Newspaper Market: Impact of advertisements on quality choices and market structure.

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Abstract

Newspaper markets are highly concentrated, with most being monopolies or duopolies within a service area. Existing literature attributes this to the network effect arising from customers deriving positive utility from advertisements. We demonstrate that newspaper markets can be concentrated due to endogenous investment in quality, when quality improvements involve fixed costs such as newsroom size. This explanation more closely aligns with empirical evidence, which shows that market concentration persists even when classified ad revenues significantly declined due to online platforms like Craigslist. Our model extends the vertical differentiation framework by incorporating the advertisement side of the newspaper market and demonstrates that several different types of market and product configurations emerge depending on advertisement levels. At low levels of advertising, the market tends toward a natural monopoly but lower-end market remains unserved. As advertising levels increase, a second firm may enter to fill the product gap at the lower end of the market. At moderate levels of advertising, the top-quality firm acts as a monopolist, while the second firm offers a free newspaper, generating revenue solely from advertisements. When advertising levels are high, the top-quality firm increases its product quality and offers a premium product at a lower price, aiming to protect its highly valuable customer base and thereby gaining a significant market share. In the face of potential new entrants, the top firm may adopt an entry deterrence strategy to drive out competitors. Our results are robust regardless of consumers getting positive or negative utility from advertisements.

***JEL* Classification:** L10, D43.

Key Words: Newspaper, market structure, product quality, vertical-differentiation, entry deterrence, 2-sided platform, media, advertisement

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1 Introduction

Newspaper markets in the USA and OECD countries are highly concentrated. While at the national level there may be multiple players with relatively equal shares, local and regional markets are often monopolies (Rosse, 1980; Dertouzos and Trautman, 1990). For instance, even though the USA has over 1,000 daily newspapers, more than 95% of cities have only one daily paper. In larger cities with two or more papers, these papers often differ in format (tabloid versus broadsheet) or political alignment (left- or right-leaning editorials). This pattern is also observed in developing countries. For example, metropolitan cities in India typically have one dominant English daily newspaper with more than a 60% market share[1].

Such market power for a leading firm is unique to print media and is not seen in other types of media. The extensive literature on print media has been driven by the need to explain this concentration, particularly the prevalence of "one-newspaper cities." Most studies attribute this to the network externality effect, which occurs when consumers derive positive utility from advertising[2]. The positive feedback loop between circulation and advertising can lead to a monopoly market, unless consumer has a strong preference for variety. In this regard, print media differs from other media such as television and radio, where consumers often view ads as a nuisance. In print media, consumers can easily ignore advertisements, and in some cases, such as classifieds, they may even welcome more ads. Several studies (Rosse, 1970; Dertouzos and Trautman, 1990; Thompson, 1989) provide empirical support for the view that consumers appreciate advertisements.

However, the theory relying purely on the network effect of advertisements fails to explain why such concentration continues to exist despite the significant decline in classified ads with the arrival of online platforms like Craigslist and Monster.com. Moreover, recent studies found that readers' attitudes toward advertisements differ across countries and regions (Sonnac, 2000; Van Cayseele and Vanormelingen, 2009; Filistrucchi et al., 2012). Readers in many European countries do not like commercial advertisements in newspapers (Gabszewicz et al., 2002).

Recent empirical evidence suggest that endogenous investment in quality could be a determinant of concentration. Berry and Waldfogel (2010) using cross-sectional data of metropolitan dailies in the USA, found empirical evidence that firms invest in quality as the market grows and that these costs are fixed in nature. They measured quality by the number of pages (more content), the number of journalist staff (more news produced rather than relying on wire reports), and the quality of staff by counting the number of Pulitzer awards. They found that when the market size increases, the number of newspapers changes relatively little, but the nature and quality of newspapers change

[1] Hindustan Times in New Delhi, Times of India in Mumbai and Bangalore, The Hindu in Chennai, and Deccan Chronicle in Hyderabad.

[2] see (Furhoff, 1973; Bucklin et al., 1989; Gabszewicz et al., 2007; Häckner and Nyberg, 2008; Chaudhri, 1998; Merrilees, 1983; Blair and Romano, 1993)

dramatically across different market sizes. This corroborates the argument by Shaked and Sutton (1987) that when the burden of quality improvement falls on fixed costs, product proliferation will not occur when market size increases. They argue that as markets grow larger in industries where quality is produced mainly through outlays on fixed costs, at least one firm will have an incentive to invest in quality. A firm producing a higher-quality product can undercut its rivals' prices and attain substantial market share[3].

Angelucci and Cage (2019) provided further evidence that quality plays a major role in the newspaper market. Using data on French dailies and an exogenous shock to newspaper advertising, they showed that as advertising revenue declines, newspapers produce less journalistic-intensive content (or quality), measured by the size of newsroom staffs.

However, the literature on the newspaper industry has very few papers that model the newspaper market structure based on the interaction of product quality choice and advertisements. Gabszewicz et al. (2012) is one such study that shows the interaction between newspaper quality and advertisement, but their primary focus is to explain the rise of free daily newspapers. Gabszewicz and Wauthy (2014) is another paper that extend the vertical differentiation model for a two-sided platform, but their model relies on the exogenous presence of network externalities across 2-sides rather than on endogenous investment in quality. They show that in the presence of cross-network externalities, if consumers are willing to pay more for a platform with a larger network size, an asymmetric equilibrium can be sustained.

This paper adopts an approach similar to Gabszewicz et al. (2012) and extends the standard vertical differentiation models (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1983) to include interaction with the advertisement side of the market. We demonstrate how different types of market structures and quality choices of players evolve as the advertisement level increases. Like other studies on vertical differentiation (see Gabszewicz and Thisse (1979); Wauthy (1996)), our results show that high-quality firms have an advantage due to their investment in quality, allowing them to attain a significant market share. However, they will not serve the lower end of the market unless consumer preference for quality is homogeneous. Since the high-quality firm does not cater to the lower-end market, it creates an opportunity for the low-quality firm to fill the product gap and serve the lower-end market, provided the advertisement level is not too low. Therefore, a natural monopoly occurs when consumer preference is homogeneous and/or the advertisement level is low. When the advertisement level is moderately high, the low-quality firm will serve the lower end of the market as a free product with the lowest quality, while the high-quality firm behaves as a monopolist without competition, similar to what is suggested by Gabszewicz et al. (2012).

[3] Berry and Waldfogel (2010) also compared their findings with the restaurant industry, where the burden of quality falls on marginal costs. They found that product variety increases with market size because the high-quality firm cannot easily undercut the low-quality firm with lower marginal costs.

A novel and interesting finding in this paper is that as the advertisement level increases further, the low-quality player can challenge the high-quality firm's market leadership. This forces the high-quality firm to significantly raise its quality—much more than the monopolist level—to protect its customer base. In extreme cases, the high-quality firm might even drive out the competitor and deter further entry. Consequently, the high-quality firm offers a premium product with a lower price-to-quality ratio[4], and both the quality and the price-to-quality ratio improve with higher advertisements, benefiting consumers. This finding aligns with the empirical evidence provided by Angelucci and Cage (2019) and Pattabhiramaiah (2014), which shows that as advertisement revenue declines, the quality of the leading newspaper decreases and the price may rise. To our knowledge, this aspect of the impact of advertisements has not been considered in any other papers.

Furthermore, we model the quality choices of duopoly players when there is a potential entry of a third player. The entry of the third player increases market competition, leading both existing players to raise their quality further and reduce their price-to-quality ratios. In fact, when advertisement levels are high, the profits of the top two players decline with increasing advertisement, which is the opposite of what happens in the duopoly model. This is similar to Donnenfeld and Weber (1995)'s finding that under vertical differentiation, product competition among duopoly incumbents leads to entry deterrence. This provides a testable case for our model.

Though our model shares some similarities with Gabszewicz et al. (2012), it is distinct in several key aspects: a) *Quality-dependent fixed costs:* This allows us to model the quality choice of firms and makes our results robust[5]; b) *Advertisers prefer affluent consumers:* to incorporate empirical findings; c) *Sequential entry for firms:* This introduces a new set of equilibria and reflects market dynamics where the leading firm has a significant advantage that can be used for preempting quality space or entry deterrence[6]; d) *Impact of third-player entry:* This has significant implications for the duopoly results. Additionally, we test the robustness of our results under simultaneous entry, different levels of consumer preference heterogeneity, and when consumers receive positive or negative utility from advertisements. Our model also shares some similarities with Lutz (1997), which is one of the few papers that model sequential entry with quality-dependent fixed costs under vertical differentiation. Our model adds advertisement side interaction to that framework and studies broader aspects of market structure, whereas Lutz (1997) focuses solely on entry deterrence. This paper makes contribution to both the newspaper market and vertical differentiation literature.

[4]We use the price-to-quality ratio to effectively represent price because standalone price could be driven by changes in quality. A lower price-to-quality ratio more directly conveys higher consumer surplus.

[5]Most models assume the cost of providing quality is L-shaped. Under such assumptions, the high-quality firm always chooses the upper bound quality, which does not capture variations in quality choices with advertisements.

[6]Simultaneous entry models do not have an equilibrium when advertisement levels are high, which is also missing in Gabszewicz et al. (2012).

2 Related Literature

An important aspect of the newspaper market is that it is two-sided, catering to two types of customers: readers and advertisers. Advertisers value circulation, so advertisement demand is linked to readers demand. At the same time, readers may like or dislike advertising, leading to interdependencies between the two sides. Initial literature used a "structural" model, deriving interdependent inverse demand equations for circulation and advertising, which are then estimated using empirical data. Rosse (1967) was one of the earlier papers to use the structural model. They estimated that there are economies of scale in production costs. However, they also indicated that returns to scale have remained fairly constant since 1939, which may not explain the rising concentration in the newspaper market. Rosse (1970) estimated positive cross-effects from advertising to consumers. Similarly, Dertouzos and Trautman (1990) used structural equations to show that there are economies of scale in production as well as positive cross-effects from advertisements to consumers. They also showed that product quality positively affects circulation demand and that circulation demands are higher in high-income markets. However, they concluded that chain newspapers do not have any advantage over independent newspapers, suggesting that this scale effect is likely localized to content and distribution. Thompson (1989) followed a similar structural equation model and found that readers appreciate advertising. They also identified that advertisers value affluent consumers, creating a tradeoff between newspaper circulation and the share of high-income readers.

Since these papers identified positive effects of advertisements on circulation, much of the theoretical literature that followed explained the "one-newspaper cities" phenomenon using network effects. Furhoff (1973) was among the first to propose the theory of the circulation spiral, which is based on the positive feedback loop between advertising and circulation. In the limit, these spirals can lead to a monopoly situation. Bucklin et al. (1989) argued that such network effects make the market prone to predatory behavior by firms with a cost advantage, driving other firms out of the market. Merrilees (1983) used a descriptive study of a price war between Sydney-based newspapers to explain a similar effect. Gabszewicz et al. (2007) analyzed the positive effect of advertising on consumers in a duopoly where consumers also have preferences for the political stance of newspapers. He showed that in such scenarios, a weaker newspaper with differentiation may still survive if the advertising intensity is not high. Similarly, Häckner and Nyberg (2008) suggested that either a monopoly equilibrium or an asymmetric market share equilibrium exists if horizontal differentiation is low or consumer preference for quality content is high, in which case advertisements play a smaller role. These two papers demonstrate that a positive valuation for advertising alone is insufficient for monopoly if consumers prefer differentiation and/or the advertisement effect is small. Therefore, smaller cities with homogeneous political preferences would have a monopoly, while larger cities with heterogeneous preferences would have a duopoly.

Chaudhri (1998) analyzed pricing when consumers have a positive valuation for advertising. He examined the two cases of monopoly and perfect competition and showed that the monopoly market has a much lower circulation price, resulting in higher circulation and social welfare. Blair and Romano (1993) focused on the monopoly case and reached a similar conclusion. However, both these papers assumed the market structure as exogenous.

Some recent studies suggest that consumers' attitudes towards advertisements in newspapers can differ across countries and regions (Sonnac, 2000). Gabszewicz et al. (2002) highlighted that newspaper readers in many European countries are ad-avoiders. Filistrucchi et al. (2012) found that Dutch readers appreciate advertising, while Van Cayseele and Vanormelingen (2009) showed that Belgian readers are ad-neutral. Incorporating these new findings, Anderson and Gabszewicz (2006) modeled the newspaper market assuming a mix of consumers, some ad-haters and some ad-lovers. They concluded that under stronger ad-attraction, concentration in the press industry should be expected, but with weaker ad-attraction, two newspapers with different horizontal characteristics can survive.

As newspapers' ad revenues declined rapidly after the advent of online platforms like Craigslist, market concentration continued to persist, casting doubt on whether the positive effect of advertisements was the primary driver of concentration. Recent empirical studies have identified that product quality plays a major role in determining newspaper market structure. Berry and Waldfogel (2010), using cross-sectional data of metropolitan dailies in the USA, found empirical evidence that firms invest in quality as the market grows and that these costs are fixed in nature. They found that when the market size increases, the number of newspapers changes relatively little (aside from horizontally differentiated suburban dailies), but the nature and quality of newspapers change dramatically across different market sizes. This highlights the vertical differentiation nature of the newspaper market. Under horizontal differentiation, the number of newspapers will increase as the market size increases. This also corroborates the argument of Shaked and Sutton (1987) that when the burden of quality improvement falls on fixed costs, product proliferation will not occur as the market size increases.

Angelucci and Cage (2019) provided further evidence that quality plays a major role in the newspaper market. Using data on French dailies and an exogenous shock to newspaper advertising, they showed that as advertising revenue declines, newspapers produce less journalistic-intensive content (or quality), measured by the size of newsroom staffs. Pattabhiramaiah (2014) showed similar evidence in the US newspaper market, demonstrating that as ad revenues decline, newspapers increase prices and reduce quality.

However, very few theoretical papers model the newspaper market using vertical differentiation. Gabszewicz et al. (2012) is one such paper, but they focused on explaining the entry of free newspapers. Gabszewicz and Wauthy (2014) extended the vertical differentiation model to a two-sided platform, but without investment in quality. They assumed the presence of cross-network externalities across two-side and that consumers are willing to pay more for a platform with a larger network size, which leads to an asymmetric equilibrium.

Most recent theoretical literature in the media market has used the platform market framework developed by Armstrong (2006) and Rochet and Tirole (2003). This framework has been extended for print media where newspapers are horizontally differentiated based on political leaning (Gabszewicz et al., 2001, 2007; Häckner and Nyberg, 2008; Anderson and Gabszewicz, 2006). For example, Anderson and Gabszewicz (2006) used the two-sided market with a Hotelling setup and assumed that viewers dislike advertisements, showing that advertisements result in newspapers locating their political opinions in the center.

This paper models the newspaper market under vertical differentiation by extending the standard vertical differentiation model to include the advertisement side. Though our model shares some similarities with Gabszewicz et al. (2012), it is distinct in several key aspects, enabling us to draw a new set of insights.

3 The model

A newspaper market consists of three types of agents: firms (newspapers), consumers (newspaper readers), and advertisers. A market has one or more firms, with each firm offering one newspaper. Firms are vertically differentiated by the choice of quality of their newspapers. The quality of a newspaper represents its effort in producing information content desired by consumers. Higher effort results in more researched and relevant content, which is perceived to be of higher quality by the readers. All firms in a market have the access to same production technology, with a constant unit printing and circulation cost, c, that is quality-independent and a fixed production cost that is a convex function of quality, $K(\theta) = \alpha\theta^2$. Each firm that enters the market chooses a subscription price (s) per reader, quality (θ), and advertisement price (p) per reader.

There are M consumers in a market. Consumers differ in their disposable income and preferences for reading news. Consumers' disposable incomes follow log-normal distribution with parameters (μ, σ^2). The consumer with higher income has a higher willingness to pay for quality. At the same time, each consumer may have a different willingness to pay due to her outside options or preference for news reading. Consumer i with income Y_i receives utility U_i from reading a newspaper with quality θ and subscription price s. U_i is represented by the utility function:

$$U_i = v_i Y_i \theta - s$$

where the product $v_i Y_i$ represents consumers' willingness to pay for quality. This multiplicative form allows us to consider both factors, income Y_i and consumer preference for quality content v_i, in determining the willingness to pay for quality. For example, some high-income consumers do not prefer subscribing to newspaper (low v_i) as they get news from alternative sources or do not like reading news. Similarly, some low income consumers have a higher willingness to pay due to their strong preference for quality news (high v_i).[7] v_i follows a uniform distribution $\sim U(0,1)$. For robustness, we also parameterize the level of heterogeneity using the distributional form $\sim U(b-1,b)$ $1 \leq b \leq 2$ (see section 8.2). The higher the b, the more homogeneous the preference. Consumers do not get utility from advertisements, which means that the advertisements do not cause nuisance as consumers can ignore advertisements, and consumers do not subscribe to a newspaper to see advertisements. The extension in section 8.3 and 8.4 discusses the result when consumers receive negative or positive utility from advertisements, respectively. A consumer subscribes to at most one newspaper, which means consumers single-home[8]. Therefore, the consumer choice problem can be represented as:

$$\max_k v_i Y_i \theta_k - s_k \text{ s.t. } U_i \geq 0$$

Advertisers are homogeneous within a market and are willing to pay βY_i to target a consumer with income Y_i. We assume β is exogenous.[9] Since advertisers are willing to pay more for high-income consumers, a newspaper that attracts disproportionately high-income consumers will have higher advertisement revenue per customer, consistent with empirical evidence (Thompson, 1989; Dertouzos and Trautman, 1990)[10]. We use a representative advertiser to model a set of homogeneous advertisers. Advertisers place ads in multiple newspapers to reach different sets of consumers, meaning advertisers multi-home.

