# Rivalrous Competition and Macroeconomic Volatility

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**Preliminary draft. Please do not circulate and or quote.**

**Abstract**

This paper presents a model in which firms make decisions on the intensive and extensive margin on a production network. Firms choose not only on prices and quantities but also their input sellers. Prices emerge from distributed local interactions between firms in segmented markets. The model generates three empirically observed features of real world economies: perennial flux at the level of individual firms, stability in certain distributional attributes of the population of firms, and sizeable fluctuation in aggregate output. More specifically, the rate of decay of aggregate volatility, with an increase in the size of the economy, is sufficiently slow to generate the empirically observed aggregate volatility in an economy as large as the United States. The model generates firm size distribution and degree distribution of the production network with significantly fatter tails than the Gaussian. The model also produces the empirically observed mean growth rate of firm sizes along with a thick right tail. In short, our model is capable of endogenously generating the empirically observed firm volatility and aggregate volatility, along with some of the intervening meso structures. Note that within our setting, distributional stability does not entail microeconomic fixity rather it emerges from incessant microeconomic change. The number of incoming and outgoing links of individual firms change but the degree distributions remains largely stable. Similarly, the sizes of individual firms vary over time but the size distribution of firms remains relatively stable. Note that these features emerge in a system <u>without</u> productivity shocks. The root cause of the dynamics is the continual injection of microeconomic disturbance because of rivalrous competition between firms.

**JEL Codes** E30, C67, D57.

**Key Words** Production Network, Aggregate Volatility, Non-equilibrium Dynamics.

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# 1 Introduction

One of the central problems of macroeconomic theory is understanding the relation between micro interactions and the time dynamics of aggregate variables. Empirical evidence suggests that the micro economy is in perennial flux. In the United States, for instance, every year tens of thousands of new firms are born and similar numbers perish[1]. The entry and exit of firms does not however capture the totality of changes within the economic system. Firms that remain within the production network too change by forming relations with new sellers of inputs and new buyers of output. The production network of modern economies is a living pulsating entity that exhibits change in its constituent parts and the relations between parts. Few economists have related these micro economic dynamics to the fluctuations in macro variables. In this paper, we develop a model in which rivalrous competition between firms generates certain structural properties of the micro economy and the observed volatility in aggregate output.

We work with a variant of Gualdi and Mandel's (2016) model of a network economy, which generalizes Acemoglu et al. (2012) model by allowing for firm entry-exit and endogenous formation of buyer-seller relations. Firms are related to each other as buyers and sellers of intermediate inputs. They produce using Cobb Douglas production functions. Firms set prices through a non-tatonnement process of local interactions between buyers and sellers in multiple markets for intermediate goods. A representative household consumes using a Cobb Douglas utility function and supplies labor to all firms. Firms also make decisions on the extensive margin by replacing existing input sellers with lower cost suppliers. Firms, therefore, form new buyer-seller relations and sever existing relations.

While each firm's decisions on the intensive and extensive margin are based on private cost calculus, these decisions influence variables well beyond their locale. When a firm adds a new seller of input and severs relation with an existing seller, the decision propagates downstream as supply shocks and upstream as demand shocks. More specifically, the firm that replaces an existing supplier with a lower cost alternative increases production and decreases the price of its output. Firms who use this output as an input into their own production processes experience a positive supply shock. This is not all. The severing of one link and adding of another generates a negative

[1]See Phillips and Kirchhoff (1989), Foster, Haltiwanger and Syverson (2008), Coad (2009), and Axtell, Guerrero and Lopez (2019) for evidence on firm birth, growth, and death.

demand shock for the former supplier and a positive demand shock for the new supplier. These demand and supply shocks propagate via buyer-seller relations generating a sequence of relative price changes that ripple through the production network. Each relative price change alters the private calculus of firms on the intensive and extensive margin. All of which means that one firm's decision to form new production relations influences other firms' decisions to do the same. Within our model, these intertwined processes are capable of generating the death of firms. We assume that a firm dies when it loses all its buyers. New firms enter the economy so as to support a stable population.

Note the *rivalrous* nature of the afore noted process of competition between firms. When a firm forges a relation with a new input-seller, it increases the flow of money to the input-seller, which the input-seller uses to purchase more of its own inputs. The only way to purchase more inputs is to outcompete existing users of those inputs in the markets for those intermediate goods. These 'outcompeted firms' must contract their outputs, which are inputs into the production processes of other firms. This is however not the only way in which the shock of from the formation of a new buyer-seller relation travels upstream. Within our setting, a firm form a new relation by severing an existing relation. Therefore, when one firm finds a new seller, another firm loses an existing buyer. The firm who lost a buyer will shrink in size, contract production, and thereby leave more of the goods it uses as inputs for competing users of the inputs, who will in turn expand production. The shock from the formation of a new buyer-seller relation also travels downstream. The firm which finds a new lower cost seller expands production, for which it demands more inputs. The suppliers of these inputs meet the increase in demand by increasing production and by reallocating its output from the firms that have not increased demand to the firm which has. The decision of one firm to change a production relation, therefore, generates long chains of reallocation of real resources across the production network. The reallocation benefits some firms and hurts others. Some firms grow, others shrink, and yet others die. The exact direction and magnitude of the reallocation of resources at each time step depends on the state of each firm and the topology of the production network.

The afore noted microeconomic dynamics of our model are capable of generating sizeable fluctuations in aggregate output. The rate of decay of aggregate volatility, with an increase in the size of the economy, is sufficiently slow to generate the empirically observed aggregate volatility in an economy as large as the United

States. The model also generates firm size distribution and degree distribution of the production network with significantly fatter tails than the Gaussian. Furthermore, the model generates the empirically observed mean growth rate of firm sizes along with a thick right tail. In short, our model is capable of endogenously generating the empirically observed firm volatility and aggregate volatility, along with some of the intervening meso structures. Note that within our setting, distributional stability does not entail microeconomic fixity rather it emerges from incessant microeconomic change. The number of incoming and outgoing links of individual firms change but the degree distribution of the production network remains stable. Similarly, the sizes of individual firms vary over time but the size distribution of firms remains stable.

The ability of our model to endogenously generate some of the empirically observed micro, meso, and macro properties of real world economies depends crucially on firms making decisions on the extensive margin. To study firm decisions on the extensive margin, we have had to develop a model in which the adjacency matrix that defines the production network changes over discrete time steps. Network models with a dynamic adjacency matrix have so far proved analytically intractable[2]. One cause of the intractability is that the changes in adjacency matrix forbids us from representing the evolution of model variables as linear transformations from one time step to another[3]. We therefore solve the model using agent-computing, i.e. we run computational experiments on a synthetic economy *in silico* and analyze the data generated by these experiments (Axtell, 2000). Such discrete and scalable systems are particularly useful to study non-equilibrium dynamics that emerge from bottom-up interactions (Arthur, 2010).

