Political Competition, Fiscal Structure and Economic Growth: Theory and Evidence from Indian States[1]

[1] This paper extends work begun with Stanley Winer and Bernie Grofman on political competition and its application to both India and Canada.

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This Draft, September 2024

Abstract

This paper uses a two period overlapping generations model with balanced growth to investigate the links arising among political competition, the effective number of political parties (ENP), fiscal structure and the growth rate of the economy. The model highlights two hypotheses with respect to political competition and ENP). First, while effective competition requires some minimum number of electorally credible parties, a rise in the effective number of parties above that minimum will fragment the credibility of opposition to the incumbent governing party, lessening effective competition and leading to operational inefficiency and excessive government size. The second hypothesis argues that as the effective number of political parties increases, a winning electoral party strategy will need to respond by offering a broader range of government services to span the wider set of options proposed by competitors. Moreover, the immediacy of electoral competition encourages the incumbent governing party to substitute consumption services for less electorally effective investment services. The combination of these factors—the income effect of excessive government size and the substitution effect of consumption for investment services—means that more fragmented party structures will be associated with lower overall rates of growth rate. We use an annual panel data set of 14 major Indian states spread over six decades to test the empirical validity of our model and the overall results suggest that the data from Indian states fits well with the predictions of the model.

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1. Introduction

In this paper we explore one aspect of the special nature of political party competition in majoritarian democracies, what has been called Duverger-Demsetz competition (Duverger 1954; Demsetz 1968, 2008), in relation to the effective number of political parties (ENP) and economic growth. To do so we follow Carmec et. Al. (2019) in using a two period overlapping generations model with balanced growth to investigate the links arising between political competition, ENP, the output mix of government services, and the growth rate of the economy.[3] The model highlights two hypotheses with respect to ENP and growth. First, while effective competition requires some minimum number of electorally credible parties, a rise in the effective number of parties above that minimum fragments the credibility of opposition to the incumbent governing party, lessening effective competition and leading to operational inefficiency and excessive government size. While this suggests that the shape of the relationship between ENP and government size is U-shaped, the implied relationship between ENP and growth has an inverted U-shape.[4] The second hypothesis argues that as the range of party platforms offered by a larger effective number of political parties increases, a winning party strategy will respond by offering a broader range of government services to span the wider set of options proposed by competitors. Moreover, the immediacy of electoral competition encourages the incumbent governing party to substitute platforms featuring consumption for less electorally effective investment services (Chhibber and Kollman, 1998; Baraldi, 2008; Lewis and Hendrawan, 2019; and Scartascini and Crain, 2021). This then feeds back into a lower growth rate. The combination of these factors—the income effect of excessive government size and the substitution effect of consumption for investment services—means that above some minimum a more fragmented party structure will be associated with a lower overall rate of economic growth.

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[3] ENP, the effective number of political parties is defined as ENPjt=1/i=1Isijt2 where si is the seat (or vote) share of party *i* in state *j* at time *t*.

[4] See Durham (1999) and Acemoglu and Robinson (2006) for analyses that posit a nonmonotonic relationship between political competition (viewed as a spectrum running from autocracy through democracies) and the rate of economic growth. This analysis considers the degree of political competition as reflected in variations in one institution of contemporary democracies—the effective number of political parties. See Ferris and Voia (2023) and Ferris and Dash (2024).

The second part of the paper tests these hypotheses on a panel of data from 14 large Indian states over the period 1959 to 2019, states that encompassed over 85 percent of the Indian population.[5] India's states provide a useful case study of political competition, government size and economic growth because the size and heterogeneity of India's population has resulted in a wide variety of political party structures.[6] For example, it is not unusual for an Indian state election to feature more than 100 parties. Even when weighting parties by the percentage of the seats won, India stands out in comparison to other Westminster parliamentary democracies as featuring a larger effective number of competing parties in both their center and state governments.[7]

2. A model of the role of political party structure on fiscal structure and economic growth in majoritarian democracies

A. The output decision, representative firm behaviour.

At time t a representative firm, i, is assumed to produce a composite output, Y_{it}, using a Cobb-Douglas technology of the form

$$Y_{it} = \theta K_{it}^{\alpha} L_{it}^{1-\alpha} (\frac{G_{it}}{L})^{1-\alpha} = \theta K_{it}^{\alpha} L_{it}^{1-\alpha} (\frac{G_{it}}{G_t} g_t)^{1-\alpha} = \theta L_{it} k_{it}^{\alpha} \left(v(N_t) g\right)^{1-\alpha} \quad (1)$$

where θ is the level of technology, G_{it} is public investment, and $\nu(N_t) = \frac{G_{it}}{G_t}$ is the share of government investment in total government spending, entering the firm's production function as an intermediate good. $\nu\left(N_t\right)$ is assumed to be decreasing in N_t, the effective number of

[5] The 14 Indian states included in our study are: Andhra Pradesh, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal. Assam was excluded because it was subdivided twice during the 70's and 80's and because it has experienced long periods of communal tension with associated outbreaks of violence. Jammu and Kashmir was excluded for a similar reason.

[6] See Ferris and Dash (2023) for a detailed discussion of the factors determining party structure across Indian states.

[7] Chhibber and Kollman (2004, Tables 1.1 to 1.4) note that over the 1960 to 2000 time period the effective number of parties in India (4 and 7) was much higher than in Canada, Britain or the U.S. (2 to 4). Dash et al. (2019) document the average effective number of parties in Indian states as between 3 and 6 over the period 1952 to 2009.

political parties.[8] Finally K_{it} is the firm's capital stock with $0 < \alpha < 1$ its factor share while L_{it} is the labour used by the firm. $k_{it} = \frac{K_{it}}{L_{it}}$ is the firm's capital to labour ratio and $g_t = G_t/L$ is per capita government services.

If capital depreciates entirely after use and N_t parametric to the firm, firms will maximize profits when,

$$\frac{\partial Y_{it}}{\partial K_{it}} = \theta\alpha K_{it}^{\alpha-1} L_{it}^{1-\alpha}(v(N_t)g_t)^{1-\alpha} = \theta\alpha(\frac{v(N_t)g_t}{k_{it}})^{1-\alpha} = R_t, \tag{2}$$

$$\frac{\partial Y_{it}}{\partial L_{it}} = \theta(1-\alpha)K_{it}^{\alpha}L_{it}^{-\alpha}(v(N_t)g_t)^{1-\alpha} = \theta(1-\alpha)k_{it}^{\alpha}(v(N_t)g_t)^{1-\alpha} = w_t,$$
(3)

where R_t is the cost of capital to the firm and w_t is the wage. As all firms are identical, all firms choose the same capital-labour ratio so that aggregate output, Y_t can be expressed as

$$Y_t = \theta L_2(\frac{K_t}{L_2})^{\alpha}(v(N_t)g_t)^{1-\alpha}, \tag{4}$$

where L_2 is the aggregate number of private firm employees. By including the number of public service employees, $L_1 = L - L_2$, output per worker, public and private can be written in per capita terms as

$y_t = \theta\frac{L_2}{L}\left(\frac{L}{L_2}\frac{K_t}{L}\right)^{\alpha}(v(N_t)g_t)^{1-\alpha}$ with $\frac{L_2}{L} = (1-\lambda)$ this becomes

$$y_t = \theta k_t^{\alpha}[(1-\lambda)v(N_t)g_t]^{1-\alpha} \tag{5}$$

Note here that $y_t = \frac{Y_t}{L}$, $k_t = \frac{K_t}{L}$ and $k_i = \frac{K_i}{L_i} = \frac{K_j}{L_j} \neq k_t$. With $1-\lambda$ as a parameter and using (5)

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[8] Winer et al (2021) argue that the public investment component of government spending is more public (non-rivalrous) in nature while public consumption is more private. Employing a swing and core voter model and using data from Indian states, they show that the share of public investment spending in total budget improves as the electoral significance of swing voters in an election increases.

$$R_t = \frac{\partial y_t}{\partial k_t} = \theta\alpha(1 - \lambda)^{1-\alpha} k_t^{\alpha-1} (v(N_t) g_t)^{1-\alpha} = \alpha \frac{y_t}{k_t}. \quad (6)$$

Note that $R_t = R$ when $\frac{y_t}{k_t}$ is a constant over time. From (3) w_t can be rewritten in per worker terms as

$$w_t = \frac{\partial y_t}{\partial(1-\lambda)} = \theta(1 - \alpha)(1 - \lambda)^{-\alpha} k_t^{\alpha} (v(N_t) g_t)^{1-\alpha} = (1 - \alpha)\frac{y_t}{(1-\lambda)}, \quad (7)$$

so that output per worker, $y_t = R_t k_t + w_t(1 - \lambda) = \alpha \frac{y_t}{k_t}. k_t + (1 - \alpha)\frac{y_t}{(1-\lambda)}. (1 - \lambda) = y_t$.

