Product innovation in a dual economy with ownership structure*

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Abstract

This paper considers a model of a dual economy with ownership structure to explore the optimality of technology transfer. The economy has two types of consumers: finitely many owners and a continuum of workers. Each consumer has a preference that follows a hierarchical demand with one basic good and two manufacturing goods $1, 2$ produced by two different firms. One of these firms have a critical technology of production. We study two kinds of ownership structures: disjointed ownership, where no owner owns shares of both manufacturing firms and common ownership, where every owner owns shares of both of these firms. When the two goods are relatively poor substitutes, not sharing the technology is superior to technology transfer under both disjointed and common ownership structures. Under common ownership, technology transfer is superior when the goods are relatively good substitutes, but under disjoint ownership, this happens only for certain parametric configurations.

Keywords: ownership structure; dual economy; technology transfer; hierarchial demand; CES utility

JEL Classification: D43, O11, O32

1 Introduction

Possible anticompetitive effects of common ownership has received attention in recent years (e.g., Azar et al., 2018; Schmalz, 2021). This paper seeks to study the effects of common ownership in a model of product innovation in the specific context of a developing or an emerging economy that has a large basic good sector and an evolving manufacturing sector.

There are several recent empirical studies that look at the relation between innovation and common ownership of firms (e.g., Li et al., 2023; Antón et al., 2024). The

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theoretical literature on the implications of ownership structures can be traced back to Demsetz (1973). More recent literature includes O'Brien and Salop (2000), López and Vives (2019) and Denicoló and Panunzi (2024). This literature has looked at issues such as R&D investment, spillovers and managerial incentives and the more general question of how ownership structures of competing firms impact product market competition.

The aim of this paper is to bring some aspects of developing or emerging economies to better understand the effects of common ownership. It is documented that concentrated ownership in industries has been a relevent feature for India, South Korea and other countries (see, e.g., Khanna and Palepu, 2005). Of specific interest is to see how concentrated ownership or common ownership of multiple firms affect diffusion of a new technology in a labor suplus economy.

To theoretically understand these issues, we consider a model of a dual economy along the lines of Lewis (1954). The economy has two sectors: a basic good sector and a manufacturing sector that produces two goods $1, 2$. The economy has two kinds of consumers: finitely many owners and a continuum of workers. All consumers of the economy, owners and workers, have a preference that is represented by a "hierarchial demand" utility function (Matsuyama, 2002).[1] This utility function specifies that the consumer exclusively cares about the basic good when its consumption falls below a certain threshold (which can be viewed as the subsistence requirement). Once this threshold level of consumption is reached for the basic good, consuming more of it is no longer useful and the consumer cares about the manufacturing goods $1, 2$. We consider goods $1, 2$ as imperfect substitutes with constant elasticity of substitution (Arrow et al., 1961). There are two firms $1, 2$ that produce goods $1, 2$.

As mentioned, the economy we consider has finitely many owners and a continuum of workers. Each owner individually owns a firm in the basic good sector. In addition, owners have shares of the manufacturing firms $1, 2$. The owners of the economy can be viewed as the landowners in the agricultural (basic good) sector. Our model can be seen as a snapshot of a specific phase in the dynamic process of the dual economy of Lewis (1954) where the owners have accumulated sufficient capital to invest in the manufacturing goods.

We study two distinct ownership structures of the manufacturing firms: (i) disjoint ownership, in which the set of owners is partitioned into two disjoint subsets, each subset owning the shares of one of two firms $1, 2$, and (ii) common ownership, in which all owners have shares of both firms $1, 2$. Thus, under the disjoint ownership structure no owner has shares of both firms, while under the common ownership structure every owner has shares of both firms.

The only input of production for both manufacturing firms is labor. One of the firms (firm 1) has a product technology that is required to produce each good. Firm 1 can transfer the technology to firm 2, in which case firms $1, 2$ compete in the product market. Alternatively, firm 1 can choose not to share the technology and become a monopolist in the product market.

The individually owned firms in the basic good sector are price takers. As in Lewis

[1]We used this utility function in a related work (Sen and Stamatopoulos, 2022). However, there we considered a representative consumer model. Rather than ownership structure, the focus of that study was to explore optimal licensing policies of patented cost-reducing technology.

(1954), in the basic good sector there is a large supply of labor at subsistence wages. Labor units that are employed by owners either at their individually owned firms in the basic good sector, or in the manufacturing sector, earns a wage that is no less the subsistence wage. Consequently these workers can meet the subsistence requirement of the basic good and can potentially have positive demand for the manufacturing goods $1, 2$. The remaining workers are self employed in the basic good sector and they earn their marginal products there that are subject to diminishing returns. Only a fraction of these workers have marginal products that exceeds the subsistence level, so there is a positive demand for goods $1, 2$ only from this fraction of workers (see Figures 1,2).

For every prices of goods $1, 2$, the demand of goods $1, 2$, profits of firms and the income of the economy is determined in equilibrium and we can find the sum of indirect utilities of all owners of each of the two manufacturing firms. The objective of each firm in the product market is to set prices to maximize the sum of indirect utilities of all of its owners. For each of the two ownership structures (disjoint and common ownerships), we determine Nash Equilibrium of this strategic interaction for two scenarios: one when there is no sharing of technology by firm 1 and another when there is technology transfer by firm 1.

Our conclusions depend on constant elasticity of subsititution (CES) parameter σ. It is shown that when goods are poor substitutes ($0 < \sigma < 1$), not sharing the technology is superiror to technology transfer for firm 1 under both disjoint and common ownership structures. On the other hand, when goods are relatively good substitutes ($\sigma > 1$ but σ is not too large), technology transfer is always superior under common ownership, but can be superior under disjoint ownership only for certain parametric configurations (Propositions 2, 4).

The paper is organized as follows. We present the model Section 2. Technology transfer under disjoint ownership is studied in Section 3. In Section 4 we study technology transfer under common ownership.

2 The model

We consider an economy with an ownership structure. The economy has two types of consumers: (i) owners and (ii) workers. In the spirit of Lewis (1954), the population of workers is large. There is a continuum of workers represented by the interval $[0, L]$, where L is a sufficiently large positive number. Each worker is a point in this interval and supplies one unit of labor. By contrast, the number of owners is a positive finite number n.

There are three goods in the economy: goods $0, 1$ and 2. Good 0 is a basic good (which is a primary commodity such as food), while goods $1, 2$ are non-basic goods which can be viewed as manufacturing goods. For simplicity we assume that all consumers in the economy, owners and workers, have the same identical preference.

The basic good 0 has two features: (i) necessity (any consumer requires a specific minimum amount of that good; if that requirement is not met, the other goods are not useful) and (ii) saturation (once the minimum threshold consumption of the basic good is attained, consuming more of it does not give additional utility). Following Matsuyama (2002), the utility function of each consumer has the feature of "hierarchical demand" with respect to the basic good. Specifically, the utility function of each consumer is given as follows, where x_i is the amount of good i for $i = 0, 1, 2$:

$$u(x_0, x_1, x_2) = \begin{cases} x_0 & \text{if } x_0 \le \underline{x}_0 \\ \underline{x}_0 + \tilde{u}(x_1, x_2) & \text{if } x_0 > \underline{x}_0 \end{cases} \tag{1}$$

The threshold $\underline{x}_0$ corresponds to the minimum critical requirement of the basic good 0. The specification (1) implies that the manufacturing goods $1, 2$ are not useful when the consumption of the basic good does not exceed this minimum level. Beyond this level, saturation of the basic good is reached and manufacturing goods are useful.

We assume that the function $\tilde{u}(x_1, x_2)$ in (1) is the constant elasticity of substitution (CES) utility function (Arrow et al., 1961) given by

$$\tilde{u}(x_1, x_2) = [x_1^\rho + x_2^\rho]^{1/\rho} \tag{2}$$

with $\rho \in (-\infty, 0) \cup (0, 1)$, i.e., $\rho < 1$ and $\rho \neq 0$. The constant elasticity of substitution of $u(x_1, x_2)$ is $\sigma \equiv 1/(1-\rho)$. Note that $\sigma > 0$ and $\sigma \neq 1$. If $0 < \rho < 1$, then $\sigma > 1$ and the goods are "good substitutes"; if $\rho < 0$, then $0 < \sigma < 1$ and the goods are "poor substitutes" (see, e.g., Black et al., 2009).[2]

2.1 Production technology

For the manufacturing goods, there are two firms $1, 2$ in the manufacturing sector, with firm i producing good i. Firms compete in prices. Firm 1 owns the basic production technology that is required to produce goods $1, 2$. If firm 1 does not share the technology with firm 2, then good 2 is not produced at all. In that case, taking $x_2 = 0$ in (1) and (2), the utility function of any consumer is simply a function of x_0, x_1, given by

$$\hat{u}(x_0, x_1) = u(x_0, x_1, 0) = \begin{cases} x_0 & \text{if } x_0 \le \underline{x}_0 \\ \underline{x}_0 + x_1 & \text{if } x_0 > \underline{x}_0 \end{cases} \tag{3}$$

On the other hand, of firm 1 licenses the production technology to firm 2, the utility function is given by (1).

