Fiscal Inflation with Incomplete Information

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We provide an explanation for fiscal inflation, not from the classic view of a passive monetary-active fiscal policy mix but from the lens of incomplete information. We characterize a unique rational expectations equilibrium (REE) with persistent fiscal inflation when the Taylor principle still holds, and the fiscal authority passively responds to the level of debt, a region where the conventional view sees only a Ricardian fiscal policy.

"*Neither the President-elect, nor I, propose this relief package without an appreciation for the country's debt burden. But right now, with interest rates at historic lows, the smartest thing we can do is act big. I believe the benefits will far outweigh the costs. — Yellen (2021)*"

I. Introduction

Generating fiscal inflation in the existing models requires the assumption of unfunded debt. Leeper (1990) shows that a unique equilibrium can exist with rational expectations under one of the two conditions: a) active monetary and passive fiscal policy where Taylor prinicple is satisfied and the government responds strongly to deviations from debt; b) passive monetary and active fiscal policy where the Taylor principle is not satisfied and the government responds weakly to debt. Under the first condition, fiscal policy is fully Ricardian and has no effect on inflation. We can only generate fiscal inflation with passive monetary policy, also known as the Fiscal Theory of Price Level (FTPL). Similarly, Bianchi, Faccini and Melosi (2023) incorporates both funded and unfunded debt in the central bank's Tarylor rule, allowing them to respond actively to funded debt and passively to unfunded debt. Only shocks to unfunded debt result in inflation. While representative agent New Keynesian (RANK) models require FTPL to generate fiscal inflation, Angeletos, Lian and Wolf (2024) show that fiscal deficit can lead to inflation without the assumption of unfunded debt under a HANK model. We show, within the RANK environment, that inflation can

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result from breaking the assumption of full information of the path of primary surpluses.

In this paper, we introduce incomplete information in a general equilibrium model to explain persistent inflation when central bank follows a standard Taylor rule. Our approach shows that persistent fiscal inflation can be generated in a model with Taylor rule and active monetary policy by deviating from the assumption of Full Information Rational Expectations (FIRE). We argue that the assumption of FIRE cannot explain most of the high post-COVID inflation. Under full information, the central bank should have largely anticipated and responded to the inflation that arised from the large fiscal stimulus. We introduce incomplete information such that economic agents cannot distinguish between fiscal spending, transfer shocks, or long run changes to primary surplus, from shocks to potential output.

Understanding fiscal inflation is important to devise optimal policy responses in the face of high inflation. After a decade and half of low inflation, U.S. economy has been experiencing a sudden and persistent increase in inflation since 2021. But most existing macroeconomic models fail to explain the rise and fall of post-COVID inflation. Ferroni, Fisher and Melosi (2024) show that the COVID shock was unusual to other shocks experienced by the economy in the past, and the large fiscal transfers in 2020 and 2021 were a significant part of the shock. We propose that in the face of high uncertainty, incomplete information about the path of potential output and primary surpluses is more representative of expectation formation process of economic agents, including the central bank.[1]

We first illustrate how inflation and output respond differently under full information and incomplete information in a simple Fisherian economy. Under incomplete information, households face a signal extraction problem where they are unable to decouple between a real interest rate shock and a primary surplus shock, breaking Ricardian equivalence. Both persistent and transitory primary surplus shock can then generate inflation as shown in Figure 2.

We then introduce a simple production economy with sticky prices. Under incomplete information, households and central bank are unable to decouple a shock to the path of primary surplus from shocks to natural output. Under FIRE, households are Ricardian and shocks to primary surplus do not affect inflation and output. However, under incomplete information households utilize the observed fiscal signal to extract information about underlying economic shocks. The signal extraction problem breaks Ricardian equivalence and both transitory and persistent fiscal shocks can generate inflation.

The paper is structured as follows. Section II introduces a simple Fisherian economy to illustrate the propogation of fiscal shocks under incomplete information. Section III introduces incomplete information in a small scale New Key-

[1]The FOMC statement in December 2021 highlights this point, “The path of the economy continues to depend on the course of the virus. Progress on vaccinations and an easing of supply constraints are expected to support continued gains in economic activity and employment as well as a reduction in inflation.”

nesian model with nominal frictions. A calibrated model shows that introducing information frictions can generate fiscal inflation in a canonical model without the assumption of unfunded shocks. Section IV presents a large-scale DSGE model with information frictions.

II. A Simple Fisherian Economy

We use a simple Fisherian economy (see Leeper (1991)) to highlight the key information frictions. The one-period nominal bond B_t, issued by the government, is sold at a price of Q_t. The gross nominal interest rate R_t is given by the inverse of Q_t, i.e., $R_t = 1/Q_t$. The steady-state gross nominal interest rate is $1/\beta$, where $\beta \in (0,1)$ is the household's time discount factor. In log-linearized form, the Fisher equation is

$$i_t = \mathbb{E}_t^{HH} \pi_{t+1} + r_t, \tag{1}$$

where i_t is the net nominal interest rate, π_t is inflation, and r_t is the *ex ante* real interest rate. The rational expectations operator $\mathbb{E}_t^{HH} = \mathbb{E}_t(\cdot|I_t^{HH})$ is conditional on the household's information set I_t^{HH}, which will be specified below. For simplicity, the real interest rate is exogenous and evolves according to

$$r_t = u_t^r + \eta_t^r,$$

where $\eta_t^r \sim N(0, \sigma_{\eta,r}^2)$ is a transitory *i.i.d.* shock and u_t^r is a persistent AR(1) shock given by $u_t^r = \rho_r u_{t-1}^r + \varepsilon_t^r$ with $\varepsilon_t^r \sim N(0, \sigma_{\varepsilon,r}^2)$ and $\rho_r \in (0,1)$.

Monetary policy follows a simple rule

$$i_t = \phi_\pi \pi_t, \tag{2}$$

where ϕ_π controls the strength with which the central bank reacts to inflation and satisfies the Taylor principle (i.e., $\phi_\pi > 1$). To focus solely on fiscal inflation, we do not introduce monetary policy shocks for now.

The government budget constraint is

$$Q_t B_t + P_t T_t = B_{t-1},$$

where T_t defines the primary surplus and $Q_t B_t / P_t$ defines the real market value of debt. Let τ_t and $s_{b,t}$ denote the log-deviations of the two variables from their corresponding steady-state values. The log-linearized government budget constraint (GBC) is

$$s_{b,t} = \beta^{-1} \left[s_{b,t-1} + i_{t-1} - \pi_t - (1-\beta)\tau_t \right]. \tag{3}$$

Recently, the elevated public debt levels have been at the center of fiscal discussions. We consider a fiscal rule that embeds a hidden time-varying debt target (also see Bianchi and Melosi (2017) and Han (2021)). Let us first consider a log-linearized surplus rule

$$\tau_t = \gamma_\tau(s_{b,t-1} - s^*_{b,t}) + \eta^\tau_t, \quad \eta^\tau_t \sim N(0, \sigma^2_{\eta,\tau}),$$

where $s^*_{b,t}$ is the log-deviation of the contemporaneous debt target from its steady-state value.[2] The parameter $\gamma_\tau > 1$ controls the strength with which the government reacts to movements of $s_{b,t-1}$ from its time-varying target $s^*_{b,t}$.

We still need to specify the law of motion for the time-varying debt target $s^*_{b,t}$. To achieve the goal, we connect to the recent literature on funded and unfunded fiscal shocks. In particular, Bianchi, Faccini and Melosi (2023) considers the following log-linearized fiscal rule

$$\tau_t = \gamma_\tau(s_{b,t-1} - s^*_{b,t-1}) + \gamma_b s^*_{b,t-1} + (\varepsilon^\tau_t + \eta^\tau_t),$$

with $0 < \gamma_b < 1 < \gamma_\tau$, $\varepsilon^\tau_t \sim N(0, \sigma^2_{\varepsilon,\tau})$, and $\eta^\tau_t \sim N(0, \sigma^2_{\eta,\tau})$. For reasons illustrated in Section II.A, Bianchi, Faccini and Melosi (2023) calls η^τ_t the "funded" shock, and ε^τ_t the "unfunded" fiscal shock when monetary policy follows an alternative rule. Equating the above two rules gives the law of motion for the time-varying debt target

$$s^*_{b,t} = (1 - \gamma_b/\gamma_\tau)s^*_{b,t-1} - \gamma_\tau^{-1}\varepsilon^\tau_t, \quad \varepsilon^\tau_t \sim N(0, \sigma^2_{\varepsilon,\tau}); \tag{4}$$

which is a stationary AR(1) process with persistence $1 - \gamma_b/\gamma_\tau \in (0, 1)$.

As we will see in Section II.A, both types of fiscal shocks are funded in our model given a standard Taylor rule (2), so it is not appropriate to name ε^τ_t the "unfunded" shock anymore. Instead, rewriting the surplus rule using the lag operator L gives

$$\tau_t = \gamma_\tau s_{b,t-1} - \gamma_\tau s^*_{b,t} + \eta^\tau_t = \gamma_\tau s_{b,t-1} + \frac{1}{1 - (1 - \gamma_b/\gamma_\tau)L}\varepsilon^\tau_t + \eta^\tau_t.$$

It is thus more precise to label ε^τ_t as the persistent fiscal shock and η^τ_t as the transitory fiscal shock.

Besides stabilizing real debt, fiscal policy also serves a major role of stabilizing the real economy. We propose a surplus rule as

$$\tau_t = \gamma_\tau(s_{b,t-1} - s^*_{b,t}) + \gamma_r u^r_t + \eta^\tau_t, \quad \eta^\tau_t \sim N(0, \sigma^2_{\eta,\tau}); \tag{5}$$

where the additional parameter, γ_r, controls the strength with which the primary surplus reacts to the persistent component of the real interest rate (i.e., u^r_t). However, even an undergraduate IS-LM model suggests that the relationship between

[2]The steady-state surplus-to-output ratio τ^* determines the steady-state real market debt s^*_b the government can finance in the long run. To see the point, imposing steady-state values in the government budget constraint leads to $s^*_b + \tau^* = s^*_b/\beta$. Consequently, $s^*_b = (\beta\tau^*)/(1-\beta) > 0$.

the real interest rate and the macro economy is complicated. For instance, if persistently high real interests rate starts to hurt the aggregate demand (i.e., the IS relation), then the government need to lower primary surpluses (i.e., decrease taxes and increase spending) to save the economy (i.e., $\gamma_r < 0$). On the other hand, higher output can also drive up real interest rates to clear the financial and bond markets (i.e., the LM relation). A higher real interest rate also increases the government's debt financing cost. Both stabilizing the economy and lowering the debt financing cost motives require the government to increase primary surpluses (i.e., $\gamma_r > 0$). For these reasons, we remain an agnostic approach and do not impose any restrictions on γ_r.

A. *Making connections to Leeper (1991) and Bianchi, Faccini and Melosi (2023)*

Before introducing incomplete information, it is worthwhile to connect to two papers in the literature, which also consider similar Fisherian economies. Setting

$$\gamma_b = \gamma_\tau, \quad \gamma_r = 0, \quad \sigma_{\varepsilon,\tau} = 0 \Rightarrow \textit{Leeper } (1991) \tag{6}$$

eliminates the time-varying debt target and the real interest rate response in the fiscal rule (5), and gives us the influential model in Leeper (1991). In the fiscal theory of price level, two separate parameter spaces deliver existence and uniqueness of a Rational Expectations Equilibrium (REE). The first region is the Active Monetary-Passive Fiscal regime (AMPF), where monetary policy responds more than one-to-one to deviations of inflation from its target ($\phi_\pi > 1$), and fiscal authority responds strongly to deviations of debt to keep it on a stable path ($\gamma_\tau > 1$). The second region is the Passive Monetary-Active Fiscal regime (PMAF), where monetary policy responds less than one-to-one to inflation ($0 \leq \phi_\pi < 1$), and fiscal authority responds weekly to debt ($0 \leq \gamma_\tau < 1$). Appendix A derives both the AMPF and PMAF solutions. As debt stability is achieved with sufficient fiscal adjustments, a distinct feature of the AMPF regime is that fiscal policy is Ricardian and inflation is independent of any fiscal shocks. In the PMAF regime, since the fiscal authority only responds to debt weakly, inflation needs to adjust to surplus shocks to stabilize government debt. Rational expectations fiscal inflation can only exist in the PMAF region in Leeper (1991). In our model, we restrict our parameter space to the AMPF regime (i.e., $\phi_\pi > 1, \gamma_\tau > 1$).

