Stochastic Choice and Unobserved Reference Points

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Introduction

- A reference point (in this case an alternative), is an alternative that can impact the choice behavior of a boundedly rational agent.
- It can do this by affecting: preferences (Status quo bias); and attention (consideration set).
- How a reference point affects such behavior leads to many possible heuristics that have been studied in the decision theory literature.
- Such work aims to provide theoretical foundations for observed patterns in decision-making different from rationality.

Stochastic Choice and Unobservable Reference Point

- We construct a model of stochastic choice with **unknown** reference points that affect attention through consideration sets.

- We include unobservable reference points in the independent random attention (IRA) framework introduced by Manzini and Mariotti(2014)(MM14).

- The reference alternatives are different to the non-reference alternatives in that they are **attention privileged** (Kovach and Sulmeynov (2023)).

A Simple Example

- Suppose you are in a food court with plenty of outlets.
- You pay attention to some subset of outlets (independently of each other), and then choose from the ones you looked at.
- However, if there is a McDonald's outlet, you always get attracted to the big M, and so, pay attention to it.
- So, the McDonald's outlet is **attention privileged**.

Contents

- We provide a characterization of an IRA model with unobservable reference points.
- Followed by identification for the reference points as well as the underlying preferences.
- We conduct a comparative static analysis by comparing our model to some existing models in the choice theory literature.
- Finally, we look at the observed behavioral patterns of choice that this model can explain.

Primitives

- Let X be the grand set of all alternatives. Choices are made from subsets of X denoted by $\mathcal{X}$ ($|\mathcal{X}| = 2^X$).
- A stochastic choice function is defined as follows:

Definition

A function $p : X^* \times \mathcal{X} \to [0, 1]$ is a stochastic choice function if:

- $p(a, A) \geq 0$
- $p(a, A) = 0$ if $a \notin A$
- $\sum_{a \in A} p(a, A) \leq 1$

- There is a default alternative denoted by a^* which could be interpretated as abstention, such that $p(a^*, A) = 1 - \sum_{a \in A} p(a, A)$ for all $A \in \mathcal{X}$.
- $X^* = X \cup \{a^*\}$, and $A^* = A \cup \{*\}$.

Independent Random Attention (IRA)

- The idea for independent random attention is that all alternatives in X are **paid attention to**, with some probability which is **menu independent**.
- The probability of choices are also determined in an independent fashion where an alternative is chosen if it is paid attention to and all the better alternatives are not.
- So, there is some attention function: $\delta : X \to (0, 1)$ which gives us the attention probability of any alternative in any menu in which it belongs.

Reference Alternatives

- We introduce reference alternatives in the above setting.
- Denote by E the set of reference points. Note that E does not affect (or is affected by the preferences), nor is it dependent upon the menu.
- In this model, an alternative $x \in E$ if and only if $\delta(x) = 1$.
- That is, all reference alternatives are attention privileged and non-reference alternatives are **not** attention privileged.

Reference Dependent Random Consideration Set Model

Here we define the reference-dependent random consideration set model:

Definition

A reference-dependent random consideration set model (RDRCM) is a stochastic choice function $p_{\succ,\delta}$ where $\succ$ is a strict total order and $\delta : X \to (0,1]$ is a map such that:

$$p_{\succ,\delta}(x,A) = \delta(x) \prod_{y \in A; y \succ x} (1 - \delta(y))$$

For all $A \in \mathcal{X}$ and for all $x \in A$.

- The reference alternatives are defined in terms of attention privilege.

Some Observations

- In this model, some alternatives can be chosen with zero probability.
- In any menu where an alternative from E is present, no alternative worse than that is chosen, giving us a **lower bound** interpretation with respect to the reference alternatives.
- We also have a **status quo bias** idea, since an alternative which is not the reference in a menu, weakly improves its chances if it was.
- There is also a property similar to **status quo irrelevance**, in that the incidence of an alternative from E in a menu does not affect the choice probability of any alternative preferred to it.

Independent Random Attention (MM14)

Manzini and Mariotti defined what they call the "random consideration set rule". We show the definition of the rule they describe below.

Definition

A random consideration set rule is a stochastic choice function p for which there exists a strict linear order $\succ$ over X and δ where $\delta : X \to (0,1)$ such that:

$$P(x, A) = \delta(x) \prod_{y \in A, y \succ x} (1 - \delta(y))$$

for all $A \in 2^X$ and for all $x \in A$.

- MM14 characterize their model with two axioms.
- The RDRCM model generalizes MM14. We provide 5 axioms to characterize the RDRCM.

R-Asymmetry

Definition

A1(R-Asymmetry): For all $a, b \in A$ with $p(a, A) > 0$ and $p(b, A) > 0$, $\frac{p(a, A \setminus \{b\})}{p(a, A)} \neq 1 \implies \frac{p(b, A \setminus \{a\})}{p(b, A)} = 1$.

R-Independence

Definition

A2(R-Independence): For all $A \in \mathcal{X}$ with $a, b \in A \cap B$ with $p(a, A) > 0, p(a, B) > 0$, $\frac{p(a,A \setminus \{b\})}{p(a,A)} = \frac{p(a,B \setminus \{b\})}{p(a,B)}$ and for all $A \in \mathcal{X}$ with $p(a^*, A) > 0, p(a^*, B) > 0$ then $\frac{p(a^*,A \setminus \{b\})}{p(a^*,A)} = \frac{p(a^*,B \setminus \{b\})}{p(a^*,B)}$.