The production technology owned by firm 1 can be viewed as *drastic* (Arrow, 1962) in regard to the manufacturing goods in that firm 2 is not capable of producing without having access to that technology. When firm 1 shares the technology with firm 2, firms $1, 2$ compete in prices in the manufacturing sector. When firm 1 does not share the tecnology, good 2 is not produced and firm 1 is a monopolist in the manufacturing sector.

[2]The CES function (2) includes several standard utility functions as special cases: (i) $\rho \to 1$ corresponds to perfect substitutes, (ii) $\rho \to 0$ gives Cobb-Douglas and (iii) $\rho \to -\infty$ gives Leontief utility functions.

2.2 The problem of an individual consumer

Consider an individual consumer z who has income $y_z > 0$ and let $p_i > 0$ be the price of good i. When all goods are produced, the utility maximization problem of this consumer is to choose $x_i \geq 0$ to maximize $u(x_0, x_1, x_2)$ given in (1) subject to the budget constraint $p_0x_0 + p_1x_1 + p_2x_2 \leq y_z$. When good 2 is not produced, the problem of this consumer is to choose $x_i \geq 0$ to maximize $\hat{u}(x_0, x_1)$ given in (3) subject to $p_0x_0 + p_1x_1 \leq y_z$. There are two possibilities.

(i) $y_z < p_0\underline{x}_0$: In this case the individual consumer's income is not sufficient to afford the minimum required level $\underline{x}_0$ of good 0. So it is optimal for the consumer to buy only good 0 and not buy goods $1, 2$ at all. The unique solution to the utility maximization problem is given by

$$x_0(p_0, y_z) = y_z/p_0, x_1(p_0, y_z) = 0 \text{ and } x_2(p_0, y_z) = 0 \tag{4}$$

(ii) $y_z \geq p_0\underline{x}_0$: In this case the income of the individual consumer is sufficient to afford the minimum level $\underline{x}_0$ of good 0. By (1) and (3), satiation for good 0 is reached at $\underline{x}_0$, so it is optimal for the consumer to buy exactly $\underline{x}_0$ units of good 0 and use the remaining income $y_z - p_0\underline{x}_0$ to buy the other available goods.

When good 2 is not produced, by (3), it is optimal for the consumer to spend the remaining income $y_z - p_0\underline{x}_0$ to purchase only good 1, so the unique solution to the utility maximization problem is

$$x_0(p_0, p_1, y_z) = \underline{x}_0, x_1(p_0, p_1, y_z) = (y_z - p_0\underline{x}_0)/p_1 \tag{5}$$

When good 2 is produced, the problem of the consumer is to spend the remaining income $y_z - p_0\underline{x}_0$ to buy goods $1, 2$ to maximize $u(x_1, x_2)$ given in (2). Denoting

$$g(p_1, p_2) := p_1^{1-\sigma} + p_2^{1-\sigma} \tag{6}$$

in this case the unique solution to the utility maximization problem is

$$x_0(p_0, p_1, p_2, y_z) = \underline{x}_0, x_1(p_0, p_1, p_2, y_z) = (y_z - p_0\underline{x}_0)/p_1^{\sigma}g(p_1, p_2) \text{ and}$$
$$x_2(p_0, p_1, p_2, y_z) = (y_z - p_0\underline{x}_0)/p_2^{\sigma}g(p_1, p_2) \tag{7}$$

2.2.1 Disposable income, individual demand and indirect utility

There are two sectors in the economy: (i) the basic good sector that produces good 0 and (ii) the manufacturing sector that produces goods $1, 2$. When a firm has the production technology, labor is the only input of production for any good and one unit of labor is needed to produce one unit of good $i = 1, 2$.

Our primary focus is to study the strategic interaction in the manufacturing sector. We assume that buyers and sellers of the basic good 0 are all price takers. Given this price taking assumption, the price of good 0 is normalized at $p_0 \equiv 1$ and the basic good can be viewed as the numeraire.

Definition 1 The *disposable income* of any individual consumer z who has income y_z is given by $\hat{y}_z) = \max\{y_z - \underline{x}_0, 0\}$. The *total disposable income* of the economy the sum of disposable incomes of all consumers (owners and workers) of the economy.

As the price of the basic good 0 is 1, any consumer having a positive disposable income has positive demand for the available non-basic goods while any consumer having zero disposable income has zero demand for these goods. Consider an individual consumer who has disposable income $\hat{y}_z$. If good 2 is not produced and the price of good 1 is p_1, by (4) and (5), its individual demand for good 1 is

$$x_1(p_1, \hat{y}_z) = \hat{y}_z/p_1 \tag{8}$$

In this case, if the total disposable income of the economy is $\hat{y}$, the total quantity demanded for good 1 is

$$X_1(p_1, \hat{y}) = \hat{y}/p_1 \tag{9}$$

Taking $x_1 = x_1(p_1, \hat{y}_z)$ in (3), the *indirect utility* (the maximized value of $\hat{u}$ at price p_1) of an individual consumer who has income y_z and therefore disposable income $\hat{y}_z = \max\{y_z - \underline{x}_0, 0\}$ is

$$v(p_1, y_z) = \begin{cases} y_z & \text{if } \hat{y}_z = 0 \\ \underline{x}_0 + \hat{y}_z/p_1 & \text{if } \hat{y}_z > 0 \end{cases} \tag{10}$$

On the other hand, if good 2 is produced and the price of good i is p_i for $i = 1, 2$, by (4) and (7), for a consumer who has disposable income $\hat{y}_z$, the individual demand for good i is

$$x_i(p_1, p_2, \hat{y}_z) = \hat{y}_z/p_i^\sigma g(p_1, p_2) \text{ for } i = 1, 2 \tag{11}$$

where $g(p_1, p_2)$ is given by (6). Therefore if the total disposable income of the economy is $\hat{y}$, the total quantity demanded for good i is

$$X_i(p_1, p_2, \hat{y}) = \hat{y}/p_i^\sigma g(p_1, p_2) \text{ for } i = 1, 2 \tag{12}$$

For $i = 1, 2$, taking $x_i = x_i(p_1, p_2, \hat{y}_z)$ in (1) and (2) and noting that $\sigma = 1/(1-\rho)$, the indirect utility of an individual consumer who has income y_z and therefore disposable income $\hat{y}_z = \max\{y_z - \underline{x}_0, 0\}$ is

$$v(p_1, p_2, y_z) = \begin{cases} y_z & \text{if } \hat{y}_z = 0 \\ \underline{x}_0 + \hat{y}_z g(p_1, p_2)^{1/(\sigma-1)} & \text{if } \hat{y}_z > 0 \end{cases} \tag{13}$$

where g is given in (6).

2.3 Ownership structure

The economy that we study has a "dual economy" structure of Lewis (1954). Recall that the economy has n owners. In both sectors of the economy (basic good and manufacturing), there are owner-operated enterprises that employ labor. In the basic good sector, each of the n owners individually own a firm of the basic good. These

n owners also own the two manufacturing firms $1, 2$. The ownership structure of firms $1, 2$ is a modified version of the structure studied in Azar and Vives (2021). We study two different kinds of ownership structures.

(i) Disjoint ownership structure: Assuming n is an even number with $n = 2k$, partition the set of owners N as $N = N_1 \cup N_2$, where $|N_1| = |N_2| = k$. Owners in N_1 own firm 1, while owners in N_2 own firm 2. For $i = 1, 2$, each owner in N_i has an equal share $1/k$ of firm i. This is a situation of a *disjoint* ownership structure, as there is no owner who owns shares of both firms $1, 2$.

(ii) Common ownership structure: For $i = 1, 2$, each owner in N has an equal share $1/n$ of firm i. This is a situation of a *common* ownership structure, as each owner owns shares of both firms $1, 2$.

Our model can be viewed as a snapshot of an economy where some agents (the n owners) have accumulated sufficient capital to invest in the manufacturing sector. For instance, consider an economy where agricultural land holdings are concentrated among a few owners and there is a large labor population. The n owners can be viewed as landowners who carry out agricultural (the basic good) production in their lands by hiring labor. If the agricultural production generates sufficient surplus, these landowners are in a position to invest in the production of manufacturing goods. Thus we have a static model to represent a specific phase of a dual economy where landowners in the basic good sector have begun investing in the manufacturing sector.

