Bureaucratic Assignment

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Abstract

We explore the application of the matching mechanism to one-sided matching markets with a social planner having multiple objectives. Specifically, we analyze the cadre allocation mechanism in the All India Services (AIS) officers of the government of India. Thakur (2021) finds aggravated systemic imbalances across cadres with the 2008 cadre allocation policy. We propose a novel strategy-proof mechanism which achieves the objectives of the social planner in the context of the current (2017) policy, which is cognitively complex and is vulnerable to manipulation by the candidates. First, We develop a mechanism for national integration that would be strategy-proof and envy-free at the zonal level. In this process, we formalize the notion of national integration. A mechanism is strategy-proof when candidates can't benefit by lying about their preferences, and it is envy-free at the zonal level if no player wants the cadre of another lower-ranked player in the same zone. Next, we design another mechanism that achieves the desired distribution of merit while being strategy-proof. Finally, we propose to merge both mechanisms to achieve the twin objective of national integration and equal distribution of merit under strategy-proof conditions.

Keywords: Matching, market design

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1. Introduction

Each year, thousands of candidates appear for an entrance exam to become a civil servant in India. The selected students are allocated to different cadres (a state or a group of states and union territories in a few cases) through a centralized procedure. The stated goal of the procedure is to "preserve national unity and integrity and to provide uniform standards of administration throughout the country."[1]

Cadre allocation to selected candidates was among the ways through which the objectives of All India Services were to be achieved (Somanathan and Natarajan, 2022). Upon selection, selected candidates are then assigned one among the various cadres. The selected officers in the All India Services have to spend most of their service years in their assigned cadres. For this reason, obtaining a cadre of choice becomes important for candidates. The assignment of cadres to selected candidates, therefore, becomes crucial. However, assigning cadres to selected candidates is a challenging task as it entails a "preference-versus-performance tradeoff"(Thakur, 2021). On the one hand, the government aims to achieve national integration, unity, and equitable development across all cadres - the performance objectives. On the other hand, it must also maintain the motivation and satisfaction of its officers to prevent high rates of exit from the civil service - the preference objectives.[2]

The government has periodically revised its cadre allocation rules to reconcile these competing objectives. One such rule, implemented in 1984 and remaining in practice until 2007, involved randomly allocating outsiders to vacant positions within a cadre. In 2008 another allocation rule came into force based on serial dictatorship, in which the order was decided based on the rank obtained in the civil services examination.[3],[4] This rule was used until 2016. However, as Thakur (2021) finds, this rule aggravated systemic imbalances across cadres, thereby impacting the objective of national unity and integrity, as well as the equitable distribution of merit among cadres. Furthermore, it has been argued that this policy led to a loss of the All-India character of the civil service and a reduction of the service to a mere regional service.[5]

The government of India, vide its notification dated 05.09.2017, changed the cadre allocation policy which was in existence for All India Services employees (GoI, 2017). As mentioned above, before this new policy, the allocation of cadres was carried out using the serial dictator mechanism in which the highest-ranked

[1] Sinha (1990) mentions that the rationale of All India Services was that these were best suited for a federal system, uniform development of the nation as a whole, national integrity and efficiency in administration.

[2] According to a news portal, Indian Mandarin, 06 IAS officers resigned from service within one month.

[3] Union Public Service Commission conducts the Civil Services Examination to select candidates for various positions in group A (officer-level) services at the central level, including two All India Services, IAS and IPS.

[4] Year-wise cadre allocation policy documents are there on DoPT, CSE website

[5] Ministry Of Home Affairs vs Himanshu Kumar Verma in the High Court of Delhi(2019)

candidate was first assigned her top choice based on the available position, and then the second highest-ranked candidate received her top choice based on the remaining seat after the allocation to the higher ranked candidate has been made. The availability of information about the seat in each cadre was of no consequence, and candidates would reveal their true preferences generating a stable, strategy-proof matching. [6] The new cadre allocation policy is cognitively complex for candidates and vulnerable to manipulation. In this paper, we study how the new cadre allocation policy combined with the rules on the domicile can lead to unfair allocation. We propose an alternative mechanism that is strategy-proof and achieves the objectives of national integration and equitable distribution of merit. However, it should be noted that our mechanism may result in justified envy. Furthermore, we demonstrate that it is impossible for any mechanism to simultaneously achieve an absence of justified envy and an equitable distribution of merit.

2. The Cadre Assignment Problem

2.1. The New Cadre Policy

The government of India notified a new cadre policy for All India Services employees in 2017 (GoI, 2017). The new policy has divided all the caders into five zones. These zones are mutually exclusive and exhaustive. The candidates now have to choose the zone in descending order. Subsequently, they are to indicate their top preference of cadre within each zone, followed by their second preference, and so on, until preferences for all cadres within each zone have been indicated. To illustrate the implementation of this process, consider for simplicity that there are three zones, z_1, z_2, and z_3, and a candidate has a preference order of $z_2 - z_1 - z_3$ among zones, and within zones z_1, z_2 and z_3, the candidate has preferences of $z_1c_1 - z_1c_2 - z_1c_3$, $z_2c_2 - z_2c_1$, and $z_3c_3 - z_3c_2 - z_3c_1$ respectively. The resulting preference order for the candidate would be $z_2c_2 - z_1c_1 - z_3c_3 - z_2c_1 - z_1c_2 - z_3c_2 - z_1c_3 - z_3c_1$.

The cadre allocation authority then runs a serial dictatorship mechanism to make final allocations based on the expressed preferences of the candidates. This mechanism takes into consideration the rank obtained in the Civil Services Examination, with candidates of higher rank being given priority in the allocation process.

The new policy differs from the existing policy in the following ways:

1. The new cadre allocation process for All India Services employees has significantly changed the options available for candidates to indicate their preferences. Previously, candidates had a total of 26! possible combinations to choose from. However, the new process has significantly reduced this num-

[6] The rule included insider and outsider priorities within the state, which meant that the candidate had to select their domicile state as a top choice to be considered for posting in that state, which renders the mechanism not exactly strategy-proof. For *inter se* preferences among cadres apart from the home cadre, the rule indeed was strategy-proof.

