Trade, foreign cash-in-advance constraints and default costs *

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December 2024
PRELIMINARY DRAFT: Comments Welcome

Abstract

I build a sovereign default model in which importers must pay for intermediate imports using foreign currency previously accumulated. This constraint helps to explain why imports and production fall during defaults and generates endogenous default costs that are increasing in the endowment. This feature helps reduce the reliance on ad-hoc default costs that rise exogenously in good times which were needed in prior quantitative sovereign default models to match the data.

JEL classification: F34, F41, G15, H63.

Keywords: Sovereign default, trade and sovereign spreads, foreign currency-in-advance constraints.

* Johri thanks Shunzhe Zhong for excellent research assistance and the Social Sciences and Humanities Research Council of Canada for research funding through grant number 435-2022-0135. This research was partially conducted while I was a visiting scientist at the planning unit at ISI Delhi in 2024.
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1 Introduction

Emerging economies often import large quantities of intermediate goods (relative to their GDP) which are priced in world markets in foreign currencies such as the US dollar. Examples include energy related products like petrol, diesel and coal as well as agriculture related products such as fertilizers and pesticides. In addition, most machinery is produced in a few developed countries, so it is likely that those contracts will also be priced in foreign currencies.[1] Importing firms in these emerging economies also tend to be relatively small and are likely forced to pay in advance for their import contracts. For these reasons, emerging economies tend to be very focused on the availability of foreign currency in the economy to facilitate imports that are essential to the functioning of the local economy. A fall in foreign exchange availability can lead to fewer imports and lower production of exportable goods which can exacerbate the currency shortage.

Keeping these observations in mind, I build a model of a small open economy that buys foreign intermediate goods and sells a locally produced final good in world markets. The economy faces a foreign-currency-in-advance constraint for its imports. As a result, it accumulates foreign currency for use in the next period. There are two incoming sources of foreign currency - i) export of the final good and ii) any proceeds from selling sovereign bonds on the international market, net of debt repayments. We assume that the local government cannot commit to repay its debt obligations, so defaults can occur. While defaults can free up resources from flowing out of the economy, they also imply that no foreign currency will flow into the economy from selling bonds while it remains in autarky. In the presence of the aforementioned constraint, if default leads to a fall in foreign currency, imports might be curtailed.

The recent default in Sri Lanka in 2022 exemplifies these points. Some excerpts from BBC news stories are useful to consider.

"Sri Lanka has defaulted on its debt for the first time in its history...A chronic shortage of foreign currency and soaring inflation have led to a severe shortage of medicines, fuel and other essentials. they [\]government] had given away virtually all the foreign exchange they

[1] Mutreja et al. (2018) point out that 80% of capita goods are produced in 10 countries.

could command [for debt repayments]...As petrol queues run for miles, with fuel being sold on the black market for eye-watering amounts, as lines for handouts of free bread get longer by the day, the island's inability to pay back debts is being painfully felt"

BBC 20th May, 2022 (https://www.bbc.com/news/business-61505842)

"Ranil Wickremesinghe said the nation urgently needs $75m (£60.8m) of foreign currency in the next few days to pay for essential imports. When Sri Lanka's foreign currency shortages became a serious problem in early 2021, its government tried to tackle the issue by banning imports of chemical fertilisers. It told farmers to use locally sourced organic fertilisers, instead. This led to widespread crop failures.."

BBC 29 March 2023 (https://www.bbc.com/news/world-61028138)

This novel consideration that defaults can lower output because imports cannot be bought creates a state-contingent equilibrium default cost which depends on the value of foreign currency to the economy. Since the demand for imported intermediate goods is increasing in the domestic endowment shock, the value of foreign exchange is also high in good times and low in bad times. Implicitly then, the model creates a mechanism where default costs are endogenously increasing in income shocks. This creates the potential to build sovereign default models less reliant on ad-hoc default costs which has been a shortcoming of the literature to date.

The idea that default costs emerge from the productivity losses that occur because defaulting economies are unable to import can be found in the seminal work of Mendoza and Yue (2012). In their model, the fall in imports is driven by the assumption that in default, not only governments but also private firms are unable to borrow, and this borrowing is essential to finance imported goods. While the model succeeds in linking default costs to a fall in imports, it leaves the question of why private firms are unable to access working capital unanswered. This is especially puzzling in default episodes that are not accompanied by banking crises which could plausibly explain why private credit dries up in the defaulting economy.[2] In contrast, the economy in my model is free to import subject to having

[2] Balteanu et al. (2011) reports that roughly 30% of default events were accompanied by bank crises.

accumulated the requisite foreign exchange in advance. As a result, depending on economic conditions, it will only infrequently be unable to acquire the desired amount of imported goods. As such, any fall in imports and consequent productivity losses due to default will be equilibrium outcomes. I note that my model implies that the economy will continue to import even when in default which is consistent with the facts noted in Mendoza and Yue (2012) but generated by assumption in their model.

A key implication of the model is that imports tend to fall when the economy defaults which is an endogenous outcome consistent with data. The model implies an occasionally binding foreign cash-in-advance constraint which plays a role in determining the extent of import declines as well as the output losses caused by defaults. Equilibrium default costs are highly state contingent. They depend on debt, endowment levels and foreign currency levels. They also influence when defaults occur – higher levels of foreign exchange in the economy imply fewer defaults and sovereign spreads relative to lower levels. I build a calibrated version of the model in order to evaluate its ability to deliver the aforementioned features as well as to account for key aspects of the data. Unlike standard default models which are parameterized such that there are zero exogenous default costs below the endowment mean but rising default costs above the mean, in this paper exogenous default costs are a constant fraction of the endowment. The total costs are then a combination of the constant exogenous costs and the state contingent endogenous costs. They display a tendency to be higher in good times, ie. above average endowment levels and this makes defaults more likely in good times relative to the same model without the constraint. A model without the constraint accumulates no foreign currency, displays constant default costs as a percentage of output, takes on more debt and pays lower spreads on average and displays no variation in the ratio of intermediate imports to GDP which is a key feature of the data.

1.1 Related literature.

This study is related to previous work on the relationship between trade and sovereign defaults. On the empirical side, Rose (2005) provides evidence that bilateral trade declines substantially upon default. More generally, several studies have provided estimates of the costs of default. For example, Kuvshinov and Zimmermann (2019) present evidence that defaults cause output to fall by 2.7% on impact. They also present evidence of particular relevance to the current paper that trade falls with imports being heavily affected by the crisis, especially when exchange rates are pegged.[3] In contrast to the mechanism in this paper, several empirical studies have attempted to link the decline in trade to a decline in trade credit, following a default (see Borensztein and Panizza (2009)for example). While plausible, the obvious endogeneity between trade and trade credit bedevils this line of work. Moreover, the reasons for the decline in trade credit to private firms remains under-explored. Other studies look for evidence of another mechanism, namely trade sanctions by creditor nations with little success (see Martinez and Sandleris (2011)).