2.4 Workers

There are $n+2$ owner-operated enterprises in the economy: n individually owned firms in the basic good sector and two firms $1, 2$ in the manufacturing sector. We assume that any labor employed by n individually owned firms in the basic good sector must be paid a wage $w_0 = \underline{x}_0$, while any labor employed by firms $1, 2$ in the manufacturing sector must be paid a wage $\hat{w} \geq \underline{x}_0$. As the price of the basic good is 1, these assumptions ensure that any worker employed in these $n + 2$ enterprises is able to consume the minimum critical level $\underline{x}_0$ of the basic good.

These minimum wages $w_0, \hat{w}$ can arise from formal labor market regulations that govern the owner-operated enterprises. Alternatively it may be the case that without consuming the minimum critical level of the basic good, a labor unit is not able to effectively contribute to production, so it is in the interests of the owners to ensure a wage that enables workers to attain this minimum critical level.

It can be noted from Definition 1 that any worker employed in the manufacturing sector has a disposable income $\hat{w} - \underline{x}_0$ after spending for the basic good, so such a worker has an individual demand for the available manufacturing goods (see (8), (11)). On the other hand, as $w_0 = \underline{x}_0$, any worker employed by one of the n individually owned firms in the basic good sector is able to consume the minimum required amount of the basic good, but has zero disposable income[3] to purchase the manufacturing goods.

[3]The assumption $w_0 = \underline{x}_0$ simplifies our analysis. It ensures that workers employed by the owner-

2.4.1 Disposable income of workers

A key aspect of the dual economy model of Lewis (1954) is when the manufacturing sector is still expanding, there is an "unlimited supply" of labor at subsistence wages. According to Lewis (1954, p.154), this happens when the labor population is sufficiently large so that:

> ...there are large sectors of the economy where the marginal productivity of labour is negligible, zero, or even negative. Several writers have drawn attention to the existence of such "disguised" unemployment in the agricultural sector, demonstrating in each case that the family holding is so small that if some members of the family obtained other employment the remaining members could cultivate the holding just as well...

As the set of workers in our model is the interval $[0, L]$, unlimited supply of labor is captured by taking L to be sufficiently large. Specifically, any labor not employed in the $n + 2$ enterprises is self employed in the basic good sector (e.g., agriculture) and simply earns its marginal product of the basic good. Because in this economy the land holdings in agriculture are concentrated in favor of a few owners, a large labor population works with small land holdings and makes minimal contribution in terms of marginal product of the basic good.

Formally, denote by $m(\ell)$ the marginal product of the ℓ-th unit of self employed labour in the basic good sector. We assume that $m(\ell)$ is continuous and decreasing, with $\lim_{\ell \downarrow 0} m(\ell) > \underline{x}_0$ and $0 < m(L) < \underline{x}_0$, so $\exists\ \underline{\ell} \in (0, L)$ such that $m(\ell) \gtreqless \underline{x}_0 \Leftrightarrow \ell \lesseqgtr \underline{\ell}$. As shown in Figure 1, this means the marginal product in the basic good sector exceeds the minimum critical level $\underline{x}_0$ only for labor units $\ell < \underline{\ell}$. Thus, a self employed labor unit in the basic good sector earns higher than $\underline{x}_0$ and have positive demand for the manufacturing goods only for labor units less than $\underline{\ell}$.

operated enterprises in the basic good sector have zero demand for goods $1, 2$. This enables us to solve the optimization problems of owners in the basic good sector independently from the problems of firms $1, 2$, as labor employment in the basic good sector does not affect the profits of firms $1, 2$.

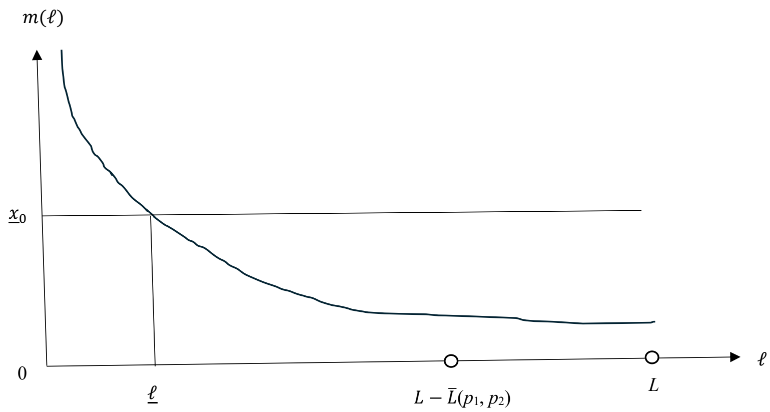

Figure 1: Marginal product of labor

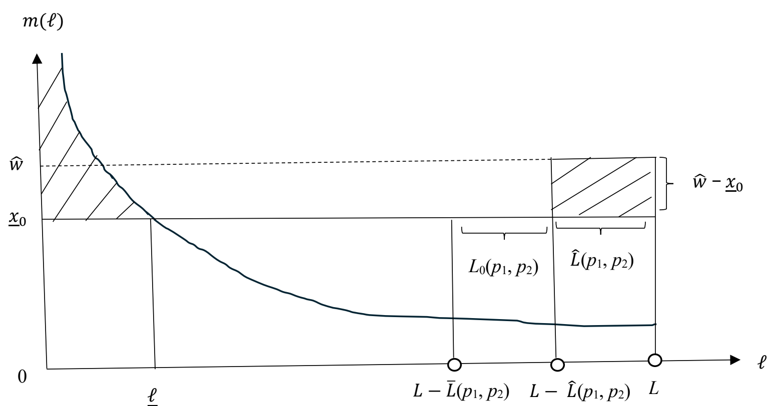

Figure 2: Disposable income of workers

Let $L_0(p_1, p_2)$ be the total labor employed by the n individually owned firms in the basic good sector and $\hat{L}(p_1, p_2)$ the total labor employed in the manufacturing sector when prices of goods $1, 2$ are p_1, p_2 (when good 2 is not produced, these are functions of only p_1). Let $\bar{L}(p_1, p_2) = L_0(p_1, p_2) + \hat{L}(p_1, p_2)$. Note that these are endogenously determined by solving optimization problems of these $n + 2$ enterprises.

To reflect "unlimited supply" of labor at subsistence wages, we assume that the total labor L is sufficiently large so that for any p_1, p_2, $\bar{L}(p_1, p_2)$ never exceeds $L - \underline{\ell}$, which ensures that the labor population that is self employed in the basic good sector always exceeds $\underline{\ell}$ (see Figure 1). Given this assumption, we can find the labor income and the demand for the manufacturing goods for the workers. According to their nature of employment, the workers can be divided into three groups as follows.

(i) The labor employed in n individually owned firms in the basic good sector is $L_0(p_1, p_2)$. Each such worker earns wage $w_0 = \underline{x}_0$. As the price of the basic good 0 is 1, each worker spends $\underline{x}_0$ for good 0 and does not have any disposable income left to buy goods 1 or 2.

(ii) The labor employed by firms $1, 2$ in the manufacturing sector is $\hat{L}(p_1, p_2)$. Each such worker earns wage $\hat{w} \geq \underline{x}_0$. As the price of the basic good 0 is 1, each worker spends $\underline{x}_0$ for good 0 and has disposable income $\hat{w} - \underline{x}_0$ to purchase goods $1, 2$. Therefore the total disposable income of workers who are employed in the manufacturing sector is

$$(\hat{w} - \underline{x}_0)\hat{L}(p_1, p_2) \tag{14}$$

This income is given by the shaded rectangle on the top right side of Figure 2.

(iii) The remaining labor population is $L - \bar{L}(p_1, p_2)$. Each of these workers are self employed in the basic good sector and earns its marginal product given by $m(\ell)$ (see Figure 1). Partition $[0, L - \bar{L}(p_1, p_2)] = A \cup B$ where $A := [0, \underline{\ell}]$ and $B := (\underline{\ell}, L - \bar{L}(p_1, p_2)]$. Each worker in A earns marginal product $m(\ell) \geq \underline{x}_0$, while each in B earns marginal product $m(\ell) < \underline{x}_0$. Thus workers in B cannot afford the minimum critical level of the basic good, having zero disposable income and consequently zero demand for goods $1, 2$. On the other hand, each worker in A spends $\underline{x}_0$ for the basic good 0 and has disposable income $m(\ell) - \underline{x}_0$ to purchase goods $1, 2$. Thus the total disposable income of the self-employed labor population $L - \bar{L}(p_1, p_2)$ is

$$\int_0^{\underline{\ell}} [m(\ell) - \underline{x}_0] \mathrm{d}\ell = M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0 \text{ where } M(\underline{\ell}) := \int_0^{\underline{\ell}} m(\ell) \mathrm{d}\ell \tag{15}$$

This income is given by the shaded region of the top left side of Figure 2.

