On the Transition to Modern Growth: Lessons from Recent Agricultural Employment*

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Abstract

We study a model where a single good can be produced using a diminishing-returns technology (Malthus) and a constant-returns technology (Solow). We map the former to agriculture and show that the share of agricultural employment declines at a constant rate and that recent observations on the share are sufficient to estimate the onset of economic transition. Our model implies that output growth is higher after the onset of transition and that the share of agricultural employment is a sufficient statistic to describe output growth during the transition. Our quantitative results are consistent with these implications.

JEL codes: O10, O13, O40
Keywords: Malthus, Solow, Agricultural employment, Economic transition

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1 Introduction

The standard of living was roughly constant prior to the 19th century despite technological change. Malthus (1798) and Ricardo (1817) accounted for this stagnation with a theory of population and diminishing returns in production. The past two centuries, however, have seen unprecedented growth in living standards, which has spawned a vast literature on the transition from stagnation to growth. Our paper belongs to a strand of the literature where a single final good can be produced with two technologies and the economy transitions from one technology to possibly both. Two prominent examples of this strand are Hansen and Prescott (2002), where the transition occurs when physical capital and total factor productivity (TFP) reach a threshold, and Tamura (2002), where endogenous human capital accumulation lowers trade costs and delivers the transition without relying on any exogenous forces.

We develop a simplified version of their models. Our economy has one good that can be produced using two technologies—Malthus and Solow. The Malthus technology is subject to diminishing returns, as land is a fixed factor, and the Solow technology has constant returns. Labor is the only factor of production. TFP in the two technologies and population grow exogenously at possibly different rates.

We map the Malthus technology to agriculture and deliver two results. First, we determine the onset of transition to modern growth using only the share of agricultural employment; we do not need information on real gross domestic product per capita (GDP, hereafter). Specifically, the share declines at a constant rate in the model during the transition, so we can use a few *recent* observations to estimate the rate of decline and project backward to determine the onset of transition. This result provides a testable implication for countries with historical data on GDP: Does GDP growth change at the agricultural employment-based onset of transition? This result is also useful for estimating the onset of transition for today's developing economies, which typically do not have a long time series of GDP.

Second, during the transition, the share of agricultural employment is a sufficient statistic for GDP dynamics in the model: GDP growth is a first-order autoregressive process with a coefficient that is pinned down by the rate of decline in the share of agricultural employment. We do not need to know the structural parameters, TFPs, or population. This result offers another testable implication: Is GDP growth since the onset consistent with the constant rate of decline in the share of agricultural employment estimated from a few recent observations?

Our economy uses only the Malthus technology initially, but it transitions to using both the Malthus and Solow technologies when the two TFPs and population reach a threshold. The onset of economic transition is when employment in the Malthus technology starts declining—labor is employed only in the Malthus technology initially, but it is employed in both technologies during the transition and increasingly more in the Solow technology. Quantitatively, we estimate the constant rate of decline in the share of agricultural employment using post-World War II data from Herrendorf, Rogerson, and Valentinyi (2014) and infer the onset of transition. For the United States the economic transition started in 1875.

Annual GDP data for the U.S. is available starting in 1800 (Delventhal, Fernández-Villaverde, and Guner, 2021). When we test the implication from our first result, we find that U.S. GDP growth is higher after 1875 than it is before.

To test our second result, we estimate the autoregressive coefficient for U.S. GDP growth from 1875 to 2016. We find that the coefficient is almost identical to the one implied by the constant rate of decline in the post-World War II share of agricultural employment. It is surprising that recent agricultural employment would account for the GDP dynamics over 140 years, especially given the structural changes since 1875.

We repeat the calculations for the United Kingdom. Our estimate of the onset of transition is 1812, which is consistent with lower U.K. GDP growth before 1812 and higher growth after. The autoregressive coefficient for GDP growth is almost the same as the one implied by post-World War II agricultural employment. We report similar results for several Western European countries.

Next, we examine the transition from Malthus to Solow for today's developing economies. We consider a sample of countries whose GDP is less than 25% of U.S. GDP in 2016. Using the share of agricultural employment, available only after 1991 for these countries, we estimate the onset of transition. For instance, the onset of transition for India is 1965. The autoregressive coefficient for GDP growth for most of these countries is consistent with the country's rate of decline in the share of agricultural employment.

This paper differs from the transition literature in a few ways. First, previous papers are concerned with the "how"—the channels that led from stagnation to growth. Their focus is not on the "when." In their quantitative implementations, the onset of economic transition is chosen using narratives or historical evidence on GDP, and the model parameters are calibrated to deliver the chosen onset of transition. For instance, Hansen and Prescott (2002) use GDP data for England, and Tamura (2002) uses it for Western Europe + Canada + the U.S. In contrast, we estimate the onset of transition without using GDP data. We then use GDP data for cross-validation. Second, our method does not require knowing the structural parameters or population. Cross-country differences in structural parameters and the processes for TFPs and population yield cross-country differences in the share of agricultural employment, which is a sufficient statistic for the onset of transition and GDP growth. Finally, because we do not rely on historical evidence on GDP or share of agricultural employment, our model is useful for studying the transition of developing economies. Thus, one could examine how much of the observed lack of cross-country income convergence is due to late transitions.