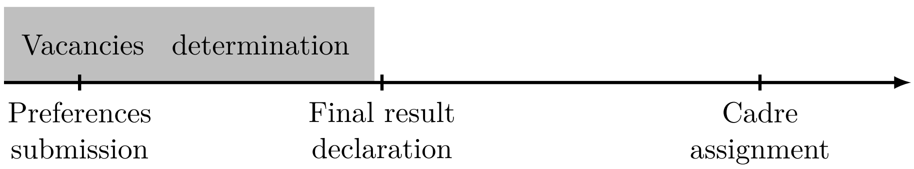
ber to 5!∗7!∗4!∗4!∗6!∗5!.[7] Although the new choice set is still substantial, it represents a reduction on the order of 10^{13} compared to the previous options.

2. The added restriction of having to indicate preferences among zones and then subsequently within zones further limits the flexibility of candidate preferences. This restriction dictates that certain preferences must come from specific zones. For example, the 6th, 11th, and 16th preferences must be from the same zone as the first preference. Similarly, the 7th, 12th, and 17th preferences must come from the zone of the second preference, and so on, making it more challenging for candidates to express their true preferences. This has also made the mechanism cognitively complex for candidates.
3. One of the most notable limitations is the inability to place preferences within the same zone consecutively. Individuals may often prefer cadres within their own zone over those in other zones, but this preference cannot be accurately reflected in the allocation process. This restriction of choice poses a significant challenge for candidates in expressing their true preferences.
4. The seemingly innocuous zoning measure introduced in the new policy creates opportunities for manipulation and undermines the integrity of the allocation process.[8]

The implicit reasons for the change in policy are to make the All-India Services live up to their ‘all-India’ mandate, to “ensure that merit is equally distributed among all the Cadres and minimizes the chances of creating a cluster of (merit or language) officers in a particular part of the country” and to emphasize national unity and integrity. [9]

Decision Timeline

Candidates provide their preferences before the result is announced without knowing the vacancies in each cadre beforehand. Vacancies in each cadre are also determined prior to the result. Following the final result, the mechanism assigns cadres to candidates based on their submitted preferences.



[7]Under the previous policy, candidates could arrange the 26 cadres in descending order of preference, resulting in 26! total possible combinations. However, the new policy, which introduces the concept of zoning, greatly reduces this number. Candidates can arrange the five zones in 5! ways; within each zone, the cadres can be arranged in $k!$ ways where k is the number of cadres in that zone. This results in a reduction in the choice set on the order of $26!/(5! * 7! * 4! * 4! * 6! * 5!) > 10^{13}$, which is a substantial decrease from the previous options.

[8]Proof of manipulation follows in a later section.

[9]Ministry Of Home Affairs vs Himanshu Kumar Verma in the High Court of Delhi(2019). The decision document can be accessed at latestlaws.

2.2. The Model

We will start with an example to make the points clearer. Consider four candidates, A, B, C, and D, ranked in ascending order and belonging to the states w, w, w and v, respectively. Furthermore, assume that the states w and x belong to zone Z_1 and that the states y and v belong to zone Z_2. These candidates have some preferences among cadres. Consider that the preference profiles of these candidates over the states are as A: w - x - y - v, B: w - x - y - v, C: w - x - y - v, and D: v - y - w - x.

The task of assigning cadres to these candidates is referred to as the cadre assignment problem. The recent policy change has altered the way candidates express their preferences. Specifically, candidates are now required to indicate their preferences among zones and then within each zone, and subsequently, their preferences are constructed as outlined in Section 2.1. Consider that the reported preferences submitted by these candidates are as follows: A: (Z1 - Z2) Z1: w - x, Z2: y - v; B: (Z1 - Z2) Z1: w - x, Z2: y - v; C: (Z1 - Z2) Z1: w - x, Z2: y - v; and D: (Z2 - Z1) Z1: w - x Z2: v - y. According to the new policy, their preference profile will be constructed as follows: A: w - y - x - v, B: w - y - x - v, C: w - y - x - v, and D: v - w - y - x. The new policy then runs a serial dictatorship mechanism on this constructed preference profile based on the ranks of candidates.

We will now outline the model formally. The cadre allocation problem consists of

1. A finite set of states $S = \{s_1, ..., s_k\}$
2. A finite set of candidates $I = \{i_1, i_2, ..., i_n\}$
   Candidates take an exam and get a score or a rank in the exam. Therefore, all states have the same preference for candidates. Without loss of generality, we can say that i_1 ranks better than i_2, i_2, in turn, ranks better than i_3, and so on.
3. Candidates have preferences over states which is denoted by $\succ_i$
4. New mechanism aggregate the sates into disjoint blocks $Z = \{Z_1, Z_2, ..., Z_m\}$, which form a partition of S.
5. Candidates submit two preferences $(Q^i, (P^{i,z})_{z\in Z})$, where Q^i denotes their preference ordering over Z and $(P^{i,z})_{z\in Z}$ denotes their ordering over states within block z.

The mechanism runs a serial dictatorship by constructing the candidates' preference $\widehat{\succ_i}$ as follows:

- The process begins by collecting the top entries of $(P^{i,z})$ for each $z \in Z$ and then organizing these states according to the order in which the corresponding blocks, to which they belong, are ranked.
- Then, it takes the second entries in $(P^{i,z})$ for each $z \in Z$ and rank these states similarly as the step above. These sets of states will have precedence below the states of the first step.
  If in any block there are no entries left, then the mechanism skips that block.
- The process follows until there are no states left.

For example, for a candidate, the mechanism will make the top state of the j^{th} ranked block the j^{th} preferred state while constructing the candidate's preferences.

Fact 1: *If the difference in the number of cadres between zones with the highest number of cadres and those with the second highest number of cadres is n, then the preference profile constructed by the mechanism will include a minimum of n and a maximum of n+1 consecutive cadres from the zone with the highest number of cadres, appearing at the end of the profile. No other consecutive cadres will be present in the profile apart from those at the end.*

A mechanism, σ, is a mapping $\sigma : (\widehat{\succ_i})_{i \in I} \to S^I$ so that no state has more than its capacity.