2.5 Problem of owners in the basic good sector

First consider the problem of the n individually owned firms in the basic good sector. We assume that each such firm has an identical production $f(\ell)$, that is, $f(\ell)$ units of the basic good can be produced by employing ℓ units of labor. Recall that for each unit of labor, a firm has to pay wage $w_0 = \underline{x}_0$. We assume the standard properties for a production function: $f(0) = 0$, f is twice continuously differentiable, increasing and concave. Moreover $\lim_{\ell \to 0+} f'(\ell) > \underline{x}_0 > \lim_{\ell \to \infty} f'(\ell)$.

As mentioned, because any worker employed in these owner-operated enterprises in the basic good sector earns wage $\underline{x}_0$, its entire income is used to purchase the minimum required amount of the basic good, so it has zero demand for goods $1, 2$. Thus, labor employment by any owner in the basic good sector does not affect the demands or profits of firms $1, 2$ in the manufacturing sector. Given this, an individual's owner's problem in the basic good sector can be solved independently of the problems of firms $1, 2$.

Because any owner seeks to maximize its utility (given by (1) if good 2 is produced and by (3) if it is not produced) and the maximized utility of an owner is an increasing function of its income, any owner employs labor with the objective of maximizing its income in the basic good sector.

Each owner is a price-taker of the basic good (the price of the basic good being 1) and it has to pay $\underline{x}_0$ for each labor unit. When it employs ℓ units of labor, its income in the basic good sector is simply the net profit $\pi(\ell) = f(\ell) - \underline{x}_0 \ell$ and its problem reduces to choosing $\ell \geq 0$ to maximize $\pi(\ell)$. Given the assumptions on $f(\ell)$, there exists a unique $\ell_0 > 0$ that maximizes $\pi(\ell)$, which satisfies the first order condition $f'(\ell_0) = \underline{x}_0$. Note that the total labor employment $L_0(p_1, p_2)$ by n owner-operated enterprises in the basic good sector is $L_0 = n\ell_0$, which does not depend on prices p_1, p_2 of the manufacturing sector.

The income of each of the n owners from the basic good sector is $\pi_0 \equiv \pi(\ell_0) = f(\ell_0) - \underline{x}_0 \ell_0$. We assume $\pi_0 > \underline{x}_0$, which ensures that each owner's income from its enterprise in the basic good sector enables it to buy the minimum required amount of the basic good and there is a positive disposable income to buy goods $1, 2$. Noting that each owner potentially has a further additional income from its ownership shares of firms 1 or 2, it follows that each of the n owners has positive demand for the available manufacturing goods.

3 Technology transfer under disjoint ownership

We begin with the case when the ownership structure is disjoint, that is, when no owner owns shares of both firms $1, 2$. First consider the case when firm 1 does not share the production technology to firm 2. In that case good 2 is not produced at all and firm 1 is a monopolist in the manufacturing sector.

3.1 Total disposable income when technology is not shared

Recall that with the production technology, one unit of labor is needed to produce one unit of a manufacturing good. Let $X_1(p_1)$ be the total quantity demanded of good 1 and $\pi_i(p_1)$ the profit of firm 1 when it sets prices p_1. Since firm 1 has to pay wage $\hat{w} \geq \underline{x}_0$ for every labor unit, the profit of firm 1 as a function of p_1 is

$$\pi_1(p_1) = (p_1 - \hat{w}) X_1(p_1) \tag{16}$$

Recall that under disjoint ownership, the set of owners is partitioned as $N = N_1 \cup N_2$, where for $i = 1, 2$, $|N_i| = k$ and each owner in N_i owns share $1/k$ of firm i. We have shown in Section 2.5 that from its individually owned operation in the basic good sector, each owner earns income π_0 (which is a constant that does not depend on p_1, p_2). In addition, any owner in N_1 earns $\pi_1(p_1)/k$ (fraction $1/k$ of the profit of firm i), so its income is $\pi_0 + \pi_1(p_1)/k$. On the other hand, because good 2 is nor produced when there is no licensing, the income of any owner in N_2 is simply π_0.

As π_0 is assumed to be more than $\underline{x}_0$ (see Section 2.5), the income of any owner is more than $\underline{x}_0$, implying that any owner in N_1 has positive disposable income $\pi_0 + \pi_1(p_1)/k - \underline{x}_0$, while any owner in N_2 has positive disposable income $\pi_0 - \underline{x}_0$. As $|N_i| = k$, in this case the total disposable income $\hat{y}_i(p_1)$ of all owners in N_i is

$$\hat{y}_1(p_1) = \pi_1(p_1) + k(\pi_0 - \underline{x}_0) \text{ and } \hat{y}_2(p_1) = k(\pi_0 - \underline{x}_0) \tag{17}$$

As $n = 2k$, the total disposable income of all owners is

$$\hat{y}_1(p_1) + \hat{y}_2(p_1) = \pi_1(p_1) + n(\pi_0 - \underline{x}_0) \tag{18}$$

By (14), (15) and (18), the total disposable income of the economy, which is the sum of disposable incomes of all owners and workers, is given by

$$\hat{Y}(p_1) = [\pi_1(p_1) + n(\pi_0 - \underline{x}_0)] + (\hat{w} - \underline{x}_0)\hat{L}(p_1) + [M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0]$$

Denoting

$$y_0 \equiv n(\pi_0 - \underline{x}_0) + M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0 \tag{19}$$

we have

$$\hat{Y}(p_1) = \pi_1(p_1) + (\hat{w} - \underline{x}_0)\hat{L}(p_1) + y_0$$

Noting that in this case the total labor employed in the manufacturing is $\hat{L}(p_1) = X_1(p_1)$, using (16), we have

$$\hat{Y}(p_1) = (p_1 - \underline{x}_0)X_1(p_1) + y_0$$

Taking $\hat{y} = \hat{Y}(p_1)$ in (9), the total quantity demanded of good 1 is $X_1(p_1) = \hat{Y}(p_1)/p_1$. Using this we have

$$\hat{Y}(p_1) = (p_1 - \underline{x}_0)\hat{Y}(p_1)/p_1 + y_0 \tag{20}$$

Note that $\hat{Y}(p_1)$ appears on both sides of (20), reflecting the general equilibrium nature of the problem. Solving (20), the total disposable income of the economy when firm 1 sets price p_1 is

$$\hat{Y}(p_1) = p_1 y_0/\underline{x}_0 \tag{21}$$

3.2 The problem of firm 1 when technology is not shared

Taking $\hat{y} = \hat{Y}(p_1)$ from (21) in (9), the total quantity demanded of good 1 is

$$X_1(p_1) = \hat{Y}(p_1)/p_1 = y_0/\underline{x}_0 \tag{22}$$

Noting that $\pi_1(p_1) = (p_1 - \hat{w})X_1(p_1)$, using (22), the total disposable income of owners in N_1 is

$$\hat{y}_1(p_1) = \pi_1(p_1) + k(\pi_0 - \underline{x}_0) = (p_1 - \hat{w})y_0/\underline{x}_0 + k(\pi_0 - \underline{x}_0) \tag{23}$$

The disposable income of each individual owner in N_1 is $\hat{y}_1(p_1)/k > 0$. Taking $\hat{y}_z = \hat{y}_1(p_1)/k$ in (10), the indirect utility of each individual owner in N_1 is $\underline{x}_0 + \hat{y}_1(p_1)/kp_1$. Since $|N_1| = k$, the sum of indirect utilities of all owners in N_1 is

$$V_1(p_1) = k\underline{x}_0 + \hat{y}_1(p_1)/p_1 = k\underline{x}_0 + [(p_1 - \hat{w})y_0/\underline{x}_0 + k(\pi_0 - \underline{x}_0)]/p_1 \tag{24}$$

When there is no licensing, good 2 is not produced at all and firm 1 is a monopolist in the manufacturing sector.

We assume prices are bounded; specifically there are positive constants $\hat{w} \leq \underline{p} < \bar{p}$ such that[4] $p_1 \in [\underline{p}, \bar{p}]$. So the problem of firm 1 is to choose $p_1 \in [\underline{p}, \bar{p}]$ to maximize the sum of indirect utilities of its owners given in (24).