B. The household consumption decision

We assume that all individuals live for two periods and can choose to work in the private or public sector. Since work takes place only in the first period and individuals must save, x_t, to spread private consumption over their lifespan. Government services are provided only when individuals are young (e.g. receive birth and education assistance) and all savings are used for private consumption in the retirement period of life. Individuals are assumed to have the same logarithmic utility functions, $U(c_1) = \ln ln\, c_1 + ln(1-v(N))g$ and $U(c_2) = ln\, c_2$ and have a common rate of time preference, $0 < \beta < 1$. Political parties allocate resources by taxing income from both labour and capital at a flat rate $0 < \tau < 1$ and by choosing the level and division of government services. They also benefit by consuming the residual not competed away by the potential competition of rival political parties. Individuals receive benefits of public consumption spending, G_{ct}, which are relatively private.

With $\frac{G_{ct}}{L} = \frac{G_{ct}}{G_t}\frac{G_t}{L} = \left(1 - v(N_t)\right) g_t$ and N_t given, where $\left(1 - v(N_t)\right)$ is the share of government consumption in total government spending, the household choice problem is

$Max\, lnc_{1t} + \ln ln\, [\left(1 - \nu(N_t)\right) g_t] + \beta \ln ln\, c_{2,\,t+1}$ subject to $c_{1.t} = \overline{w}_t - x_t$,

$c_{2,t+1} = \overline{R}_{t+1} x_t$ and $\left(1 - v(N_t)\right) g_t$ given.

Here $c_{1,t}$ and $c_{2,t+1}$ refer to private consumption in the first and second periods and x_t is the chosen level of savings. $\overline{R}_{t+1}$ is the return to savings net of taxes in the second period so that $\overline{R}_{t+1} = (1 - \tau)\left(1 + r_{t+1}\right)$ where $r_{t+1} > 0$ is the interest rate, and $\overline{w}_t = (1 - \tau)w_t$ is the wage net of taxes and $\overline{w}_t > 0$. Substituting the constraints into the objective function, the problem becomes

Max $L\left(x_t\right) =$

$$\ln ln\left((1 - \tau)w_t - x_t\right) + \ln ln\left(1 - v(N_t)\right) g_t + \beta \ln ln\left((1 - \tau)\left(1 + r_{t+1}\right)x_t\right) \quad (8)$$

The first order condition for an optimal choice is $\frac{\partial L}{\partial x_t} = \frac{-1}{(1-\tau)w_t - x_t} + \beta\frac{(1-\tau)\left(1+r_{t+1}\right)}{(1-\tau)\left(1+r_{t+1}\right)x_t} = 0,$

which simplifies to $x_t = \frac{\beta(1-\tau)}{(1+\beta)}w_t.$ (9)

Substituting $w_t = (1 - \alpha)\frac{y_t}{(1-\lambda)}$ from (7) we find $k_{t+1} = x_t = \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}y_t.$ (10)

Using the budget constraint and (10) to solve for the optimal consumption choices, we find

$$c_{1,t} = \overline{w}_t - x_t = (1 - \tau)w_t - \frac{\beta(1-\tau)}{(1+\beta)}w_t = \frac{(1+\beta)(1-\tau) - \beta(1-\tau)}{(1+\beta)}w_t = \frac{(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}y_t$$
(11)

Note that period 2 consumption and savings are both linear functions of per capita income, y_t. Under balanced growth $\frac{y}{k}$ is constant over time so that period 2 consumption will also be a linear function of y_t. That is,

$$c_{2,t+1} = \overline{R}_{t+1}x_t = R_{t+1}\frac{\beta(1-\tau)^2(1-\alpha)}{(1+\beta)(1-\lambda)}y_t = \frac{\alpha\,\beta(1-\tau)^2(1-\alpha)y_t}{(1+\beta)(1-\lambda)}\left(\frac{y_{t+1}}{k_{t+1}}\right)$$

$$= \frac{\alpha\,\beta(1-\tau)^2(1-\alpha)}{(1+\beta)(1-\lambda)}y_t\theta[\frac{(1-\lambda)v(N_t)g_t}{k_t}]^{1-\alpha} = \frac{\alpha(1-\alpha)\beta(1-\tau)^2\theta^2((1-\lambda)v(N_t)g_t)^{2-2\alpha}}{(1+\beta)k_t} \quad (12)$$

Note that from (5) government investment services remain a constant proportion of the per capita capital stock. That is,

$$\frac{y_t}{k_t} = \theta(1 - \lambda)^{1-\alpha} k_t^{\alpha-1} (v(N_t) g_t)^{1-\alpha} = \theta[(1 - \lambda)(\frac{v(N_t) g_t}{k_t})]^{1-\alpha} \quad (13)$$

C. The government budget constraint

Political parties compete for votes by offering a level of government services, g_t, and a division of these services between households, $(1 - v(N_t))$, and firms, $v(N_t)$. These are paid for by a unit tax on all income, τ. As discussed earlier, L_1 is the number of public sector employees and their wage, w_t, is the same as private sector's market determined wage. Government pays its employees out of taxes. We assume that the government budget must be balanced so that tax revenues, $T_t = \tau(Y_t + L_1 w_t) = G_I + G_c + L_1 w_t + z(N_t) Y_t$ where $z(N_t) Y_t$ is the cost of a political system with N_t effective parties. In the following section $z(N_t) Y_t$ is introduced as the agency cost of government and assumed to depend upon the degree of political competition. The size of the government sector is then larger than G_t and more accurately measured by T_t.

Rewriting the budget constraint in per capita terms, $\tau\left(\frac{Y_t}{L}\right) = \frac{G_t}{L} + (1 - \tau)\frac{L_1}{L} w_t + z(N_t) y_t$ or $g_t = \tau y_t - (1 - \tau)\lambda w_t - z(N_t) y_t$. Substituting in $w_t = (1 - \alpha)\frac{y_t}{(1-\lambda)}$ from (7) we find

$$g_t = \tau y_t - (1 - \tau)\lambda(1 - \alpha)\frac{y_t}{(1-\lambda)} - z(N_t) y_t = \left[\tau - z(N) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right] y_t \quad (14)$$

If we substitute g_t from (14) back into (5), we find,

$$y_t = \theta k_t^{\alpha} \left\{(1 - \lambda) v(N_t)\left[\tau - z(N) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right] y_t\right\}^{1-\alpha}$$

or $y_t = \theta^{\frac{1}{\alpha}} \left\{v(N_t)\left[(\tau - z(N_t))(1 - \lambda) - (1 - \tau)(1 - \alpha)\lambda\right]\right\}^{\frac{1-\alpha}{\alpha}} k_t.$ (15)

Note that in equation (15), the private output per capita is a linear function of per capita capital generating features resembling the classic '*AK*' model. For $y_t > 0$, it must be the case that

$[(\tau - z(N_t))(1 - \lambda) - (1 - \tau)(1 - \alpha)\lambda] > 0$ or $\tau > \frac{z(N_t)(1-\lambda)+(1-\alpha)\lambda}{[(1-\lambda)+(1-\alpha)\lambda]}$.