Turning to the quantitative sovereign default literature, unlike the present model, most sovereign default models with trade assume that default costs are entirely exogenous. An early example can be found in Cuadra and Sapriza (2006), who study the impact of exogenous terms of trade fluctuations on debt and default decisions. Defaults lead to an exogenous fall in the productivity of domestic production. Like my model, the productivity loss is a constant percentage decline. In contrast, Kikkawa and Sasahara (2020) provide a sovereign default model of international trade with endogenously determined terms of trade but no production. Their focus is on the gains from trade and the interaction with default. Defaults are assumed to reduce the domestic endowment when they are near or above the mean value but not at all below this threshold level. This formulation of default costs is typical in the sovereign default literature that follows early work by Arellano (2008) and Aguiar and Gopinath (2006) and is in contrast to the models in my paper. Most relevant to the current model, Serfaty (2024) builds a model with exogenously given iceberg trade costs. These trade costs are assumed to increase when a sovereign defaults on its debt. In addition, the model contains productivity costs to the domestic consumption good.[4] As previously mentioned, Mendoza and Yue (2012) develops a model where a fall in imports is generated by an assumed inability

[3] While this study focuses on default costs linked to a decline in trade, other studies have highlighted the importance of banking crises as the source of output declines in the defaulting country. Kuvshinov and Zimmermann (2019) find that default costs are high when defaults are combined with banking crises but only find 11 such events in their dataset with a total of 92 default events over the 1970-2010 period. Given this, it appears important to explore the role that trade drops can play in understanding default episodes.

[4] There are other differences between the models. Imports are needed to produce the domestic good in my work, hence any endogenous decline in imports creates an endogenous productivity loss. Obviously the occasionally binding constraint imposed by the presence of sufficient foreign exchange stocks in order to buy foreign intermediate goods is also a major difference.

of firms to obtain working capital subsequent to a sovereign default. The model delivers an endogenous default cost that is increasing in good times ie., when output is high. See also papers such as Niemann and Pichler (2020) that build on this framework.

My model is also related to a developing literature on sovereign default models with international reserves such as Bianchi et al. (2018) and Bianchi and Sosa-Padilla (2024). Economies in these models accumulate a risk-free asset along with borrowing using long term debt because these reserves provide some insurance against rollover risk in the former and macroeconomic stabilization for aggregate demand in the latter. Unlike these models, here the economy accumulates foreign exchange because it is essential to buy imported intermediate goods. As a result of the novel benefit built into the model, the economy will choose positive amounts of foreign exchange to hold even when bonds last for only one period (see Alfaro and Kanczuk (2009) for a discussion of why the optimal level of reserves is zero in this situation).[5]

Finally there are other quantitative sovereign default models that attempt to alter default incentives using equilibrium devices. Sosa-Padilla (2018) builds a quantitative sovereign default model where banking crises endogenize default costs through a credit crunch. Alamgir and Johri (2022) show that the proportion of the population in absolute poverty can influence default risk when it interacts with political constraints on the ability of the government to transfer resources to the poor. Alamgir et al. (2023) build a model with corrupt bureaucrats and show that defaults create a novel benefit that is particularly potent in bad times – leakages from the government budget are reduced. Finally Johri et al. (2023) builds a default model with endogenous reelection probabilities and show that default incentives interact with reelection probabilities.

Layout. The rest of the article proceeds as follows. Section 2 introduces our benchmark model of sovereign borrowing and default with occaisonally binding constraints Section 3 discusses the calibration of the model and the main results and quantitative implications of the theory as well as shows the robustness of the framework to changing some assumptions. Section 4 concludes. An appendix provides information about the data, computation

[5]While foreign reserves pay interest while foreign exchange in my model does not, I conjecture that this is not the source of the difference in results. I also note that international currency is counted as part of reserves in the IMF definition.

methods, and further robustness exercises.

2 The Model Environment

The economy is made up of a sovereign government, a representative household and international lenders. The sovereign cannot commit to repay its debt so that risk neutral lenders will charge an interest rate at a level above the risk-free world interest rate in order to be compensated for the probability of default, as is standard in models that follow Eaton and Gersovitz (1981) and especially the quantitative implementations that build on Arellano (2008) and Aguiar and Gopinath (2006). Despite many recent improvements to the core framework of these studies, a concern that remains with the literature is that the quantitative results rely to a large extent on exogenous default costs that embody a specific ad hoc structure without great justification. In particular, many studies incorporate default costs that are high in good times and non-existent in bad times. Building on insights offered by Mendoza and Yue Mendoza and Yue (2012), I will endogenise default costs in terms of a fall in traded intermediate inputs. Unlike that paper, I will build a model where the fall in imported inputs is an endogenous equilibrium outcome due to the presence of an occasionally binding foreign cash-in-advance requirement.

In what follows we use recursive notation, where *un-primed* variables (e.g. x) represent current values, while *primed* variables (e.g. x') represent next-period values. Time is discrete and goes on forever: $t = 0, 1, 2, \dots$.

2.1 The Government and household

The sovereign is infinitely lived and maximizes the welfare of the representative household. The government borrows from a large number of international lenders by issuing long-duration bonds. As in Hatchondo and Martinez (2009), a bond issued in period t promises an infinite stream of coupons, which decrease at a constant rate δ.[6] Specifically, a bond issued in period t promises to pay $\kappa(1-\delta)^{j-1}$ units of the tradable good in period $t+j$,

[6] Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016) allow for issuance of both short-term and long-term debt, while studying optimal maturity.

for all $j \geq 1$. Hence, the evolution of debt can be written as follows:

$$b' = (1 - \delta)b + \nu,$$

where b coupons are due at the beginning of the period, ν refers to new long-term bonds issued in the period, and b' to coupons due at the beginning of next period while κ controls the size of the per-bond coupon payment. This payment structure summarizes all future obligations that emerge from past debt issuances into a single state variable: b.

The flow budget constraint of the government is:

$$T = \begin{cases} [b' - (1-\delta)b]\, q(b', y) - \kappa b, & \text{if debt repaid} \\ 0, & \text{if defaulted} \end{cases} \tag{1}$$

Here q is the price at which sovereign bonds can be sold and T refers to any transfers or taxes made to households in the economy.

2.1.1 Households

The per-period utility of the household depends on consumption and is given by:

$$u(c) = \frac{c^{1-\xi}}{1-\xi} \tag{2}$$

Households recieve a random endowment of a domestic good, d each period which is assumed to have compact support $\mathcal{D}$ and to follow a Markov process with transition function $\mu(d', d)$. These domestic goods can be combined with foreign intermediate goods f to produce final output y using a constant returns to scale production technology:

$$y = y(d, f) \tag{3}$$

The budget constraint of the household is

$$M - p_f f + y - c + T = M'. \tag{4}$$

Here M refers to the amount of foreign funds available in a period for importing intermediate goods while T refers to the lump sum tax or transfer received by the household from the government. p_f refers to the price of foreign goods relative to the domestic goods.[7] The foreign funds cash-in-advance constraint provides the rationale for saving M' and may be written as

$$p_f f \leq M. \tag{5}$$

For consistency with the standard sovereign default model, I assume that the household makes no economic decisions and cedes the authority to choose the pair (M', f) to the sovereign government. As a result, we can combine equations (4) and (1) to obtain

$$M' + p_f f - M + c - y(d,f) = \begin{cases} [b' - (1-\delta)b]\, q(b',y) - \kappa b, & \text{if debt repaid} \\ 0, & \text{if defaulted} \end{cases} \tag{6}$$

For a given amount of resources and given the debt and default decisions to be discussed below, the sovereign faces an interesting intertemporal choice between increasing consumption today and loosening the constraint in the future, which in turn potentially allows additional production in the future. This potential depends on future expected values of d which may determine if the constraint is binding or not in the future. Note that this intertemporal decision is also in operation in autarky which also differentiates this model from the standard sovereign default models.