Proposition 1 *Under disjoint ownership, when the production technology is not shared, firm 1 is a monopolist and it is optimal for firm 1 to set $p_1 = \bar{p}$. For $i = 1, 2$, the sum of indirect utilities $V_i(\bar{p})$ of all owners in N_i is given by*

$$V_1(\bar{p}) = k\underline{x}_0 + [(\bar{p} - \hat{w})y_0/\underline{x}_0 + k(\pi_0 - \underline{x}_0)]/\bar{p} \text{ and } V_2(\bar{p}) = k\underline{x}_0 + k(\pi_0 - \underline{x}_0)]/\bar{p} \tag{25}$$

Proof Note from (24) that $\partial V(p_1)/\partial p_1 = [\hat{w}y_0/\underline{x}_0 - k(\pi_0 - \underline{x}_0)]/p_1^3$. Since $\hat{w} \geq \underline{x}_0$, by (19) we have

$$\hat{w}y_0/\underline{x}_0 \geq y_0 = [n(\pi_0 - \underline{x}_0) + M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0] > n(\pi_0 - \underline{x}_0) = 2k(\pi_0 - \underline{x}_0) > k(\pi_0 - \underline{x}_0)$$

Thus $\partial V(p_1)/\partial p_1 > 0$ for all $p_1 \in [\underline{p}, \bar{p}]$, so the unique maximum of $V(p_1)$ is attained at $p_1 = \bar{p}$. For owners in N_1, the expression $V_1(\bar{p})$ of (25) follows by taking $p_1 = \bar{p}$ in (24). The disposable income of each owner in N_2 is the constant $\pi_0 - \underline{x}_0 > 0$. Taking $p_1 = \bar{p}$ and $\hat{y}_z = \pi_0 - \underline{x}_0$ in (10), the indirect utility of each owner in N_2 is $\underline{x}_0 + (\pi_0 - \underline{x}_0)/\bar{p}$, so the sum of indirect utilities is $k\underline{x}_0 + k(\pi_0 - \underline{x}_0)/\bar{p}$, completing the proof of (25). ■

3.3 Total disposable income under technology transfer

Now consider the case where firm 1 transfers its production technology to firm 2. In practice, firm 1 can charge a payment from firm 2 (based on fees or royalties or their combinations) for the technolgy transfer and the terms of such a payment has to be optimally determined. However, to keep the analysis simple, as a starting point we look at technology transfer without any payments.

When firm 1 transfers the production technology to firm 2, each firm $i = 1, 2$ can produce one unit of good i by employing one unit of labor. Let $X_i(p_1, p_2)$ be the total quantity demanded of good i and $\pi_i(p_1, p_2)$ the profit of firm i when firms 1, 2 set prices

[4] The positive lower bound $\underline{p}$ is needed to ensure the demand of good is always positive. The upper bound $\bar{p}$ is needed to ensure the existence of equilibrium.

p_1, p_2. Since each firm has to pay wage $\hat{w}$ for every labor unit, we have

$$\pi_i(p_1, p_2) = (p_i - w)X_i(p_1, p_2) \text{ for } i = 1, 2 \tag{26}$$

Recall that under disjoint ownership, the set of owners is $N = N_1 \cup N_2$, where for $i = 1, 2$, $|N_i| = k$ and each owner in N_i owns share $1/k$ of firm i. We have shown in Section 2.5 that from its individually owned operation in the basic good sector, each owner earns income π_0 (which is a constant that does not depend on p_1, p_2). In addition, any owner in N_i earns $\pi_i(p_1, p_2)/k$ (fraction $1/k$ of the profit of firm i), so its income is $\pi_0 + \pi_i(p_1, p_2)/k$.

As π_0 is assumed to be more than $\underline{x}_0$ (see Section 2.5), the income of any owner is more than $\underline{x}_0$, so any owner in N_i has positive disposable income $\pi_0 + \pi_i(p_1, p_2)/k - \underline{x}_0$. As $|N_i| = k$, the total disposable income of all owners in N_i is

$$\hat{y}_i(p_1, p_2) = \pi_i(p_1, p_2) + k(\pi_0 - \underline{x}_0) \text{ for } i = 1, 2 \tag{27}$$

As $n = 2k$, the total disposable income of all owners is

$$\hat{y}_1(p_1, p_2) + \hat{y}_2(p_1, p_2) = \pi_1(p_1, p_2) + \pi_2(p_1, p_2) + n(\pi_0 - \underline{x}_0) \tag{28}$$

By (14), (15) and (28), the total disposable income of the economy, which is the sum of disposable incomes of all owners and workers, is given by

$$\begin{aligned}\hat{Y}(p_1, p_2) &= [\pi_1(p_1, p_2) + \pi_2(p_1, p_2) + n(\pi_0 - \underline{x}_0)] + (\hat{w} - \underline{x}_0)\hat{L}(p_1, p_2) + [M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0] \\ &= \pi_1(p_1, p_2) + \pi_2(p_1, p_2) + (\hat{w} - \underline{x}_0)\hat{L}(p_1, p_2) + y_0\end{aligned}$$

where the positive constant y_0 is given in (19). For each good $i = 1, 2$, the production of one unit of good requires one unit of labor, the labor employed by firm i to meet the demand of good i is $X_i(p_1, p_2)$ (the quantity demanded of good i). Therefore the total labor employed in the manufacturing sector is $\hat{L}(p_1, p_2) = X_1(p_1, p_2) + X_2(p_1, p_2)$ and by (26), we have

$$\hat{Y}(p_1, p_2) = (p_1 - \underline{x}_0)X_1(p_1, p_2) + (p_2 - \underline{x}_0)X_2(p_1, p_2) + y_0$$

Taking $\hat{y} = \hat{Y}(p_1, p_2)$ in (12), in this case the total quantity demanded of good i is

$$X_i(p_1, p_2) = \hat{Y}(p_1, p_2)/p_i^\sigma g(p_1, p_2) \text{ for } i = 1, 2 \tag{29}$$

where g is given by (6). Using this we have

$$\hat{Y}(p_1, p_2) = (p_1 - \underline{x}_0)\hat{y}(p_1, p_2)/p_1^\sigma g(p_1, p_2) + (p_2 - \underline{x}_0)\hat{Y}(p_1, p_2)/p_2^\sigma g(p_1, p_2) + y_0 \tag{30}$$

Note that $\hat{Y}(p_1, p_2)$ appears on both sides of (30), reflecting the general equilibrium nature of the problem. Solving (30), we have

$$\hat{Y}(p_1, p_2) = y_0 p_1^\sigma p_2^\sigma g(p_1, p_2)/(p_1^\sigma + p_2^\sigma)\underline{x}_0 \tag{31}$$

By (29) and (31), the total quantity demanded of goods $1,2$ are

$$X_1(p_1,p_2) = y_0 p_2^\sigma/(p_1^\sigma + p_2^\sigma)\underline{x}_0 \text{ and } X_2(p_1,p_2) = y_0 p_1^\sigma/(p_1^\sigma + p_2^\sigma)\underline{x}_0 \tag{32}$$

Using (32) in (26) and (27), the total disposable incomes of two groups of owners are

$$\hat{y}_1(p_1,p_2) = (p_1 - \hat{w})y_0 p_2^\sigma/(p_1^\sigma + p_2^\sigma)\underline{x}_0 + k(\pi_0 - \underline{x}_0) \text{ and}$$
$$\hat{y}_2(p_1,p_2) = (p_2 - \hat{w})y_0 p_1^\sigma/(p_1^\sigma + p_2^\sigma)\underline{x}_0 + k(\pi_0 - \underline{x}_0) \tag{33}$$

3.4 Strategic interaction between firms $1,2$

We are now in a position to study the strategic interaction between firms $1,2$. For $i = 1,2$, the objective of firm i is to maximize the sum of indirect utilities of its owners. As each owner in N_i (the set of owners of firm i) has disposable income $\hat{y}_i(p_1,p_2)/k > 0$, taking $\hat{y}_z = \hat{y}_i(p_1,p_2)/k$ in (13), the indirect utility of each such owner is

$$\underline{x}_0 + \hat{y}_i(p_1,p_2)g(p_1,p_2)^{1/(\sigma-1)}/k$$

As $|N_i| = k$, the sum of indirect utilities of all owners in N_i is

$$v_i(p_1,p_2) = k\underline{x}_0 + \hat{y}_i(p_1,p_2)g(p_1,p_2)^{1/(\sigma-1)} \text{ for } i = 1,2$$

Using (33), it follows that the sum of indirect utilities of each group of owners is

$$v_1(p_1,p_2) = k\underline{x}_0 + [(p_1 - \hat{w})y_0 p_2^\sigma/(p_1^\sigma + p_2^\sigma)\underline{x}_0 + k(\pi_0 - \underline{x}_0)]g(p_1,p_2)^{1/(\sigma-1)}$$
$$v_2(p_1,p_2) = k\underline{x}_0 + [(p_2 - \hat{w})y_0 p_1^\sigma/(p_1^\sigma + p_2^\sigma)\underline{x}_0 + k(\pi_0 - \underline{x}_0)]g(p_1,p_2)^{1/(\sigma-1)} \tag{34}$$

where g is given in (6). Therefore the strategic interaction between firms reduces to a two person game Γ_d (d stands for disjoint ownership) with firms $1,2$ where firms choose prices $p_1,p_2 \in [\underline{p},\bar{p}]$ and the payoff of firm $i = 1,2$ is given by $v_i(p_1,p_2)$ in (34). The next proposition chracterizes Nash Equilibrium (NE) of Γ when goods $1,2$ are poor substitutes or not sufficiently good substitutes.