## 1.1 Related literature

Most economists think of macroeconomic fluctuations as comparative statics of perfectly coordinated systems (Bausor, 1986). Within this analytical paradigm, the central problem is that of amplifying the influence of the distribution of firm productivity on aggregate output so that each new draw of firm productivity relates to a sufficiently different level of aggregate output (Klette and Kortum, 2004). An amplification mechanism becomes necessary because the sum of independent idiosyncratic productivity

[2]Schelling (1971) model of segregation and Elliott, Golub and Jackson's (2014) model of financial contagion belong to this class of network models.

[3]See Mandel and Veetil (2021) for some analytical results of a variant of this model with a fixed adjacency matrix.

shocks decays too fast to generate sizeable aggregate volatility in large economies (Lucas, 1977). Early work in this direction emphasized the production network itself (Acemoglu et al., 2012), while later work noted the role of attributes like input-specificity and price rigidity which in conjunction with the production network can amplify productivity shocks[4]. Within a network economy, the level of output is not a simple sum of firm productivity but involves a transformation of each firm's productivity defined by the adjacency matrix of the network. In so far as the degree distribution is sufficiently skewed, the transformation effected by the adjacency matrix generates sizeable difference in the aggregate output associated with different distributions of firm productivity. One could then interpret idiosyncratic productivity shocks as generating sizeable aggregate volatility as measured by the difference in equilibrium output consistent with each draw of firm productivity. This is what may be called 'equilibrium aggregate volatility'. The nature of aggregate volatility generated by our system is profoundly different from the afore noted aggregate volatility exhibited by equilibrium systems with idiosyncratic productivity shocks. Unlike in equilibrium network models, the macro fluctuations generated by our model reflects microeconomic miscoordination. The miscoordination emerges from the propagation of disturbances that arise from the finite sized firms making new decisions on the extensive margin well before the system could adapt to old entry-exit decisions by making numerous adjustments on the intensive margin.

Our model builds on the firm entry-exit literature a la Hopenhayn (1992). This literature has been extended by Bilbiie, Ghironi and Melitz (2012), Rocha and Pujolas (2011), and Carvalho (2019) to study the macroeconomic consequences of firm entry-exist. Their work, however, does not consider the buyer-seller relations between firms. From this point of view, our paper is closely related to Baqaee (2018) and Taschereau-Dumouchel (2020), who show that firm entry-exit amplifies idiosyncratic productivity shocks with a network setting. This paper differs from Baqaee (2018) and Taschereau-Dumouchel (2020) in that we show firm entry-exit can generate sizeable macroeconomic fluctuation even in the absence of productivity shocks. In this sense, our paper echoes the subtitle of Bonart, Jean-Philippe and Augustin (2014): "aggregate volatility without idiosyncratic shocks". Our model is a thematic cousin of the models developed by Axtell (1999) and Richiardi (2006) to study labor dynamics.

[4]See, for instance, Barrot and Sauvagnat (2016), Pasten, Schoenle and Weber (2018), Bigio and La'o (2020), Acemoglu and Tahbaz-Salehi (2020), Elliott, Golub and Leduc (2022), Miranda-Pinto, Silva and Young (2023), and Baqaee and Rubbo (2022).

All three models view mesoscopic order as emerging from incessant micro economic change within systems that exhibit non-equilibrium dynamics. Overall, this paper is a part of the small but growing group of models that explain macroeconomic fluctuations as emerging endogenously from market interactions (Dessertaine et al., 2022).

## 1.2 Organization of the paper

Section 2 computes equilibrium aggregate volatility using a data set with more than 600,000 buyer-seller relations between more than 150,000 US entities. Our estimates suggest that equilibrium aggregate volatility is an order of magnitude lower than the empirically observed magnitude, thereby motivating a different approach to viewing macroeconomic fluctuations. Section 3 presents the model. We enumerate the sequence of interactions between finite sized firms in discrete time. Section 4 compares equilibrium aggregate volatility with disequilibrium and non-equilibrium volatility generated by our dynamic production network model. Our computations show that disequilibrium volatility is two orders of magnitude lower than the empirically observed aggregate volatility. Non-equilibrium dynamics, with firm entry-exist and without idiosyncratic shocks, is capable of generating the empirically observed aggregate volatility for certain ranges of parameter values. Section 5 presents the micro distributional properties that emerge from our model's dynamics. The degree distribution of the production network and the size distribution of firms that emerge from the model have fatter tails than the Gaussian. Furthermore, the model generates a firm volatility distribution with the empirically observed mean and a fat right tail. Section 6 concludes the paper. Model code is available at bitbucket.org/VipinVeetil/networkeconomy.

# 2 Computing equilibrium aggregate volatility

Empirical studies suggest that in the US economy firm volatility is on the order of 10% whereas aggregate volatility is on the order of 1% (Gabaix, 2011). According to the Central Limit Theorem, the rate of decay of aggregate volatility is given by $m^{0.5}$ with $m$ denoting the number of firms. The US economy has on the order of $10^6$ firms (Axtell, 2001). Therefore, according to the Central Limit Theorem, firm idiosyncratic productivity shocks generate an aggregate volatility of 0.01%, i.e. about a hundredth of the empirically observed magnitude. The rate of decay of aggregate volatility within a network economy, however, can be slower than the

CLT rate because of the interdependence between firms (Carvalho, 2014). More specifically, Acemoglu et al. (2012) show that the scaling of aggregate volatility in a network economy is given by $m^{\frac{1-\alpha}{\alpha}}$, where $\alpha$ is the exponent of the powerlaw which characterizes the degree distribution of the production network. This result is derived using an equilibrium model of the network economy, and therefore we shall refer to $m^{\frac{1-\alpha}{\alpha}}$ as the 'equilibrium rate of decay' of aggregate volatility with an increase in the number of firms.

The empirical significance of the result depends on the exact value of the powerlaw exponent of the degree distribution of the production network. For instance, at $\alpha = 1.2$ the equilibrium network amplification mechanism accounts of nearly all of the empirically observed aggregate volatility, whereas at $\alpha = 1.5$ the mechanism accounts for about a tenth of it. More generally, the elasticity of aggregate volatility to powerlaw exponent is $\frac{1-\alpha}{\alpha}m^{(1-2\alpha)/\alpha}$. Despite the high sensitivity of the equilibrium production network mechanism to the powerlaw exponent, no one has so far measured the empirical significance of the mechanism using granular data. Acemoglu et al. (2012), Contreras (2014), Caliendo et al. (2018), and Pasten, Schoenle and Weber (2018) present empirical estimates based on linkage data from sectoral networks which contain on the order of $10^2$ entities. There is little reason to presume that the degree distribution at such an aggregate level retains properties of more disaggregate degree distribution. This is important. Any measure of the share of aggregate volatility accounted for by firm volatility must depend on reasonable estimates of the powerlaw exponent of firm buyer-seller network not sectoral network.