Definition 1: *A mechanism σ is strategy-proof if $\forall i, \widehat{\succ_{-i}}, \nexists s_1, s_2,$ s.t. $s_1 \succ_i s_2$ & $s_2 \widehat{\succ}_i s_1$, where $\widehat{\succ_i}$ is derived from candidate i's reported $(Q^i, (P^{i,z})_{z \in Z})$.*
*Note**: Do we need $\widehat{\succ_{-i}}$ in the above definition?*
A mechanism σ is strategy-proof at zonal level if $\forall i, \widehat{\succ_{-i}}, \nexists s_1, s_2, z,$ s.t. $s_1, s_2 \in z, s_1 \succ_i s_2$ & $s_2 \widehat{\succ}_i s_1$, where $\widehat{\succ_i}$ is derived from candidate i's reported $(Q^i, (P^{i,z})_{z \in Z})$. The concept of strategy-proof at the zonal level is stronger than the concept of strategy-proof.

From the above definition, it can be noticed that if a mechanism is strategy-proof, it is also strategy-proof at the zonal level. And if a mechanism is not strategy-proof at the zonal level, then it is not strategy-proof. [10]

Proposition 1: *The new mechanism is not strategy-proof.*

Proof. If we prove that the new mechanism is not strategy-proof at the zonal level, it is also proved that the mechanism is not strategy-proof.
Consider four candidates, A, B, C and D, having their ranks in ascending order (i.e. A being the top-ranked candidate) belonging to state w, w, w, and v, respectively. Furthermore, consider that states w,x belong to zone Z_1 and states y, v belongs to Z_2. All the states w, x, y and v have one seat each. Consider the preference $\succ$ profile of the candidate over the states as follows:

- A: w - x - y - v
- B: w - x - y - v
- C: w - x - y - v
- D: v - y - w - x

Consider again the reported preferences submitted by candidates as follows:

- A: (Z_1 - Z_2)
  Z_1: w - x
  Z_2: y - v
- B: (Z_1 - Z_2)

[10] There might arise a case where a mechanism is not strategy-proof, but it is strategy-proof at the zonal level.

Z_1: w - x
Z_2: y - v

- C: (Z_1 - Z_2)
  Z_1: w - x
  Z_2: y - v
- D: (Z_2 - Z_1)
  Z_1: w - x
  Z_2: v - y

The present mechanism creates the preference profile as follows:

- A: w - y - x - v
- B: w - y - x - v
- C: w - y - x - v
- D: v - w - y - x

The mechanism then runs the serial dictatorship and assigns w to A, y to B, x to C, and v to D. As can be seen from this, candidate B gets state y, her third preferred state (in her preference over state $\succ$), despite having a better rank than candidate C of the same zone, who got state x which is the second preference of candidate B.
Now consider that all else remains the same except candidate B altered her reported preference as follows:

B: (Z_1 - Z_2)
Z_1: x - w
Z_2: y - v

The mechanism then creates her preference profile as B: x - y - w - v. And, the resulting assignment will be w to A, x to B, v to C, and y to D. Therefore, we see that there exists $w, x \in Z_1, w \succ_B x$ & $x \widehat{\succ}_B w$ which violates the condition of strategy-proof. ■

If the candidates know their rank before giving their preferences, then it is possible for B to manipulate her choice. However, if the rank is not known and one has to give her preference *ex-ante*, then is it possible to manipulate?
We find that it is still possible to manipulate if the students want to minimize her chance of getting into a zone she doesn't like and maximize the chance into a zone she likes. Consider in the above example that all the rankings have equal probability for each candidate. Then while reporting true preferences, D will get v (since none of A, B, and C has v in their first three choices). Since the preference order of A, B, and C is the same and the probability of ranking is also equal, the result will also be symmetric for them. Now, each of A, B, and C will be assigned one of the states w, x, and y; they will get these states with probability 1/3. Now consider the case when B manipulates her preference (in a way as stated above), and all others stick to their true preference. Then the constructed preference order in the new mechanism would be

- A: w - y - x - v

- B: x - y - w - v
- C: w - y - x - v
- D: v - w - y - x

In this scenario, D is also guaranteed to receive state v. As for B, if her rank is 1, she will receive state x. If her rank is 2, she will still receive state x since only one of w or v will be taken. If her rank is 3, she will still receive state x since either w and y or w and v will be occupied. If her rank is 4, she will still receive state x, as all of w, y, and v will be occupied. Thus, in all possible cases, B will receive state x if she manipulates her preference while the other candidates state their preferences truthfully.

Definition 2: *A mechanism σ is envy-free if $\forall i \nexists j$ s.t. i ranks better than j and $\sigma(j) \succ_i \sigma(i)$.*
A mechanism σ is envy-free at the zonal level if $\forall i \in z \nexists j \in z$ s.t. i ranks better than j and $\sigma(j) \succ_i \sigma(i)$. The concept of being envy-free at the zonal level is stronger than being envy-free
A mechanism σ is envy-free at the state level if $\forall i \in s \nexists j \in s$ s.t. i ranks better than j and $\sigma(j) \succ_i \sigma(i)$. The concept of being envy-free at the state level is stronger than being envy-free at the zonal level and is strongest among these three concepts of envy-free.

As we can see from the definition of envy-free, if a mechanism σ is envy-free, then it is also envy-free at the zonal level and envy-free at the state level. It is also to be noticed that if a mechanism is not envy-free at the state level, it is not envy-free at the zonal level and not envy-free.

Proposition 2: *The new mechanism is not envy-free.*

Proof. If we can prove that the new mechanism is not envy-free at the state level, it is also confirmed that it is not envy-free. From the proof of Proposition 1, see that candidates B and C, where B has a better rank than C, belong to the same state w and have the same preference order, w - x - y - v. However, the mechanism constructs their preference as w - y- x - v. Given that B has a better rank than C, the new mechanism assigns B to state y and C to state x. And we see that B preferred x over y and yet was assigned to state y despite a better rank than C, who is assigned to state x. Thus, the new mechanism, which constructs the preference from using preference within zones and preference among zones overriding true preferences of the candidates, is not envy-free at the state level and hence not envy-free.

