V_f as θ_E approaches 1: if the derivative is negative, then V_f must be maximized at some $0 < \theta_E < 1$. Note that

$$\lim_{\theta_E \to 1} \frac{dV_f}{d\theta_E} = N \left(\widehat{c}(1) + \lim_{\theta_E \to 1} \frac{d\widehat{c}(\theta_E)}{d\theta_E} \right).$$

Recall from the proof of Proposition 5 that for $\theta_E \in (0,1)$ and $c > 0$, $\widehat{c}(\theta_E)$ solves

$$G_2(\theta_E, c) = \sum_{n=1}^{N} \binom{N-1}{n-1} (\theta_E)^{n-1} (1-\theta_E)^{N-n} \pi(\theta_E, n) = 0.$$

From the total differential of $dG_2 = 0$ along the path of $\widehat{c}(\theta_E)$, we can derive $d\widehat{c}(\theta_E)/d\theta_E = -(\partial G_2/\partial\theta_E)/(\partial G_2/\partial c)$. Further,

$$dG_2/dc = \sum_{n=1}^{N} \binom{N-1}{n-1} (\theta_E)^{n-1} (1-\theta_E)^{N-n} \frac{d\pi(\theta_E, n)}{dc} = -1,$$

which gives us $d\widehat{c}(\theta_E)/d\theta_E = (dG_2/d\theta_E)$, and

$$\lim_{\theta_E \to 1} \frac{dV_f}{d\theta_E} = N \left(\xi(N) - 1 + \lim_{\theta_E \to 1} \frac{dG_2(\theta_E, c)}{d\theta_E} \right). \tag{A.8}$$

Differentiating $G_2(\theta_E, c)$ with respect to θ_E, term by term, and taking the limit as $\theta_E \to 1$, we get

$$\lim_{\theta_E \to 1} \frac{dG_2(\theta_E, c)}{d\theta_E} = \left[(N-1) \lim_{\theta_E \to 1} \pi(\theta_E, N) + \lim_{\theta_E \to 1} \frac{d\pi(\theta_E, N)}{d\theta_E} \right] - \left[(N-1) \lim_{\theta_E \to 1} \pi(\theta_E, N-1) \right], \tag{A.9}$$

where the first square-bracketed term arises from the derivative of the last term in the summation series, and the second square-bracketed term comes from the derivative of the second-to-last term of the series; Because $d\pi(\theta_E, n)/d\theta_E$ is finite for any n, it can be easily shown that the derivatives of all other terms approach zero in the limit as θ_E approaches 1.

Note that as $\xi(N) > 1$, from (10), we get $\lim_{\theta_E \to 1} d\pi(\theta_E, N)/d\theta_E = -1$. Further, as $\theta_E \to 1$, $\widehat{c}(\theta_E) \to \widehat{c}(1) = \underline{c}(N) = \xi(N) - 1$. Therefore,

$$\lim_{\theta_E \to 1} \pi(\theta_E, N) = \xi(N) - 1 - \widehat{c}(1) = 0, \text{ and}$$

$$\lim_{\theta_E \to 1} \pi(\theta_E, N-1) = v \cdot \mathbf{1}_{\{N=2\}} + \xi(N-1) - 1 - \widehat{c}(1) = v \cdot \mathbf{1}_{\{N=2\}} + \xi(N-1) - \xi(N)$$

where $\mathbf{1}_{\{N=2\}}$ is an indicator function that takes the value 1 if $N = 2$, and 0 otherwise.

Replacing the limiting values in the right-hand-side of (A.9), we get

$$\lim_{\theta_E \to 1} \frac{dG_2(\theta_E, c)}{d\theta_E} = (N-1)\left(\xi(N) - \xi(N-1) - v \cdot \mathbf{1}_{\{N=2\}}\right) - 1.$$

Further, replacing the limiting value of $dG_2(\theta_E, c)/d\theta_E$ in (A.8) and using the fact that $(N-1)\,v \cdot \mathbf{1}_{\{N=2\}} = v \cdot \mathbf{1}_{\{N=2\}}$, we can express

$$\begin{aligned}\lim_{\theta_E \to 1} \frac{dV_f}{d\theta_E} &= N\left[N\xi(N) - (N-1)\,\xi(N-1) - 2 - v \cdot \mathbf{1}_{\{N=2\}}\right] \\ &= N\left[\xi(N-1)(1-Nq) - 2 - v \cdot \mathbf{1}_{\{N=2\}}\right].\end{aligned}$$

Therefore, (15) implies that $\lim_{\theta_E \to 1} dV_f/d\theta_E < 0$ and it provides a sufficient condition for having an interior maximum. □

Proof of Proposition 8. It follows from (17) that $V_{inv} = 0$ at $\theta_E = 0$, and $V_{inv} > 0$ at $\theta_E = 1$, implying that V_{inv} reaches its maximum at some $\theta_E > 0$.