(16)

If we now substitute (15) back into (10), we find that,

$$k_{t+1} = \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}\theta^{\frac{1}{\alpha}}\left\{v\left(N_t\right)\left[\tau - z(N_t)\right](1-\lambda) - (1-\alpha)\lambda\right]\right\}^{\frac{1-\alpha}{\alpha}} k_t = \Psi\left(\tau, \nu(N_t), z\left(N_t\right)\right) k_t \tag{17}$$

where α, β and λ are parameters. The linear difference equation of per capita capital defined in equation (17) will result a positive growth rate, γ, when

$$\gamma = \Psi\left(\tau, \nu\left(N_t\right), z\left(N_t\right); \alpha, \beta, \lambda\right) - 1. \tag{18}$$

provided $\Psi\left(\tau, \nu\left(N_t\right), z\left(N_t\right); \alpha, \beta, \lambda\right) > 1$. This growth rate is (given N_t) a constant, launching the economy on a balanced growth path at $t = 0$.

D. Political Parties and Competition

Political parties compete to govern by proposing a policy platform that consists of a level of government services, g, split between investment, $v(N_t)$ and consumption services, $1 - v(N_t)$, and a tax rate, τ such that the government budget constraint is met. We assume that there are also private party benefits derived from being the governing party, $z(N_t)$, and these agency costs of providing government services are assumed to be controlled by the degree of political competition which in turn is a nonmonotonic function of N.[9] We assume that the effective number of competing political parties, N_t, is determined by the costs and benefits of party participation (political institutions and customs determined outside of the model). In any given political environment as summarized by N_t, $v(N_t)$ and $z(N_t)$ are both constants.

In choosing which party to support, the representative household prefers the party offering a wider range of consumption services where the breadth of these offerings is an increasing function of number of effective rivals. That is, because parties offer overlapping but distinctive

[9] Recent research suggests that z(N) has a U shape, first falling as a larger number of effective parties offsets the joint incentive that monopolistically competitive parties have to collude at the expense of the electorate. However, as entry continues, the winner-take-all nature of a majoritarian election means that more effective parties will tend to decrease the likelihood that any one challenger will be a credible rival to the incumbent. That is, above a minimum further entry fragments the vote among parties reducing the credibility of competing parties as effective monitors of the behaviour of the governing party. See Ferris, Winer and Grofman (2016), Ferris and Voia (2023) and Ferris and Dash (2024).

services and because households value parties offering a larger variety of consumption services, the winning party is led to offer more consumption services as the effective number of competing political parties. Hence as N_t increases $(1 - v(N_t))$ increases and $v(N_t)$ falls.

More formally, the winning party is assumed to maximize a political support function is based on the utility received by current voters. Using $W(U(c_{1t}, c_{2t}, 1 - v(N)_t)$ to represent the political support function, the appropriate strategy that maximizes welfare of the representative household is to

$Max\, W\left(g_t, \tau_t; N_t\right)$ subject to the budget constraint $g_t = \left[\tau - z(N) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right]y_t$ as defined in equation (14). Using a Lagrangian,

$$W\left(g_t, v\left(N_t\right), \tau\right) = lnc_{1t} + lnc_{2t} + ln\left[\left(1 - v\left(N_t\right)\right)g_t\right] + \mu\left\{\left[\tau - z\left(N_t\right) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right]y_t - g_t\right\}$$

, (19)

where from (11) and (12) $c_{1t} = \frac{(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}y_t$, and $c_{2t} = \frac{\alpha(1-\alpha)\beta(1-\tau)^2\theta^2((1-\lambda)v(N_t)g_t)^{2-2\alpha}}{(1+\beta)k_t}$.

With $y_t = \theta(1 - \lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha}$ from (15) $c_{1t} = \frac{(1-\tau_t)(1-\alpha)\theta(1-\lambda)^{-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha}}{(1+\beta)}$.

Using $X = \left[\tau - z(N) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right] > 0$ to simplify presentation, the first order conditions for an internal optimum become:

$$\frac{\partial W}{\partial g_t} = \frac{4-3\alpha}{g_t} + \mu\left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N_t))^{1-\alpha}g_t^{-\alpha}\right) = 0 \qquad (20)$$

$$\frac{\partial W}{\partial \tau} - \frac{3}{(1-\tau_t)} - \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha} = 0 \qquad (21)$$

$$\frac{\partial W}{\partial \mu_t} = g_t - \left[\tau - z\left(N_t\right) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right]\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha} = 0$$

(22)

These three first order conditions capture the optimal political trade-offs arising under the winning electoral strategy. First, overall government spending will be increased until the marginal value of that additional spending (coming directly from the consumption value of government consumption services and indirectly from the additional private consumption permitted by additional income produced by greater government investment) falls into line with the rising cost of the higher taxes needed to fund the government budget. Secondly, the tax rate will be increased as long as the marginal value of private consumption lost due to higher taxes is smaller than the marginal value of a larger level of government services permitted by a budget surplus. Given the conditions needed for an internal optimum, these conditions are sufficient to solve for optimal values for g_t^* and τ^* as a function of the initial per capita capital stock, k_t, and N_t.

By totally differentiating equations (20) – (22) with respect to N, we can solve for a sufficient condition for an internal optimum and thus solve for the general equilibrium effects of a change in ENP on the supply of government services, g_t, and tax rate, τ_t. The mechanics of doing so is reported in the accompanying appendix which shows that $\frac{dg}{dN} < 0$ while $\frac{d\tau}{dN}$ is indeterminant. Intuitively, any increase in ENP above the optimal competitive level leads the incumbent party to substitute more consumption for investment services. This in turn leads private output to fall along with levels of private consumption. The fall in output also leads to the loss of tax revenue as the cost of government, $z(N)$, is rising. Together these effects lead the governing party to respond by cutting back somewhat on its overall supply of government services. The effect on the tax rate is ambiguous because the higher tax revenues needed because of the increase in $z(N)$ is countered by the fall in tax revenue needed to supply a now fewer number of government services overall. The net effect will depend upon the elasticities of substitution in both production and consumption.

Using these results, we can solve for the effect of an increase in ENP, N, on the growth rate, Ψ. From (17) and (18),

$$\frac{d\Psi}{dN} = \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}\theta^{\frac{1}{\alpha}}\left\{\left(\frac{1-\alpha}{\alpha}\right)\left[v'(N)\frac{\tau(1-\alpha\lambda)-(1-\alpha)\lambda-(1-\lambda)z(N)}{(1-\lambda)} - v(N)z'(N)\right]^{\frac{1-2\alpha}{\alpha}}\right\}$$

$$+ \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}\theta^{\frac{1}{\alpha}}\left\{\frac{1-\alpha}{\alpha}\left[v(N)\frac{1-\alpha\lambda}{1-\lambda}\right]^{\frac{1-2\alpha}{\alpha}} - \left[v(N)\frac{\tau(1-\alpha\lambda)-(1-\alpha)\lambda-(1-\lambda)z(N)}{(1-\lambda)(1-\tau)}\right]^{\frac{1-\alpha}{\alpha}}\right\}\frac{d\tau}{dN}.$$

(23)

The first term in (23) represents the impact effect of an increase in N on the growth rate. It has two parts. The first is the direct effect of excessive party competition on the composition of government services, where fewer government investment services directly decrease private output and household savings so that the future capital stock falls relative to what it would have been. The second part of the term reflects the decreased incentive the governing party has to control agency costs under fragmented party competition producing a larger size of government. This in turn requires higher taxes with negative effects on both private output and current savings. Both impact effects are then negative in their effect on the growth rate.