Two remarks are in order here. First, in models such as Hansen and Prescott (2002) and Tamura (2002) the onset of economic transition is, *by definition*, when the Malthusian share of employment starts declining, no matter what the details of the model are. Without using any specific model, one could make an *ad-hoc assumption* that the rate of decline is constant and estimate the onset of transition. Our approach has two advantages: (i) The ad-hoc assumption, without more structure, would have no further implications for GDP dynamics, and (ii) we provide a framework where the constant rate of decline is a result, not an assumption.

Second, we have left out several forces from our model that have been included in the transition literature. For instance, in the human capital models of Becker, Murphy, and Tamura (1990) and Galor and Weil (2000), the economy transitions from stagnation and high fertility to growth and lower fertility. In Goodfriend and McDermott (1995), exogenous population growth allows for increasing returns to specialization, and the economy transitions from household production and stagnation to market production and growth. In Jones (2001), the evolution of population and ideas delivers technological progress that helps the economy transition. Forces such as human capital accumulation, technological change, demographic transition, etc., undoubtedly offer rich implications for economies during the transition to modern growth. However, our simple framework delivers the onset of transition and GDP dynamics based on just agricultural employment. Our analysis begs the question: Do the richer frameworks yield similar quantitative implications?

2 Model

Time is continuous and represented by $t \geq 0$. There are two technologies, denoted M (for Malthus) and S (for Solow), producing a single consumption good. Technology M uses land (fixed and normalized to 1) and labor, and exhibits diminishing returns to labor. Technology S uses only labor and exhibits constant returns to scale. Outputs at date t from the two technologies are denoted Y_t^M and Y_t^S:

$$\begin{aligned} Y_t^M &= \left(Z_t^M H_t^M\right)^{1-\alpha}, \quad \alpha \in (0,1), \\ Y_t^S &= Z_t^S H_t^S, \end{aligned}$$

where Z_t^M and Z_t^S are exogenous labor-augmenting productivities, and H_t^M and H_t^S are employment in M and S, respectively. Total gross domestic product is

$$Y_t = Y_t^M + Y_t^S.$$

The working population is exogenous and denoted by P_t.

In what follows, we adopt the following two notations. First, we use lowercase letters to denote a variable per unit of population: $x_t \equiv X_t/P_t$. Second, we use a dot notation to denote the *growth rate* of a variable:

$$\dot{X}_t \equiv d\ln(X_t)/dt.$$

We assume that Z_t^S, Z_t^M, and P_t grow at constant but potentially different rates $\dot{Z}^S$, $\dot{Z}^M$, and $\dot{P}$, respectively.

$$Z_t^S = Z_0^S \exp\left(t\dot{Z}^S\right),\; Z_t^M = Z_0^M \exp\left(t\dot{Z}^M\right),\; \text{and}\;\; P_t = P_0 \exp\left(t\dot{P}\right),$$

where Z_0^S, Z_0^M, and P_0 are initial conditions.

The working population is allocated between the two technologies,

$$H_t^M + H_t^S = P_t.$$

Labor is perfectly mobile across the two technologies. Thus, the optimal, i.e., output-maximizing, allocation of labor requires:

$$Z_t^S \leq (1-\alpha)\left(Z_t^M\right)^{1-\alpha}\left(H_t^M\right)^{-\alpha}, \tag{1}$$

with equality whenever $H_t^M < P_t$. The left-hand side is the marginal product of labor in technology S, and the right-hand side is the marginal product of labor in technology M. Figure 1 represents the optimal allocation of labor. If Z_t^S is low enough (e.g., $Z_{t,\text{Low}}^S$), the marginal product of labor in technology M exceeds that in technology S even if the entire working population is allocated to technology M. In this case Equation (1) holds with a strict inequality.

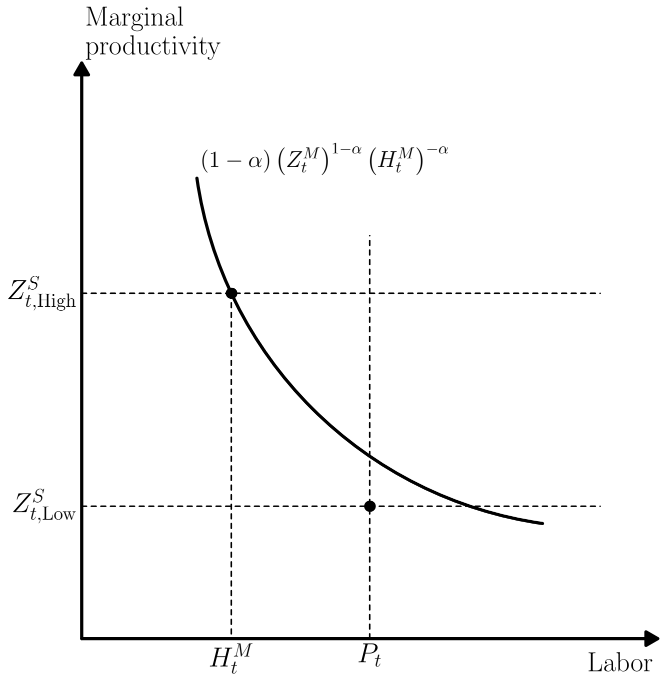


Figure 1: The optimal allocation of labor

When Z_t^S is sufficiently high (e.g., $Z^S_{t,\text{High}}$), the two marginal products are equalized and $H_t^M < P_t$.