Regularity

Definition

A3(Regularity): $A \subset B$ implies $p(a, A) \geq p(a, B)$ for all $a \in A^*$

In MM14 i-Asymmetry and i-Independence implied Regularity, that is not the case here. Consider an example where $X = \{x, y\}$, P(x,{x,y})=1, P(x,{x})=2/3 and $P(y, \{y\}) = 1/3$. Here A1,A2,A4,A5 works, but A3 does not.

Non-Trivial and Undominated Default

Definition

A4(Non-Trivial): $p(x, \{x\}) > 0$ for all $x \in X$.

Definition

A5(Undominated Default): For all $A \in \mathcal{X}$ such that $p(a^*, A) = 0$ there exists an alternative $x \in A$ such that $p(a^*, \{x\}) = 0$.

Theorem

Theorem

A stochastic choice function p is representable by a reference-dependent random consideration set model if and only if it satisfies A1-A5.

Identification

- We can identify the set of all reference alternatives using singleton menus.
- If we do not have data from singleton menus, we can still identify all references under reasonable richness restrictions.
- We will also be able to recover the underlying preferences among all alternatives, reference or otherwise.

Random Utility Model Representation

A stochastic choice function $p : X^* \times \mathcal{X}$ has a **Random Utility Model (RUM)** representation if there exists a distribution $\Delta(\mathcal{L}(X))$ with typical element ν over the set of all linear orders on X^* denoted by $\mathcal{L}(X)$ with typical element R such that:

$$p(x, A) = \sum_{R \in \mathcal{L}(X) : xRy \forall y \in A^*} \nu(R)$$

for all $x \in A^*$.

RUM Representation

- Similar to MM14, the general model also has a RUM representation.
- The set of references restricts the orders with positive weight in the distribution ν.
- This allows for a comparative statics analysis of adding/removing specific alternatives from the set of references.
- We provide the specific representation in the next two slides.

RUM Representation of MM14

- For any alternative $a \in X$, we have, $\nu(\{R : aRa^*\}) = \delta(a)$.

- If $a \succ b$, then consider an R such that bRa, aRa^* and bRa^*, we will have $\nu(R) = 0$.

- For any two alternatives a and b, $\nu(\{R : aRa^* \text{ and } bRa^*\}) = \delta(a)\delta(b)$.

RUM Representation of RDRCM

- An reference, if present is always considered. So, we have $\nu(\{R : a^* R a\}) = 0$ for all $a \in E$.

- Suppose $a \in E$ and $a \succ b$, then $\nu(\{R : b \succ a\}) = 0$.

- As we add alternatives to E more and more orders are given zero weight in a systematic way.

RUM Representation of RDRCM

- Adding an alternative to E, means every alternative below (above) it in $\succ$ is below (above) it for all R's with positive weight.
- For all alternatives in E only those R's have positive weight that conform to $\succ$ between them.
- As E becomes equal to X, only the R coinciding with $\succ$ gets the positive weight equal to 1. We have reached preference maximization.
- Adding a reference **weakly improves** the choices of the DM.

Menu Dependent Reference Interpretation

- In our model, the reference alternatives are menu-independent.
- However, there exists a menu-dependent reference specification that is behaviourally equivalent to the RDRCM.
- This is when there is a reference assignment function from each menu to the set of alternatives, and the assignment is preference conforming with respect to the set E.
- We call the most preferred reference in a menu as the **effective reference alternative**.
- This way our data could be specified where each menu has one reference (or no reference) similar to the status quo bias literature.

Behavioral Implications

- MM14 is able to accomodate violations of Stochastic Transitivity and menu effects, we discuss some more patterns that our model can accomodate.
- **Status Quo Monotonicity:** Clearly, an alternative being a reference is more likely to be chosen since its paid more attention to. There are more implications.
- We can observe extreme choice reversals among non reference alternatives.
- Adding a reference to a menu does not affect the probabilities of any alternative better than it. (categorical interpretation)

Choice Overload

- Choice Overload in the context of references means the tendency to choose the reference should increase as the menu size increases.
- A model of independent attention means that in larger menus, the attention parameter does not change which means that accommodating choice overload is difiucult.
- Considering the menu-dependent interpretation, each menu has an **effective reference point**.
- Adding an alternative to a menu that changes the effective reference point, will weakly increase the probability of choosing the effective reference point.

General Attention Model

- The attention parameters in the model are independent of menu and other alternatives.
- The general attention model would just have a distribution over subsets of menus which would specify the probability that each subset is the one considered.
- **Attention privilege** there would mean that any subset where the reference is not considered is assigned zero weight in such a distribution.
- The next step is to provide identification results for the general attention framework.

Literature

- **Reference Point:** Masatlioglu and Ok (2005,2013); Tapki (2007); Dean et al. (2017); Sagi (2006); Apesteguia and Ballester (2009); Bossert and Sprumont (2007).
- **Unknown Reference:** Ok et al. (2015); Tserenjigmid (2019).
- **Stochastic Choice and Consideration:** Manzini and Mariotti (2014); Aguiar (2015), Horan (2019); Kovach and Sulmeynov (2023); Brady and Rehbeck (2016).

Thank You!