Agglomeration and Coordination Failures: Lessons for Industrial Policy*

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September 20, 2023

Abstract

When firm location decisions exhibit strong strategic complementarities, certain equilibria emerge where firms coordinate their entry, while others encounter coordination failures. Many large-scale, big-push-style, place-based industrial policies designed to stimulate economic growth aim to guide the production sector towards pareto dominant equilibrium. In this ongoing project, I present an econometric framework to: a) estimate agglomeration forces in the presence of multiplicity, b) identify instances of multiplicity, and c) estimate firm beliefs as encoded in the equilibrium selection rule. This framework, under appropriate instruments, can be used to decompose the causal impact of policy shocks, distinguishing between effects stemming from movement along equilibrium and those arising from equilibrium switching. I apply this framework to study the evolution of the economic activity distribution in India, as well as its response to various Industrial Policy initiatives.

*I am grateful to Nikhil Aggarwal, David Atkin, Abhijit Banerjee, Glenn Ellison, Dave Donaldson, Tobias Saiz, participants at the MIT Trade, Development and IO lunch, and the participants at the Globalization Seminar at Dartmouth College for their helpful comments and feedback. I also acknowledge support from the George and Obie Shultz Fund at MIT.

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1 Introduction

Industrial policy is ubiquitous. Even developed countries are engaged in Industrial Policy whether it is to address green transition, build resilient supply chains or tackle geopolitical tensions with China. Yet, as researchers, we don't quite understand when and how Industrial Policy works. Industrial Policy has traditionally been understudied due to data limitations and lack of exogenous variation. Even though some of the existing work (juhasz lane hanlon) has isolated exogenous variation to study the impact of protection, the mechanisms through which such policies work still remain unclear.

The theoretical foundations of industrial policy are well-established, grounded in the broader rationale often invoked in policymaking – addressing market failures. A recent review article by Juhász, Lane, and Rodrik (2023) identifies externalities and coordination failures as the top two justifications for industrial policy. In this paper, I aim to provide an econometric framework for dissecting the impact of industrial policy into two fundamental components: movement along equilibria and shift in equilibria. To illustrate, consider an analogy of firms engaging in an entry game. A policy shock can influence the entry outcomes through three channels. First, the shock may directly increase entry by enhancing expected payoffs, such as through tax incentives, making entry more financially attractive. Second, it can affect entry through spillover effects; for example, tax incentives targeting one industry may stimulate entry in another, creating a positive feedback loop that boosts the initial industry's profitability and encourages further entry. Lastly, the policy may stimulate entry by inducing a shift in equilibria. Even if no relevant payoff variables change, the policy can "advertise" a region as a future industrial hub, attracting firms to establish themselves there. Alternatively, the policy may introduce a big of temporary incentives that makes it the dominant strategy for all firms to enter - if the equilibria is sticky, such big push can be temporary. I label the first two channels as part of "movement along the equilibria" and the third as "equilibrium switching." I present a robust framework for decomposing the effects of industrial policy into these three channels.

My contribution is threefold. First, I provide estimates of the spillover parameter even in a setting where multiple equilibria are played in the data. The literature on empirical estimation of games that estimating games is hard when there is a possibility of multiplicity. Otsu, Pesendorfer, and Takahashi (2016) provide tests for when the data across different markets can be pooled in the estimation of games. Aguirregabiria and Mira (2019) provide an identification argument for the equilibrium selection rule assuming finite and small support for unobserved heterogeneity. leverage the trade setting to provide a methodology that can be used to estimate the parameter even with underlying multiplicity and continuous unobserved heterogeneity. Estimation of spillover parameters is absolutely key in informing trade policy.

Secondly, we explore whether real-world industrial policy primarily operates through movement along equilibria or equilibrium switching. In other words, we introduce a method to quantify the impact of policy shocks on the selection of equilibria. This not only enriches our understanding of historical industrial policy instances but also provides insights into the types of policy interventions capable of shifting equilibria. Importantly, even though there has been some work on identifying underlying mechanisms for effects of Industrial Policy (Hanlon (2020), Juhász, Squicciarini, and Voigtländer (2020)), no prior work has attempted to quantify the influence of industrial policy on equilibrium selection.

Lastly, while the paper primarily focuses on firm location decisions and place-based industrial policy, the methodology presented here can be applied more broadly. It can extend to the estimation of network games with strategic complementarities, opening the door to comprehensive investigations in various domains.

2 Model

I model the distribution of economic activity across space, as resulting from the entry decisions of firms in various industries. With some abuse of notation, I will denote the set and the cardinality of the set interchangeably. There are K firm-groups that can be interpreted as industries but in general, can refer to any firm group that can be assigned to potentially anonymous firms across various markets.