Bianchi, Faccini and Melosi (2023) also generates persistent fiscal inflation, but requires a different monetary rule. Setting

$$\gamma_r = 0, \quad i_t = \phi_\pi(\pi_t - \pi_t^F) + \phi^F \pi_t^F \Rightarrow Bianchi, Facciniand\ Melosi\ (2023) \tag{7}$$

yields the simple model in Bianchi, Faccini and Melosi (2023). While we consider a standard Taylor rule, Bianchi, Faccini and Melosi (2023) considers a modified Taylor rule in which monetary authority reacts differently to funded and unfunded shocks. In particular, they require $0 \leq \phi^F < 1$ and derive a π_t^F that would arise in a shadow economy in which the PMAF policy mix is always in place. In other words, the central bank is always willing to accommodate unfunded shocks and let inflation arise to dilute the real debt.

Both papers rely on the central idea of the fiscal theory of price level (i.e., the PMAF regime) to generate fiscal inflation. Furthermore, both work assume a Full Information Rational Expectations (FIRE) environment. The immediate implication of the FIRE assumption, however, suggests that the central bank should largely predict the persistent COVID inflation that would arise from the large-scale fiscal stimuli. This implication seems to contradict the observation that the Fed publicly communicated that “inflation is transitory” in June 2021. In our model, we do not need FIRE economic agents, neither a passive center bank willing to accommodate fiscal inflation. The information friction-incomplete information- is sufficient to drive persistent fiscal inflation in our framework.

B. *Introducing incomplete information*

We now introduce incomplete information formally while preserving the assumption of rational expectations. Full-information models assume economic agents can observe all shocks in the economy directly. The coexistence of persistent and transitory real interest shocks, and the persistent and transitory fiscal shocks, naturally introduces a signal extraction to the household.

Let $\mathcal{M}$ denotes the model structure, which include all structural parameters and the equilibrium conditions (1)-(5). As in all rational expectations models, we first assume $\mathcal{M}$ is common knowledge to the household. Consequently, our information friction is different than the imperfect information channel considered in learning models (e.g., Eusepi and Preston (2018)).

We assume the household can observe the entire history of real interest rates $\{r_{t-k}|k \geq 0\}$.[3] Rewriting the surplus rule (5) as

$$\tau_t - \gamma_\tau s_{b,t-1} = -\gamma_\tau s^*_{b,t} + \gamma_r u^r_t + \eta^\tau_t. \tag{8}$$

It is natural to assume household can also observe the entire histories of surpluses $\{\tau_{t-k}|k \geq 0\}$ and the realized real market debt $\{s_{b,t-1-k}|k \geq 0\}$. It follows from (8) that $-\gamma_\tau s^*_{b,t} + \gamma_r u^r_t + \eta^\tau_t$, which is a linear combination of persistent and transitory fiscal shocks, and the persistent real interest rate shock, is also in the household information set.

Let us define

$$\mathcal{I}^{HH}_t = \{r_{t-k}, (-\gamma_\tau s^*_{b,t-k} + \gamma_r u^r_{t-k} + \eta^\tau_{t-k}), \mathcal{M}|k \geq 0\}. \tag{9}$$

Given the information set, we can formulate the signal extraction problem be-

[3] Alternatively, we can assume household can observe i_t and derive r_t from the Fisher equation (1) by subtracting the expectation $\mathbb{E}^{HH}_t \pi_{t+1}$.

tween the household's observables $\mathbf{x}_t$ and the exogenous state variables $\mathbf{s}_t$ as

$$\underbrace{\begin{bmatrix} r_t \\ -\gamma_\tau s^*_{b,t} + \gamma_r u^r_t + \eta^\tau_t \end{bmatrix}}_{\mathbf{x}_t} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ \gamma_r & 0 & -\gamma_\tau & 1 \end{bmatrix} \underbrace{\begin{bmatrix} u^r_t \\ \eta^r_t \\ s^*_{b,t} \\ \eta^\tau_t \end{bmatrix}}_{\mathbf{s}_t}. \tag{10}$$

In contrast, the FIRE information set is

$$\mathcal{I}^{FIRE}_t = \{\varepsilon^r_{t-k}, \eta^r_{t-k}, \varepsilon^\tau_{t-k}, \eta^\tau_{t-k}, \mathcal{M} | k \geq 0\}. \tag{11}$$

It is noteworthy that even though the fiscal authority is responding to u^r_t in (5), it does not need to have full information. For example, an alternative interpretation of the transitory fiscal shock η^τ_t is to treat it as an measurement error of $\gamma_r u^r_t$.

C. Solving the model

The model setup introduces an additional parameter γ_r. Before solving the incomplete information model, we first analyze how γ_r changes the existence and uniqueness properties of the REE. Fortunately, we know from Blanchard, L'Huillier and Lorenzoni (2013) and Chahrour and Ulbricht (2022) that changing from $\mathcal{I}^{FIRE}_t$ to $\mathcal{I}^{HH}_t$ does not alter the determinacy regions. Appendix B solves the FIRE version of the model and shows

PROPOSITION 1: *Given the model structure $\mathcal{M}$ and assume $\phi_\pi > 1$, $0 < \gamma_b < 1 < \gamma_\tau$. For any γ_r, the model permits a unique rational expectations equilibrium (REE).*

The additional parameter γ_r thus does not impact the REE's existence and uniqueness. Intuitively, when $\phi_\pi > 1$ satisfies the Taylor principle, monetary policy anchors a stationary inflation process. Similarly, the restrictions $\gamma_\tau > 1$ and $\gamma_b < \gamma_\tau$ guarantees that both the real market debt and the time-varying debt target are stable.

We solve the model following the algorithm in Blanchard, L'Huillier and Lorenzoni (2013). Let $\mathbf{y}_t$ denote the vector of endogenous state variables and $\mathbf{s}_t$ the exogenous state variables. The equilibrium conditions (1)-(5) can be written as

$$\mathbf{F}\mathbb{E}^{\mathbf{HH}}_{\mathbf{t}}\mathbf{y_{t+1}} + \mathbf{G}\mathbf{y_t} + \mathbf{H}\mathbf{y_{t-1}} + \mathbf{M}\mathbf{s_t} + \mathbf{N}\mathbb{E}^{\mathbf{HH}}_{\mathbf{t}}\mathbf{s_{t+1}} = \mathbf{0}, \tag{12}$$

where $\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{M}, \mathbf{N}$ are coefficient matrices. Solving (12) gives a law of motion

$$\mathbf{y_t} = \mathbf{P}\mathbf{y_{t-1}} + \mathbf{Q}\mathbf{s_t} + \mathbf{R}\mathbf{s_{t|t}}. \tag{13}$$

The $\mathbf{s_{t|t}}$ denotes households' perceived exogenous state variables, which can be obtained from the Kalman recursion once we establish the household's signal extraction problem (10) as a state-space model.[4]

For illustration, we fix a set of parameters. The time discount factor β is 0.99, suggesting a quarterly model. Other parameters are

$$\begin{aligned} \phi_\pi = 1.5, \quad \rho_r = 0.9, \quad \sigma_{\varepsilon,r} = 1, \quad \sigma_{\eta,r} = 3, \\ \gamma_\tau = 2, \quad \gamma_b = 0.1, \quad \sigma_{\varepsilon,\tau} = 3, \quad \sigma_{\eta,\tau} = 1; \end{aligned}$$

The persistent real interest rate shock u_t^r has a auto-correlation of 0.9. The implied persistence of the time-varying debt target $s_{b,t}^*$ is $1 - \gamma_b/\gamma_\tau = 0.95$. The standard deviations are chosen to be roughly in line with the existing estimates (see Smets and Wouters (2007) and Leeper, Plante and Traum (2010)).

Figure 1 plots the impulse responses of inflation (quarterly, non-annualized) to either a persistent real interest rate shock $\varepsilon_1^r = \sigma_{\varepsilon,r}$ or a transitory real interest rate shock $\eta_1^r = \sigma_{\eta,r}$. Under full information, due to a Ricardian fiscal policy, inflation is entirely driven by interest rate shocks and is independent of γ_r. As higher real interest rates drive up inflation, the impulse responses are either persistent or transitory, depending on the type of the underlying interest rate shock. Under incomplete information, as γ_r enters the household's signal extraction problem (10), even a transitory shock η_1^r can generate some persistent inflation responses as households erroneously (but rationally) attribute movements in their observables to a combination of both types of interest rate shocks. When a persistent shock ε_1^r hits the economy, the initial impact is small compared to FIRE. Households gradually learn ε_1^r over time, and the responses converge to its FIRE counterpart. The hump-shaped impulse responses has become a hallmark of incomplete information, rational expectations models.

Interestingly, only the magnitude (i.e., the absolute value) of γ_r, not its sign, matters for these impulse responses in Figure 1. It is because the magnitude of γ_r determines how households should trust the signal $-\gamma_\tau s_{b,t}^* + \gamma_r u_t^r + \eta_t^\tau$. As the absolute value of γ_r increases, households can figure out the underlying real interest rate shocks more easily. In the extreme cases of $\gamma_r = \pm\infty$, the incomplete information impulse responses of interest rate shocks are identical to their FIRE counterparts.

Figure 2 plots the impulse responses of inflation to a one-standard-deviation primary surplus shock. To see the effects of fiscal stimulus, we consider either $\varepsilon_1^\tau = -\sigma_{\varepsilon,\tau}$ or $\eta_1^\tau = -\sigma_{\eta,\tau}$ so that the government is decreasing primary surpluses at $t = 1$. The FIRE model generates identically zero fiscal inflation impulse responses as expected. Noticeably,

PROPOSITION 2: *Given the model structure $\mathcal{M}$ and $\mathcal{I}_t^{HH}$. Assume $\phi_\pi > 1$, $0 < \gamma_b < 1 < \gamma_\tau$. If $\gamma_r = 0$, then fiscal policy is Ricardian.*

The fiscal impulse responses, thus, are still identically zero when $\gamma_r = 0$. It is

[4]For technical details of the Kalman recursion, see Hamilton (1994).

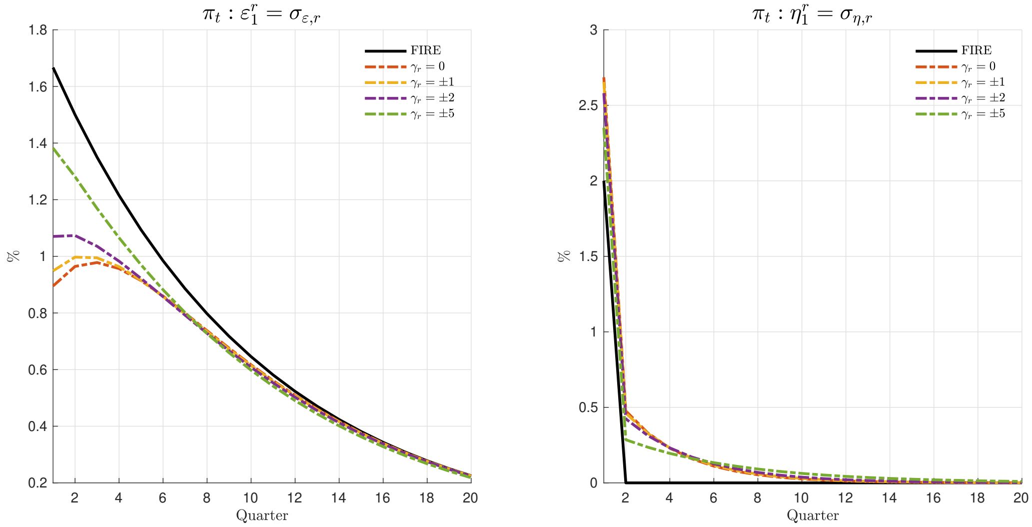

Figure 1. Impulse responses of inflation to positive real interest rate shocks.

because the household is facing a decoupled signal extraction problem as $\gamma_r = 0$. While the signal r_t always only contains information on real interest rate shocks, the fiscal signal now only contains information on primary surplus shocks as it becomes $-\gamma_\tau s^*_{b,t} + \eta^\tau_t$. When an $\varepsilon^\tau_1 = -\sigma_{\varepsilon,\tau}$ hits the economy, households know there are no real interest rates since $r_{1-k} \equiv 0$. As the signal extraction only exists among persistent and transitory shocks, fiscal policy is still Ricardian in the incomplete information model when $\gamma_r = 0$.

