Greener Merger Guidelines: When Do They Matter?

Ana Espinola-Arredondo; Felix Munoz-Garcia[†], and Kiriti Kanjilal[‡]

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Abstract

We study mergers in polluting industries where a public firm is present. We evaluate how merger approvals are affected when the competition authority's guidelines consider environmental criteria. As expected, we show that more mergers are approved, yielding less pollution, and higher welfare. However, we find that firms only have incentives to submit a merger request if the public firm is mostly privatized; otherwise, no merger requests are filed in equilibrium, rendering changes in guidelines inconsequential. We, then, examine how merger approvals are affected by the presence of the public firm and pollution. We also conduct robustness checks allowing for the manager of the public firm to ignore emissions, the merger to yield cost-reducing effects, and convex production costs.

KEYWORDS: Mergers, Public Firm, Pollution, Mixed Oligopoly, Cost-reduction effects. JEL CLASSIFICATION: L13, L32, L44.

^{*}Address: 101B Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: anaespinola@wsu.edu.

[†]Address: 103H Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: fmunoz@wsu.edu.

[‡]Address: Indraprastha Institute of Information Technology, Address: B-208, Research and Development Block, Delhi 110020, India, E-mail: kanjilal@iiitd.ac.in.

1 Introduction

Merger guidelines have been under intense scrutiny in recent years and competition authorities (CAs) are facing more pressure to consider pollution and environmental criteria. The European Commission (EC), for instance, revised its guidelines in June 2023, including a new section on mergers that can yield sustainability benefits; see EC Communication (2023), section 9.3. Subsequently, EU Merger Regulation stated that, in order to support the objectives of the European Green Deal, merger enforcement should take into account environmental benefits from mergers; see EC Competition Merger Brief (2023).¹ Initiatives in other countries include Japan, Australia, Germany, Austria, the Netherlands, and Greece.²

Similarly, the UK's Competition and Markets Authority issued a green agreement guidance in October 2023, describing "the circumstances in which an otherwise potentially anticompetition agreement may be exempt from competition rules on the basis of the sustainability benefits it brings," see Luoma et al. (2023). The US Federal Trade Commission is still debating whether to include environmental considerations in competition rules, but recommendations to change its merger guidelines abound; see, for instance, Hearn et al. (2023) and Hanawalt et al. (2024).

In this paper, we analyze the above change in CA guidelines, first showing that it can yield more merger approvals. As expected, mergers help reduce aggregate output and its associated emissions, thus giving rise to welfare gains. When all firms are private, this new guideline can lead to more incentives to file merger requests, as firms anticipate more likely approvals. When a public firm is present, however, mergers become less profitable, thus reducing firm incentives to file a merger.

Our results imply that fewer merger requests should be observed when a publicly owned firm is active than otherwise, leaving market structures unaffected. In this context, policy initiatives to reform CA guidelines are likely inconsequential. Our finding is emphasized when pollution becomes more severe and firms' production is more efficient. In contrast, when the public firm is mostly privatized, pollution is not severe, and firms are relatively inefficient, our results are ameliorated, implying that changes in CA guidelines can trigger more merger requests, followed by more approvals, and an overall welfare gain.

Our paper considers a polluting industry where a public firm and several private firms compete à la Cournot (mixed oligopoly), which are common in energy and mining markets.³ To study how merger approval decisions are affected by the presence of pollution and a public firm, we consider a sequential-move game where, in the first stage, the merging entity submits a merger request to the CA; in the second stage, the CA responds approving or blocking the merger; and, in the third stage,

¹Recent examples of merger approvals citing environmental considerations include Norsk Hydro and Alumetal in the aluminum industry, and Sika/MBCC in the chemical admixture industry used in cement and concrete, see EC Competition Merger Brief (2023).

²For more details on each country, see Japan Fair Trade Commission (2023), OECD (2020), Bundeskartellamt (2020), Kartellgesetz (2005), Autoriteit Consument and Markt (2021), and Hellenic Competition Commission (2020), respectively. The OECD also explored the role of environmental protection in competition policy in several roundtables; see, for instance, OECD (2010) and more recently in OECD (2020).

³Examples of public firms in the oil and gas market include Electricité de France (EDF) in France, Equinor in Norway, Enel in Italy, ENAP in Chile, Aramco in Saudi Arabia, and Gazprom in Russia.

firms compete. For generality, we allow for the CA to evaluate mergers according to traditional criteria (the consumer surplus and the welfare criterion, but ignoring environmental damages) and according to new guidelines (welfare criterion considering environmental damages).

In the last stage, we identify the public firm's best response function, first showing that it shifts downwards, thus producing fewer units, when this firm is more nationalized or when pollution is more severe. These are "good news" for its rivals, who benefit from higher prices and profits than when all firms are private. However, we also find that the public firm's best response function becomes steeper. This implies that, when the aggregate output of the merging firms decreases due to the merger, the public firm responds increasing its output more significantly than when all firms are private, i.e., "bad news" for the merging entity.

Overall, the negative effect of the merger on profits, stemming from outsiders increasing their output, is stronger when a public firm is present in the industry than otherwise, making the merger less likely to be profitable. Technically, Salant et al. (1983) 80-percent rule becomes more stringent in this context, with merging firms requiring larger market shares to find the merger profitable. If this negative effect is sufficiently strong, which occurs when the government holds a significant stake at the public firm and pollution is severe, the merging entity finds the merger unprofitable regardless of its market share in the industry. In this case, the sheer presence of the public firm prevents mergers from being filed in the first place. This lack of requests may facilitate the CA's task when the merger would have been welfare reducing. Nonetheless, it can also lead to welfare-enhancing mergers not being filed in equilibrium, leading to unambiguous welfare losses, just because of the public firm's presence.

We study different CA guidelines, and how they are affected by the presence of a public firm.⁴ When the CA evaluates mergers according to the consumer-surplus criterion, all mergers are blocked. A similar outcome emerges when considering the welfare criterion, with mergers still being blocked if the CA ignores environmental damages. When the CA considers environmental damages, however, it may approve the merger if it accounts for a small market share (small welfare loss) or when pollution is severe (large welfare gain).

In this context, the presence of a public firm induces the CA to more likely block mergers since this agency anticipates that the public firm will partially address market failures. Hence, the reduction in pollution from approving the merger becomes less necessary when a public firm is active than otherwise, yielding fewer mergers being approved and, ultimately, filed in equilibrium. The opposite argument applies when emissions are more damaging, since the CA uses merger approvals as a policy tool to reduce aggregate pollution.

Our results help measure the welfare gain from changing the CA's guidelines to require the consideration of environmental damages. This welfare gain is significant when the public firm

⁴In recent years, some mixed oligopolies have experienced mergers between private firms. Examples include FedEx merging with Genco in 2015 and with TNT Express in 2016, where the public company USPS also operates; Lundin Mining merging with Lumina Copper in Chile on March, 2023, where the government-owned Codelco keeps approximately 29 percent of the national copper production; and Czech brewery Radegast merging with Velke Popovice and Plzensky Prazdroj in 2002, where Budweiser Budvar-Brewery is state-owned.

is largely privatized, pollution is severe, and production is relatively efficient. In this setting, the CA becomes more likely to approve the merger according to the new guidelines, lowering emissions, and yielding large welfare gains. Otherwise, the public firm corrects a larger portion of the environmental externality, thus making the output reduction from a merger approval less necessary. Nonetheless, this context may lead to no merger requests in equilibrium: when the public firm is more nationalized, the merging entity has no incentives to merge, as discussed above, making the CA's guidelines inconsequential.

For robustness, we consider three alternative settings. First, we allow for the public firm to ignore pollution in its welfare function, which can arise if regulations only consider consumer and producer surplus. In this context, the public firm produces more output and emissions, inducing the CA to more likely approve mergers than when the public firm considers environmental damages. We show that this approval differential increases when the public firm is less privatized and pollution becomes more severe. Second, we allow for the merger to generate cost-reducing effects for the merging entity, showing that, as expected, mergers are more likely to become profitable. Costreducing effects yield a larger output and, while they also generate more pollution, the merger is more likely to become welfare improving than in the absence of cost-reducing effects, thus leading to more merger approvals. Third, we consider cost convexities, showing that changes in the CA guidelines are less likely to affect merger approvals in this context.

1.1 Related literature

Our paper contributes to the literature on mixed oligopolies, allowing for mergers in polluting industries. This literature has been applied to different fields, such as R&D investments (Delbono and Denicolò, 1993), banking competition (La Porta et al. 2002), and models of open economies (Chang, 2005; Chao and Yu, 2006; Matsumura and Tomaru, 2012), among others. Some articles in this literature consider polluting markets, as in our model (see, for instance, Barcena-Ruiz and Garzón, 2006; Pal and Saha, 2015; and Lee and Park 2021), but do not allow for mergers, thus not examining firms' incentives to merger, or how different CA guidelines are affected by the presence of a public firm.

In the field of environmental economics, several articles have also studied the welfare effects of mergers. This literature seeks to identify under which conditions the merger, despite lowering aggregate output, can become welfare improving because of reducing emissions. These include studies assuming industries with two firms, Fikru and Gautier (2016, 2017); three firms, Lambertini and Tampieri (2014); several firms, Fikru and Gautier (2020); or trade liberalization, Chaudhuri and Benchekroun (2012). These articles, however, consider that all firms are privately owned, whereas we study how the presence of a public firm alters firms' incentives to merge and, subsequently, the CA's decision.

This paper is, then, closer to the literature studying mergers in mixed oligopolies without environmental externalities, including Barcena-Ruiz and Garzón (2003), Nakamura and Inoue (2007), Méndez-Naya (2008), and Kanjilal et al. (2022). We consider a polluting production process affecting both the welfare function of the public firm's manager (who may consider or ignore these emissions) and that of the CA when evaluating merger approvals. For generality, we also allow for mergers to not affect production costs, as in Barcena-Ruiz and Garzón (2003) and Méndez-Naya (2008); or to generate cost-reducing effects, as in Nakamura and Inoue (2007) and Kanjilal et al. (2022). Unlike the existing literature, our paper helps identify how the presence of a public firm in polluting industries hinders firms' incentives to merge, and the CA's merger approval decision.

The paper is organized as follows. Section 2 presents the model and section 3 identifies equilibrium behavior in each stage. Section 4, then, analyzes how our findings are affected by changes in modeling assumptions, and section 5 concludes.

2 Model

Consider an industry with $n \ge 3$ firms (one public and n-1 private firms) competing à la Cournot, facing inverse demand function p(Q) = 1 - Q, where $Q \equiv q + \sum_{i=1}^{n-1} q_i$ denotes aggregate output, qrepresents the public firm's output, and q_i is the output of private firm i = 1, 2, ..., n-1, and every firm faces marginal cost c, where $1 > c \ge 0$.

Every private firm maximizes its profits

$$\pi_i = p(Q)q_i - cq_i \tag{1}$$

while the public firm maximizes a combination of social welfare and profits

$$V = (1 - \alpha)\pi + \alpha W \tag{2}$$

where the public firm's profit is $\pi = p(Q)q - cq$; and social welfare is given by W = CS + PS - Env. In particular, $CS = \frac{Q^2}{2}$ denotes consumer surplus, $PS = \pi + \sum_{i=1}^{n-1} \pi_i$ measures producer surplus, and $Env = dQ^2$ represents the environmental damage, where every unit of output generates a unit of emissions. This damage is increasing and convex in aggregate emissions, where d > 1/2 as in Poyago-Theotoky (2007) and Lambertini et al. (2017) among others. Parameter α represents the weight that the manager of the public firm assigns to welfare, while $1 - \alpha$ is the weight she assigns to profits; as in Matsumura (1998) and Fujiwara (2007), among others.⁵ For simplicity, we assume that the severity of pollution is not excessive, $d < \frac{1}{2\alpha(n-1)}$, as otherwise the public firm's nationalization is not excessive, $\alpha < \frac{1}{2d(n-1)} \equiv \overline{\alpha}$.

When evaluating a merger request, we allow for the CA to consider two merger guidelines: (i) the consumer surplus criterion, where a merger is approved if it increases aggregate output; and (ii) the welfare criterion, where merger are approved if they are welfare enhancing.

⁵Alternatively, parameter α can be interpreted as the government's share in the public firm and $1 - \alpha$ denotes the degree of privatization. When $\alpha = 1$, the firm is not privatized at all, while $\alpha = 0$ implies that it is fully privatized.

Therefore, our model embodies previous studies as special cases: Matsumura (1998) when $\alpha = d = 0$ (all firms are private and no external effects arise); Kanjilal et al. (2022) when $\alpha > 0$ and d = 0 (public firm is present, but no external effects exist); or Denicolò (1999) and Strandholm et al. (2018, 2023) when $\alpha = 0$ and d > 0 (all firms are private, but external effect are allowed).

The time structure of the game is the following:

- 1. In the first stage, the k private firms that seek to merge choose whether to submit, as an entity, a merger approval request to the CA, where k satisfies $n 1 \ge k \ge 2$.
- 2. In the second stage, the CA responds approving or blocking the merger.
- 3. In the third stage, firms observe the CA's decision, and compete à la Cournot.

We solve this sequential-move game by backward induction, first finding output levels in the third stage.

3 Equilibrium Analysis

3.1 Third stage - No merger

If a merger does not ensue, firms compete à la Cournot, with every private firm *i* maximizing (1), which yields best response function $q_i(q) = \frac{1-c}{n} - \frac{1}{n}q$. This function is unaffected by externalities, *d*, or by the public firm's privatization level, α .⁶

The public firm instead maximizes (2), yielding best response function

$$q(q_i) = \frac{1-c}{2+\alpha(2d-1)} - \frac{1+2\alpha d}{2+\alpha(2d-1)} \sum_{i=1}^{n-1} q_i$$
(3)

where its vertical intercept decreases in c, α , and d. Intuitively, when the public firm is more nationalized (higher α) and pollution becomes more severe (higher d), this firm decreases its production.

Similarly, the slope of (3) is unambiguously decreasing in α and d, indicating that, as private firms increase their output, the public firm responds reducing its own output to a larger extent to curb pollution.⁷ When external effects are absent (d = 0 but $\alpha > 0$), equation (3) simplifies to $q(q_i) = \frac{1-c}{2-\alpha} - \frac{1}{2-\alpha} \sum_{i=1}^{n-1} q_i$, thus coinciding with that in Kanjilal et al. (2022). In this context, a more public firm (higher α) responds to the decrease in output from a merger by increasing its own output more significantly, since the market failure from underproduction in oligopoly is strenghten.

⁶Every private firm's best response function, nonetheless, shifts downwards in the initial marginal cost c and in the number of private firms competing in the industry, n; and becomes flatter as n increases, thus indicating that output competition is softened.