Players and Action Space. Assume there are N_k potential entrants in each sector k, choosing to locate between L locations. Denote the set of all potential entrants by $N = \cup_k N_k$. Each potential entrant decides whether and where to locate her firm, therefore there at $L + 1$ actions available to each potential entrant.

Markets and Locations. By market, I mean an instance of the game played between players. A market will have a set of locations that the firms in that market are choosing from. When I discuss the model, I am discussing how the game is played within any given market. Therefore, the market-level notation is suppressed initially and introduced only when I discuss data generating process.

Payoffs. Payoff for a firm in sector k for locating her firm in location l is given by

$$\pi_{ikl} = \tilde{v}_{ikl}(\mathbf{x}, \mathbf{a_{-i}}, \boldsymbol{\omega})$$

I allow profits to depend on observed characteristics of all locations l denoted by $\mathbf{x} \equiv (x_l)$, actions of other agents within and outside industry denoted by a_{-ikl} and a_{-kl} respectively, $\mathbf{a_{-i}} \equiv (a_{-i,kl}, a_{-k,l})_{l \in L}$. I also allow for the profits to depend on unobserved but payoff-relevant characteristics denoted by $\boldsymbol{\omega}$.

There is an idiosyncratic part of the pay-off that is private information to each entrant, denoted by $\boldsymbol{\epsilon}_{ik}$. That is, before making the entry decision, $\boldsymbol{\epsilon}_{ik}$ is observed by firm i only, although everyone knows the joint distribution of $(\boldsymbol{\epsilon}_{ik})_{i\in N_k,k\in K}$. I assume, in line with the literature that studies estimation of non-cooperative incomplete-information games, that this part is additive separable, independent of all payoff and non-payoff relevant observed and unobserved characteristics, and has a known continuously differentiable distribution.

That is,

$$\pi_{ikl} = v_{ikl}(\mathbf{x}, \mathbf{a_{-i}}, \boldsymbol{\omega}) + \epsilon_{ikl}, \text{where}$$

Assumption 1. *$(\epsilon_{ikl})_{i\in N_k,k\in K,l\in L+1}$ is i.i.d across $i \in N$. $(\epsilon_{ikl})_{l\in L+1}$ is drawn from a known continuously-differentiable distribution G.*

Given this information structure, players play according to the Bayes-Nash equilibrium. Player i's beliefs about player j's actions are denoted by $P_{ij}(., \mathbf{x}, \boldsymbol{\omega}) \in \Delta^{L+1}$. Players will choose a location so as to maximize expected profits given their beliefs about what other players are playing. If i's beliefs of player j strategy are independent across all players $j \in N-1$, then i's strategy, conditional on observing $\boldsymbol{\epsilon}_{ik}$ is given by

$$\sigma_{ik}(\mathbf{x}, \boldsymbol{\omega}, \boldsymbol{\epsilon}_{ik}) = \arg\max_{a_l\in L+1} \epsilon_{ikl} + \sum_{a'_j\in L+1, j\in N-1} \Pi_{j\neq i} P_{ij}(a'_j, \mathbf{x}, \boldsymbol{\omega}) v_{ikl}(\mathbf{x}, a'_{-i}, \boldsymbol{\omega})$$

Assumption 1 along with consistency of beliefs implies that beliefs indeed are going to be independent and equal to the ex-ante choice probabilities:

$$P_{ij}(a, \mathbf{x}, \boldsymbol{\omega}) = \int \mathbf{1}(\sigma_j(\mathbf{x}, \boldsymbol{\omega}, \boldsymbol{\epsilon}) = a) G(\boldsymbol{\epsilon})$$

Definition 1. *Let $\boldsymbol{x} \in X$ and $\boldsymbol{\omega} \in \Omega$. The best-response conditional choice probability function $\Psi_{ik} : X \times \Omega \times (\Delta^{L+1})^{N-1} \to \Delta^{L+1}$ for player i in industry k is given by*

$$\Psi_{ikl}(\boldsymbol{x}, \boldsymbol{\omega}, \boldsymbol{P}) = \int \boldsymbol{1}\left(\arg\max_{a_l\in L}\left(\epsilon_{ikl} + \sum_{a'_{-i}\in(L+1)^{N-1}} \Pi_{j\neq i,k\in K} P_{jk}(a'_j, \boldsymbol{x}, \boldsymbol{\omega}) v_{ikl}(\boldsymbol{x}, a'_{-i}, \boldsymbol{\omega})\right) = a_l\right) G(\boldsymbol{\epsilon})$$

where the l-th element of the vector Ψ_{ik} is ik's probability of playing action a_l.