2.2 Import elasticity and constraints

Figure 1, displays optimal unconstrained and constrained imports for different levels of domestic goods endowment when imports are complements to domestic goods in production. The blue line illustrates the case where foreign funds M are sufficient to purchase the unconstrained optimal amount of imports f at all levels of endowment d. At the other extreme, when M is very low, the red line displays that imports are constrained at all possible levels of d. The intermediate possibilities are illustrated by the green line, where the economy has sufficient M at low levels of d but eventually imports are increasingly constrained. While obvious, the figure illustrates an important aspect of the model – the elasticity of imports to changes in domestic endowment is highly state contingent and variable even though the

[7] Clearly, M may also be thought of as saving using a zero return asset which, in the absence of the cash-in-advance constraint would not be optimal. I verify this point later.

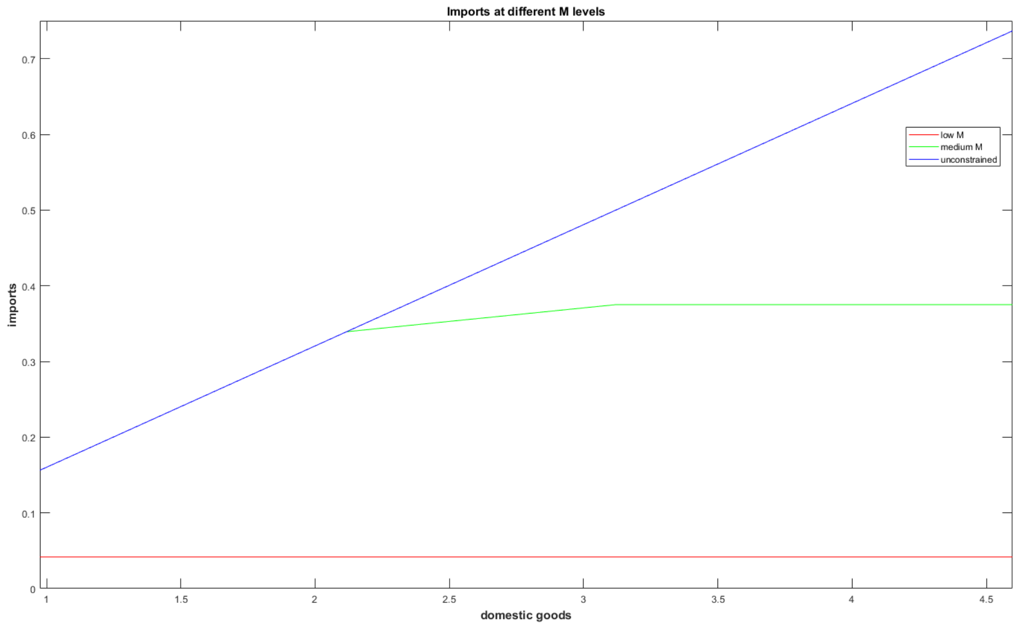

Figure 1

production technology is of the constant elasticity form. The intermediate case is particularly illustrative because the elasticity falls as d increases even though M remains constant.

We now turn to the choice of debt and default and discuss how these choices will affect how much M will be available in the economy to buy imports. This will in turn affect final goods output in the economy and lead to an endogenous relationship between debt and default choices and output.

2.3 Debt and Default

Each period, conditional on being in good financial standing, the sovereign government chooses whether to honor its outstanding foreign debt or default. Default involves temporary exclusion from international financial markets and depressed output levels but opens up fiscal room for transfers to households since debt is not repaid.

For a given level of foreign resources M, let $V(b, d, M)$ denote the government's value function when it has access to credit markets. It begins the period with a level of debt obligations b, and a realized value of d and a stock of foreign funds M. Let $V^R(b, d, M)$ represent the value associated with the government's decision to repay its debt, and $V^D(d, M)$

the value function when it decides to default.[8] The decision problem can be expressed as follows:

$$V(b,d,M) = \max\{V^R(b,d,M), V^D(d,M)\}. \tag{7}$$

When the government repays, its value function is given by:

$$V^R(b,d,M) = \max_{b',M'}\left\{u(c) + \beta \int_{y'} V(b',d',M')\mu(d',d)dd'\right\}, \tag{8}$$

subject to equations (5) -(6).

When the government defaults on its debt obligations, the problem is:

$$V^D(d,M) = \max_{M'}\left\{u(c) + \beta\left(\theta \int_{d'} V(0,d',M')\mu(d',d)dd' + (1-\theta)\int_{d'} V^D(d',M')\mu(d',d)dd'\right)\right\} \tag{9}$$

Equation 9 reflects that the economy faces the stochastic exclusion from international credit markets following default. Market access is regained with probability θ zero debt and some level of M, which is captured by the value function $V(0,d',M')$. Alternatively, the economy may remain in autarky with probability $1-\theta$.

Unlike typical models in the literature where default costs take the general form specified by Chatterjee and Eyigungor (2012), the loss caused due to default is specified as follows:

$$y_a = a_1 y, a_1 \leq 1$$

These losses are intended to capture all disruptions caused by default other than a fall in imports. Potent examples include credit crunches less than full blown banking crises as discussed in Sosa-Padilla (2018). Unlike Chatterjee and Eyigungor (2012), this specification does not allow for an asymmetry in default costs, implying higher losses in high-revenue periods. If output losses do display this pattern then they are an equilibrium outcome of the model. When $a_1 = 1$, there is no exogenous default cost.

[8] Note that the value function under default still depends on the endogenous state M unlike a standard sovereign default model.

The default policy of the sovereign is characterized by:

$$d(b,d,M)=\begin{cases}0 & \text{if } V^R(b,d,M)\geq V^D(d,M)\\ 1 & \text{otherwise.}\end{cases} \tag{10}$$

Let $\mathcal{D}(b,M)$ represent the set of revenue realizations for which the sovereign finds it optimal to default, given a debt level b and foreign funds level M:

$$\mathcal{D}(b,M)=\{d\in\mathcal{Z}:d(b,d,M)=1\},$$

hence, the next-period default probability of the sovereign is

$$\lambda(b',d,M)=\int_{\mathcal{D}(b,M)}\mu(d',d)dy'\ .$$

2.4 Foreign Lenders

Foreign lenders are risk neutral and can borrow funds at the risk free rate r_f. Lenders have perfect information about the endowment process of the small open economy and observe the level of M. Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition yielding:

$$q(b',d,M')=E_{d'|d}\frac{(1-d(b',d',M'))(\kappa+(1-\delta)q(b'',d'))}{1+r_f} \tag{11}$$

where $d(b',d',M')$ and $q(b'',d',M'')$ represent the government's default decision and equilibrium bond price in the next period.