The slope of the best response function in (3), $s \equiv -\frac{1+2\alpha d}{2+\alpha(2d-1)}$, satisfies $\frac{\partial s}{\partial \alpha} = -\frac{1+2d}{[2+\alpha(2d-1)]^2} < 0$, $\frac{\partial s}{\partial d} = -\frac{2\alpha(1-\alpha)}{[2+\alpha(2d-1)]^2} < 0$, and $\frac{\partial^2 s}{\partial \alpha \partial d} = \frac{2\alpha(3+2d)-4}{[2+\alpha(2d-1)]^3} < 0$ if $\alpha < \frac{2}{3+2d}$, where $\frac{2}{3+2d} > \overline{\alpha}$ because $\alpha < \overline{\alpha}$ by definition, implying that $\frac{\partial^2 s}{\partial \alpha \partial d} < 0$ for all admissible parameters.

When environmental effects are present as well $(d > 0 \text{ and } \alpha > 0)$, best response function $q(q_i)$ shifts downwards and becomes steeper. To understand this result, note that, before the merger, the public firm seeks to address two market failures, pollution and underproduction, lowering its output when pollution is more severe. When a merger ensues, aggregate output from private firms decreases, making underproduction a relatively more severe market failure than that stemming from pollution. As a consequence, the merger helps alleviate the severity of environmental damages, allowing the public firm to increase its output more intensively than before the merger. In addition, this effect is emphasized when d increases, i.e., equation (3) becomes steeper in d. However, when all firms are private, $\alpha = 0$, equation (3) becomes $q(q_i) = \frac{1-c}{2} - \frac{1}{2} \sum_{i=1}^{n-1} q_i$, implying that the public firm ignores both market failures.

The next lemma identifies the equilibrium output levels.

Lemma 1. In the third stage, if the merger is does not ensue, the equilibrium output of the public firm is $q^{NM} = \frac{(1-c)[1-2\alpha d(n-1)]}{1+2\alpha d+(1-\alpha)n}$, and that of every private firm i is $q_i^{NM} = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)n}$, where $q_i^{NM} > q^{NM} > 0$. Output q^{NM} is unambiguously decreasing in d, α , c, and n. Output q_i^{NM} is unambiguously increasing in d and α , but decreasing in c and n.

When pollution becomes more severe (higher d), the public firm decreases its output to help curb emissions, while every private firm responds increasing its production; a finding that is emphasized when a larger share of the public firm is nationalized (higher α).

While private firms take advantage of more severe pollution by increasing their output, the next lemma shows that the decrease in the public firm's output dominates, yielding an overall decrease in aggregate output, $Q^{NM} \equiv q^{NM} + (n-1) q_i^{NM}$.

Lemma 2. Aggregate output under no merger, $Q^{NM} = \frac{(1-c)[n(1-\alpha)+\alpha]}{1+2\alpha d+n(1-\alpha)}$, is unambiguously decreasing in α , d and c, but increasing in n.

When the public firm is completely privatized, $\alpha = 0$, aggregate output simplifies to $Q^{NM} = \frac{n(1-c)}{n+1}$, as in a standard Cournot model with *n* private firms. Since all firms ignore pollution, aggregate output is unaffected by parameter *d* in this context. In contrast, when $\alpha > 0$ aggregate output decreases, which is emphasized by more severe pollution.

The next corollary evaluates the "efficiency ratio" under no merger, $ER^{NM} \equiv \frac{Q^{NM}}{Q^{SO}}$, where $Q^{SO} = \frac{1-c}{2d+1}$ denotes the socially optimal output.⁸ Ratios closer to one indicate that aggregate output is close to the social optimum, while ratios above (below) one suggest that aggregate output is socially excessive (insufficient).

Corollary 1. The efficiency ratio under no merger is $ER^{NM} = \frac{(2d+1)[n(1-\alpha)+\alpha]}{1+2\alpha d+(1-\alpha)n} > 1$, which is increasing in d and n, but decreasing in α .

Therefore, aggregate output is socially excessive (i.e., $ER^{NM} > 1$). This inefficiency is partially corrected when the public firm becomes more nationalized (higher α). In contrast, more severe

⁸This output arises from maximizing social welfare W = CS + PS - Env, as defined in section 2.

pollution (higher d) or more firms (higher n) yields more output and emissions, increasing ER^{NM} , and worsening output inefficiencies.

3.2 Third stage - Merger

After the merger, n - k + 1 firms operate in the industry: (i) the merged entity, with k "insider" firms; and (ii) the unmerged private firms, (n - 1) - k, "outsiders" along with the public firm.

Lemma 3. In the third stage, if the merger is approved, the equilibrium output of the merged entity and every private outsider is

$$q_I^M = q_O^M = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d + (1-\alpha)(n-k+1)},\tag{4}$$

and that of the public firm is $q^M = \frac{(1-c)[1-2\alpha d(n-k)]}{1+2\alpha d+(1-\alpha)(n-k+1)}$. All output levels are positive, decreasing in c and n, and increasing in k; but the output of the public (private) firm is decreasing (increasing, respectively) in α and d.

If k = 1, the results in Lemma 3 coincide with those under no mergers (Lemma 1). When the number of merging firms, k, increases, every firm produces more units, but the public firm responds more intensively to increases in k than every private firm does. As shown in equation (3), the public firm seeks to partially compensate for the output reduction of the merger when $\alpha > 0$, by increasing its output more intensively than outsiders do. However, when all firms are private, $\alpha = 0$, every firm produces $\frac{1-c}{n-k+2}$ after the merger, which is unaffected by pollution considerations.

Overall, when the merger accounts for a larger market share (higher k), the public firm anticipates a more concentrated market, and the distribution of output in equilibrium shifts from private to the public firm. In contrast, when pollution is more severe (higher d) or the public firm is less privatized (higher α), this company seeks to reduce emissions to a larger extent, shifting the distribution of output towards private firms. The public firm's output decrease dominates, as under no mergers, which we confirm in the next lemma.

Lemma 4. Aggregate output under the merger, $Q^M = \frac{(1-c)[1+(1-\alpha)(n-k)]}{1+2\alpha d+(1-\alpha)(n-k+1)}$, is unambiguously decreasing in c, α , d, k, but increasing in n.

As expected, Q^M coincides with Q^{NM} when k = 1; otherwise, it decreases in k, implying that $Q^M < Q^{NM}$ for all $k \ge 2$. In addition, Q^M decreases in the severity of pollution, d, and the weight on welfare, α . Evaluating Q^M at extreme values of α , we find that, when all firms are private $(\alpha = 0)$, aggregate output coincides with that in standard merger models, $Q^M = \frac{(1-c)(n-k+1)}{n-k+2}$, thus being unaffected by pollution. When the weight on welfare is positive, however, aggregate output decreases.

The next corollary evaluates the efficiency ratio under the merger, $ER^M = Q^M/Q^{SO}$.

Corollary 2. The efficiency ratio under the merger is $ER^M = \frac{(1+2d)[1+(1-\alpha)(n-k)]}{1+2\alpha d+(1-\alpha)(n-k+1)}$, which is increasing in d and n, but decreasing in α and k, and satisfies $ER^{NM} > ER^M > 1$.

Aggregate output is still socially excessive, $ER^M > 1$, with this inefficiency worsening as pollution becomes more severe (higher d) or more firms compete in the industry (higher n). In contrast, the inefficiency is ameliorated, lowering ER^M , when more firms merge (higher k), and when the public firm is more nationalized (higher α) since this firm lowers its output, helping Q^M approach the social optimum.

In addition, the efficiency ratio ER^M coincides with ER^{NM} when k = 1, but otherwise ER^M decreases in the number of merged firms, k. Therefore, ER^M is closer to one than ER^{NM} for all $k \geq 2$, implying that the merger is "efficiency enhancing."⁹ This result holds when all firms are private, $\alpha = 0$, since the merger helps reduce polluting emissions; and it is strengthened when a public firm exists, $\alpha > 0$, because this firm helps further reduce aggregate output and pollution, bringing ER^M closer to one.

3.3 Second stage

The CA anticipates the aggregate equilibrium outputs Q^{NM} and Q^M , and approves a merger according to different criteria. First, if the CA only considers consumer surplus (CS-criterion), it approves the merger if $Q^M > Q^{NM}$. However, as shown in Lemma 4, aggregate output decreases in the number of merging firms, k, entailing that $Q^M < Q^{NM}$, and the merger is blocked. Second, if the CA uses welfare (W-criterion), it approves the merger if it is welfare improving, where welfare is defined as $W = CS + PS - Env_{CA}$, and $Env_{CA} = \beta \times Env$ represents environmental damages and $\beta \in [0, 1]$. This welfare function allows for different settings: the CA can ignore pollution when evaluating mergers, $\beta = 0$; it can consider pollution but not fully, $0 < \beta < 1$; or consider the same welfare function as the manager of the public firm, $\beta = 1$.¹⁰

Lemma 5. When the CA considers the CS-criterion, it blocks the merger under all parameter conditions. However, when the CA considers the W-criterion, it blocks the merger when $\beta < \underline{\beta}$ under all parameters; otherwise, it approves the merger if and only if $k < \min\{n-1, \overline{k}(\beta)\}$, where cutoff $\overline{k}(\beta)$ is increasing in β and satisfies $n-1 > \overline{k}(\beta) \ge 2$ for all $\underline{\beta} \le \beta < \overline{\beta}$. Cutoffs $\overline{k}(\beta), \underline{\beta},$ and $\overline{\beta}$ are presented, for compactness, in the appendix.

Our results give rise to three regions, depicted in figure 1: (i) when the CA assigns a relatively low weight to pollution, $\beta < \underline{\beta}$, it blocks the merger for all parameter conditions (see Region A); (ii) when the CA assigns a moderate weight to pollution, $\beta \leq \beta < \overline{\beta}$, it blocks the merger if it

⁹Alternatively, this implies that the merger is welfare enhancing, as defined for the public firm. As the next section discusses, the CA can exhibit a different welfare function, entailing that mergers are not necessarily welfare improving.

 $^{^{10}}$ The government may assign more weight on pollution than the CA because the former may seek re-election while the term of CA officials typically exceeds the electoral cycle; see Prat et al. (2022). For completeness, we also explore the opposite case, where governments, as captured by the public firm's objective function, ignore pollution in one of our extensions.

accounts for a large market share, $k > \overline{k}(\beta)$, but approves it otherwise (shaded area in Region *B*); and (iii) when the CA assigns a high weight on environmental damages, $\beta \ge \overline{\beta}$, it approves the merger for all parameters (Region *C*).¹¹

Region A includes $\beta = 0$ as a special case, where the CA ignores pollution in its welfare function, and mergers are blocked regardless of their market share. This is a well-known result, where the loss in consumer surplus dominates the increase in aggregate profits and, as a consequence, total welfare decreases after the merger. When the CA considers environmental damages, in Region B, it weighs two market failures: the market concentration from approving the merger (negative welfare effect) against the reduced pollution that the merger brings (positive effect). When the number of merging firms is relatively low, $k \leq \overline{k}(\beta)$, the positive effect dominates, and the CA responds approving the merger, as illustrated in the shaded area of this region. Intuitively, the CA uses mergers as a tool to curb pollution. Finally, in Region C the CA assigns a high weight on environmental damages, making the merger welfare improving regardless of its market share, i.e., for all values of k.



Fig. 1. Merger approvals with the welfare criterion.

Figure 2a shows how our above results are affected by changes in α . When the public firm is more nationalized (higher α), cutoff $\overline{k}(\beta)$ shifts downwards, shrinking the shaded region in which mergers are approved. Intuitively, the CA anticipates that the public firm will be addressing market failures to a larger extent (from underproduction in oligopoly and from pollution), making the merger less necessary. Figure 2b, in contrast, shows that cutoff $\overline{k}(\beta)$ shifts upwards when pollution is more severe (higher d), expanding the shaded region in which mergers are approved. Since the environmental damage function Env_{CA} increases in β or d, the merger becomes more

¹¹Figure 1 considers $\alpha = 0.12$, d = 1/2, and n = 6, since α must satisfy $\alpha < \overline{\alpha}$ by definition, where $\overline{\alpha} = 1/5$ in this setting. In addition, cutoffs $\underline{\beta}$ and $\overline{\beta}$ are defined in the proof of Lemma 5 which, in the context of figure 1, become $\beta = 0.20$ and $\overline{\beta} = 0.34$.

necessary, since it can help decrease aggregate pollution.



Fig. 2a. Merger approvals - Changes in α .

Fig. 2b. Merger approvals - Changes in d.

3.4 First stage

In the first stage, the merging entity anticipates the CA's decision, approving the merger following Lemma 5, and submits a request if its profits satisfy $\pi_I^M \ge k \pi_i^{NM}$, as we next show.

Proposition 1. The merging entity finds the merger profitable when $n < \overline{n}$ and its market share, $\frac{k}{n}$, satisfies

$$\frac{k}{n} \ge \frac{\hat{k}}{n} \equiv \frac{3 + 2n + \alpha[4(d-1) - \alpha(4d-1) - 2n(2-\alpha)] - \theta}{2n(1-\alpha)^2}$$
(5)

where $\theta \equiv [(1-\alpha)^3 [5+\alpha(8d-1)+4n(1-\alpha)]]^{1/2}$ and $\overline{n} \equiv \frac{5+2\alpha[4d-3+\alpha[1-2d(1-d)]]}{(1-\alpha)^2}$. In contrast, when $n \geq \overline{n}$, the merger is unprofitable for all values of k. Cutoff $\frac{\widehat{k}}{n}$ is increasing in α and d, and $\overline{k}(\beta) > \widehat{k}$ if and only if $\beta > \widehat{\beta}$, where $\widehat{\beta} \equiv \frac{[1+\alpha(2d-1)][2-2\alpha(2-d+\alpha(d-1))+\theta]}{2d(1-\alpha)[1+4n+\alpha(2-\alpha(3+4d^2-4n)-8n)]}$.

When all firms are private, $\alpha = 0$, cutoff $\frac{\hat{k}}{n}$ simplifies to $\frac{\hat{k}}{n} = \frac{3+2n-[4n+5]^{1/2}}{2}$, as in Salant et al. (1983), indicating that the merger must account for more than 80 percent of the market share to be profitable. When a public firm exists, our results show that cutoff $\frac{\hat{k}}{n}$ increases, implying that the merger requires an even larger market share to be profitable, which may not arise under any parameter values (i.e., $n \geq \overline{n}$).

A similar argument applies to more severe pollution. As shown in section 3.1, an increase in α or d makes the public firm's best response function steeper, entailing that the output reduction of the merging entity is now responded with a larger increase in output by the public firm. This emphasizes the negative effect of the merger on profits, ultimately making the merger less attractive when a public firm is present than otherwise.

Figure 3 superimposes cutoff \hat{k} on figure 1. This cutoff is not a function of parameter β and, as shown in Proposition 1, \hat{k} lies above $\bar{k}(\beta)$ for all $\beta \leq \hat{\beta}$, but below otherwise; giving rise to four regions, each yielding different equilibrium behavior. First, in region (PM, B), the merger is profitable for the merging entity, denoted as PM, since $k > \hat{k}$, but merger requests are blocked by the CA, represented by B, since $k > \bar{k}(\beta)$. In this context, the merging entity anticipates the CA's rejection, and does not submit a merger request in equilibrium. In region (PM, A) the merger is still profitable, PM, but approved, A, because $k < \bar{k}(\beta)$. Anticipating an approval, the merging entity submits a request. This is the only region where merger requests should be observed in equilibrium. In region (UPM, B), the merger now becomes unprofitable, UPM, since $k < \hat{k}$, and blocked because it reduces welfare, B, given that $k > \bar{k}(\beta)$, implying that no merger filings should be observed.