If initial conditions are such that

$$Z_0^S < (1-\alpha)\left(Z_0^M\right)^{1-\alpha}(P_0)^{-\alpha}, \tag{2}$$

then all labor is allocated to technology M at date 0. If inequality (2), evaluated at t, continues to hold, then all labor continues to be allocated to technology M at t. Some labor will be allocated to technology S at some date t^* if Equation (1) is satisfied with equality at t^*. For this to occur, it must be the case that

$$\dot{Z}^S > (1-\alpha)\dot{Z}^M - \alpha\dot{P}. \tag{3}$$

We assume conditions (2) and (3) are satisfied for the rest of the paper.

The date at which technology S starts operating satisfies

$$Z_0^S e^{t^* \dot{Z}^S} = (1-\alpha)\left(Z_0^M e^{t^* \dot{Z}^M}\right)^{1-\alpha}\left(P_0 e^{t^* \dot{P}}\right)^{-\alpha}.$$

That is, the onset of transition is given by

$$t^* = \frac{\ln\left[(1-\alpha)\left(Z_0^S\right)^{-1}\left(Z_0^M\right)^{1-\alpha}\left(P_0\right)^{-\alpha}\right]}{\dot{Z}^S-(1-\alpha)\dot{Z}^M+\alpha\dot{P}}. \tag{4}$$

2.1 Analysis

It is convenient to analyze the economy in two parts: before and after t^*.

Before the transition When $t \leq t^*$, the analysis is straightforward. The entire working population is employed in technology M and output per capita is that of technology M. Using our notations defined earlier,

$$\begin{aligned}
\text{Share of agricultural employment} &: \; h_t^M = 1, && (5)\\
\text{GDP} &: \; y_t = y_t^M = \left(Z_t^M\right)^{1-\alpha}\left(P_t\right)^{-\alpha}, && (6)\\
\text{GDP growth} &: \; \dot{y} = \dot{y}^M = (1-\alpha)\dot{Z}^M - \alpha\dot{P}. && (7)
\end{aligned}$$

During the transition When both technologies operate, Equation (1) holds with equality and employment in technology M is

$$H_t^M = (1-\alpha)^{1/\alpha}\frac{1}{Z_t^S}\left(\frac{Z_t^M}{Z_t^S}\right)^{(1-\alpha)/\alpha}.$$

It follows that the *employment share* of technology M and its growth rate are

$$\begin{aligned}
h_t^M &= (1-\alpha)^{1/\alpha}\frac{1}{Z_t^S P_t}\left(\frac{Z_t^M}{Z_t^S}\right)^{(1-\alpha)/\alpha}, && (8)\\
\dot{h}^M &= \frac{1-\alpha}{\alpha}\dot{Z}^M - \frac{1}{\alpha}\dot{Z}^S - \dot{P}. && (9)
\end{aligned}$$

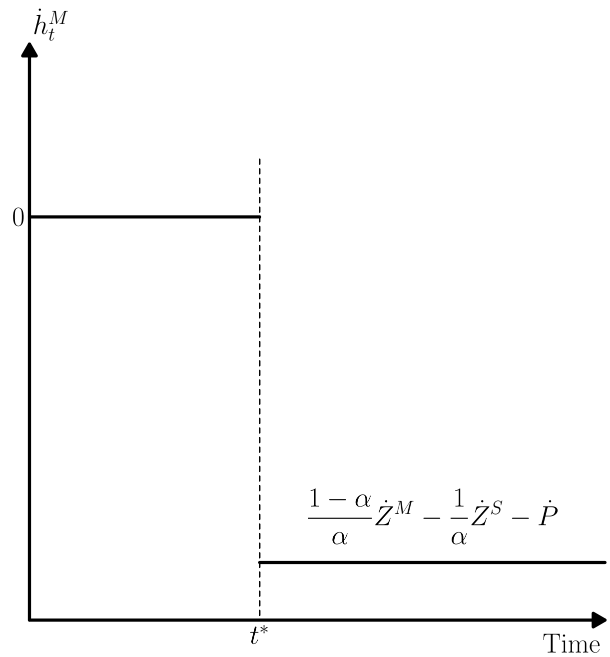


Figure 2: Growth rate of employment share in the Malthus technology

Note: This is an illustrative path of the growth rate of employment share in the Malthus technology under the assumption that the exogenous variables grow at constant, but potentially different, rates.

Two observations are worth making here. First, the share of labor in the Malthus technology is the only endogenous variable in our model. The evolution of the share is the solution to a sequence of static problems. The dynamics in the model are entirely due to the evolution of Z^S, Z^M, and P. Second, condition (3) implies $\dot{h}^M < 0$ after t^*. Equation (9) delivers the result that the employment share of technology M decreases at a constant rate after t^* (see Figure 2).

The economy's GDP is

$$y_t = Z_t^S + \frac{\alpha}{1-\alpha} Z_t^S h_t^M. \tag{10}$$

The growth rate of y_t is

$$\dot{y}_t = \frac{(1-\alpha)(1-h_t^M)}{1-\alpha+\alpha h_t^M}\dot{Z}^S + \frac{(1-\alpha)h_t^M}{1-\alpha+\alpha h_t^M}\dot{Z}^M - \frac{\alpha h_t^M}{1-\alpha+\alpha h_t^M}\dot{P}. \tag{11}$$

See Appendix A for the derivation of Equations (10) and (11).