■

3. Mechanism for National Integration and Equal Distribution of Merit

In this section, we will first define a matrix and two functions which will be useful in formulating the mechanism.
Consider an m x m matrix $\mathcal{M}$ whose ij-th (i-th row and j-th column) entry is the number of candidates from zone Z_i getting allocation in Z_j. Then the sum of all the elements of the matrix, $\mathcal{M}$, gives the total number of selected candidates. Further, consider the following two functions defined as follows:

- Capacity function, $C(.) : 2^S \rightarrow \mathbb{N} \cup \{0\}$, is a function that, when given any subset of S as an argument, will give the capacity of that subset of S. In simple words, the capacity function describes the number of vacancies in a set of states. When a zone is given as an argument, this function tells how many vacancies are in that zone.
- Availability function, $A(.) : 2^S \rightarrow \mathbb{N} \cup \{0\}$, is a function which, when given any subset of S as an argument, will give the number of candidates selected from that subset. Put simply, the availability function tells about the number of candidates selected from a set of states. When given a zone as an argument, this function tells how many candidates are selected from that zone.

Then, the sum of elements of the i-th row of $\mathcal{M}$ will equal $A(Z_i)$ and the sum of elements of the j-th column of $\mathcal{M}$ will equal $C(Z_j)$.
Given $C(Z_i)$ and $A(Z_i)$ for each $i \in \{1, 2, ..., m\}$, we can construct a matrix $\mathcal{M}$. We add one row and one column to the last to show the capacity function, C (.) and the availability function, A (.), respectively. We call this matrix $\mathcal{M}^+$.[11] The value of the last row and the last column of the matrix $\mathcal{M}^+$ works as the constraint for matrix $\mathcal{M}$. This means that once the capacity and availability of each zone are known, the entry of matrix $\mathcal{M}$ will be such that the sum of the elements of each column will add up to the capacity of that zone. Similarly, the sum of the elements of each row will add up to the availability of that zone. In the table below, labelled Allocation Table, we have shown one such $\mathcal{M}^+$ matrix. This is a 6x6 matrix which means that there are five zones. The last (i.e., 6th) row and the last column (i.e., 6th) in this matrix show the capacity and availability of each zone. The greyed cell, containing 3, is in the fourth row and the third column, indicating that three candidates from Zone-IV have been assigned to Zone-III. The red cell containing 37 indicates that Zone-II has a capacity of thirty-seven candidates, meaning there are thirty-seven vacancies in Zone-II. The green cell, containing 40, indicates that Zone II has forty available candidates, which is to say that forty candidates got selected from Zone-II. The last element, that is, the element in the sixth row and sixth column, of matrix $\mathcal{M}^+$ gives the total number

[11] The matrix $\mathcal{M}^+$ defined in this way will be of m+1 rows and m+1 columns and where the value of the entry at $(m+1)$-th row and $(m+1)$-th column is the total seats available. Apart from this, other elements of the last row will give the zone's capacity, whereas the entry of the last column will give the number of candidates available from that zone.

of selected candidates.

	Zone-I	Zone-II	Zone-III	Zone-IV	Zone-V	A(.)
Zone-I	9	10	9	9	8	45
Zone-II	9	9	7	7	8	40
Zone-III	8	7	8	5	7	35
Zone-IV	3	4	3	8	2	20
Zone-V	8	7	9	6	10	40
C(.)	37	37	36	35	35	Total = 180

Table 1: Allocation Table

3.1. Mechanism for National Integration

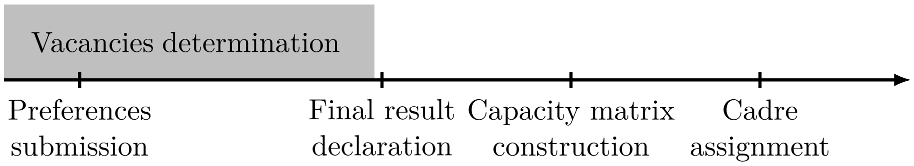
National integration refers to the distribution of candidates from each zone to all other zones. As can be enunciated from the policy and court judgments document, the goal is to have a proportional mix of candidates in each zone, similar to the mix in other zones. After the results of the examination are declared, the planning authority gets to know the number of candidates selected from each state and thus from each zone. Vacancies in each state and therefore in each zone are also known to them in advance. Thus, the value of the last row and the last column of the matrix $\mathcal{M}^+$ is known. Once the availability and capacity of each zone are known, we can define a new mechanism that will produce the assignment with zonal constraints.

Definition 3: *A mechanism, $\bar{\sigma}$, is a mapping $\bar{\sigma} : ((\succ_i)_{i \in I} \times \mathcal{M}) \rightarrow S^I$, such that*

- ***i)*** *for each $a, b \in Z$, the number of candidates assigned from zone a to zone b is equal to $\mathcal{M}_{ab}$ (where $\mathcal{M}_{ab}$ denotes the entry of a^{th} row and b^{th} column of matrix $\mathcal{M}$),*
- ***ii)*** *no states get more than its capacity, and*
- ***iii)*** *serial dictatorship is followed for assignment and priority is decided on the basis of the ranking of candidates.*

Decision Timeline

Now, the decision timeline will change a little as the authority needs to construct an allocation matrix before the mechanism will rum to allocate the cadre. Candidates submit their preferences as in the existing mechanism before the result is out without prior knowledge of cadre vacancies. Vacancies in each cadre are determined before the result. Following the final result declaration, the authority constructs an allocation matrix, which requires knowledge of the candidates selected from each cadre. The mechanism utilizes the allocation matrix and candidates' preferences to allocate cadres.



Proposition 3: *$\bar{\sigma}$ respects the constraint of the matrix $\mathcal{M}$ and is strategy-proof.*

Proof. First, $\bar{\sigma}$ respects the constraint of matrix M, which follows from the definition of $\bar{\sigma}$. Now, we have to prove that $\bar{\sigma}$ is strategy-proof. If a mechanism is strategy-proof, it means that truth-telling is a weakly dominant strategy for candidates. That is to say, if a candidate manipulates her preference instead of giving her true preference, the resulting allocation will be worse or equal to what she would receive if she had provided her true preference. For the first-ranked candidate, the best strategy is to report truthfully, as by reporting truthfully, she will get the best among the available cadres. So the first-ranked candidate does not have any incentive to lie. The best strategy for the second-ranked candidate is to report truthfully, as she will also get the best among the remaining available seats. Therefore, she also does not have any incentive to lie. If all candidates before candidate A reported truthfully, then the best strategy for candidate A is also to report truthfully; otherwise, she may get the cadre below her preference than the cadre she would receive in truth-telling. Therefore, truth-telling is the best strategy for everyone. Thus, the mechanism is strategy-proof.
We see that if candidates know their ranking, truth-telling is the best strategy given the mechanism $\bar{\sigma}$. However, if candidates do not know their rankings, then also truth-telling is the best strategy. To see this, consider candidate B, who manipulates her preference by reporting s_j before s_i, whereas, in reality, she prefers s_i over s_j. The following can happen when her turn arrives:

Case 1: both s_i and s_j have the capacity,[12]

Case 2: only s_j has the capacity,

Case 3: only s_i has the capacity, and

Case 4: none of the states, s_i and s_j, has the capacity.