A similar argument applies in region (UPM, A) where the merger is approved by the CA, A, but it is unprofitable for the merging entity since $k < \hat{k}$. In this context, it would be welfare improving to subsidize firms to merge because, intuitively, pollution receives a high weight on the CA's decision and the merger does not account for a large market share.



Fig. 3. Equilibrium results.

Figure 3 considers that $n < \overline{n}$, meaning that, graphically, cutoff \hat{k} lies below the upper bound of the vertical axis, n - 1, thus giving rise to the four regions identified above. If, instead, $n \ge \overline{n}$, cutoff \hat{k} lies above n - 1, implying that only regions (UPM, B) and (UPM, A) can be sustained in equilibrium, with mergers being unprofitable for all (k, β) -pairs, and thus no firms submitting requests to the CA. Intuitively, when the merger accounts for a small market share, $n \ge \overline{n}$, it becomes unprofitable regardless of pollution considerations $(d \text{ and } \beta)$ and independently on whether a public firm exists or not (α) .

In contrast, when the merger accounts for a large market share, $n < \overline{n}$, all four regions can be supported in equilibrium, and are affected by these parameters. In particular, a more privatized public firm (lower α) produces an upward shift in cutoff $\overline{k}(\beta)$, as shown in Lemma 5, but a downward shift in \hat{k} , as identified in Proposition 1. This expands region (PM, A), shrinks (UPM, B), but yields an ambiguous effect in regions (PM, B) and (UPM, A). Table I evaluates the areas of the four regions at a baseline setting $\alpha = 0$, d = 1/2 and n = 6, and then allows for changes in each parameter at a time. The table shows that the downward shift in cutoff \hat{k} dominates, producing an overall increase in region (PM, B) while shrinking (UPM, A). The expansion of region (PM, A), the only context where a merger is filed, highlights a novel role of privatizations, namely, providing firms with stronger incentives to submit merger requests that are not only profitable but welfare improving. In other words, privatizations facilitate welfare-enhancing mergers, which become more likely to arise in equilibrium.

However, more severe pollution (higher d) produces an upward shift in both $\overline{k}(\beta)$ and \hat{k} , shrinking both (PM, B) and (UPM, A), but giving rise to ambiguous effects in regions (PM, A) and (UPM, B). As shown in Table I, the upward shift in $\overline{k}(\beta)$ dominates, yielding an overall increase in region (PM, A) while shrinking (UPM, B).

			Regions										
		(PM, B)	(UPM, B)	(PM, A)	(UPM, A)								
Baseline		0.058	1.001	3.318	3.807								
Higher α	$\alpha = 0.05$	0.034	1.067	3.227	3.826								
	$\alpha = 0.08$	0.017	1.111	3.166	3.838								
	$\alpha = 0.10$	0.004	1.142	3.123	3.846								
Higher d	d = 0.6	0.049	0.834	3.586	3.974								
	d = 0.7	0.042	0.715	3.780	4.093								
	d = 0.8	0.037	0.625	3.927	4.183								

Table I. Areas of regions.

3.5 Welfare comparisons

No public firm. Table II illustrates our results in previous sections. For presentation purposes, we first consider a setting where all firms are private, $\alpha = 0$. The top row (baseline scenario) assumes $c = \beta = d = 1/2$, and n = 6, and subsequent rows change one parameter at a time.¹² The first three columns evaluate aggregate output, environmental damages, and welfare under no merger, NM; and the second set of columns measure these outcomes if a merger occurs, M. The next three columns examine the changes in these outcomes due to the merger: (i) the increase in aggregate output, $\Delta Q \equiv Q^M - Q^{NM}$; (ii) the decrease in environmental damage, $\Delta Env \equiv Env^{NM} - Env^M$; and (iii) the increase in overall welfare, $\Delta W \equiv W^M - W^{NM}$. The last three columns report the cutoff for which the merger is: (i) profitable, $k > \hat{k}$, as identified in Proposition 1; (ii) welfare

¹²Cutoff \hat{k} is, in this context, 4.807, indicating that only sufficiently large mergers are profitable, in line with the 80-percent rule. Otherwise, the equilibrium in Proposition 1 cannot be sustained. Hence, we consider k = 5 firms in the baseline scenario.

	No merger			Merger				Changes		Cutoffs		
	Q^{NM}	Env^{NM}	W^{NM}	Q^M	Env^M	W^M	ΔQ	ΔEnv	ΔW	\widehat{k}	$\overline{k}(\beta)$	\widehat{eta}
Baseline	0.429	0.046	0.077	0.333	0.028	0.083	-0.095	0.018	0.007	4.807	6.091	0.295
d = 0.6	0.429	0.055	0.067	0.333	0.033	0.078	-0.095	0.022	0.010	4.807	6.353	0.246
d = 0.7	0.429	0.064	0.058	0.333	0.039	0.072	-0.095	0.025	0.014	4.807	6.531	0.211
d = 0.8	0.429	0.073	0.049	0.333	0.044	0.067	-0.095	0.029	0.018	4.807	6.659	0.185
$\beta = 0.7$	0.429	0.064	0.058	0.333	0.039	0.072	-0.095	0.025	0.014	4.807	6.531	0.295
$\beta = 0.8$	0.429	0.073	0.049	0.333	0.044	0.067	-0.095	0.029	0.018	4.807	6.659	0.295
$\beta = 1$	0.429	0.092	0.031	0.333	0.056	0.056	-0.095	0.036	0.025	4.807	6.833	0.295
c = 0.4	0.514	0.066	0.110	0.400	0.040	0.012	-0.114	0.026	0.010	4.807	6.091	0.295
c = 0.3	0.600	0.090	0.150	0.467	0.054	0.163	-0.133	0.036	0.013	4.807	6.091	0.295
c = 0.2	0.686	0.118	0.196	0.533	0.071	0.213	-0.152	0.046	0.017	4.807	6.091	0.295

enhancing, $k < \overline{k}(\beta)$, as found in Lemma 5; and (iii) can be sustained in equilibrium, $\beta > \hat{\beta}$, as defined in Proposition 1.

Table II. Equilibrium outcomes when $\alpha = 0$.

While the merger entails the same change in output (ΔQ is unaffected by parameters d and β), it gives rise to larger reductions in environmental damage when either of these parameters increases, ΔEnv , ultimately enhancing its welfare gain ΔW .¹³ Therefore, when pollution becomes more severe or the CA guidelines assign a larger weight on environmental damages, profitable mergers are more likely to be approved.

Alternatively, ΔW measures the welfare gain of changing the CA's guidelines, from one in which it only considers consumer surplus or welfare, but ignoring pollution, to another in which the CA takes the environmental effects of mergers into account. In the former guidelines, no merger is approved under all parameter values (see column ΔQ), as shown in Lemma 5, giving rise to welfare W^{NM} . In the latter guideline, however, mergers can be approved, yielding welfare W^M , and a welfare gain ΔW .¹⁴

Public firm is present. Table III examines how our results are affected by the presence of a public firm. For comparison purposes, we consider the same parameters as in Table II and $\alpha = 0.1$, which is compatible with initial condition $\alpha < \overline{\alpha} = 1/5$. Cutoff \hat{k} lies below 5 firms while $\overline{k}(\beta) > 6$, for most parameter values, supporting the equilibrium where mergers are profitable and approved. However, when pollution becomes more severe (higher d), cutoff $\hat{k} > 5$, implying that no merger is profitable (see region (UPM, A) in figure 3). Intuitively, the presence of the public firm in a

¹³This is confirmed by cutoff $\overline{k}(\beta)$ being increasing in d and β , making the CA more likely to approve mergers according to the W-criterion.

 $^{^{14}}$ When firms become more efficient (lower c), the merger entails a larger change in aggregate output, producing more substantial environmental benefits, and ultimately yielding larger welfare gains from the merger.

context of severe pollution leads this firm to respond by significantly increasing its output after the merger, thus ameliorating its profitability.

Overall, the welfare gain from changing the CA's guidelines are the largest when the public firm is mostly privatized (low α) and firms are efficient (low c). Otherwise, the public firm corrects a larger portion of the environmental externality, implying that the output reduction from approving the merger becomes less necessary. In contrast, when firms are more efficient, this output reduction is more critical to curb pollution, making the merger welfare improving under larger conditions.

	No merger			Merger			Changes			Cutoffs		
	Q^{NM}	Env^{NM}	W^{NM}	Q^M	Env^M	W^M	ΔQ	ΔEnv	ΔW	\widehat{k}	$\overline{k}(\beta)$	\widehat{eta}
Baseline	0.423	0.045	0.077	0.328	0.027	0.083	-0.095	0.018	0.006	4.989	6.056	0.331
$\alpha = 0.07$	0.425	0.045	0.077	0.329	0.027	0.083	-0.095	0.018	0.006	4.930	6.068	0.319
$\alpha = 0.04$	0.426	0.046	0.077	0.331	0.027	0.083	-0.095	0.018	0.007	4.875	6.079	0.309
$\alpha = 0$	0.429	0.046	0.077	0.333	0.028	0.083	-0.095	0.018	0.007	4.807	6.091	0.295
d = 0.6	0.422	0.053	0.069	0.325	0.032	0.078	-0.096	0.022	0.009	5.006	6.335	0.283
d = 0.7	0.420	0.062	0.060	0.323	0.037	0.073	-0.097	0.025	0.013	5.025	6.528	0.248
d = 0.8	0.419	0.070	0.051	0.321	0.041	0.068	-0.098	0.029	0.016	5.043	6.669	0.222
$\beta = 0.7$	0.423	0.063	0.059	0.328	0.038	0.073	-0.095	0.025	0.013	4.989	6.563	0.331
$\beta = 0.8$	0.423	0.072	0.050	0.328	0.043	0.067	-0.095	0.029	0.017	4.989	6.711	0.331
$\beta = 1$	0.423	0.089	0.033	0.328	0.054	0.056	-0.095	0.036	0.024	4.989	6.909	0.331
c = 0.4	0.508	0.064	0.111	0.393	0.034	0.120	-0.115	0.026	0.009	4.989	6.056	0.331
c = 0.3	0.592	0.088	0.151	0.459	0.053	0.163	-0.134	0.035	0.012	4.989	6.056	0.331
c = 0.2	0.677	0.115	0.198	0.524	0.069	0.213	-0.153	0.046	0.015	4.989	6.056	0.331

Table III. Equilibrium outcomes when $\alpha > 0$.

4 Extensions

4.1 The public firm ignores pollution

In this section, we examine how our results are affected when the manager of the public firm ignores pollution. In particular, she still maximizes a combination of social welfare and profits

$$V = (1 - \alpha)\pi + \alpha W$$

but social welfare is now given by W = CS + PS, thus ignoring environmental damages. This can be due to legal or administrative mandates that prohibit public firms from considering pollution when making their output decisions, or just to personal preferences of the manager running this company at the time. In this context, when the CA also ignores pollution, $\beta = 0$, the objective functions of CA and the public firm become more aligned, but when the CA considers pollution, $\beta > 0$, their objective functions are misaligned.

In this context, the public firm's best response function becomes

$$q(q_i) = \frac{1-c}{2-\alpha} - \frac{1}{2-\alpha} \sum_{i=1}^{n-1} q_i$$
(6)

which is flatter than when this firm considers environmental damages (equation 3). Intuitively, a given decrease in the private firms' aggregate output after the merger is now responded less intensively by the public firm when it ignores pollution, increasing its output less significantly.

We solve the sequential-move game by backward induction, presenting the technical details in Appendix 1. In the third stage, aggregate output is $Q^{NM} = \frac{(1-c)[n(1-\alpha)+\alpha]}{1+n(1-\alpha)}$ without the merger and decreases to $Q^M = \frac{(1-c)[1+(1-\alpha)(n-k)]}{1+(1-\alpha)(n-k+1)}$ with the merger, both being increasing in α . In this context, when the public firm becomes more nationalized (higher α), it seeks to produce more output, since it ignores pollution, shifting output from private to public firms, but producing an overall increase in aggregate output.

In the second stage, the CA exhibits a similar decision rule as that in Lemma 5, blocking mergers when using the CS-criterion. Similarly, when the CA uses the W-criterion, it blocks mergers when it assigns a low weight to pollution, but approves them otherwise if, in addition, their market share is relatively low, i.e., $k < \overline{k}^{IP}(\beta)$, where superscript *IP* denotes that the manager of the public firm ignores pollution,

$$\bar{k}^{IP}(\beta) \equiv \frac{(1-\alpha)[3-\alpha+2n(1-\alpha)]-2d\beta[1-\alpha(\alpha-2)+4n(1-\alpha)+2n^2(1-\alpha)^2]}{(1-\alpha)[1-\alpha-2d\beta(1+\alpha+2n(1-\alpha))]}$$
(7)

and, as in Lemma 5, cutoff $\overline{k}^{IP}(\beta)$ is unambiguously increasing in β .

In the first stage, the merging entity anticipates the CA's merger approval decision, and submits a request if it accounts for a sufficiently large market share, that is,

$$\frac{k}{n} \ge \frac{\hat{k}^{IP}}{n} \equiv \frac{3 - \alpha + 2n(1 - \alpha) - (1 - \alpha)^{1/2}\gamma}{2n(1 - \alpha)},\tag{8}$$

where, for compactness, $\gamma \equiv [5 - \alpha + 4n(1 - \alpha)]^{1/2} > 0$. As in Proposition 1, cutoff $\frac{\hat{k}^{IP}}{n}$ simplifies to $\frac{\hat{k}}{n} = \frac{3+2n-[5+4n]^{1/2}}{2n}$ when $\alpha = 0$, as in Salant et al. (1983); but when $\alpha > 0$ this cutoff shifts upwards, indicating that the merger must represent a larger market share to be profitable. As in the main model, the public firm is "bad news" for the merging entity, since its presence shifts output from private to the public firm after the merger, making it less profitable. However, as shown in equation (6), this output shift is smaller when the public firm ignores pollution, implying that mergers become more profitable than when this firm considered environmental damages.

Comparing cutoffs $\overline{k}^{IP}(\beta)$ and \widehat{k}^{IP} , we find that $\overline{k}^{IP}(\beta) > \widehat{k}^{IP}$ for all $\beta > \widehat{\beta}^{IP}$, where $\widehat{\beta}^{IP} \equiv$

 $\frac{1}{2d\left(\frac{\gamma}{(1-\alpha)^{1/2}}-2\right)}$. Recall that when $\overline{k}^{IP}(\beta) > \widehat{k}^{IP}$, an equilibrium can be sustained where firms file for a merger and the CA responds approving it, but otherwise no mergers are requested. In addition, cutoff $\widehat{\beta}^{IP}$ is unambiguously decreasing in α , d, and n. Hence, the presence of a public firm makes the merger more likely to be unprofitable. A similar argument applies when pollution is more severe and the industry is more competitive (higher d and n, respectively).