We measure the empirical significance of the aggregate volatility generated by firm shocks using a novel data set of buyer-seller relations between firms in the US economy. Our data set consists of $659,869$ relations between $164,195$ entities in the US economy. The data set comes from Standard and Poor's Capital IQ. It contains economic relations formed between the years 2005 and 2017 without information on the exact year in which the link was formed. The Capital IQ data set contains an order of magnitude more firms and linkages than the data on relations between publicly traded firms reported by Atalay et al. (2011) and three orders of magnitude more entities than the US Input-Output Table.

Economists before us have reported the degree distribution of the buyer-seller network between publicly listed firms in the US and near-universe of firms in Japan (Atalay et al., 2011; Konno, 2009). One of their empirical claims is that the degree distribution follows a powerlaw. Our estimates, using Maximum Likelihood, suggests that powerlaw is a better fit than exponential and lognormal for the out-degree distribution[5]. For the in-degree distribution, the powerlaw is a better fit than the exponential but worse than the lognormal[6]. Figure 1a presents the counter CDF of the degree distribution of the US network.

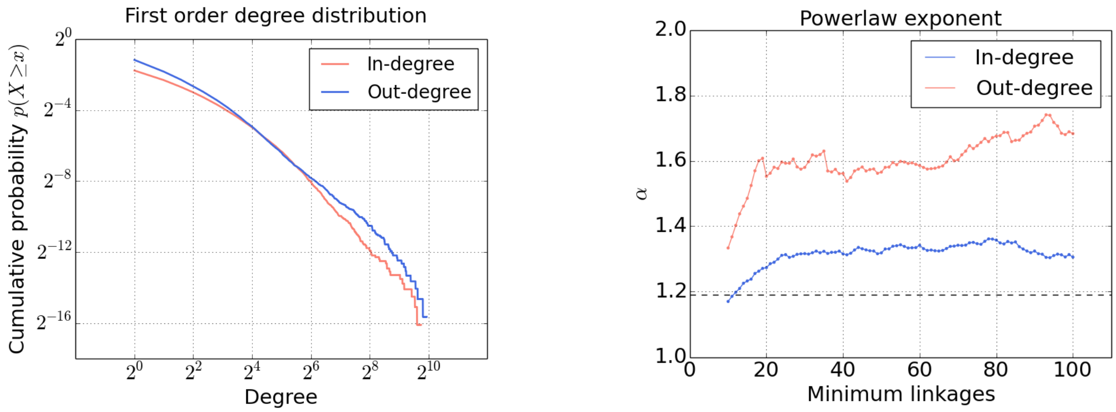
Powerlaw occurs in the tails. Naturally then, the size of tail chosen impacts the value of the powerlaw exponent (Chicheportiche and Bouchaud, 2012). Figure 1b presents estimates of the powerlaw exponent $\alpha$ for different sizes tails. The x-axis of Figure 1b marks the cutoff in terms of the minimum number of buyer-seller linkages for a firm to be included in the tail. The powerlaw exponent of the firm degree distribution within our data set differs sizeably from the exponent (approximately 1.4) derived from the IO table by Acemoglu et al. (2012)[7]. The dotted horizontal line in Figure 1b marks the powerlaw exponent $\alpha$ necessary for idiosyncratic productivity shocks to generate the empirically observed aggregate volatility within an equilibrium setting. More specifically, with firm volatility of 0.1 and aggregate volatility of 0.01, in an economy with about $10^6$ firms, $\alpha$ must be about 1.2 to generate empirically observed aggregate volatility. The values of $\alpha$ within our data set are systematically greater than 1.2. Put differently, for a wide range of values of the in-degree exponent and for all values of the out-degree exponent, the equilibrium model generates significantly lower aggregate volatility than the empirically observed magnitude[8].

[5] The likelihood ratio test favors powerlaw over exponential with likelihood ratio $R = 13$ and $p$-value of less than 0.01. The likelihood ratio test favors powerlaw over lognormal with $R = 0.5$ and $p$-value of less than 0.1.

[6] The likelihood ratio test favors powerlaw over exponential with $R = 8$ and $p$-value of less than 0.01. The likelihood ratio test favors lognormal over powerlaw with $R = -1.9$ and $p$-value of less than 0.1.

[7] The powerlaw exponent of the second order degree distributions are also greater than those reported by Acemoglu et al. (2012) using the IO table. And unlike in the US Input-Output network, the second-order exponents are greater than the first order exponents within our firm network. The second-order powerlaw exponent therefore does not provide tighter bounds for the equilibrium rate of decay of aggregate volatility.

[8] There are two reasons to emphasize the aggregate volatility computed using the out-degree exponent. The first of which is that the powerlaw exponent of the in-degree distribution is less significant because the in-degree is better fit by a lognormal distribution than a powerlaw. Second, within an equilibrium setting, we typically think of an idiosyncratic productivity shock to one firm affecting other firms through changes in the quantity of inputs.




(a) (b)

Figure 1

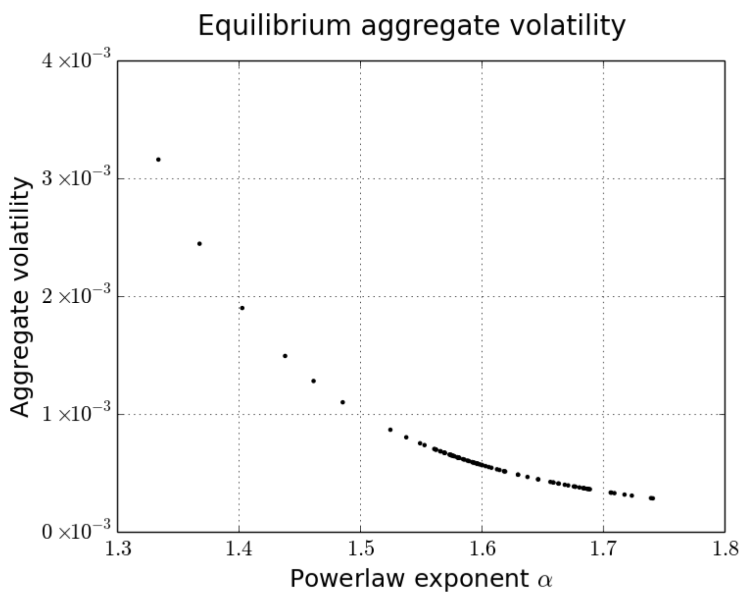



Figure 2: Equilibrium aggregate volatility for different values of the powerlaw exponent, with $10^6$ firms and firm volatility at 0.1.