A probability vector $\boldsymbol{P} \in (\Delta^{L+1})^N$ is an equilibrium conditional choice probability (CCP) vector given $(\boldsymbol{x}, \boldsymbol{\omega})$ iff it satisfies $\boldsymbol{P} = \Psi(\boldsymbol{x}, \boldsymbol{\omega}, \boldsymbol{P})$.

By Brower's Fixed Point Theorem, under assumption there is at least one equilibrium, but there can be many.

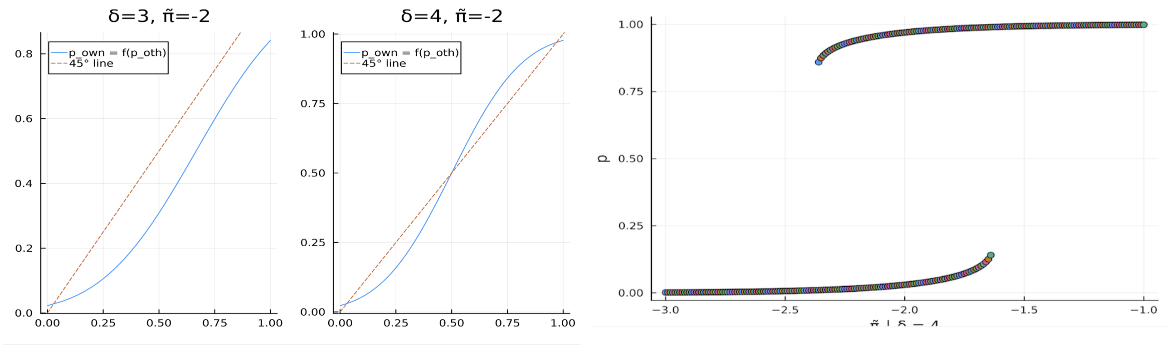

Figure 1: Left Panel (a): $\tilde{\pi} = -2; \delta = 3$, (b): $\tilde{\pi} = -2; \delta = 4$, Right Panel: $\delta = 4$

2.0.1 Example

Suppose $L = 1$, so the location decision of firms boils down to the entry decision. The payoff of a firm i in industry k is given by

$$\pi_{ik1} = \tilde{\pi}_k + \sum_{k' \in K} \delta_{kk'} Q_{k'} + \epsilon_{ik1}; \qquad \pi_{ik0} = \epsilon_{ik0}$$

Here, $Q_k = \frac{\sum_{i \in N_k} a_{ik}}{N_k}$, where a_{ik} is the binary variable for whether firm i belonging to industry k enters. $\boldsymbol{\epsilon}_{ik}$ is the private information of firm i. Let $\epsilon_{ik} \perp \epsilon_{jk'} \forall i, j \in N, k, k' \in K$, implying $E_{\boldsymbol{\epsilon}_{k'}}(P_{k'} \mid \epsilon_{ik}) = E_{\boldsymbol{\epsilon}_{k'}} P_{k'm}$. That is, independence of private information means that firm i's private information shocks are not informative of firm j's private information shocks.

Lets assume that all firms in a group play symmetric strategies. Firm i chooses the action that maximises the ex-ante profits, which are given by the same expression above if we replace the actual probability Q_k with the conditional choice probability P_k. Lets assume that $(\epsilon_{ik0}, \epsilon_{ik0})$ follow a Type-I Extreme Value distribution. Under the distributional assumption on $\boldsymbol{\epsilon}_{ik}$, the conditional choice probability of firms in group k is given by

$$P_k = \frac{\exp \bar{\pi}_{k1}}{1 + \exp \bar{\pi}_{k1}}; \text{where} \bar{\pi}_{k1} = \tilde{\pi}_k + \sum_{k' \in K} \delta_{kk'} P_{k'}$$

Single Industry. In the case of single industry, the equilibrium choice probability is given by the fixed point of the mapping $P \to \frac{\exp \tilde{\pi} + \delta P}{1 + \exp \tilde{\pi} + \delta P}$. This is visualized in the left panel figure 2.0.1 where for a low value of δ there is a single equilibrium, while for a high value of δ, three equilibria emerge.

Multiple equilibria exists in the intermediate range of $\tilde{\pi}$. For a low value of $\tilde{\pi}$, the dominant strategy for everyone is to not enter, and for a high level of $\tilde{\pi}$, firms are going to enter irrespective of the entry probability. This can be visualised in the right panel of figure 2.0.1 where multiplicity arises when $\tilde{\pi} \in [-2.4, -1.4]$