The second term describes the indirect or general equilibrium consequences of an increase in ENP on the growth rate. It also has two parts that in this case have opposite effects on the growth rate. In addition, as we have seen, the sign of $\frac{d\tau}{dN}$ is itself ambiguous. Hence it is unclear whether the indirect effects complement or counter the impact effects. What can be said is that the effect of ENP on growth will be negative, $\frac{d\Psi}{dN} < 0$, if the general equilibrium consequences of a change in N on the tax rate are either complementary or small relative to the magnitude of the first order impact effects. In the empirical section that follows, we assume this is the case.

3. Empirical Implementation

i. Testing strategy

In this section we test two predictions that follow from the above analysis. First, an increase in ENP above some minimum increases excessive government size which in turn decreases the growth rate of income per capita. This follows from the assumption that ENP is a nonmonotonic measure of the degree of effective political competition, first standing for an increase in the degree of party competitiveness resulting from a reduction in the degree of oligopolistic party power and then reflecting the reduction in effective party competition coming from greater electoral fragmentation.[10] Second, an increase in ENP is argued to change the composition of government spending away from investment towards consumption services and reducing the

[10] In a majoritarian political system, the winner-take-all nature of electoral competition means that a larger effective number of parties will fragment the likelihood that any competitor will be a credible rival thus lowering contestability and the level of effective political competition. In our earlier work we have referred to this as Duverger-Demsetz competition. On the other hand, as the effective number of parties falls towards 2 collusion can arise among the smaller number of effective rivals allowing them to promote party specific goals at the expense of the electorate. See Ferris, Winer and Grofman (2016), Ferris and Voia (2023), and Ferris and Dash (2024).

rate of growth of per capita income.[11] This follows from the assumption that a larger ENP broadens the variety of current policy options offered by competitors and requires a winning electoral strategy to promise a larger range of consumption services as opposed to investment oriented government services than would otherwise have been the case.

Our test has three stages. The first stage follows the analysis of Ferris and Dash (2024) who used election year data from Indian states to test the hypothesis that political competition (as measured by ENP) and government expenditure size have a U-shaped relationship with the minimum point representing the level of ENP that minimizes government agency costs through party competition. Here we use a panel of annual Indian state data to reassess that finding. Recognizing that state government size, ENP(Seat) and real income per capita (Rypc) are variables with different degrees of stationary, the analysis uses the error correction form of an autoregressive distributed lag (ARDL) model to separate long and short run influences on government size. The second stage of the test is to confirm the existence of a positive relationship between the share of government consumption to investment expenditures (net of interest) and ENP. The final stage tests the combined hypothesis that the rate of growth of per capita income and ENP(Seat) have an inverted U-shaped relationship, consistent with political party competition peaking at a value of ENP(Seat) that is neither too small (2 or below) nor too large (6 or above), and that higher ratios of government consumption to investment serves reduce the growth rate. Confirmation would then provide evidence consistent with the more general hypothesis that political party competition has a positive effect on economic growth.

ii. Variables used in the tests

The sources of the Indian panel data used in the tests of the model's predictions and their descriptive statistics are provided in the Data Appendix to the paper. The descriptive statistics themselves indicate considerable variation across states. For example, in 2019/2020 real per capita income in Karnataka was more than four and a half times larger than its counterpart in Bihar and similar variability is present in literacy rates, urbanization and the ratio of

[11] As is the case with the effect of ENP on government size, the positive link between ENP and effective party competition diminishes beyond some minimum so that the increase in the consumption to investment mix diminishes with increases in ENP.

consumption to investment expenditures in state budgets. Variation arises not only across states but over time with differences in the time series properties of variables playing an important role in our tests. Except for the percentage of the population that is older than sixty (Old), all variables are either I(1), growing stochastically over time, or I(0), stationary through time.[12] This is the feature that motivates the use of autoregressive distributed lag regressions in the first two stages of our test.

The dependent variable in the first stage of our test, government size (GovSize), is defined as the ratio of aggregate noninterest government expenditure to state GDP. The ratio of government consumption to investment expenditures (Cons_ratio) in the second stage is defined as the ratio of noninterest state government revenue expenditure to capital expenditure and economic growth is defined as the growth rate of state real income per capita (Grypc). Our primary variable of interest, ENP(Seat) is defined as one over the sum of each party's seat share of the state legislature. The control variables used account for heterogeneity across the factors with a potential influence on the three dependent variables include: real state income per capita (Rypc), the percentage of the population that is literate (literacy), the percentage of the population older than 60 (Old), the share of agriculture in state GDP (Agriculture_share), the percentage of the state population living in urban areas (Urban), the average population size of state constituencies (in 1000s, Density), the percentage of seats reserved for disadvantaged groups (Reservation), the fraction of noninterest state expenditure financed by intergovernmental grants (Grant_share), and whether or not the state has a fiscal rule restraining the size of the budget deficit (Fiscal_rule).

iii. Stage 1 test: The relationship between ENP(Seat) and GovSize

In Table 1 we show two versions of the error correction form of an ARDL model of government size. Column (1) presents the dynamic fixed effects version of that model, where ARDL analysis imposes common covariant coefficient across states in both the long and the short run. Column

[12] Regressing nonstationary variables raises the possibility of estimating spurious relationships. In our dataset the variable Old is I(2), increasing stochastically at an increasing rate. Hence Old appears either as a first or second difference in our tests.

(2) assumes a common long run but allows for variation across states in the short run by presenting the pooled mean coefficient values.[13]

-- insert Table 1 about here --

For our purposes, the important questions to be answered by the ARDL models are whether shocks to the system in the short run converge back to the estimated long run time path (is the estimated long run model stable?) and whether the inclusion of a quadratic effect for ENP(Seat) is significant and the estimated shape consistent with the hypothesized role of political competition in relation to government size. For convergence, the error correction term estimates need to be both negative and significantly less than one. Table 1 indicates this for both cases. The estimated error correction terms are relatively small in absolute size indicating that the time frame for correction back to the long run can be as long as five years. The mean pooled regression suggests a somewhat shorter period of readjustment than does the dynamic fixed effects model.

The coefficients in Table 1 that are in bold indicate the effect of ENP(Seat) on government size. Both sets of coefficients are highly significant, and the negative/positive sequencing of values indicates a U-shaped relationship with GovSize. This is consistent with larger effective party sizes first enhancing political competition and lowering GovSize before resulting greater fragmentation, less competition and the ineffective monitoring of government agency costs. To better illustrate the effect of ENP(Seat) on government size we follow Leonida et al (2013, 2015) and test for nonmonotonicity using fractional polynomial (fp) analysis. The *fp* procedure in Stata uses 44 combinations of the powers of k = (-2 -1 -.5 0 .5 1 2 3) to find the best fitting second degree fractional polynomial of government (as a share of GDP) within a regression of ENP(Seat) on our six control variables[14],

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[13] Although the Hausman test does not distinguish between the two forms of the model, the significant long run coefficient estimates can be seen to be broadly similar.

[14] Using the coefficients of a quadratic equation to illustrate the effect of ENP(Seat) on GovSize imposes symmetry on the shape of the relationship and will bias the shape if the true shape is asymmetric about the minimum point. The *fp* procedure allows for a wide range of shapes that allows for the determination of the best fitting flexible form without predetermining its shape (as done in the quadratic case in Table 1).

$$Gov\_Size_t = \alpha_0 + \sum_{j=1}^{j=2} \gamma_j ENPSeats_{jt}^{k} + \sum \delta_l Z_{lt} + \epsilon_t,$$

where the Z_l are l control variables. The results of the test and the best fitting fractional polynomial are presented in Table 2 together with a graph of the estimated form (and confidence interval) of the relationship between government expenditure size and ENPSeats. The results show (a) that monotonicity is rejected relative to non-monotonicity and (b) that the best fitting relationship between government size and ENPSeats has an inverted U-shape that is skewed to the left and reaches a minimum at about ENPSeats = 4.3. The analysis is then broadly consistent with the Duverger-Demsetz view of political party competition but with an optimal degree of interparty competition on government size arising at a value much larger than 2.