No-trivial fiscal inflation can emerge when $\gamma_r \neq 0$. We first discuss the case $\gamma_r < 0$. When the only shock $\varepsilon^\tau_1 = -\sigma_{\varepsilon,\tau}$ hits the economy, the real interest signals r_1 stays at zero, while the fiscal signal $-\gamma_\tau s^*_{b,1} + \gamma_r u^r_1 + \eta^\tau_1$ turns negative.[5] Given a $\gamma_r < 0$, a rational expectations household's perceived persistent real interest rate shock, denoted by $u^r_{1|1}$, needs to be positive, while the perceived transitory real interest rate shock, $\eta^r_{1|1} = r_1 - u^r_{1|1}$, is negative. The inflationary pressure of a higher $u^r_{1|1}$ dominates the deflationary pressure of a negative $\eta^r_{1|1}$, leading to an increase in inflation at $t = 1$. Furthermore, due to the persistence of $s^*_{b,t}$, fiscal inflation is also persistent and gradually converges to zero. On the other hand, when $\gamma_r > 0$, a negative primary surplus shock $\varepsilon^\tau_1 = -\sigma_{\varepsilon,\tau}$ will lead to deflation in the incomplete information model.

We conclude the Fisherian economy by briefly discussing the right panel of Figure 2. While the impact responses of the transitory fiscal shock at $t = 1$ are similar to their persistent counterparts, the subsequent impulse responses also display an interesting "stepping on a rake" pattern (i.e., monetary price

[5]A negative primary surplus shock $\varepsilon^\tau_1 = -\sigma_{\varepsilon,\tau}$ increases the hidden debt target $s^*_{b,t}$.

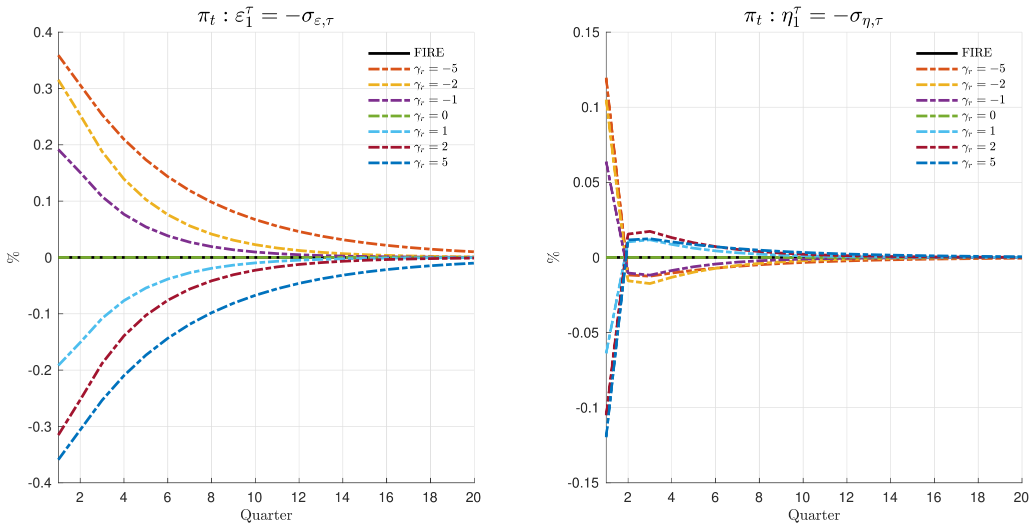

FIGURE 2. IMPULSE RESPONSES OF INFLATION TO NEGATIVE PRIMARY SURPLUS SHOCKS.

puzzle) documented in Sims (2011). Berkelmans (2011) explains the pattern and shows that a monetary tightening can raise the initial price level using a similar incomplete information friction. In our model, depending on the sign of γ_r, the fiscal price puzzle can go both ways.

III. A Benchmark Production Economy with Sticky Price

A key drawback of the Fisherian economy is that we have assumed an exogenous real interest rate process r_t. We now consider a benchmark production economy that features price stickiness (i.e., Calvo pricing) where real interest rate can be determined endogenously.

The representative household maximizes

$$\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{L_t^{1+\chi}}{1+\chi} \right],$$

where $\beta \in (0,1)$ is the deterministic discount factor and $1/\chi$ is the Frisch elasticity of labor supply L_t. During each period, the household earns real wage income $W_t L_t$ and receives real lump-sum dividend D_t from firms. They spend income on the final consumption goods C_t and have access to the one-period, nominal government bond B_t. They also need to pay real lump-sum net taxes T_t (i.e., primary surplus). Denote R_t the nominal interest rate. The household's flow budget constraint is

$$P_t C_t + Q_t B_t + P_t T_t = P_t W_t L_t + B_{t-1} + P_t D_t,$$

where P_t is the aggregate price level and $Q_t = 1/R_t$ is the price of the bond.

The household's inter-temporal Euler equation pins down the price of bond as

$$Q_t = \beta \mathbb{E}_t^{HH} \frac{P_t}{P_{t+1}} \frac{C_t}{C_{t+1}}.$$

The household's intra-temporal Euler equation determines the real wage as

$$W_t = C_t L_t^{\chi}.$$

There is a perfectly competitive sector of final good firms that produces the final consumption good Y_t by combining a unit measure of intermediate differentiated inputs. Final good is aggregated according to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\eta_t^p + u_t^p}} \right)^{1+\eta_t^p + u_t^p}, \tag{14}$$

where $\eta_t^p \sim N(0, \sigma_{\eta,p}^2)$ is an *i.i.d.* price mark-up shock that changes the elasticity of substitution among intermediate goods. Besides η_t^p, there is also a highly persistent cost-push shock, u_t^p.[6] The exogenous u_t^p evolves according to

$$u_t^p = \rho_p u_{t-1}^p + \varepsilon_t^p, \quad \varepsilon_t^p \sim N(0, \sigma_{\varepsilon,p}^2),$$

with $\rho_p \in (0,1)$ and $\rho_p \approx 1$. The near-unit-root process is meant to capture low-frequency movements of inflation.

There are a continuum of intermediate firms indexed by $j \in [0,1]$. Firms are endowed with a linear production $Y_t(j) = L_t(j)$, where $Y_t(j)$ is the production level and $L_t(j)$ is the amount of labor hired by firm j at time t. Denote $\theta \in (0,1)$ the price stickiness. During each period, there is a constant probability $(1-\theta)$ that a firm can reset its price to maximize its current market value of profit generated while that price remains effective (i.e., Calvo pricing). Cost minimization implies that firms are subject to the same real marginal cost $MC_t = W_t$. Since households own the firms and the real marginal costs are the same across firms, we assume all firms share the same information set as the household, that is, $\mathcal{I}_t^{HH} = \mathcal{I}_t^{Firm}$ and $\mathbb{E}_t^{HH}(\cdot) = \mathbb{E}_t^{Firm}(\cdot)$.[7]

Monetary policy follows the same Taylor rule as in the Fisherian economy. Let τ_t and $s_{b,t}$ denote the log-deviations of primary surplus-to-output T_t/Y_t and the log-deviations of real market debt-to-output ratio $(Q_t B_t / P_t Y_t)$ from their

[6] An equivalent alternative is to introduce u_t^p as a persistent labor disutility shock that enters the household's preference directly and η_t^p as a transitory wage mark-up shock.

[7] Technically, the symmetric information assumption eliminates the higher-order beliefs between different economic agents and allows us to both solve and estimate the quantitative model timely. Han, Ma and Mao (2022) considers the firm's pricing problem and show how dispersed information among firms shapes inflation and inflation expectations. Han (2024) introduces asymmetric information among households, firms, and the central bank to generate misaligned inflation expectations.

steady state values, respectively. The log-linearized equilibrium conditions of the benchmark production economy are

$$\begin{aligned}
&\textit{IS equation:} && y_t = \mathbb{E}_t^{HH} y_{t+1} - (i_t - \mathbb{E}_t^{HH}\pi_{t+1}) \\
&\textit{Phillips curve:} && \pi_t = \beta\mathbb{E}_t^{HH}\pi_{t+1} + \kappa(1+\chi)y_t + \kappa(u_t^p + \eta_t^p) \\
&\textit{Monetary policy:} && i_t = \phi_\pi \pi_t \\
&\textit{GBC:} && s_{b,t} = \beta^{-1}\left[s_{b,t-1} + (y_{t-1} - y_t) + i_{t-1} - \pi_t - (1-\beta)\tau_t\right]
\end{aligned}$$

with $\kappa = (1-\theta)(1-\beta\theta)/\theta$.

Fiscal policy now follows a more standard rule (see Leeper, Plante and Traum (2010)) that respond to both the real market debt and output gap. Let

$$\tau_t = \underbrace{\gamma_\tau(s_{b,t-1} - s_{b,t}^*)}_{\textit{debt stabilization}} + \underbrace{\phi_{\tau,y}(y_t - y_t^n)}_{\textit{output gap stabilization}} + \eta_t^\tau, \tag{15}$$

where y_t^n denote the natural output and $\eta_t^\tau \sim N(0, \sigma_{\eta,\tau}^2)$. The parameter $\phi_{\tau,y} \geq 0$ control the strengths with which the primary surplus reacts to the output gap (i.e., automatic stabilizer). The time-varying debt target, $s_{b,t}^*$, still follows the AR(1) rule (4).

Consistent with the DSGE literature, natural output y_t^n is defined as the output that "would prevail under flexible prices and wages in the absence of the 'mark-up' shocks" (see Smets and Wouters (2007)). It follows $y_t^n = -(1+\chi)^{-1}u_t^p$ is only driven by the persistent cost-push shock.

We are now ready to specify the incomplete information set $\mathcal{I}_t^{HH}$. Consistent with the Fisherian economy, we assume only the total cost-push shock, $u_t^p + \eta_t^p$, are observable to households. Rewriting the fiscal rule (15) as

$$\tau_t - \gamma_\tau s_{b,t-1} - \phi_{\tau,y} y_t = -\gamma_\tau s_{b,t}^* + \frac{\phi_{\tau,y}}{1+\chi}u_t^p + \eta_t^\tau$$

indicate that the right side variables are also known to households. We can define the household's information set as follows-

$$\mathcal{I}_t^{HH} = \{(u_{t-k}^p + \eta_{t-k}^p), (-\gamma_\tau s_{b,t-k}^* + \frac{\phi_{\tau,y}}{1+\chi}u_{t-k}^p + \eta_{t-k}^\tau), \mathcal{M} | k \geq 0\}.$$

For the production benchmark, we fix an inverse of Frisch elasticity $\chi = 2$. We follow Nakamura and Steinsson (2008) and fix $\theta = 0.6$. The moderate price stickiness implies on average firms reset prices every 7.5 months. We set $\rho_p = 0.98$. The standard deviations of price mark-up and cost-push shocks are $\sigma_{\varepsilon,p} = 5$ and $\sigma_{\eta,p} = 10$. Other parameters $\{\beta, \phi_\pi, \gamma_\tau, \gamma_b, \sigma_{\varepsilon,\tau}, \sigma_{\eta,\tau}\}$ remain the same as in the Fisherian economy. We vary the values of $\phi_{\tau,y} = \{0, 1, 2, 5\}$ to show its effect on output and inflation.