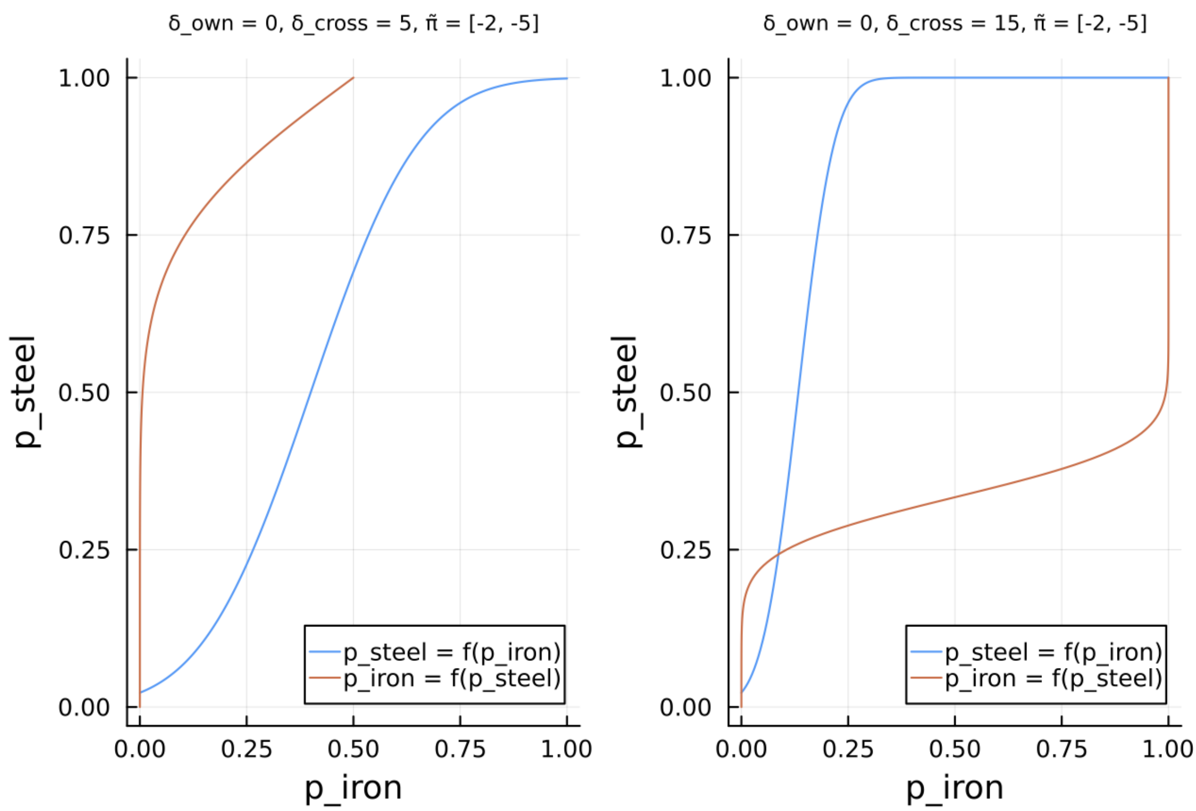

Figure 2: Red: Best Response Curve of Iron; Blue: Best Response Curve of Steel

Multiple Industries. Suppose there are two industries - iron and steel. $\tilde{\pi}_k$ for $k \in \{Iron, Steel\}$ is $-2, -5$ respectively. There are no cross-industry spillovers, but the spillovers across industries are parameterised by δ_{cross}.

The left panel of the figure 2.0.1 plots the best response curves $P_{-k} \to \Psi_k(\tilde{\pi}, P_{-k})$ for $k \in \{Iron, Steel\}$. The best response curves intersect only once for a low δ, but they intersect thrice for high δ.

2.1 Data Generating Process

As an econometrics, we have data from M independent markets. We observe the market-level characteristics and choices played in each market, denoted by $\mathbf{x}_m = (x_{lm})_{l \in L}$ and $a_m = (a_{iklm})_{k \in K, i \in N_k, l \in L+1}$ respectively. The researcher does not observe $\boldsymbol{\omega}_m$ which we call unobserved heterogeneity across markets. I assume that $\mathbf{x}_m$ and $\boldsymbol{\omega}_m \mid \mathbf{x}_m$ are iid across markets. Denote the conditional distribution of $\boldsymbol{\omega}_m \mid \mathbf{x}_m$ by $F(. \mid .)$. Denote the support of $\boldsymbol{\omega} \mid \mathbf{x}$ as $\Omega(\mathbf{x})$

Define the set of equilibria $\Lambda(\mathbf{x}_m, \boldsymbol{\omega}_m) = \{\mathbf{P} | \mathbf{P} - \Psi(\mathbf{x}_m, \boldsymbol{\omega}_m, \mathbf{P}) = 0\}$. Denote the selected equilibria by a random variable $\tau_m \in 1, 2, \ldots, \#\Lambda(\mathbf{x}_m, \boldsymbol{\omega}_m)$, with distribution $\lambda(. \mid \mathbf{x}_m, \omega_m)$. Lets index the τth equilibrium by $\mathbf{P}^{\tau}(x_m, \omega_m)$. Denote the conditional choice probability for a market by $\mathbf{P}_m = (P_{klm})_{k \in K, l \in L} = \mathbf{P}^{\tau_m}(x_m, \omega_m)$