3 Quantitative analysis

The onset of modern growth in our model is when technology S starts operating at date t^*. While Equation (4) pins down t^*, the expression is not quantitatively useful, since it depends on unobserved variables. In this section, we show that the employment share in the Malthus technology is sufficient to determine t^*. Quantitatively, we map the employment share in the Malthus technology in the model to the share of employment in agriculture in the data. Similarly, the level and growth rate of GDP in Equations (10) and (11) depend on unobserved TFPs and their growth rates. We also show that the rate of decline of the share of employment in agriculture is a sufficient statistic for the dynamics of GDP growth during the transition.

3.1 Determining the onset of modern growth

Under the assumption in Section 2 that Z^M, Z^S, and P grow at constant (but potentially different) rates, Equation (9) implies that $\dot{h}^M$ is constant. So, $h_t^M = \exp((t-t^*)\dot{h}^M)$ at any date $t \geq t^*$, or

$$t^* = t - \frac{\ln\left(h_t^M\right)}{\dot{h}^M}, \qquad \text{for } t \geq t^*. \tag{12}$$

Equation (12) has several implications. First, the reasons that different countries have different shares or growth rates of agricultural employment do not matter for estimating the onset of modern growth. Countries could differ in their structural parameters, and the levels and growth rates of TFPs and population. In our model, these differences manifest themselves in different h_t^M and $\dot{h}_t^M$ (see Equations 8 and 9). Second, both level and growth rate of the share of agricultural employment are needed to estimate the onset of transition. Countries with the same share of agricultural employment at a point in time could have started their transitions at different times. Third, the onset of economic transition can be determined without using GDP data, which means we can test whether the onset of transition implied by the share of agricultural employment coincides with a change in GDP growth. Finally, since the model implies that the growth rate of the share of agricultural employment is constant, a few recent (presumably more reliable) observations are sufficient to determine the onset of transition. We do not need historical time series on the share of agricultural employment.

To operationalize Equation (12), consider the specification below for country i:

$$\ln(h_{t,i}^M) = \beta_{0,i} + \beta_{1,i} t, \tag{13}$$

which implies $\beta_{1,i} = \dot{h}_i^M$. The onset of transition is

$$\begin{aligned} t_i^* &= -\frac{\beta_{0,i}}{\beta_{1,i}}, \\ &= -\frac{\ln\left(h_{t,i}^M\right) - \beta_{1,i} t}{\beta_{1,i}} = t - \frac{\ln\left(h_{t,i}^M\right)}{\dot{h}_i^M}, \end{aligned} \tag{14}$$

which is the same as Equation (12).

3.2 Dynamics of GDP during the transition

From Equation (10) it is easy to see that the long-run path of GDP is that of Z_t^S. The relative deviation of GDP from its long-run path, which we denote by $\hat{y}_t$, is then

$$\hat{y}_t \equiv \frac{y_t - Z_t^S}{Z_t^S} = \frac{\alpha}{1-\alpha} h_t^M, \quad \text{for} \quad t \geq t^*. \tag{15}$$

First, Equation (15) implies that $\hat{y}_t$ grows at rate $\dot{h}^M$. As the share of agricultural employment declines, $\hat{y}_t$ approaches zero and the paths of GDP and Z^S converge. Second, using (15) at two instants t and $t+\omega$, we get

$$\ln(1+\hat{y}_{t+\omega}) - \ln(1+\hat{y}_t) + \ln(Z_{t+\omega}^S) - \ln(Z_t^S) = \ln y_{t+\omega} - \ln y_t.$$

We then approximate $\ln(1+\hat{y}_{t+\omega}) - \ln(1+\hat{y}_t) \simeq \hat{y}_{t+\omega} - \hat{y}_t$.[1] This yields

$$\hat{y}_{t+\omega} - \hat{y}_t + \ln(Z_{t+\omega}^S) - \ln(Z_t^S) \simeq \ln y_{t+\omega} - \ln y_t,$$

which implies

$$\exp(\omega \dot{h}^M)\hat{y}_t - \hat{y}_t + \omega \dot{Z}^S \simeq \ln y_{t+\omega} - \ln y_t, \text{ since } \dot{\hat{y}}_t = \dot{h}^M.$$

Similarly, at $t+\omega$,

$$\begin{aligned}
\exp(\omega \dot{h}^M)\hat{y}_{t+\omega} - \hat{y}_{t+\omega} + \omega \dot{Z}^S &\simeq \ln y_{t+2\omega} - \ln y_{t+\omega} \\
\Rightarrow \exp(\omega \dot{h}^M)\left(\exp(\omega \dot{h}^M)\hat{y}_t - \hat{y}_t\right) + \omega \dot{Z}^S &\simeq \ln y_{t+2\omega} - \ln y_{t+\omega}.
\end{aligned}$$

Substituting, rearranging, and evaluating at $\omega = 1$, we get

$$\ln y_{t+2} - \ln y_{t+1} \simeq \exp(\dot{h}^M)\left(\ln y_{t+1} - \ln y_t\right) + \dot{Z}^S\left(1 - \exp(\dot{h}^M)\right). \tag{16}$$

The rate of decline in the share of agricultural employment is thus a sufficient statistic to describe the dynamics of GDP growth after t^*.

Note that (16) is a result, not just an accounting formula. While, as noted earlier, one could estimate t^* by *assuming* $\dot{h}^M$ *is constant*, the ad-hoc assumption would not imply (16). In deriving (16) we have used the model's optimal allocation of labor in the two technologies, which depends on the model's pa-

[1] We are approximating $\ln(1+\hat{y}_{t+\omega}) - \ln(1+\hat{y}_t)$ with $\hat{y}_{t+\omega} - \hat{y}_t$, not $\ln(1+\hat{y}_t)$ with $\hat{y}_t$.

rameters.