Let us first consider the case $\underline{c}(N) > 0$, which occurs if $\xi(N) > 1$. In this case, $\theta_E \leq \xi(n)$ for all $\theta_E \in [0,1]$ and $n \leq N$, and therefore, by Proposition 1, $\theta_I = \theta_E$ and $V_{inv} = N\theta_E$, which is increasing in θ_E. Hence, the designer prefers full entry.

Next, consider $\underline{c}(N) = 0$. Then, as $\theta_E \to 1$, we have $\widehat{c}(\theta_E) \to \widehat{c}(1) = \underline{c}(N) = 0$. We will derive a sufficient condition for an interior maximum by examining the derivative of V_{inv} as θ_E approaches 1: if the derivative is negative, then V_{inv} must be maximized at some $0 < \theta_E < 1$. Note that

$$\lim_{\theta_E \to 1} \frac{dV_{inv}(\theta_E)}{d\theta_E} = N \lim_{\theta_E \to 1} \frac{d}{d\theta_E}\left[\sum_{n=1}^{N} \binom{N-1}{n-1} (\theta_E)^{n-1} (1-\theta_E)^{N-n}\, \theta_I(n)\right].$$

Differentiating $V_{inv}(\theta_E)$ with respect to θ_E, term by term, and taking the limit as $\theta_E \to 1$, we get

$$\lim_{\theta_E \to 1} \frac{dV_{inv}(\theta_E)}{d\theta_E} = N\left[(N-1)\lim_{\theta_E \to 1} \theta_I(N) + \lim_{\theta_E \to 1} \frac{d\theta_I(N)}{d\theta_E}\right] - N\left[(N-1)\lim_{\theta_E \to 1} \theta_I(N-1)\right], \tag{A.10}$$

where the first square-bracketed term arises from the derivative of the last term in the summation series, and the second square-bracketed term comes from the derivative of the second-to-last term of the series; Because $d\theta_I(n)/d\theta_E$ is finite for any n, it can be easily shown that the derivatives of all other terms approach zero in the limit as θ_E approaches 1.

To find $\lim_{\theta_E \to 1} d\theta_I(N)/d\theta_E$, observe that $\theta_I(N)$ solves

$$f(\theta_I, \theta_E) := v(\alpha-1)\, q\left(1-(q\theta_I/\theta_E)\right)^{N-1} - \theta_I = 0.$$

Therefore, from the total differential of $df = 0$ along the path of $\theta_I(N)$, we can derive $d\theta_I(N)/d\theta_E = -(\partial f/\partial\theta_E)/(\partial f/\partial\theta_I)$. Further,

$$\begin{aligned}\frac{\partial f}{\partial \theta_E} &= (N-1)\, v(\alpha-1)\, q\left(1-(q\theta_I/\theta_E)\right)^{N-2}\left(\frac{q\theta_I}{\theta_E^2}\right), \text{ and}\\ \frac{\partial f}{\partial \theta_I} &= -(N-1)\, v(\alpha-1)\, q\left(1-(q\theta_I/\theta_E)\right)^{N-2}\left(\frac{q}{\theta_E}\right) - 1,\end{aligned}$$

which give us

$$\lim_{\theta_E \to 1} \frac{d\theta_I(N)}{d\theta_E} = \frac{(N-1)\, v(\alpha-1)\, q^2\left(1-q\hat{\theta}\right)^{N-2}\hat{\theta}}{(N-1)\, v(\alpha-1)\, q^2\left(1-q\hat{\theta}\right)^{N-2}+1}, \tag{A.11}$$

where $\hat{\theta} := \lim_{\theta_E \to 1} \theta_I(N)$. Because $f(\theta_I, \theta_E)$ is continuous in θ_I and $\theta_E > 0$, $\hat{\theta}$ satisfies $\hat{\theta} = v(\alpha-1)\, q\left(1-q\hat{\theta}\right)^{N-1}$. Therefore, (A.11) can be simplified as

$$\lim_{\theta_E \to 1} \frac{d\theta_I(N)}{d\theta_E} = \frac{(N-1)\, q\hat{\theta}^2}{(N-1)\, q\hat{\theta} + \left(1-q\hat{\theta}\right)} = \frac{(N-1)\, q\hat{\theta}^2}{(N-2)\, q\hat{\theta}+1} \in (0,1). \tag{A.12}$$

We define $\hat{\hat{\theta}} := \lim_{\theta_E \to 1} \theta_I(N-1)$, which solves $v(\alpha-1)\, q(1-q\theta)^{N-2} - \theta = 0$. It can be easily verified that$\hat{\hat{\theta}} > \hat{\theta}$.

Using (A.12), we can simplify (A.11) as

$$\lim_{\theta_E \to 1} \frac{dV_{inv}(\theta_E)}{d\theta_E} = N\left[(N-1)\left(\hat{\theta} - \hat{\hat{\theta}}\right) + \frac{(N-1)\, q\hat{\theta}^2}{(N-2)\, q\hat{\theta}+1}\right].$$

Therefore, (18) implies that $\lim_{\theta_E \to 1} dV_{inv}/d\theta_E < 0$ and it provides a sufficient condition for having an interior maximum. □

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