Proposition 2 *Suppose either $0 < \sigma < 1$ or $1 < \sigma < \bar{p}/(\bar{p} - \hat{w}) \equiv \sigma_0$.*

(i) *The game Γ_d has a unique NE: $(p_1 = \bar{p}, p_2 = \bar{p})$. At the NE, the sum of indirect utilities $v_i(\bar{p},\bar{p})$ of all owners of firm i is as follows:*

$$v_1(\bar{p},\bar{p}) = v_2(\bar{p},\bar{p}) = k\underline{x}_0 + [(\bar{p} - \hat{w})y_0/2\underline{x}_0 + k(\pi_0 - \underline{x}_0)]2^{1/(\sigma-1)}/\bar{p} \tag{35}$$

(ii) *When $0 < \sigma < 1$, not sharing the technology is superior to technology transfer for firm 1. When $1 < \sigma < \sigma_0$, $\exists\ \hat{\sigma} > 2$ such that*

  (a) *If $\bar{p} \geq 2\hat{w}$, then $\bar{p}/(\bar{p} - \hat{w}) \equiv \sigma_0 \leq 2$ and technology transfer is superior to not sharing the technology for firm 1 for all $\sigma \in (1,\sigma_0)$.*

(b) *If $\bar{p} < 2\hat{w}$, then $\sigma_0 > 2$. If $\hat{\sigma} \geq \sigma_0$, then technology transfer is superior to not sharing the technology for firm 1 for all $\sigma \in (1, \sigma_0)$. If $\hat{\sigma} < \sigma_0$, then not sharing the technology is superior to technology transfer for $\sigma \in (1, \hat{\sigma})$ and technology transfer is superior for $\sigma \in (\hat{\sigma}, \sigma_0)$.*

Proof See the Appendix. ■

Proposition 2 shows that when goods $1, 2$ are poor substitutes ($0 < \sigma < 1$), not sharing the technology is superior, but when these goods are relatively good substitutes ($1 < \sigma < \sigma_0$), either technology transfer is always superior, or it is superior for relatively small values of σ. One point of interest is the possible non-monotonicity of the superiority of technology transfer with regard to σ.

4 Technology transfer under common ownership

We now consider the case when the ownership structure is common, that is, every owner owns an equal share of each of the firms $1, 2$. When firm 1 does not share the production technology to firm 2, good 2 is not produced at all and firm 1 is a monopolist in the manufacturing sector.

4.1 Total disposable income when technology is not shared

Recall that with the production technology, one unit of labor is needed to produce one unit of a manufacturing good. Let $X_1(p_1)$ be the total quantity demanded of good 1 and $\pi_1(p_1)$ the profit of firm 1 when it sets prices p_1. The profit of firm 1 as a function of p_1 is given by (16). Under common ownership, each of the n owners owns share $1/n$ of firm i. We have shown in Section 2.5 that from its individually owned operation in the basic good sector, each owner earns income π_0 (which is a constant that does not depend on p_1, p_2). In addition, each owner earns $\pi_1(p_1)/n$ (fraction $1/n$ of the profit of firm i), so its income is $\pi_0 + \pi_1(p_1)/n$. Note that since in this case good 2 is not produced, the profit of firm 2 is 0, so owners do not earn any income from firm 2.

As π_0 is assumed to be more than $\underline{x}_0$ (see Section 2.5), the income of any owner is more than $\underline{x}_0$, implying that any owner has positive disposable income $\pi_0 + \pi_1(p_1)/n - \underline{x}_0$. As there are n ownes, the total disposable income of all owners is

$$\pi_1(p_1) + n(\pi_0 - \underline{x}_0) \tag{36}$$

By (14), (15) and (36), the total disposable income of the economy, which is the sum of disposable incomes of all owners and workers, is given by

$$\begin{aligned}\hat{Y}(p_1) &= [\pi_1(p_1) + n(\pi_0 - \underline{x}_0)] + (\hat{w} - \underline{x}_0)\hat{L}(p_1) + [M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0] \\ &= \pi_1(p_1) + (\hat{w} - \underline{x}_0)\hat{L}(p_1) + y_0\end{aligned}$$

where y_0 is given in (19). Noting that in this case the total labor employed in the manufacturing is $\hat{L}(p_1) = X_1(p_1)$, using (16), we have

$$\hat{Y}(p_1) = (p_1 - \underline{x}_0)X_1(p_1) + y_0$$

Taking $\hat{y} = \hat{Y}(p_1)$ in (9), the total quantity demanded of good 1 is $X_1(p_1) = \hat{Y}(p_1)/p_1$. Using this we have

$$\hat{Y}(p_1) = (p_1 - \underline{x}_0)\hat{Y}(p_1)/p_1 + y_0$$

Solving the equation above, the total disposable income of the economy when firm 1 sets price p_1 is given by (21).

4.2 The problem of firm 1 when technology is not shared

Taking $\hat{y} = \hat{Y}(p_1)$ from (21) in (9), the total quantity demanded of good 1 is given by (22) and the total disposable income of owners in is

$$\hat{y}(p_1) = \pi_1(p_1) + n(\pi_0 - \underline{x}_0) = (p_1 - \hat{w})y_0/\underline{x}_0 + n(\pi_0 - \underline{x}_0) \tag{37}$$

The disposable income of each individual owner is $\hat{y}(p_1)/n > 0$. Taking $\hat{y}_z = \hat{y}(p_1)/n$ in (10), the indirect utility of each individual owner in N_1 is $\underline{x}_0 + \hat{y}(p_1)/np_1$. Since ther are n owners, the sum of indirect utilities of all owners is

$$V(p_1) = n\underline{x}_0 + \hat{y}(p_1)/p_1 = n\underline{x}_0 + [(p_1 - \hat{w})y_0/\underline{x}_0 + n(\pi_0 - \underline{x}_0)]/p_1 \tag{38}$$

When there is no technology transfer, good 2 is not produced at all and firm 1 is a monopolist in the manufacturing sector. The problem of firm 1 is to choose $p_1 \in [\underline{p}, \bar{p}]$ to maximize the sum of indirect utilities of the owners given in (38).

Proposition 3 *Under common ownership, when the production technology is not shared, firm 1 is a monopolist and it is optimal for firm 1 to set $p_1 = \bar{p}$. The sum of indirect utilities $V(\bar{p})$ of all owners is given by*

$$V(\bar{p}) = n\underline{x}_0 + [(\bar{p} - \hat{w})y_0/\underline{x}_0 + n(\pi_0 - \underline{x}_0)]/\bar{p} \tag{39}$$

Proof Same as the proof of Proposition 1. ■

4.3 Total disposable income under technology transfer

When firm 1 transfers the production technology to firm 2, each firm $i = 1, 2$ can produce one unit of good i by employing one unit of labor. The profit $\pi_i(p_1, p_2)$ of firm $i = 1, 2$, is given by (26). We have shown in Section 2.5 that from its individually owned operation in the basic good sector, each owner earns income π_0 (which is a constant that does not depend on p_1, p_2). In addition, any owner earns $\pi_1(p_1, p_2)/n$ $+\pi_2(p_1, p_2)/n$ (fraction $1/k$ of the sum of profits of firms $1, 2$), so its income is $\pi_0 + \pi_1(p_1, p_2)/n$ $+\pi_2(p_1, p_2)/n$.

As π_0 is assumed to be more than $\underline{x}_0$ (see Section 2.5), the income of any owner is more than $\underline{x}_0$, so any owner has positive disposable income $\pi_0+\pi_1(p_1, p_2)/n$ $+\pi_2(p_1, p_2)/n-$

$\underline{x}_0$. As there are n owners, the total disposable income of all owners is

$$\pi_1(p_1,p_2)+\pi_2(p_1,p_2)+n(\pi_0-\underline{x}_0) \tag{40}$$

By (14), (15) and (40), the total disposable income of the economy, which is the sum of disposable incomes of all owners and workers, is given by

$$\hat{Y}(p_1,p_2)=\pi_1(p_1,p_2)+\pi_2(p_1,p_2)+(\hat{w}-\underline{x}_0)\hat{L}(p_1,p_2)+y_0$$

where the positive constant y_0 is given in (19). The total labor employed in the manufacturing sector is $\hat{L}(p_1,p_2)=X_1(p_1,p_2)+X_2(p_1,p_2)$ and by (26), we have

$$\hat{Y}(p_1,p_2)=(p_1-\underline{x}_0)X_1(p_1,p_2)+(p_2-\underline{x}_0)X_2(p_1,p_2)+y_0$$

Taking $\hat{y}=\hat{Y}(p_1,p_2)$ in (12), in this case the total quantity demanded of good i is given by (29). Using this we have

$$\hat{Y}(p_1,p_2)=(p_1-\underline{x}_0)\hat{y}(p_1,p_2)/p_1^\sigma g(p_1,p_2)+(p_2-\underline{x}_0)\hat{Y}(p_1,p_2)/p_2^\sigma g(p_1,p_2)+y_0$$