Each market is characterized by a set of exogenous market factors (M, μ, σ^2), production technology (c, α), and advertisement level β. Each firm in this market chooses endogenous parameters: subscription price (s), newspaper quality (θ), and advertisement price (p) per unit of circulation. We use the price-to-quality ratio to effectively represent firms' pricing strategies, as this ratio is more directly related to consumer surplus. Standalone price changes could be driven by changes in quality.

Firms enter the market sequentially. The timing of their decisions is as follows:

Entry Stage: Firms enter sequentially and each firm chooses the quality before the next firm makes entry decision.

[7] Alternatively, we could have chosen the additive form $v_i + Y_i$, which changes the form of the demand function but does not change the result.

[8] This follows from the vertical differentiation. If a consumer subscribes to two newspapers, say $\{1,2\}$, then their utility is given by $\max(\theta_1, \theta_2) - s_1 - s_2$. So unless the low-quality newspaper is free, the utility maximizing consumer will never buy both products.

[9] This is equivalent to endogenous price when advertiser's utility is linear in the number of users; higher demand market has higher β.

[10] This has been observed in Indian market as well. English dailies that target affluent customers has advertisement rates 3 times that of other dailies.

Price Stage: Firms that have entered the market simultaneously set their subscription prices (s) and advertisement prices (p).

We solve for the pure strategy Subgame Perfect Nash Equilibrium. We also assume that firms do not incur fixed entry cost other than the cost to establish quality.

3.1 Key assumptions and rationale

We make following assumptions in our model.

1 *Firms are only vertically differentiated.* This paper focuses on the impact of advertising and consumer heterogeneity on the quality of newspapers, which is one of the key factors determining market structure. Therefore, it is natural to focus on vertical differentiation and abstract away from the variety due to horizontal differentiation. In many contexts, horizontal differentiation can be treated as a separate market, in which case our results will still hold. This applies when the consumers of two differentiated products do not overlap or when they do not have to choose one product over the other. For example, in the Indian context, English and Vernacular newspapers can be considered different markets with distinct competitive dynamics and consumer profiles. Similarly, a financial newspaper can be considered a separate market from general dailies when a consumer's choice of a general newspaper does not preclude her from subscribing to a financial newspaper. However, this does not apply in cases of horizontal differentiation due to partisanship, in which case the demand function is a mixture of pure vertical models, and the product variety will increase. See Gabszewicz and Thisse (1986) and Neven and Thisse (1989) for the models that use both horizontal and vertical differentiation, though not in a newspaper context.

2 *Quality improvement is through fixed cost of production.* The content quality of a newspaper is primarily determined by the number of journalists and the quality of staff (e.g., award-winning journalists), which are part of the fixed costs. While quality could also be related to the quality of printing and/or the number of pages, both of which impact variable costs, empirical evidence clearly suggests that newspaper quality is primarily driven by fixed costs (Reddaway, 1963; Berry and Waldfogel, 2010; Angelucci and Cage, 2019).

3 *Income follows a log-normal distribution.* The income distribution of a population is widely modeled using a log-normal distribution, as it fits many income datasets (see reference). The Pareto distribution is another commonly used model for income, but its moments are restricted for certain parameter ranges, making it unsuitable for some situations, specifically at lower income levels. The log-normal distribution also has the advantageous property that if the pre-tax income is log-normal and the tax schedule is progressive in the form ax^b, then the disposable income also follows a log-normal distribution. The log-normal distribution allows for calculating the Gini-coefficient, a standard measure of income inequality, using the single parameter σ, which is given by $G(\sigma) = 2\Phi(\frac{\sigma}{\sqrt{2}}) - 1$. This enables us to calibrate σ across markets with varying levels of income-inequality.

4 *Consumers are neutral to advertisements.* This has been widely discussed in the literature. Some studies find that newspaper and magazine readers appreciate advertisements, specifically classifieds (Rosse, 1970; Thompson, 1989; Filistrucchi et al., 2012; Dertouzos and Trautman, 1990). However, other studies argue that readers are ad-neutral as they can easily ignore advertisements (Gabszewicz et al., 2001). Fan (2013) and Van Cayseele and Vanormelingen (2009) find empirical evidence supporting ad-neutrality. Sonnac (2000) conducts a cross-country analysis and finds that readers' attitudes vary across countries. Nonetheless, most structural analyses of newspaper and magazine markets model readers as being indifferent to advertising (Fan, 2013; Gabszewicz et al., 2012; Gentzkow et al., 2014). We assume readers are ad-neutral in our base model; however, we discuss the impact of consumers being ad-lovers or ad-haters on our results in the robustness section.

5 *Consumers' willingness to pay depends on both income and preference for quality content.* We consider both factors to generalize the demand function and compare markets with varying income and reader characteristics. Additionally, we parameterize the level of consumer heterogeneity in a market using the distribution $U(b-1, b)$. The literature typically considers only one factor, which is either income (Gabszewicz and Thisse, 1979) or preference for quality (Wauthy, 1996; Gabszewicz et al., 2012).

6 *Advertisers are homogeneous in a market:* Advertiser heterogeneity is not modeled for simplicity, as it does not impact our results. Our model is equivalent to one with heterogeneous advertisers having linear utility functions: an advertiser of type μ receives utility $\mu N_i - p_i$ by advertising in newspaper (i) with N_i readers and an advertisement price p_i. The newspaper's profit from advertisements in such cases is equivalent to a constant unit advertising price, which in our case is β (see Gabszewicz et al. (2012)). Due to single-homing customers, all firms charge monopoly pricing to advertisers; hence, advertiser heterogeneity does not change competitive dynamics. We also don't analyze the advertisers welfare except calculating the social planner's choice of quality.

7 *Firms enter sequentially.* Since quality improvement is achieved through fixed costs, the vertical differentiation model confers an endogenous advantage to the higher-quality firm. Therefore, the first mover gains a significant advantage by preempting the profitable higher-quality niche. The sequential entry model captures this dynamic. Additionally, sequential entry ensures that a pure strategy equilibrium exists across all parameter values. Shaked and Sutton (1987) pointed out that sequential entry guarantees a pure strategy equilibrium at the product choice stage if the price stage has an equilibrium, which is not necessarily true for simultaneous entry. However, for completeness, we also provide the simultaneous entry results in section 8.1 for comparison.

4 Benchmark: Social Planner's Problem

Consider a social planner who sets the subscription price s and the quality θ such that $(s, \theta) \in \mathbb{R}^2_+$ to maximize the total surplus, which includes subscription profit, consumer surplus, and advertiser surplus. A consumer i will subscribe to the newspaper if and only if she derives non-negative utility:

$$U_i = v_i Y_i \theta - s \geq 0 \text{ which implies } v_i \geq \frac{s}{Y_i \theta}$$

For uniform distribution of v and log-normal income distribution $ln(Y) \sim N(\mu, \sigma^2)$, the demand function $N(s, \theta)$ is given by[11]:

$$N(s, \theta) = \begin{cases} M \int_0^\infty Pr(v \geq \frac{s}{Y\theta}) dF(Y) = M(1 - \frac{s}{\kappa\theta}) & \text{if } 0 \leq s \leq \kappa\theta, \\ 0 & \text{if } s \geq \kappa\theta \end{cases} \tag{1}$$

$$\text{where } \kappa = e^{\mu - \frac{1}{2}\sigma^2}$$

Equation (1) highlights that the market will be fully covered only when the subscription price is zero, as some consumers do not value reading a newspaper. When the market is uncovered, the demand is higher in markets with higher median income (μ), ceteris paribus[12], and lower in markets with higher income inequality (σ), ceteris paribus. A higher σ signifies a higher proportion of consumers in the lower tail of the income distribution, who do not subscribe to the newspaper in an uncovered market.

Given the demand function $N(s, \theta)$, the social planner's objective function $W(s, \theta)$ is given by:

$$W(s, \theta) = \underbrace{N(s, \theta)(s - c)}_{\text{subscription net revenue}} + \underbrace{M \int_0^\infty \Big(\int_{\frac{s}{Y\theta}}^1 (vY\theta - s) dv \Big) dF(Y)}_{\text{consumer surplus}} + \underbrace{M\beta \int_0^\infty Y(1 - \frac{s}{Y\theta}) dF(Y)}_{\text{advertiser surplus}} - \underbrace{\alpha\theta^2}_{\text{fixed cost}}. \tag{2}$$

Equation (2) simplifies to[13]:

$$W(s, \theta) = N(s, \theta)(s - c + \kappa\beta) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) - \alpha\theta^2 \tag{3}$$

$$\text{where } \kappa = E[\frac{1}{Y}] = e^{\mu - \frac{\sigma^2}{2}} \text{ and } \delta = E[Y] = e^{\mu + \frac{\sigma^2}{2}}$$

Proposition 1. *A social planner that optimizes the total welfare would set optimal subscription price s_{SP} and quality θ_{SP} such that:*

$$s_{SP} = \max(0, c - \kappa\beta) \tag{4}$$

[11] $\int_0^\infty Pr(v \geq \frac{s}{\theta Y}) dF(Y) = \int_0^\infty (1 - \frac{s}{\theta Y}) dF(Y) = \int_0^\infty dF(Y) - \frac{s}{\theta} \int_0^\infty \frac{1}{Y} dF(Y) = 1 - \frac{s}{\theta} E(\frac{1}{Y}) = 1 - \frac{s}{\theta} e^{-\mu + \frac{1}{2}\sigma^2}$

[12] higher μ implies a higher κ, which implies a higher $1 - \frac{s}{\kappa\theta}$

[13] see Appendix A.1

$$\theta_{SP} = \begin{cases} \frac{M\delta}{4\alpha} \ \textit{if } s_{SP=0} \\ \theta^* \ \textit{that solves } 4\alpha\kappa\theta^3 - Me^{2\mu}\theta^2 + M(c-\kappa\beta)^2 = 0, \ \textit{otherwise} \end{cases} \tag{5}$$

Further, if $c - \kappa\beta$ is sufficiently high then the social planner will not serve the market.

Proof of Proposition 1 follows from the first order condition of (3) (see Appendix A.1). Proposition 1 highlights that the social planner sets the subscription price to cover the variable costs of printing and circulation (c), less offset from advertiser's benefit ($\kappa\beta$). If the advertiser's benefit fully covers the variable costs, then the planner will offer the newspaper for free, achieving full market coverage. Please note that $\kappa\beta$ in true sense is the advertiser's benefit from the marginal consumer and not from every consumer. The total advertiser's benefit is $N(\kappa\beta) + M\beta(\delta - \kappa)$

Definition 1. *We refer the term $\kappa\beta - c$ where $\kappa = e^{\mu - \frac{\sigma^2}{2}}$ as 'advertisement intensity', and use the invertible function $\phi : \beta \to \mathbb{R} \equiv \kappa\beta - c$ to compute it.*[14]

The advertisement intensity measures the contribution from advertisement per new subscriber, net of the variable cost of printing and circulation. This term arises because advertises are willing to pay higher for higher-income consumers. It increases with the median market income (μ) and the advertiser's willingness to pay (β), and decreases with the income inequality (σ). It's important to note that high income inequality reduces advertisement intensity because high proportion of customers are in the lower end of the income.

Note: Since ϕ is a strictly increasing invertible function of β both ϕ and β can be used interchangeably to represent the advertisement level. We describe propositions in terms of exogenous β, whereas equations and cutoff values are defined in terms of ϕ as it simplifies the expressions.

5 Monopolist

Consider that there is only one firm in the market that sets the non-negative subscription price s, quality θ, and advertisement price p per unit of circulation to maximize its profit. A consumer i will subscribe to the newspaper if and only if she receives non-negative utility, meaning:

$$U_i = v_i Y_i \theta - s \geq 0 \text{ implies } v_i \geq \frac{s}{Y_i \theta}$$

Since the consumer decision in this scenario is similar to that of the social planner case, the monopolist's demand function for the uniform distribution of v and log-normal income distribution will be given by (1). The corresponding monopolist's profit function $\Pi(s, \theta, p)$ is:

$$\Pi(s,\theta,p) = \underbrace{N(s,\theta)}_{\text{Unit demand}} \underbrace{(s - c + p)}_{\text{per-unit margin}} - \underbrace{\alpha\theta^2}_{\text{fixed cost}} \tag{6}$$

[14] $\phi(\beta)$ is an invertible function as it is a a well defined strictly increasing function of β.

Therefore, the monopolist's problem can be written as

$$\max_{s,\theta,p} N(s,\theta)(s-c+p)-\alpha\theta^2 \text{ s.t.}$$

$$pN(s,\theta) \le M\int_0^\infty \beta Y(1-\frac{s}{Y\theta})dF(Y) \tag{7}$$

$$s \ge 0, \theta \ge 0$$

(7) is the participation constraint (non-negative surplus)[15] of the advertiser[16].

Monopolist will set p such that the constraint (7) is binding i.e. it will extract full surplus. If not then the monopolist can increase price p by a small amount and increase the profit while still meeting the participation constraint[17]. This implies that

$$pN(s,\theta) = M\beta(E(Y)-\frac{s}{\theta}) = M\beta(\delta-\frac{s}{\theta}) \tag{8}$$

Replacing p from (8) in (6) and using $\phi \equiv \kappa\beta - c$ (Definition 1) we get

$$\Pi(s,\theta) = N(s,\theta)(s+\phi) + M\beta(\delta-\kappa) - \alpha\theta^2 \tag{9}$$

$$\text{where } \delta = e^{\mu+\frac{\sigma^2}{2}},\ \kappa = e^{\mu-\frac{\sigma^2}{2}},\ N(s,\theta) \text{ given by (1)}$$

It is important to note that the term $M\beta(\delta-\kappa)$ arises due to income inequality and is a fixed rent that the monopolist earns by serving customers in the right tail of income distribution. If there is no income-inequality, i.e. $\sigma = 0$, then this term vanishes. The monopolist problem is to choose s and θ that maximizes its profit function (9) subjected to non-negative profit:

$$\max_{s,\theta} \Pi(s,\theta) \text{ s.t. } \Pi(s,\theta) \ge 0, s \ge 0, \theta \ge 0 \tag{10}$$

See Appendix A.2 for the solution of (10). The results are summarized below.

Lemma 1. *Suppose the exogenous parameters are such that the monopolist cover the market partially (interior solution of (10)), then the equilibrium quality θ_{MP} is given by the unique solution of (11)*

$$8\alpha\kappa\theta^3 - M\kappa^2\theta^2 + M\phi^2 = 0 \ s.t.\ \theta > \frac{M\kappa}{12\alpha} \tag{11}$$

and the subscription price s_{MP} is given by (12)

$$s_{MP} = \frac{1}{2}\kappa\theta_{MP} - \frac{1}{2}\phi \tag{12}$$

---

[15]The advertiser gets surplus of βY_i from a subscribing consumer with income Y_i

[16]Note that there is one representative advertiser

[17]This is a standard result in platform market theory when consumers single-home and advertisers multi-home

The quality set by monopolist is increasing in market size (M) and income-level (μ), decreasing in quality cost(α) and income-inequality (σ), and non-monotonic in β and c with single-peak at $\phi = 0$.

Proof of Lemma 1 follows from the first order condition of (9) and implicit function theorem on (11) (see Appendix A.2). The condition $\theta > \frac{M\kappa}{12\alpha}$ meets the necessary second order condition, and the condition (13) below ensures that the (11) has a solution[18].

$$|\phi| < \frac{M\kappa^2}{12\sqrt{3}\alpha} \tag{13}$$

(12) highlights that the higher advertisement intensity, $\phi \equiv \kappa\beta - c$, reduces the subscription price for the consumer. In other words, consumers are subsidized for the externality they exert on advertisers, which is a standard results in the platform market (Armstrong, 2006).

We now identify the critical conditions for corners solutions when the monopolist will not serve the market (zero market coverage) or serve the market with zero subscription price and hence the full market coverage [19].

Proposition 2. *There exist $(\underline{\beta}, \overline{\beta})$ with $0 \leq \underline{\beta} < \overline{\beta}$ such that*

- *a If $\beta < \underline{\beta}$, the monopolist will not serve the market; the $\underline{\beta}$ is positive only if the marginal cost c is sufficiently high.*
- *b If $\beta > \overline{\beta}$, the monopolist will set the subscription price $s_{MP} = 0$ and cover the full market, but will produce the lowest quality $\theta_{MP} = 0$.*
- *c If $\beta \in [\underline{\beta}, \overline{\beta}]$ then the monopolist will set the subscription price and quality as provided in Lemma 1, and the market remains uncovered with coverage increasing in advertisement intensity phi.*

Further $\overline{\beta}$ is increasing in the market size (M) and marginal cost (c), and decreasing in quality cost (α), and the relationship is reverse for $\underline{\beta}$[20]. Relationship of $\overline{\beta}$ is ambiguous with μ and σ and depends on c[21], while $\underline{\beta}$ is decreasing in μ and increasing in σ

Proof of Proposition 2 is given in the Appendix A.2. Figure 1 depicts the result graphically.

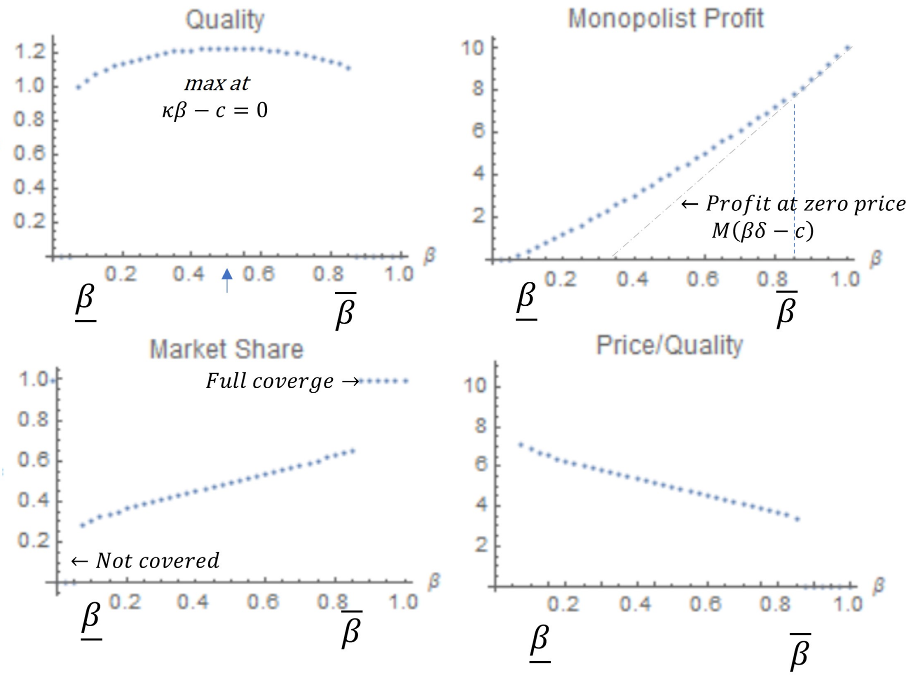
[18] Appendix A.2 shows that (11) has a unique solution whenever there is an interior solution.