Equilibrium Types. For notational convenience, let's denote all the pay-off relevant variables for a market m as $\tilde{\pi}_m \equiv (\mathbf{x}_m, \boldsymbol{\omega}_m)$. τ not only indexes equilibria conditional on $\tilde{\pi}_m$ but also has a meaning as we vary $\tilde{\pi}_m$. τ denotes the "equilibrium type" - a notion I borrow from Aguirregabiria and Mira (2019). Informally, $P^{\tau'}(\tilde{\pi}_1) \in \Lambda(\tilde{\pi}_1)$ and $P^{\tau''}(\tilde{\pi}_2) \in \Lambda(\tilde{\pi}_2))$ belong to the same type τ, that is, they will be indexed by same $\tau = \tau' = \tau''$, iff there exists a continuous path $P[t], t \in [0,1], P[t] \in \Lambda(t\tilde{\pi}_1 + (1-t)\tilde{\pi}_2)$ that connects $P^{\tau'}$ and $P^{\tau''}$.

For example, in the right panel of figure 2.0.1, the lower curve in the graph belongs to one equilibrium type, as any two probabilities on the curve can be joined by the continuous path of CCPs on the curve. Similarly, the upper curve constitutes another equilibrium type.

Likelihood Function. The likelihood of observing actions $\mathbf{a}_m$ in market m, conditional on $\mathbf{x}_m$ is given by

$$\mathcal{L}(\mathbf{a}_m \mid \mathbf{x}_m) = \int_{\boldsymbol{\omega}\in\Omega(\mathbf{x}_m)} \sum_{\tau\in 1,2,\ldots,\Lambda(\mathbf{x}_m,\boldsymbol{\omega}_m)} \Pi_{j\in N_k, k\in K} P^{\tau}_{jk}(\mathbf{a}_j, \mathbf{x}_m, \boldsymbol{\omega})\lambda(\tau \mid \boldsymbol{\omega}, \mathbf{x}_m) dF(\boldsymbol{\omega} \mid \mathbf{x}_m) \tag{1}$$

Additionally, if we make assumptions about symmetric payoffs and strategies within a firm-group, we can derive joint likelihood for the number of entrants from each firm group in every location $\mathbf{n}_m \equiv (n_{klm})_{k\in K, l\in L+1}$, as a function of the marginal likelihood.

Assumption 2 (Within-group Symmetry). *1. Within Group Identical Ex-ante Payoff:*

$$\nu_{ikl}(\boldsymbol{x}_m, \boldsymbol{a}_{-im}, \boldsymbol{\omega}_m) = \upsilon_{kl}(\boldsymbol{x}_m, \bar{a}_{km}, \bar{a}_{-km}, \boldsymbol{\omega}_m)$$

for all $i \in N_k$

2. Within Group Symmetric Equilibria:

$$\lambda(\tau \mid \boldsymbol{x}_m, \boldsymbol{\omega}_m) > 0 \implies P_{ik}(\boldsymbol{x}_m, \boldsymbol{\omega}_m) = P_{i'k}(\boldsymbol{x}_m, \boldsymbol{\omega}_m) \tag{2}$$

for all $i, i' \in N_k, k \in K$

The first part of assumption 2 says that the ex-ante pay-offs are identical for all firms in a group, and they depend on the group-level entry outcomes. Note that symmetric pay-offs does not, in general, guarantee symmetric strategies. The second part of the assumption imposes that all players play symmetric strategies in an equilibrium that is played with positive probability. Under the symmetry assumption, we can express the likelihood of entry in different locations in different markets as -

$$\mathcal{L}(\mathbf{n}_m \mid \mathbf{x}_m) = \int_{\boldsymbol{\omega}\in\Omega(\mathbf{x}_m)} \sum_{\tau\in 1,2,\ldots,\Lambda(\mathbf{x}_m,\boldsymbol{\omega}_m)} \Pi_{k\in K}\mathcal{L}(\mathbf{n}_{km} \mid \tau, x_m, \omega)\lambda(\tau \mid \boldsymbol{\omega}, \mathbf{x}_m) dF(\boldsymbol{\omega} \mid \mathbf{x}_m) \tag{3}$$

where $\mathbf{n}_k \mid \tau, \mathbf{x}, \boldsymbol{\omega} \sim Mult(N_k, P_k^\tau(., \mathbf{x}, \boldsymbol{\omega}))$