-- inset Table 2 about here --

iv. Stage 2 test: The Consumption/Investment ratio (Cons_ratio) and ENP(Seat)

The results of the test for the effect of ENP(Seat) on the ratio of government consumption to investment services are presented in Table 3. Two versions of the pooled mean group regression are presented: the first with fixed (state) effects and one that adds a correction for the possibility of a time trend unaccounted for by the model's variables. The results are consistent with a stable long run relationship (the error correction terms are both significantly less than one) and the large absolute size of their coefficients (roughly .75) indicates a relatedly quick transitional adjustment to the long run. Note that the presence of a fiscal rule not only has been found to add fiscal accountability to state budgets but are found here to consistent with increasing the investment component of state budgets (see Chakraborty and Dash, 2017). Both features are complementary with higher economic growth.

-- insert Table 3 about here --

For our purposes the results indicate an inverted U-shaped relationship between Cons_ratio and ENP(Seats). This implies that larger numbers of effective competing parties are associated with the ratio of consumption to investment government services increasing at a decreasing rate before peaking at a value of ENP(Seats) of about 3.5. While this is consistent with the model's predicted effect, it also implies that the effect of ever larger types of rival party policy alternatives wears off relatively quickly in its effect on the winning party's electoral strategy. At that point the cost of responding to current policies targeting ever small segments of the electorate exceeds the benefit of additional investment and the promise of higher future consumption to a broader section of the electorate.

v. Stage 3 Test: The effect of ENP(Seat) and Cons_ratio on Growth

In Table 4 we present the results of a fixed effects regression testing for an inverted U-shaped relationship between ENP(Seats) and the growth rate of real per capita income and a negative relationship between Cons_ratio and per capita growth. Two fixed effects versions are presented: one with and one without a time trend while both include a lagged per capita growth variable to account for persistence in the growth rate (the latter indicating convergence back to an average growth rate of 3.5% over our time period). The two models can explain over fifty percent of the variation in state growth rates and highlight the role of urbanization and agriculture share of GDP in contributing to growth. Even controlling for the effects of ENP(Seats) and Cons_ratio on growth, changes on government size have their own significant independent relationship with changes in government size. The negative coefficient estimate is consistent with a significant countercyclical response by state governments to fluctuations in the business cycle.

-- insert Table 4 about here --

In terms of our variables of interest, ENP(Seats) does exhibit the predicted inverted U-shaped relationship with real per capita growth. The sequencing of significant positive and negative coefficients is consistent with increases in effective party numbers first increasing party competitiveness at a decreasing rate (and through this the growth rate) before peaking in its effect at an ENP(Seat) value of 3. Beyond that level further increases in ENP(Seat) increase party fragmentation consistent with the hypothesis of reducing the intensity of political competition and undermining the growth rate. The data is also consistent with the hypothesis that higher values of ENP(Seat) represents a wider range of policy options offered in the upcoming election and requiring a winning party strategy to widen its range of currently provided services at the expense of services and programs directed at future consumption. Although the coefficient estimate on Cons_ratio is highly significant, its absolute value is relatively small. Together this suggests that while there is evidence of excessive party competition biasing policy towards shorter rather than longer term objectives, the primary effect of excessive effective party numbers is to increase government size and divert resources that would otherwise be available for capital accumulation and economic growth.

4. Conclusion

In this paper we have argued that changes in the effective number of political parties (ENP) have consequences for the form and intensity of political competition and through this for government size, the composition of government services, and economic growth. The key hypothesis is that ENP is a nonmonotonic measure of party competition such that increases in ENP from a low level produce a breakdown of political oligopolist behavior at the expense of the electorate. At some point, however, further increases in ENP peak in their effectiveness such that further increases fragment rival credibility, reducing effective competition and allowing the governing party to benefit at the expense of voters. In the model that begins the paper increases in ENP beyond its most competitively effective level increase the agency costs of government and requires a winning electoral strategy that substitutes more consumption services for government activity supportive of capital investment. Both features reinforce in leading to a decline in private investment activity and hence lower growth.

The empirical section tests for the channels by which ENP is hypothesized to affect economic growth through political competition. First, we find the data are consistent with the hypothesis that, controlling for demographic and other influences on government size, increases in ENP do have a nonmonotonic U-shaped effect on government size. Second, we find that increases in ENP have an inverted U-Shaped effect on the ratio of consumption to investment elements in the government's budget, again consistent with the nonmonotonic relationship of ENP and political competitiveness. Finally, we test for the combined effects of ENP and the consumption to investment ratio on growth and find that the data are consistent with the predicted inverted

U-shape. To the extent that ENP is nonmonotonic measure of political party competitiveness, the data is then consistent with greater political party competitiveness enhancing economic growth.

Data Sources

The panel data used cover 14 major Indian states: Andhra Pradesh, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal and cover the fiscal years from 1959-60 to 2019-20. Variables are collected from a variety of sources as detailed below.

Public finance variables: The *Reserve Bank of India Bulletin* provides the longest time-series public finance data at the state level. All expenditure variables are net of interest. Various issues of the *RBI Bulletin* were used to collate this dataset.

Political variables: The *Election Commission of India* (*ECI*) publishes details of both parliamentary and assembly elections on their website (http://eci.nic.in/eci/eci.html). Information available in *ECI's* reports is used to prepare the coding of the qualitative variables: election year, political alignment, party names, seat shares.

Economic and demographic variables: Data for these variables are obtained from the *National Accounts Statistics*. Time-series data for variable state domestic product in constant prices (2004-05 rupees) is not readily available for the entire period. The base year changes approximately once in every decade, and the method of back-ward splicing is used to account for base year adjustment.

Descriptive Statistics for 14 Indian States: 1959/60 – 2019/20

Variable name	Definition	Obs.	Mean	Standard Deviation	Fisher Test for panel unit root
GovSize	Noninterest aggregate state expenditure/state GDP	831	13.61	4.23	χ^2 = 28.7 Prob =.424 D(.) χ^2=625 Prob = 0
Cons_ratio	Noninterest state current expenditure/ Noninterst state capital expenditure	846	8.70	19.38	χ^2 = 192.8 Prob = 0
ENP(Seat)	1 divided by the sum of party seat shares squared	799	2.71	1.01	χ^2 = 124.3 Prob = 0
Rypc	Real state income per capita (1000's)	830	21.11	17.39	χ^2 = 0 Prob = 1 D(.) χ^2=57.1 Prob= .0009
Grypc	Growth rate of real state income per capita	816	.035	.069	χ^2=365.3 Prob = 0
Density	Average population size of state constituency (in 1000's)	831	365.84	246.13	χ^2= 44.9 Prob = .02 D(.)χ^2= 159.9 Prob = 0
Old	Percentage of the state population over 60	806	7.12	1.43	χ^2= 2.00 Prob = 1 D2(.) χ^2= 158.1 Prob = 0
Literacy	Percentage of the state population that is literate	806	54.03	19.43	χ^2= 20.1 Prob .86 D(.)χ^2= 47.9 Prob = .01
Urbanization	Percentage of the state population living in urban areas.	806	26.07	9.75	χ^2=2.19 Prob = 1 D2(.) χ^2= 40.1 Prob = .06
Fiscal_rule	1 if a fiscal rule adopted, 0 otherwise	846	.255	.436	
Reservation	(reserved seats/assembly size)*100	844	22.4	7.6	χ^2=122.8 Prob = 0
Agriculture_ share	Agriculture's share of state GDP	831	36.35	14.12	χ^2 = 17.6 Prob .93

					D(.)χ^2= 463.3 Prob = 0
Grant_size	Intergovernmental transfers/ noninterest government expenditure	846	13.71	5.96	χ^2= 117.1 Prob = 0

D(.) {D2(.)} first and second difference operators.