[19] When subscription price is zero all consumers get non-negative utility

[20] Expressions for $\overline{\beta}$ and $\underline{\beta}$ is given by: $\phi^{-1}(\frac{M\kappa^2}{27\alpha})$ and $max(0, \phi^{-1}(-\frac{M\kappa^2}{27\alpha}))$ respectively.

[21] If μ increases and σ decreases, both advertisement revenue and subscription revenue increases with the opposing effect. If c is small the first effect outweigh and $\overline{\beta}$ increases, reverse otherwise.

Figure 1: Monopoly equilibrium at different value of β



The profit and the market coverage of the monopolist is given by:

$$\Pi(\theta) = \begin{cases} 0 & \text{if } \beta \leq \underline{\beta}, \\ \frac{M}{4\kappa\theta}(\kappa\theta + \phi)^2 + M\beta(\delta - \kappa) - \alpha\theta^2 & \text{if } \beta \in [\underline{\beta}, \overline{\beta}], \\ M(\beta\delta - c) & \text{if } \beta > \overline{\beta} \end{cases} \tag{14}$$

$$\frac{N}{M} = \begin{cases} 0 & \text{if } \beta < \underline{\beta}, \\ \frac{1}{2}(1 + \frac{\phi}{\kappa\theta_{MP}}) & \text{if } \beta \in [\underline{\beta}, \overline{\beta}], \\ 1 & \text{if } \beta > \overline{\beta} \end{cases} \tag{15}$$

Intuitively, when $\beta \in [\underline{\beta}, \overline{\beta}]$, the monopolist faces a trade-off between acquiring a marginal customer through price reduction (or higher quality) and incurring revenue loss (or higher quality cost) from existing customers. If β increases, enhancing the value of a marginal customer, the monopolist will adjust the price, quality, or both, depending on the elasticity of demand with respect to price and quality. If $\phi < 0$, the marginal customer is acquired by both reducing the price and improving the quality. If $\phi = 0$, the marginal customer is acquired solely through a price reduction. If $\phi > 0$, the marginal customer is acquired by reducing the price but with an offsetting reduction in quality to lower costs. Therefore, the quality is non-monotonic in β even though the market coverage increases monotonically with β. The monopolist covers one-third the market when $\beta = \underline{\beta}$, one-half the market when $\kappa\beta - c = 0$, and two-third the market when $\beta = \overline{\beta}$.

When $\beta > \overline{\beta}$ the potential loss of advertisement revenue from all non-subscribing customer outweighs the subscription revenue through higher-priced quality product. Consequently, the monopolist opts to forego all subscription revenue in order to capture the full advertisement revenue, which means it covers the market by setting the price to zero and offering the minimum quality[22].

Remark: There is a tension between advertisement and subscription revenue. Optimizing subscription revenue means maintaining a quality product with a positive subscription price, which leads to many low-end customers not subscribing, resulting in a loss of advertisement revenue. This tension generates a corner solution when advertisement levels are high, as the firm offers a free product and foregoes subscription revenue to capture the full advertisement revenue.

Corollary 1. *Monopolist strictly under provisions quality relative to the social planner $\theta_{MP} < \theta_{SP}$ and weakly covers less market.*

Proof: Using implicit function theorem on (5) and (11), we can show that $\theta_{MP} < \theta_{SP}$[23]. Comparison between (4) and (12) shows the the $s_{MP} > s_{SP}$ when $\beta < \overline{\beta}$. Higher price and lower quality results in lower market coverage by the monopolist when $\beta < \overline{\beta}$. When $\beta \geq \overline{\beta}$, both a monopolist and the social planner will cover full market but the monopolist will provide a very low quality product.

□

The comparison between the social planner and the monopolist is shown in Figure 2.

Figure 2: Comparison between social planner and monopolist

Corollary 2. *Smaller market (lower M) has lower quality for all values of β. The same is true for market with lower median income (μ) and higher income inequality (σ), provided that c is not critically high.*

The proof derives from two observations: a) when the monopolist sets a positive quality level, i.e., $\beta \in [\underline{\beta}, \overline{\beta}]$, the quality level increases with M and μ, and decreases with σ (as stated in Lemma 1); b) the interval $[\underline{\beta}, \overline{\beta}]$ expands at both ends if either μ or M increase or σ decreases (as per

[22]The minimum quality or zero quality refers to the product which do not invest in building quality by hiring editorial staff but rather use wire services. Metro or 20 minutes are examples of such newspapers in Europe.

[23]Desired solution of $a\theta^3 - b\theta^2 + c^2 = 0$ is decreasing in a and increasing in b . (11) has higher a and lower b relative to (5) and hence $\theta_{MP} < \theta_{SP}$, when $s_{SP} > 0$. When $s_{SP} = 0$, the social planner sets the quality $\frac{M\delta}{4\alpha}$. which is strictly greater than the maximum quality set by monopolist for any parameter values

Proposition 2). An exception occurs when c is critically high, causing $\overline{\beta}$ to decrease with μ. In such scenarios, for some values of β, the monopolist may transition from positive to zero quality if μ increases and/or σ decreases.

To summarize, the key aspects of the monopolist market are: a) the monopolist under-provides quality relative to the social planner and sets prices higher than the social planner; b) the market remains uncovered unless the advertisement level is high, i.e., $\beta > \overline{\beta}$; c) when the advertisement level is low to moderate, i.e., $\beta \in [\underline{\beta}, \overline{\beta}]$, higher advertising subsidizes consumers as they pay a lower subscription price. However, when the advertisement level becomes sufficiently high, i.e. $\beta > \overline{\beta}$, consumers are offered very low-quality products and lose all their surplus. Thus, high advertising revenue in a non-competitive market does not necessarily entail high investment in quality and can result in the undesirable outcome of a poor-quality product; d) Corollary 2 demonstrates that consumers with similar preferences in a smaller market may receive a lower quality product, consistent with Berry and Waldfogel (2010)'s empirical observation. It also shows that higher income inequality leads to a lower quality product, as a higher proportion of consumers falls into the lower tail of the income distribution, prompting the monopolist to lower both price and quality to capture a sufficient market share.

6 Duopoly

Now we consider competition in a market but with a restriction that maximum two firms, denoted as $k \in \{1, 2\}$, can enter. The timing for the sequential entry of firms is as follows:

Stage 1: Firm 1 (or leader) makes the entry decision and choose the quality θ_1

Stage 2: Firms 2 (or follower) makes the entry decision and choose the quality θ_2.

Stage 3: If both firms enter they simultaneously choose the price, otherwise Firm 1 sets price as a monopolist.

We conjecture four distinct types of equilibrium in such a market.

Definition 2. *We call the equilibrium Type A (natural monopoly) when only one firm enters the market and sets prices and quality at monopolistic levels.*

Definition 3. *We call the equilibrium Type B (uncovered or interior solution) when both firms enter and set strictly positive subscription prices and quality levels, that is,* $(s_k, \theta_k) \in \mathbb{R}^2_{++}$ *for* $k \in \{1, 2\}$*, and the market remains uncovered.*

Definition 4. *We call the equilibrium Type C (corner solution) when both firms enter, and the follower firm (Firm 2) sets both price and quality to zero* $(s_2, \theta_2) = (0, 0)$*, while the leader (Firm 1) sets monopolistic price and quality* $(s_1, \theta_1) = (s_{MP}, \theta_{MP})$*. Under this equilibrium, the market is fully covered.*

Let us denote quality infinitesimally greater than θ as θ_+ that is $\theta_+ \equiv \theta + \epsilon$ where $\epsilon \to 0$.

Definition 5. *Define $\theta_c : \phi \to \mathbb{R}_{++}$ such that if Firm 1 chooses $\theta_1 = \theta_c$ then Firm 2 is indifferent between choosing $\theta_2 = 0$ and $\theta_2 = \theta_{c+}$.*

Definition 6. *We call the equilibrium type D (contestable) when both firms enter and the leader (Firm 1) sets the quality θ_c, and the follower (Firm 2) sets its quality to zero, that is $(\theta_1, \theta_2) = (\theta_c, 0)$. Firm 1 sets the price as a monopolist would for θ_c quality, and Firm 2 sets the price to zero.*

First, we characterize the equilibrium types B, C, and D. Subsequently, we will identify the conditions under which each type of equilibrium exists. Type A equilibrium is equivalent to the monopoly equilibrium described in the monopolist section 5.

6.1 Type B Equilibrium

Let's assume that the entering firm sets lower quality than the leading firm under equilibrium, $0 < \theta_2 < \theta_1$, which we will validate. This implies, $s_2 < s_1$; otherwise, all consumers will switch to the high-quality newspaper (Firm 1) and the low-quality newspaper (Firm 2) will make a negative profit. The utility of a consumer (i) who is indifferent between the two newspapers will be given by:

$$U_i = v_i y_i \theta_2 - s_2 = v_i y_i \theta_1 - s_1 \Rightarrow v_i y_i = \frac{s_1 - s_2}{\theta_1 - \theta_2}$$

Consumers with a higher preference for quality than that of the indifferent consumer will buy the high-quality newspaper, while those with a lower preference will buy low-quality newspaper provided they receive non-negative utility. Therefore, the demand functions for the two firms are given by:

$$N_1(s_1, s_2, \theta_1, \theta_2) = M \int_0^\infty \int_{\frac{s_1 - s_2}{y(\theta_1 - \theta_2)}}^1 dv dy \quad = \quad M(1 - \frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)}) \tag{16}$$

$$N_2(s_1, s_2, \theta_1, \theta_2) = M \int_0^\infty \int_{\frac{s_2}{y\theta_2}}^{\frac{s_1 - s_2}{y(\theta_1 - \theta_2)}} dv dy \quad = \quad M(\frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)} - \frac{s_2}{\kappa\theta_2}) \tag{17}$$

$$\text{where } \kappa = E[\frac{1}{Y}] = e^{\mu - \frac{1}{2}\sigma^2}$$

The firms will set the advertisement prices such that they capture the full surplus from the advertisers, because each firm provides a unique, non-overlapping set of consumers. Therefore, the advertisement prices are determined as follows:

$$p_1 N_1 \quad = \quad M\beta \int_0^\infty (1 - \frac{s_1 - s_2}{y(\theta_1 - \theta_2)}) y dy \Rightarrow p_1 N_1 = M\beta(\delta - \frac{s_1 - s_2}{\theta_1 - \theta_2})$$

$$p_2 N_2 \quad = \quad M\beta \int_0^\infty (\frac{s_1 - s_2}{y(\theta_1 - \theta_2)} - \frac{s_2}{y\theta_2}) y dy \Rightarrow p_2 = \kappa\beta$$

$$\text{where } \delta = E[Y] = e^{\mu + \frac{1}{2}\sigma^2}$$

Using above advertisement prices and $\phi \equiv \kappa\beta - c$ (Definition 1), we derive the profit functions:

$$\begin{aligned} \Pi_1(s_1, \theta_1, s_2, \theta_2) &= N_1(s_1 + \phi) + M\beta(\delta - \kappa) - \alpha\theta_1^2 && (18) \\ \Pi_2(s_1, \theta_1, s_2, \theta_2) &= N_2(s_2 + \phi) - \alpha\theta_2^2 && (19) \end{aligned}$$

Since this is a two-stage strategic game. We use backward induction to find the sub-game perfect Nash equilibrium. The first-order conditions provide the reaction functions in the price stage (stage 2):

$$\begin{aligned} s_1(s_2) &= \frac{1}{2}s_2 + \frac{1}{2}\kappa(\theta_1 - \theta_2) - \frac{1}{2}\phi && (20) \\ s_2(s_1) &= \frac{\theta_2}{2\theta_1}s_1 - \frac{1}{2}\phi && (21) \end{aligned}$$

Since both profit functions are strictly concave for $\theta_2 < \theta_1$ [24], the first-order conditions are also sufficient. Equilibrium prices, given by the unique solution of the above two linear equations:

$$\begin{aligned} s_1 &= \frac{2\kappa\theta_1(\theta_1 - \theta_2)}{4\theta_1 - \theta_2} - \frac{3\theta_1}{4\theta_1 - \theta_2}\phi && (22) \\ s_2 &= \frac{\kappa\theta_2(\theta_1 - \theta_2)}{4\theta_1 - \theta_2} - \frac{2\theta_1 + \theta_2}{4\theta_1 - \theta_2}\phi && (23) \end{aligned}$$

The subscription price of both firms decreases with the advertisement intensity ϕ, as firms reduce their subscription prices to acquire marginal customers and increase advertisement revenue.

Substituting (22)-(23) in (18)-(19) we can derive the expression of the profit function of each player at stage 1 as a function of quality:

$$\begin{aligned} \Pi_1(\theta_1, \theta_2) &= M\frac{\theta_1 - \theta_2}{\kappa(4\theta_1 - \theta_2)^2}(2\kappa\theta_1 + \phi)^2 + M\beta(\delta - \kappa) - \alpha\theta_1^2 \\ \Pi_2(\theta_1, \theta_2) &= M\frac{\theta_1(\theta_1 - \theta_2)}{\kappa\theta_2(4\theta_1 - \theta_2)^2}(\kappa\theta_2 + 2\phi)^2 - \alpha\theta_2^2 \end{aligned}$$

Notice that only the high-quality firm earns a fixed rent $M\beta(\delta - \kappa)$, which arises from serving consumers in the right tail of the income distribution. If we define $\gamma \equiv \frac{\theta_2}{\theta_1}$, then the above equation can be rewritten as follows:

$$\begin{aligned} \Pi_1 &= M\frac{1 - \gamma}{\kappa\theta_1(4 - \gamma)^2}(2\kappa\theta_1 + \phi)^2 + M\beta(\delta - \kappa) - \alpha\theta_1^2 && (24) \\ \Pi_2 &= M\frac{1 - \gamma}{\kappa\theta_2(4 - \gamma)^2}(\kappa\theta_2 + 2\phi)^2 - \alpha\theta_2^2 && (25) \end{aligned}$$

[24] $\frac{\partial^2\Pi_1}{\partial s_1^2} = -\frac{2M}{(\theta_1 - \theta_2)}$ and $\frac{\partial^2\Pi_2}{\partial s_2^2} = -\frac{2M\theta_1}{\theta_2(\theta_1 - \theta_2)}$

The corresponding market shares and the price-to-quality ratios of firms are given by:

$$\frac{N_1}{M} = \frac{2}{4-\gamma} + \frac{1}{\kappa\theta_1(4-\gamma)}\phi \tag{26}$$

$$\frac{N_2}{M} = \frac{1}{4-\gamma} + \frac{2}{\kappa\theta_2(4-\gamma)}\phi \tag{27}$$

$$\frac{s_1}{\theta_1} = 2\frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_1}\frac{3}{4-\gamma}\phi \tag{28}$$

$$\frac{s_2}{\theta_2} = \frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_2}\frac{2+\gamma}{4-\gamma}\phi \tag{29}$$

As advertisement intensity ϕ increases, both firms lower the price-to-quality ratio to acquire marginal customers and increase market share. However, the price-to-quality ratio of the lower-quality firm is more responsive because it has a smaller market share, making the reduction in prices for existing customers less costly.

The first-order conditions below provide the reaction functions for firms.[25]

$$\frac{4M\kappa}{(4-\gamma)^3}(1+\frac{\phi}{2\kappa\theta_1})\big[(2\gamma^2-3\gamma+4) - \frac{\phi}{2\kappa\theta_1}(4-7\gamma)\big] = 2\alpha\theta_1 \tag{30}$$

$$\frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})\big[4-7\gamma - \frac{2\phi}{\kappa\theta_2}(2\gamma^2-3\gamma+4)\big] = 2\alpha\theta_2 \tag{31}$$

Firm 1 will find the most profitable choice of its own quality after considering the reaction function of Firm 2. The characteristics of the Type B equilibrium, if it exists, are described in the following lemmas, and their proofs are provided in Appendix A.3. We first show that the leading firm will take the high quality position so that the follower enters with lower quality.

Lemma 2. *Firm 1 will set the quality so that the Firm 2 enters with lower quality i.e.* $\theta_2 < \theta_1$.

Intuitively, the high-quality firm (Firm 1) has an inherent advantage, so as a first mover, Firm 1 will preempt that position. Firm 1 will attract consumers who have higher willingness to pay for the quality and thus has ability to charge higher price (see (22)-(23)). In addition, it earns extra rent through advertisement by serving the right tail customer in the income distribution, the term $M\beta(\delta-\kappa)$. Next we consider the equilibrium solution with the benchmark case $\phi = 0$.