Figure 3 plots the impulse responses of output and inflation to either a 1-$\sigma_{\varepsilon,p}$

cost-push shock or a 1-$\sigma_{\eta,p}$ price mark-up shock. Consistent with conventional wisdom, both shocks lower output and increase inflation on impact in both full- and incomplete-information models, thus can be categorized as supply shocks. Since fiscal policy is Ricardian under FIRE, the magnitude of $\phi_{\tau,y}$ does not impact the full-information model's impulse responses: The responses are persistent to the AR(1) cost-push shock, and are transitory to the *i.i.d.* price mark-up shock. Under incomplete information, since households also utilize a fiscal signal that involves $\phi_{\tau,y}$ to extract information about the underlying shocks, different values of $\phi_{\tau,y}$ generate various impulse responses. Given the current parameterization, a persistent cost-push shock $\varepsilon_1^p = \sigma_{\varepsilon,p}$ generates hump-shaped output responses, and has a much larger inflationary impact if $\phi_{\tau,y}$ is small.

Figure 4 plots the impulse responses of output and inflation to negative primary surplus shocks. Consistent with the Fisherian economy, both output and inflation display non-trivial impulse responses when $\phi_{\tau,y} > 0$.[8] Fiscal policy thus is non-Ricardian under incomplete information. Importantly, output increases on impact to either a $\varepsilon_1^\tau = -\sigma_{\varepsilon,\tau}$ or a $\eta_1^\tau = -\sigma_{\eta,\tau}$, so these negative primary surplus shocks (i.e., more lump-sum transfers) are indeed expansionary. Inflation, on the other hand, also increases on impact. These findings support the classic view that fiscal shocks, in general, are demand shocks and move output and inflation in the same direction. Crucially, persistent fiscal inflation can arise to the persistent fiscal shock $\varepsilon_1^\tau = -\sigma_{\varepsilon,\tau}$ (or equivalently, a time-varying debt target shock). As $\phi_{\tau,y}$ increases, the stabilization role plays a larger role in shaping the endogenous variables' impulse responses. For example, when $\phi_{\tau,y} = 5$, both output and inflation's initial impulse responses are smaller compared to the case $\phi_{\tau,y} = 2$.

IV. A Quantitative Model

We now embed the incomplete information structure to a medium-scale DSGE New Keynesian framework. Following Leeper, Traum and Walker (2017) and Bianchi, Faccini and Melosi (2023), the model includes a large set of real and nominal frictions and a rich fiscal block. These features are included to improve the model's empirical fit and match US business cycle dynamics. Since the model structure is standard, we describe here its main ingredients and leave the details and the log-linearization to the Appendix C.

There are two types of households in the economy, savers and hand-to-mouth consumers. Both households are subject to external habit formation and a discount factor shock $d_t = u_t^d + \eta_t^d$, where u_t^d is the persistent and η_t^d is the transitory component of d_t. They receive wage income by providing labor to firms, and are subject to various taxes. They also receive lump-sum transfers from the government. Savers have access to both short- and long-term government bonds, and also can accumulate capital subject to variable capacity utilization and adjustment costs in investment. Hand-to-mouth households consume all of their

[8]As in Proposition 2, the impulse responses are still trivially zero in the incomplete-information model when $\phi_{\tau,y} = 0$.

disposable, after-tax income and do not save. We include both sticky wages and wage indexation in the wage setting equations. Households also face a persistent labor disutility shock, u_t^l.

The final goods firms remain the same as in the production benchmark and aggregate intermediate inputs using the technology (14). The total cost-push shock is $\eta_t^p + u_t^p$, where we interpret η_t^p as a transitory price mark-up shock and u_t^p as a persistent cost-push shock due to external forces such as international trade.

The intermediate goods firms are subject to Calvo pricing and price indexation. To allow balanced growth, the labor-augmenting technology A_t follows an exogenous process that is stationary in the growth rate. Specifically, the log-deviation of A_t/A_{t-1}, denoted by a_t, evolves according to $a_t = u_t^a + \eta_t^a$, where u_t^a is the persistent and η_t^a is the transitory component of a_t.

Monetary policy follows a generalized Taylor rule

$$\hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + (1-\rho_r)(\phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t) + u_t^m, \tag{16}$$

which embeds interest rate inertia and satisfies the Taylor principle (i.e., $\phi_\pi > 1$). It also responds to deviation of inflation $\hat{\pi}_t$ and output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$. As in the production benchmark, we derive a natural output $\hat{y}_t^n = -(1+\chi)^{-1} u_t^p$ where χ is the inverse of Frisch elasticity. The existence of the persistent monetary policy shock, u_t^m, prevents households from learning u_t^p perfectly from the monetary policy rule (16).

Fiscal policy consists of a set of rules. Let $s_{b,t}$ denote the real market debt-to-output ratio. The fiscal authority adjusts government spending $\hat{g}_t$, transfers $\hat{z}_t$, and tax rates on capital income, labor income, and consumption $\hat{\tau}_J$ as follows:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\left[\gamma_G\left(\hat{s}_{b,t-1} - \hat{s}_{b,t}^*\right) + \phi_{g,y}\tilde{y}_t\right] + u_t^g, \tag{17}$$

$$\hat{z}_t^b = \rho_Z \hat{z}_{t-1}^b - (1-\rho_Z)\left[\gamma_Z\left(\hat{s}_{b,t-1} - \hat{s}_{b,t}^*\right) + \phi_{z,y}\tilde{y}_t\right] + u_t^z, \tag{18}$$

$$\hat{\tau}_{J,t} = \rho_J \hat{\tau}_{J,t-1} + (1-\rho_J)\gamma_J \hat{s}_{b,t-1}, \text{ for } J \in \{K, L, C\}; \tag{19}$$

where γ_G, γ_Z, and $\gamma_J > 0$ are large enough to guarantee that debt remains on a stable path. As in the Fisherian economy, the time-varying debt target, $s_{b,t}^*$, follows a stationary AR(1) process

$$\hat{s}_{b,t}^* = \rho_b \hat{s}_{b,t-1}^* + \varepsilon_t^b, \quad \varepsilon_t^b \sim N(0, \sigma_{\varepsilon,b}^2),$$

with $\rho_b \in (0,1)$. It is noteworthy we let all tax rates respond neither to the hidden debt target $s_{b,t}^*$, nor the output gap $\tilde{y}_t$. Such a modeling choice follows Leeper, Traum and Walker (2017) and Bianchi, Faccini and Melosi (2023). We introduce incomplete information by assuming all economic agents can observe the entire histories of $\{d_t, (u_t^p + \eta_t^p), a_t\}$, but cannot disentangling their persistent and transitory components. Since monetary and fiscal policy rules (16), (17), and (18) all involves the output gap $\tilde{y}_t$, these policy instruments can also serve a signaling role (see Melosi (2017)) to the private sector in revealing the natural output $\hat{y}_t^n$ and the hidden time-varying debt target $s^*_{b,t}$. Appendix C.C5 shows that we can define $\{s_t^m, s_t^g, s_t^z\}$ as a set of exogenous signals the policy instruments can convey to the private sector. We do not introduce incomplete information to any other shocks in the economy. More formally, we define an incomplete information set as-

$$\mathcal{I}_t^{HH} = \{d_{t-k}, a_{t-k}, (u_{t-k}^p + \eta_{t-k}^p), u_{t-k}^i, u_{t-k}^l, \eta_t^w, s_{t-k}^m, s_{t-k}^g, s_{t-k}^z, \mathcal{M}|k \geq 0\},$$

where u_t^i is a persistent investment adjustment cost shock, u_t^l is a persistent labor disutility shock, and η_t^w is a transitory wage mark-up shock.

V. Conclusions

In this paper, we generate fiscal inflation without the assumption of a passive central bank and unfunded shocks, both of which are required under the existing class of FTPL models. Our model breaks Ricardian equivalence by introducing information frictions in the expectations formation process of economic agents. Households, firms, and the central bank only observe composite signals of policy response and use those signals to extract information about the underlying shock. In a small-scale model with price stickiness, we show that a positive shock to primary surplus increase output and inflation, consistent with the effects of a fiscal policy shock in a textbook IS-LM framework.

In ongoing work, we are extending our analysis with the following- a) Bayesian estimation of the large scale DSGSE model presented in Section IV under incomplete information, b) comparing model fit of FIRE and incomplete information estimation for different U.S. sample periods, c) out-of-sample estimation for the high inflation and subsequent disinflation post-COVID.

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APPENDIX

Recovering AMPF and PMAF dynamics in Leeper (1991)

This section recovers the classic *Active Monetary-Passive Fiscal* (AMPF) and *Passive Monetary-Active Fiscal* (PMAF) inflation dynamics of the simple model in Leeper (1991). For simplicity, we focus on fiscal inflation and shut down the real interest rate shocks (i.e., $\sigma_{\varepsilon,r} = 0, \sigma_{\eta,r} = 0$). Since there is also no persistent fiscal shock (i.e., $\sigma_{\varepsilon,\tau} = 0$), the only shock in the economy is the transitory fiscal shock η_t^τ. The simplified equilibrium conditions are

(A1) Fisher equation: $i_t = \mathbb{E}_t^{HH}\pi_{t+1}$

(A2) Monetary policy: $i_t = \phi_\pi \pi_t$

(A3) Gov. budget constraint: $s_{b,t} = \beta^{-1}\left[s_{b,t-1} + i_{t-1} - \pi_t - (1-\beta)\tau_t\right]$

(A4) Fiscal policy: $\tau_t = \gamma_\tau s_{b,t-1} + \eta_t^\tau$

Denote the inflation dynamics $\pi_t = \Pi(L)\eta_t^\tau = \sum_{j=0}^{\infty}\Pi_j\eta_t^\tau$. We utilize the frequency domain techniques (see Kasa, Walker and Whiteman (2014)) to derive the functional form of $\Pi^{FI}(L)$. Combining the Fisher equation (A1) and the Taylor rule (A2) and applying the Wiener-Kolmogorov formula lead to

$$L^{-1}\left[\Pi(L) - \Pi_0\right] = \phi_\pi \Pi(L).$$

Rearranging the terms yield

$$\Pi(L) = \frac{\Pi_0}{1 - \phi_\pi L}. \tag{A5}$$

If $\phi_\pi > 1$ (i.e., active monetary policy), the above equation defines a stationary inflation process if and only if $\Pi_0 = 0$. It follows the $\Pi(L) = 0$ and $\pi_t = 0$ under active monetary policy. Combining the government budget constraint (A3) and the fiscal rule (A4) and plugging in $i_t = 0, \pi_t = 0$ yield

$$s_{b,t} = \beta^{-1}\left[1 - (1-\beta)\gamma_\tau\right]s_{b,t-1} - \beta^{-1}\left[(1-\beta)\right]\eta_t^\tau. \tag{A6}$$

If $0 < \beta^{-1}\left[1 - (1-\beta)\gamma_\tau\right] < 1$, or equivalently, $\gamma_\tau > 1$ (i.e., passive fiscal policy), then the $s_{b,t}$ defined by (A6) is always a stationary process. We summarize the AMPF inflation and debt dynamics as

(A7)

$$\text{AMPF:} \quad \pi_t = 0, \quad s_{b,t} = -\frac{\beta^{-1}(1-\beta)}{1 - \beta^{-1}\left[1 - (1-\beta)\gamma_\tau\right]L}\eta_t^\tau, \text{ when } \phi_\pi > 1, \gamma_\tau > 1.$$

If $0 < \phi_\pi < 1$ (i.e., passive monetary policy), then equation (A5) always defines a stationary AR(1) inflation process, where Π_0 is a free parameter. Since $i_t =$

$\phi_\pi \Pi_0/(1-\phi_\pi L)\varepsilon_t, \pi_t = \Pi_0/(1-\phi_\pi L)\varepsilon_t$, it follows

$$i_{t-1} - \pi_t = \frac{\Pi_0 \phi_\pi L}{1-\phi_\pi L}\eta_t^\tau - \frac{\Pi_0}{1-\phi_\pi L}\eta_t^\tau = -\Pi_0 \eta_t^\tau.$$