# 3 The model

We consider an economy with a finite number of firms and a representative household. The firms and the differentiated goods they produce are indexed by $M = \{1, \cdots, m\}$. The household has index 0 and $N = \{0, \cdots, m\}$ denotes the set of agents. The representative household inelastically supplies 1 unit of labor and has a Cobb-Douglas utility function of the form:

$$u(x_1, \cdots, x_m) := \prod_{j \in M} x_i^{a_{j0}} \tag{1}$$

where for all $j \in M$, $a_{j0} \in \mathbb{R}_+$ is the share of good $j$ in the household's consumption expenditure, therefore $\sum_{j \in M} a_{j0} = 1$.

Each firm $i \in M$ has a Cobb-Douglas production function of the form

$$f_i(x_0, \cdots, x_m) := \lambda_i \prod_{j \in N} x_j^{a_{ji}} \tag{2}$$

where $\lambda_i \in \mathbb{R}_{++}$ is a productivity parameter. For all $j \in N$, $a_{ji} \in \mathbb{R}_+$ is the share of good $j$ in firm $i$'s expenditure on inputs, thus $\sum_{j \in N} a_{ji} = 1$ and there are constant return to scale. We assume each firm uses a non-zero quantity of labor in its production process, i.e. for all $i \in M$, $a_{0i} > 0$. The production structure of the network economy is characterized by the adjacency matrix $A = (a_{ij})_{i,j \in N}$, in which $a_{ij} > 0$ if and only if agent $i$ is a supplier of agent $j$. The network economy denoted by $\mathcal{E}(A, \lambda)$ is irreducible and aperiodic[9]. Given irreducibility and aperiodicitiy, the unique general equilibrium of the network economy (up to price normalization) $\mathcal{E}(A, \lambda)$ can be defined as follows:

**Definition 1.** *A collection $(\overline{p}_i, \overline{x}_i, \overline{q}_i)_{i \in N} \in \mathbb{R}_+^N \times (\mathbb{R}_+^N)^N \times \mathbb{R}_+^N$ of prices, intermediary inputs vectors and outputs is a general equilibrium of $\mathcal{E}(A, \lambda)$ if and only if:*

$$\forall i \in M, \;\; \overline{q}_i = \lambda_i \prod_{j \in N} \overline{x}_{ij}^{a_{ji}} \qquad \textit{feasibility} \tag{3}$$

$$\forall i,j \in N, \;\; \overline{x}_{ji} := a_{ji} \frac{\overline{p}_i \overline{q}_i}{\overline{p}_j} \qquad \textit{profit and utility maximization} \tag{4}$$

$$\forall i \in N, \;\; \overline{q}_i = \sum_{j \in N} \overline{x}_{ij} \qquad \textit{market clearing} \tag{5}$$

[9] The existence of a representative household that buys goods from all firms and sells labor to all firms, along with the fact that at least one firm buys input from another firm, guarantees irreducibility and aperiodicitiy. For more details see Mandel and Veetil (2021).

## 3.1 Time dynamics with fixed network

Time is discrete and indexed by $t \in \mathbb{N}$. The following sequence of events occur at every time step:

- Each agent $i \in N$ receives the nominal demand $\sum_{j\in N} a_{ij} w_j^t$.
- Firms adjust prices towards market-clearing values according to:

$$p_i^t = (1-\tau)\overline{p}_i^t + \tau p_i^{t-1} \tag{6}$$

  where $\tau \in [0,1]$ measures price-stickiness. Given the nominal demand $\sum_{j\in N} a_{ij} w_j^t$ and stock of output $q_i^t$, the market clearing price $\overline{p}_i^t$ for firm $i$ is given by:

$$\overline{p}_i^t = \frac{\sum_{j\in N} a_{ij} w_j^t}{q_i^t}. \tag{7}$$

- Markets do not clear if $\tau > 0$. In case of excess demand, buyers are rationed proportionally to their demand. In case of excess supply, $\overline{q}_i^t := \frac{\sum_{j\in N} a_{ij} w_j^t}{p_i^t}$ is sold. The remaining output is carried as inventory, which we denote by $I^t = q_i^t - \overline{q}_i^t$.
- The household sets market-clearing wage:

$$p_0^t = \sum_{i\in M} a_{0i} w_i^t \tag{8}$$

- Working capitals are updated on the basis of revenues, for all $i \in N$:

$$w_i^{t+1} = \sum_{j\in N} a_{ij} w_j^t \tag{9}$$

- Firms produce their output for the next period, for all $i \in M$:

$$q_i^{t+1} = I^t + \lambda_i \prod_{j\in N} \Big(\frac{a_{ji} w_i^t}{p_j^t}\Big)^{a_{ji}} \tag{10}$$

- Labor supply is replenished to 1.

In the absence of exogenous shocks, the out-of-equilibrium dynamics detailed above converges to the general equilibrium of the network economy $\mathcal{E}(A,\lambda)$. The speed of convergence to equilibrium is determined by the second largest eigenvalue of the production network (Golub and Jackson, 2012)[10]. Each time step of the model represents one month, the model is run for 20 years for it to reach sufficiently close the equilibrium. Firms sizes in all simulations begin with random assignments, they adjust to the network because within our model firm sizes are determined by the topology of the production network.

## 3.2 Evolution of the production network

The production network evolves because of rivalrous competition between firms for low cost suppliers of inputs. More specifically, at the end of the every time step, each firm receives an independent opportunity to change one of its suppliers with probability $\rho \in [0, 1]$. If this opportunity materializes for firm $i$ in period $t$, then firm $i$ randomly selects one of its suppliers $j$ and another firm $k$ among those to which it is not connected. Firm $i$ severs its connection with firm $j$ and adds a connection to firm $k$ if the price charged by $k$ is less than the price charged by $j$. The weight of the connection of $i$ to $k$ is assumed to be the same as the weight of the connection from $i$ to $j$.