Lemma 3. *If* $\phi = 0$ *then there exist a unique solution such that two firms choose quality in the ratio* $\gamma = 0.195064$, *which is a constant for all* (μ, σ^2, α). *The market share of the high quality firm is twice that of the low quality firm. The high-quality firm sets quality lower than the monopoly level but cover larger market share.*

The corresponding θ_1 and θ_2 can be derived from (31).

$$\theta_1 = \rho\frac{M\kappa}{\alpha} \text{ and } \theta_2 = \rho\gamma\frac{M\kappa}{\alpha} \text{ where}$$

[25] Necessary second order conditions are verified while finding the solutions.

$$\gamma = 0.195064, \rho = \frac{4-7\gamma}{2\gamma(4-\gamma)^3} = 0.1226$$

The corresponding market shares of the two players using (26) and (27) yields:

$$\frac{N_1}{M} = \frac{2}{4-\gamma} = 52.56\%, \frac{N_2}{M} = \frac{1}{4-\gamma} = 26.28\%$$

The market remains uncovered, with the total market coverage 78.84%. Compare this to the monopolist case, where the market is only 50% covered when $\phi = 0$. The entry of a low-quality player results in the expansion of market coverage by targeting consumers with lower valuations. Furthermore, the high-quality firm lowers both its price-to-quality ratio and its quality relative to that of a monopolist[26]. Therefore, the competitive entry reduces the price set by a monopolist and expands overall market coverage, resulting in higher consumer surplus.

The constant ratio, $\frac{4}{7}$, is established in the vertical differentiation literature when there are no fixed or variable costs of quality (see Choi and Shin (1992) and Lutz (1997)). In our model, due to the convex quality costs, the ratio is significantly smaller.

Lemma 4. *There exist $\phi(\underline{\beta}) < \phi_{il} < 0$ and $0 < \phi_{ir} < \phi(\overline{\beta})$ such that an interior solution exist iff $\phi \in (\phi_{il}, \phi_{ir})$ and this solution is unique for a given ϕ.*

Recall $[\underline{\beta}, \overline{\beta}]$ is the interval in which a monopolist will serve the market with positive quality and price (i.e., interior solution). We can easily observe that the first-order conditions do not have a solution if ϕ is sufficiently negative or sufficiently positive. When $\frac{\phi}{\kappa\theta_2} \to -\frac{1}{2}$, the left side of (31) approaches zero, indicating that no positive solution for θ_2 is possible. Similarly, when $\phi > 0$, Firm 1 reduces the price-to-quality ratio with higher ϕ until $\frac{s_2}{\theta_2} \to 0$ (see (29)), at which point a positive θ_2 is not optimal. Additionally, the interval (ϕ_{il}, ϕ_{ir}) is a subset of the interval $(\phi(\underline{\beta}), \phi(\overline{\beta}))$ in which the monopolist chooses positive quality [27].

However, the existence of an interior solution is a necessary but not sufficient condition for Type B equilibrium because: a) Firm 2 may have negative profit and will not enter and Type A equilibrium occurs (see lemma 5); b) Firm 2 may have profitable deviation to $\theta_2 = 0$ and Type C equilibrium occurs (see lemma 11); or c) Firm 2 may have profitable deviation to $\theta_2 = \theta_1+$ (see lemma 10).

Lemma 5. *There exist a critical $\phi_0 \in (\phi_{il}, 0)$ such that for all $\phi \in (\phi(\underline{\beta}), \phi_0)$ only one firm enters the market (Type A equilibrium).*

Intuitively, if the advertisement revenue per customer is sufficiently small, the low-quality firm may need to significantly raise its prices to achieve a positive contribution margin per customer.

[26] $\theta_1 = \frac{0.1226M\kappa}{\alpha} < \frac{M\kappa}{8\alpha} = \theta_{MP}$ when $\phi = 0$

[27] The cutoff value approximates to $\phi_{il} = -\frac{M\kappa^2}{187\alpha}, \phi_{ir} = \frac{M\kappa^2}{255\alpha}$. We don't have close form solution of these cutoff values, so we identified the cut-off values using numerical methods with precision $0.005\frac{M\kappa^2}{\alpha}$

However, this firm might not attract enough demand at these higher prices to cover the costs associated with the required quality. Consequently, the low-quality firm may choose not to enter the market, resulting in a natural monopoly (Type A equilibrium). Note that for $\phi < \phi(\underline{\beta})$, the market does not sustain even a single player, as stated in Proposition 2. It is also important to note that if the cost parameter c is low such that $\phi(0) > \phi_0$, then both firms would enter for all $\beta \geq 0$.

Lemma 6. *γ decreases continuously with ϕ for $\phi \in [0, \phi_{ir}]$, and the high-quality firm sets a lower quality than that of a monopolist for all $\phi \in (\phi_{il}, \phi_{ir})$.*

As the advertisement revenue becomes more valuable, the low-quality firm expands market by attracting customers with lower valuation. It is more optimal for this firm to reduce quality with this expansion to relax the competition. The result that the high-quality firm set lower quality than that of monopolist is due to the sequential entry (or Stackelberg) model. The high-quality firm crowds out the quality space of the low-quality firm by reducing quality. In simultaneous entry model (see section 8.1), the high-quality firm sets higher quality higher than the monopolist level in this interval of ϕ.

Lemma 7. *For any given ϕ, the equilibrium quality level of both firms increase with M and μ, and decreases with α and σ, ceteris paribus.*

The increase in M or μ or a decrease in σ increases marginal revenue (as indicated in the LHS of (30)-(31)), while a reduction in α decreases the marginal cost. Consequently, both firms in the Type B equilibrium have higher quality levels, which increases the marginal cost (RHS of (30)-(31)) to match the marginal revenue.

Lemma 8. *Profit and the market coverage of both firms increases continuously with ϕ under Type B equilibrium.*

As ϕ increases, there is a direct effect of increased advertisement revenue for both firms. In addition, there is a positive strategic effect due to relaxed competition when $\phi \geq 0$ because γ decreases[28]. Therefore, profits of both firms increase. Higher ϕ leads to market expansion because marginal customers become more attractive, prompting both firms to lower their price-to-quality ratios to acquire these customers. The market share of the two firms at the right extreme of the interior solution, when $\phi \to \phi_{ir}$ [29]:

$$\frac{N_1}{M} = 52.75\% \quad \frac{N_2}{M} = 38.86\%$$

Comparing the above numbers with those at $\phi = 0$, we can infer that the main impact of increase in advertisement is the increase in market coverage of the low quality product.

[28] Even when γ increases with ϕ, which occurs near the Type A cutoff (ϕ_0), the direct effect still outweighs the negative strategic effect.

[29] Evaluating θ_1 and θ_2 with $\phi_{ir} \approx \frac{M\kappa^2}{255\alpha}$ and substituting in (26)-(27)

6.2 Type C Equilibrium

In Type C equilibrium, the low-quality firm (Firm 2) chooses zero quality and the high-quality firm (Firm 1) chooses monopolist level quality.

Lemma 9. *Suppose low-quality firm (Firm 2) chooses $\theta_2 = 0$ and $\phi \in [0, \frac{M\kappa^2}{12\sqrt{3}\alpha})$. Then, the best response form Firm 1 is to set $\theta_1 = \theta_{MP}$. The corresponding subscription prices would then be $s_2 = 0, s_1 = s_{MP}$.*

Proof is shown in Appendix A.3. The condition $\phi >= 0$ ensures that Firm 2 has positive profit with $(s_2, \theta_2) = (0, 0)$, and $0 \leq \phi \leq \frac{M\kappa^2}{12\sqrt{3}\alpha}$ satisfies condition (13) so that the $\theta_{MP} > 0$.

Given $\theta_2 = 0$ and Lemma 9, Firm 1 behaves as a monopolist with the profit function (14) and the quality given by (11). Firm 2 serves all the customers not served by Firm 1 and earns revenue solely from advertisements. Its profit function Π_{2c} is given by:

$$\Pi_{2c} = \frac{M}{2}(1 - \frac{\phi}{\kappa\theta_{MP}})\phi \tag{32}$$

Under type C equilibrium, the market is fully covered and the market share of Firm 1 increases with ϕ while the market share of Firm 2 decreases with ϕ.

$$\frac{N_1}{M} = \frac{1}{2}(1 + \frac{\phi}{\kappa\theta_{MP}}) \text{ and } \frac{N_2}{M} = 1 - \frac{N_1}{M} = \frac{1}{2}(1 - \frac{\phi}{\kappa\theta_{MP}}) \tag{33}$$

Type C equilibrium does not exist if $\phi < 0$ because Firm 2 will have negative profits. Type C equilibrium will also break if ϕ is high enough that the Firm 2 can earn higher profit by setting a quality infinitesimally greater than θ_{MP}, denoted as θ_{MP+}.

Suppose Firm 1 sets $\theta_1 = \theta_{MP}$ and Firm 2 responds by setting $\theta_2 = \theta_{MP+}$. Since the quality is infinitesimally close (firms are not differentiated), the prices in stage 2 will be zero for both firms, $s_1 = 0, s_2 = 0$ (as per Equations (22)-(23)). Firm 2 with higher quality but same price will capture the full market and the full advertisement revenue, $M(\beta\delta - c)$[30], but will have no subscription revenue. Its profit function is given by:

$$\Pi_{2d} = M(\beta\delta - c) - \alpha\theta_{MP}^2 = M\phi + Mc(e^{\sigma^2} - 1) - \alpha\theta_{MP}^2 \tag{34}$$

The indifference point for Firm 2 to choose between $\theta_2 = 0$ and $\theta_2 = \theta_{MP+}$ is given by the solution of following equation:

$$F(\phi) = \Pi_{2d} - \Pi_{2c} = M\phi e^{\sigma^2} + Mc(e^{\sigma^2} - 1) - \alpha\theta_{MP}^2 - \frac{M}{2}(1 - \frac{\phi}{\kappa\theta_{MP}})\phi \equiv 0 \tag{35}$$

[30] δ is expected income per consumer

We verify that $F(\phi)$ is a continuous and strictly increasing function of ϕ if $\phi \geq 0$[31], and $F(\frac{M\kappa^2}{12\sqrt{3}\alpha}) > 0$[32]. Therefore, (35) will have a non-negative solution iff

$$F(0) \leq 0 \rightarrow c(e^{\sigma^2} - 1) \leq \frac{M\kappa^2}{64\alpha}$$

We introduce few additional parameters ϕ_2 and σ_c, as follows:

Definition 7. *Define ϕ_2 such that $F(\phi_2) = 0$, which means that if $\phi = \phi_2$ and the Firm 1 chooses $\theta_1 = \theta_{MP}$ then Firm 2 is indifferent between choosing $\theta_2 = 0$ and $\theta_2 = \theta_{MP+}$.*

Lemma 10. *The necessary condition for type C Equilibrium is $\phi \in [0, \phi_2]$ and this interval is non-empty iff $c(e^{\sigma^2} - 1) \leq \frac{M\kappa^2}{64\alpha}$.*

Proof follows from the facts: a) Type C equilibrium does not exist if $\phi < 0$, as Firm 2 will have negative profit and b) if $\phi > \phi_2$, $\theta_2 = 0$ is not the best response of Firm 2 when $\theta_1 = \theta_{MP}$. In addition, $\phi_2 \geq 0$ iff $c(e^{\sigma^2} - 1) \leq \frac{M\kappa^2}{64\alpha}$.

The condition outlined in Lemma 10 is not sufficient for establishing a Type C equilibrium, as Firm 2's best response might be a strictly positive θ_2 when Firm 1 chooses $\theta_1 = \theta_{MP}$, which is a Type B equilibrium. Conversely, the presence of an interior solution does not necessarily lead to a Type B equilibrium, as Firm 2 may find a profitable deviation to $\theta_2 = 0$, thereby sustaining a Type C equilibrium. This condition is identified in Lemma 11 (see Appendix A.3 for proof).

Lemma 11. *There exists a critical $\phi_1 \in (0, \phi_{ir})$ such that for $\phi \in (\phi_1, \phi_2]$ there exists a unique Type C equilibrium, and for $\phi \in (\phi_0, \phi_1)$ there exist a unique Type B equilibrium.*

Note that ϕ_0 is cutoff below which only one firm enters (Type A equilibrium). Also ϕ_2 could be lower than ϕ_1, making the interval $\phi \in (\phi_1, \phi_2]$ empty. If $\phi_2 \geq \phi_{ir}$, then there is a non-empty interval $[\phi_{ir}, \phi_2]$ where a Type C equilibrium is guaranteed to occur. From the implicit function theorem on $F(\phi_2) = 0$, we get that ϕ_2 is decreasing in σ. We define a critical σ_c at which $\phi_2 = \phi_{ir}$

Definition 8. *Define $\sigma_c : (c, \mu, \alpha) \rightarrow \mathbb{R}_{++}$ such that if $\sigma = \sigma_c$, then $\phi_2 = \phi_{ir}$.*

Next we make the following assumption so that the interval $[\phi_{ir}, \phi_2]$ is not empty, which ensures that Type C equilibrium will exist for some ϕ. Later we highlight the implications when this assumption does not hold.

Assumption 1. *$\sigma < \sigma_c$ so that $\phi_2 > \phi_{ir}$*

[31] $\frac{dF}{d\phi} = Me^{\sigma^2} - \frac{M}{2}(1 - \frac{2\phi}{\theta_{MP}}) - (2\alpha\theta_{MP} + \frac{M}{2}(\frac{\phi}{\kappa\theta_{MP}})^2)\frac{d\theta_{MP}}{d\phi} > 0$ because $\frac{d\theta_{MP}}{d\phi} \leq 0$ when $\phi \geq 0$ (Lemma 1)

[32] If $\phi = \frac{M\kappa^2}{12\sqrt{3}\alpha}$ then $\theta_{MP} = \frac{M\kappa}{12\alpha}$ (see Appendix A.3) and we can easily verify that $F > 0$

6.3 Type D Equilibrium

Now we consider the equilibrium characteristics when the advertisement intensity is sufficiently high, i.e. $\phi > \phi_2$, so that Type C equilibrium is not sustained (Lemma 10).

If $\phi > \phi_2$ Firm 2 can contest the leadership of Firm 1. It can get higher profit by marginally exceeding the quality of Firm 1 when Firm 1 sets $\theta_1 = \theta_{MP}$, in which case Firm 1 will make negative profit. Anticipating this, Firm 1 will set the quality level high enough that Firm 2 does not find profitable to adopt such strategy. θ_c as specified in Definition 5 is this quality level of Firm 1 that makes Firm 2 indifferent between choosing the strategy of maximal differentiation, $\theta_2 = 0$, and maximal competition $\theta_2 = \theta_{c+}$. θ_c is implicitly defined by (36):

$$G(\theta_c(\phi)) = \underbrace{M\phi e^{\sigma^2} + Mc(e^{\sigma^2} - 1) - \alpha\theta_c^2}_{\Pi_2 \text{ if } \theta_2=\theta_{c+} \text{ and } \theta_1=\theta_c} - \underbrace{\frac{M}{2}(1 - \frac{\phi}{\kappa\theta_c})\phi}_{\Pi_2 \text{ if } \theta_2=0 \text{ and } \theta_1=\theta_c} \equiv 0 \tag{36}$$

We can easily verify that θ_c is increasing in ϕ and that $\theta_c(\phi_2) = \theta_{MP}$.

Lemma 12. *There exist a unique type D equilibrium if $\phi > \phi_2$. In this equilibrium, Firm 1 sets its quality θ_c that increases with ϕ and Firm 2 sets its quality to zero.*

Note: The proof of all lemmas under duopoly is provided in Appendix A.3.

6.4 Duopoly Market Configurations

Proposition 3 states our main result, which includes all types of possible market configurations. It uses the parameters: $\beta_0 \equiv \phi^{-1}(\phi_0), \beta_1 \equiv \phi^{-1}(\phi_1), \beta_2 \equiv \phi^{-1}(\phi_2)$. The proof follows directly from Lemmas 5, 6, 11, and 12.

Proposition 3. *There exist $(\beta_0, \beta_1, \beta_2)$ with $\underline{\beta} < \beta_0 < \beta_1 < \beta_2 < \overline{\beta}$ such that*

a *If $\beta \in (\underline{\beta}, \beta_0)$, then a unique Type A equilibrium exists. One firm enters, setting the monopolistic price and quality as given by (12) and (11), and the market remains uncovered.*

b *If $\beta_0 < \beta < \beta_1$, then a unique Type B equilibrium exists. Both firms set positive and differentiated qualities, and the quality differential increases with β if $\phi(\beta) \geq 0$. The market remains uncovered.*

c *If $\beta_1 < \beta \leq \beta_2$, then a unique Type C equilibrium exists. The high-quality firm sets monopolistic price and quality levels, while the low-quality firm offers a free product of minimum quality. The market is fully covered.*

d *If $\beta > \beta_2$, then a unique Type D equilibrium exists. The high-quality firm chooses a premium product with quality higher than the monopolistic level, which increases with β, while the low-quality firm provides a free product of minimum quality. The market is covered.*

Figure 3 depicts endogenous parameters under duopoly graphically.