Table 1

Autoregressive Distributed Lag Models of Government Size

14 Indian States: 1959 – 2019

(standard errors in brackets)

Error Correction Form	Dynamic Fixed Effects Regression (state clustered) (1)	Pooled Mean Group Regression (2)
Long Run: Real income per capita (rypc)	-.234*** (.046)	-.170*** (.028)
D(Old)	-7.96 (8.68)	-16.6*** (4.65)
Literacy	.150*** (.033)	.142*** (.031)
Density	.006*** (.002)	.007*** (.513)
Fiscal Rule	1.75** (.760)	.677 (.513)
Urbanization	.515*** (.081)	.477*** (.098)
Central Grants as a percentage of noninterest Government spending	.022 (.080)	-.005 (.032)
Enp(Seat)	**-3.25*** (.676)**	**-3.38*** (.635)**
Enp(Seat)_squared	**.401*** (.088)**	**.407*** (.094)**
Short Run: Error Correction Term	-.244*** (.034)	-.330*** (.045)
Growth rate of real income per capita (grypc)	-9.22*** (1.01)	-7.92*** (1.32)
D2(Old)	-1.00 (2.61)	-1.64 (3.18)
D(Density)	-.011*** (.002)	.006 (.031)
D(Urban)	-.329* (.192)	.434 (.661)
D(Grant_size)	-.019 (.019)	-.038 (.026)
Constant	.511 (.785)	.423 (.709)
Observations Fixed Effects Log Likelihood ENP(seat) value that minimizes government size	778 Yes **4.05**	778 Yes -1165.0 **4.15**

* (**) [***] indicates significantly different from zero at 10% (5%) [1%]. D(.){D2(.)} first {second} difference operator.

Table 2

Fractional Polynomial Regression (comparison of 44 fitted models)

ENP(seat)	Test Df	Deviance	Residual std. dev.	Deviance difference	P	Powers
Omitted	4	3318.18	2.08	47.33	.000	
Linear	3	3298.55	2.06	27.71	.000	1
m = 1	2	3288.44	2.04	17.59	.000	-1
m = 2	0	3270.681	2.02	0.000		3 3

Test df is degrees of freedom, and P = P > F is significance level for tests comparing models vs. model with m = 2 based on deviance difference, F(df, 762).

*(**)[***] report significance at 10%(5%)[1%]

Best fitting regression: F(9,13) = 69.9; Prob > F = 0.000; xtFisher on equation residual = 73.3 Prob = 0

GovSize = - 1.42 - .179***Rypc - 13.5D(Old) +.148***Literacy +.005***Density +.273Fiscal Rule

+.399***Urban +.006Grant_size -.136***ENP_Seat_1 +.073***ENP_Seat_2

Component plot of best fitting fractional polynomial with 95% confidence interval

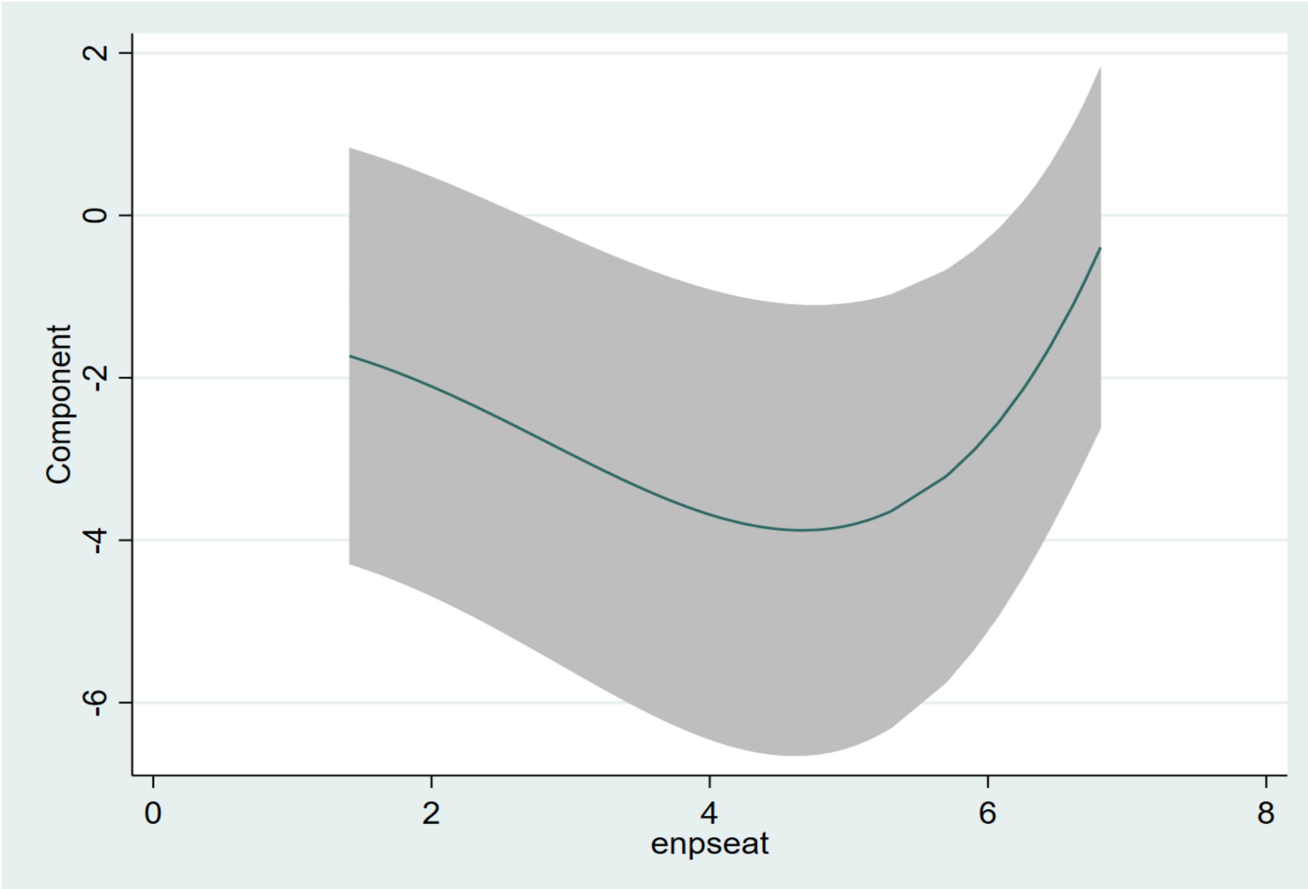

Table 3
Autoregressive Distributed Lag Models of the Government Consumption Ratio (of Investment)
14 Indian States: 1956 – 2018
(robust standard errors in brackets)

Error Correction Form	Pooled Mean Group Regression (2)	Pooled Mean Group Regression (with Year trend) (3)
Long Run:		
Real income per capita	.006 (.029)	-.018 (.028)
D(Old)	-2.26 (6.00)	4.47 (5.48)
Literacy	.196*** (.042)	.007 (.071)
Density	-.003 (.004)	-.008** (.0036)
Fiscal Rule Dummy	-2.00*** (.617)	-2.08*** (.591)
Urbanization	-.387*** (.132)	-.656*** (.167)

Central Grants as a percentage of noninterest Government spending	.002 (.003)	.005 (.003)
Enp(seat)	**2.22***** **(.803)**	**2.65***** **(.756)**
Enp(seat)_squared	**-.318**** **(.127)**	**-.368***** **(.122)**
Yearid		.317*** (.112)
Short Run: Error Correction Term	-.746*** (.085)	-.760*** (.085)
Growth rate of real income per capita	-15.28 (11.45)	-15.70 (11.49)
D2(Old)	5.91 (11.41)	4.83 (11.69)
D(Literacy)	1.26 (.886)	1.09 (.794)
D(Density)	.313 (.324)	.281 (.325)
D(Urban)	-6.86 (4.79)	-6.80 (4.94)
D(Grant_size)	.015 (.015)	-.013 (.015)
Constant	4.83** (2.38)	11.30*** (3.02)
Observations Fixed Effects Log Likelihood ENP(seat) when Cons_Ratio peaks	778 Yes -2059.7 3.49	778 Yes -2056.4 3.60

* (**) [***] indicates significantly different from zero at 10% (5%) [1%]. D(.) [D2()] first [second] difference operator.