The afore noted process of changes in network linkages means that some firms may lose all their buyers. We assume that a firm that loses all its buyers dies and exits the market. Such an exit is capable of generating cascades of exits. When Firm $A$ dies, its input seller Firm $B$ loses an output buyer. If $A$ was the only buyer of $B$, then $B$ too dies and exits the market. And so on for other firms who sell to $B$. Note that no firm is allowed to sell only to the household, to remain in the market each firm must sell to at least one other firm. Therefore, at the end of each time step, after the exit of each firm, we run a *recursive algorithm* which ensures that all firms who lose all sellers of intermediate inputs exit the production network. When a firm loses an input seller, it renormalizes the weights of the Cobb Douglas production function towards remaining suppliers. At the end of each time step, as many firms enter the economy as the number that exited so as to maintain a stable population of firms. Each new firm $i$ enters the economy with the following attributes:

- The number of suppliers $s_i$ and the number of buyers $b_i$ of firm $i$ are independently

[10] For more details on the stability of these dynamics see Gualdi and Mandel (2016). These out-of-equilibrium dynamics are akin to the question of stability considered by Fisher (1989) and others, with the difference that our system involves interaction within a network setting.

drawn from a binomial distribution $B(p,m)$. Where $m$ is the number of firms in the economy and $p$ is the probability of success of the binomial distribution. The probability of success $p$ is set such that the mean of the binominal distribution $p \times m$ equals the mean degree of the production network before the beginning of the network evolution process.

- $s_i$ firms are drawn from the population of firms in the economy and each is connected as an input-seller to firm $i$. Firm $i$ divides expenditures equally among all its intermediate input suppliers, i.e. the Cobb Douglas exponents of firm $i$ equals $\frac{1-a_{0i}}{s_i}$.

- $b_i$ firms are drawn from the population of firms in the economy and each is connected as an output-buyer to firm $i$. Each output-buyer $j \in b_i$ renormalizes its Cobb Douglas exponents so that $j$ spends equally on all its input-sellers.

- The price, wealth, and output of firm $i$ is initialized using a uniform(0,1) distribution.

# 4 Macroeconomic volatility

## 4.1 Computing disequilibrium aggregate volatility

**Definition 2.** *Disequilibrium aggregate volatility is the volatility of GDP with stochastic firm productivity and fixed production network, i.e. there is no entry-exit of firms or changes in buyer-seller linkages.*

- Firms are placed on a production network whose degree distribution follows a powerlaw (we use the powerlaw exponent computed in Section 2). We do not allow for firm entry-exit, i.e. the evolution of the network as described in Section 3.2 does not occur. Firms interact with each other on a fixed network by choosing quantities and prices.

- The productivity of firms is stochastic. More specifically, Equation 2 is modified by setting $\lambda_i = e^{z_i}$. Therefore, the production function of each firm $i$ becomes:

$$f_i(x_0, \cdots, x_m) := e^{z_i} \prod_{j \in N} x_j^{a_{ji}} \tag{11}$$

$z_i$ is distributed as a Gaussian $N(0,\gamma)$ with 0 mean and $\gamma$ standard deviation.

- We measure aggregate volatility as the standard deviation of ln(GDP). This definition of aggregate volatility is consistent with our usage of firm volatility within the model. More specifically, firm volatility in reality is measured as the standard deviation of firm growth rates. Since we are implementing firm volatility as the standard deviation of the log of firm output, we must make the appropriate adjustment in computing aggregate volatility[11].
- We set the price-stickiness parameter $\tau = 0$, therefore prices are fully flexible.
- We compute GDP by summing the output of all firms that goes to the final consumer which is the representative household. Output is valued at equilibrium prices as in Green and Laffont (1981).
- Each time step of our model represents a month. Therefore, yearly firm volatility $\gamma$ is scaled to the monthly level as $\gamma/12$, after which monthly GDP is summed to the yearly GDP to compute aggregate volatility.
- The model is first run for 20 years, without productivity shocks, so that the economy reaches a steady state. After this the model is run for 100 years with productivity shocks to generate the data with which to compute disequilibrium aggregate volatility.

## 4.2 Computing non-equilibrium aggregate volatility

**Definition 3.** *Non-equilibrium aggregate volatility is the volatility of GDP with deterministic firm productivity and endogenously evolving production network.*

- We assume that the productivity of firms is deterministic.
- We allow for the production network to evolve endogenously through changes in buyer-seller linkages and the entry-exit of firms as described in Section 3.2.

[11] See equation 6 in Acemoglu, Akcigit and Kerr (2016) for the idea in terms of rates of change of $z_i$. See Acemoglu, Akcigit and Kerr (2016, pp. 278-285) for Hicks-neutral productivity shocks and Acemoglu et al. (2012) for Harrod-neutral. The two implementations merely shift the level of GDP, not the standard deviation of log GDP.

- GDP is computed as the sum of the quantity of output of different firms that goes to the representative household valued at the prices prevailing at each time step. Note that GDP cannot be computed using equilibrium prices because equilibrium does not exist in the system with an endogenously evolving production network. Prices are constantly changing in response to the entry-exit of firms and in response to the change in buyer-seller relations.

- We assume prices are sticky, i.e. $\tau > 0$. The introduction of price-stickiness is important to compute GDP for the following reason. Price setting within our model is such that for each firm $i$, the fully flexible price is the inverse of its quantity (see Equation 7). Therefore, the product of price and quantity is constant. Price-stickiness allows for the product of price and quantity to change with changes in either price or quantity, thereby registering changes in GDP. These changes in GDP are necessary to compute aggregate volatility.

- Each computational experiment begins with firms on an Erdos-Renyi random graph. The system is run for 20 years for the transient changes to decay. During the transient period the degree distribution of the graph becomes more fat tailed than the Erdos-Renyi random graph. After the transient, the system is run for 100 years to generate data with which to compute aggregate volatility.

- We compute aggregate volatility using the regular definition which is the standard deviation of changes in yearly GDP (Comin and Philippon, 2005, p. 167-168). Since the non-equilibrium setting does not include productivity shocks, there is no need to redefine aggregate volatility to bring about a consistency between the empirical measure of firm volatility and its implementation with the model.

## 4.3 Equilibrium, disequilibrium, and non-equilibrium aggregate volatility

Figure 3 presents the scaling of the aggregate volatility with an increase in the number of firms. For an economy as large the United States, equilibrium volatility accounts for less than a tenth of the empirically observed aggregate volatility. Furthermore, there is reason to suspect that this measure may be overstating the aggregate volatility generated by idiosyncratic productivity shocks. More specifically, equilibrium aggregate volatility is based on the unrealistic assumption that new productivity shocks occur only after the economy has adjusted to old shocks. Within an economy in which new productivity shocks arise before the system has fully adjusted to old shocks, disequilibrium aggregate volatility is a more sensible measure than equilibrium aggregate volatility. Figure 3 shows that disequilibrium volatility is significantly lower than equilibrium volatility.