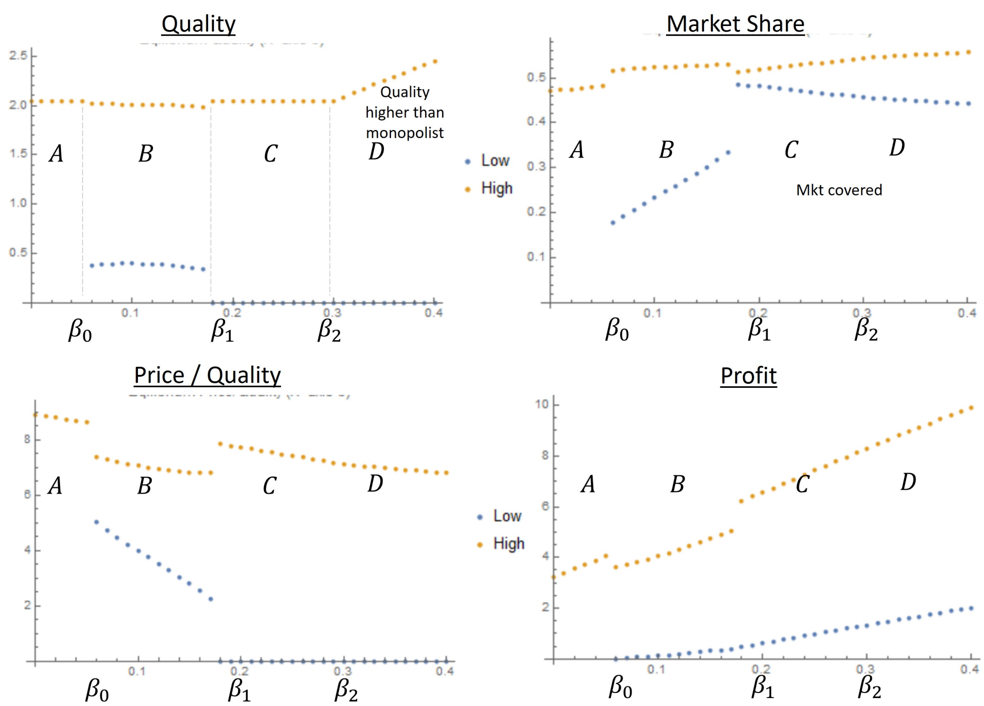


Figure 3: Quality, Market Share, Price and Profit of two firms under Duopoly

If the advertisement level is low, $\beta < \beta_0$, the market does not support two players and becomes a natural monopoly with a higher price-to-quality ratio. The market is partially covered, as many consumers with a low value for quality content do not subscribe. Note that β_0 could be zero when c is low enough or μ or M is high enough, in which case the Type A equilibrium does not exist.

As the advertisement level increases, the market transitions from Type A to Type B, where both players enter and compete. This competition lowers the price-to-quality ratio of the high-quality firm, while the low-quality firm attracts new subscribers at the lower end. Both higher market coverage and lower price-to-quality ratios increase consumer surplus. Although market coverage expands significantly, the market still remains partially covered. This expansion is primarily driven by the low-quality firm, which fills a product gap at the lower end. The differentiation between the two firms increases with advertisement as the low-quality firm strives to capture the niche lower-end market. Higher advertisement increases the value of marginal consumers, and the low-quality firm lowers its price-to-quality ratio to acquire these customers. However, it also reduces quality to lessen competition. Thus, while advertisement enables higher market coverage and lower prices, it also leads to a reduction in quality.

With a further increase in the advertisement level, the market transitions from Type B to Type C, where the second firm enters offering a free product of the lowest quality. As advertisement increases, the low-quality firm faces a tension between subscription revenue and advertisement revenue. When β increases above β_1, the potential advertisement revenue from non-subscribers becomes more valuable than the subscription revenue from its current subscribers. As a result, this firm switches to a free product to capture these non-subscribers and foregoes all subscription revenue. Although the market is fully covered — enabling advertisers to reach all consumer — consumer surplus decreases because the low-quality firm provides the lowest quality product, yielding zero surplus to its consumers. Additionally, competition reduces, and the high-quality firm behaves like a monopolist with a higher price-to-quality ratio, which also reduces consumer surplus. As a result consumer surplus becomes non-monotonic in advertisement level β, as illustrated in Figure 4.

With very high advertisement levels, i.e., $\beta > \beta_2$, the low-quality firm can contest the high-quality firm's leadership if the high-quality firm continues to set the monopolist level quality. It becomes profitable for the low-quality firm to marginally exceed the monopolist level quality. This is because advertisement revenue is sufficient to cover the cost of high quality, even after losing subscription revenue due to aggressive price competition. This compels the high-quality firm to enhance its quality beyond the monopolistic level to protect its market share. As advertisement levels increase, so does competitive pressure, leading to higher quality and a lower price-to-quality ratio (i.e., better reach) for the premium product. In other words, higher advertisement levels drive the premiumization of the market.

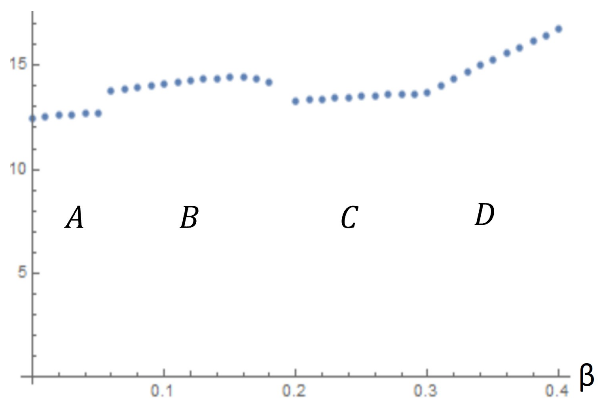


Figure 4: Consumer Surplus

Next we observe how equilibrium changes with exogenous parameter:

Proposition 4. *The equilibrium cut-off points β_0, β_1, β_2 changes when M, α, μ or σ increases, ceteris paribus, as shown in the table below:*

	β_0	β_1	β_2
$M \Uparrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
$\alpha \Uparrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$
$\mu \Uparrow$	$\Downarrow$	*depends on c*	$\Uparrow$
$\sigma \Uparrow$	$\Uparrow$	*depends on c*	$\Downarrow$
$c \Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$

Intuitively, an exogenous change that either increases subscription demand or reduces quality cost —such as an increase in M or μ, or a decrease in σ or α— increases the quality level of the high-quality firm across all β values, thus relaxing competition. This combined effect of increased demand (or reduced cost) and diminished competition boosts the profitability of both firms. The higher profitability of the low-quality firm reduces β_0, the threshold for non-negative profit for the low-quality firm. Further, as the high-quality firm raises its quality, β_2 also increases, making it more costly for the low-quality firm to contest its leadership. Meanwhile, β_1 marks the point at which the low-quality firm is indifferent between subscription earnings from current subscribers and potential advertising revenue from non-subscribers. Increases in M or decreases in α enhance the former without affecting the latter, thus raising β_1. However, changes in μ or σ impact both earnings, leading to an ambiguous effect on β_1. If the variable cost of circulation c is small, the first effect dominates and β_1 tends to increase with higher μ or lower σ.

Corollary 3. *For any given β, the quality produced by the high-quality firm increases when M or μ increases, and decreases when α or σ increases.*

The statement follows from Lemmas 1 and 7. The factors that raise marginal revenue or reduce marginal cost for the firm increase its quality.

Corollary 4. *If the median income (μ) or market size (M) is sufficiently low, a Type C equilibrium does not exist. The same is true if α or σ is sufficiently high.*

The above result highlights the situation when Assumption 1 is violated[33]. When μ or M decrease, or α or σ increase, the reduction in β_2 (Proposition 4) is much larger than that in β_1, hence the observed result. These changes lead to a reduction in the quality of the high-quality firm, which in turn makes it easier for the low-quality player to contest, thereby lowering β_2. Since the cost is convex in quality, these changes have a higher impact on β_2. Conversely, the impact on β_1 is smaller and arises through an indirect competitive effect on the low-quality firm.

6.5 Implications of Duopoly Result

The duopoly results highlight how equilibrium characteristics and market configurations change with the advertisement level (β). The tension between subscription and advertisement revenue leads to many corner solutions, creating different market configurations. Key aspects of the duopoly results include:

[33] A decrease in μ or M, or an increase in α, reduces σ_c.

1. *Natural monopoly:* The market becomes a natural monopoly when the advertisement level is low relative to the marginal cost of printing and circulation. Smaller or lower-income markets are more likely to be monopolies. In section 8.2, we will show that a market can also become a natural monopoly at higher advertisement levels if consumer preferences are homogeneous.

2. *Market expansion:* Advertisements reduce the subscription price[34]. They also increase market coverage by attracting lower-end consumers through price reductions or the entry of a low-quality firm targeting these consumers.

3. *Concentrated market even when consumers are ad-neutral:* In our model, endogenous fixed investment in quality drives market concentration and advantages to the leading firm. This occurs irrespective of whether there is a positive feedback effect of advertisement on the consumer side.

4. *Non-monotonic consumer surplus:* Higher market coverage does not necessarily mean higher consumer surplus. Specifically, consumer surplus decreases when the market configuration changes from Type B to Type C (see Figure 4), even though advertisers reach more consumers. Policies that subsidize unit costs (e.g., subsidized postal rates) to increase consumer welfare can have the opposite effect if the market shifts from Type B to Type C. Anderson and Peitz (2020) termed such an effect as the "see-saw effect," where a change in market fundamentals causes one side to lose and the other to gain, highlighting the need for careful consideration in policy-making.

5. *Entry of Free newspapers:* Moderately high advertisement levels (i.e. $\beta \in (\beta_1, \beta_2)$) lead to the entry of second firm as a free newspaper with minimal quality (e.g., relying on wire reports instead of editorial staff). Gabszewicz et al. (2012) first explained this phenomenon. They noted that the rise of free newspapers like Metro or 20 Minutes in Europe and Boston Metro and Philadelphia Metro in the USA accompanied increased ad revenue or reduced printing costs.

6. *Premium products:* Higher advertisement levels (i.e., $\beta > \beta_2$ or Type D equilibrium) challenge the high-quality firm's leadership, prompting it to raise its quality beyond the monopolistic level, potentially reaching the social planner level in the limit. At the same time, the price-to-quality ratio decreases, reducing the market power of the high-quality firm. This aligns with empirical evidence from Angelucci and Cage (2019), which shows that newspapers reduce quality when advertisement revenue declines.

However, a question could be raised about the feasibility of contesting leadership in the newspaper industry, given the effort required to build a consumer base and quality. In other words, is the threat of a challenge credible? This may require further empirical investigation.

[34] In two-sided markets, one side benefits if it has a positive effect on the other side

However, one piece of information that supports the feasibility of such a challenge is that most newspapers are owned by national-level chains, which often have leadership in one market while being the second player in another market.[35] These chains have the resources and technology to establish quality.

7. *Income inequality effect on firms*: Higher income inequality reduces the quality and profit of both firms, contrary to the effect of higher median income. With higher inequality, more consumers place lower value on quality, forcing firms to lower the price-to-quality ratio to retain marginal consumers, thereby lowering subscription revenue. However, the high-quality firm's advertisement revenue increases due to more affluent consumer base, making it more attractive for the low-quality firm to challenge the high-quality firm's leadership. This leads to a more likely occurrence of Type D equilibrium (i.e., β_2 decreases).

In a vertical differentiation model without advertisement (Gabszewicz and Thisse, 1979; Wauthy, 1996) we would not see Type C or Type D equilibrium. It is the advertisement revenue that makes free product attractive, or makes it profitable to contest the leadership of high-quality firm even if it means loosing all subscription revenue. The vertical differentiation model without advertisement also has a corner configuration with full market coverage (similar to Type C), but that type of equilibrium arises when consumer preference is homogeneous which we discuss in section 8.2.

7 Third player entry

In this section, we evaluate how the market configuration changes if a third firm is allowed to enter[36]. Suppose a maximum of three firms can enter the market, denoted as $k \in \{1, 2, 3\}$. Firms enter sequentially, with each firm making its quality choice, θ_k, before the next firm makes its entry decision. In the price stage, firms that have entered the market simultaneously set their subscription price s_k and p_k.

Since with three players there could be several different combinations of corner solutions, we focus our analysis on specific β intervals. The first interval is near the neighborhood of $\phi = 0$, where the advertisement intensity is not high, and the equilibrium is likely to be an interior solution (see Proposition 5). The second interval is when the advertisement intensity is large enough that the third firm can contest the two high-quality players. This includes all $\beta > \beta_1$ (see Proposition 6).

Proposition 5. *There exists $\epsilon > 0$ such that for all $\phi \in (-\epsilon, \epsilon)$, there exists a unique equilibrium in which Firm 3 enters with the lowest but positive quality. Relative to the duopoly equilibrium:*

[35] For example, the Times of India and Hindustan Times, where they have challenged each other in the Delhi and Mumbai markets

[36] Finding equilibrium qualities and market structures under free entry is intractable in vertical differentiation models, so we draw some inferences from the entry of a third firm.

- *The quality of Firm 1 and Firm 2 increases.*

- *Firm 1 and Firm 2 are placed closer to each other, i.e.,* $\frac{\theta_2}{\theta_1}$ *increases.*

- *The prices of both Firm 1 and Firm 2 decrease, and their market coverage increases.*

The steps to prove Proposition 5 is given in Appendix A.5. The entry of the third firm pushes up the quality of Firm 1 and Firm 2 as they try to reduce the business-stealing effect of Firm 3 by differentiation. However, firms are placed closer (quality ratios are higher) as the quality space is reduced, which leads to reduced prices and profits. Higher quality and lower prices increase the market coverage of Firm 1 and Firm 2. Additionally, Firm 3 attracts more lower-end consumers to the market, further increasing total market coverage. Total market coverage increase from 79% to 92% as shown below:

$$\text{Duopoly: } \frac{N_1}{M} = 52.6\% \ \frac{N_2}{M} = 26.3\%$$

$$\text{Three Firms: } \frac{N_1}{M} = 53.3\% \ \frac{N_2}{M} = 27.3\% \ \frac{N_3}{M} = 11.4\%$$

Next, we consider what happens if β (or corresponding ϕ) is such that Firm 3 does not enter with positive quality. Such a point will exist in the interval $(\phi^{-1}(0), \beta_1)$.[37] Let's denote the market share of Firm 1 and Firm 2 when they choose positive prices as N_1 and N_2, and as N_{1c} and N_{2c} when Firm 2 sets a zero price. Let A represent advertisement revenue net of marginal cost when a firm captures all demand. The expressions for these variables are given by (26), (27), (33), and (34).

$$\begin{aligned}
N_1 &= \frac{2}{4-\gamma} + \frac{1}{\kappa\theta_1(4-\gamma)}\phi \\
N_2 &= \frac{1}{4-\gamma} + \frac{1}{\kappa\theta_2(4-\gamma)}\phi \\
N_{1c} &= \frac{1}{2}(1 + \frac{1}{\kappa(\theta_1-\theta_2)}\phi) \\
N_{2c} &= \frac{1}{2}(1 - \frac{1}{\kappa(\theta_1-\theta_2)}\phi) \\
A &= M(\beta\delta - c)
\end{aligned}$$

Since Firm 3 can contest the duopoly leadership of both Firm 1 and Firm 2, causing them to lose their demand, Firm 1 and Firm 2 will protect their profits by ensuring the following constraints

[37] As β increases beyond $\phi^{-1}(0)$, Firm 2 decreases its price-to-quality ratio, leaving very little of the market uncovered, making it suboptimal for Firm 3 to enter, except with a free newspaper earning advertisement revenue from residual demand. This point is before β_1, when Firm 2, with a larger market share, finds it suboptimal to produce a quality newspaper.

are met while making quality decisions[38]:

$$
\begin{aligned}
&(I) \qquad && MN_{2c}\,\phi - \alpha\theta_2^2 - M(1-N_1-N_2)\phi \leq 0 \\
&(II) \qquad && A - \alpha\theta_1^2 - M(1-N_1-N_2)\phi \leq 0 \\
&(III) \qquad && MN_{2c}\phi - \alpha\theta_2^2 \geq 0
\end{aligned}
$$

Constraint (I) implies that Firm 3 should not get higher profit if it contests Firm 2 by setting $\theta_3 = \theta_{2+}$[39]. Constraint (II) implies that Firm 3 should not get higher profit if it contests Firm 1 by setting $\theta_3 = \theta_{1+}$. Constraint (III) implies that Firm 2 gets non-negative profit when its price reaches zero and Firm 3 is driven out[40]. We solve the duopoly problem with these three constraints. Proposition 6 states the result.

Proposition 6. *Suppose three firms can enter and $\beta > \beta_1$. Then:*

- *Firm 2 will set a positive quality which increases with β*
- *Firm 1 will set higher quality and lower price, and will earn lower profit than in a duopoly, with its profit declining as β increases.*
- *If β is sufficiently high, Firm 2 will either earn zero profit or will not enter the market.*

Figure 5 depicts this result graphically. It shows how Firm 1 (high-quality) and Firm 2 (low-quality) product choices, prices, and profits change when Firm 3 can enter. 'D' refers to duopoly and 'T' refers three firms case. The yellow (blue) line represents Firm 1 (Firm 2) under duopoly, and the red (green) line represents Firm 1 (Firm 2) with three firms. The top left box of Figure 5 shows that the quality of both Firm 1 and Firm 2 increases. Under duopoly, Firm 2 was producing the lowest quality newspaper, but with Firm 3's entry, it differentiates by increasing quality. This also means that it does not necessarily set a zero price, as shown in the bottom left box. Firm 1 increases quality for two reasons: a) Firm 2 has higher quality, so the competitive response is to differentiate and reduce the business-stealing effect, b) To ensure that the Firm 3 does not contest its product choice. Firm 3 has a lower profit than Firm 2 under duopoly, meaning higher gains from setting $\theta_3 = \theta_{1+}$. Higher competition from Firm 2 also lowers the price of Firm 1, thus reducing its profit. Under duopoly, Firm 1's profit was rising with β, but with Firm 3's entry, it decreases with β.

Finally, when β is sufficiently high, Firm 2 sets a zero price while having positive quality[41]. In this case, there is no demand left for Firm 3, and it will exit the market. Firm 2 covers a large market but earns zero profit. If Firm 2 decides not to enter, there is a loss of consumer surplus as Firm 1 will only partially cover the market.

[38] Note that when Firm 3 produces a free newspaper with zero quality, the market is effectively a duopoly.

[39] Firm 3's profit when it sets zero price and other firms set positive prices is $M(1-N_1-N_2)\phi$, which is residual demand times per unit contribution.

[40] If Firm 2 sets zero price and positive quality, then there is no residual demand for Firm 3.