A couple of remarks are in order. First, note that conditional on all observable and unobservable pay-off relevant characteristics, if there is a unique equilibrium being played in the data, entry in different sectors in different should be independent. However, conditional on observable characteristics, the correlation in location decisions across sectors could arise from either unobserved heterogeneity ω or multiplicity of equilibria. Second, conditional on market-level characteristics, entry for each sector follows a multinomial distribution under a unique equilibria, but follows a finite mixture of multinomial distributions under multiplicity. This idea is formalised in the next proposition:

Proposition 1. 1. *If $\mid \Lambda(\boldsymbol{x}, \boldsymbol{\omega}) \mid = 1$, and assumption 2 holds, then $(\boldsymbol{n}_k)_{k \in K} \mid (\boldsymbol{\omega}, \boldsymbol{x})$ are pair-wise independent, and $\boldsymbol{n}_k \mid (\boldsymbol{\omega}, \boldsymbol{x})$ follows a multinomial distribution.*

2. *If $\mid \Lambda(\boldsymbol{x}_m, \boldsymbol{\omega}_m) \mid = T > 1$, $\boldsymbol{n}_k \mid (\boldsymbol{\omega}, \boldsymbol{x})$ follows a mixture of at most T distinct multinomial distributions.*

The result in this proposition can be used to derive a test for multiplicity as in De Paula and Tang (2012). Correlation in firm entry across markets similar on observed fundamentals will be suggestive of either multiplicity or correlated unobserved fundamentals. Similarly, bi-modality of the distribution of entry will be suggestive of either underlying multiplicity or bi-modal unobserved heterogeneity.

2.1.1 Example Contd.

Going back to the two-industry example depicted in figure 2.0.1, across markets that are identical on both observables and unobservables, we will see a correlation in the number of entrants in iron and steel only if there are multiple equilibria.

3 Indentification

The approaches in the game estimation literature assume a finite number of players, and identification and inference relies on $M \to \infty$. But in a macroeconomic setting where we are interested in aggregate outcomes and general equilibrium, we might have many potential firms per firm group. Along with the assumption that they all play the same strategies, we can estimate the conditional choice probabilities without relying on the variation across markets. As $N_k \to \infty$, data from a single market will give a consistent estimate P_{klm} as $\frac{n_{klm}}{N_k m}$, therefore we don't need to pool data across markets. Once we have the market-specific choice probabilities, we can back out π_{klm} which we can project on x_m and P_{-klm} along with appropriate exclusion restrictions. Note that within-group spillovers are harder to identify in general, without a full solution method.

3.1 Identification of CCPs

Denote the ex-ante profit function with $\bar{\nu}_k(\mathbf{x}_m, \mathbf{P}_m, \boldsymbol{\omega}_m) = E_{\mathbf{P}_m}\nu_k(\mathbf{x}_m, \mathbf{a}_{-im}, \boldsymbol{\omega}_m)$

Proposition 2. *Under assumptions 1 and 2, along with $N_{km} \to \infty$ for all $k \in K, m \in M$, conditional choice probabilities p_{km} and ex-ante profits $\pi_{km} = \bar{\nu}_k(\boldsymbol{x}_m, \boldsymbol{P}_m, \boldsymbol{\omega}_m)$ are identified.*

Proof. Group -specific CCP p_{km} is identified from the Law of Large Numbers since all the players in a group k and market m play symmetric strategies (Assumption 2). π_{km} can be recovered from p_{km} using Assumption 1 and applying the Proposition 1 in Hotz and Miller 1993. □

Denote the mapping between p_k and π_k with $\pi_k = \Gamma(p_k)$. For example, with T1EV ϵs:

$$p_{kl} = \Gamma^{-1}(\pi_k) = \frac{\exp \pi_{kl}}{\sum_{l'} \exp \pi_{kl'}}$$

3.2 Identification of Pay-off Function

Even though we have recovered π_{km}, the structural relationship between π_k and $(\mathbf{x}_m, \mathbf{a}_{-im}, \boldsymbol{\omega}_m)$ is still unknown. However, having recovered the values of all the variables that enter the structural relationship, we can estimate the relationship using appropriate instruments.

Proposition 3. *If*

- $\nu_k(\boldsymbol{x}, \boldsymbol{a}_{-i}, \boldsymbol{\omega})$ *is additively separable in* $\boldsymbol{\omega}_k$.
- *There exist a set of instruments z such that* $E(\omega \mid z) = 0$
- *(Newey-Powell'03 condition) For all $\delta(\boldsymbol{x}, \boldsymbol{P}_{-i})$ with finite expectation,* $E(\delta(\boldsymbol{x}, \boldsymbol{P}_{-i})) \mid z) = 0 \implies \delta(\boldsymbol{x}, \boldsymbol{P}_{-i})) = 0,$

then, $\nu_k(.)$ is identified.