2.5 Recursive equilibrium definition

Definition 1. *The recursive equilibrium for this economy is characterized by*

1. *a set of value functions V, V^R, and V^D,*
2. *a default policy rule d, and a borrowing policy rule b' for the sovereign government,*
3. *policy rule for a foreign exchange accumulation m' and imports f by the household,*
4. *a bond price function q,*

such that:

(a) *given the default and borrowing policy functions of the government and policy rules of the household, V, V^R, and V^D satisfy equations* (7), (8) *and* (9) *when the government can trade bonds at q;*

(b) *given the default and borrowing policy functions, and the foreign exchange accumulation policy of the household, the bond price function q is given by equation* (11)*;*

(c) *the default and borrowing policy functions d and b' solve the dynamic programming problem defined by equations* (7), (8), (9) *and* (10) *when the government can trade bonds at q and given the household's policy rules*

(d) *given the default and borrowing policy rules and the bond price function, the household's policy rules solves the household's problem.*

3 Quantitative Analysis

3.1 Parameterization

We solve the model numerically using value function iteration[9]. A period in the model is one year. Some of our parameters are taken from the literature. We will discuss these first. We assume a coefficient of relative risk aversion of 2, and set the risk-free rate to 4% , which are standard values used in the quantitative sovereign default literature. We set $\beta = 0.8$ which is equivalent to a quarterly discount factor of 0.95. The probability of reentry into international debt markets was set to 0.25, so that the government remains in exclusion for a period of 4 years following a default episode. This is the median value reported by Gelos et al. (2011). We set $\delta = 0.1$ which produces an average duration of approximately 6.5 years, similar to what is found in Bai et al. (2017) who report an average debt duration of 6.7 years in a panel of 11 economies.[10] We use the Macaulay definition of duration which, with our coupon structure, is given by $D = (1 + i^*)/(\delta + i^*)$, where i^* denotes the constant per-period yield delivered by the bond. Turning to the the domestic endowment process parameters, we calibrate them as follows. We assume that d follows a log-normal AR(1) process:

$$\log(d_t) = (1 - \rho)\mu + \rho \log(d_{t-1}) + \epsilon_t,$$

[9] Further details of our procedure can be found in the Appendix.

[10] Using a sample of 27 countries, Cruces et al. (2002)) find an average duration of 4.77 years, with a standard deviation of 1.52 years.

with $|\rho| < 1$, and $\epsilon_t \sim N(0, \sigma^2)$. The persistence and volatility parameters are estimated for log detrended real GDP from 1988 to 2022 for each of the 27 countries in our dataset. We then calibrate to the mean of these values to obtain $\rho = 0.93$ and $\sigma = 0.06$. Recall that output is an endogenous object in the model, so the endowment process parameters are chosen so that the persistence and volatility of output match the means in the data. Unlike Chatterjee and Eyigungor (2012) where default costs are assumed to be increasing in the endowment state, here I assume a default cost that is a constant proportion of the endowment. This form of default costs can be found in the quantitative sovereign default literature (see Hatchondo and Martinez (2009) for example with endowment losses of 10% to 50 % compared to a loss of 8% in this paper). They show (see Table 2), that this cost has a sizable impact on debt to GDP ratios. Therefore the default cost parameter $a_1 = 0.92$ is chosen to match the average value of the country mean debt to GDP ratio in the data.

Table 1: Parameter values.

d autocorr.	ρ	0.89	estimated mean
SD of ϵ	σ	0.065	estimated mean
Borrower's risk aversion	ξ	2	standard
Risk-free rate	r^*	0.04	standard
Discount factor	β	0.8	Calibrated
Duration of defaults	θ	0.25	Prior literature
Coupon decay rate	δ	0.1	Calibrated
Import share	λ	0.285	Calibrated
Default cost parameter	a_1	.92	Calibration

We use a Cobb Douglas form for the final good technology in the Benchmark model and later show sensitivity to varying the elasticity of substitution using a CES form. The technology is

$$y_t = d_t^{1-\lambda} f_t^{\lambda},$$

where λ refers to the share of intermediate imports. The mean value of intermediate import shares for all the countries in our data is 0.24. This is calculated for each country by subtracting all consumption good imports from total imports and then expressing as a fraction of GDP. We use $\lambda = 0.3$ such that this share is close to the data value.

3.2 Key moments

Table 2: Targeted and non-targeted moments

	Data	Benchmark Model	Non Binding
Targeted moments			
Mean debt/GDP (%)	23	23.3	25.5
Range	(5 - 62)		
Mean import share	0.23	0.23	0.285
Non-targeted moments (median values)			
Mean spread (%)	3.75	2.0	1.44
Range	(1.3 - 8.0)		
Mean M/GDP (%)		0.23	0
$\sigma(s)$	1.72	6.2	4.96
$\sigma(C)/\sigma(GDP)$	1.1	1.02	0.97
$\sigma(Imports)/\sigma(GDP)$	0.48	0.07	0
Range	(0.06 - 0.68)		
Corr(s, GDP)	-0.45	-0.08	-.07
Corr(Imports, GDP)	0.59	0.94	0.99

Note: $\sigma(x)$ and $\rho(x, z)$ denote the standard deviation of variable x and the correlation coefficient between variables x and z, respectively. C is private consumption, TB is the trade balance, and s is the sovereign spread. For GDP, private consumption, and government spending we report statistics for the deviations from a log-linear trend; for the spread, we use its level.

In order to get a sense of the ability of the calibrated model to deliver the key co-movement properties associated with sovereign spreads as discussed in the literature, we report some non-targeted measures of volatility and co-movement in Table 4 as well as the targeted moments discussed above.

Since our model makes predictions for the co-movement of imported intermediate goods with GDP, I also report these moments. The first column in Table 4 reports the observed value of the moments in our panel data. We calculate these moments for each country separately in our dataset when information was available for the entire time period from 1995-2015 and then report the mean value. The corresponding moments from the simulated benchmark model with long bonds and Cobb-Douglas production technology are reported in column two. These are calculated by simulating the benchmark model for 5000 periods and then discarding the first 100 periods as burn-in. Real GDP and consumption are logged and linearly de-trended prior to calculating the moments. The model delivers a mean spread that is lower than the average across countries in our panel but it is well within the range of observed mean spreads. The average amount of foreign currency as a percent of GDP in the economy is also substantial, at 24.7 percent. The model also delivers key features of emerging economy business cycles – spreads are negatively correlated with GDP, and imports are positively correlated. As well, consumption is slightly more volatile than output.

The final column of the table reports corresponding moments for a version of the benchmark model, where the foreign currency in advance constraint is removed. This model is called the Non Binding model. A few facts emerge from column three. First, in the absence of the constraint, the economy no longer wishes to hold onto foreign currency. Second, the import share now remains a constant fraction of GDP as foreshadowed by the discussion above. As a result, import volatility relative to GDP is zero. Third, the mean spread is lower and debt to GDP ratio higher than in the benchmark model. I will discuss the policy rules from the two models below to provide a clearer comparison of the role of the constraint.

3.3 The role of foreign currency

Figure 2 displays the default region for the two models discussed above. The green line pertains to the Non Binding model, while the red (low) and blue (high) lines refer to the Benchmark model at two different levels of M. Holding M constant, each line separates the state space into two regions: below the curve is the set of states in (z,debt) space such that the economy chooses to default, while above the line it chooses to repay its debt. It is immediately obvious from the figure that the Benchmark model results in greater default risk than the Non Binding model with the same parameters. It is also clear that having more foreign currency, M, lowers default risk, ceteris paribus. Since default risk is increasing in debt in quantitative sovereign default models, one way to judge the additional default risk across models is to hold debt constant, as is done with the vertical black line at $b = 0.3$, which is arbitrarily chosen to illustrate the point. Looking vertically, at the distance between the green and red line for instance, reveals that the Benchmark economy will default at much higher levels of domestic endowment z (and hence y) than the Non Binding economy. Obviously, for sufficiently low levels of debt, there will be no defaults at any level of z. This additional default risk is reflected in the menu of spreads on offer to the two economies for different levels of debt. Holding debt near the mean value of both economies, Figure 3 displays the relationship between spreads and z. The labels pick out a common value of the endowment that is just above mean for comparison. In the Non Binding economy (green line), the spread is essentially zero while in the Benchmark economy it is 117 basis points. Since spreads rise rapidly for lower values of z, the figure is cut off at 190 basis points for visual clarity.