The reason for why disequilibrium volatility is sizeably lower than equilibrium volatility is as follows. Suppose the distribution of the productivity of firms at time steps 1, 2, and 3 is $D^1$, $D^2$, and $D^3$. Let the equilibrium GDP corresponding to each of these distributions of firm productivity be $y^1$, $y^2$, and $y^3$. Equilibrium aggregate volatility marks the jump from $y^1$ to $y^2$ and $y^2$ to $y^3$. Consider the productivity shock at $t = 3$, when the economy is still adjusting to the productivity shock at $t = 2$. While the economy is adjusting to the shock received at $t = 2$, suppose firm $i$ with low productivity under the $D^1$ temporarily acquires a large share of inputs[12]. At that very movement $D^3$ arrives, which happens to increase the productivity of firm $i$. This will generate a jump in GDP, which may be larger than the difference between $y^2$ and $y^3$. More generally, there exist various peculiar draws of productivity along with various disequilibrium states of the economy, such that disequilibrium changes can be larger than equilibrium changes.

But the probability of these peculiar draws of productivity along with various disequilibrium states being realized declines rapidly with an increase in the size of the economy. So much so that in a sufficiently large economy, the productivity changes brought about by $D^3$ will tend to act as noise upon the productivity changes brought about by $D^2$. More specifically, as an economy is moving from $y^1$ towards $y^2$, $D^3$ will tend to make the economy move to $\hat{y}$. And $|y^2 - \hat{y}|$ will tend to be less than $|y^2 - y^3|$ because idiosyncratic firm level shocks at one time step will tend to average out the idiosyncratic shocks from the pervious time steps. This averaging becomes more pronounced with an increase in the size of the economy because of the Law of Large Numbers. Put differently, the joint-probability of productivity draws and states of the economy necessary for succeeding disequilibrium states to be more different than succeeding equilibrium states declines with an increase in the number of firms. Our computational experiments show that this averaging of idiosyncratic firm level shocks over time is sufficient to generate an order of magnitude difference between

[12]Disturbances to the network economy generates complex time sequences of changes in relative prices, which in turn produces temporariy changes in the working capital possessed by different firms, see Mandel, Taghawi-Nejad and Veetil (2019) and Mandel and Veetil (2021) for more details.

equilibrium and disequilibrium aggregate volatility.

Figure 3 shows that non-equilibrium aggregate volatility is sizeably greater than equilibrium and disequilibrium volatility. Furthermore, non-equilibrium aggregate volatility is on the same order as the empirically observed aggregate volatility. Figure 4 shows that the model is capable of generating the empirically observed aggregate volatility for a range of parameter values. Figure 4a plots the non-equilibrium aggregate volatility for different values of mean degree of the production network $d$ and different probabilities of a change in input sellers $\rho$. The figure shows that aggregate volatility decreases with an increase in mean degree. This is because within our setting the probability of a firm changing one of its links is independent of the number of input sellers, therefore as the number of input sellers grow the proportion of links changed by firms declines. Figure 4b shows that non-equilibrium aggregate volatility increases with an increase in price-stickiness. Figures 4a and 4b show that non-equilibrium aggregate volatility increases with the probability of link change.

**Result 1.** ***Non-equilibrium aggregate volatility***
*Firm decisions on production levels, production relations, and entry-exit generate sizeable aggregate volatility in a large economy, without idiosyncratic productivity shocks.*

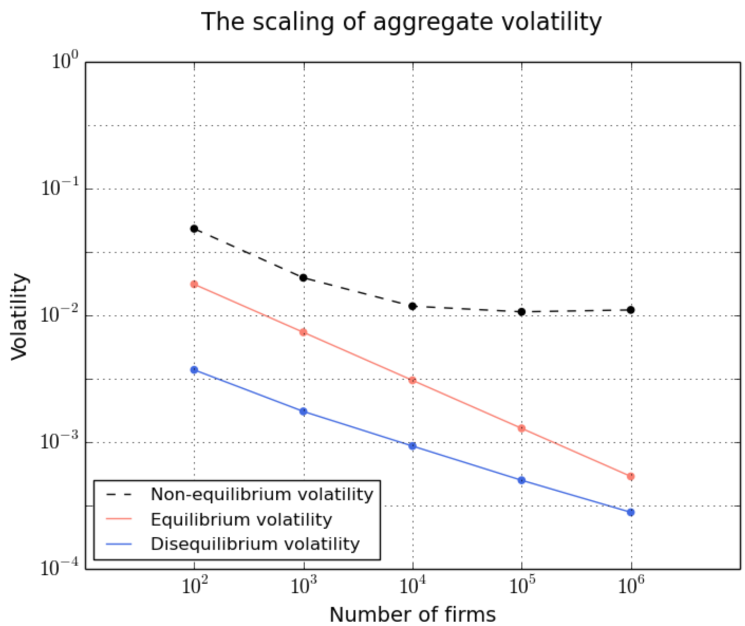
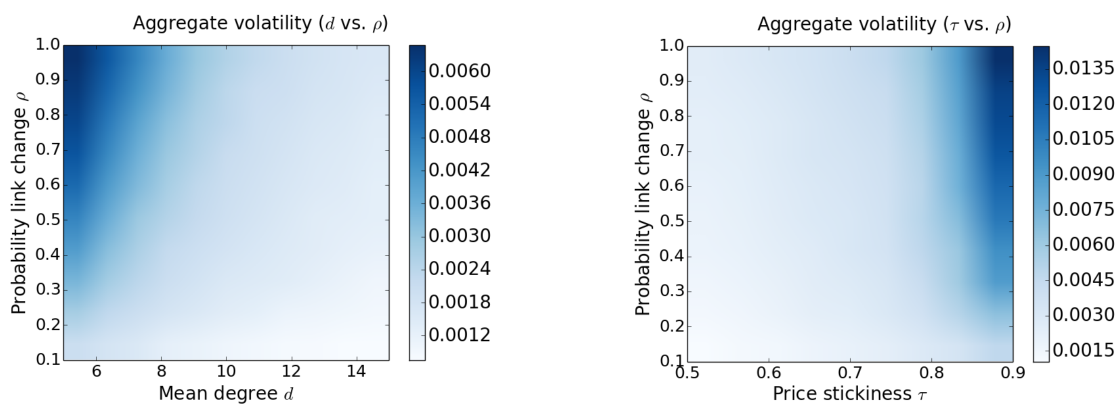



Figure 3: Equilibrium, disequilibrium, and non-equilibrium aggregate volatility for different sized economies. For disequilibrium, the production network is powerlaw with an exponent of 1.19 and $\tau = 0$. For disequilibrium and non-equilibrium : $a_{0i} = 0.3 \forall i$. For non-equilibrium: $\tau = 0.9$, $\rho = 1$, $d = 10$.