[41] In this case all three constraints are binding

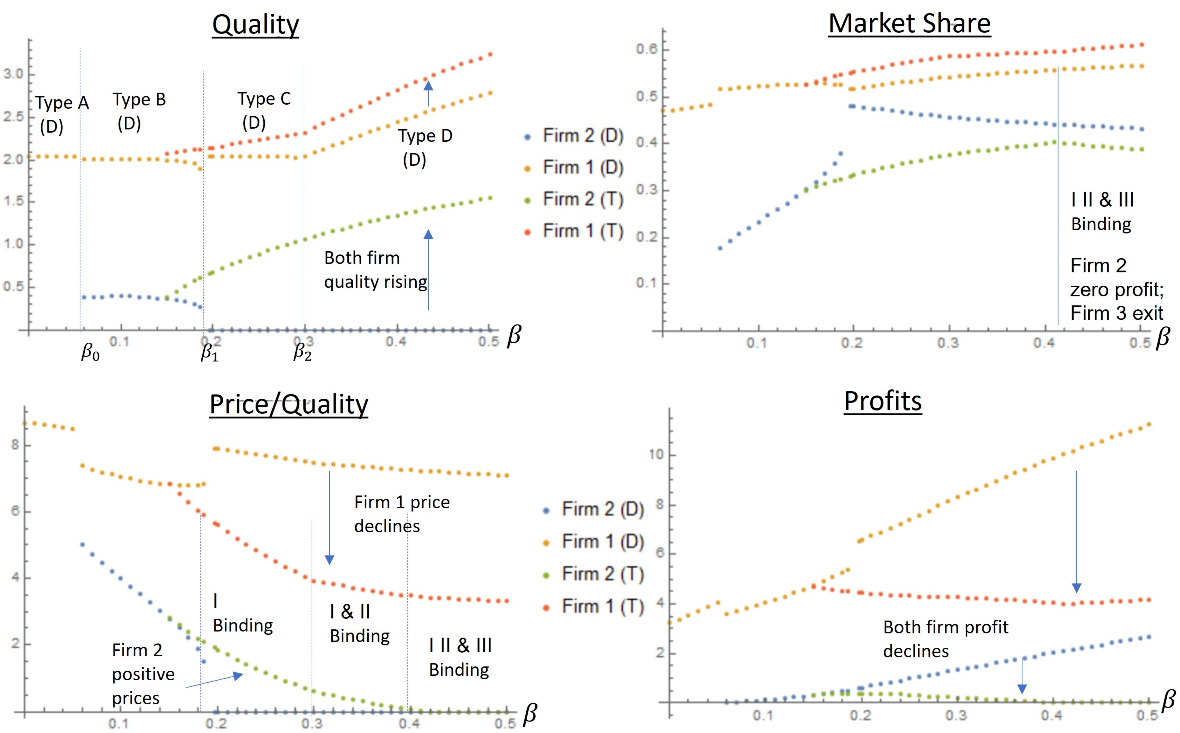


Figure 5: Market configurations with Three Firms

Note: Figure 5 does not show Firm 3's endogenous values to reduce clutter and to focus on the impact of Firm 3's entry on Firm 1 and Firm 2. Firm 3 will be a free newspaper with zero quality if $\beta > \beta_1$, and it will exit the market in the region where Firm 2 has zero price and profit.

To summarize this section, the entry of the third firm has two key aspects. First, it expands the market where it was previously uncovered (Proposition 5). Even after the entry of three players, the market is not fully covered, allowing for further entry. When the advertisement does not provide a sufficiently positive per unit contribution (i.e., in the small neighborhood of $\phi = 0$), the tension between advertisement and subscription revenue is diminished, and we don't see corner solutions. The market will support multiple but finite numbers of vertically differentiated firms (Shaked and Sutton, 1983).

More importantly, the entry of the third firm increases competition and raises product qualities when the advertisement level is high (Proposition 6). Both Firm 1 and Firm 2 offer much higher quality newspapers at lower prices, thus increasing consumer surplus. Additionally, their profits decrease with the advertisement level. In contrast, their profits were increasing with advertisement under duopoly. In this sense, advertisements make the newspaper industry more efficient even when the market is concentrated with few players. This is consistent with the empirical evidence provided by Angelucci and Cage (2019) and Pattabhiramaiah (2014), which shows that with the decline in advertisement, firms raised their prices and lowered the quality. However, it is of empirical importance to verify if the profits of these firms also increased when the advertisement revenue declined. This is similar to Donnenfeld and Weber (1995)'s finding that under vertical differentiation, product competition among duopoly incumbents leads to entry deterrence.

8 Robustness

In this section, we consider the robustness of our duopoly results under different assumptions such as: a) Firms make entry decisions and quality decisions simultaneously; b) The distribution of consumer preferences for reading is more homogeneous; c) Consumers are not ad-neutral. We analyze our results with these new assumptions only with respect to duopoly and, in some cases, monopoly, but not for the three-firm scenario to reduce complexity. However, it can be easily inferred that none of these assumptions change the result for three-firm case except when consumer preferences are homogeneous, in which case Type D equilibrium could vanish.

8.1 Duopoly with Simultaneous Entry

We change the timing of the game so that the firms enter and choose quality simultaneously:

> *Stage 1:* Both firms make entry decisions and simultaneously choose the quality (θ) of their own product.
>
> *Stage 2:* Firms simultaneously set subscription price (s) and advertisement price (p).

The solution of the simultaneous entry model is detailed in Appendix A.4. Lemmas 13 summarizes how a Type B equilibrium under simultaneous entry compares to that under sequential entry, and Lemma 15 shows that no Type D equilibrium exist in simultaneous entry model. There are no changes in Type A and Type C equilibria, as in these cases the high-quality firm behaves like a monopolist. We continue to maintain Assumption 1 so that $\phi_2 > \phi_{ir}$, which guarantees the existence of Type C equilibrium.

Lemma 13. *Suppose a Type B equilibrium exists for a given ϕ. In this equilibrium:*

1. *The high-quality firm sets a higher quality and price-to-quality ratio but has lower market coverage and profit compared to those under sequential entry. This quality is higher than the monopolist level.*

2. *The low-quality firm sets a higher quality and price-to-quality ratio, but earns higher profits despite lower market shares compared to those under sequential entry.*

3. *The quality ratio $\gamma \equiv \frac{\theta_2}{\theta_1}$ is lower than that in sequential entry/*

The competition reduces under simultaneous entry as firms are located farther (lower γ), and therefore both firms achieve a higher price-to-quality ratio, which results in lower market coverage.

Under sequential entry, the high-quality firm lower quality to crowd out the quality space (closer substitute) of the low-quality firm, making its entry less profitable. This is reversed in simultaneous entry, and the high-quality firm sets higher quality, even higher than the monopoly, to distance itself from the entrant. This results in higher profit for the low-quality firm, but the profit of the high-quality firm reduces as it loses first mover advantage. Simultaneous entry also lowers consumer surplus due to both higher price and lower market coverage. This comparison is exactly similar to the comparison between the Cournot equilibrium (simultaneous) and the Stackelberg equilibrium (sequential). Next we evaluate how the cutoff points ϕ_0 and ϕ_1 change relative to that of sequential entry.

Lemma 14. *There exist critical cut-off points ϕ_0 and ϕ_1 such that a unique Type B equilibrium exists if $\phi \in (\phi_0, \phi_1)$. Further, ϕ_0 is lower and ϕ_1 is higher than those in the sequential entry model.*

Notice that the length of the interval (ϕ_0, ϕ_1) increases from both sides because the lower competition results in higher profit for the low-quality firm. Conversely, this implies that the leading firm in sequential entry is able to deter the entrant for some advertisement level. The cut-off point ϕ_2 does not change between the two models as this point depends on the θ_{MP}.

Lemma 15. *There does not exist any pure strategy equilibrium if $\phi > \phi_2$.*

The proof of Lemma 15 is provided in Appendix A.4. Intuitively, when the advertisement intensity exceeds ϕ_2, the vertical differentiation strategy breaks, as both firms compete aggressively to capture the full advertisement revenue, each setting quality levels marginally higher than the other's. This intense competition results in the absence of a pure strategy equilibrium, similar to the one observed in the Hotelling model (D'Aspremont et al., 1979).

Proposition 7 outlines the duopoly market configurations under simultaneous entry. The proof is derived directly from Lemmas 5, 11,14 and 15. We define: $\beta_0 \equiv \phi^{-1}(\phi_0), \beta_1 \equiv \phi^{-1}(\phi_1), \beta_2 \equiv \phi^{-1}(\phi_2)$.

Proposition 7. *There exist $(\beta_0, \beta_1, \beta_2)$ with $\underline{\beta} < \beta_0 < \beta_1 < \beta_2 < \overline{\beta}$ such that*

a *If $\beta \in (\underline{\beta}, \beta_0)$ then unique equilibrium of type A exist. One firm enters and sets the monopolist price and quality as given by (12) and (11) and the market remains uncovered.*

b *If $\beta_0 < \beta < \beta_1$ then unique equilibrium of type B exist. Both firm set positive and differentiated quality and the quality differential increases with β if $\phi(\beta) \geq 0$. Market remains uncovered.*

c *If $\beta_1 < \beta \leq \beta_2$ then unique equilibrium of type C exist. The high quality firm sets the monopolist level price and quality, and the low quality firm provides a free product with minimum quality. Market is fully covered.*

d *There does not exist any pure strategy equilibrium if $\beta > \beta_2$.*

Note: In the small right-side neighborhood of β_1, there may not exist an equilibrium as firms switch between Type B and Type C configuration (see Appendix A.4 for explanation).

Figure 6 graphically depicts the endogenous parameters across all values of β under a duopoly with simultaneous entry.

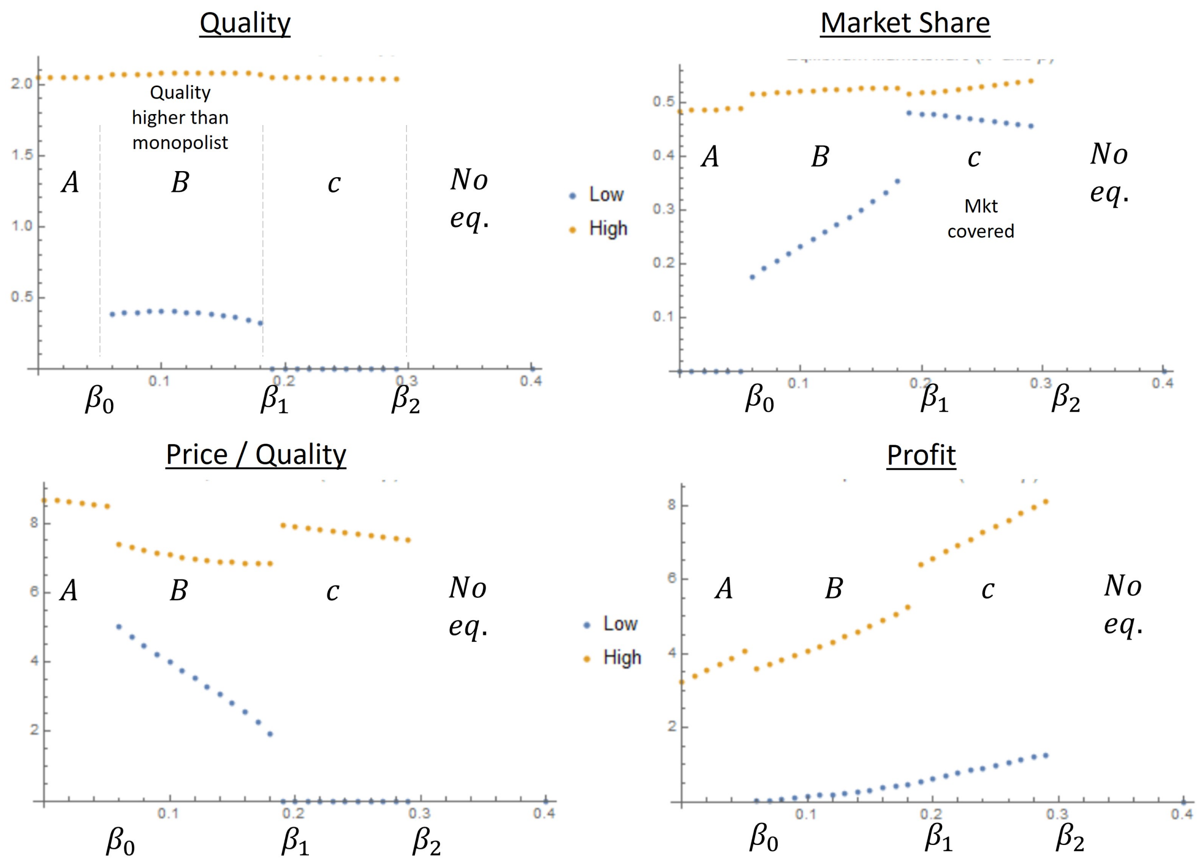


Figure 6: Quality, Market Share, Price and Profit of two firms under Simultaneous Entry Duopoly

To summarize, the simultaneous entry model differs from the sequential entry model in three aspects. First, under the Type B equilibrium (i.e., $\beta \in (\beta_0, \beta_1)$), there is less competition in simultaneous entry. As a result, both firms raise their price-to-quality ratio, which reduces consumer surplus and market coverage. This lower competition benefits the low-quality firms, but the high-quality firm, losing its first-mover advantage, earns lower profits. Second, under sequential entry, the first-mover firm deters entry for some parameter values close to β_0. This deterrence is absent in simultaneous entry, allowing some markets to transition from Type A to Type B, which has a higher market coverage and consumer surplus. Finally, the simultaneous entry model lacks a pure strategy equilibrium for $\beta > \beta_2$ and in the vicinity of β_1, which is not an issue under sequential entry. Shaked and Sutton (1987) pointed out that in a vertical differentiation model, if the price equilibrium exist on the second stage then sequential entry guarantees the existence of a pure strategy equilibrium, while such existence problem may arise on the product choice stage under simultaneous entry.

8.2 Duopoly and consumer preference heterogeneity

In this section, we parameterize consumer preferences to reflect varying degrees of heterogeneity (or conversely, homogeneity). Suppose the consumer's preference for quality content, v, follows a uniform distribution $v \sim U(b-1, b)$ where $1 \leq b \leq 2$. The ratio $\frac{b}{b-1}$ measures the heterogeneity across consumers, and it decreases when b increases. The lower the b, the more heterogeneous is the preference. In the previous section, we assumed $b = 1$, which implies maximum heterogeneity.

Our findings in this section demonstrate that as consumer preferences become more homogeneous (i.e., b increases), the market becomes more concentrated. Specifically, the high-quality firm becomes more dominant and captures a larger share of the market. As b approaches 2, the market evolves into a natural monopoly, irrespective of the advertisement level. This is a standard result in the vertical differentiation literature (Gabszewicz and Thisse, 1979; Wauthy, 1996). Wauthy (1996) used a duopoly model under vertical differentiation and showed that as consumer preferences become more homogeneous, the market transitions from an uncovered configuration to a covered configuration and finally to a monopoly. Our model validates the same result, even in the presence of advertising. Higher advertisement levels facilitate the market's transition from uncovered (Type B) to covered configuration (Type C) at a lower level of homogeneity. In addition, when preferences are sufficiently homogeneous, we observe only the Type C market configuration. In contrast, with the heterogeneous preferences of our base model, we observed four distinct types of market configurations, depending on the advertisement level.

We first examine how a monopolist's behavior changes as b increases, since this factor plays a crucial role in determining the structure of the duopoly.

Monopoly

The demand function when $v \sim U(b, b-1)$:

$$N(s,\theta) = M(\mathbf{b} - \frac{s}{\kappa\theta}) \qquad \text{if } (\boldsymbol{b}-1)\kappa\theta \leq s \leq \mathbf{b}\kappa\theta \tag{37}$$

(37) indicates that the demand curve shifts outwards when b increases. If the monopolist problem has an interior solution, i.e. market is uncovered, the quality, θ_{MP} is given by the unique solution of (38).

$$8\alpha\kappa\theta^3 - M\boldsymbol{b^2}\kappa^2\theta^2 + M\phi^2 = 0 \text{ s.t. } \theta > \frac{M\kappa\boldsymbol{b^2}}{12\alpha} \tag{38}$$

The subscription price and the profit function when there is an interior solution:

$$\begin{aligned} s_{MP} &= \frac{1}{2}\boldsymbol{b}\kappa\theta_{MP} - \frac{1}{2}\phi && (39) \\ \Pi_{MP} &= \frac{M}{4\kappa\theta_{MP}}(\boldsymbol{b}\kappa\theta_{MP} + \phi)^2 + M\boldsymbol{b}\beta(\delta - \kappa) - \alpha\theta_{MP}^2 && (40) \end{aligned}$$

If the monopolist problem has a corner solution then

$$\begin{aligned} \theta_{MP} &= (b-1)\frac{M\kappa}{2\alpha} && (41) \\ s_{MP} &= (b-1)\kappa\theta_{MP} && (42) \\ \Pi_{MP} &= (b-1)^2\frac{M^2\kappa^2}{4\alpha} + Mb\beta(\delta-\kappa) + M\phi && (43) \end{aligned}$$

The cutoff advertisement level when the monopolist does not serve the market, $\underline{\beta}$, and when it opts for a corner solution, $\overline{\beta}$, are given by:

$$\phi(\underline{\beta}) = -\boldsymbol{b^3}\frac{M\kappa^2}{27\alpha} \qquad \phi(\overline{\beta}) = \rho(b)\frac{M\kappa^2}{27\alpha} \text{ where } \rho \text{ is strictly increasing in } b \text{ and } \rho \in [1, 3\sqrt{6}]$$

We can infer from the above equations that as b increases, quality, price-to-quality, and market coverage also increase. This is because a higher b causes the demand curve to shift outward. Consider the benchmark case where $\phi = 0$: $\theta_{MP} = b^2\frac{M\kappa}{8\alpha}$ increases quadratically with b, and the market coverage $\frac{N}{M} = \frac{1}{2}b$ increases linearly with b. The market is fully covered when $b = 2$. Additionally, the cutoff $\underline{\beta}$ decreases as b increases, enabling more markets with lower advertisement level to be served by a monopolist. Proposition 8 states this result.