It is worth noting that the increased default risk emerges in good times in the Benchmark economy, ie. when z is above mean whereas most default risk occurs below the mean in the Non Binding economy. In this context, Tomz and Wright (2007) show that about one-third of default episodes occurred in good times. [11]

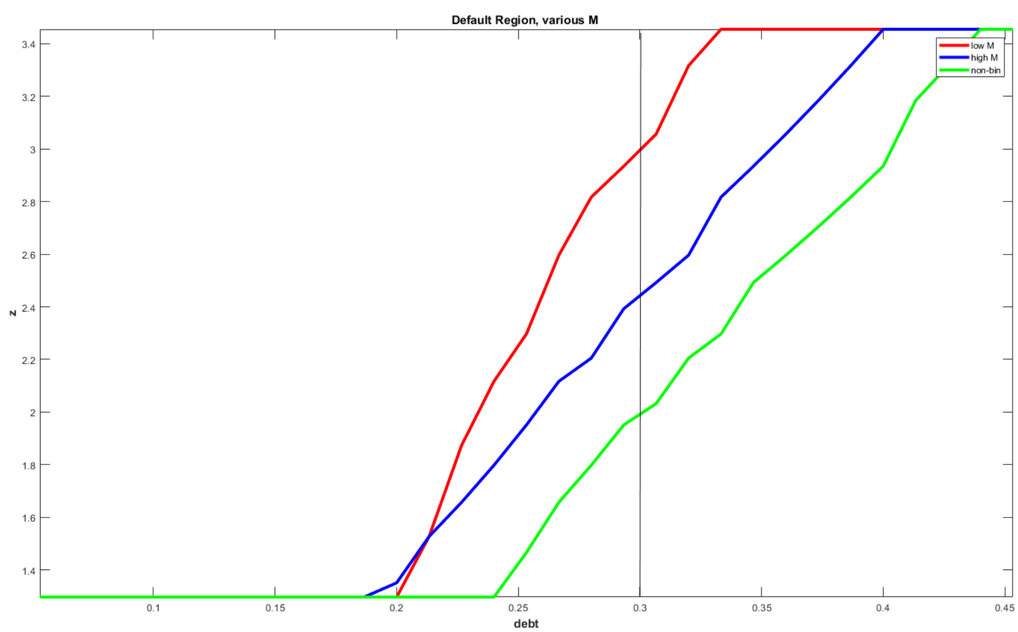

Figure 2

[11] Park (2017) builds a model with physical capital accumulation with this feature. Moreover, default incentives are high when the stock of capital is low but fall as capital is accumulated. Beyond a certain level though, default risk increases again. An important difference is that Park assumes that the defaulting economy cannot import goods from abroad while that is allowed in the Benchmark model as long as foreign currency is available.

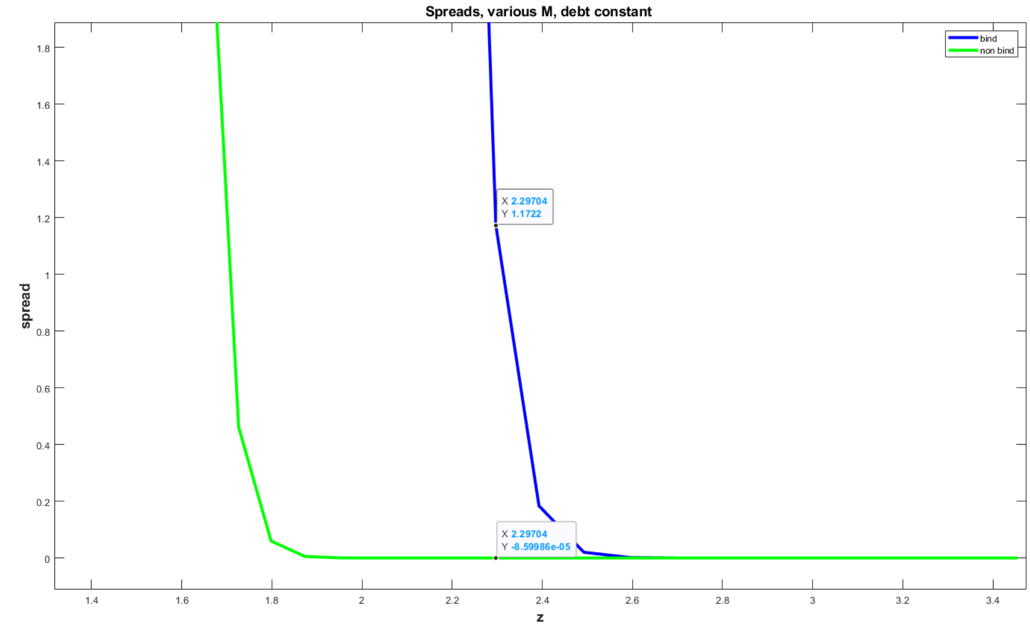

Figure 3

3.4 Endogenous and exogenous default costs

So far we have seen that the model with occasionally binding foreign cash-in-advance constraints can generate additional default risk. We now turn to the mechanism that lies behind this result. Recall that most sovereign default models assume an ad-hoc default cost that is increasing in the output of the economy. For example, in Johri et al. (2023), following the formulation of default costs in Chatterjee and Eyigungor (2012), a quadratic loss function is imposed on income during a default episode:

$$\Phi(y) = \max\left\{y\left[\lambda_0 + \lambda_1[y - \mathbb{E}(y)]\right], 0\right\}$$

where y is an exogenous endowment, $\mathbb{E}(y)$ is its mean and $\lambda_0 = 0.125$ and $\lambda_1 = 1.15$. Note that the loss function is increasing in income beyond the mean and is zero below the mean. This is imposed in order to obtain reasonable equilibrium debt levels and spreads. By contrast, the loss caused due to default is specified in all the model variants here as a constant proportion of the domestic endowment (8%). As a result, in these models, output will fall in default both due to the exogenous fall in domestic goods as well as any fall in equilibrium imports by the economy. Thus, any variation over the state space in the percentage fall of output due to default will occur due to the changing tightness of the foreign cash-in-advance constraint. Figure 4 displays default costs as z deviations from mean vary on the horizontal axis for the Non Binding (green) and Benchmark (blue) models. In the former model, the constraint is never binding by assumption so imports rise in constant proportion to z. As a result, defaults lead to a fixed percentage fall in output. In the Benchmark model, default costs which combine endogenous and exogenous elements are often higher but also sometimes lower than in the Non Binding model and also vary with z. The tendency for costs to rise with z are clearly visible in the linear regression line. These lines are drawn holding both b and M constant and the position of the blue line would shift as they change. Since costs are highly variable with the state vector, in Figure 5, I take the average over all the z states and plot the relationship between default costs and debt. The declining relationship between default costs and debt is clearly visible. With higher debt, the burden on repaying the loans keeps increasing until further borrowing leads to resources flowing out of the economy rather than into the economy. As a result, the economy accumulates lower levels of foreign cash holdings and thus is more constrained in their ability to import goods when repaying debt. This fall in ability to import, lowers the gap between output when repaying debt compared to when defaulting. Figure 7 displays the pattern discussed above. As debt increases, the choice of how much foreign cash to save for the next period, M' is displayed when averaged over z. The blue line refers to a lower level of current M while the red line to a higher level.