To estimate the autoregressive coefficient of the growth rate of GDP after t^*, we specify the data-generating process for GDP for country i as

$$\ln y_{t,i} = \gamma_{0,i} + \gamma_{1,i} t + \gamma_{2,i} \exp(\gamma_{3,i} t), \quad \text{for } t \geq t_i^*. \tag{17}$$

Suppressing the country notation, this process has the property that

$$\dot{y}_t \equiv \frac{d \ln y_t}{dt} = \gamma_1 + \gamma_2 \gamma_3 \exp(\gamma_3 t).$$

It is easy to see that

$$\dot{y}_{t+1} - \gamma_1 = (\dot{y}_t - \gamma_1) \exp(\gamma_3),$$

which implies

$$\dot{y}_{t+1} = \exp(\gamma_3) \dot{y}_t + \text{constant}.$$

Hence, the autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.

Equations (16) and (17) thus represent a testable implication: The autoregressive coefficient for GDP growth after t^* is the exponential of the growth rate of the share of agricultural employment. Estimate of the latter is $\exp(\beta_1)$ from (13).

In sum, the share of agricultural employment is sufficient to pin down both the onset of transition and modern GDP dynamics.[2]

In the next two subsections, we use recent data on the share of agricultural employment and estimate the onset of transition for the United States and the

[2] In the quantitative exercises below, our model's h^M is measured by the share of agricultural employment in the data. So, one interpretation is that agricultural goods are produced using the Malthus technology and non-agricultural goods are produced using the Solow technology. With such an interpretation, a relative price is involved in the GDP calculation. However, TFP in our model does not map to the traditional measure; it could include the relative price of non-agricultural good and inputs other than labor. For instance, the evolution of Z^S could capture the dynamics of the other inputs and relative price.

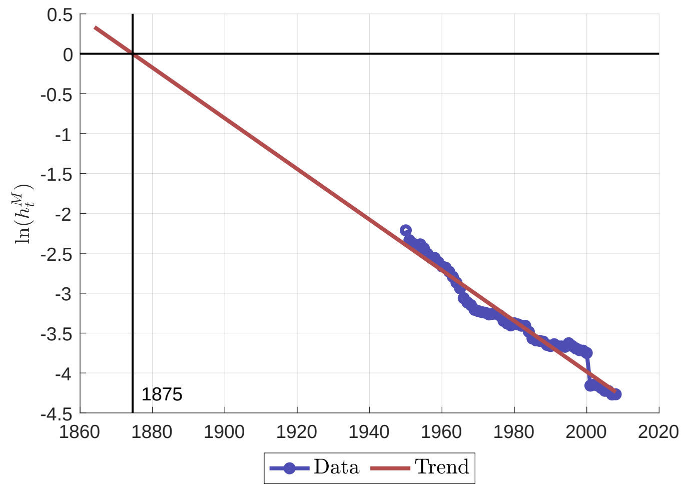


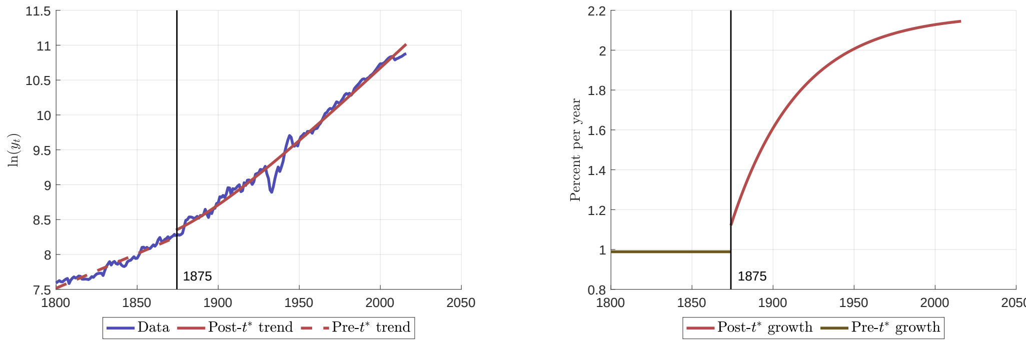
Figure 3: Onset of modern growth in the United States

Source: Herrendorf et al. (2014) and authors' calculations.

United Kingdom. Both countries have long time series of annual observations on GDP. We first validate our estimate using GDP data: GDP growth before the onset of transition is less than that after the onset of transition. Second, we estimate the autoregressive coefficient of the growth rate of GDP during the transition and show that the coefficient matches the rate of decline in the recent share of agricultural employment. Finally, we estimate the onset of transition for several countries in Western Europe and show that the evolution of GDP for each country is consistent with its share of agricultural employment.

3.3 The U.S.

We estimate β_0 and β_1 in (13) with post-World War II annual data on the share of agricultural employment for the U.S. We find t^* to be 1875 from Equation (14). This is illustrated in Figure 3.