Proposition 8. *As consumer preferences for quality content become more homogeneous (i.e. b increase), the market coverage by the monopolist also increases. The market becomes fully covered when $b = 2$. Furthermore, $\underline{\beta}$ decreases with b, enabling the monopolist to serve more markets that have lower advertisement levels.*

Also note that the corner solution has a positive quality and subscription revenue if $b > 1$. This occurs because a monopolist can cover the full market at a higher price. For $b = 2$, the monopolist chooses the same quality across all values of β. This implies that as consumer preferences become more homogeneous, the monopolist does not significantly reduce quality when $\beta > \overline{\beta}$. Figure 7 displays monopolist quality and market coverage for three different levels of b.

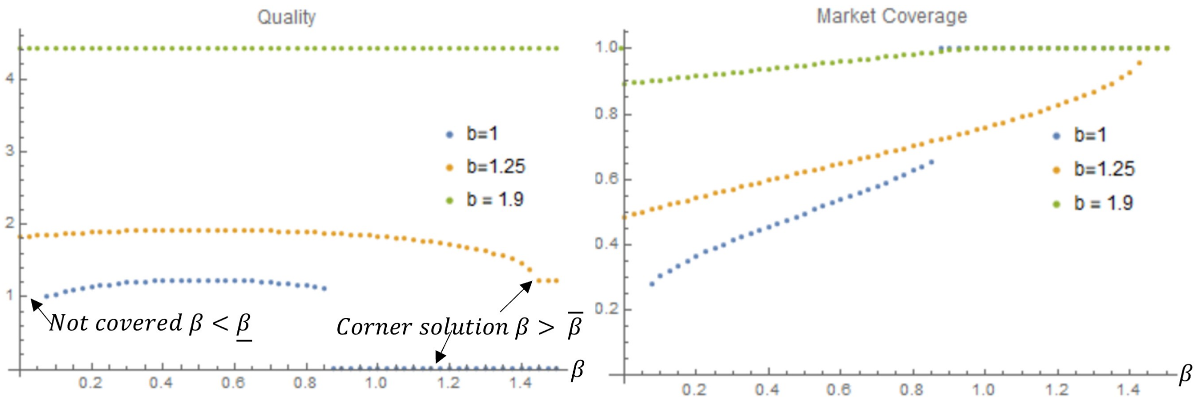


Figure 7: Monopolist's quality and market coverage for different levels of consumer preference heterogeneity(b)

As depicted in Figure 7, three distortions of the monopolist model from section 5 are partially addressed when consumer preferences become more homogeneous: a) market coverage increases; b) markets that are not served earlier are served when b increases; and c) the monopolist does not significantly reduce quality, even when the advertisement level is high.

8.2.1 Duopoly

The duopoly market also demonstrates that the dominance of the high-quality firm increases as consumer preference becomes more homogeneous (b increases), transitioning the market toward a natural monopoly when $b = 2$. All markets shift to Type C when b is sufficiently high[42], irrespective of the advertisement level.

Type B equilibrium (uncovered market)

Under Type B equilibrium (interior solution), both firm enters and the market remain uncovered. The corresponding equations for demand, profit, market share, and price-to-quality are shown

[42]In Type C market, the high-quality firm chooses monopolist level quality while the low-quality firm serves the residual demand at lowest quality

below. The parameter **b** is shown in the bold to highlight difference from the model in section 6.

$$N_1 = M(\mathbf{b} - \frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)}) \tag{44}$$

$$N_2 = M(\frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)} - \frac{s_2}{\kappa\theta_2}) \tag{45}$$

$$\Pi_1 = M\frac{1-\gamma}{\kappa\theta_1(4-\gamma)^2}(2\mathbf{b}\kappa\theta_1 + \phi)^2 + M\mathbf{b}\beta(\delta - \kappa) - \alpha\theta_1^2 \tag{46}$$

$$\Pi_2 = M\frac{1-\gamma}{\kappa\theta_2(4-\gamma)^2}(\mathbf{b}\kappa\theta_2 + 2\phi)^2 - \alpha\theta_2^2 \tag{47}$$

$$\frac{N_1}{M} = \frac{2\mathbf{b}}{4-\gamma} + \frac{1}{\kappa\theta_1(4-\gamma)}\phi \tag{48}$$

$$\frac{N_2}{M} = \frac{\mathbf{b}}{4-\gamma} + \frac{2}{\kappa\theta_2(4-\gamma)}\phi \tag{49}$$

$$\frac{s_1}{\theta_1} = 2\mathbf{b}\frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_1}\frac{3}{4-\gamma}\phi \tag{50}$$

$$\frac{s_2}{\theta_2} = \mathbf{b}\frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_2}\frac{2+\gamma}{4-\gamma}\phi \tag{51}$$

$$\text{where } \gamma = \frac{\theta_2}{\theta_1}, \kappa = e^{\mu - \frac{1}{2}\sigma^2}, \delta = e^{\mu + \frac{1}{2}\sigma^2}$$

We characterize the Type B equilibrium by solving the first-order conditions of both firms under sequential entry, using the exact same approach as detailed in Appendix A.3. The result is stated in Lemmas 16. The proofs can be easily derived from Equations (44)-(51); however, we discuss the intuitions behind the results.

Lemma 16. *Suppose a Type B equilibrium exists for a given β and b. If b increases marginally, indicating more homogeneous consumer preferences, then the quality, market share, price-to-quality and profit of both firm increases.*

The above results stem directly from the demand curve shifting outward due to an increase in b. These effects benefit both firms as the market is uncovered. However, once the market becomes fully covered, it transitions to a Type C equilibrium, which involves different dynamics as stated in Lemma 17.

Type C equilibrium (corner configuration)

Under a Type C equilibrium, the market is covered, so any increase in the market share of the high-quality firm will lower the market share of the low-quality firm. In other words, the low-quality firm serves the residual demand. This implies that higher demand does not necessarily benefit the low-quality firm, as stated in Lemma 17. We also modify the definition of Type C, as defined in Definition 4, to allow the low-quality firm to set non-zero quality. The equations for price, market share, and profit under Type C are given by:

$$
\begin{aligned}
s_1 &= \frac{1}{2}\mathbf{b}\kappa\theta_1 - \frac{1}{2}\kappa\theta_2 - \frac{1}{2}\phi && (52)\\
s_2 &= (\mathbf{b}-1)\kappa\theta_2 && (53)\\
\frac{N_1}{M} &= \frac{\mathbf{b}\theta_1 - \theta_2}{2(\theta_1-\theta_2)} + \frac{1}{2\kappa(\theta_1-\theta_2)}\phi && (54)\\
\frac{N_2}{M} &= \frac{(2-\mathbf{b})\theta_1 - \theta_2}{2(\theta_1-\theta_2)} - \frac{1}{2\kappa(\theta_1-\theta_2)}\phi && (55)\\
\Pi_1 &= \frac{M}{4\kappa(\theta_1-\theta_2)}(\kappa(\mathbf{b}\theta_1-\theta_2)+\phi)^2 + M\mathbf{b}\beta(\delta-\kappa) - \alpha\theta_1^2 && (56)\\
\Pi_2 &= \underbrace{M\,(\frac{\kappa((2-\mathbf{b})\theta_1-\theta_2)-\phi}{2\kappa(\theta_1-\theta_2)})}_{\text{Market share}}\ \underbrace{((\mathbf{b}-1)\kappa\theta_2+\phi)}_{\text{Revenue per subscriber}}\ -\alpha\theta_2^2 && (57)
\end{aligned}
$$

Lemma 17. *Suppose a Type C equilibrium exists for a given β and b. Under this equilibrium:*

a) The market share of the high-quality firm increases with b, while the market share of the low-quality firm decreases with b.

b) The quality and the profit of high-quality firm increase with b.

c) The quality and profit of the low-quality firm are non-monotonic in b, initially increasing and then decreasing.

d) When $b = 2$, the low-quality firm is driven out of the market.

As observed in Type B, the quality, profit, and market share of the high-quality firm increase with b due to the outward shift of the demand curve. However, since the low-quality firm serves the residual demand, its market share decreases as the market share of the high-quality firm increases (see (54)-(55)). The quality and profit of the low-quality firm are concave and non-monotonic in b due to two opposing effects when b increases:

a) The market share decreases (first term in (57)), which reduces revenue and lowers both the equilibrium quality and profit.

b) Revenue per subscriber increases (second term in (57)) as the valuation of the lowest-value customer rises. This raises both equilibrium quality and profit.

The latter effect predominates when b is small, while the former effect prevails when b is large, making the quality and profit non-monotonic. When $b = 2$, the high-quality firm fully covers the market, leaving no residual demand for the low-quality firm. Proposition 9 states our main result.

Proposition 9. *For a given β:*

a) If the market is Type A at $b = 1$, then there exist two thresholds b_0 and b_1 with $1 < b_0 < b_1 < 2$ such that the market transitions to Type B when $b > b_0$ and to Type C when $b > b_1$.

b) *If the market is Type B at $b = 1$, then there exists a threshold $b_1 \in (1, 2)$ such that the market transitions to Type C when $b > b_1$.*

c) *If the market is Type D at $b = 1$, then there exists a threshold $b_2 \in (1, 2)$ such that the market transitions to Type C when $b > b_2$.*

d) *If $b = 2$, the market becomes a natural monopoly.*

Further, b_0 and b_1 are decreasing in β, and b_2 is increasing in β.

If b increases, then the demand curve shifts outward, which in turn increases the profit of both firms, including that of a potential entrant, provided that the market is not uncovered (see Lemma 16). b_0 represents the threshold b at which the entrant just starts making a positive profit, marking the transition from a Type A to a Type B market. If b continues to increase, market coverage expands until b reaches the threshold b_1, at which point the market is just covered.

An increase in b raises θ_{MP}, the monopolist's quality level. This hardens the constraint for a Type D equilibrium, wherein the low-quality firm must make higher profit by marginally exceeding θ_{MP} when the high-quality firm chooses θ_{MP} (see Definition 7). When $b > b_2$, this constraint is violated, and the market moves to Type C.

When β increases and moves closer to the cut-off points β_0 (for Type A) or β_1 (for Type B), smaller increases in b are required for the transition, hence b_0 and b_1 decreases with β. Conversely, the transition from Type D to Type C requires a reduction in β to move closer to the cut-off point β_2, hence b_2 increases with β_2.

Figure 8 displays endogenous parameters when market transitions from Type A to Type B to Type C as b increases.

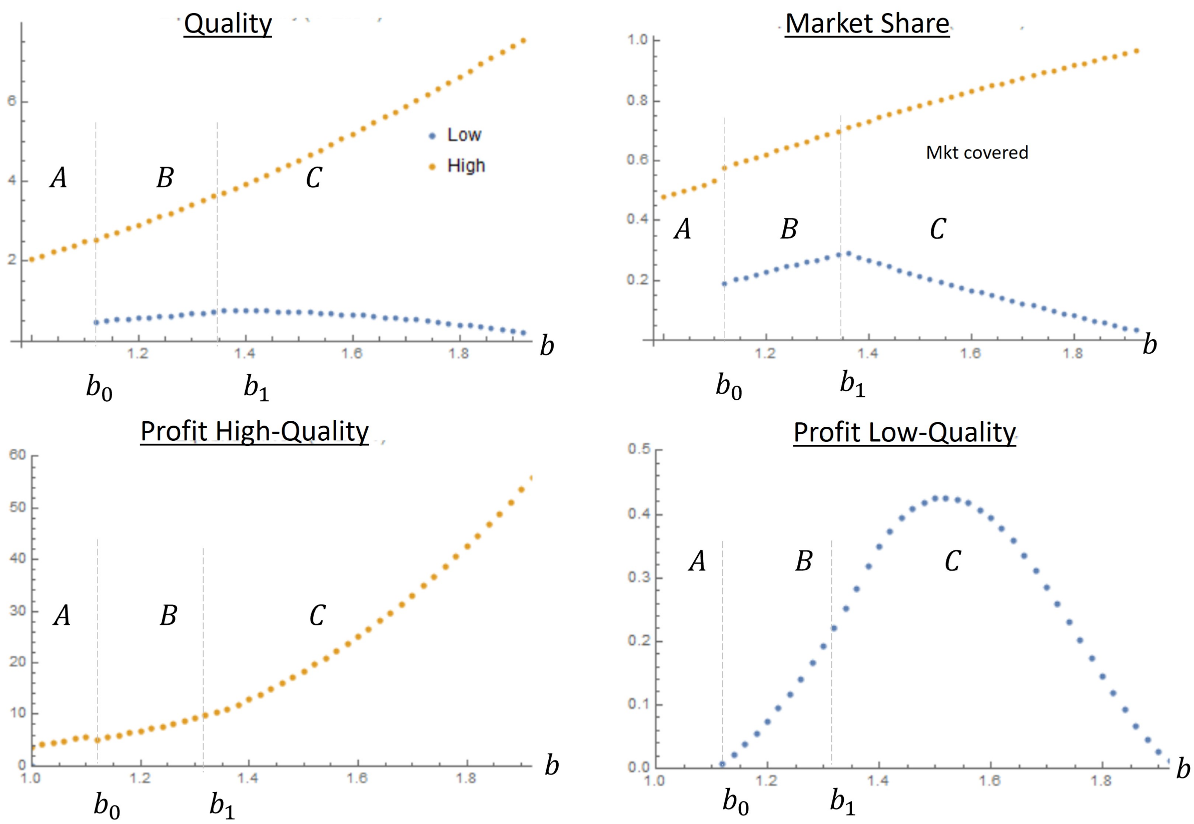


Figure 8: Quality, market share and profit of firms when b increases, showing market transitions from Type A to B to C

8.3 Consumers see advertisement as nuisance

Let's assume that consumers see advertisements as a nuisance and experience negative utility from them. This utility loss is proportional to the advertisement level, as shown below:

$$U_i = v_i Y_i \theta - \eta\beta - s \text{ where } \eta > 0 \tag{58}$$

We refer to the parameter η as the nuisance factor. We first summarize our findings of this section before detailing the model. The key results when η increases are:

a) Total market coverage decreases because consumers with lower utility from reading drop off from subscribing. Even in Type C or Type D markets, where the low-quality product is free, the market is not fully covered.

b) Profits for both firms decrease across all market configurations, with a larger impact on the low-quality firm. Marginal consumers of the low-quality firm are deciding between subscribing and not subscribing, while marginal consumers of the high-quality firm are deciding between the two firms' products, both of which include advertisements. The low-quality firm reduces its price-to-quality ratio to compensate for the utility loss and retain some consumers, prompting a competitive response from the high-quality firm to also lower price-to-quality ratio to retain its customers, leading to reduced profits for both firms. The lower profit of the low-profit firm also results in a lower β_0, as marginal firms exit the market.

c) The quality of both firms is higher under Type C and Type D configurations, which is counter-intuitive given the lower profits. According to the utility function (58), setting a zero quality is no longer optimal since demand drops to zero even when subscriptions are free. To maintain its customer base and earn advertisement revenue, the low-quality firm must raise its quality when η increases, triggering a competitive response from the high-quality firm to also increase quality. However, rising quality without subscription revenue (price is zero) disproportionately reduces the low-quality firm's profit under Type C and Type D.

d) The range of advertisement levels, (β_1, β_2), for which Type C configuration exists decreases because β_1 increases and β_2 decreases. Lower profits for the low-quality firm under Type C raise the cut-off point β_1 for switching to a corner solution. The decreased profit in Type C also relaxes the constraint for Type D equilibrium, making it more attractive for the low-quality firm to contest the leadership of high-quality firm, thus lowering β_2.

These results suggest that the high-quality firm becomes even more dominant relative to the low-quality firm.

Next, we detail the model with the modified utility function (58). Since all four types of market configurations are possible, we highlight the changes in each configuration.

Type A equilibrium (monopoly)

The demand function of monopolist will be given by:

$$N(s,\theta) = M\left(1 - \frac{s + \boldsymbol{\eta\beta}}{\kappa\theta}\right) \qquad \text{if } 0 \leq s \leq \kappa\theta - \eta\beta \tag{59}$$

(59) indicates that the demand curve shifts inward when η increases. If the monopolist's problem has an interior solution, i.e., the market is uncovered, the quality, θ_{MP}, is given by the unique solution of: (38).

$$8\alpha\kappa\theta^3 - M\kappa^2\theta^2 + M(\phi - \eta\beta)^2 = 0 \text{ s.t. } \theta > \frac{M\kappa}{12\alpha} \tag{60}$$

The subscription price and the profit function when there is an interior solution:

$$s_{MP} = \frac{1}{2}\kappa\theta_{MP} - \frac{1}{2}(\phi + \eta\beta) \tag{61}$$

$$\Pi_{MP} = \frac{M}{4\kappa\theta_{MP}}(\kappa\theta_{MP} + \phi - \boldsymbol{\eta\beta})^2 + M\beta(\delta - \kappa) - \alpha\theta_{MP}^2 \tag{62}$$

If the monopolist's problem has a corner solution then

$$
\begin{aligned}
s_{MP} &= 0 && (63)\\
\theta_{MP} &= \sqrt[3]{\frac{M\phi\eta\beta}{2\alpha\kappa}} && (64)\\
\Pi_{MP} &= M(1-\frac{\eta\beta}{\kappa\theta_{MP}})\phi + M\beta(\delta-\kappa) - \alpha\theta_{MP}^2 && (65)
\end{aligned}
$$

The cutoff advertisement level when the monopolist does not serve the market, $\underline{\beta}$, and when it opts for a corner solution, $\overline{\beta}$, are given by:

$$\phi(\underline{\beta}) = \eta\beta - \frac{M\kappa^2}{27\alpha} \qquad \phi(\overline{\beta}) = \eta\beta + \frac{M\kappa^2}{27\alpha}$$

We can make following inferences from the above equations:

a) As the nuisance factor η increases, price-to-quality ratio, profit, and market coverage decrease. This is the direct effect of the inward shift of the demand curve. Consumers need to be compensated for the loss of utility through price reduction and/or quality increase.

b) The market is not fully covered even when the subscription price is zero (see (65)) because consumers who derive very little utility from reading will not subscribe even free product.

c) The cutoff point $\underline{\beta}$ for not serving a market increases due to lower profit, as some marginally profitable markets will not be served.

d) The cutoff point for a corner solution $\overline{\beta}$ also increases because of the lower profit under corner solution due to both lower demand and the higher cost of quality (note: quality in a corner solution is not zero).