While Figure 5 holds M constant, debt is held constant in Figure 6. As before, default costs are expressed as a percent and averaged over all z levels and the relationship with M is displayed for two different debt levels. At very low levels of M, the constraint is almost always binding so import levels are always very limited. In default, since z falls by 8%, the optimal level of f also falls thus the constraint is less binding in default leading to low default costs. As M increases the gap between imports in repay states and default states first increases then decreases for low debt states but stays roughly constant for high debt states. The three figures together paint a rich picture of the highly state contingent nature of default costs in the Benchmark model.

We now turn to the underlying cause of endogenous default costs – imports. Figure 8

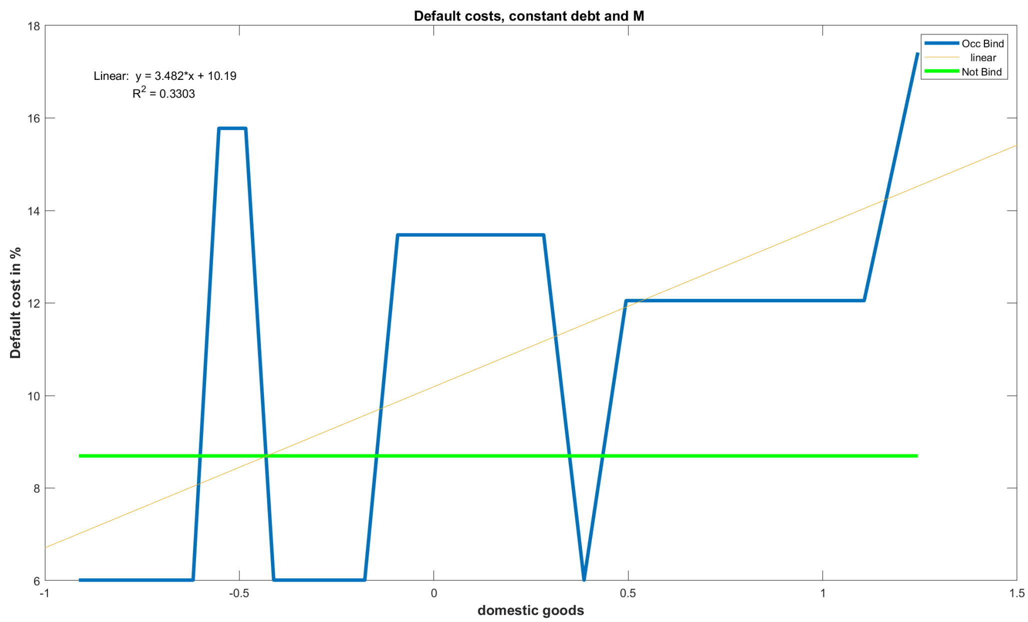

Figure 4

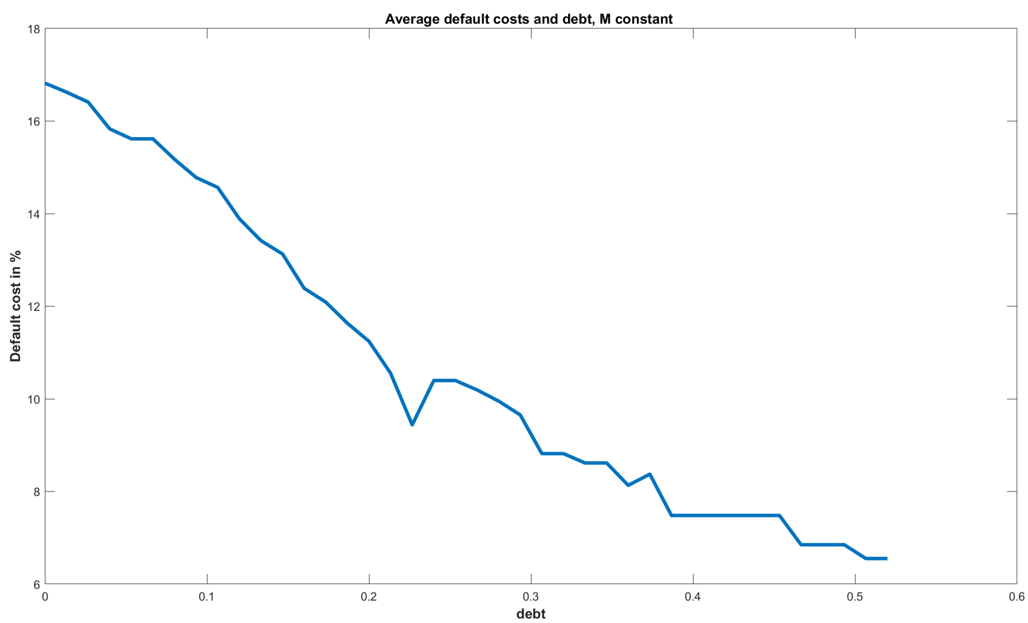

Figure 5

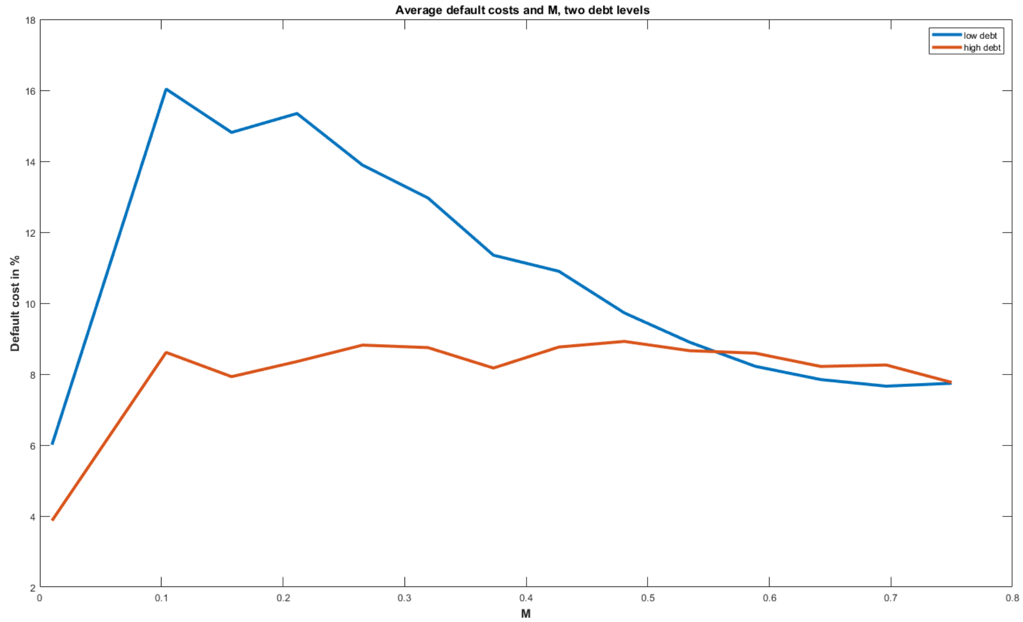

Figure 6

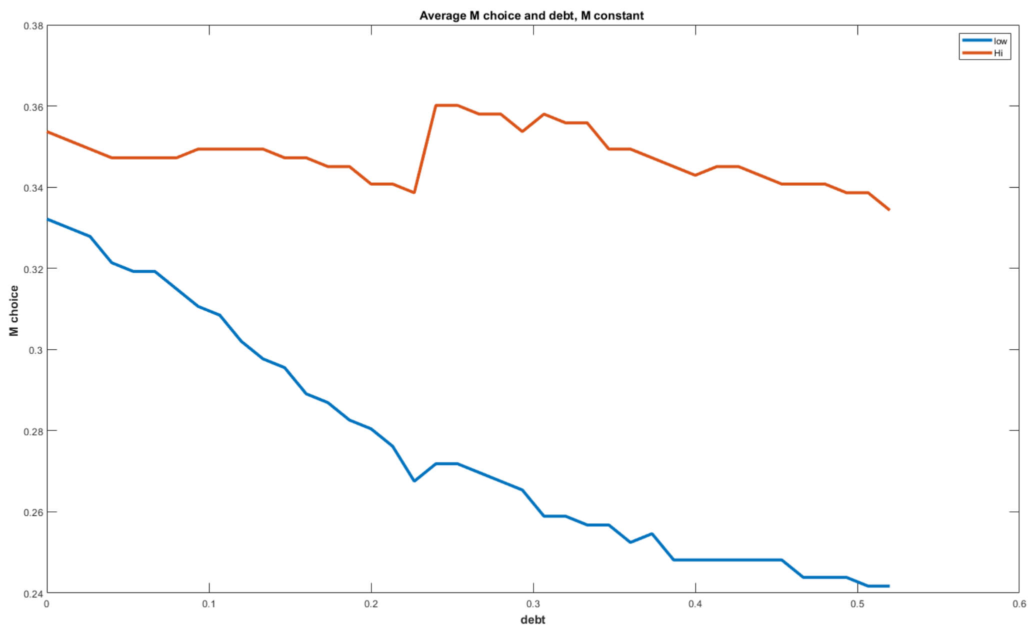

Figure 7

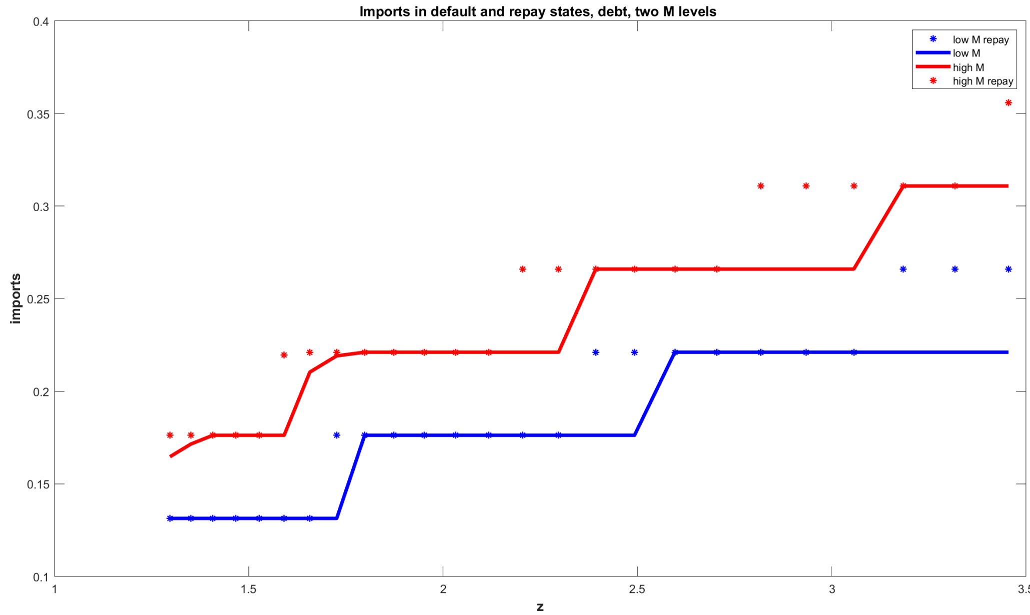

Figure 8

plots imports in the economy as the domestic endowment varies for two different levels of M holding debt constant. Lines with stars refer to the import policy when the economy repays its debt while the solid line is the optimal policy whether defaults occur or not. A number of features of the model are illustrated in the figure. Comparing the red to the blue line reveals that the economy imports more when there is more M. Areas where imports are not increasing in endowment reveals that the foreign cash-in-advance constraint is binding, whether in default or in repay states. Finally, the gap between the stars and the solid line in default displays regions where default causes imports to fall due to a tightening constraint since bond sales no longer bring in foreign funds. The figure illustrates the behaviour of imports that I discussed above.