Solving the equation above, $\hat{Y}(p_1,p_2)$ is given by (31). By (29) and (31), the total quantity demanded of goods $1,2$ are given by (32). Using (32) in (26) and (27), the total disposable income of all owners is

$$\hat{y}(p_1,p_2)=(p_1-\hat{w})y_0p_2^\sigma/(p_1^\sigma+p_2^\sigma)\underline{x}_0+(p_2-\hat{w})y_0p_1^\sigma/(p_1^\sigma+p_2^\sigma)\underline{x}_0+n(\pi_0-\underline{x}_0) \tag{41}$$

4.4 Strategic interaction between firms $1,2$

For $i=1,2$, the objective of firm i is to maximize the sum of indirect utilities of its owners. As each owner has disposable income $\hat{y}(p_1,p_2)/n>0$, taking $\hat{y}_z=\hat{y}(p_1,p_2)/n$ in (13), the indirect utility of each such owner is

$$\underline{x}_0+\hat{y}(p_1,p_2)g(p_1,p_2)^{1/(\sigma-1)}/n$$

The sum of indirect utilities of all owners is

$$v(p_1,p_2)=n\underline{x}_0+\hat{y}(p_1,p_2)g(p_1,p_2)^{1/(\sigma-1)}$$

Using (41), it follows that the sum of indirect utilities of each group of owners is

$$\begin{aligned} v(p_1,p_2)&=n\underline{x}_0+[(p_1-\hat{w})y_0p_2^\sigma/(p_1^\sigma+p_2^\sigma)\underline{x}_0\\ &+(p_2-\hat{w})y_0p_1^\sigma/(p_1^\sigma+p_2^\sigma)\underline{x}_0+n(\pi_0-\underline{x}_0)]g(p_1,p_2)^{1/(\sigma-1)}\\ &=n\underline{x}_0+[y_0p_1^\sigma p_2^\sigma g(p_1,p_2)/\underline{x}_0(p_1^\sigma+p_2^\sigma)-\hat{w}y_0/\underline{x}_0+n(\pi_0-\underline{x}_0)]g(p_1,p_2)^{1/(\sigma-1)} \end{aligned} \tag{42}$$

where g is given in (6). Therefore the strategic interaction between firms reduces to a two person game Γ_c (c stands for common ownership) with firms $1,2$ where firms choose prices $p_1,p_2\in[\underline{p},\bar{p}]$ and the payoff of firm $i=1,2$ is given by $v(p_1,p_2)$ in (42). The next proposition chracterizes Nash Equilibrium (NE) of Γ_c.

Proposition 4 *Suppose either $0 < \sigma < 1$ or $\sigma > 1$.*

(i) *The game Γ_c has a unique NE: $(p_1 = \bar{p}, p_2 = \bar{p})$. At the NE, the sum of indirect utilities $v(\bar{p}, \bar{p})$ of all owners of firm is as follows:*

$$v(\bar{p}, \bar{p}) = n\underline{x}_0 + [(\bar{p} - \hat{w})y_0/\underline{x}_0 + n(\pi_0 - \underline{x}_0)]2^{1/(\sigma-1)}/\bar{p} \tag{43}$$

(ii) *If $\sigma > 1$, then technology transfer is superior to not sharing the technology for firm 1 and if $0 < \sigma < 1$, then not sharing the technology is superior to technology transfer for firm 1.*

Proof See the Appendix. ■

Proposition 4 shows that technology transfer is superior goods are relatively good substitutes ($\sigma > 1$ but σ not too large). On the other hand, not sharing the technology when goods are relatively poor substitutes ($0 < \sigma < 1$).

5 Concluding remarks

In this paper we study technology transfer of a product innovation in a model of a dual economy with ownership structure. One conclusion that emerges from our analysis is that together with ownership structure, the nature of product market is an important factor to determine the diffusion of production technology. In our model, the nature of the product market is characterized by the CES parameter σ. When the two manufacturing goods are poor substitutes ($0 < \sigma < 1$), not sharing the technology is superior for firm 1 under both disjoint and common ownership structures. However, when the two manufacturing goods are relatively good substitutes ($\sigma > 1$) technology transfer is superior under common ownership, but may not be always superior under disjoint ownership.

Two main limitations of our analysis at this point are: (i) under all scenariors, the prices at the product market are always the maximum prices, so the price effect of the goods on the indirect utilities are not adequately captured and (ii) we consider two ownership structures (disjoint and common), but more general ownership structures (for example, some owners have shares of both firms and others exclusively have shares of only one firm) are also plausible.

Appendix

Proof of Proposition 2 (i) Denoting $y_0/\underline{x}_0 \equiv \tau$ and $k(\pi_0 - \underline{x}_0) \equiv \theta$, from (34) we note that

$$\partial v_1(p_1, p_2)/\partial p_1 = h(p_1, p_2)g(p_1, p_2)^{(2-\sigma)/(\sigma-1)}/p_1^\sigma(p_1^\sigma + p_2^\sigma)^2 \text{ where}$$

$$h(p_1, p_2) := \tau p_1^{2\sigma-1}p_2[\sigma\hat{w} + (1-\sigma)p_1]$$
$$+ \tau p_1^\sigma p_2^\sigma(p_2 - \sigma p_1 + \sigma\hat{w}) + [\hat{w}\tau p_2^{2\sigma} - \theta(p_1^{2\sigma} + p_2^{2\sigma})] + p_1^\sigma p_2^\sigma(\hat{w}\tau - 2\theta) \tag{44}$$

Thus $\partial v_1(p_1,p_2)/\partial p_1 \gtreqless 0 \Leftrightarrow h(p_1,p_2) \gtreqless 0$.

In what follows, we show that $v_1(p_1,p_2)$ is increasing in p_1 for all $0 < p_1 \le p_2$. Recall from (19) that $y_0 = n(\pi_0 - \underline{x}_0) + M(\underline{\ell}) - \underline{\ell}\,\underline{x}_0$. As $n = 2k$, we have $y_0 > n(\pi_0 - \underline{x}_0) = 2k(\pi_0 - \underline{x}_0) = 2\theta$. Since $\hat{w} \ge \underline{x}_0$, it follows that $\hat{w}\tau \ge \underline{x}_0\tau = y_0 > 2\theta$. Thus $\hat{w}\tau > 2\theta$. Noting that $\sigma > 0$, this shows that the last term of (44) is positive for all $p_1, p_2 > 0$.

Consider the second last term $\hat{w}\tau p_2^{2\sigma} - \theta(p_1^{2\sigma} + p_2^{2\sigma})$. As $\theta > 0$, for $0 < p_1 \le p_2$, we have

$$\hat{w}\tau p_2^{2\sigma} - \theta(p_1^{2\sigma} + p_2^{2\sigma}) \ge \hat{w}\tau p_2^{2\sigma} - 2\theta p_2^{2\sigma} = (\hat{w}\tau - 2\theta)p_2^{2\sigma} > 0$$

So the second last term of (44) is positive for all $0 < p_1 \le p_2$.

Next consider the term $\tau p_1^{\sigma} p_2^{\sigma}(p_2 - \sigma p_1 + \sigma\hat{w})$. When $0 < p_1 \le p_2$, for $0 < \sigma < 1$, we have $\sigma p_1 < p_1$, so that $p_2 - \sigma p_1 > 0$ and this term is also positive. So consider $\sigma > 1$. When $0 < p_1 \le p_2$, we have

$$p_2 - \sigma p_1 + \sigma\hat{w} \ge p_2 - \sigma p_2 + \sigma\hat{w} = \sigma\hat{w} - (\sigma - 1)p_2$$

$$\ge \sigma\hat{w} - (\sigma - 1)\bar{p} = \bar{p} - \sigma(\bar{p} - \hat{w}) > 0 \text{ for } 1 < \sigma < \sigma_0 \equiv \bar{p}/(\bar{p} - \hat{w})$$

This shows that the term $\tau p_1^{\sigma} p_2^{\sigma}(p_2 - \sigma p_1 + \sigma\hat{w})$ is positive for $0 < p_1 \le p_2$ when either $0 < \sigma < 1$ or $1 < \sigma < \sigma_0$.

Finally consider the term $\tau p_1^{2\sigma-1} p_2[\sigma\hat{w} + (1 - \sigma)p_1]$. For $0 < \sigma < 1$, this term is clearly positive. For $\sigma > 1$, we have

$$[\sigma\hat{w} + (1 - \sigma)p_1] = \sigma\hat{w} - (\sigma - 1)p_1 \le \sigma\hat{w} - (\sigma - 1)\bar{p} = \bar{p} - \sigma(\bar{p} - \hat{w}),$$

which is positive for $1 < \sigma < \sigma_0 \equiv \bar{p}/(\bar{p} - \hat{w})$.