In case 1, by manipulating, she will get s_j, a worse outcome for her. In case 2, her outcome will not be better as s_i is already filled, and she will get s_j which she would receive if she reported truthfully. In case 3, she will receive s_i, which she would receive even when she had reported truthfully. In case 4, as none of s_i and s_j has the capacity, it would be of no better consequence to misreport s_j over s_i, so truthful reporting is equally good. Therefore, truthful reporting is a weakly dominant strategy in all possible cases. We conclude that truth-telling is a weakly dominating strategy in the scenario when the rank is not known. Thus, the mechanism $\bar{\sigma}$ is strategy-proof. ■

Proposition 4: *$\bar{\sigma}$ is envy-free at the zonal level.*[13]

[12] Here, state s_i has capacity means that state has the capacity and by filling which matrix $\mathcal{M}$ is not getting violated.

[13] The national integration policy demands that people from one zone go to other zones, and

Proof. The mechanism $\bar{\sigma}$ runs the serial dictatorship over the true preference of candidates over states. Suppose that candidates A and B belong to the same zone and that A has a better rank than B, and A envies the allocation of B, that is, $\bar{\sigma}(B) \succ_A \bar{\sigma}(A)$. Since A's turn come first, $\bar{\sigma}(B)$ still has the capacity; otherwise, candidate B will not get $\bar{\sigma}(B)$ as her turn comes later. Therefore, either A does not have a better rank than B, or A does not envy the allocation of B, which is a contradiction. Thus, the mechanism $\bar{\sigma}$ is envy-free. ■

The planning authority has information on the demand for candidates (capacity) in each zone and the number of candidates selected (availability) from each zone once the selection list is available. With this information, the planning authority can construct the matrix $\mathcal{M}^+$. Suppose the vacancies in Zone-I to Zone-V are 37, 37, 36, 35, and 35, respectively, and the number of candidates selected from each zone is 45, 40, 35, 30, and 40, respectively. These constraints can be used to construct a 6x6 matrix $\mathcal{M}^+$, as shown in the Allocation Table. This matrix is among many feasible matrices that could be constructed with these constraints..

Proposition 5: *The planner can achieve maximum feasible national integration.*[14],[15]

Proof. The proof of this proposition is trivial. From the selected candidate list, the planner has the availability of candidates, that is, $A(Z_i)$ for each zone. The planner also has every cadre's capacity (or requirement) sourced from the cadre, so each zone's capacity is available. Thus, the planner has $C(Z_i)$ and $A(Z_i)$ for each $z_i \in Z$. With the help of $C(Z_i)$ and $A(Z_i)$, the planner can construct the matrix $\mathcal{M}$ under the constraint of row sum and column sum. Many such matrices $\mathcal{M}$ can be constructed with these constraints. From all these matrices, the planner will choose one that would maximize national integration. ■

The planner can achieve the maximum feasible national integration. The mechanism sigma bar will produce an assignment that will be envy-free at the zonal level, and being strategy-proof, it also incentivises candidates to reveal their actual preference ordering. Now we claim that given the matrix $\mathcal{M}$ that the planner has chosen for the national integration, mechanism $\bar{\sigma}$ produces a matching(assignment of candidates to cadres) which is Pareto efficient.

Proposition 6: *For a given matrix $\mathcal{M}$, the mechanism $\bar{\sigma}$ is Pareto efficient.*

Proof. Suppose that for a given matrix $\mathcal{M}$, there exists a mechanism σ' that Pareto dominates $\bar{\sigma}$. Then,

i) the assignment produced by σ' will be at least as acceptable as

in each zone, the mix of people from all other zones should be there. Any mechanism ensuring these cannot be one without justified envy. At best, a mechanism that can achieve these will be one without justified envy at the zonal level, which our mechanism also does.

[14] We are not going into a discussion about what that maximum would be as this is a policy decision as to what mix would yield a maximum National Integration and would be best left for the policymakers to decide.

[15] We say the feasible national integration because the planner can only maximize the NI in the limit of $C(Z_i)$ and $A(Z_i)$ as these are given to planners. According to the policy, $C(Z_i)$ is provided by each state and $A(Z_i)$ is based on the result.

the assignment produced by $\bar{\sigma}$, and

ii) there exists at least one candidate for which the cadre she receives in σ' is strictly better than the cadre she receives in $\bar{\sigma}$.

Define a function $\mathcal{V}(.) : (Z_1 \times Z_2) \rightarrow \mathbb{N} \cup 0$, as the number of candidates from zone Z_1 who are assigned to a cadre in zone Z_2. This is the element at the matrix $\mathcal{M}$'s 1st row and 2nd column. Also, define $\mathcal{Z}(.) : (I \cup S) \rightarrow Z$ as

$$\mathcal{Z}(x) = \begin{cases} \text{zone from where candidate } x \text{ belongs }, & \text{if } x \text{ is a candidate,} \\ \text{zone in which cadre } x \text{ lies }, & \text{if } x \text{ is a cadre.} \end{cases}$$