The discussion above has shown that the amount of foreign cash brought into the period, M has an impact on the optimal policies of the economy. There are obvious advantages of having more resources overall. It supplements current production and can be used for consumption, debt repayment and to buy imports, where this mix is chosen optimally. As a result, default is less likely and since M is observed by lenders, they offer better loan terms as seen in Figure 9. To round out the discussion of optimal policy, Figure 10 illustrates that

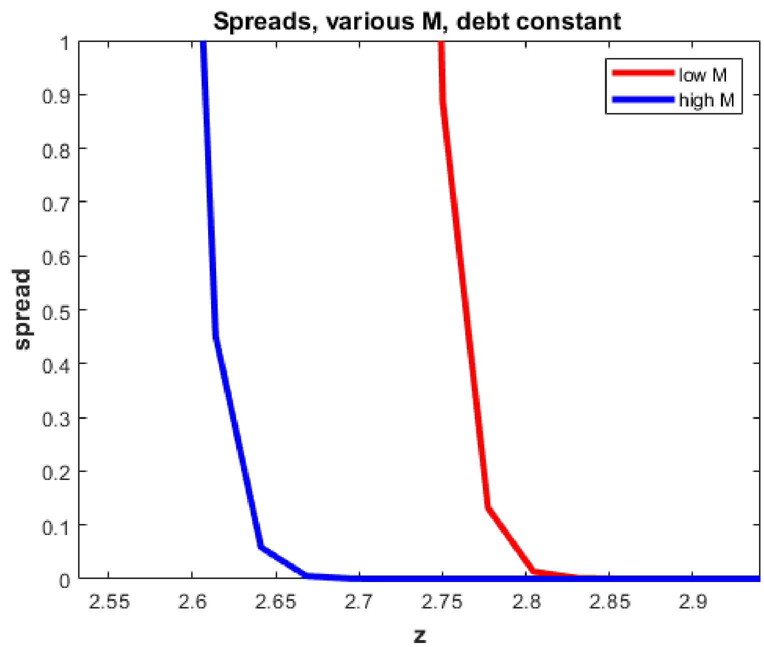

Figure 9

lending terms get worse the higher the current debt burden.

3.5 The impact of η

In this section, I explore the impact of varying the elasticity of substitution in the production technology while holding all other parameters at their Benchmark model values. For this purpose I employ a CES production technology to combine domestic and foreign goods:

$$y = ((1-\lambda) * d^{\eta-1/\eta} + \lambda * f^{\eta-1/\eta)})^{\eta/(\eta-1)} \tag{12}$$

where η governs the elasticity of substitution. Table 3 reports the impact of varying $\eta = [.8, 1.8]$ as well as the Benchmark results for Cobb Douglas production. The table reflects the negative relationship between debt and spreads. It also displays a trade-off between import share and import volatility. Not surprisingly, as η rises, domestic goods become better substitutes for foreign goods leading to lower imports on average.