Combining the government budget constraint (A3) and the fiscal rule (A4) and plugging in $i_{t-1} - \pi_t = \Pi_0 \eta_t^\tau$ yield

$$s_{b,t} = \beta^{-1}\left[1-(1-\beta)\gamma_\tau\right] s_{b,t-1} - \beta^{-1}\left[\Pi_0 + (1-\beta)\right]\eta_t^\tau. \tag{A8}$$

If $0 < \gamma_\tau < 1$ so that $\beta^{-1}\left[1-(1-\beta)\gamma_\tau\right] > 1$ (i.e., active fiscal policy), we can rewrite equation (A8) using the lag operator L as

$$\left(1 - \beta^{-1}\left[1-(1-\beta)\gamma_\tau\right] L\right) s_{b,t} = -\beta^{-1}\left[\Pi_0 + (1-\beta)\right]\eta_t^\tau \tag{A9}$$

Evaluating the above equation at $L = z_1 = \beta\left[1-(1-\beta)\gamma_\tau\right]^{-1}$ pins down

$$\Pi_0 = -(1-\beta),$$

leading to the PMAF inflation and debt dynamics as

$$\text{PMAF:} \quad \pi_t = \frac{\beta - 1}{1-\phi_\pi L}\eta_t^\tau, \quad s_{b,t} = 0, \text{ when } 0 < \phi_\pi < 1, 0 < \gamma_\tau < 1. \tag{A10}$$

Deriving determinacy condition of the Fisherian economy

This section derives the determinacy condition of the Fisherian economy on γ_r given $\phi_\pi > 1$ and $0 < \gamma_b < 1 < \gamma_\tau$. Since $\mathcal{I}_t^{HH}$ and $\mathcal{I}_t^{FIRE}$ share the same determinacy region, we consider the FIRE model in this section. For convenience, we shut down both the persistent and transitory fiscal shocks (i.e., $\sigma_{\varepsilon,\tau} = 0, \sigma_{\eta,\tau} = 0$) and the transitory interest rate shock (i.t., $\sigma_{\eta,r} = 0$) as these simplifications do not change the determinacy region. The simplified equilibrium conditions are

$$\begin{aligned}
&\text{(B1)} \quad && \text{Fisher equation:} \quad && i_t = \mathbb{E}_t^{FIRE}\pi_{t+1} + u_t^r \\
&\text{(B2)} \quad && \text{Monetary policy:} \quad && i_t = \phi_\pi \pi_t \\
&\text{(B3)} \quad && \text{Gov. budget constraint:} \quad && s_{b,t} = \beta^{-1}\left[s_{b,t-1} + i_{t-1} - \pi_t - (1-\beta)\tau_t\right] \\
&\text{(B4)} \quad && \text{Fiscal policy:} \quad && \tau_t = \gamma_\tau s_{b,t-1} + \gamma_r u_t^r
\end{aligned}$$

Again, we use the frequency domain techniques to derive the REE. Denote the inflation dynamics $\pi_t = \Pi(L)\varepsilon_t^r = \sum_{j=0}^\infty \Pi_j \varepsilon_t^r$. Denote $R(L) = \frac{1}{1-\rho_r L}$. Combining the Fisher equation (B1) and the Taylor rule (B2) and applying the Wiener-Kolmogorov formula lead to

$$L^{-1}\left[\Pi(L) - \Pi_0\right] + R(L) = \phi_\pi \Pi(L).$$

Rearranging the terms yield

$$\Pi(L) = \frac{\Pi_0 - R(L)L}{1 - \phi_\pi L}. \tag{B5}$$

Since $\phi_\pi > 1$, the above equation defines a stationary inflation process if and only if $\Pi_0 = R(\phi_\pi^{-1})\phi_\pi^{-1}$. It follows

$$\Pi(L) = \frac{1}{\phi_\pi - \rho_r}\frac{1}{1 - \rho_r L},$$

and

$$i_{t-1} - \pi_t = \frac{1}{\phi_\pi - \rho_r}\frac{\phi_\pi L - 1}{1 - \rho_r L}\varepsilon_t^r.$$

Combining the government budget constraint (B3) and the fiscal rule (B4) yields

$$s_{b,t} = \beta^{-1}\left[1 - (1-\beta)\gamma_\tau\right] s_{b,t-1} + \beta^{-1}(i_{t-1} - \pi_t) - \beta^{-1}(1-\beta)\gamma_r u_t^r \tag{B6}$$

If $0 < \beta^{-1}\left[1 - (1-\beta)\gamma_\tau\right] < 1$, or equivalently, $\gamma_\tau > 1$ (i.e., passive fiscal policy), then the $s_{b,t}$ defined by (B6) is always a stationary process. It follows γ does not change the determinacy region as long as $\phi_\pi > 1$ and $0 < \gamma_b < 1 < \gamma_\tau$.

The Medium-scale DSGE Model Environment

C1. Households

There are two types of households in the economy and their measures sum up to one. Among these households, a fraction of μ are hand-to-mouth consumers and the remaining $1 - \mu$ are savers.

Savers

Savers, each indexed by j, derive utility from the consumption of a composite good, $C_t^{*S}(j)$, which comprises private consumption $C_t^S(j)$ and government consumption G_t such that $C_t^{*S}(j) = C_t^S(j) + \alpha_G G_t$. The parameter α_G governs the substitutability between private and government consumption. When α_G is negative (positive), these goods are complements (substitutes). External habits in consumption imply that utility is derived relative to the previous period value of aggregate savers' consumption of the composite good hC_{t-1}^{*S}, where $h \in [0,1]$ is the habit parameter. Saver households also derive disutility from the supply of differentiated labor services from all its members, indexed by $l, L_t^S(j) = \int_0^1 L_t^S(j,l)dl$. The period utility function is given by

$$U_t^S(j) = d_t\left(\ln\left(C_t^{*S}(j) - hC_{t-1}^{*S}\right) - u_t^l \frac{L_t^S(j)^{1+\chi}}{1+\chi}\right), \tag{C1}$$

where $d_t = u_t^d + \eta_t^d$ is a discount factor shock, u_t^l is a labor disutility shock, and $1/\chi$ is the Frisch elasticity of labor supply.

Savers accumulate wealth in the form of physical capital $\bar{K}_t^S$. The law of motion for physical capital is given by

$$\bar{K}_t^S(j) = (1-\delta)\bar{K}_{t-1}^S(j) + u_t^i \left[1 - s\left(\frac{I_t^S(j)}{I_{t-1}^S(j)}\right)\right] I_t^S(j), \tag{C2}$$

where δ is the depreciation rate, u_t^i is a shock to the marginal efficiency of investment and s denotes an investment adjustment cost function that satisfies the properties $s\left(e^{\varkappa}\right) = s'\left(e^{\varkappa}\right) = 0$ and $s''\left(e^{\varkappa}\right) > 0$, where $\varkappa$ is a drift parameter capturing the logarithm of the growth rate of technology in steady state.

Households derive income from renting effective capital $K_t^S(j)$ to the intermediate firms. Effective capital is related to physical capital according to following law of motion,

$$K_t^S(j) = \nu_t(j)\bar{K}_{t-1}^S, \tag{C3}$$

where $\nu_t(j)$ is the capital utilization rate. In steady state, the utilization rate $\nu(j)$ is 1. The cost of utilizing one unit of physical capital is given by the function $\Psi\left(\nu_t(j)\right)$ that satisfies the following properties: $\Psi(1) = 0$, and $\frac{\Psi''(1)}{\Psi'(1)} = \frac{\psi}{1-\psi}$, where $\psi \in [0, 1)$. We denote the gross rental rate of capital as $R_{K,t}$ and the tax rate on capital rental income as $\tau_{K,t}$.

The household can also save through purchasing one-period government bonds in zero net supply and a more general portfolio of long-term government bonds in non-zero net supply. The one-period bonds promising a nominal payoff B_t at time $t+1$ can be purchased at the present discounted value $R_{n,t}^{-1}B_t$, where $R_{n,t}$ is the gross nominal interest rate set by the central bank. The long-term bond B_t^m mimics a portfolio of bonds with average maturity m and duration $(1-\beta\rho)^{-1}$, where $\rho \in [0, 1]$ is a constant rate of decay. This bond can be purchased at price P_t^m, which is determined by the arbitrage condition

$$R_{n,t} = \mathbb{E}_t^{HH}[(1 + P_{t+1}^m)/P_t^m]\, e^{-u_t^{rp}},$$

where the wedge u_t^{rp} can be interpreted as a risk premium shock.

In every period, households receive after-tax nominal labor income, after-tax revenues from renting capital to the firms, lump-sum transfers from the government Z_t^S, and dividends from the firms D_t. These resources can be spent to consume and to invest in physical capital and bonds. Omitting the index j to simplify the notation, we can write the nominal budget constraint for the saver household as

$$\begin{aligned} & P_t\left(1+\tau_{C,t}\right)C_t^S+P_tI_t^S+P_t^mB_t^m+R_{n,t}^{-1}B_t \\ &=\left(1+\rho P_t^m\right)B_{t-1}^m+B_{t-1}+\left(1-\tau_{L,t}\right)\int_0^1 W_t(l)L_t^S dl \\ &\quad+\left(1-\tau_{K,t}\right)R_{K,t}\nu_t\bar{K}_{t-1}^S-\Psi\left(\nu_t\right)\bar{K}_{t-1}^S+P_tZ_t^S+D_t, \end{aligned} \tag{C4}$$

where $W_t(l)$ denotes the wage rate faced by all household members, and $\tau_{C,t}$ and $\tau_{L,t}$ denote the tax rates on consumption and labor income, respectively. The household maximizes expected utility $\mathbb{E}_0^{HH}\sum_{t=0}^{\infty}\beta^tU_t^S$ subject to the sequence of budget constraints in equation (C4) and the law of motion of capital accumulation (C2).

For ease of notations, we drop the index j in the following. The first-order optimality conditions (F.O.Cs) with respect to consumption, labor supply, one-period bond, investment, capital, and capital utilization are

$$(\partial C_t^{*S}) \quad \Lambda_t^S=d_t(C_t^{*S}-hC_{t-1}^{*S})^{-1} \tag{C5}$$

$$(\partial L_t^S) \quad u_t^lL_t^{S,\chi}=\Lambda_t^S(1-\tau_{L,t})\frac{W_t^h}{P_t} \tag{C6}$$

$$(\partial B_t) \quad \Lambda_t^S=\beta R_{nt}\mathbb{E}_t^{HH}\left[\frac{\Lambda_{t+1}^S}{\pi_{t+1}}\right] \tag{C7}$$

$$\begin{aligned}(\partial I_t) \quad 1=Q_t^k\mu_t&\left[1-s\left(\frac{I_t}{I_{t-1}}\right)-s'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\ &+\beta\mathbb{E}_t^{HH}\left\{\frac{\Xi_{t+1}^K}{\Lambda_t^S}\left[Q_{t+1}u_{t+1}^is'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]\right\}\end{aligned} \tag{C8}$$

$$(\partial\bar{K}_t)\ Q_t=\beta\mathbb{E}_t^{HH}\left\{\frac{\Xi_{t+1}^K}{\Lambda_t^S}\left[((1-\tau_{K,t})r_{K,t}\nu_{t+1}-\Psi(\nu_{t+1}))+Q_{t+1}(1-\delta)\right]\right\} \tag{C9}$$

$$(\partial\nu_t) \quad (1-\tau_{K,t})r_{K,t}=\Psi'(\nu_t) \tag{C10}$$

where Λ_t^S and Ξ_{t+1}^K are the Lagrange multipliers associated with the budget and capital accumulation constraints, respectively, and $Q_t=\frac{\Xi_t^K}{\Lambda_t^S}$ is the Tobin's Q and equals to one in the absence of adjustment costs.

Hand-to-Mouth Households

In every period, hand-to-mouth households derive disposable, after-tax income from labor supply and government transfers and consume all of them. They supply differentiated labor services, and set their wage to be equal to the average wage that is optimally chosen by the savers, as described below. Hand-to-mouth households face the same tax rates on consumption and labor income as savers.