This shows that when $0 < \sigma < 1$ or $1 < \sigma < \sigma_0$, then $h(p_1,p_2) > 0$ and hence $v_1(p_1,p_2)$ is increasing in p_1 for all $0 < p_1 \le p_2$. Due to the symmetric nature of the problem, note that sign $v_2(p_1,p_2) =$ sign $h(p_2,p_1)$. Thus $v_2(p_1,p_2)$ is increasing in p_2 for all $0 < p_2 \le p_1$, so we cannot have any equilibrium at which either $p_1 < p_2$ (firm 1 can gain by slightly raising its price from p_1) or $p_2 < p_1$ (firm 2 can gain by slightly raising its price from p_2). Also we cannot have an equilibrium at which $p_1 = p_2 < \bar{p}$ (one of the firms can gain by slightly raising its price).

Therefore the only candidate for equilibrium is $p_1 = p_2 = \bar{p}$. We have shown that for $i = 1, 2$, and $i \ne j$, $v_i(p_1,p_2)$ is increasing in p_i when $0 < p_i \le p_j$. Thus, in particular, $v_i(p_1,p_2)$ is increasing in p_i when $0 < p_i = p_j$. This shows that $v_1(p_1,\bar{p}) < v_1(\bar{p},\bar{p})$ for all $0 < p_1 < \bar{p}$ and $v_2(p_1,\bar{p}) < v_2(\bar{p},\bar{p})$ for all $0 < p_2 < \bar{p}$. This shows that $(p_1 = \bar{p}, p_2 = \bar{p})$ is indeed an equilbrium and this is the unique equilibrium when $0 < \sigma < 1$ or $1 < \sigma < \sigma_0$.

(ii) Comparing $V_1(\bar{p})$ with $v_1(\bar{p},\bar{p})$, we see that

$$v_1(\bar{p},\bar{p}) \gtreqless V_1(\bar{p}) \Leftrightarrow 2^{1/(\sigma-1)}A \gtreqless B \text{ where}$$

$$A \equiv [(\bar{p} - \hat{w})y_0/\underline{x}_0 + 2k(\pi_0 - \underline{x}_0)] \text{ and } B \equiv [2(\bar{p} - \hat{w})y_0/\underline{x}_0 + 2k(\pi_0 - \underline{x}_0)]$$

Note that $0 < A < B < 2A$. For $0 < \sigma < 1$, we have $2^{1/(\sigma-1)} < 1$. Thus $2^{1/(\sigma-1)}A < A < B$, so in this case no licensing is superior to free licensing for firm 1.

For $\sigma > 1$, $2^{1/(\sigma-1)}$ is decreasing in σ. For $\sigma = 2$, we have $2^{1/(\sigma-1)} = 2$ and

$2^{1/(\sigma-1)}A = 2A > B$, so free licensing is superior when $\sigma = 2$. As $2^{1/(\sigma-1)}$ is decreasing in σ, it follows that $v_1(\bar{p}, \bar{p}) > V_1(\bar{p})$ for all $\sigma \in (1, 2)$. Thus, if $\sigma_0 \leq 2$, then free licensing is superior to no licensing for all $\sigma \in (1, \sigma_0)$. This completes the proof of (ii)(a).

We also observe that $\lim_{\sigma\to\infty} 2^{1/(\sigma-1)} = 0$, it follows that $\exists\ \hat{\sigma} > 2$ such that

$$v_1(\bar{p}, \bar{p}) \gtreqless V_1(\bar{p}) \Leftrightarrow \sigma \lesseqgtr \hat{\sigma}$$

Specifically, $2^{1/(\hat{\sigma}-1)}A = B$, so that $[1/(\hat{\sigma} - 1)] \log 2 = \log(B/A)$ implying that $\hat{\sigma} = 1 + [\log 2]/[\log(B/A)]$. Comparison of δ_0 and $\hat{\delta}$ yields the conclusion of (ii)(b). ■

Proof of Proposition 4 (i) Denote $Z \equiv \hat{w}y_0/\underline{x}_0 - n(\pi_0 - \underline{x}_0)$. Note from (**??**) that $y_0 > n(\pi_0 - \underline{x}_0)$. As $\hat{w} > \underline{x}_0$, it follows that $Z > 0$.

Note from (42) that

$$\partial v(p_1, p_2)/\partial p_1 = \psi(p_1, p_2)g(p_1, p_2)^{(2-\sigma)/(\sigma-1)}/\underline{x}_0 p_1(p_1^\sigma + p_2^\sigma)^2 \text{ where}$$

$$\psi(p_1, p_2) := \sigma y_0 p_1 p_2 (p_1^{\sigma-1} + p_2^{\sigma-1})(p_2 - p_1) + \underline{x}_0 Z(p_1^{\sigma+1} + 2p_1 p_2^\sigma + p_1^{1-\sigma} p_2^{2\sigma})$$

As $Z > 0$, we note that $\psi(p_1, p_2) > 0$ for all $0 < p_1 \leq p_2$. Thus $\partial v(p_1, p_2)/\partial p_1 > 0$ and hence $v(p_1, p_2)$ is increasing in p_1 for all $0 < p_1 \leq p_2$ (this holds for all $0 < \sigma < 1$ as well as $\sigma > 1$). Due to the symmetric nature of the problem, it follows that $v(p_1, p_2)$ is increasing in p_2 for all $0 < p_2 \leq p_1$.

This shows that we cannot have any equilibrium at which either $p_1 < p_2$ (firm 1 can gain by slightly raising its price from p_1) or $p_2 < p_1$ (firm 2 can gain by slightly raising its price from p_2). Also we cannot have an equilibrium at which $p_1 = p_2 < \bar{p}$ (one of the firms can gain by slightly raising its price).

Therefore the only candidate for equilibrium is $p_1 = p_2 = \bar{p}$. We have shown that for $i, j = 1, 2$, $i \neq j$, $v(p_1, p_2)$ is increasing in p_i when $0 < p_i \leq p_j$. Thus, in particular, $v(p_1, p_2)$ is increasing in p_i when $0 < p_i = p_j$. Thus $v(p_1, \bar{p}) < v(\bar{p}, \bar{p})$ for all $0 < p_1 < \bar{p}$ and $v(p_1, \bar{p}) < v_2(\bar{p}, \bar{p})$ for all $0 < p_2 < \bar{p}$. This shows that $(p_1 = \bar{p}, p_2 = \bar{p})$ is indeed an equilbrium and this is the unique equilibrium when $0 < \sigma < 1$ or $\sigma > 1$.

Part (ii) follows by comparing (43) and (39). ■

References

Antón, M., Ederer, F., Giné, M., Schmalz, M.C. 2024. Innovation: the bright side of common ownership? Management Science.

Arrow, K.J, Chenery, H., Minhas, B., Solow, R. 1961. Capital-labor substitution and economic efficiency. Review of Economics and Statistics, 43:225-250.

Arrow, K.J., 1962. Economic welfare and the allocation of resources for invention. In: R.R. Nelson (Ed.), The Rate and Direction of Inventive Activity: Economic and Social Factors, Princeton University Press.

Azar, J., Schmalz, M.C., Tecu, I. 2018. Anticompetitive effects of common ownership. Journal of Finance, 73:1513-1565.

Azar, J., Vives, X. 2021. General equilibrium oligopoly and ownership structure. Econometrica, 89:999-1048.

Black, J., Hashimzade, N., Myles, G. 2009. A Dictionary of Economics, 3rd Edition. Oxford University Press.

Demsetz, H. 1973. Industry structure, market rivalry, and public policy. Journal of Law & Economics, 16:1-9.

Denicoló, V., Panunzi, F. (2024). Common ownership, competition, and corporate governance. Management Science.

Khanna, T., Palepu, K.G. 2005. The evolution of concentrated ownership in India: broad patterns and a history of the Indian software industry. In: R.R. Morck (Ed.), The History of Corporate Governance around the World: Family Business Groups to Professional Managers, University of Chicago Press.

Lewis, W.A. 1954. Economic development with unlimited supplies of labour. Manchester School 22:139-191.

Li, X., Liu, T., Taylor, L.A. 2023. Common ownership and innovation efficiency. Journal of Financial Economics 147:475-497.

López, Á.L., Vives, X. 2019 Overlapping ownership, R&D spillovers, and antitrust policy. Journal of Political Economy 27:2394-2437.

Matsuyama, K. 2002. The rise of mass consumption societies. Journal of Political Economy 110: 1035-1070.

Schmalz, M.C. 2021. Recent studies on common ownership, firm behavior, and market outcomes. Antitrust Bulletin 66: 12-38.

O'Brien, D.P., Salop, S.C. 2000. Competitive effects of partial ownership: financial interest and corporate control. Antitrust Bulletin 67:559-614.