A – Level of GDP

B – Growth rate of GDP

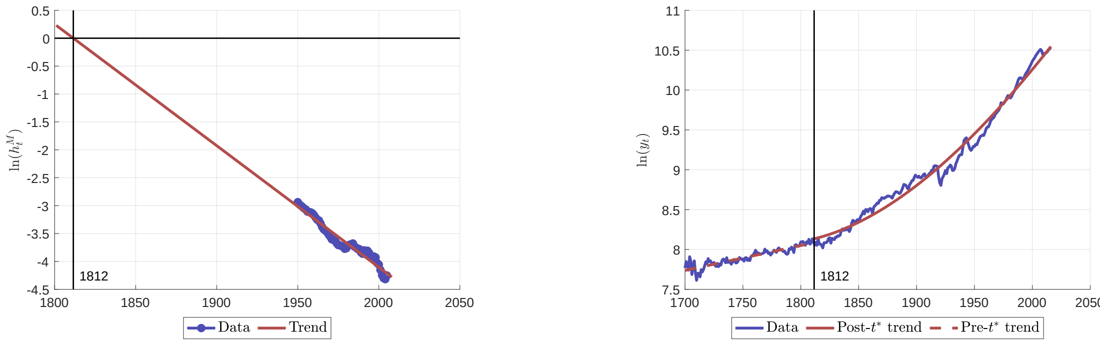
Figure 4: GDP and its growth in the United States: 1800-2016

Note: For GDP we do not use interpolated observations. Instead, we use only consecutive annual observations, which start in 1800. In Panel A, the pre-1875 trend is based on the best linear fit of the GDP time series from 1800 to 1875. The post-1875 trend is based on the estimated coefficients for the specification in (17).
Source: Delventhal et al. (2021) and authors' calculations.

Next, we estimate the coefficients in (17) using post-t^* GDP data for the U.S. We find $\exp(\gamma_3) = 0.977$. The fit is illustrated in Panel A of Figure 4. We check whether our estimated coefficient is consistent with agricultural employment dynamics—i.e., whether $\exp(\gamma_3)$ approximately equals $\exp(\beta_1)$. We find $\exp(\beta_1) = \exp(\dot{h}^M) = 0.969$.

One way to validate our estimate of t^* is to check whether GDP growth is higher after t^*. (Recall that we did not use GDP data to estimate the onset of transition.) It is clear in Panel B of Figure 4 that the GDP growth rate is higher after 1875.

Thus, our estimate of 1875 as the year when the U.S. transitioned from Malthus to Solow, based only on agricultural employment, is consistent with (a) the change in GDP growth at 1875 and (b) autoregressive coefficient for GDP growth after 1875.



A – Share of agricultural employment: 1949-2016

B – GDP: 1700-2016

Figure 5: Onset of transition and GDP dynamics in the United Kingdom

Note: For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations, which start in 1700. The pre-1812 trend is based on the best linear fit of the GDP time series from 1700 to 1812. The post-1812 trend is based on the estimated coefficients for the specification in (17).
Source: Herrendorf et al. (2014), Delventhal et al. (2021), and authors' calculations.

3.4 The U.K. and Western European Countries

As in the case of the U.S., we estimate the coefficients in (13) with post-World War II annual data on the share of agricultural employment for the U.K. From Equation (14), we find t^* to be 1812 for the U.K. This is illustrated in Panel A of Figure 5.

Panel B of the figure illustrates GDP dynamics. We estimate the coefficients in (17) using GDP data post-t^*. The autoregressive coefficient of GDP growth $\exp(\gamma_3) = 0.997$. The coefficient is almost equal to the (exponential of the) growth rate of the share of agricultural employment: $\exp(\dot{h}^M) = 0.979$.

Table 1 reports our results for several countries in Western Europe. We do not have data on the share of agricultural employment going back to 1949 for all of these countries as we did for the U.S. and the U.K. We use the share data from the World Bank, available from 1991 to 2022, to estimate the onset of transition.

Table 1: Onset of transition and GDP dynamics: Western Europe

	t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$
Austria	1881	0.9768	0.9830
Belgium	1900	0.9619	0.9570
Denmark	1890	0.9704	0.9583
Finland	1905	0.9712	0.8499
France	1889	0.9718	0.9625
Germany	1908	0.9610	0.8805
Greece	1912	0.9791	0.9374
Italy	1880	0.9760	0.9690
Netherlands	1861	0.9761	0.9933
Norway	1918	0.9616	0.8665
Portugal	1925	0.9719	0.8449
Spain	1920	0.9669	0.9064
Sweden	1851	0.9762	0.9575
Switzerland	1865	0.9771	0.9052

Note: For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations after t^*. For the share of agricultural employment we use data from 1991 to 2022. The onset of transition is at t^*, the rate of decline in the share of agricultural employment is $\dot{h}^M$, and the autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.
Source: World Bank, Delventhal et al. (2021), and authors' calculations.

The fit is remarkable: More than a century's worth of GDP growth in Western Europe is consistent with the rate of decline in the share of agricultural employment over the past 30 years. Ex-ante one would not expect the recent observations on agricultural employment to account for GDP dynamics over the 20th century, during which the composition of GDP in Western Europe has changed dramatically.

Our estimate of the onset of transition might be late by a few decades for some countries. An example is France, a leading industrial nation in the 19th century, for which the past 30 years of the share of agricultural employment imply the onset of transition was 1889. The share in France, however, is an anomaly. We have sporadic observations starting in 1856 (Herrendorf et al., 2014). Around 1860, the U.S. and France had almost the same share: 52% and 51%, respectively. In 1954, the U.S. share had declined to 9%, while France's share was almost three times higher at 26%. One reason for the anomaly in France could be the Méline tariffs (see Golob, 1944). We do not have a mechanism in our model to study the effect of such distortions.