Proof. Additive separability implies that the ex-ante profits can be written as

$$\bar{\nu}_k(\mathbf{x}, \mathbf{P}_{-i}, \boldsymbol{\omega}) = \beta_k(\mathbf{x}, \mathbf{P}_{-i}) + \boldsymbol{\omega}_k$$

Directly applying Proposition 2.1 in Newey and Powell (2003) to the equation above gives the result.

□

The first assumption is without loss of generality - I do allow for profits of industry k to arbitrarily depend on ω_{-k}. The only requirement is that $\nu_k(.)$ is additively separable in $\mathbf{P}_{-i}$ and $\boldsymbol{\omega}$. To see this, under additive separability, I can rewrite the ex-ante profit function as

$$\begin{aligned}\bar{\nu}_k(\mathbf{x}, \mathbf{P}_{-i}, \boldsymbol{\omega}) &= \sum_{a_{-i}} P_{-i}(a_{-i})\nu_k(a_{-i}, \mathbf{x}, \boldsymbol{\omega}) \\ &= \underbrace{\eta_k(\mathbf{x}, \boldsymbol{\omega})}_{\equiv \omega_k} + \underbrace{\sum_{a_{-i}} P_{-i}(a_{-i})\beta_k(\mathbf{a}_{-i}, \mathbf{x})}_{\equiv \beta_k(\mathbf{P}_{-i}, \mathbf{x})}\end{aligned}$$

and redefine ω_k to be the additive part.

The second and third assumption require that we find shifters for $\mathbf{x}$ and $\mathbf{P}_{-i}$ that are mean independent of $\boldsymbol{\omega}$. More concretely, we need characteristics $z_{k'}$ that enter into the profit equation of $k' \neq k$, and hence are shifters for $\mathbf{P}_{k'}$, but are independent of ω_k.

3.3 Identification of Equilibrium Selection Rule

We saw that the reduced form relation between market characteristics $(\mathbf{x}, \boldsymbol{\omega})$ and (p_k, π_k) will be a correspondence under multiplicity -

$$\begin{aligned}p_k(\mathbf{x}, \boldsymbol{\omega}) &\in \{\mathbf{P}_k : \mathbf{P} \in \Lambda(\mathbf{x}, \boldsymbol{\omega})\} \\ \pi_k(\mathbf{x}, \boldsymbol{\omega}) &\in \{\bar{\nu}_k(\mathbf{x}, \mathbf{P}, \boldsymbol{\omega}); \mathbf{P} \in \Lambda(\mathbf{x}, \boldsymbol{\omega})\}\end{aligned}$$

Recall that τ indexes an equilibrium type. By definition, if both τth and τ'th equilibrium types exist conditional on $(\mathbf{x}, \boldsymbol{\omega})$, $\tau \neq \tau' \implies \mathbf{P}^{\tau}(x, \omega) \neq \mathbf{P}^{\tau'}(x, \omega)$.

Note that, once we recover $\bar{\nu}_k(\mathbf{x}, \mathbf{P}, \boldsymbol{\omega})$, we can recover the equilibrium set $\Lambda(\mathbf{x}, \boldsymbol{\omega})$ as the set of fixed points of function $\mathbf{P} \to \Gamma^{-1}(\bar{\nu}(\mathbf{x}, \mathbf{P}, \boldsymbol{\omega}))$.

Proposition 4. *If*

1. $\pi_m = \nu(\boldsymbol{x}_m, \boldsymbol{P}_m, \boldsymbol{\omega}_m)$ *can be inverted to get* $\boldsymbol{\omega}_m = r(\pi_m, \boldsymbol{x}_m, \boldsymbol{P}_m)$*, and*

2. $\boldsymbol{P}_m = p^{\tau_m}(\boldsymbol{x}_m, \boldsymbol{\omega}_m)$ *can be inverted to get* $\tau_m = s(\boldsymbol{P}_m, \boldsymbol{x}_m, \boldsymbol{\omega}_m)$*;*

then $\lambda(\tau \mid \boldsymbol{x}, \boldsymbol{\omega})$ *is identified.*

Proof. x_m is data, and $\mathbf{P}_m$ is identified in each market from $N_k \to \infty$. Hotz-Miller inversion gets us $\boldsymbol{\pi}_m = \Gamma(\mathbf{P}_m)$. Assumption 1 tells us that we can recover ω_m. Once ω_m is recovered, we can recover τ_m from Assumption 2. The distribution of τ_m conditional on $\mathbf{x}_m, \boldsymbol{\omega}_m$ gives us $\lambda(\tau \mid \mathbf{x}, \boldsymbol{\omega})$. $\square$

The above result does not assume functional form for the profit function, but it is helpful to think about the parametric case. Suppose $\bar{\nu}_k(\mathbf{x}, \mathbf{P}, \boldsymbol{\omega}) = \beta_k(\mathbf{x}, \mathbf{P}) + \boldsymbol{\omega}_k$ is known up to some parameter θ_k. With some abuse of notation, lets say $\pi_{km} = \beta_k(\mathbf{x}_m, \mathbf{P}_m; \theta_k) + \boldsymbol{\omega}_{km}$ where $\beta_k(.)$ is known.