Finally, we compare cutoff $\hat{\beta}^{IP}$ against its counterpart, $\hat{\beta}$ identified from Proposition 1, finding the "approval differential" that can be attributed to the public firm ignoring pollution:

$$AD \equiv \widehat{\beta} - \widehat{\beta}^{IP}.$$

When AD > 0, the CA is more likely to approve mergers when the manager of the public firm ignores pollution than otherwise since $\hat{\beta} > \hat{\beta}^{IP}$. Graphically, the region above cutoff $\hat{\beta}^{IP}$ is larger than that above $\hat{\beta}$, thus triggering more merger approvals. In contrast, when AD < 0, the CA is less likely to approve mergers. Figure 4a depicts AD as a function of n, considering d = 1/2 and allowing for different values of α .

When n is low, AD is positive, reflecting that the CA approves more mergers when the public firm ignores pollution. Intuitively, the CA seeks to compensate the large output that the public firm produces by approving more mergers, as they help reduce output and pollution. When nincreases, however, this approval differential shrinks, making the CA's decision more similar when the public firm considers or ignores pollution.



Fig. 4a. AD, changes in α .

Fig. 4b. AD, changes in d.

When all firms are private, $\alpha = 0$, AD collapses to zero, implying that there is no approval differential, which holds for all parameters. When α increases, AD shifts upwards. This occurs because, in this context, a more nationalized public firm produces a larger output, which the CA seeks to compensate by approving mergers under larger conditions. A similar argument applies when pollution becomes more severe (higher d), as depicted in figure 4b. The large output levels of the public firm are now more damaging for the environment, leading the CA to more likely approve

mergers, i.e., the AD curve shifts upwards.

4.2 Allowing for cost-reducing mergers

This extension analyzes how our results are affected when the merger lowers the merging entity's production cost, from c to c - x, where $x \ge 0$ denotes the cost-reducing effect of the merger, or "synergies," as in Williamson (1968) and Perry and Porter (1985). When x = 0, our equilibrium results coincide with those in section 3; but when x > 0, the merging entity benefits from a cost advantage relative to outsiders and the public firm, thus providing firms with more incentives to submit a merger request.

Appendix 2 presents technical details, while here we highlight how equilibrium results are affected by cost-reducing parameter x. When a merger does not ensue, equilibrium output coincides with that in Lemma 1, since no firm benefits from cost reductions. When a merger occurs, however, equilibrium output levels are

$$q_I^M = \frac{(1-c)[1+\alpha(2d-1)] + [(1-\alpha)(n-k) + 2\alpha d + 1]x}{1+2\alpha d + (1-\alpha)(n-k+1)},$$

$$q_O^M = \frac{(1-c)[1+\alpha(2d-1)] - x(1-\alpha)}{1+2\alpha d + (1-\alpha)(n-k+1)}, \text{ and } q^M = \frac{(1-c)[1-2\alpha d(n-k)] - x(1+2\alpha d)}{1+2\alpha d + (1-\alpha)(n-k+1)}$$

where the merging entity's output, q_I^M , is unambiguously positive and increasing in the merger's cost-reducing effect, x; whereas that of every outsider and the public firm's, q_O^M and q^M , are decreasing in their cost disadvantage, x, and remain positive as long as the merger does not provide an excessive cost-reducing benefit, $x < x_P \equiv \frac{(1-c)[1-2\alpha d(n-k)]}{1+2\alpha d}$. For simplicity, we hereafter assume that $x < x_P$, as otherwise the public firm, the outsiders, or both, would stay inactive after the merger.¹⁵

In this context, aggregate output after the merger becomes $Q^M = \frac{(1-c)[1+(1-\alpha)(n-k)]+(1-\alpha)x}{1+(1-\alpha)(n-k+1)+2\alpha d}$, which coincides with that in Lemma 4 when cost-reducing effects are absent, x = 0, but otherwise unambiguously increases in x. Aggregate output, then, increases after the merger, $Q^M > Q^{NM}$, if cost-reducing effects are sufficiently strong, $x > x_{CA} \equiv \frac{(1-c)(k-1)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)}$. Cutoff x_{CA} satisfies $x_{CA} < x_P$, implying that output-enhancing mergers are feasible. When all firms are private, $\alpha = 0$, this cutoff collapses to $x_{CA} = \frac{(1-c)(k-1)}{n+1}$, but otherwise increases when the public firm is more nationalized, α , and pollution is more severe, d. Intuitively, the public firm produces less output when its welfare consideration increases, either because it is less privatized or environmental damage is more significant. Therefore, the CA requires a more significant cost-reducing effect to approve the merger.¹⁶ In this setting, the merger is profitable if and only if $x > x_{\pi}$ (where cutoff x_{π} is

¹⁵In addition, cutoff x_P simplifies to $x_P = 1 - c$ when all firms are private, $\alpha = 0$, and is decreasing in α and d, implying that condition $x < x_P$ becomes more demanding when the public firm is more nationalized (higher α) or pollution is more severe (higher d). In both cases, the public firm seeks to reduce its output more significantly to curb pollution, thus becoming inactive under larger parameter conditions.

¹⁶This result can also be shown by considering the differential $x_P - x_{CA}$, where the equilibrium with merger approvals can be sustained. This differential is decreasing in α since cutoff x_{CA} (x_P) increases (decreases) in α , making the equilibrium less likely to arise. A similar argument applies to increases in d.

defined, for compactness, in Appendix 2), with cutoffs satisfying $x_{\pi} < x_{CA} < x_P$ for all admissible parameters. Hence, all mergers that are approved when the CA uses the CS-criterion, $x > x_{CA}$, are also profitable, $x > x_{\pi}$.

When the CA uses, instead, the W-criterion, it approves a merger request if and only if $W^M > W^{NM}$, where W^M (W^{NM}) denotes the welfare with (without) the merger. These welfare levels yield large, highly nonlinear expressions, and we provide numerical simulations in Table IV considering, for consistency, the same parameter values as in the baseline of Table III.¹⁷

			No merger		Merger		Changes			
		Q^{NM} Env^{NM}		W^{NM}	Q^M	Env^M	W^M	ΔQ	ΔEnv	ΔW
Baseline	x = 0	0.423	0.045	0.077	0.328	0.027	0.083	-0.095	0.018	0.006
Higher x	x = 0.1	0.423	0.045	0.077	0.359	0.032	0.107	-0.064	0.013	0.030
	x = 0.2	0.423	0.045	0.077	0.389	0.038	0.143	-0.033	0.007	0.066
	x = 0.3	0.423	0.045	0.077	0.421	0.044	0.191	-0.002	0.001	0.114
	x = 0.4	0.423	0.045	0.077	0.452	0.051	0.252	0.029	-0.006	0.175

Table IV. Equilibrium outcomes with cost-reducing effects.

When cost-reducing effects are absent, x = 0, our baseline yields the same results as in the baseline of Table III (see top row). However, when cost-reducing effects are present, x > 0, aggregate output Q^M increases while Q^{NM} remains unaffected, allowing for the merger to increase aggregate output, $\Delta Q > 0$, if its cost-reducing effect is sufficiently large, $x > x_{CA} = 0.308$. In this setting, an equilibrium arises where a merger is requested and approved. When the CA considers total welfare, mergers with nil or low cost-reducing effects are approved, since they improve environmental conditions, increasing welfare ($\Delta W > 0$); but also mergers with strong cost-reducing effects, x = 0.4. In this case, while environmental damage increases, $\Delta Env < 0$, the benefit for consumers dominates, yielding an increase in overall welfare, $\Delta W = 0.175$.

4.3 Allowing for non-linear costs

In this extension, we allow every firm's cost function to be $TC = cq_i + \frac{h}{2}(q_i)^2$, with associated marginal cost $MC_i = c + hq_i$, where $c \in [0, 1]$ and $h \ge 0$. When h = 0, marginal costs are constant in output, as in the main model; but otherwise, marginal costs are increasing, giving rise to diseconomies of scale, potentially making mergers more profitable.

For compactness, Appendix 3 identifies equilibrium output and profits, while here we focus on how convexities (as captured by parameter h) affect our equilibrium results in previous sections. In

¹⁷In particular, $c = d = \beta = 1/2$, $\alpha = 1/10$, n = 6, and k = 5. In this context, the above cutoffs become $x_{\pi} = -0.001$, $x_{CA} = 0.308$, and $x_P = 0.409$, implying that mergers are profitable for all values of x. In addition, these results entail that, when the CA uses the CS-criterion, an equilibrium where mergers are profitable and approved arises for all $x \in [0.308, 0.409]$. Similar results apply under different parameter combinations, as shown in Table A.1 in Appendix 2.

the third stage, we find that, as production costs are more convex (higher h), output competition is softened (i.e., flatter best response functions). In addition, when the merger does not ensue, the equilibrium output of every firm i under no merger is

$$\begin{split} q^{NM} &= \frac{(1-c)[1+h-2\alpha d(n-1)]}{1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)}, \\ q^{NM}_i &= \frac{(1-c)[1+h+\alpha(2d-1)]}{1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)}, \end{split}$$

and aggregate output is $Q^{NM} = \frac{(1-c)[n(1-\alpha+h)+\alpha]}{1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)}$, all being unambiguously positive. When costs are linear, h = 0, the above expressions simplify to those in Lemma 1 and 2.

When the merger occurs, equilibrium output become

$$q^{M} = \frac{(1-c)[1+h-2\alpha d(n-k)]}{(1+h)(n-k+h+2) - \alpha[n-k+h+1-2d(1+h)]},$$
$$q_{O}^{M} = q_{I}^{M} = \frac{(1-c)[1+h+\alpha(2d-1)]}{(1+h)(n-k+h+2) - \alpha[n-k+h+1-2d(1+h)]}$$

and aggregate output is $Q^M = \frac{(1-c)[1+h+(1+h-\alpha)(n-k)]}{(1+h)(n-k+h+2)-\alpha[n-k+h+1-2d(1+h)]}$, which are also unambiguously positive. When costs are linear, h = 0, the above equilibrium output coincide with those in Lemma 3 and 4.

In the second stage, if the CA uses the CS-criterion, it blocks the merger since $Q^M < Q^{NM}$ for all $k \ge 2$. If, instead, the CA uses the W-criterion, it approves the merger if and only if $W^M > W^{NM}$, which holds if $k < \overline{k}(\beta, h)$. Cutoff $\overline{k}(\beta, h)$ is highly nonlinear and figure 5 depicts it at different values of h, considering the same parameter values as in figure 1 for comparison purposes. When h = 0, this cutoff coincides with that in figure 1, but otherwise it shifts downwards. This suggests that, as costs become more convex, the CA anticipates less aggregate output and emissions, requiring a higher weight on pollution (higher β) to approve the merger. In other words, mergers become a

less necessary tool to reduce total emissions when production costs are convex than otherwise.



Fig 5. Cutoff $\overline{k}(\beta, h)$ evaluated at different values of h.

In the first stage, the merging entity finds the merger profitable if and only if $\frac{k}{n} > \frac{\hat{k}(h)}{n}$, where cutoff $\frac{\hat{k}(h)}{n}$ is presented, for compactness, in Appendix 3. When all firms are private, $\alpha = 0$, cutoff $\frac{\hat{k}(h)}{n}$ simplifies to $\frac{\hat{k}(h)}{n} = \frac{3+2(n+h)-[5+4(n+h)]^{1/2}}{2n}$, which unambiguously increases in h, thus implying that cost convexities provide firms with less incentives to merge.¹⁸ Intuitively, cost convexities reduce production costs for the merging entity, giving them more incentives to merge; but also attenuate the positive effect of the merger on profits (the output reduction becomes smaller, leading to a smaller price increase). Overall, the latter effect dominates, making mergers less likely to be profitable.

When $\alpha > 0$, the comparative statics of cutoff $\frac{\hat{k}(h)}{n}$ are intractable, but numerical simulations show that it is still increasing in h, thus exhibiting the same comparative statics as when all firms are private. Therefore, the only region where mergers are profitable and welfare improving in equilibrium, (PM, A), unambiguously shrinks when production costs become more convex; while region (UPM, B) unambiguously expands.¹⁹

5 Discussion

Presence of public firm. When all firms are private, a change in merger guidelines, requiring the CA to consider environmental effects, makes mergers more likely to be approved; a result that is emphasized when pollution becomes more severe and firms are more efficient. When a public

¹⁸In particular, $\frac{\partial \left(\frac{\hat{k}(h)}{n}\right)}{\partial h} = \frac{[5+4(n+h)]^{1/2}-1}{n[5+4(n+h)]^{1/2}} > 0$ since $n \ge 3$. When h = 0, this cutoff simplifies to that in Salant et al. (1983).

¹⁹We also find that, numerically, region (PM, B) expands whereas (UPM, A) shrinks, entailing that welfare improving mergers that firms find unprofitable are less likely to arise when production costs become more convex.

firm is present, it seeks to curb pollution, changing output competition. In particular, this firm responds more significantly to a given decrease in its rivals' output than a private firm would, thus ameliorating the profitability of a merger. This strengthens Salant et al.'s (1983) 80-percent rule, making mergers more likely to be unprofitable when a public firm exists than otherwise. Then, fewer merger requests should be filed in polluting industries where a public firm is present, especially when pollution is severe and costs are low.

Changing CA guidelines? The CA is less willing to approve mergers when a public firm is present than otherwise, since this firm already addresses part of the environmental externality. In this context, the reduction in pollution that the merger approval entails is ameliorated by the public firm, decreasing the potential welfare gain of the merger. This result entails that changing the CA's guidelines, considering pollution in its merger assessments, may be important in industries where all firms are private. In this context, the CA would use merger approvals as a new, although imperfect, tool to curb pollution. However, in industries with a public firm, the CA's decision does not significantly change when considering or ignoring emissions in its guidelines, suggesting that these changes not impactful in markets such as oil, gas, and mining.

Unprofitable mergers. Our results also underscore how the presence of the public firm makes mergers less profitable. When the CA ignores pollution, firms anticipate the merger request will be denied and do not submit a request regardless of its profitability. When the CA considers pollution, however, mergers are more likely to be approved, but are also more likely unprofitable, especially when pollution is severe, entailing that mergers are not filed in equilibrium. Our findings, then, suggest that changes in CA guidelines would not lead to different merger requests or approvals, especially when pollution is severe or the public firm is present, making the new guidelines inconsequential. In contrast, when pollution is not severe and the public firm is mostly privatized, guideline changes would lead to more merger requests being submitted and approved in equilibrium, yielding welfare gains.