3.5 Developing economies

An advantage of our approach, based on recent agricultural employment, is that we can estimate the onset of transition from Malthus to Solow for today's developing economies. These economies typically do not have time series of GDP data long enough to determine the onset of transition. Table 2 reports the onset of transition and the autoregressive coefficient of GDP growth post transition. As in the previous subsection, the share of agricultural employment is from the World Bank, 1991 to 2022. Using (14) and the coefficients in (13), we estimate the onset of transition for each country. The estimates of the coefficients in (17) yield $\exp(\gamma_3)$. The sample is the set of countries (i) for which we have annual GDP observations after their onset of transition and (ii) whose GDP was below 25% of U.S. GDP in 2016.

Again, our approach fits the data well: $\exp(\gamma_3) \approx \exp(\dot{h}^M)$ for most countries. Note that the countries in Table 2 are in different stages of development. In 2000, China is 8 times as rich as Mozambique. The countries are also in different stages of structural transformation. The share of agricultural employment in Burkina Faso in 2000 is 85%, but in Sri Lanka it is 38%. Despite these differences, GDP dynamics after the transition is pinned down by agricultural employment dynamics.

Table 2: Onset of transition and GDP dynamics: Developing economies

	t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$		t^*	$\exp(\dot{h}^M)$	$\exp(\gamma_3)$
Afghanistan	1969	0.9845	0.9913	Liberia	1954	0.9862	0.9949
Bangladesh	1971	0.9807	0.9839	Malawi	1952	0.9932	0.9301
Benin	1970	0.9774	0.9710	Mongolia	1963	0.9776	0.9929
Bolivia	1941	0.9840	0.9914	Mozambique	1971	0.9929	0.9288
Burkina Faso	1975	0.9933	0.9846	Myanmar	1963	0.9870	0.9887
Burundi	1969	0.9969	0.8756	Namibia	1959	0.9751	0.9926
Cambodia	1987	0.9718	0.7524	Nepal	1960	0.9921	0.9767
Cameroon	1976	0.9815	0.9908	Peru	1943	0.9823	0.8789
Chad	1967	0.9931	0.9851	Philippines	1955	0.9787	0.8289
China	1977	0.9683	0.9834	Rwanda	1992	0.9802	0.3045
Comoros	1971	0.9791	0.8064	Senegal	1974	0.9709	0.9802
Congo	1967	0.9897	0.9920	Sierra Leone	1976	0.9841	0.9889
Ethiopia	1959	0.9933	0.9933	Sri Lanka	1938	0.9837	0.9890
Gambia	1956	0.9888	0.8081	St. Lucia	1953	0.9649	0.9917
Ghana	1964	0.9830	0.9872	Syria	1951	0.9722	0.9908
India	1965	0.9846	0.9857	Tanzania	1978	0.9897	0.9801
Indonesia	1950	0.9834	0.8880	Togo	1976	0.9746	0.9849
Laos	1980	0.9887	0.9819	Yemen	1968	0.9734	0.8709
Lesotho	1963	0.9779	0.9909	Zambia	1958	0.9916	0.9926

Note: For GDP, we do not use interpolated observations. Instead, we use only consecutive annual observations after t^*. For the share of agricultural employment we use data from 1991 to 2022. The sample is the set of developing economies (i) for which we have annual GDP observations after their onset of transition and (ii) whose GDP was below 25% of U.S. GDP in 2016. The onset of transition is at t^*, the rate of decline in the share of agricultural employment is $\dot{h}^M$, and the autoregressive coefficient on GDP growth is $\exp(\gamma_3)$.
Source: World Bank, Delventhal et al. (2021), and authors' calculations.

4 Conclusion

In our model, a single good can be produced using two technologies: Malthus (diminishing returns) and Solow (constant returns). TFPs and population are exogenous. The economy's GDP exhibits three stages: (i) stagnation, (ii) transition with higher growth, and (iii) constant growth in the long run. We map the Malthus technology to agriculture and show that agricultural employment is sufficient to determine both the onset of economic transition and the dynamics of GDP during the transition. Specifically, we show that GDP growth during the transition follows a first-order autoregressive process and that the autoregressive coefficient is pinned down by the rate of decline in the share of agricultural employment.

Quantitatively, we use recent data on agricultural employment to estimate the onset of transition for the U.S., U.K., and several Western European countries. Our estimate does not rely on GDP data but is consistent with lower growth before the onset of transition and higher growth after. The autoregressive coefficient of GDP growth during the transition is practically the same as that implied by the rate of decline in the share of agricultural employment. There is no a priori reason that agricultural employment over a recent few years would pin down GDP dynamics over two centuries that were characterized by large structural changes.

Our method is especially useful in the context of developing economies, which do not have historical data. Again, we find that the share of agricultural employment is sufficient to determine the onset of transition and GDP growth during the transition.

Our model is a model of economic transition, not demographic transition, as the processes for TFPs and population are exogenous. Endogenizing one or more of these processes could potentially deliver more testable implications. However, the fact remains that our simple quantitative framework, using only agricultural employment, is remarkably consistent with the onset of transition and dynamics of GDP. Thus, richer frameworks have to bear the burden of delivering our quantitative results.