Table 4

The Effect of ENP(Seat) and Consumption Ratio on Growth

14 Indian States: Annual 1956 - 2018

(standard error adjusted for clusters in brackets)

	Growth Rate of Income per capita (1)	Growth Rate of Income per capita (2)
Lagged_Growth Rate	-.051* (.026)	-.124*** (.016)
D(Literacy)	.004 (.003)	-.001 (.002)
D(Urban)	.026*** (.006)	.006* (.003)
D(Agriculture_share)	.014*** (.001)	.013*** (.001)
Reservation	-.0003 (.002)	-.003*** (.001)

D(GovSize)	-.010*** (.002)	-.008*** (.002)
Yearid		.001*** (.00002)
Consumption Ratio	**-.0001**** **(.00004)**	**-.0001***** **(.00002)**
ENP(Seat)	**.025***** **(.008)**	**.013**** **(.006)**
ENP(Seat)_squared	**-.004***** **(.001)**	**-.0022***** **(.0007)**
Constant	-.002 (.049)	.061 (.020)
Statistics		
Number of Obs.	760	760
Fixed Effects	Yes	Yes
Overall R^2	.563	.552
F	101.6***	454.3***
ENP(seats) peak	3.13	3.0

*(**)[***], significantly different from zero at 10% (5%) [1%]. D(.) first difference

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Appendix

Solve for the effects of a change in ENP on the supply of government services and tax rate.

$$W\left(g_t,\ v\left(N_t\right),\ \tau\right) = lnc_{1t} + lnc_{2t} + ln\left[\left(1 - v\left(N_t\right)\right)g_t\right] + \mu\left\{g_t - \left[\tau - z\left(N_t\right) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right]y_t\right\}) \tag{19}$$

Where from (11) and (12) $c_{1t} = \frac{(1-\tau)(1-\alpha)}{(1+\beta)(1-\lambda)}y_t$, and $c_{2t} = \frac{\alpha(1-\alpha)\beta\left(1-\tau_t\right)^2\theta^2(1-\lambda)^{-\alpha}\left(v(N)g_t\right)^{2-2\alpha}}{(1+\beta)k}$ and

$y_t = \theta(1 - \lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)g_t\right)^{1-\alpha}$ so that $c_{1t} = \frac{(1-\tau_t)(1-\alpha)\theta(1-\lambda)^{-\alpha}k_t^{\alpha}\left(v(N)g_t\right)^{1-\alpha}}{(1+\beta)}$.

Using $X = \left[\tau - z(N) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right] > 0$, the first order conditions for an internal optimum are:

$$\frac{\partial W}{\partial g_t} = \frac{4-3\alpha}{g_t} + \mu\left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N_t)\right)^{1-\alpha}g_t^{-\alpha}\right) = 0 \tag{20}$$

$$\frac{\partial W}{\partial \tau_t} = -\frac{3}{(1-\tau_t)} - \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)g_t\right)^{1-\alpha} = 0 \tag{21}$$

$$\frac{\partial W}{\partial \mu_t} = g_t - \left[\tau - z\left(N_t\right) - \frac{(1-\tau)(1-\alpha)\lambda}{(1-\lambda)}\right]\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)g_t\right)^{1-\alpha} = 0 \tag{22}$$

To solve for the effect of a change in N on g and τ, totally differentiate the system (20) – (22) to find

$$\begin{aligned}
&-\left[\frac{4-3\alpha}{g_t^2} - \alpha\mu\left(X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)\right)^{1-\alpha}g^{-\alpha-1}\right)\right]dg - \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)\right)^{1-\alpha}g^{-\alpha}\,d\tau \\
&\quad + \mu\left[-z'(N)(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}v(N)^{1-\alpha}g_t^{-\alpha} + X(1-\alpha)^2\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)g_t\right)^{-\alpha}v'(N)\right]dN \\
&\quad + \left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)\right)^{1-\alpha}g^{-\alpha}\right)d\mu = 0,
\end{aligned} \tag{20)'$$

$$\begin{aligned}
&-\mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)\right)^{1-\alpha}g_t^{-\alpha}dg + \frac{3}{(1-\tau)^2}d\tau - \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(g_t\right)^{1-\alpha}v(N)^{-\alpha}v'(N)dN \\
&-\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}\left(v(N)g_t\right)^{1-\alpha}d\mu = 0,
\end{aligned} \tag{21)'$$

and $\left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N_t))^{1-\alpha}g_t^{-\alpha}\right)dg - \left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha}d\tau + 0\,d\mu$

$$z'(N)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha} - X\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}v(N)^{-\alpha}g_t^{1-\alpha}v'(N)\,dN = 0. \quad (22)'$$

written in matrix form

$$\Bigg[-\left[\frac{4-3\alpha}{g_t^2} - \alpha\mu\left(X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha-1}\right)\right] - \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha}\left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha}\right) -$$

$$= \Big[\mu\Big[- z'(N)(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}v(N)^{1-\alpha}g_t^{-\alpha} + X(1-\alpha)^2\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{-\alpha}v'(N)\Big] - \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}(g_t)^{1-\alpha}v(N)^{-\alpha}v'(N)\,z'(N)\theta(1 -$$

The sign of the determinant of the left-hand side matrix appears to be ambiguous, that is, $D = |- \ - \ + \ - \ + \ - \ + \ - \ 0| = -|+ \ - \ - \ 0| + |- \ - \ + \ 0| + |- \ + \ + \ -|$ = (+) + (+) + (+ -) = ? However, a necessary and sufficient condition for a maximum is for the determinant of the bordered Hessian, $D > 0$. Using this we can sign the comparative static effects of a change in N (with $z'(N) > 0$ and $v'(N) < 0$). Using Cramer's rule, we can determine the sign of the comparative static effects as

$$\frac{dg}{dN} = \frac{\left|-\mu\left[-z'(N)(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}v(N)^{1-\alpha}g_t^{-\alpha} - X(1-\alpha)^2\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{-\alpha}v'(N)\right]\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha}\left(1-X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha}\right) - \mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}(g_t)^{1-\alpha}v(\right.}{D>0}$$

$$= \frac{|-ve -ve +ve +ve +ve -ve +ve -ve\ 0\,|}{D>0} = \frac{-ve|+ve -ve -ve\ 0\,| - (-ve)|+ve -ve +ve\ 0\,| + (+ve)|+ve +ve +ve -ve\,|}{+ve} = \frac{(+ve) + (+ve) + (-ve\ + -ve)}{D>0)} = \ ? \ > 0.$$

That is, the first two terms are positive while the third is negative. If the first two terms dominate then an increase in ENP would lead to a larger supply of government services. This would also imply that with both g and z larger and because y most likely to fall (since v(N) is smaller), the tax rate would also need to rise.

$$\frac{d\tau}{dN} = \frac{\left|-\left[\frac{4-3\alpha}{g_t^2} - \alpha\mu\left(X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha-1}\right)\right]\mu\left[-z'(N)(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}v(N)^{1-\alpha}g_t^{-\alpha} + X(1-\alpha)^2\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{-\alpha}v'(N)\right]1-X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha} - \mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}(\right.}{}$$

$$= \frac{|-ve -ve +ve -ve +ve -ve +ve +ve\ 0\,|}{D>0} = \frac{-|+ - + 0\,| + |- - + 0\,| + |- + + +\,|}{+ve} = \frac{(-ve) + (+ve) + (-ve)}{+ve} = \ ? \ .$$

An increase in fragmentation (again N > N*) leads to a lower level of government services and a lower tax rate.