Robustness checks. Our findings are robust to changes in modeling assumptions. First, we consider a setting where the public firm ignores pollution in its welfare function, which can arise when this firm's regulations consider consumer and producer surplus alone. In this context, we show that the public firm produces more output than when accounting for environmental damages. Anticipating this larger output, the CA is more likely to approve mergers, since this "approval differential" helps reduce aggregate pollution –a differential that grows as the public firm is less privatized and pollution becomes more severe. Second, we allow for the merger to yield cost-reducing effects, showing that mergers are more likely to become profitable, since the merging entity benefits from a cost advantage in this setting. When pollution is not severe, mergers are also more likely to be welfare improving, because cost-reducing effects yield a larger aggregate output; but when pollution is severe, mergers are more probably blocked. Third, cost convexity makes the merger less likely to be welfare enhancing, thus requiring large weights on pollution for the CA to approve the merger.

Further research. We consider that firms are cost symmetric before the merger, while the

merging entity may benefit from cost-reducing effects. Alternatively, one could allow for firms to be cost asymmetric before the merger, with the public firm exhibiting different marginal costs than private firms. Similarly, we study mergers between private firms, but one could allow for mergers between the public firm and k private firms. Finally, an extension could consider that the CA does not accurately observe pollution damages in an incomplete information environment.

6 Appendix

6.1 Appendix 1 - Public firm ignoring pollution

Third stage, No merger. Every private firm *i* still exhibits the same best response function as in the main model, $q_i(q) = \frac{1-c}{n} - \frac{1}{n}q$. However, that of the public firm becomes

$$q(q_i) = \frac{1-c}{2-\alpha} - \frac{1}{2-\alpha} \sum_{i=1}^{n-1} q_i$$

Simultaneously solving for q and q_i in best response functions $q(q_i)$ and $q_i(q)$, we obtain

$$q^{NM} = \frac{1-c}{1+n(1-\alpha)}$$
 and $q_i^{NM} = \frac{(1-c)(1-\alpha)}{1+n(1-\alpha)}$,

which are both positive, and satisfy $\frac{\partial q^{NM}}{\partial \alpha} = \frac{n(1-c)}{[1+n(1-\alpha)]^2} > 0$ and $\frac{\partial q_i^{NM}}{\partial \alpha} = -\frac{1-c}{[1+n(1-\alpha)]^2} < 0$, thus indicating that, as the public firm is more nationalized, its output increases while that of every private firm decreases. Aggregate output under no merger is $Q^{NM} = \frac{(1-c)[n(1-\alpha)+\alpha]}{1+n(1-\alpha)}$, which increasing in α because $\frac{\partial Q^{NM}}{\partial \alpha} = \frac{n(n+1)(1-c)}{[1+n(1-\alpha)]^2} > 0$. In this context, the efficiency ratio becomes $ER^{NM} = \frac{n(1-\alpha)+\alpha}{n(1-\alpha)+1}$, which satisfies $ER^{NM} < 1$ since $\alpha < 1$, thus showing underproduction relative to social optimal.

Third stage, Merger. Using Lemma 3, equilibrium output in this scenario is

$$q^M = \frac{1-c}{1+(1-\alpha)(n-k+1)}$$
 and $q_I^M = q_O^M = \frac{(1-c)(1-\alpha)}{1+(1-\alpha)(n-k+1)}$

which are both positive, and satisfy $\frac{\partial q^M}{\partial \alpha} = \frac{(1-c)(n-k+1)}{[1+(1-\alpha)(n-k+1)]^2} > 0$ and $\frac{\partial q_i^{NM}}{\partial \alpha} = -\frac{1-c}{[1+(1-\alpha)(n-k+1)]^2} < 0$, thus entailing an output shift towards the public firm. Aggregate output, then, becomes $Q^M = \frac{(1-c)[1+(1-\alpha)(n-k+1)]}{1+(1-\alpha)(n-k+1)}$, which coincides with Q^{NM} when k = 1, and decreases in k since $\frac{\partial Q^M}{\partial k} = -\frac{(1-c)(1-\alpha)^2}{[2-\alpha+(1-\alpha)(n-k)]^2} < 0$, implying that $Q^M < Q^{NM}$ for all $k \ge 2$. Aggregate output Q^M is increasing in α since $\frac{\partial Q^{NM}}{\partial \alpha} = \frac{1-c}{[1+(1-\alpha)(n-k+1)]^2} > 0$. In this setting, the efficiency ratio is $ER^M = \frac{1+(1-\alpha)(n-k)}{1+(1-\alpha)(n-k+1)}$, which satisfies $ER^M < 1$, also indicating underproduction. Comparing ER^M and ER^{NM} , we obtain that $ER^M - ER^{NM} = -\frac{(k-1)(1-\alpha)^2}{[(n(1-\alpha)+1)](1+(1-\alpha)(n-k+1)]} < 0$, implying that output efficiency worsens after the merger.

Second stage. First, when the CA uses consumer surplus to evaluate the merger request, it blocks the merger since $Q^M < Q^{NM}$ for all $k \ge 2$ for all parameters. Second, when the CA uses welfare to evaluate mergers but ignores environmental damages, W = CS + PS, welfare without the merger is

$$W_{NP}^{NM} = \frac{(1-c)^2 [n-\alpha(n-1)][2-\alpha+n(1-\alpha)]}{2[1+n(1-\alpha)]^2}$$

and that after the merger is

$$W_{NP}^{M} = \frac{(1-c)^{2}[1+(n-k)(1-\alpha)][3-2\alpha+(n-k)(1-\alpha)]}{2[2-\alpha+(n-k)(1-\alpha)]^{2}}$$

where W_{NP}^{M} coincides with W_{NP}^{NM} when k = 1. The merger improves welfare if and only if $W_{NP}^{M} - W_{NP}^{NM} > 0$, where

$$W_{NP}^{M} - W_{NP}^{NM} = -\frac{(1-\alpha)^{3} (1-c)^{2} (k-1)[3-\alpha+(2n-k)(1-\alpha)]}{2[2-\alpha+(n-k)(1-\alpha)]^{2}[1+n(1-\alpha)]^{2}} < 0$$

holds for all admissible parameters, implying that the CA blocks the merger. Third, when the CA considers environmental damage in its welfare function, $W = CS + PS - Env_{CA}$, welfare before the merger is

$$W^{NM} = \frac{(1-c)^2 [n-\alpha(n-1)][2+n-\alpha(n+1)+2d\beta(\alpha(n-1)-n)]}{2[1+n(1-\alpha)]^2}$$

while that after the merger is

$$W^{M} = \frac{(1-c)^{2}[1+(n-k)(1-\alpha)][3-2\alpha+(n-k)(1-\alpha)-2d\beta(1+(n-k)(1-\alpha))]}{2[2-\alpha+(n-k)(1-\alpha)]^{2}}.$$

Defining the welfare gain from the merger as $\Delta W \equiv W^M - W^{NM}$, we obtain that $\Delta W = 0$ at $k = \overline{k}(\beta)$, where cutoff $\overline{k}^{IP}(\beta)$ is

$$\overline{k}(\beta) \equiv \frac{(1-\alpha)[3-\alpha+2n(1-\alpha)]-2d\beta[1-\alpha(\alpha-2)+4n(1-\alpha)+2n^2(1-\alpha)^2]}{(1-\alpha)[1-\alpha-2d\beta(1+\alpha+2n(1-\alpha))]}.$$

Analyzing cutoff $\overline{k}^{IP}(\beta)$, we first find that, when evaluated at $\beta = 0$, this cutoff collapses to $\overline{k}^{IP}(0) = \frac{3+2n(1-\alpha)-\alpha}{1-\alpha}$, which lies above n, $\overline{k}^{IP}(0) > n$, since $n - \overline{k}^{IP}(0) = -\frac{3+n(1-\alpha)+\alpha}{1-\alpha} < 0$ for all admissible parameters. Then, this cutoff unambiguously increases in β since

$$\frac{\partial \overline{k}^{IP}(\beta)}{\partial \beta} = \frac{4d[1+n(1-\alpha)]^2}{(1-\alpha)[1-\alpha-2d\beta(1+\alpha+2n(1-\alpha))]^2} > 0$$

and reaches a vertical asymptote at $\beta = \tilde{\beta} \equiv \frac{1-\alpha}{2d[1+\alpha+2n(1-\alpha)]}$. Further increases in β then entail

that cutoff $\overline{k}^{IP}(\beta)$ crosses the horizontal axis, $\overline{k}^{IP}(\beta) = 0$, at

$$\underline{\beta} \equiv \frac{(1-\alpha)[3-\alpha+2n(1-\alpha)]}{2d[1+\alpha(2-\alpha)+4n(1-\alpha)+2n^2(1-\alpha)^2]}$$

and keeps increasing in β , so it solves $\overline{k}^{IP}(\beta) = n - 1$ at

$$\overline{\beta} \equiv \frac{(1-\alpha)[2(2-\alpha)+n(1-\alpha)]}{2d[2+5n+\alpha\left[2(1-\alpha)-n(8-3\alpha)\right]]}$$

where $\overline{\beta} > 0$ since $\alpha < 1$, and cutoffs $\overline{\beta}$ and $\underline{\beta}$ satisfy $\overline{\beta} > \underline{\beta}$ since cutoff $\overline{k}^{IP}(\beta)$ is unambiguously increasing in β .

First stage. Using the above outputs under no merger, we find the equilibrium profits for each private firm, $\pi_i^{NM} = \left(\frac{(1-c)(1-\alpha)}{1+n(1-\alpha)}\right)^2$. The equilibrium profit for merging firms (insiders) is $\pi_I^M = \left(\frac{(1-c)(1-\alpha)}{1+(1-\alpha)(n-k+1)}\right)^2$.

Hence, firms merge if and only if

$$\left(\frac{(1-c)(1-\alpha)}{1+(1-\alpha)(n-k+1)}\right)^2 \ge k \left(\frac{(1-c)(1-\alpha)}{1+n(1-\alpha)}\right)^2$$

Solving for k, and dividing both sides by n, yields

$$\frac{k}{n} \geq \frac{\widehat{k}}{n} \equiv \frac{3 - \alpha + 2n(1 - \alpha) - (1 - \alpha)^{1/2}\gamma}{2n(1 - \alpha)}$$

where $\gamma \equiv [5 - \alpha + 4n(1 - \alpha)]^{1/2} > 0$ since $\alpha < 1$. When $\alpha = 0$, cutoff $\frac{\hat{k}}{n}$ simplifies to $\frac{\hat{k}}{n} = \frac{3+2n-[5+4n]^{1/2}}{2n}$, as in Salant et al. (1983); but when $\alpha > 0$ this cutoff satisfies

$$\frac{\partial \left(\frac{\hat{k}}{n}\right)}{\partial \alpha} = \frac{(1-\alpha)^{1/2}\gamma - (1-\alpha)}{n(1-\alpha)^{5/2}\gamma} > 0$$

since $(1 - \alpha)^{1/2} > (1 - \alpha)$ and $\alpha < 1$.

For the merger to be profitable and approved, we need that $\overline{k}^{IP}(\beta) - \hat{k} > 0$, where cutoff $\overline{k}^{IP}(\beta)$ was found in the second stage above. When $\alpha = \beta = 0$, this difference is $\frac{3+2n+[5+4n]^{1/2}}{2} > 0$. When $\beta = 0$, the difference becomes

$$\overline{k}^{IP}(0) - \widehat{k} = \frac{3 - \alpha + 2n(1 - \alpha) + (1 - \alpha)^{1/2}\gamma}{2(1 - \alpha)}$$

which is positive for all admissible values. Finally, when $\alpha, \beta > 0$, the difference $\overline{k}^{IP}(\beta) - \widehat{k} > 0$ for all $\beta > \widehat{\beta}$, where $\widehat{\beta} \equiv \frac{1}{2d\left(\frac{\gamma}{(1-\alpha)^{1/2}}-2\right)}$. Cutoff $\widehat{\beta}$ decreases in d and satisfies $\frac{\partial \widehat{\beta}}{\partial \alpha} = -\frac{1}{d(1-\alpha)^{1/2}\gamma[\gamma-2(1-\alpha)^{1/2}]^2} < 0$ and $\frac{\partial \widehat{\beta}}{\partial n} = -\frac{(1-\alpha)^{3/2}}{d\gamma[\gamma-2(1-\alpha)^{1/2}]^2} < 0$.

6.2 Appendix 2 - Mergers with cost-reducing effects

Third stage. When no merger ensues, equilibrium output coincides with that in Lemma 1, $q^{NM} = \frac{(1-c)[1-2\alpha d(n-1)]}{1+2\alpha d+(1-\alpha)n}$ and $q_i^{NM} = \frac{(1-c)[1-\alpha(1-2d)]}{1+2\alpha d+(1-\alpha)n}$, while aggregate output coincides with that in Lemma 2, $Q^{NM} = \frac{(1-c)[n(1-\alpha)+\alpha]}{1+2\alpha d+n(1-\alpha)}$.

When a merger occurs, however, the merging entity benefits from a lower cost, c-x, so it solves

$$\max_{q_I^M \ge 0} \pi_I^M = \left[1 - \left(q^M + q_I^M + \sum_{i=1}^{n-1-k} q_{O,i}^M \right) \right] q_I^M - (c-x) q_I^M$$

which yields best response function

$$q_{I}^{M}\left(q^{M}, q_{O}^{M}\right) = \frac{1 - (c - x)}{2} - \frac{q^{M} + (n - k - 1)q_{O}^{M}}{2}$$

Every unmerged private firm still faces marginal cost c, solving

$$\max_{q_O^M \ge 0} \pi_O^M = \left[1 - \left(q^M + q_I^M + \sum_{i=1}^{n-1-k} q_{O,i}^M \right) \right] q_O^M - c q_O^M$$

which yields best response function

$$q_O^M\left(q^M, q_I^M\right) = \frac{1-c}{n-k} - \frac{q^M + q_I^M}{n-k}$$

The public firm also faces marginal cost c, solving

$$\max_{q^M \ge 0} V^M = \alpha W^M + (1 - \alpha) \pi^M$$

where $W^M = \int_0^{Q^M} (1-y)dy - cQ^M - d(Q^M)^2$, $\pi^M = (1-Q^M)q^M - cq^M$, and $Q^M = q^M + q_I^M + (n-k-1)q_O^M$. Differentiating and solving for q^M , we obtain best response function

$$q^{M}\left(q_{I}^{M}, q_{O}^{M}\right) = \frac{1-c}{2+\alpha(2d-1)} - \frac{(1+2\alpha d)\left[q_{I}^{M} + \sum_{i=1}^{n-k-1} q_{O_{i}}^{M}\right]}{2+\alpha(2d-1)}$$

Letting s_P denote the slope of the above best response function, it satisfies $\frac{\partial s_P}{\partial \alpha} = -\frac{(1+2\alpha d)\left[\sum_{i=1}^{n-k-1} q_{O_i}^M - q_I^M\right]}{(2\alpha(2d-1)^2)} < 0$ if and only if $\sum_{i=1}^{n-k-1} q_{O_i}^M > q_I^M$ and $\frac{\partial s}{\partial d} = -\frac{2\alpha(1-\alpha)\left[\sum_{i=1}^{n-k-1} q_{O_i}^M - q_I^M\right]}{(2\alpha(2d-1)^2)} < 0$ if and only if $\sum_{i=1}^{n-k-1} q_{O_i}^M > q_I^M$.