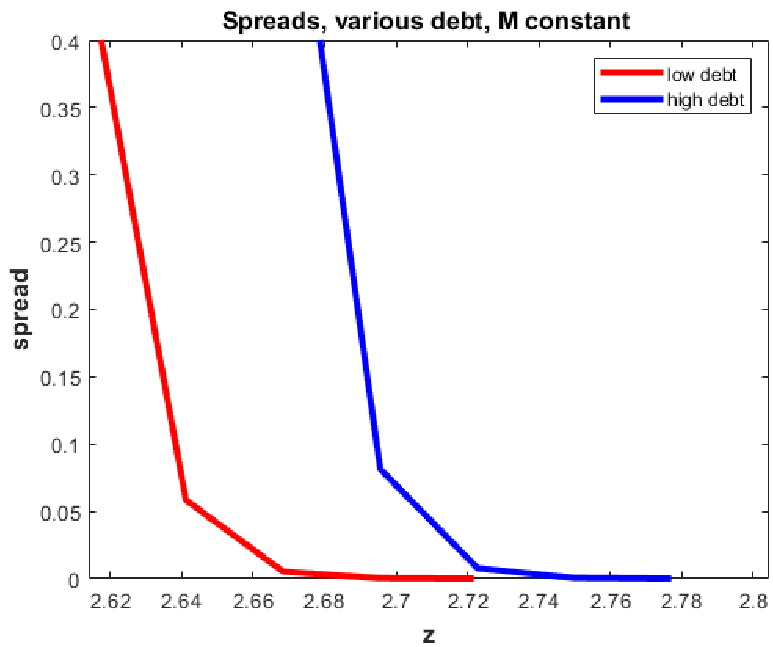

Figure 10

Table 3: The impact of η

	$\eta = .8$	Cobb Douglas	$\eta = 1.8$
Mean debt/GDP (%)	17.5	22.45	30.0
Mean import share	0.34	0.24	0.09
Mean spread (%)	0.93	2.29	2.57
$\sigma(Imports)/\sigma(GDP)$	0.36	0.07	0.015

Note: $\sigma(x)$ and $\rho(x, z)$ denote the standard deviation of variable x and the correlation coefficient between variables x and z, respectively. C is private consumption, TB is the trade balance, and s is the sovereign spread. For GDP, we report statistics for the deviations from a log-linear trend; and for the spread, we use its level.

3.6 One period bonds

In this section, we show that the foreign cash-in-advance constraint creates a motive for the economy to hold foreign funds even when bonds last for one period only. Recall that Alfaro and Kanczuk (2009) show that it is not optimal to hold foreign reserves in an economy with one period debt. To do this, we simply impose $\delta = 1$ in the Cobb Douglas economy. The results with a comparison to the long bond case are displayed in Table 4. The main impact is that mean spread falls while other moments look very similar.

Table 4: Long and Short Bonds

	long	Short
Mean debt/GDP (%)	22.45	22.0
Mean import share	0.24	0.24
Mean spread (%)	2.29	1.52
Mean M/GDP (%)	0.24	0.24
$\sigma(Imports)/\sigma(GDP)$	0.07	0.07

Note: $\sigma(x)$ and $\rho(x, z)$ denote the standard deviation of variable x and the correlation coefficient between variables x and z, respectively. C is private consumption, TB is the trade balance, and s is the sovereign spread. For GDP, we report statistics for the deviations from a log-linear trend; and for the spread, we use its level.

4 Conclusions

to be written

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5 Appendix under construction

5.1 List of countries

Argentina, Brazil, Bulgaria, xChile, China, Colombia, Costa Rica, xCote D'Ivoire, xCroatia, Dominican Republic, Ecuador, Egypt, Arab Rep., El Salvador, xEthiopia, Gabon, xGhana, Guatemala, xHonduras, xHungary, India, Indonesia, xIraq, xKazakhstan, xKenya, Jamaica, Jordan, xLatvia, Lebanon, xLithuania, xMalaysia, Mexico, xMongolia, Morocco, xMozambique, xNamibia, xNigeria, Pakistan, xPanama, xParaguay, Peru, Philippines, xPoland, xRomania, Russian Federation, xSenegal, xSerbia, xSlovak Republic, South Africa, xSri Lanka, xTanzania, Thailand, xTrinidad And Tobago, xTunisia, Turkey, Ukraine, xUruguay, xVenezuela RB, xVietnam, xZambia

5.2 Construction of Country Averages Data: incomplete

We took country averages of all observations between 2006 and 2015 for each of the series. We chose 2006 because we did not have any data from the Enterprise Survey prior to this year.

The following data transformations were performed to obtain the series used in the paper. We computed "External debt to GDP" by dividing "External debt stocks, public and publicly guaranteed (PPG) (DOD, current US$)" by "GDP (current US$)". "General government final consumption expenditure" was computed by multiplying "General government final consumption expenditure (% of GDP)" by "GDP (current US$)". The country average of "General government final consumption expenditure" was multiplied by the country average "Value of gift expected to secure a government contract (% of contract value)" in order to compute the average "Diversion level" by country. The "Rule of Law" variable from The World Governance Indicators ranges from -2.5 to +2.5. We rescaled this variable by adding 2.5 to the average value of each country to ensure that it is always positive. "Total Resources" was computed by adding "External debt to GDP" to "Revenue, excluding grants (% of GDP)" and multiplying everything by "GDP (current US$)". "External debt to tax revenue" is computed by dividing " External debt to GDP " by " Revenue, excluding grants (% of GDP)".

5.3 Construction of Annual Dataset

1. The country-specific consumer goods import data are retrieved from the WITS. Consumer Goods Imports By Country and Region for all available countries (171). 2. GDP, total imports, and final consumption are retrieved from the WDI for all countries and merged with the consumer goods import data (154). All variables are in current dollars which can be downloaded from https://databank.worldbank.org/reports.as px?source=2&series=NY.GDP.MKTP.CD&country=#. 3. The non-consumer goods imports are calculated by subtracting the consumer goods imports for each country from total import. This is the proxy to be used for intermediate imports. The non-consumer goods import share is calculated by dividing the non-consumer goods import by the country's GDP. 4. Spread and debt data is obtained from Farzana Alamgir using the dataset in Alamgir et al. (2023) from 1995-2015. This is merged with the import data leaving 28 countries. Spread data comes from J.P. Morgan's EMBI Global Index and consists of weekly observations ranging from the first week in 1995 to May 29, 2015. There are a total of 67 countries in this dataset. In order to convert to annual frequency, we computed annual averages for each country. 5. First, second moment, and correlation

The data sources are: J.P. Morgan's EMBI Global Index, and The World Development Indicators. Aside from the series mentioned in the Country Averages Dataset, we also used the following series: "GDP per capita (constant 2010 US$) - NY.GDP.PCAP.KD", "Exports of goods and services (% of GDP) - NE.EXP.GNFS.ZS", "Total reserves (includes gold, current US$) - FI.RES.TOTL.CD", and "Imports of goods and services (% of GDP) - NE.IMP.GNFS.ZS".

The following data transformations were performed. "Log Y deviations from trend" was computed by logging "GDP per capita (constant 2010 US$)" and then removing the linear trend. We computed "Reserves to GDP" by diving "Total reserves (includes gold, current US$)" by "GDP (current US$)". "Net exports to revenue" was computed by subtracting "Imports of goods and services (% of GDP)" from "Exports of goods and services (% of GDP)" and dividing it by "Revenue, excluding grants (% of GDP)".