Consider that the candidate a_1 is the highest ranking candidate who gets improvement in her assigned cadre under σ' compared to her assignment under $\bar{\sigma}$, i.e. $\sigma'(a_1) \succ_{a_1} \bar{\sigma}(a_1)$. Consider again $\mathcal{V}(\mathcal{Z}(a_1), \mathcal{Z}(\sigma'(a_1))) = n$. Then, there are $n-1$ other candidates from zone $\mathcal{Z}(a_1)$ who have been assigned to zone $\mathcal{Z}(\sigma'(a_1))$ by the mechanism σ'. Also, $\bar{\sigma}$ must have assigned n candidates from zone $\mathcal{Z}(a_1)$ to the zone $\mathcal{Z}(\sigma')$. By construction, a_1 is not among that n candidates as a_1 has improved on her assign under σ'. Also, a_1 has a lower rank than these n candidates otherwise a_1 would have got $\mathcal{Z}(\sigma'(a_1))$ under $\bar{\sigma}$ but a_1 has got an improvement under σ' and then received $\mathcal{Z}(\sigma'(a_1))$. So, there is a candidate, among the above n candidates (candidates of the zone $\mathcal{Z}(a_1)$ who have been assigned to the zone $\mathcal{Z}(\sigma'(a_1))$ under $\bar{\sigma}$), who has not been assigned to $\mathcal{Z}(\sigma'(a_1))$ under σ', calling this candidate a_2. a_2 also have a better rank and then a_1 and $\sigma'(a_2) \succeq_{a_2} \bar{\sigma}(a_2)$ but $\sigma'(a_2) \neq \bar{\sigma}(a_2)$ therefore, $\sigma'(a_2) \succ_{a_2} \bar{\sigma}(a_2)$ but this is a contradiction, as a_1 is the best-ranked candidate who received an improvement.

■

3.2. Mechanism for Equal Distribution of Merit

One of the objectives of cadre management is to ensure an equitable balance of quality across cadres (Thakur, 2019). The sentiment that the meritorious candidate should be appropriately distributed across cadres to the extent possible is also preferred by the Government of India in *Ministry Of Home Affairs vs Himanshu Kumar Verma in the High Court of Delhi(2019)*. In this paper, we call this policy objective an equal distribution of merit among cadres.

In practice, equal distribution of merit cannot be achieved, and what can instead be done is to minimize the uneven distribution of merit, proxied by rank. There can be many ways through which planners can achieve this. This can be done by assigning the candidates to the cadres in a way that will lead to the average rank of receiving candidates in each cadre being equal [16]. However, this will not always be feasible, so to reduce inequality, the planner will minimize the maximum difference between the average rank of any two cadres. The solution to this will be unique up to the point that any two cadres can exchange the same number of candidates having the same rank-sums (which is to say that the number of candidates between the two exchanging cadres will be the same and the sum of the ranks of

[16] Here, only those cadres are considered which have a vacancy this year

the exchanging candidates of each of these two will equal). In this case, there will be justified envy, as the solution will dictate the assignment, and the preference of candidates does not have much significance. Preference can only be used when an exchange is possible. Furthermore, the national integration achieved through this assignment will also depend on the solution.

Another way through which unevenness of merit can be reduced is by selecting the first-k candidates, in order of their ranks, and assigning them to all the cadres based on their preferences so that each cadre will get a proportional number of accepting candidates.[17] In the next round, k+1st to 2kth candidate will be selected. They will again be assigned proportionately to the accepting caders. This procedure will be followed until there are no candidates left. We call this mechanism σ_k which runs as follows:

Step 1: Given a k by the planner, the mechanism quantifies the quota for each cadre for every round based on a proportional system. [18]

Step 2.1: The first batch of candidates is selected and assigned to the cadres according to the quota of each cadre in the first round. The assignment will be followed one by one based on the serial dictatorship of these candidates. The mechanism will notify the last candidate for whom the assignment is done

Step 2.2: Here, the mechanism will follow the same assignment procedure as in Step 2.1 for the 2nd batch of candidates.

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Step 2.Last: The mechanism will assign the final remaining candidates to the cadre following the same assignment procedure as step 2.1 and terminates.

Proposition 7: *The mechanism σ_k is strategy-proof.*

Proof. Suppose candidate B knows her rank, m, and the value of k that the planner has decided. Now consider that candidate B manipulates her preference by reporting s_j before s_i, while in reality, she prefers s_i over s_j. It must be that $(n-1)k < m <= n * k$ for some $n >= 1$. Then the assignment for B will be considered in the nth round when the candidate from $(n-1)k+1$ to $n * k$ will be considered for assignment. The mechanism will follow the serial dictatorship as it has done in all the previous rounds. The following can happen when B's turn arrives:

1. both s_i and s_j have capacity [19]
2. only s_j has the capacity

[17] In case the proportional number that comes is not an integer, then we will take the nearest integer, and we adjust the same in the next round.

[18] How the quota for each round for each cadre is fixed is explained in Appendix 4, along with an example.

[19] Here, state s_i has capacity means that state has the capacity and by filling which the quota decided for that round is not getting violated.

3. only s_i has the capacity
4. none of the states, s_i and s_j, has the capacity.

By manipulating, she may get the state s_j (a worse outcome which will occur in cases 1 and 2). In contrast, stating the truth, she will get s_i if there is capacity in that. In case 4 above, the same things will repeat for the other two pairs of states until B gets some allocation. Therefore, truth-telling is a weakly dominating strategy when the rank is known and the value of k is known. Mechanism σ_k is ex post strategy-proof. Since the mechanism is strategy-proof ex post, this will also be strategy-proof ex ante. ■

Proposition 8: *The mechanism σ_k will have justified envy at the state level.*

Proof. Suppose that there are eight vacancies, two in each state w,x,y, and v, and eight candidates are selected for these. Of these eight candidates A, B and C are three candidates who have secured 1st, 3rd and 5th rank, respectively. Consider the preference profile $\succ$ of these three candidates (A, B, and C) over the states w, x, y, and v as follows:

- A: w - x - y - v
- B: w - x - y - v
- C: w - x - y - v

For equal distribution of merit, authority implements σ_4 mechanism. This mechanism will assign w to A and C, whereas B will be assigned some state other than w. Even though B (ranked 3rd)has a better rank than C (ranked 5th) $\sigma_4(C) \succ_B \sigma_4(B)$. That is, the mechanism σ_4 is not envy-free. Since we have not assumed anything about the candidates' states, B and C can very well be from the same states, making the mechanism with justified envy at the state level. ■

There will be some rare cases where there is no justified envy. Consider set $\mathcal{A}$ be the set of assignments which will be justified-envy free. These knife-edge cases will be those assignments when the σ_k mechanism leads to an assignment from the set $\mathcal{A}$. In the example above example (discussed in the proof above), if the choice of candidates from rank one to rank eight will be as under

- Rank 1: w - x - y - v
- Rank 2: x - w - y - v
- Rank 3: y - x - w - v
- Rank 4: v - x - y - w
- Rank 5: w - x - y - v
- Rank 6: x - w - y - v
- Rank 7: y - x - w - v
- Rank 8: v - x - y - w

Now, the mechanism σ_4 will assign w to the first and fifth ranked candidates, x to the second and sixth ranked candidates, y to rank the third and seventh ranked candidates, and v to rank the fourth and eighth ranked candidates. In this case, there will not be any justified envy.