The specification of period-by-period utility for hand-to-mouth households is the same as that of savers, i.e.,

$$U_t^N(j) = d_t \left(\ln \left(C_t^{*N}(j) - hC_{t-1}^{*N} \right) - \frac{u_t^l L_t^N(j)^{1+\chi}}{1+\chi}) \right). \tag{C11}$$

Their budget constraint is

$$\left(1+\tau_{C,t}\right) P_t C_t^N = \left(1-\tau_{L,t}\right) \int_0^1 W_t(l) L_t^N(l) dl + P_t Z_t^N, \tag{C12}$$

where the superscript N indicates the variables for hand-to-mouth households.

The hand-to-mouth households maximize the discounted utility $\mathbb{E}_0^{HH} \sum_{t=0}^{\infty} \beta^t U_t^N$ subject to the budget constraint (C12). The F.O.Cs are

$$(\partial(C_t^{*N})) \quad \Lambda_t^N = d_t (C_t^{*N} - hC_{t-1}^{*S})^{-1}, \tag{C13}$$

$$(\partial L_t^N) \quad u_t^l L_t^{S,\chi} = \Lambda_t^N (1-\tau_{L,t}) \frac{W_t}{P_t}. \tag{C14}$$

C2. Firms

Final good producers

There is a perfectly competitive sector of final good firms that produces the homogeneous good Y_t at time t by combining a unit measure of intermediate differentiated inputs using the aggregation technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\eta_t^p + u_t^p}} di \right)^{1+\eta_t^p + u_t^p}, \tag{C15}$$

where η_t^p is the *i.i.d.* price mark-up shock. The variable u_t^p is a cost-push shock, and is assumed to follow a near-unit-root process. The highly persistent cost-push shock is meant to capture other external forces, such as international trade, that can generate low-frequency movements of inflation. Profit maximization yields the demand function for intermediate goods as

$$Y_t(i) = Y_t \Big(\frac{P_t(i)}{P_t} \Big)^{-\frac{1+\eta_t^p + u_t^p}{\eta_t^p + u_t^p}}, \tag{C16}$$

where $P_t(i)$ is the price of the differentiated good i and P_t is the price of the final good.

Intermediate good producers

There is a unit measure of intermediate firms who produce goods according to the production function

$$Y_t(i) = K_t(i)^{\alpha} \left(A_t L_t(i)\right)^{1-\alpha} - A_t \Omega, \tag{C17}$$

where Ω is a fixed cost of production that grows with the rate of labor-augmenting technological progress A_t, and $\alpha \in [0,1]$ is the capital share. The labor-augmenting technological progress A_t follows an exogenous process that is stationary in the growth rate. Specifically, we assume that $a_t = \ln\left(A_t/A_{t-1}\right) - \varkappa = u_t^a + \eta_t^a$. Intermediate firms rent capital and labor from perfectly competitive capital and labor markets, respectively. As described in the following, L_t is a bundle of all the differentiated labor services supplied in the economy, which are aggregated into a homogeneous input by a labor agency. Intermediate firms' cost minimization implies the same nominal marginal cost for all firms

$$MC_t = (1-\alpha)^{\alpha-1} \alpha^{-\alpha} \left(R_{K,t}\right)^{\alpha} W_t^{1-\alpha} A_t^{-1+\alpha}. \tag{C18}$$

Intermediate producers reset price in the spirit of the Calvo pricing. At time t, a firm i can optimally reset its price with probability ω_p. Otherwise, it adjusts the price with partial indexation to the previous period inflation rate according to the rule

$$P_t(i) = \left(\Pi_{t-1}\right)^{\xi_p} (\Pi)^{1-\xi_p} P_{t-1}(i), \tag{C19}$$

where $\xi_p \in [0,1]$ is a parameter, $\Pi_{t-1} = P_{t-1}/P_{t-2}$, and Π denotes the aggregate rate of inflation at steady state. Intermediate producers that are allowed to reset their price maximize the expected discounted stream of nominal profits,

(C20)

$$\max \mathbb{E}_t^{Firm} \sum_{s=0}^{\infty} (\beta\omega_p)^s \frac{\Lambda_{t+s}^S}{\Lambda_t^S} \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\xi^p} \Pi^{1-\xi^p} \right) P_t(i) Y_{t+s}(i) - MC_{t+s} Y_{t+s}(i) \right],$$

subject to the demand function (C16), with Λ_t^S denoting the marginal utility of the savers.

The F.O.C is given by

(C21)

$$\max \mathbb{E}_t^{Firm} \sum_{s=0}^{\infty} (\beta\omega_p)^s \frac{\Lambda_{t+s}^S}{\Lambda_t^S} Y_{t+s}(i) \left[\frac{-1}{\eta_t^p + u_t^p} X_{t,s}^P P_t(i) + \frac{1 + \eta_t^p + u_t^p}{\eta_t^p + u_t^p} MC_{t+s} \right] = 0,$$

where

$$X_{t,s}^P = \begin{cases} 1 & \text{for } s = 0 \\ \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\xi^p} \Pi^{1-\xi^p} \right) & \text{for } s = 1, \ldots, \infty \end{cases} \tag{C22}$$

C3. Wage Settings

Both savers and hand-to-mouth households supply a unit measure of differentiated labor service, indexed by l. In each period, a saver household has probability ω_w to optimally re-adjust the wage rate that applies to all of its workers, $W_t(l)$. If the wage cannot be re-optimized, it will be increased at the geometric average of the steady-state rate of inflation Π and of last period inflation Π_{t-1}, according to the rule

$$W_t(l) = W_{t-1}(l)\left(\Pi_{t-1}e^{\varkappa}\right)^{\xi_w}\left(\Pi e^{\xi}\right)^{1-\xi_w}, \tag{C23}$$

where $\xi_w \in [0,1]$ captures the degree of nominal wage indexation. All households, including both savers and non-savers, sell their labor service to a representative, competitive agency that transforms it into an aggregate labor input, according to the technology

$$L_t = \left(\int_0^1 L_t(l)^{\frac{1}{1+\eta_t^w}} dl\right)^{1+\eta_t^w}, \tag{C24}$$

where η_t^w is an *i.i.d.* exogenous wage mark-up shock. The agency rents labor type $L_t(l)$ at price $W_t(l)$ and sells a homogeneous labor input to the intermediate producers at price W_t. The static profit maximization problem yields the labor demand function

$$L_t(l) = L_t\left(W_t(l)/W_t\right)^{-(1+\eta_t^w)/\eta_t^w}. \tag{C25}$$

Labor unions take this marginal rate of substitution as the cost of the labor services in their negotiations with the labor packers. The markup above the marginal disutility is distributed to the households. For those that can adjust, the problem is to choose a wage $W_t(l)$ that maximizes the discounted total wage income in the future subject to (C23) and (C25),

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \frac{\Lambda_{t+s}^S P_t}{\Lambda_t^S P_{t+s}}\left[W_{t+s}(l) - W_{t+s}^h\right] L_{t+s}(l). \tag{C26}$$

The F.O.C. becomes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \frac{\Lambda_{t+s}^S P_t}{\Lambda_t^S P_{t+s}} L_{t+s}(l)\left[\left(X_{t,s}^W W_t(l) - W_{t+s}^h\right)\left(-\frac{1+\eta_{w,t+s}}{\eta_{w,t+s}}\right) - X_{t,s}^W W_t(l)\right] = 0, \tag{C27}$$

where

$$X^W_{t,s} = \begin{cases} 1 & \text{for } s = 0 \\ \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\xi^w} \Pi^{1-\xi^w}\right) & \text{for } s = 1, \ldots, \infty \end{cases} \tag{C28}$$

C4. Monetary and Fiscal Policy

We have assumed the government supplies one-period bonds that are in zero net supply and both types pf households receive the same amount of transfers. It follows the government nominal budget constraint is

$$P^m_t B^m_t + \tau_{K,t} R_{K,t} K_t + \tau_{L,t} W_t L_t + \tau_{C,t} P_t C_t = \left(1 + \rho P^m_t\right) B^m_{t-1} + P_t G_t + P_t Z_t, \tag{C29}$$

where C_t and Z_t denote aggregate consumption and total transfers, respectively. Their expressions are the following

$$C_t = \mu C^N_t + (1-\mu) C^S_t, \tag{C30}$$

$$Z_t = \int_0^1 Z_t(j) dj. \tag{C31}$$

The budget constraint (C29) implies that the fiscal authority finances government expenditures, transfers, and the rollover of expiring long-term debt by issuing new long-term debt obligations as well as by raising labor, capital, and consumption taxes.

The aggregate resource constraint is given by

$$Y_t = C_t + I_t + G_t + \Psi_t(\nu_t) \bar{K}_{t-1}. \tag{C32}$$

We rescale the variables entering the fiscal rules as $g_t = G_t/A_t$ and $z_t = Z_t/A_t$. For each variable $x_t, \widehat{x}_t$ denotes the percentage deviation from its balanced-growth steady state. Additionally, we use $\tilde{y}_t$ to denote the percentage deviation of output from the natural output y^n, i.e., $\tilde{y}_t = \hat{y}_t - \hat{y}^n_t$.

Let $s_{b,t} = (P^m_t B^m_t)/(P_t Y_t)$ denote the real market debt-to-GDP ratio. The fiscal authority adjusts government spending $\hat{g}_t$, transfers $\hat{z}_t$, and tax rates on capital income, labor income, and consumption $\hat{\tau}_J, J \in \{K, L, C\}$ as follows:

$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\left[\gamma_G\left(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}\right) + \phi_{g,y}\tilde{y}_t\right] + u^g_t, \tag{C33}$$

$$\hat{z}^b_t = \rho_Z \hat{z}^b_{t-1} - (1-\rho_Z)\left[\gamma_Z\left(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}\right) + \phi_{z,y}\tilde{y}_t\right] + u^z_t, \tag{C34}$$

$$\hat{\tau}_{J,t} = \rho_J \hat{\tau}_{J,t-1} + (1-\rho_J)\gamma_J \hat{s}_{b,t-1}, \tag{C35}$$

where γ_G, γ_Z, and $\gamma_J > 0$ are large enough to guarantee that debt remains on a stable path. The time-varying debt target, $s^*_{b,t}$, follows a stationary AR(1) process

$$\hat{s}^*_{b,t} = \rho_b \hat{s}^*_{b,t-1} + \varepsilon^b_t, \quad \varepsilon^b_t \sim N(0, \sigma^2_{\varepsilon,b}),$$

with $\rho_b \in (0, 1)$.

Finally, the central bank follows a Taylor rule and adjusts the short-term interest rate R_t^n in response to deviations of inflation and output gap. The linearized monetary policy rule is the following:

$$\hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + (1 - \rho_r)(\phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t) + u_t^m, \tag{C36}$$

where u_t^m is the monetary policy shock and $\phi_\pi > 1$ satisfies the Taylor principle.

C5. *Specifying Shocks and Introducing Incomplete Information*

Throughout the paper, we use the notation u_t^x to denote a persistent shock and η_t^x to denote a transitory shock. For each $x \in \{d, i, rp, p, a, g, z, m\}$, we specify a stationary AR(1) process for the persistent shock u_t^x as

$$u_t^x = \rho_x u_{t-1}^x + \varepsilon_t^x, \quad \varepsilon_t^x \sim N(0, \sigma_{\varepsilon,x}^2);$$

with $\rho_x \in (0, 1)$. For $x \in \{d, p, a, w\}$, we let the transitory shock $\eta_t^x \sim N(0, \sigma_{\eta,x}^2)$ follows an *i.i.d.* process.

We now introduce incomplete information by assuming all households, intermediate firms, and final good firms share the same information set, denoted by $\mathcal{I}_t^{HH}$. We assume agents can only observe the entire histories of the discount factor shock $d_t = u_t^d + \eta_t^d$, the labor-augmenting technological process $a_t = u_t^a + \eta_t^a$, and the total cost-push shock $u_t^p + \eta_t^p$, rather than their persistent and transitory components.