Solving simultaneously for q^M , q_I^M , and q_O^M in the above best response functions yields

$$q^{M} = \frac{(1-c)[1-2\alpha d(n-k)] - x(1+2\alpha d)}{1+2\alpha d + (1-\alpha)(n-k+1)}, \ q^{M}_{O} = \frac{(1-c)[1+\alpha(2d-1)] - x(1-\alpha)}{1+2\alpha d + (1-\alpha)(n-k+1)}$$

and
$$q_I^M = \frac{(1-c)[1+\alpha(2d-1)] + [(1-\alpha)(n-k) + 2\alpha d + 1]x}{1+2\alpha d + (1-\alpha)(n-k+1)}$$

where q_I^M is unambiguously positive; $q^M > 0$ if and only if $x < x_P \equiv \frac{(1-c)[1-2\alpha d(n-k)]}{1+2\alpha d}$, which holds by definition; and $q_O^M > 0$ if and only if $x < x_O \equiv \frac{(1-c)[1+\alpha(2d-1)]}{1-\alpha}$. Cutoffs x_O and x_P satisfy $x_O - x_P = \frac{2\alpha d(1-c)[2+(n-k)(1-\alpha)+\alpha(2d-1)]}{(1-\alpha)(2\alpha d+1)} > 0$, implying that $x_O > x_P$, which entails that the initial condition $x < x_P$ implies that all firms are active after the merger.

Both cutoffs x_P and x_O simplify to (1 - c) when evaluated at $\alpha = 0$. In addition, cutoff x_P satisfies $\frac{\partial x_P}{\partial d} = -\frac{2d(1-c)(n-k+1)}{(1+2\alpha d)^2} < 0$ and $\frac{\partial x_P}{\partial \alpha} = -\frac{2\alpha(1-c)(n-k+1)}{(1+2\alpha d)^2} < 0$, implying that condition $x < x_P$ becomes more demanding when the public firm is more nationalized (higher α) or pollution is more severe (higher d).

 $\begin{array}{l} Comparative statics. \text{ The public firm's output, } q^M, \text{ satisfies } \frac{\partial q^M}{\partial c} = -\frac{1+2\alpha d(n-k)}{1+(1-\alpha)(n-k+1)+2\alpha d} < 0, \\ \frac{\partial q^M}{\partial \alpha} = -\frac{(n-k+1)[(1-c)[2d(n-k+1)-1]+x+2dx]}{[1+2d+(1-\alpha)(n-k+1)]^2} < 0, \\ \frac{\partial q^M}{\partial d} = -\frac{2(n-k+1)(\alpha(1-c)[1+(n-k)(1-\alpha)]+(1-\alpha)\alpha x)}{[1+2d+(1-\alpha)(n-k+1)]^2} < 0, \\ \text{and } \frac{\partial q^M}{\partial n} = \frac{(1+2\alpha d)[(1-c)(\alpha(2d-1)-1]-x(1-\alpha)}{1+2d+(1-\alpha)(n-k+1)]^2} < 0 \text{ if and only if } x < x_O, \text{ which holds given that } \\ x < x_P \text{ by definition and cutoff } x_O \text{ satisfies } x_O > x_P. \text{ Every outsider's output, } q^M_O, \text{ satisfies } \frac{\partial q^M_O}{\partial c} = -\frac{1+\alpha(2d-1)}{1+(1-\alpha)(n-k+1)+2\alpha d} < 0, \\ \frac{\partial q^M_O}{\partial \alpha} = \frac{(1-c)[2d(n-k+1)-1]+x+2dx}{(1+2d+(1-\alpha)(n-k+1)]^2} > 0, \\ \frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]+2(1-\alpha)\alpha x}{[1+2d+(1-\alpha)(n-k+1)]^2} > 0, \\ 0, \text{ and } \frac{\partial q^M_O}{\partial n} = \frac{(1-c)[2d(n-k+1)-1]+x(1-\alpha)}{(1+2d+(1-\alpha)(n-k+1))]^2} < 0 \text{ if and only if } x < x_O, \text{ which holds since } x < x_P \\ \text{by assumption. Finally, the merged entity's output, } q^M_I, \text{ satisfies } \frac{\partial q^M_O}{\partial c} = -\frac{1+\alpha(2d-1)}{(1+2d+(1-\alpha)(n-k+1))]^2} < 0, \\ \frac{\partial q^M_O}{\partial a} = \frac{(1-c)[2d(n-k+1)-1]+x+2dx}{(1+2d+(1-\alpha)(n-k+1))]^2} > 0, \\ \frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]+2(1-\alpha)\alpha x}{(1+2d+(1-\alpha)(n-k+1))]^2} > 0, \\ \frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]+2(1-\alpha)\alpha x}{(1+2d+(1-\alpha)(n-k+1))]^2} > 0, \\ \frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]+2(1-\alpha)\alpha x}{(1+2d+(1-\alpha)(n-k+1))]^2} > 0, \\ \frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]+2(1-\alpha)\alpha x}{(1+2d+(1-\alpha)(n-k+1))]^2} > 0, \\ \frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]+2(1-\alpha)\alpha x}{(1+2d+(1-\alpha)(n-k+1))]^2} > 0, \\$

The aggregate output after the merger becomes

$$Q^{M} = q^{M} + (n - k - 1)q_{O}^{M} + q_{I}^{M}$$

=
$$\frac{(1 - c)[1 + (1 - \alpha)(n - k)] + (1 - \alpha)x}{1 + (1 - \alpha)(n - k + 1) + 2\alpha d}$$

which is unambiguously positive. In addition, Q^M satisfies $\frac{\partial Q^M}{\partial c} = -\frac{(n-k)(1-\alpha)+1}{1+(1-\alpha)(n-k+1)+2\alpha d} < 0;$ $\frac{\partial Q^M}{\partial \alpha} = \frac{(1-c)[1-2d(n-k+1)]-x(1+2d)}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$ since $d > \frac{1}{2}$ by definition; $\frac{\partial Q^M}{\partial d} = -\frac{2\alpha[(1-c)[1+(n-k)(1-\alpha)]+(1-\alpha)x]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$; and $\frac{\partial Q^M}{\partial k} = -\frac{(1-\alpha)[(1-c)[1+\alpha(2d-1)]-x(1-\alpha)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$ if and only if $x < x_O \equiv \frac{(1-c)[1+\alpha(2d-1)]}{1-\alpha}$, which holds given that $x < x_P$ by definition, so Q^M is decreasing in k for all admissible parameters. Similarly, Q^M is increasing in n because $\frac{\partial Q^M}{\partial n} = \frac{(1-\alpha)[(1-c)[1+\alpha(2d-1)]-x(1-\alpha)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} > 0$ if and only if $x < x_O$, which is satisfied for all admissible parameters.

Second stage. When the CA uses consumer surplus to evaluate mergers, $Q^M > Q^{NM}$ holds if and only if $x > x_{CA} \equiv \frac{(1-c)(k-1)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)}$, where $x_P - x_{CA} = \frac{(1-c)[1-2\alpha d(n-1)][2+(n-k)(1-\alpha)+\alpha(2d-1)]}{(1+2\alpha d)[1+2\alpha d+n(1-\alpha)]} > 0$, entailing that $x_P > x_{CA}$ for all admissible parameter values. In addition, cutoff x_{CA} collapses to $x_{CA} = \frac{(1-c)(k-1)}{n+1}$ when all firms are private, $\alpha = 0$, and satisfies $\frac{\partial x_{CA}}{\partial k} = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)} > 0$, $\frac{\partial x_{CA}}{\partial n} = -\frac{(1-c)(k-1)(1-\alpha)[1+\alpha(2d-1)]}{[1+2\alpha d+n(1-\alpha)]^2} < 0$, $\frac{\partial x_{CA}}{\partial c} = -\frac{(k-1)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)} < 0$, $\frac{\partial x_{CA}}{\partial \alpha} = \frac{(1-c)(k-1)(k-1)(2dn-1)}{[1+2\alpha d+n(1-\alpha)]^2} > 0$.

First stage. Using outputs from Lemma 1, we find the equilibrium profits for each private

firm, $\pi_i^{NM} = \left(\frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)}\right)^2$. The equilibrium profits for merging firms (insiders) is

$$\pi_I^{NM} = \left(\frac{[1+\alpha(2d-1)](1-c) + [1+2\alpha d + (1-\alpha)(n-k)]x}{1+2\alpha d + (1-\alpha)(n-k+1)}\right)^2$$

Hence, firms find the merger profitable if and only if

$$\left(\frac{[1+\alpha(2d-1)](1-c)+[1+2\alpha d+(1-\alpha)(n-k)]x}{1+2\alpha d+(1-\alpha)(n-k+1)}\right)^2 \ge k\left(\frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)}\right)^2$$

Applying square roots on both sides of the above inequality, yields

$$\frac{(1-c)\left[1+\alpha(2d-1)\right]}{1+2\alpha d+(1-\alpha)(n-k+1)} + \frac{1+2\alpha d+(1-\alpha)(n-k)}{1+2\alpha d+(1-\alpha)(n-k+1)}\frac{x}{1-c} \ge \sqrt{k}\frac{(1-c)\left[1+\alpha(2d-1)\right]}{1+2\alpha d+n(1-\alpha)(n-k+1)}$$

and solving for x, we find that

$$x \ge \frac{(1-c)\left[1+\alpha(2d-1)\right]}{1+2\alpha d+(1-\alpha)(n-k)} \left[\sqrt{k}\frac{\left[1+2\alpha d+(1-\alpha)(n-k+1)\right]}{1+2\alpha d+n(1-\alpha)} - 1\right] \equiv x_{\pi}$$

where cutoff x_{π} collapses to $x_{\pi} = (1-c)\left(\frac{\sqrt{k}}{n+1} - \frac{1}{n-k+2}\right)$ when all firms are private, $\alpha = 0$, and satisfies

$$x_{\pi} - x_{CA} = -\frac{(1-c)[1+\alpha(2d-1)](\sqrt{k}-1)\sqrt{k}[2+(n-k)(1-\alpha)+\alpha(2d-1)]}{[1+2\alpha d + (n-k)(1-\alpha)][1+2\alpha d + n(1-\alpha)]}$$

which is unambiguously negative since $\sqrt{k} > 1$ because $k \ge 2$. Therefore, the ranking of cutoffs is $x_{\pi} < x_{CA} < x_P$ for all admissible parameters.

Table A.1 considers the same parameter values as Table III in the main text, allowing for x = 0.10, and changes one parameter at a time, to test how equilibrium results and welfare are affected by each parameter.

	No merger				Merger			Changes			Cutoffs		
	Q^{NM}	Env^{NM}	W^{NM}	Q^M	Env^M	W^M	ΔQ	ΔEnv	ΔW	x_{π}	x_{CA}	x_P	
Baseline	0.423	0.045	0.077	0.328	0.027	0.083	-0.095	0.018	0.006	-0.001	0.308	0.409	
$\alpha = 0.07$	0.425	0.045	0.077	0.361	0.326	0.106	-0.063	0.012	0.029	-0.007	0.301	0.434	
$\alpha = 0.04$	0.426	0.046	0.077	0.363	0.033	0.106	-0.063	0.012	0.029	-0.013	0.294	0.461	
$\alpha = 0$	0.429	0.046	0.077	0.366	0.034	0.106	-0.062	0.012	0.029	-0.021	0.286	0.500	
d = 0.6	0.422	0.053	0.069	0.356	0.038	0.101	-0.066	0.015	0.032	0.001	0.313	0.393	
d = 0.7	0.420	0.062	0.060	0.354	0.044	0.095	-0.067	0.018	0.035	0.003	0.318	0.377	
d = 0.8	0.419	0.070	0.051	0.351	0.049	0.089	-0.068	0.021	0.038	0.005	0.323	0.363	
$\beta = 0.7$	0.423	0.063	0.059	0.359	0.045	0.094	-0.064	0.018	0.035	-0.001	0.308	0.409	
$\beta = 0.8$	0.423	0.072	0.050	0.359	0.051	0.088	-0.064	0.020	0.037	-0.001	0.308	0.409	
$\beta = 1$	0.423	0.089	0.033	0.358	0.064	0.075	-0.064	0.025	0.042	-0.001	0.308	0.409	
c = 0.4	0.508	0.064	0.111	0.424	0.045	0.147	-0.084	0.019	0.036	-0.001	0.369	0.491	
c = 0.3	0.592	0.088	0.151	0.490	0.060	0.194	-0.103	0.028	0.042	-0.001	0.431	0.572	
c = 0.2	0.677	0.115	0.198	0.555	0.077	0.247	-0.122	0.038	0.050	-0.001	0.492	0.655	
k = 4	0.423	0.045	0.077	0.392	0.038	0.102	-0.031	0.006	0.024	0.058	0.231	0.364	
k = 3	0.423	0.045	0.077	0.413	0.043	0.097	-0.010	0.002	0.020	0.066	0.154	0.318	
k = 2	0.423	0.045	0.077	0.427	0.046	0.094	0.004	-0.001	0.017	0.046	0.077	0.273	

Table A.1. Equilibrium outcomes with cost-reducing effects - Robustness checks.

6.3 Appendix 3 - Allowing for non-linear costs

Third stage, No merger. Each private firm i solves

$$\max_{q_i \ge 0} \pi_i^{NM} = \left[1 - \left(q^{NM} + q_i^{NM} + \sum_{j \ne i}^{n-2} q_j^{NM} \right) \right] q_i^{NM} - \left(c + \frac{h}{2} q_i^{NM} \right) q_i^{NM}$$

yielding best response function

$$q_i^{NM}\left(q^{NM}\right) = \frac{1-c}{n+h} - \frac{1}{n+h}q^{NM}.$$

The intercept term of this function, $\frac{1-c}{n+h}$, is decreasing in h; while its slope, $-\frac{1}{n+h}$, is increasing in h. Therefore, as production costs are more convex (higher h), every firm produces fewer units, but output competition is softened (i.e., flatter best response functions). When costs are linear, h = 0, this best response function simplifies to $q_i^{NM}(q^{NM}) = \frac{1-c}{n} - \frac{1}{n}q^{NM}$, coinciding with that in the main body of the paper.