3.3. Mechanism for Twin Policy Objectives: National Integration and Equal Distribution of Merit

After separately considering the two policy objectives of the recent cadre allocation policy, national integration and equal distribution of merit, we will consider the two objectives together. We appropriately modify the mechanism $\bar{\sigma}$ to accommodate the need for equal distribution of merit along with national integration. We call this new mechanism $\bar{\sigma_k}$. The mechanism $\bar{\sigma_k}$ runs as follows:

Step 1: Using $C(Z_i)$ and $A(Z_i)$ for each $i \in \{1, 2, ..., m\}$, the planner constructs an $m \times m$ matrix $\mathcal{M}$ whose ij-th (i-th row and j-th column) entry is the number of candidates from zone Z_i receiving allocation in Z_j. The planner can construct this matrix to achieve the maximum feasible national integration.[20]

Step 2: Given a k by the planner, the mechanism quantifies the quota for each cadre for every round based on a proportional system.

Step 3.1: The first batch of candidates is selected and assigned to the cadres based on the quota of the first round. The assignment will be followed one by one based on the serial dictatorship of these k candidates. After the initial assignment for each candidate, it will be checked if the constraint of matrix $\mathcal{M}$ is respected or not. If the constraint is fulfilled, then the assignment will be final; otherwise, the next choice of the same candidate will be considered till the matrix $\mathcal{M}$ is respected, and then the assignment is final.

Step 3.2: Here, the mechanism will follow the same assignment procedure as in Step 3.1 for the second batch of candidates. [21]

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Step 3.Last: The mechanism will assign the final remaining candidates to the cadre following the same assignment procedure as in Step 3.1 and ends.

The mechanism $\bar{\sigma_k}$ can achieve desirable national integration (under the feasibility constraint dictated by $C(Z_i)$ and $A(Z_i)$) and reduce the unequal distribution of merit. Since the steps that mechanism $\bar{\sigma_k}$ follows are the same as that of σ_k but with one added constraint of matrix $\mathcal{M}$, the mechanism $\bar{\sigma_k}$ will also have justified envy as σ_k. And only in the rare knife-edge cases there will not be any justified envy, but that would be due to the preference ordering of candidates but not because of the mechanism. However, the mechanism $\bar{\sigma_k}$ is strategy-proof.

Proposition 9: *The mechanism $\bar{\sigma_k}$ is strategy-proof.*

Proof. The proof of this proposition is the same as the proof of *proposition 7*. ■

[20] $C(Z_i)$ and $A(Z_i)$ are defined in section 3

[21] How the quota for each round for each cadre is fixed is explained in section 4Appendix, along with an example.

The mechanism $\bar{\sigma_k}$ achieves desirable national integration and minimizes the unequal distribution of merit while being strategy-proof. However, the mechanism leads to assignment having justified envy. Can we do better? That is, can we have a mechanism that will meet both the objectives of national integration and equal distribution of merit and, at the same time, be strategy-proof and not have justified envy? But we have not been able to find a mechanism which will both minimizes the unequal distribution of merit and be justified envy-free. We claim that no such mechanism is feasible which will attain both the equal distribution of merit (or minimizes the unequal distribution of merit) and be justified envy-free.

Proposition 10: *There does not exist a justified envy-free mechanism which leads to equal distribution of merit.*

Proof. Consider a mechanism σ_e, which is an envy-free one, and that also minimises unequal distribution of merit. Now consider there are m cadres represented by $s_1, s_2....s_m$ having n vacancies each. The mechanism σ_e will assign selected m∗n candidates among these m cadres so that there won't be justified envy as well as it will minimise the unequal distribution of merit. Suppose each of the m∗n candidates has the same preference order, which is $s_1 \succ s_2 \succ ...... \succ s_n$. Now to be an envy-free mechanism, the first n ranked candidates will be assigned s_1 the next n ranked candidates (i.e. candidates ranked from n+1 to 2n) will be assigned s_2, and following the same way, all the candidates are assigned so that the last n ranked candidates (i.e. candidates ranked from (m-1)n+1 to m∗n) will be assigned s_n. This is the only assignment where there won't be justified envy. Any other assignment other than the one described above will have justified envy. But this is also the assignment with the most unequal distribution of merit possible. Therefore, we see that the mechanism σ_e does not meet the criteria of being justified envy-free and minimising the unequal distribution of merit. Since the choice of σ_e is arbitrary, we can say that no such mechanism exists with both the property.

2nd Proof. Consider a mechanism σ_f, which minimises unequal distribution of merit and is justified-envy free. The mechanism has assigned the cadres to candidates in a way that minimises the unequal distribution of merit and is justified envy-free. Now, suppose there are two candidates, m and n, where m has a better ranking than n. Then the cadre allocated to n will be available to m since there is no justified envy. Now, say m changes his preference such that $\sigma_f(n) \succ_m \sigma_f(m)$. Then in this new preference scenario to minimise the unequal distribution of merit allocation will be the same as when m has not changed its preference i.e. $\sigma_f(m) \succ_m \sigma_f(n)$. This is to say m will be assigned $\sigma_f(m)$ and n will be assigned $\sigma_f(n)$. But in the changed preference scenario, we have $\sigma_f(n) \succ_m \sigma_f(m)$. That is, there is justified envy. But if we change the assignment to such that m will be assigned $\sigma_f(n)$ and n will be assigned $\sigma_f(m)$, then we are losing in minimisation of unequal distribution of merit.

■

4. Conclusion

The Government of India aimed to change the cadre allocation policies based on the following two key aspects:

- National Integration (NI): Ensuring a diverse mix of candidates from all zones within each zone, promoting national unity.
- Equal Distribution of Merit (EDM): Ensuring an even distribution of merit across different cadres.

Both the points above concern the design of policy cadre allocation to selected candidates. In pursuance of these, the government changed the existing cadre policy in 2017 and brought a new policy. However, as shown above, the new policy also falls short of meeting the desired objectives. We propose a one-sided mechanism that can achieve the objective of the government precisely by solving the issues arising out of earlier cadre allocation policy. This novel mechanism has a one-sided preference and is directly relevant to policy in the Indian context, where only candidates have preferences over cadres, and cadres do not have any preferences over candidates.