A lesson from our model is for the well-known lack of cross-country convergence in GDP. One reason for the lack of convergence could be that some countries transitioned from Malthus to Solow later and some transitioned earlier. If we had reliable historical GDP data for all countries then we could check whether today's poor countries have more recent transition dates compared with rich countries. Our approach, however, does not require historical GDP data; a few recent observations on the share of agricultural employment will suffice. Using a level and the rate of decline of the share, we can determine the onset of transition and, hence, examine how much of the lack of income convergence is due to late versus early transitions.

References

G. S. Becker, K. M. Murphy, and R. Tamura. Human capital, fertility, and economic growth. *Journal of Political Economy*, 98(5, Part 2):S12–S37, 1990.

M. J. Delventhal, J. Fernández-Villaverde, and N. Guner. Demographic transitions across time and space. *NBER Working Paper Series*, (29480), 2021.

O. Galor and D. N. Weil. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90(4):806–828, 2000.

E. O. Golob. *The Méline tariff: French agriculture and nationalist economic policy.* Columbia University Press, 1944.

M. Goodfriend and J. McDermott. Early development. *American Economic Review*, 85(1):116–133, 1995.

G. D. Hansen and E. C. Prescott. Malthus to Solow. *American Economic Review*, 92(4):1205–1217, 2002.

B. Herrendorf, R. Rogerson, and A. Valentinyi. Growth and structural transformation. In P. Aghion and S. N. Durlauf, editors, *Handbook of Economic Growth*, volume 2, chapter 6, pages 855–941. Elsevier, 2014.

C. I. Jones. Was an industrial revolution inevitable? Economic growth over the very long run. *The BE Journal of Macroeconomics*, 1(2), 2001.

T. R. Malthus. *An Essay on the Principle of Population.* J Johnson, London, 1798.

D. Ricardo. *On the principles of political economy and taxation.* J Murray, London, 1817.

R. Tamura. Human capital and the switch from agriculture to industry. *Journal of Economic Dynamics and Control*, 27(2):207–242, 2002.

A Derivation of Equations (10) and (11)

Using the solution for H_t^M for $t \geq t^*$, the output of technology M when both technologies operate is

$$Y_t^M = \left(Z_t^M\right)^{1-\alpha} \left[(1-\alpha) \frac{\left(Z_t^M\right)^{1-\alpha}}{Z_t^S}\right]^{(1-\alpha)/\alpha} = (1-\alpha)^{(1-\alpha)/\alpha} \left(\frac{Z_t^M}{Z_t^S}\right)^{(1-\alpha)/\alpha},$$

implying that output per capita and its growth rate are

$$y_t^M = = \frac{1}{1-\alpha} Z_t^S h_t^M, \tag{A.1}$$

$$\dot{y}^M = \dot{Z}^S + \dot{h}^M. \tag{A.2}$$

For technology S, output per capita is

$$y_t^S = Z_t^S(1-h_t^M) = Z_t^S - (1-\alpha) y_t^M, \tag{A.3}$$

and its rate of growth is

$$\dot{y}_t^S = \frac{d \ln Z_t^S}{dt} \frac{Z_t^S}{y_t^S} - (1-\alpha) \frac{d \ln y_t^M}{dt} \frac{y_t^M}{y_t^S} = \frac{1}{1-h_t^M} \dot{Z}^S - \frac{h_t^M}{1-h_t^M} \dot{y}^M. \tag{A.4}$$

The economy's GDP is $y_t = y_t^S + y_t^M$. Using (A.1) and (A.3), this is

$$y_t = Z_t^S + \frac{\alpha}{1-\alpha} Z_t^S h_t^M.$$

The GDP growth rate is

$$\dot{y}_t = \frac{d \ln y_t^S}{dt} \frac{y_t^S}{y_t} + \frac{d \ln y_t^M}{dt} \frac{y_t^M}{y_t} = \left(1 - \frac{y_t^M}{y_t}\right) \left(\dot{Z}^S \frac{1}{1-h_t^M} - \dot{y}^M \frac{h_t^M}{1-h_t^M}\right) + \frac{y_t^M}{y_t} \dot{y}^M.$$

where

$$\frac{y_t^M}{y_t} = \frac{\frac{1}{1-\alpha} Z_t^S h_t^M}{Z_t^S + \frac{\alpha}{1-\alpha} Z_t^S h_t^M} = \frac{h_t^M}{1-\alpha+\alpha h_t^M}.$$

It follows that

$$
\begin{aligned}
\dot{y}_t &= \frac{1 - y_t^M / y_t}{1 - h_t^M} \dot{Z}^S + \left(\frac{y_t^M}{y_t} - \frac{h_t^M}{1 - h_t^M} \left(1 - \frac{y_t^M}{y_t} \right) \right) \dot{y}^M, \\
&= \frac{1-\alpha}{1-\alpha+\alpha h_t^M} \dot{Z}^S + \frac{\alpha h_t^M}{1-\alpha+\alpha h_t^M} \left(\dot{Z}^S + \frac{1-\alpha}{\alpha} \dot{Z}^M - \frac{1}{\alpha} \dot{Z}^S - \dot{P} \right), \\
&= \frac{(1-\alpha)(1-h_t^M)}{1-\alpha+\alpha h_t^M} \dot{Z}^S + \frac{(1-\alpha) h_t^M}{1-\alpha+\alpha h_t^M} \dot{Z}^M - \frac{\alpha h_t^M}{1-\alpha+\alpha h_t^M} \dot{P}.
\end{aligned}
$$