Special Note on simplifying the terms in the matrix:

A. for signing the two corner terms a_{13} and a_{31}: Since $y_t = \theta(1-\lambda)^{1-\alpha} k_t^{\alpha}(v(N)g_t)^{1-\alpha}$

Note that $(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha} = \frac{(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{1-\alpha}}{g} = \frac{(1-\alpha)y}{g}$ and $X = \frac{g}{y}$ so that

$$\left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha}\right) = 1 - \frac{g}{y}\left(\frac{(1-\alpha)y}{g}\right) = 1-(1-\alpha) = \alpha > 0.$$

B. For term a_{12} and a_{21} $-\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\alpha)(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g_t^{-\alpha} =- \mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\alpha)\frac{y}{g}\right)$

C. For terms a_{23} and a_{32} $\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N)g_t)^{1-\alpha} = \left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y$

D. For a_{11}

$$-\left[\frac{4-3\alpha}{g_t^2} - \alpha\mu\left(X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{-\alpha-1}\right)\right] =- \left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N))^{1-\alpha}g^{1-\alpha}\right)}{g_t^2}\right] =- \left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2}\right]$$

E. For a_{13} and a_{31} $\left(1 - X(1-\alpha)\theta(1-\lambda)^{1-\alpha}k_t^{\alpha}(v(N_t))^{1-\alpha}g_t^{-\alpha}\right) = 1 - X\frac{(1-\alpha)y_t}{g_t} = \alpha$

F. For the right-hand matrix (labeled B) b_{11} $\mu(1-\alpha)\frac{y_t}{g_t}\left[-z'(N) + v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right]$

G. For b_{12} $-\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)$

H. For b_{13} $z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)$

$$D = \left| -\left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2}\right] \quad -\mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\alpha)\frac{y}{g}\right)\alpha \quad -\mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\alpha)\frac{y}{g}\right) \quad \frac{3}{(1-\tau)^2} \quad \left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y \quad \alpha \quad \left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y \quad 0 \right|$$

$$D = \left[\frac{4-3\alpha-\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2}\right]\left[\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 - 2\alpha(1-\alpha)\,\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)^2\frac{y^2}{g} - \alpha^2\frac{3}{(1-\tau)^2}$$

$$= \left[\frac{4-3\alpha-\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2} - \frac{2\alpha(1-\alpha)\,\mu}{g}\right]\left[\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 - \alpha^2\frac{3}{(1-\tau)^2}$$

$$= \left[\frac{4-3\alpha-\alpha\mu\left((1-\alpha)g_t\right)}{g_t^2} - \frac{2\alpha\mu(1-\alpha)\,g}{g^2}\right]\left[\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 - \alpha^2\frac{3}{(1-\tau)^2}$$

$$D = \left[\frac{4-3\alpha(1+\mu(1-\alpha)g)}{g_t^{2}}\right]\left[\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 - \alpha^2\frac{3}{(1-\tau)^2}$$

That is, a sufficient condition for $D < 0$ is if $\left[\frac{4-3\alpha(1+\mu(1-\alpha)g)}{g_t^{2}}\right]$ or $4 - 3\alpha(1 + \mu(1 - \alpha)g < 0.$

Assuming that the sufficient condition holds, the comparative static effects of a change in N become

$$\frac{dg}{dN} = \frac{\left|\mu(1-\alpha)\frac{y_t}{g_t}\left[-z'(N)+v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right]-\mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\alpha)\frac{y}{g}\right)\alpha-\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)\ \frac{3}{(1-\tau)^2}\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\ z'(N)y_t-\frac{X(1-\alpha)y_t}{v(N)}v'(N)\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\ 0\right|}{D<0}$$

where the numerator is

$$\begin{aligned}
D_{gN} =& -\mu(1 - \alpha)\frac{y_t}{g_t}\left[- z'(N) + v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right]\left[\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 \\
&- \mu\left(\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1 - \alpha)\frac{y}{g}\right)\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right] \\
&+ \alpha\left\{- \mu\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y - \frac{3}{(1-\tau)^2}\left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
=& -\mu(1 - \alpha)\frac{y_t}{g_t}\left[- z'(N) + v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right]\left[\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 \\
&- \mu\left(\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1 - \alpha)\frac{y}{g}\right)\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right] \\
&- \mu\alpha\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y - \frac{3\alpha}{(1-\tau)^2}\left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right]
\end{aligned}$$

$$\begin{aligned}
=&\left[- z'(N) + v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right] \\
&\left\{- \mu(1 - \alpha)\frac{y_t}{g_t}\left[\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\right]^2 - \mu\left(\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1 - \alpha)\frac{y}{g}\right)\left(1 + \frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y - \frac{3\alpha}{(1-\tau)^2}\right\}
\end{aligned}$$

$$-\mu\alpha\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y.$$

Then with $z'(N) > 0$ and $v'(N) < 0$, D_{gN} can be signed as $D_{gN} = (-ve(-ve) - (-ve) > 0.$

From this it follows that if the sufficient condition ($3\alpha(1+\mu(1-\alpha)g > 4$) holds then

$$\frac{dg}{dN} = \frac{D_{gN}>0}{D<0} < 0.$$

Intuitively, an increase in ENP (above N*) reallocates government services out of production into consumption leading to a fall in output and private consumption. Combined with the higher cost of government, government services will be cut back somewhat and partially reallocated back into investment services. That is, the second order effects moderate somewhat the initial response to more government consumption services.

$$\frac{d\tau}{dN} = \frac{D_{\tau N} = \left| -\left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2}\right]\mu(1-\alpha)\frac{y_t}{g_t}\left[-z'(N)+v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right]\alpha \ -\mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\alpha)\frac{y}{g}\right) -\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y \ \alpha \ z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)} v'(N) \ 0 \right|}{D<0}$$

Where

$$D_{\tau N} = \left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2}\right]\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right] + \alpha\mu(1-\alpha)\frac{y_t}{g_t}\left[-z'(N)+v'(N)\frac{X(1-\alpha)}{v(N)g_t}\right]\left[1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right]y + \left\{-\alpha\mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\right.\right.$$

$$= \left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right]\left\{\left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2}\right]\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y + \alpha\mu(1-\alpha)\frac{y_t}{g_t}\left[1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right]y - \alpha\mu\left(\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)(1-\alpha)\frac{y}{g}\right)\right\} + \alpha^2\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)$$

$$= \left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)y\left[z'(N)y_t - \frac{X(1-\alpha)y_t}{v(N)}v'(N)\right]\left\{\left[\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu X\left((1-\alpha)y_t\right)}{g_t^2}\right]y + \alpha\mu(1-\alpha)\frac{y_t-1}{g_t}\right\} + \alpha^2\mu\left(1+\frac{(1-\alpha)\lambda}{(1-\lambda)}\right)\frac{(1-\alpha)y_t}{v(N)}v'(N)$$

Then even if the sufficient condition holds, so that $\frac{4-3\alpha}{g_t^2} - \frac{\alpha\mu\left(X(1-\alpha)y_t\right)}{g_t^2} < 0$

$D_{\tau N} = \{(+\ ve)(+\ ve)\}\{(-\ ve)\ +\ (+\ ve)\} + (-\ ve)$ is ambiguous in sign. The reallocation of government services from investment to consumption means that y must fall and even though g overall will be lower, the level of taxes needed to fund the budget (and thus the level of τ) could rise of fall depending on the size of the coefficients.