The civil services across the Indian sub-continent, spanning Bangladesh, India, and Pakistan, follow a consistent pattern: higher-level bureaucrats are chosen by the federal government and subsequently allocated to prominent administrative positions within provinces. Our proposed mechanism offers valuable applicability in these contexts. Moreover, it holds potential for utilization in organizational settings where individuals' preference is based on their career aspirations. On the other hand, organizations want to balance merit and diversity.

The implications of our model extend significantly to candidates as well. In the current policy framework, candidates face the arduous task of meticulously filling out their cadre preference form to secure a more desirable cadre. They contemplate their anticipated ranks and vacancies across various cadres. In contrast, our suggested strategy-proof mechanism releases them from this complex burden. Expressing their genuine preference order would be in their best interest. This mechanism ensures the authenticity of generated cadre preference data, which can subsequently guide the evaluation of less preferred cadres. By designing incentives, the government can enhance the appeal of these cadres, fostering a more balanced distribution.

The Indian constitutional framework provides affirmative action, a vital consideration within this context. Sönmez and Yenmez (2022) has proposed a novel mechanism for facilitating the proper execution of affirmative action policies. Conjointly, this mechanism can seamlessly be integrated with the framework proposed by Sönmez and Yenmez (2022) to implement the cadre allocation with the intended objective while meeting the mandates of affirmative action.

We propose two mechanisms and integrate them to achieve the desired result. Furthermore, it can be evaluated if the proposed mechanism can achieve EDM through NI only; then, a mechanism with NI will be sufficient. As our solution is based on the serial dictatorship mechanism, it is Pareto optimal and comparable to any strategy-proof mechanism that eliminates justified envy. The efficacy of our novel mechanism can be validated through simulation, comparing it with the current government-used mechanism. Thakur (2021) analyzes the issues emerging due to the 2008 allocation policy and suggests a scope of market design approach with a two-sided market assumption to solve them. We propose a one-sided mechanism that can achieve the objective of the government precisely by solving the issues arising out of earlier cadre allocation policy. This novel mechanism has a one-sided preference and is directly relevant to policy in the Indian context, where only candidates have preferences over cadres, and cadres do not have any preferences over candidates. This also offers a potential way through which the government can achieve its desired level of NI (in terms of allocating senior bureaucrats from one part of the country to different parts of the country), given the constraints of vacancy and selection from each cadre.

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Appendix: Allocation of seats to each cadre

This appendix explains how the seats are allocated to each cadre, given k, the number of seats to be considered in each round for equal distribution of merit.

Seat allocation method

For a given number of seats to be considered in each round k and capacity $C(S)$, there will be a maximum of $\lceil C(S)/k \rceil$ rounds.[22] Define a Quota function, $Q_i(s) : S \to \mathbb{R}_+$, which determines the number of seats to be assigned to cadre s in round i, where i can take integer value up to $\lceil C(S)/k \rceil$.

$$Q_i(s) = \begin{cases} 0.5 & i = 0 \\ \{Q_{i-1}(s)\} + C(s) * k/C(S) & 1 <= i <= [C(S)/k] \\ C(s) - \sum_{j=0}^{[C(S)/k]} [Q_j(s)] & i > [C(S)/k] \end{cases}$$

where $\{Q_i(s)\}$ is the fractional part of $Q_i(s)$, $C()$, is the capacity function, and k is given by the authority, which denotes the number of seats to be considered for each round.[23] Further, $[C(S)/k]$ denotes an integer part of $C(S)/k$. For each round i, the total number of seats that would be allocated to a cadre s will be $[Q_i(s)]$ and the total number of seats that would be considered for allocation to all the cadres will be $seat_i = \sum_{s \in S} [Q_i(s)]$.

An illustrative example

Consider six cadres $s_1, s_2, s_3, s_4, s_5, s_6$ with capacities 6, 4, 5, 5, 4 and 1, respectively. There is a total of 25 vacancies. For equal distribution of merit, let's say the authority decided to allocate four seats in each round, i.e. $k = 4$. Table below explain the allocations in each round. In round 1, $Q_1(s_1) = Q_0(s_1) + C(s_1) * k/C(S)$ = 0.5+6*4/25 = 1.46. Therefore, seat allocated to s_1 in round 1 is $[Q_1(s_1)] = 1$. Similarly, in round 3, $Q_3(s_3) = \{Q_2(s_3)\} + C(s_3) * k/C(S) = 0.1 + 5 * 4/25 = 0.9$ and consequently seat allocated to s_3 in that round is 0.

[22] $\lceil . \rceil$ denotes the ceiling function.

[23] This may so happen that number of seats considered in each round will differ from k and that will depend on this mechanism

		s_1	s_2	s_3	s_4	s_5	s_6	S	Rank considered
Round	Capacity ->	6	4	5	5	4	1	25	
	$Q_0(s)$	0.5	0.5	0.5	0.5	0.5	0.5		
Round 1	$Q_1(s)$	1.46	1.14	1.3	1.3	1.14	0.66		
	$seat_1(s)$	1	1	1	1	1	0	5	1 to 5
Round 2	$Q_2(s)$	1.42	0.78	1.1	1.1	0.78	0.82		
	$seat_2(s)$	1	0	1	1	0	0	3	6 to 8
Round 3	$Q_3(s)$	1.38	1.42	0.9	0.9	1.42	0.98		
	$seat_3(s)$	1	1	0	0	1	0	3	9 to 11
Round 4	$Q_4(s)$	1.34	1.06	1.7	1.7	1.06	1.14		
	$seat_4(s)$	1	1	1	1	1	1	6	12 to 17
Round 5	$Q_5(s)$	1.3	0.7	1.5	1.5	0.7	0.3		
	$seat_5(s)$	1	0	1	1	0	0	3	17 to 20
Round 6	$Q_6(s)$	1.26	1.34	1.3	1.3	1.34	0.46		
	$seat_6(s)$	1	1	1	1	1	0	5	21 to 25
Round 7	$Q_7(s)$	0	0	0	0	0	0		
	$seat_7(s)$	0	0	0	0	0	0	0	none

Table 2