The public firm solves

$$\max_{q^{NM} \ge 0} \quad V^{NM} = \alpha W^{NM} + (1 - \alpha) \pi^{NM}$$

where $\pi^{NM} = \left[1 - \left(q^{NM} + \sum_{i=1}^{n-2} q_i^{NM}\right)\right] q^{NM} - \left(c + \frac{h}{2} q^{NM}\right) q^{NM}$ and W^{NM} is defined as in the main body of the paper. This yields best response function

$$q^{NM}\left(q_{i}^{NM}\right) = \frac{1-c}{2+\alpha(2d-1)+h} - \frac{1+2\alpha d}{2+\alpha(2d-1)+h}\sum_{i=1}^{n-1}q_{i}^{NM}$$

The vertical intercept, $\frac{1-c}{2+h-\alpha(2d-1)}$, is decreasing in h; while its slope, $-\frac{1+2\alpha d}{2+h+\alpha(2d-1)}$, is increasing in h, thus exhibiting similar comparative statics as the best response function of every private firm. When costs are linear, h = 0, this best response function simplifies to

$$q^{NM}\left(q_{i}^{NM}\right) = \frac{1-c}{2+\alpha(2d-1)} - \frac{1+2\alpha d}{2+\alpha(2d-1)}\sum_{i=1}^{n-1}q_{i}^{NM}$$

which coincides with equation (3) in the main body of the paper. Simultaneously solving for q^{NM} and q_i^{NM} , we obtain equilibrium output levels

$$q^{NM} = \frac{(1-c)[1+h-2\alpha d(n-1)]}{1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)} \text{ and} q_i^{NM} = \frac{(1-c)[1+h+\alpha(2d-1)]}{1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)}.$$

Output q_i^{NM} is unambiguously positive, while q^{NM} is positive iff $\alpha < \frac{1+h}{2d(n-1)}$ which is true by assumption since cutoff $\frac{1+h}{2d(n-1)}$ satisfies $\frac{1+h}{2d(n-1)} > \overline{\alpha}$. Aggregate output is given by $Q^{NM} = \frac{(1-c)[n(1-\alpha+h)+\alpha]}{1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)}$ which is unambiguously positive. When costs are linear, h = 0, the above expressions simplify to

$$q^{NM} = \frac{(1-c)[1-2\alpha d(n-1)]}{1+2\alpha d+(1-\alpha)n} \qquad q_i^{NM} = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)n} \qquad Q^{NM} = \frac{(1-c)[n(1-\alpha)+\alpha]}{1+2\alpha d+n(1-\alpha)}$$

coinciding with those in Lemma 1 and 2. Aggregate output satisfies

$$\frac{\partial Q^{NM}}{\partial h} = \frac{(1-c)\left[(n-1)\alpha[2(1+h)+\alpha(2d-1)]-(1+h)^2n\right]}{[1+2\alpha d(h+1)+h(n+h+1)+(1-\alpha)(n+h)]^2} < 0$$

if and only if $d < \frac{1}{2\alpha(n-1)} \left(\frac{(1+h)[n(1+h)-2\alpha(n-1)]+\alpha^2(n-1)}{\alpha} \right)$, where $\frac{(1+h)[n(1+h)-2\alpha(n-1)]+\alpha^2(n-1)}{\alpha} > 1$ for all admissible parameters. Since $d < \frac{1}{2\alpha(n-1)}$ by definition, Q^{NM} unambiguously decreases in h.

Third stage, Merger. When there is a merger, the best response function of the public firm is

$$q^{M}\left(q_{I}^{M}, q_{O}^{M}\right) = \frac{1-c}{2+h+\alpha(2d-1)} - \frac{(1+2\alpha d)\left[q_{I}^{M} + \sum_{i=1}^{n-k-1} q_{O_{i}}^{M}\right]}{2+h+\alpha(2d-1)}$$

that of the merging entity (insiders) is

$$q_{I}^{M}\left(q^{M}, q_{O}^{M}\right) = \frac{1 - (c - x)}{2 + h} - \frac{q^{M} + (n - k - 1)q_{O}^{M}}{2 + h}$$

and that of every outsider private firm is

$$q_O^M(q^M, q_I^M) = \frac{1-c}{n-k+h} - \frac{q^M + q_I^M}{n-k+h}.$$

For the above best response functions, all vertical intercept terms are decreasing in h and slopes are increasing in h. When costs are linear, h = 0, these best response function simplify to

$$\begin{split} q^{M}\left(q_{I}^{M}, q_{O}^{M}\right) &= \frac{1-c}{2+\alpha(2d-1)} - \frac{\left(1+2\alpha d\right)\left[q_{I}^{M}+\sum_{i=1}^{n-k-1}q_{O_{i}}^{M}\right]}{2+\alpha(2d-1)},\\ q_{I}^{M}\left(q^{M}, q_{O}^{M}\right) &= \frac{1-(c-x)}{2} - \frac{q^{M}+(n-k-1)q_{O}^{M}}{2}, \text{ and}\\ q_{O}^{M}\left(q^{M}, q_{I}^{M}\right) &= \frac{1-c}{n-k} - \frac{q^{M}+q_{I}^{M}}{n-k}. \end{split}$$

Simultaneously solving, we find that equilibrium quantities are

$$q^{M} = \frac{(1-c)[1+h-2\alpha d(n-k)]}{(1+h)(n-k+h+2) - \alpha[n-k+h+1-2d(1+h)]} \text{ and}$$
$$q^{M}_{O} = q^{M}_{I} = \frac{(1-c)[1+h+\alpha(2d-1)]}{(1+h)(n-k+h+2) - \alpha[n-k+h+1-2d(1+h)]}$$

Output q_O^M and q_I^M are unambiguously positive while q^M is positive if $\alpha < \frac{1+h}{2d(n-k)}$ which is true by assumption since cutoff $\frac{1+h}{2d(n-k)}$ satisfies $\frac{1+h}{2d(n-k)} > \overline{\alpha}$. Finally, aggregate output is $Q^M = \frac{(1-c)[1+h+(1+h-\alpha)(n-k)]}{(1+h)(n-k+h+2)-\alpha[n-k+h+1-2d(1+h)]}$, which is also unambiguously positive. Aggregate output satisfies

$$\frac{\partial Q^M}{\partial h} = -\frac{(1-c)\left[1+(n-k)[1-\alpha(2(1+\alpha d)-\alpha)]+h^2(n-k+1)+2h(1+(n-k)(1-\alpha)]\right]}{[(n+1)(n-k+2+h)-\alpha(n-k+1+h-2d(1+h))]^2} < 0$$

if and only if $d < \frac{1}{2\alpha} \left(\frac{1-\alpha(2-\alpha)}{\alpha} \right)$, where $\frac{1-\alpha(2-\alpha)}{\alpha} > 1$ for all admissible values of α , implying that $\frac{1}{2\alpha} < \frac{1}{2\alpha} \left(\frac{1-\alpha(2-\alpha)}{\alpha} \right)$. Since $d < \frac{1}{2\alpha(n-1)}$ by assumption, condition $d < \frac{1}{2\alpha} \left(\frac{1-\alpha(2-\alpha)}{\alpha} \right)$ holds, and Q^M is unambiguously decreasing in h.

When costs are linear, h = 0, the above equilibrium output simplify to

$$q^{M} = \frac{(1-c)[1-2\alpha d(n-k)]}{1+2\alpha d+(1-\alpha)(n-k+1)} \text{ and } q^{M}_{I} = q^{M}_{O} = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)(n-k+1)}$$

coinciding with those in Lemma 3; while aggregate output becomes, $Q^M = \frac{(1-c)[1+(1-\alpha)(n-k)]}{1+2\alpha d+(1-\alpha)(n-k+1)}$ also coinciding with that in Lemma 4.

Finally, when k = 1, $Q^M = Q^{NM}$; and Q^M satisfies $\frac{\partial Q^M}{\partial k} = -\frac{(1-c)(1-\alpha+h)(1+h)[1+\alpha(2d-1)+h]}{[(1+h)(n-k+2+h)-\alpha(1+h-2d(1+h)+n-k)]^2} < 0$, implying that $Q^M < Q^{NM}$ for all $k \ge 2$. Therefore, when the CA uses the CS-criterion, the merger is blocked for all admissible parameter values.

In the first stage, the merging entity finds the merger profitable if and only if $\frac{k}{n} \geq \frac{\hat{k}(h)}{n}$, where

$$\frac{\widehat{k}(h)}{n} \equiv \frac{[3+2(n+h)](1+h)^2 + \alpha^2[1+2(n+h) - 4d(1+h)] - 4\alpha(1+h)[n+(1-d)(1+h)] - \theta(h)}{2n(1-\alpha+h)^2}$$

and $\theta(h) \equiv \left[(1 - \alpha + h)^3 [(1 + h)(5 + 4(n + h)) - \alpha(1 + 4(n + h) - 8d(1 + h)] \right]^{1/2}$. When costs are linear, h = 0, this cutoff simplifies to $\frac{\hat{k}}{n} \equiv \frac{3 + 2n + \alpha[4(d-1) - \alpha(4d-1) - 2n(2-\alpha)] - \theta}{2n(1-\alpha)^2}$, as identified in Proposition 1.

6.4 Proof of Lemma 1

Simultaneously solving for q and q_i in best response functions $q(q_i)$ and $q_i(q)$, we obtain

$$q^{NM} = \frac{(1-c)[1-2\alpha d(n-1)]}{1+2\alpha d+(1-\alpha)n} \text{ and} q_i^{NM} = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)n}.$$

where q^{NM} is positive if and only if $\alpha < \frac{1}{2d(n-1)} \equiv \overline{\alpha}$, and satisfies $\frac{\partial q^{NM}}{\partial d} = \frac{2\alpha(1-c)[\alpha(n-1)-n]n}{[n+1-\alpha(n-2d)]^2} < 0;$ $\frac{\partial q^{NM}}{\partial \alpha} = \frac{n(1-c)(1-2dn)}{[n+1-\alpha(2d+n)]^2} < 0;$

$$\frac{\partial q^{NM}}{\partial n} = \frac{(1-c)(1-2\alpha d)[1+\alpha(2d-1)]}{\left[1+2\alpha d+(1-\alpha)n\right]^2} < 0$$

if and only if $\alpha < \frac{1}{2d}$, which holds since $\alpha < \overline{\alpha}$; and $\frac{\partial q^{NM}}{\partial c} = \frac{1+2\alpha d(n-1)}{1+2\alpha d+(1-\alpha)n} > 0$ for all admissible parameters.

Similarly, q_i^{NM} is positive since $d > \frac{1}{2}$, and satisfies $\frac{\partial q_i^{NM}}{\partial d} = \frac{2\alpha(1-c)[n-\alpha(n-1)]}{[1+2\alpha d+n(1-\alpha)]^2} > 0$; $\frac{\partial q_i^{NM}}{\partial \alpha} = \frac{(1-c)(1+2dn)}{[1+2\alpha d+n(1-\alpha)]^2} > 0$; $\frac{\partial q_i^{NM}}{\partial n} = -\frac{(1-\alpha)(1-c)[1+\alpha(2d-1)]}{[1+2\alpha d+n(1-\alpha)]^2} < 0$; $\frac{\partial q_i^{NM}}{\partial c} = -\frac{\alpha(2d-1)+1}{1+2\alpha d+n(1-\alpha)} < 0$.

6.5 Proof of Lemma 2

Using Lemma 1, we obtain that aggregate output under no merger is

$$Q^{NM} = q^{NM} + (n-1) q_i^{NM} = \frac{(1-c) [n(1-\alpha) + \alpha]}{1 + 2\alpha d + n(1-\alpha)}$$

which satisfies $\frac{\partial Q^{NM}}{\partial \alpha} = -\frac{(1-c)(2dn-1)}{[1+2\alpha d+n(1-\alpha)]^2} < 0; \ \frac{\partial Q^{NM}}{\partial d} = -\frac{2\alpha(1-c)[n(1-\alpha)+\alpha]}{[1+2\alpha d+n(1-\alpha)]^2} < 0; \ \frac{\partial Q^{NM}}{\partial n} = \frac{(1-\alpha)(1-c)[1+\alpha(2d-1)]}{[1+2\alpha d+n(1-\alpha)]^2} > 0$ since $d > \frac{1}{2}$ by definition; and $\frac{\partial Q^{NM}}{\partial c} = -\frac{n(1-\alpha)+\alpha}{1+2\alpha d+n(1-\alpha)} < 0.$

6.6 Proof of Corollary 1

Dividing $Q^{NM} = \frac{(1-c)[n(1-\alpha)+\alpha]}{1+2\alpha d+n(1-\alpha)}$ over $Q^{SO} = \frac{1-c}{1+2d}$, and rearranging, yields $ER^{NM} = \frac{(2d+1)[n(1-\alpha)+\alpha]}{1+2\alpha d+n(1-\alpha)}$, which satisfies $ER^{NM} > 1$ because $(2d+1)[n(1-\alpha)+\alpha] > 1+2\alpha d+n(1-\alpha)$ simplifies to $d > \frac{1}{2n}$, which holds since $d > \frac{1}{2}$ by definition and $n \ge 3$.

which holds since $d > \frac{1}{2}$ by definition and $n \ge 3$. In addition, ER^{NM} satisfies $\frac{\partial ER^{NM}}{\partial d} = \frac{2(1-\alpha)(n+1)[n(1-\alpha)+\alpha]}{[1+2\alpha d+n(1-\alpha)]^2} > 0$; $\frac{\partial ER^{NM}}{\partial \alpha} = \frac{(1+2d)(1-2dn)}{[1+2\alpha d+n(1-\alpha)]^2} < 0$ if $d > \frac{1}{2n}$, which holds since $d > \frac{1}{2}$ by definition; and $\frac{\partial ER^{NM}}{\partial n} = \frac{(1-\alpha)(1+2d)[1+\alpha(2d-1)]}{[1+2\alpha d+n(1-\alpha)]^2} > 0$ since $d > \frac{1}{2}$ by assumption.

6.7 Proof of Lemma 3

After the merger is approved, the merging entity's solves

$$\max_{q_I^M \ge 0} \ \pi_I^M = \left[1 - \left(q^M + q_I^M + \sum_{i=1}^{n-1-k} q_{O,i}^M \right) \right] q_I^M - c q_I^M$$

which yields best response function

$$q_I^M(q^M, q_O^M) = \frac{1-c}{2} - \frac{q^M + (n-k-1)q_O^M}{2}$$

Every unmerged private firm solves

$$\max_{q_O^M \ge 0} \pi_O^M = \left[1 - \left(q^M + q_I^M + \sum_{i=1}^{n-1-k} q_{O,i}^M \right) \right] q_O^M - c q_O^M$$

which yields best response function

$$q_O^M\left(q^M, q_I^M\right) = \frac{1-c}{n-k} - \frac{q^M + q_I^M}{n-k}$$

The public firm solves

$$\max_{q^M \ge 0} V^M = \alpha W^M + (1 - \alpha) \pi^M$$

where $W^M = \frac{(Q^M)^2}{2} + (\pi_I^M + \pi_O^M + \pi^M) - d(Q^M)^2$, $\pi^M = (1 - Q^M) q^M - cq^M$, and $Q^M = q^M + q_I^M + (n - k - 1)q_O^M$. Differentiating and solving for q^M , we obtain best response function

$$q^{M}\left(q_{I}^{M}, q_{O}^{M}\right) = \frac{1-c}{2+\alpha(2d-1)} - \frac{\left(1+2\alpha d\right)\left[q_{I}^{M}+\sum_{i=1}^{n-k-1}q_{O_{i}}^{M}\right]}{2+\alpha(2d-1)}$$

Letting s_P denote the slope of the above best response function, it satisfies $\frac{\partial s_P}{\partial \alpha} = -\frac{(1+2\alpha d)\left[\sum_{i=1}^{n-k-1} q_{O_i}^M - q_I^M\right]}{(2\alpha(2d-1)^2)} < 0$

 $0 \text{ if and only if } \sum_{i=1}^{n-k-1} q_{O_i}^M > q_I^M \text{ and } \frac{\partial s}{\partial d} = -\frac{2\alpha(1-\alpha)\left[\sum_{i=1}^{n-k-1} q_{O_i}^M - q_I^M\right]}{(2\alpha(2d-1)^2)} < 0 \text{ if and only if } \sum_{i=1}^{n-k-1} q_{O_i}^M >$ q_I^M .