Once θ_k is recovered using appropriate exclusion restrictions a'la Newey and Powell 2003, Assumption 1 in Proposition 4 is satisfied: $\boldsymbol{\omega}_{km} = \pi_{km} - \beta_k(\mathbf{x}_m, \mathbf{P}_m; \theta_k)$. Moreover, since θ_k is identified, the set of equilibria $\Lambda(\mathbf{x}, \boldsymbol{\omega})$ can be solved for as the set of fixed points of $\mathbf{P} \to \Gamma^{-1}(\beta_k(\mathbf{x}, \mathbf{P}; \theta_k) + \boldsymbol{\omega}_k)$. In practice, I follow the following algorithm in the assignment of equilibrium type to different equilibria:

Start with some initial values of $\tilde{\pi}_0 = (x_0, \omega_0)$. Solve for the set $\Lambda(\tilde{\pi}_0)$. Assign indices randomly from 1 to $\#\Lambda(\tilde{\pi}_0)$. Denote the assignment rule by $\tau(p)$.

1. For nth point $\tilde{\pi}_n$, denote the τ-th equilibrium by p_n^τ, and the set of assignments by T_n
2. Perturb $\tilde{\pi}_{n+1} = (x_{n+1}, \omega_{n+1}) = (x_n + \Delta, \omega_n + \Delta)$, and find $\Lambda(\tilde{\pi}_{n+1})$.
3. Given the total number of equilibrium types $K \in [\#\Lambda(\tilde{\pi}_n), \#\Lambda(\tilde{\pi}_n) + \#\Lambda(\tilde{\pi}_{n+1})]$, search for the assignment of equilibrium types by minimizing the distance between equilibria of the same type:

$$\mathcal{C}(K) = \min_{\tau(p)\in 1\ldots K : p \in \Lambda(x_{n+1}, \omega_{n+1})} \sum_{\tau \in T_n \cap T_{n+1}} || \, p_n^\tau - p_{n+1}^\tau \, ||^2$$

4. Determine the optimal number of K using AIC or BIC, as in K-means clustering.

Repeat the process till the whole range of (x, ω) is covered.

Now that we know the relation $(\mathbf{x}_m, \omega_m, \tau_m) \to \mathbf{P}_m$, observed $\mathbf{P}_m$ can be inverted to recover the equilibrium selection rule τ_m. Once τ_m is recovered for each market, then estimating the impact of *any* policy shock on τ_m is standard and can be estimated using parallel trends assumption.

4 Conclusion

In this short paper, I provide an econometric framework to identify spillovers and the equilibrium selection rule in a game that features multiplicity. Importantly, I provide methods to identify both the objects of interest with minimal restrictions on the unobserved heterogeneity. This is important because the causal impact of many policy shocks masks movement along the equilibria and equilibrium switching. My method can be applied to all those settings to decompose the causal impact into the two channels. Currently, I am working on applying my methods to decompose the causal impact of Industrial Parks in India into the one due to agglomeration spillovers and the one due to equilibrium switching.

References

Aguirregabiria, Victor and Pedro Mira (2019). “Identification of games of incomplete information with multiple equilibria and unobserved heterogeneity”. In: *Quantitative Economics* 10.4, pp. 1659–1701.

De Paula, Aureo and Xun Tang (2012). “Inference of signs of interaction effects in simultaneous games with incomplete information”. In: *Econometrica* 80.1, pp. 143–172.

Hanlon, W Walker (2020). “The persistent effect of temporary input cost advantages in shipbuilding, 1850 to 1911”. In: *Journal of the European Economic Association* 18.6, pp. 3173–3209.

Juhász, Réka, Nathan J Lane, and Dani Rodrik (2023). *The New Economics of Industrial Policy*. Tech. rep. National Bureau of Economic Research.

Juhász, Réka, Mara P Squicciarini, and Nico Voigtländer (2020). *Technology adoption and productivity growth: Evidence from industrialization in France*. Tech. rep. National Bureau of Economic Research.

Newey, Whitney K and James L Powell (2003). “Instrumental variable estimation of nonparametric models”. In: *Econometrica* 71.5, pp. 1565–1578.

Otsu, Taisuke, Martin Pesendorfer, and Yuya Takahashi (2016). “Pooling data across markets in dynamic Markov games”. In: *Quantitative Economics* 7.2, pp. 523–559.