We define the natural output $y_t^n = Y_t^n / A_t$ as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks, η_t^p and η_t^w. Furthermore, we make the simplifying assumptions that fiscal policy is not distortionary, and a simplified monetary rule with $\rho_r = \phi_y = 0$, and no monetary shocks $u_t^m \equiv 0$. We allow the potential output in the medium-scale model $\hat{y}_t^n$ to depend on persistent components of technology shock, cost-push shock, and the labor disutility shock, $y_t^n = \delta_a u_t^p + \delta_p u_t^p + \delta_l u_t^l$. The fact that the natural output is driven by only technology, cost-push, and labor disutility shock is consistent with the core idea in macroeconomics that while monetary and fiscal shocks can affect aggregate demand in the short run, in the long run the natural output level is only driven by the aggregate supply shocks.

Since $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n = \hat{y}_t + (\delta_a u_t^p + \delta_p u_t^p + \delta_l u_t^l)$, both the monetary policy rule (C36) and the fiscal rules (C33) and (C34) indicate policy variables can also serve as signals to households. For instance, rewriting the monetary rule (C36) as

$$\hat{r}_t^n - \rho_r \hat{r}_{t-1}^n - (1 - \rho_r)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right) = \underbrace{(1 - \rho_r)\phi_y(\delta_a u_t^p + \delta_p u_t^p + \delta_l u_t^l) + u_t^m}_{s_t^m}$$

indicates the history of the right side variables, s_t^m, is also known to households.

Similarly, rewriting the fiscal rules (C33) and (C34) as

$$\hat{g}_t - \rho_G \hat{g}_{t-1} + (1-\rho_G)\left[\gamma_G \hat{s}_{b,t-1} + \phi_{g,y}\hat{y}_t\right] = \underbrace{(1-\rho_G)\left[\gamma_G \hat{s}^*_{b,t} + \phi_{g,y}(\delta_a u^p_t + \delta_p u^p_t + \delta_l u^l_t)\right] + u^g_t}_{s^g_t},$$

$$\hat{z}_t - \rho_Z \hat{z}_{t-1} + (1-\rho_Z)\left[\gamma_Z \hat{s}_{b,t-1} + \phi_{z,y}\hat{y}_t\right] = \underbrace{(1-\rho_Z)\left[\gamma_Z \hat{s}^*_{b,t} + \phi_{z,y}(\delta_a u^p_t + \delta_p u^p_t + \delta_l u^l_t)\right] + u^z_t}_{s^z_t},$$

suggests that the two right side variables, s^g_t and s^z_t, are also in the household's information set.

We do not introduce incomplete information to any other shocks in the model and assume they can be observed perfectly by the households. These shocks are meant to improve the empirical fit of the medium-scale DSGE model. Formally, we define the incomplete-information set $\mathcal{I}^{HH}_t$ as

$$\mathcal{I}^{HH}_t = \{d_{t-k}, a_{t-k}, (u^p_{t-k} + \eta^p_{t-k}), u^i_{t-k}, u^l_{t-k}, \eta^w_t, s^m_{t-k}, s^g_{t-k}, s^z_{t-k}, \mathcal{M} | k \geq 0\}.$$

C6. Deriving the log-linearized equilibrium conditions

In order to make the model stationary, we detrend the non-stationary variables to account for the unit root in the labor-augmenting technology process. We define the following variables: $y_t = \frac{Y_t}{A_t}, c^*_t = \frac{C^{*S}_t}{A_t}, c^S_t = \frac{C^S_t}{A_t}, c^N_t = \frac{C^N_t}{A_t}, k_t = \frac{K_t}{A_t}, g_t = \frac{G_t}{A_t}, z_t = \frac{Z_t}{A_t}, b_t = \frac{P^m_t B^m_t}{P_t A_t}, sb_t = \frac{P^m_t B^m_t}{P_t Y_t}, w_t = \frac{W_t}{P_t A_t}$, and $\lambda^S_t = \Lambda^S_t A_t$. In what follow, $e^{\varkappa}$ denotes the steady-state growth of the technology process. That is, $e^a = e^{\varkappa}$.

Production function:

$$\hat{y}_t = \frac{y+\Omega}{y}\left[\alpha\hat{k}_t + (1-\alpha)\hat{L}_t\right]. \tag{C37}$$

Capital-labor ratio:

$$\hat{r}_{K,t} - \hat{w}_t = \hat{L}_t - \hat{k}_t. \tag{C38}$$

Marginal cost:

$$\hat{m}_t = \alpha\hat{r}_{K,t} + (1-\alpha)\hat{w}_t. \tag{C39}$$

Phillips curve:

$$\hat{\pi}_t = \frac{\beta}{1+\xi_p\beta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\xi_p}{1+\xi_p\beta}\hat{\pi}_{t-1} + \kappa_p\hat{m}_t + \kappa_p\hat{tp}_t, \tag{C40}$$

where $\kappa_p = \left[(1-\beta\omega_p)(1-\omega_p)\right] / \left[\omega_p(1+\beta\xi_p)\right]$.

Saver household's FOC for consumption:

$$\hat{\lambda}_t^S = \hat{d}_t - \frac{h}{e^z - h}\hat{a}_t - \frac{e^z}{e^z - h}\hat{c}_t^{*S} + \frac{h}{e^z - h}\hat{c}_{t-1}^{*S} - \frac{\tau^C}{1+\tau^C}\hat{\tau}_t^C \tag{C41}$$

Cost-Push shock Process:

$$\hat{tp}_t = \hat{u}_t^p + \hat{\eta}_t^p \tag{C42}$$

Demand shock:

$$\hat{d}_t = \hat{u}_t^d + \hat{\eta}_t^d \tag{C43}$$

Labor disutility shock:

$$\hat{u}_t^L = \rho_l \hat{u}_{t-1}^L + \varepsilon_t^l \tag{C44}$$

Technology shock:

$$\hat{a}_t = \hat{u}_t^a + \hat{\eta}_t^a \tag{C45}$$

where $\hat{a}_t = \ln\left(\frac{A_t}{A_{t-1}}\right) - \varkappa$.

Public/private consumption in utility:

$$\hat{c}_t^* = \frac{c^S}{c^S + \alpha_G g}\hat{c}_t^S + \frac{\alpha_G g}{c^S + \alpha_G g}\hat{g}_t. \tag{C46}$$

Euler equation:

$$\hat{\lambda}_t^S = \hat{r}_{n,t} + \mathbb{E}_t\hat{\lambda}_{t+1}^S - \mathbb{E}_t\hat{\pi}_{t+1} - \mathbb{E}_t\hat{a}_{t+1} \tag{C47}$$

Maturity structure of debt:

$$\hat{r}_{n,t} + \hat{P}_t^m = \frac{P_m}{1+P_m}\mathbb{E}_t\hat{P}_{t+1}^m \tag{C48}$$

where $P_m = \frac{\rho}{R_n - \rho}$.

Saver household's FOC for capacity utilization:

$$\hat{r}_{K,t} - \frac{\tau_K}{1-\tau_K}\hat{r}_{K,t} = \frac{\psi}{1-\psi}\hat{v}_t. \tag{C49}$$

Saver household's FOC for capital:

(C50)

$$\hat{q}_t = \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}_{n,t} + \beta e^{-\varkappa}(1-\tau_K)r_K\mathbb{E}_t\hat{r}_{K,t+1} - \beta e^{-\varkappa}\tau_K r_K\mathbb{E}_t\hat{\tau}_{K,t+1} + \beta e^{-\varkappa}(1-\delta)\mathbb{E}_t\hat{q}_{t+1}$$

Saver household's FOC for investment:

$$\hat{i}_t = \frac{1}{(1+\beta)se^{2\varkappa}}\hat{q}_t + \hat{u}_t^i + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{i}_{t+1} - \frac{1}{1+\beta}\hat{a}_t + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{a}_{t+1} + \frac{1}{1+\beta}\hat{i}_{t-1} \tag{C51}$$

Effective capital:

$$\hat{k}_t = \hat{v}_t + \hat{\bar{k}}_{t-1} - \hat{a}_t. \tag{C52}$$

Law of motion for capital:

$$\hat{\bar{k}}_t = (1-\delta)e^{-\varkappa}\left(\hat{\bar{k}}_{t-1} - \hat{a}_t\right) + \left[1-(1-\delta)e^{-\varkappa}\right]\left[(1+\beta)se^{2\varkappa}\hat{u}_t^i + \hat{i}_t\right]. \tag{C53}$$

Hand-to-mouth household's budget constraint:

$$\tau_C C_N \hat{\tau}_{C,t} + (1+\tau_C)C_N\hat{C}_t^N = (1-\tau_L)wL\left(\hat{w}_t + \hat{L}_t\right) - \tau_L wL\hat{L}_{L,t} + z\hat{z}_t. \tag{C54}$$

Wage equation:

$$\begin{aligned}\hat{w}_t = {} & \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - u_t^l - \chi\hat{L}_t - \hat{d}_t + \lambda_t^S - \frac{\tau_L}{1-\tau_L}\hat{\tau}_{L,t}\right] + \frac{\chi_w}{1+\beta}\hat{\pi}_{t-1} \\ & - \frac{1+\beta\chi_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\chi_w}{1+\beta}\hat{a}_{t-1} - \frac{1+\beta\chi_w}{1+\beta}\hat{a}_t - \frac{\beta}{1+\beta}\mathbb{E}_t\hat{a}_{t+1} + \kappa_w\eta_t^w\end{aligned} \tag{C55}$$

where $\kappa_w \equiv [(1-\beta\omega_w)(1-\omega_w)] \,/ \left[\omega_w\left(1+\frac{(1+n^w)\xi}{\eta^w}\right)\right]$.

Aggregate households' consumption:

$$c\hat{c}_t = c^S(1-\mu)\hat{c}_t^S + c^N\mu\hat{c}_t^N. \tag{C56}$$

Aggregate resource constraint:

$$y\hat{y}_t = c\hat{c}_t + i\hat{i}_t + g\hat{g}_t + \psi'(1)k\hat{k}_t. \tag{C57}$$

Defining output gap:

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^n \tag{C58}$$

Defining potential output:

$$\hat{y}_t^n = -(1+\xi)^{-1}u_t^p \tag{C59}$$

Government budget constraint:

(C60)
$$\frac{b_t}{y} + \tau_K r_K \frac{k}{y}\left[\hat{r}_{K,t} + \hat{r}_{K,t} + \hat{k}_t\right] + \tau_L w \frac{L}{y}\left[\hat{\tau}_{L,t} + \hat{w}_t + \hat{L}_t\right] + \tau_C \frac{C}{y}\left(\hat{\tau}_{C,t} + \hat{c}_t\right)$$
$$= \frac{1}{\beta}\frac{b}{y}\left[\hat{b}_{t-1} - \hat{\pi}_t - \hat{P}^m_{t-1} - \hat{a}_t\right] + \frac{b}{y}\frac{\rho}{ye^{\varkappa}}\hat{P}^m_t + \frac{g}{y}\hat{g}_t + \frac{z}{y}\hat{z}_t,$$

where we define $b_t = \frac{P^m_t B_t}{P_t A_t}$ so that $s_{b,t} = \frac{P^m_t B_t}{P_t Y_t} = \frac{b_t}{y_t}$. It follows

(C61)
$$s_{b,t} = \hat{b}_t - \hat{y}_t.$$

Fiscal Rules:

(C62)
$$\hat{g}_t = \rho_G \hat{g}_{t-1} - (1-\rho_G)\left[\gamma_G(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}) + \phi_{g,y}\tilde{y}_t\right] + u_{g,t},$$
(C63)
$$\hat{z}_t = \rho_Z \hat{z}_{t-1} - (1-\rho_Z)\left[\gamma_Z(\hat{s}_{b,t-1} - \hat{s}^*_{b,t}) + \phi_{z,y}\tilde{y}_t\right] + u_{z,t},$$
(C64)
$$\hat{\tau}_{L,t} = \rho_L \hat{\tau}_{L,t-1} + (1-\rho_L)\gamma_L \hat{s}_{b,t-1},$$
(C65)
$$\hat{\tau}_{K,t} = \rho_K \hat{\tau}_{K,t-1} + (1-\rho_K)\gamma_K \hat{s}_{b,t-1},$$
(C66)
$$\hat{\tau}_{C,t} = \rho_C \hat{\tau}_{C,t-1} + (1-\rho_C)\gamma_C \hat{s}_{b,t-1},$$
(C67)

where the time-varying debt target follows an AR(1) process

(C68)
$$s^*_{b,t} = \rho_b s^*_{b,t-1} + \varepsilon^b_t.$$

Monetary Rule:

(C69)
$$\hat{r}_{n,t} = \rho_r \hat{r}_{n,t-1} + (1-\rho_r)\left[\phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t\right] + u^m_t.$$

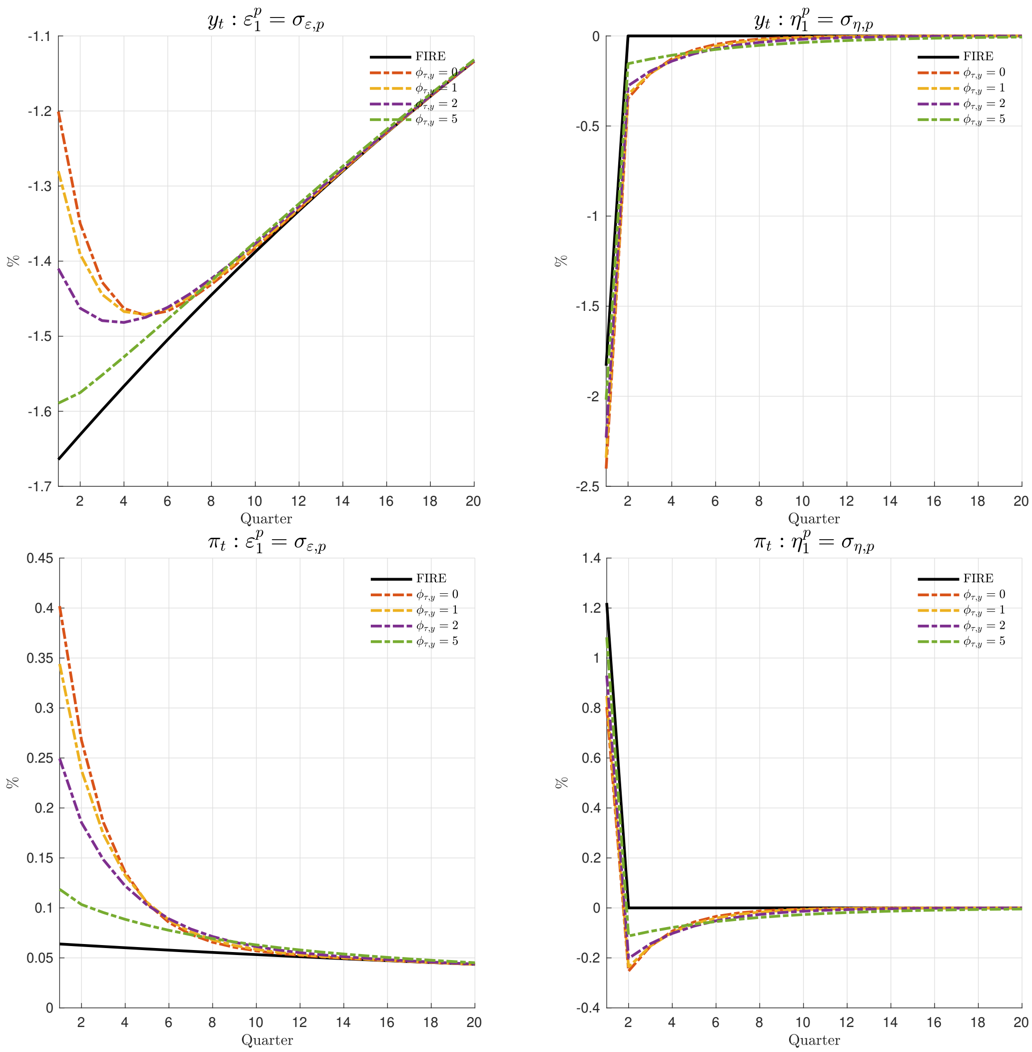

Figure 3. Impulse responses of output and inflation to positive mark-up and cost-push shocks.

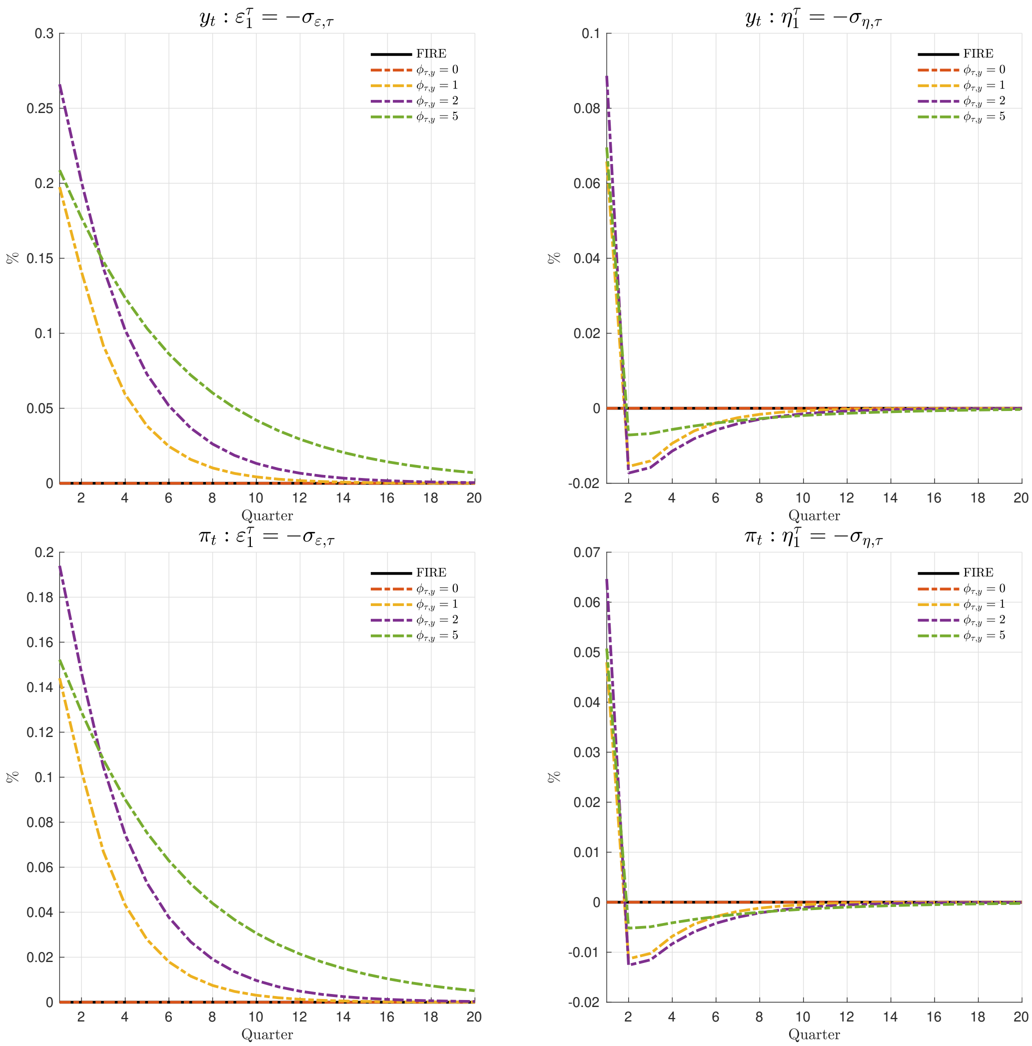

FIGURE 4. IMPULSE RESPONSES OF OUTPUT AND INFLATION TO NEGATIVE PRIMARY SURPLUS SHOCKS.

Putting the Incomplete Information Model Solution to the State-space representation for estimation

This section establishes the incomplete information model solution as a state-space representation used in the estimation. We follow the notation and algorithm of Blanchard, L'Huillier and Lorenzoni (2013) closely. The signal extraction problem is defined by a pair of equations

$$\mathbf{x_t} = \mathbf{A}\mathbf{x_{t-1}} + \mathbf{B}\nu_\mathbf{t}, \tag{D1}$$

$$\mathbf{s_t} = \mathbf{C}\mathbf{x_t} + \mathbf{D}\nu_\mathbf{t}, \tag{D2}$$

where ν_t is an n_ν-dimensional vector of mutually independent *i.i.d.* shocks. The dimensions of the exogenous variables $\mathbf{x_t}$ and signals $\mathbf{s_t}$ are n_x and n_s. Let $\mathbf{y_t}$ denote a vector of endogenous state variables of size n_y. The economic model can be described in terms of the stochastic difference equation

$$\mathbf{F}\mathbb{E}_\mathbf{t}[\mathbf{y_{t+1}}] + \mathbf{G}\mathbf{y_t} + \mathbf{H}\mathbf{y_{t-1}} + \mathbf{M}\mathbf{s_t} + \mathbf{N}\mathbb{E}_\mathbf{t}[\mathbf{s_{t+1}}] = \mathbf{0}, \tag{D3}$$

where $\mathbb{E}_t(\cdot)$ is the rational expectations operator under incomplete information and $\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{M}, \mathbf{N}$ are matrices of parameters. The solution of the model can be described as

$$\mathbf{y_t} = \mathbf{P}\mathbf{y_{t-1}} + \mathbf{Q}\mathbf{s_t} + \mathbf{R}\mathbf{x_{t|t}}, \tag{D4}$$

where the matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ can be found by solving the three matrix equations

$$\mathbf{F}\mathbf{P^2} + \mathbf{G}\mathbf{P} + \mathbf{H} = \mathbf{0}, \quad (\mathbf{F}\mathbf{P} + \mathbf{G})\mathbf{Q} + \mathbf{M} = \mathbf{0},$$
$$(\mathbf{F}\mathbf{P} + \mathbf{G})\mathbf{R} + [\mathbf{F}(\mathbf{Q}\mathbf{C} + \mathbf{R}) + \mathbf{N}\mathbf{C}]\,\mathbf{A} = \mathbf{0}.$$

We know from the Kalman recursion that the law of motion of $\mathbf{x_{t|t}}$ can be written as

$$\mathbf{x_{t|t}} = \mathbf{K}\mathbf{C}\mathbf{A}\mathbf{x_{t-1}} + (\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A}\mathbf{x_{t-1|t-1}} + \mathbf{K}(\mathbf{C}\mathbf{B} + \mathbf{D})\nu_\mathbf{t},$$

where $\mathbf{K}$ is the Kalman gain matrix of size $n_x \times n_s$. It follows we can write the model solution $\mathbf{y}_t$ as

$$\mathbf{y_t} = \mathbf{P}\mathbf{y_{t-1}} + (\mathbf{Q} + \mathbf{R}\mathbf{K})\mathbf{C}\mathbf{A}\mathbf{x_{t-1}} + \mathbf{R}(\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A}\mathbf{x_{t-1|t-1}} + (\mathbf{Q} + \mathbf{R}\mathbf{K})(\mathbf{C}\mathbf{B} + \mathbf{D})\nu_\mathbf{t}$$

Define the extended state variables as $\left[\mathbf{y_t}, \mathbf{x_t}, \mathbf{x_{t|t}}\right]'$. The state-space representation of the incomplete information model solution can be written as

$$\begin{bmatrix} \mathbf{y_t} \\ \mathbf{x_t} \\ \mathbf{x_{t|t}} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & (\mathbf{Q} + \mathbf{R}\mathbf{K})\mathbf{C}\mathbf{A} & \mathbf{R}(\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}\mathbf{C}\mathbf{A} & (\mathbf{I} - \mathbf{K}\mathbf{C})\mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{y_{t-1}} \\ \mathbf{x_{t-1}} \\ \mathbf{x_{t-1|t-1}} \end{bmatrix} + \begin{bmatrix} (\mathbf{Q} + \mathbf{R}\mathbf{K})(\mathbf{C}\mathbf{B} + \mathbf{D}) \\ \mathbf{B} \\ \mathbf{K}(\mathbf{C}\mathbf{B} + \mathbf{D}) \end{bmatrix} \nu_\mathbf{t}.$$