(a) (b)

Figure 4: Non-equilibrium aggregate volatility for different values of price stickiness ($\tau$), probability of link change $\rho$, and mean degree $d$. Results reported in (a) use $\tau = 0.5$ and results reported in (b) use $d = 10$. All results are from experiments with $10^5$ firms. The mean degree varies from 5 to 10 with increments of 1. $\tau$ varies from 0.5 to 0.9 with increments of 0.05. And $\rho$ varies from 0.1 to 1 with increments of 0.1.

# 5 Meso stability amidst micro flux

## 5.1 Emergent distributions

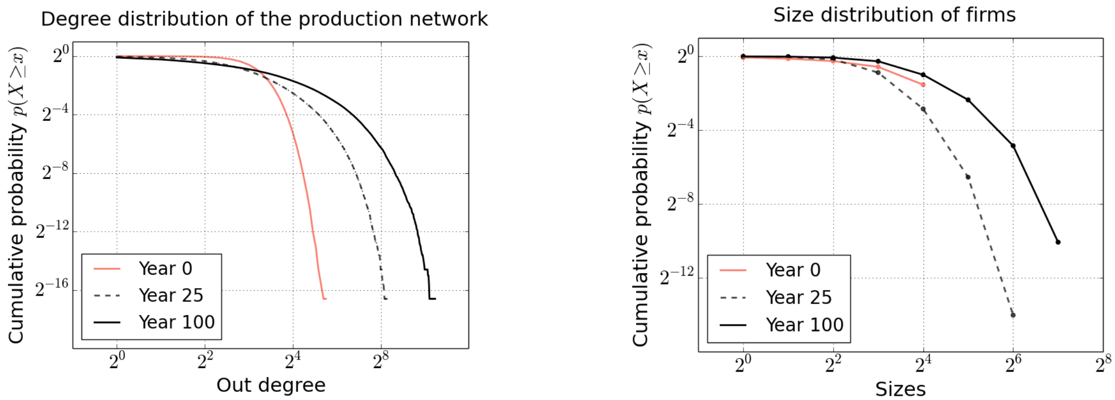
Figure 6 shows the degree distribution and size distribution at three time steps: the initial distribution at Year 0, interim distribution at Year 25, and final distribution at Year 100. Figure 5a shows that beginning from an Erdos Renyi random graph, the degree distribution of the production network grows fat-tails as the model dynamics unfold over time. Figure 5b shows that beginning from a uniform random distribution, the size distribution of firms grows fatter tails as the model dynamics unfold.

Finally, Figure 6 marks the convergence of the degree and size distribution. Figure 6 plots the distance of the distribution at each time step from the final distribution. The distance at Year $t$ denoted by $\omega_t$ is defined as follows:

$$\omega_t = \sum_{i \in E} |CDF(x_i^t) - CDF(x_i^T)| \tag{12}$$

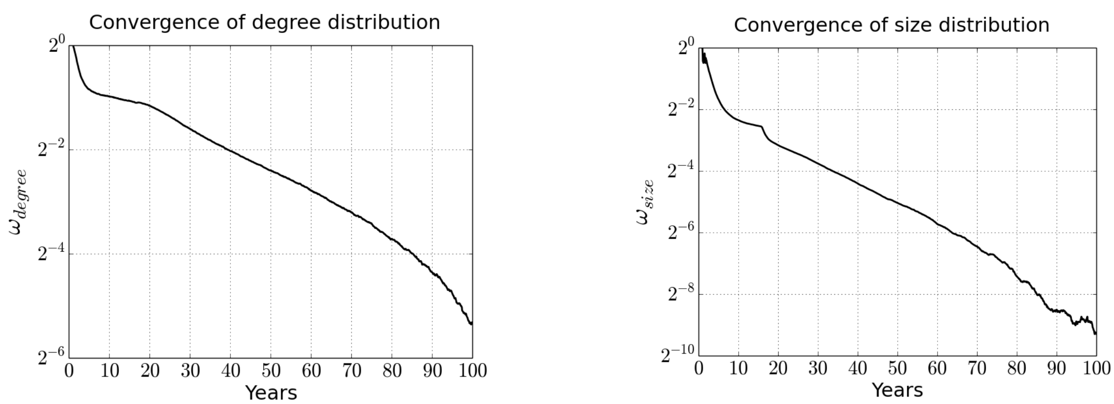
where CDF denotes the empirical cumulative distribution function. $E$ is the set of events, meaning degree in case of degree distribution and sizes in case of size distributions. Figure 6 plots the distances for each for the 100 years beginning from Year 0, normalized to the distance in the first year. Figure 6a shows that the degree distribution converges as the distance to the distribution at Year 100 decreases at each time step. Figure 6b shows that the size distribution convergences as the distance to the distribution at Year 100 decreases at each time step.

**Result 2.** ***Degree distribution and size distribution***
*Firm decisions on production levels, production relations, and entry-exit generate degree distribution and size distribution with fatter tails than a normal distribution.*




(a) (b)

Figure 5: Log-log plots of the degree and size distribution of firms. $m = 10^5$, $\rho = 1$, $\tau = 0.9$, $d = 10$.These are distribution that emerge after 100 years of firm interactions, where the first year begins after 20 years of transient.



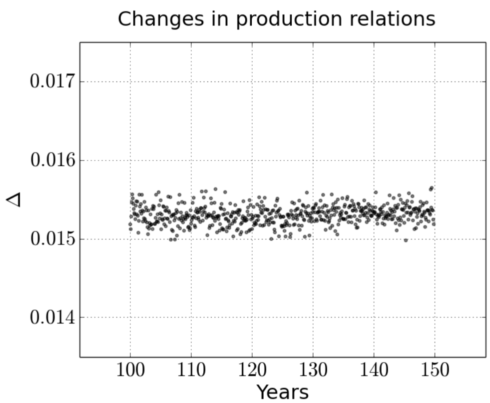

(a) (b)

Figure 6: $m = 10^5$, $\rho = 1$, $\tau = 0.9$, $d = 10$.