Solving simultaneously for q^M , q_I^M , and q_O^M in the above three best response functions yields output levels

$$q^{M} = \frac{(1-c)[1-2\alpha d(n-k)]}{1+2\alpha d+(1-\alpha)(n-k+1)} \text{ and } q^{M}_{I} = q^{M}_{O} = \frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)(n-k+1)}$$

where q_I^M and q_O^M are unambiguously positive, and $q^M > 0$ if and only if $\alpha < \frac{1}{2d(n-k)}$. However, the initial assumption $\alpha < \frac{1}{2d(n-1)} \equiv \overline{\alpha}$ is more demanding that $\alpha < \frac{1}{2d(n-k)}$ because $\frac{1}{2d(n-1)} < \frac{1}{2d(n-k)}$ since $k \geq 2$, implying that $q^M > 0$ holds for all admissible parameters, i.e., $\alpha < \overline{\alpha}$.

Since $\kappa \leq 2$, implying that $q \geq 0$ noids for all admissible parameters, i.e., $\alpha < \overline{\alpha}$. *Comparative statics.* The public firm's output, q^M , satisfies $\frac{\partial q^M}{\partial c} = -\frac{1-2\alpha d(n-k)}{1+2\alpha d+(1-\alpha)(n-k+1)} < 0$ which holds since $\alpha < \overline{\alpha} \equiv \frac{1}{2d(n-1)}$, $\frac{\partial q^M}{\partial \alpha} = -\frac{(n-k+1)(1-c)[2d(n-k+1)-1]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$, $\frac{\partial q^M}{\partial d} = -\frac{2\alpha(1-c)(n-k+1)[1+(n-k)(1-\alpha)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$, $\frac{\partial q^M}{\partial d} = -\frac{(1-c)(1+2\alpha d)[\alpha(2d-1)+1]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$, $\frac{\partial q^M}{\partial k} = -\frac{(1-c)(1+2\alpha d)[\alpha(2d-1)+1]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} > 0$. Every outsider's output, q^M_O , satisfies $\frac{\partial q^M_O}{\partial c} = -\frac{1+\alpha(2d-1)}{1+2\alpha d+(1-\alpha)(n-k+1)} < 0$, $\frac{\partial q^M_O}{\partial \alpha} = \frac{(1-c)[2d(n-k+1)-1]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} > 0$, $\frac{\partial q^M_O}{\partial d} = \frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} > 0$, $\frac{\partial q^M_O}{\partial n} = \frac{(1-\alpha)(1-c)[\alpha(1-2d-1)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$, and $\frac{\partial q^M_O}{\partial k} = \frac{(1-\alpha)(1-c)[\alpha(2d-1)+1]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} > 0$. Finally, the merged entity's output, q^M_I , exhibits the same comparative statics as q^M_O .

6.8 Proof of Lemma 4

The aggregate output is

$$Q^{M} = q^{M} + (n - k - 1)q_{O}^{M} + q_{I}^{M}$$

=
$$\frac{(1 - c)[1 + (1 - \alpha)(n - k)]}{1 + 2\alpha d + (1 - \alpha)(n - k + 1)}$$

which is unambiguously positive. In addition, Q^M satisfies $\frac{\partial Q^M}{\partial c} = -\frac{1+(n-k)(1-\alpha)}{1+2\alpha d+(1-\alpha)(n-k+1)} < 0;$ $\frac{\partial Q^M}{\partial \alpha} = \frac{(1-c)[1-2d(n-k+1)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0$ since $d > \frac{1}{2}$ by definition; $\frac{\partial Q^M}{\partial d} = -\frac{2\alpha(1-c)[1+(n-k)(1-\alpha)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0;$ $\frac{\partial Q^M}{\partial k} = -\frac{(1-\alpha)(1-c)[1+\alpha(2d-1)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} < 0;$ and $\frac{\partial Q^M}{\partial n} = \frac{(1-\alpha)(1-c)[1+\alpha(2d-1)]}{[1+2\alpha d+(1-\alpha)(n-k+1)]^2} > 0.$

6.9 Proof of Corollary 2

Dividing Q^M from Lemma 4 over $Q^{SO} = \frac{1-c}{1+2d}$, and rearranging, yields

$$ER^{M} = \frac{(1+2d)[1+(1-\alpha)(n-k)]}{1+2\alpha d + (1-\alpha)(n-k+1)},$$

which satisfies $ER^M > 1$ if and only if $d > \frac{1}{2(n-k+1)}$. Since $d > \frac{1}{2}$ by definition and $n \ge 3$, condition $d > \frac{1}{2(n-k+1)}$ holds for all admissible parameters, implying that $ER^M > 1$.

 $\frac{d}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N} \frac{1}{2(n-k+1)} \text{ for all attributes parameters, } \lim_{n \to \infty} \sum_{j=0}^{N}$

$$\frac{\partial ER^{NM}}{\partial k} = -\frac{(1-\alpha)(1+2d)[1+\alpha(2d-1)]}{[1+2\alpha d + (1-\alpha)(n-k+1)]^2} < 0.$$

6.10 Proof of Lemma 5

No environmental damages. As shown in Lemma 4, aggregate output satisfies $Q^M < Q^{NM}$ under all parameter values. When the CA ignores environmental damages, welfare without the merger is

$$W_{NP}^{NM} = \frac{(1-c)^2 [n-\alpha(n-1)] [2+\alpha(4d-1)+n(1-\alpha)]}{2[1+2\alpha d+n(1-\alpha)]^2}$$

and that after the merger is

$$W_{NP}^{M} = \frac{(1-c)^{2}[1+(n-k)(1-\alpha)][3+(n-k)(1-\alpha)+2\alpha(2d-1)]}{2[2+\alpha(2d-1)+(n-k)(1-\alpha)]^{2}}$$

where W_{NP}^{M} coincides with W_{NP}^{NM} when k = 1. The merger improves welfare if and only if $W_{NP}^{M} - W_{NP}^{NM} > 0$, where

$$W_{NP}^{M} - W_{NP}^{NM} = -\frac{(1-\alpha)\left(1-c\right)^{2}\left[1+\alpha(2d-1)\right]^{2}(k-1)\left[3+(2n-k)(1-\alpha)+\alpha(4d-1)\right]}{2\left[2+(n-k)(1-\alpha)+\alpha(2d-1)\right]^{2}\left[1+2\alpha d+n(1-\alpha)\right]^{2}} < 0$$

holds for all admissible parameters.

Environmental damages. When the CA considers environmental damage in its welfare function, welfare before the merger is

$$W^{NM} = \frac{(1-c)^2 [n-\alpha(n-1)] [2-n(2d\beta-1)+\alpha(2d(2+(n-1)\beta)-(n+1))]}{2[1+2\alpha d+n(1-\alpha)]^2}$$

while that after the merger is

$$W^{M} = \frac{(1-c)^{2}[1+(n-k)(1-\alpha)][3+(n-k)(1-\alpha)+2\alpha(2d-1)-2d\beta(1+(n-k)(1-\alpha))]}{2[2+(n-k)(1-\alpha)+\alpha(2d-1)]^{2}}.$$

Defining the welfare gain from the merger as $\Delta W \equiv W^M - W^{NM}$, we obtain that $\Delta W = 0$ at $k = \overline{k}(\beta)$, where cutoff $\overline{k}(\beta)$ is

$$\overline{k}(\beta) \equiv \frac{[1+\alpha(2d-1)][3+\alpha(4d-1)+2n(1-\alpha)]-2d\beta[1+\alpha(2(1+d)+\alpha(2d-1))]}{(1-\alpha)[1+\alpha(2d-1)-2d\beta(1+\alpha+2\alpha d+2n(1-\alpha))]}.$$

Analyzing cutoff $\overline{k}(\beta)$, we first find that, when evaluated at $\beta = 0$, this cutoff collapses to $\overline{k}(0) = \frac{3+\alpha(4d-1)+2n(1-\alpha)}{1-\alpha}$, which lies above n, $\overline{k}(0) > n$, since $n - \overline{k}(0) = -\frac{3+\alpha(4d-1)+n(1-\alpha)}{1-\alpha} < 0$ for all admissible parameters. Then, this cutoff unambiguously increases in β since

$$\frac{\partial \overline{k}(\beta)}{\partial \beta} = \frac{4d[1 + \alpha(2d - 1)][1 + 2\alpha d + n(1 - \alpha)]^2}{(1 - \alpha)[1 + \alpha(2d - 1) - 2d\beta(1 + \alpha + 2\alpha d + 2n(1 - \alpha))]^2} > 0$$

and reaches a vertical asymptote at $\beta \equiv \tilde{\beta} \equiv \frac{1+\alpha(2d-1)}{2d[1+\alpha(2d+1)+2n(1-\alpha)]}$. Further increases in β then entail that cutoff $\bar{k}(\beta)$ crosses the horizontal axis, $\bar{k}(\beta) = 2$, since $k \geq 2$ by assumption, at

$$\underline{\beta} \equiv \frac{[1 + \alpha(2d - 1)][1 + \alpha(4d + 1) + 2n(1 - \alpha)]}{2d[\alpha(2 + \alpha - 2d(1 - 3\alpha)) - 1 + 4\alpha n(1 - \alpha)(1 + d) + 2n^2(1 - \alpha)^2]}$$

and keeps increasing in β , so it solves $\overline{k}(\beta) = n - 1$ at

$$\overline{\beta} \equiv \frac{[1+\alpha(2d-1)][4+2\alpha(2d-1)+n(1-\alpha)]}{4d[1+\alpha(2d+1-\alpha)]+2dn(1-\alpha)[5+\alpha(2d-3)]}$$

where cutoffs $\overline{\beta}$ and $\underline{\beta}$ satisfy $\overline{\beta} > \underline{\beta}$ since cutoff $\overline{k}(\beta)$ is unambiguously increasing in β .

Therefore, for all $\beta < \tilde{\beta}$, $\Delta W > 0$ for all $k > \overline{k}(\beta)$. However, since cutoff $\overline{k}(\beta)$ lies unambiguously above n, condition $k > \overline{k}(\beta)$ does not hold, and the merger is blocked under all parameters. In contrast, for all $\beta \ge \tilde{\beta}$, $\Delta W > 0$ for all $k < \overline{k}(\beta)$. In this setting, $\overline{k}(\beta)$ crosses the horizontal axis at $\underline{\beta}$, increases in β , and crosses n at $\overline{\beta}$, implying that for $\beta \ge \hat{\beta}$ the merger is approved if and only if $k < \min\{n, \overline{k}(\beta)\}$.

6.11 Proof of Proposition 1

Using outputs from Lemma 1, we find the equilibrium profits for each private firm, $\pi_i^{NM} = \left(\frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)}\right)^2$. The equilibrium profits for merging firms (insiders) is $\pi_I^M = \left(\frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)(n-k+1)}\right)^2$. Hence, firms merge if and only if

 $\left(\frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+(1-\alpha)(n-k+1)}\right)^2 \ge k \left(\frac{(1-c)[1+\alpha(2d-1)]}{1+2\alpha d+n(1-\alpha)}\right)^2$

Applying square roots on both sides of the above inequality, and dividing both sides by $(1 - c)[1 + \alpha(2d - 1)]$, yields

$$\frac{1}{1+2\alpha d + (1-\alpha)(n-k+1)} \ge \sqrt{k} \frac{1}{1+2\alpha d + n(1-\alpha)}$$

Solving for k, and dividing both sides by n, yields

$$\frac{k}{n} \ge \frac{\hat{k}}{n} \equiv \frac{3 + 2n + \alpha[4(d-1) - \alpha(4d-1) - 2n(2-\alpha)] - \theta}{2n(1-\alpha)^2}$$

where $\theta \equiv [(1-\alpha)^3 [5+\alpha(8d-1)+4n(1-\alpha)]]^{1/2}$. When $\alpha = 0$, cutoff $\frac{\hat{k}}{n}$ simplifies to $\frac{\hat{k}}{n} = \frac{3+2n-[4n+5]^{1/2}}{2n}$, as in Salant et al. (1983), but when $\alpha > 0$ this cutoff satisfies

$$\frac{\partial\left(\frac{k}{n}\right)}{\partial\alpha} = \frac{(1+2d)[\alpha(2-\alpha)-1+\theta]}{n(1-\alpha)^{2\theta}} > 0$$

since $\theta > 1$ for all admissible parameters; and

$$\frac{\partial \left(\frac{\hat{k}}{n}\right)}{\partial d} = -\frac{2\alpha \left[\frac{(1-\alpha)^2}{\theta} - 1\right]}{n(1-\alpha)} > 0$$

because $\frac{(1-\alpha)^2}{\theta} < 1$ since $\theta > 1$ and $\alpha < \overline{\alpha}$.

In addition, cutoff \hat{k} satisfies $\hat{k} < n-1$, thus being binding, if and only if $n < \overline{n} \equiv \frac{5+2\alpha[4d-3+\alpha[1-2d(1-d)]]}{(1-\alpha)^2}$, giving rise to two regions: (i) when $n < \overline{n}$, a merger is profitable if, furthermore, $k > \hat{k}$; otherwise, a merger is not profitable for any value of k. Cutoff \overline{n} is increasing in both α and d since $\frac{\partial \overline{n}}{\partial \alpha} = \frac{2[2(1+2d)+\alpha(4d^2-1)]}{(1-\alpha)^3} > 0$ and $\frac{\partial \overline{n}}{\partial d} = \frac{4\alpha[2+\alpha(2d-1)]}{(1-\alpha)^2} > 0$.

For the merger to be profitable and approved, we need that $\overline{k}(\beta) - \widehat{k} > 0$, where cutoff $\overline{k}(\beta)$ was found in Lemma 5. When $\alpha = \beta = 0$, this difference is $\frac{3+2n+[4n+5]^{1/2}}{2} > 0$. When $\beta = 0$, the difference becomes

$$\overline{k}(0) - \widehat{k} = \frac{3 + 2n + \alpha[4(d-1) - \alpha(4d-1) - 2n(2-\alpha)] + \theta}{2(1-\alpha)^2}$$

which is positive for all admissible values of α and d. Finally, when $\alpha, \beta > 0$, the difference $\overline{k}(\beta) - \widehat{k} > 0$ for all $\beta > \widehat{\beta}$, where

$$\widehat{\beta} \equiv \frac{[1 + \alpha(2d - 1)] \left[2 - 2\alpha(2 - d + \alpha(d - 1)) + \theta\right]}{2d(1 - \alpha)[1 + 4n + \alpha(2 - \alpha(3 + 4d^2 - 4n) - 8n)]}.$$

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