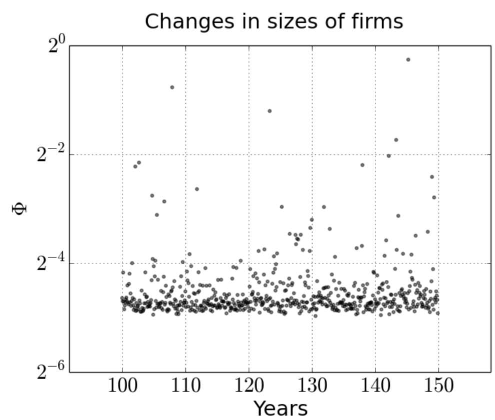
## 5.2 Flux at the level of individual firms

While the degree and size distribution converge and thereby come to exhibit stability, the economy at the level of individual firms is in perennial flux. Figure 7a plots the changes in buyer-seller linkages at every time step as a proportion of total links in the economy. Figure 7a shows that the rate of change in buyer-seller relations is sizeable even after the transient of the first 20 years. Figure 7b plots the changes in the firm sizes from one time step to the next as measured by: $\varphi_t = \frac{1}{m}\sum_{i=1}^{m} \frac{|s_i^t - s_i^{t-1}|}{s_i^{t-1}}$. Figure 7b shows that there are sizeable changes in firm sizes at each time step.

**Result 3.** ***Micro variables***
*The evolution of the production network generates sizeable changes the in buyer-seller relations between firms and the sizes of individual firms.*




(a)




(b)

Figure 7

## 5.3 Firm level volatility

Figure 8 plots the distribution of annual growth rate in firm sales: $v_i^t = \frac{s_i^t - s_i^{t-1}}{s^{t-1}}$, for each firm $i$. The figure plots the distribution of growth rate in the last year of the model run. The plot shows that firm growth rate is skewed to the right. Firm level volatility as measured by the standard deviation of the distribution is on the same order of magnitude as the empirically observed figure.

**Result 4.** ***Idiosyncratic productivity change***
*The firm volatility generated by the model is approximately equivalent to the empirically observed magnitude. The distribution of firm growth rate has a fat right tail.*

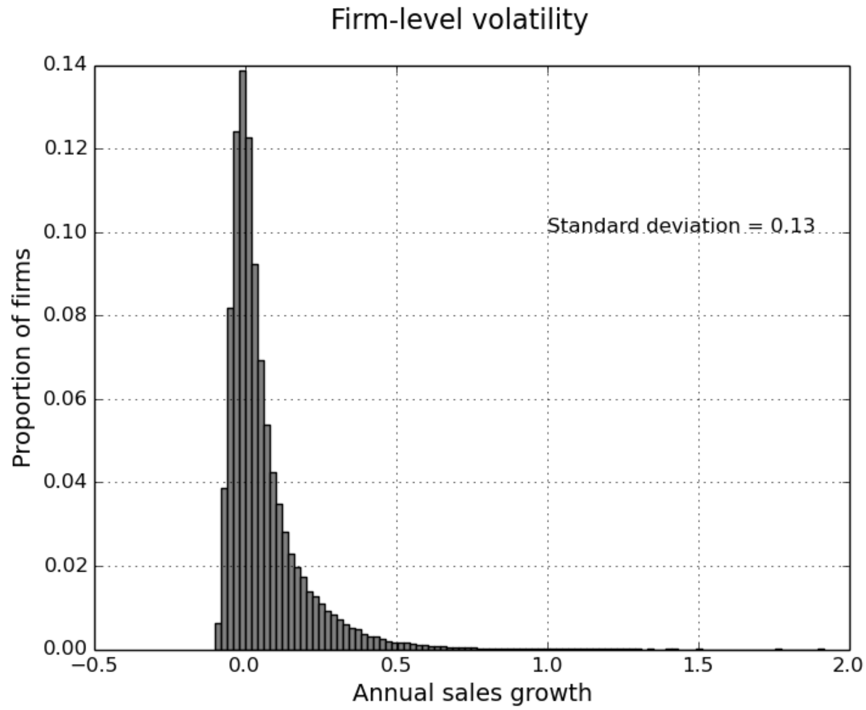



Figure 8: firm volatility as measured by annual sales growth of firms from one time step to the next. $m = 10^5$, $\rho = 1$, $\tau = 0.9$, $d = 10$. This is the sales growth distribution that emerge after 100 years of firm interactions, where the first year begins after 20 years of transient.

# 6 Concluding thoughts

Frisch (1933) argued that macroeconomic fluctuations arise from "certain exterior impulses" that "hit the economic mechanism". He divided business cycle theory into the *impulse problem* and the *propagation problem*. Where by 'impulse problem' he meant the origins of the external impulses that hit the economy. And by 'propagation mechanism' he meant the process by which the exterior impulse is amplified, dampened, and more generally carried by the exchange mechanism. Most business cycle models are founded on the Frischian dichotomy between impulse and propagation. They differ merely in the origins of the impulse and sources of propagation. This paper presents an alternative to Frisch's basic architecture. The Frischian dichotomy does not hold within our system wherein the impulse is the propagation. More specifically, it is precisely the propagation of old changes that provides the impulse for new action. Decisions to switch from one input seller to another, or to enter-exit the production network, generates a whole sequence of relative price changes across the economic system. These price changes become the incentive for new decisions on prices, quantities, production relations, and entry-exit. Our system exhibits non-equilibrium dynamics as it remains away from equilibrium due to forces endogenous to the model (Prigogine and Lefever, 1975; Vickers, 1985). The system, however, converges to equilibrium if firms do not make decisions on the extensive margin. This is because while firms' decisions on the extensive margin are miscoordinating, decisions on the intensive margins are coordinating. Macroeconomic dynamics emerge from the interplay between the forces of coordination and miscoordination.

The difference between Frisch's architecture and ours stems from disparate conceptions of market competition. The Frischian architecture presume market competition is always coordinating. In so far as competition is coordinating, the economic system exhibits perfect stability in the absence of exogenous impulses. Each day is a mundane repetition of the day before. This is what Mises called 'the evenly rotating economy', Schumpeter 'the circular flow', and Arrow-Debrue 'general equilibrium'. Matters are wholly different within systems in which competition is rivalrous process (Hayek, 1948; Wagner, 2001). Ours is one such setting. One firm's decision to switch to a lower cost provider of an input may temporarily improve its position, but the decision hurts its former supplier. Furthermore, each firm's decision to change an input seller propagates upstream as positive or negative demand shocks, and downstream as positive or negative supply shocks. Some firms are hurt by these shocks whereas others benefit, all of which depends on their network positions. There is no senses in which competition on the extensive margin is neutral or coordinating. Rivalry between firms for input sellers incessantly injects microeconomic disturbances. We have shown that such a rivalrous process of competition proves sufficient to generate sizeable macroeconomic volatility in large economies in the absence of exogenous productivity shocks. Our approach suggests that it would be mistaken to treat firm-volatility as an exogenous variable to be amplified by the skewed size of distribution of firms or the skewed degree distribution of the production network. Indeed we have shown that the degree distribution of the production network, the size distribution of firms, and firm volatility are interdependent variables that co-emerge amidst rivalrous competition.

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