Note: Quality may increase or decrease with η depending on the advertisement level. It remains single-peaked with respect to β, with the peak at $\phi = \eta\beta$ (see (60)). With higher η, the peak shifts to the right, but the maximum quality level remains at $\frac{M\kappa}{8\alpha}$.

Type B (uncovered configuration) equilibrium

Demand for firms are derived using the utility of indifferent customers and is shown below:

$$
\begin{aligned}
N_1 &= M(1-\frac{s_1-s_2}{\kappa(\theta_1-\theta_2)}) && (66)\\
N_2 &= M(\frac{s_1-s_2}{\kappa(\theta_1-\theta_2)} - \frac{s_2+\boldsymbol{\eta\beta}}{\kappa\theta_2}) && (67)
\end{aligned}
$$

Note that the demand for Firm 1 does not directly depend on η because the marginal consumers are deciding between the products of Firm 1 and Firm 2, and they incur this utility loss with both firms. In contrast, the marginal consumers for Firm 2 are deciding between subscribing and not subscribing. However, the strategic actions of Firm 2 may change the demand for Firm 1 and consequently its profit, price-to-quality ratio, and market coverage, as shown in equations (68)-(75).

$$
\begin{aligned}
s_1 &= \frac{2\kappa\theta_1(\theta_1-\theta_2)}{4\theta_1-\theta_2} - \frac{3\theta_1}{4\theta_1-\theta_2}\phi - \boldsymbol{\frac{\theta_1-\theta_2}{4\theta_1-\theta_2}\eta\beta} && (68)\\
s_2 &= \frac{\kappa\theta_2(\theta_1-\theta_2)}{4\theta_1-\theta_2} - \frac{2\theta_1+\theta_2}{4\theta_1-\theta_2}\phi - \boldsymbol{\frac{2(\theta_1-\theta_2)}{4\theta_1-\theta_2}\eta\beta} && (69)\\
\Pi_1 &= M\frac{1-\gamma}{\kappa\theta_1(4-\gamma)^2}(2\kappa\theta_1 + (\phi - \boldsymbol{\eta\beta}))^2 + M\beta(\delta-\kappa) - \alpha\theta_1^2 && (70)\\
\Pi_2 &= M\frac{1-\gamma}{\kappa\theta_2(4-\gamma)^2}(\kappa\theta_2 + 2(\phi - \boldsymbol{\eta\beta}))^2 - \alpha\theta_2^2 && (71)\\
\frac{N_1}{M} &= \frac{2}{4-\gamma} + \frac{1}{\kappa\theta_1(4-\gamma)}(\phi - \boldsymbol{\eta\beta}) && (72)\\
\frac{N_2}{M} &= \frac{1}{4-\gamma} + \frac{2}{\kappa\theta_2(4-\gamma)}(\phi - \boldsymbol{\eta\beta}) && (73)\\
\frac{s_1}{\theta_1} &= 2\frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_1}\frac{3}{4-\gamma}\phi - \boldsymbol{\frac{1}{\theta_1}\frac{1-\gamma}{4-\gamma}\eta\beta} && (74)\\
\frac{s_2}{\theta_2} &= \frac{k(1-\gamma)}{4-\gamma} - \frac{1}{\theta_2}\frac{2+\gamma}{4-\gamma}\phi - \boldsymbol{\frac{2}{\theta_1}\frac{1-\gamma}{4-\gamma}\eta\beta} && (75)
\end{aligned}
$$

$$\text{where } \gamma = \frac{\theta_2}{\theta_1}, \kappa = e^{\mu-\frac{1}{2}\sigma^2}, \delta = e^{\mu+\frac{1}{2}\sigma^2}$$

We solve the first-order conditions of both firms under sequential entry and determine the equilibrium θ_1 and θ_2 and thereby prices and profits, using the exact same approach as detailed in Appendix A.3. The result is stated in Lemmas 18.

Lemma 18. *Suppose the market is in a Type B equilibrium for a given exogenous parameter values. If the nuisance factor η increase, then price-to-quality, market coverage and profit of both firm decreases.*

As η decreases, the marginal customers of low-quality firm (Firm 2), who have a lower value for quality content, need to be compensated for the utility loss from advertisements either by reducing price or increasing quality (i.e., lowering the price-to-quality ratio), or they will not subscribe. Firm 2 lowers the price-to-quality ratio to retain some of these customers, depending on elasticity, but not all. The lower price and market share reduce the profit of Firm 2. As Firm 2 lowers its price, Firm 1 (high-quality firm) loses some of its consumers. To protect its customer base, Firm 1 also lowers its price-to-quality ratio, but to a smaller extent than Firm 2. This reduces Firm 1's profit and market coverage.

Type C (corner) market configuration

$$
\begin{aligned}
s_1 &= \frac{1}{2}\kappa(\theta_1-\theta_2) - \frac{1}{2}\phi \ ; \ s_2 = 0 & (76)\\
\frac{N_1}{M} &= \frac{1}{2} + \frac{1}{2\kappa(\theta_1-\theta_2)}\phi & (77)\\
\frac{N_2}{M} &= \frac{1}{2} - \frac{1}{2\kappa(\theta_1-\theta_2)}\phi - \boldsymbol{\frac{\eta\beta}{\kappa\theta_2}} & (78)\\
\Pi_1 &= \frac{M}{4\kappa(\theta_1-\theta_2)}(\kappa(\theta_1-\theta_2)+\phi)^2 + M\beta(\delta-\kappa) - \alpha\theta_1^2 & (79)\\
\Pi_2 &= M\underbrace{(\frac{1}{2} - \frac{\phi}{2\kappa(\theta_1-\theta_2)} - \boldsymbol{\frac{\eta\beta}{\kappa\theta_2}})}_{\text{Market share}}\phi - \alpha\theta_2^2 & (80)
\end{aligned}
$$

Two important differences to note in the above equations compared to the Type C configuration in section 6 are: a) The market is never fully covered if $\eta > 0$, as the lowest value consumers ($\frac{\eta\beta}{\kappa\theta_2}$ term) will not subscribe even at a zero price; the higher the η, the greater the loss of subscribers. b) The low-quality firm will set a non-zero quality in Type C, and this quality increases with η.

Lemma 19. *Suppose the market is in a Type C equilibrium for given exogenous parameter values. If the nuisance factor η increases, then:*

a) The price-to-quality ratio and profit of both firms decrease.

b) The quality of both firms increases.

c) The market share of the high-quality firm increases, while that of the low-quality firm decreases.

As η increases, more and more consumers with a low value for quality content decide not to subscribe. Since the price cannot go below zero, the low-quality firm (Firm 2) raises its quality to retain some of these consumers. The higher cost of quality and a reduced consumer base decrease its profit. In a competitive response to Firm 2's action, Firm 1 lowers its price-to-quality ratio, partially by lowering the price and partially by raising quality to differentiate from the competition, thereby reducing its own profit.

Type D equilibrium

The quality set by the high-quality firm, θ_c, is implicitly defined by the equation:

$$
\underbrace{M(1-\frac{\eta\beta}{\kappa\theta_c})\phi + M\beta(\delta-\kappa) - \alpha\theta_c^2}_{\text{Firm 2 Profit if it marginaly exceeds } \theta_c} - \underbrace{[\frac{M}{2}(1-\frac{2\eta\beta}{\kappa\theta_2} - \frac{\phi}{\kappa(\theta_c-\theta_2)})\phi - \alpha\theta_2^2]}_{\text{Firm 2 profit it opts for corner solution}} = 0
$$

$$
\Rightarrow \frac{M}{2}\phi + M(\frac{\eta\beta}{\theta_2} - \frac{\eta\beta}{\theta_c}) + \alpha(\theta_2^2-\theta_c^2) + M\beta(\delta-\kappa) + \frac{M}{2}\frac{1}{2\kappa(\theta_c-\theta_2} = 0 \qquad (81)
$$

Lemma 20. *Suppose a Type D equilibrium exists for a given set of parameters. If the nuisance factor η increases, the quality set by the high-quality firm θ_c increases.*

When η increases, both θ_2 and $\frac{\eta\beta}{\theta_2}$ increase as per Lemma 19. Therefore, we can infer from (81) that the left side of the equation increases, which means θ_c has to increase to equate it to zero.

Proposition 10. *If the nuisance factor η increases, the profit of both firms and the total market coverage decrease. Further, β_0 and β_1 increase, while β_2 decreases*

β_0 increases as the profit of the low-quality firm increases. β_1 increases because the profit of Firm 2 under the corner solution is more significantly reduced by an increase in η compared to the profit under the uncovered configuration (Lemma 19). β_2 decreases because the profit under the corner configuration decreases, which relaxes the Type D constraint that the low-quality firm must achieve higher profit when it deviates by exceeding the quality of the high-quality firm. Two implications of Proposition 10 are: a) The range of the interval (β_0, β_1) when Type C equilibrium exists decreases; b) The advertisement nuisance increases the quality of the premium product under Type D.

8.4 Consumers get positive utility from advertisements

Let's assume that consumers get positive utility from advertisements as they learn about new products, or search classifieds for new business opportunities. This utility gain is proportional to the advertisement level as shown below:

$$U_i = v_i Y_i \theta + \lambda\beta - s \text{ where } \lambda > 0 \tag{82}$$

This model will be similar to what we discussed in the previous section, with η replaced by $-\lambda$, so we will not discuss it in detail. The key difference is that the low-quality firm will set a positive price $s_2 = \lambda\beta$ in the corner solution when $\theta_2 = 0$. The effect will be opposite of what we discussed when ads were a nuisance. More specifically:

- The profit and the market coverage of both firms increase as the demand curve shifts outward.
- The interval (β_1, β_2) increases, i.e., the range on which Type C equilibrium exists increases.
- The high-quality firm sets lower quality under Type D because the constraint for Type D becomes difficulty to satisfy. This implies that the high-quality firm need not raise quality as much under Type D equilibrium to protect its market share.

When advertisements were a nuisance, some customers dropped off, but the competition for the remaining consumers increased, reducing prices and profits. Whereas, when consumers get positive utility from advertisements, demand increases, and competition relaxes.

Another important point to note is that in our model, the market is concentrated irrespective of whether there is a positive effect of advertisements on consumers.

9 Concluding Remark

We developed a model to analyze two-sided newspaper markets based on vertical differentiation. Our analysis shows that the tension between advertisement and subscription revenue leads to various market configurations. High advertisement levels can make the market highly competitive, as firms are willing to sacrifice subscription revenue to compete for advertisement revenue. As a result, the leading firm provides a very high-quality product at a lower price to protect its customer base. This market configuration results from our assumptions of a sequential entry model and quality-dependent fixed costs. We also demonstrated that our results are robust to varying consumer attitudes towards advertisements.

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A Appendix

A.1 Social planner objective function and optimal choice

We simplify (2) and we get

$$W(s,\theta) = N(s,\theta)(s-c) + M\int_0^\infty \left[Y\theta\frac{v^2}{2} - sv\right]_{\frac{s}{Y\theta}}^{1} dF(Y) + M\beta(E[Y] - \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M\int_0^\infty (\frac{Y\theta}{2} - s + \frac{s^2}{2Y\theta})dF(Y) + M\beta(E[y] - \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}E[Y] + \frac{s^2}{2\theta}E[\frac{1}{Y}] - s) + M\beta(E[Y] - \frac{s}{\theta}) - \alpha\theta^2$$

Substituting $E[Y] = e^{\mu+\frac{\sigma^2}{2}} = \delta$ and $E[\frac{1}{Y}] = e^{-\mu+\frac{\sigma^2}{2}} = \frac{1}{\kappa}$ for log-normal distribution we get:

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa + \kappa + \frac{s}{\theta}) - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) + \underbrace{M\beta\kappa(1 - \frac{s}{\theta\kappa})}_{\kappa\beta N(s,\theta)} - \alpha\theta^2$$

$$W(s,\theta) = N(s,\theta)(s-c+\kappa\beta) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) - \alpha\theta^2 \tag{83}$$

First order condition after putting the value of $N(s,\theta)$:

$$\frac{\partial W}{\partial s} = 0 \rightarrow M(-\frac{1}{\theta\kappa})(s-c+\kappa\beta) + M(1 - \frac{s}{\theta\kappa}) - M(1 - \frac{s}{\theta\kappa}) = 0$$

$$s = c - \kappa\beta \tag{84}$$

Notice that s is independent of θ.

Case I $s = c - \kappa\beta \leq 0$: We get corner solution as demand $N(s,\theta) = M$. $W(s,\theta)$ becomes:

$$W(s,\theta) = M(s-c+\kappa\beta) + M(\frac{\theta}{2}\delta + \frac{s^2}{2\theta\kappa} - s) + M\beta(\delta - \kappa) - \alpha\theta^2$$

Since $\frac{\partial W}{\partial s} = M\frac{s}{\theta\kappa}$ is increasing in s, therefore the social planner will set the price $s = 0$ and the objective function becomes:

$$W(s,\theta) = \frac{1}{2}M\delta\theta + M\beta(\delta - \kappa) - \alpha\theta^2$$

and therefore the optimal quality θ_{SP} is given by:

$$\frac{\partial W}{\partial \theta} = 0 \Rightarrow \frac{1}{2} M\delta = 2\alpha\theta_{SP}$$

$$\theta_{SP} = \frac{M\delta}{4\alpha} \tag{85}$$

Case II $s = c - \kappa\beta > 0$:. We will have interior solution and the optimal quality is given by the first order condition:

$$\frac{\partial W}{\partial \theta} = 0 \to M\frac{s}{\kappa\theta^2}(s - c + \kappa\beta) + \frac{M}{2}(\delta - \frac{s^2}{\kappa\theta^2}) - 2\alpha\theta = 0$$

$$\text{Using (84) we have: } 4\kappa\alpha\theta^3 - M\delta\kappa\theta^2 + M(c - \kappa\beta)^2 = 0$$

$$4\alpha\kappa\theta^3 - Me^{2\mu}\theta^2 + M(c - \kappa\beta)^2 = 0 \tag{86}$$

We need to check second order condition. Hessian is given by:

$$H_w = \begin{vmatrix} \frac{-M}{\kappa\theta} & \frac{M}{\kappa\theta^2}(s - c + \kappa\beta) \\ \frac{M}{\kappa\theta^2}(s - c + \kappa\beta) & \frac{Ms}{\kappa\theta^3}(2(c - \kappa\beta) - s) - 2\alpha \end{vmatrix}$$

The sufficient second order condition for interior solution demands that

$$\theta > \frac{M\delta}{6\alpha}$$

Implicit function theorem on (86) implies that the optimal quality decreases with higher price $s = c - \kappa\beta$ when the second order condition is satisfied.

Now, we check the critical quality level that demand approaches zero i.e. $\frac{s}{\theta} = \kappa$. Substituting this value in (86) we get

$$\theta = \frac{M\delta}{4\alpha}(1 - e^{-\sigma^2})$$

This implies that if the solution of (86) lies in the interval $(\frac{M\delta}{4\alpha}(1 - e^{-\sigma^2}), \frac{M\delta}{4\alpha}]$ then the social planner will serve the market with positive price. Further, if $s = c - \kappa\beta$ is sufficiently high then the social planner will not serve the market.

A.2 Solution of Monopolist problem

a) First we characterize the interior solution that is the solution with $N(s,\theta) \in (0, M)$ (uncovered market). Replacing $N(s,\theta)$ from the demand function (1) in the profit function (9) we have

$$\Pi(s,\theta) = M(1 - \frac{s}{\kappa\theta})(s + \phi) + M\beta(\delta - \kappa) - \alpha\theta^2 \tag{87}$$

The first order conditions:

$$\frac{\partial \Pi}{\partial s} = 0 \Rightarrow -\frac{M}{\kappa\theta}(s+\phi) + M(1-\frac{s}{\kappa\theta}) = 0$$

$$s = \frac{1}{2}\kappa\theta - \frac{1}{2}\phi \tag{12}$$

$$\frac{\partial \Pi}{\partial \theta} = 0 \Rightarrow -\frac{Ms}{\kappa\theta^2}(s+\phi) - 2\alpha\theta = 0$$

Replacing the value of s from (12) and with $\theta > 0$ for interior solution we get:

$$8\alpha\kappa\theta^3 - M\kappa^2\theta^2 + M\phi^2 = 0 \tag{11}$$

The sufficient condition for the first order condition to have a local maximum requires that the Hessian H_m is negative definite at the solution of (12) and (11).

$$H_m = \begin{vmatrix} \frac{-2M}{\kappa\theta} & \frac{M}{\kappa\theta^2}(2s+\phi) \\ \frac{M}{\kappa\theta^2}(2s+\phi) & -\frac{2Ms}{\kappa\theta^3}(s+\phi) - 2\alpha \end{vmatrix}$$

Putting the value of s from (12) we get

$$H_m = \begin{vmatrix} \frac{-2M}{\kappa\theta} & \frac{M}{\theta} \\ \frac{M}{\theta} & -\frac{M}{2\kappa\theta^3}(\kappa^2\theta^2 - \phi^2) - 2\alpha \end{vmatrix}$$

If θ_{MP} is the solution of (11) then the sufficient condition for local maximum (i.e. negative definite Hessian) requires

$$\theta_{MP} > \frac{M\kappa}{12\alpha}$$

Using implicit function theorem on (11) we can identify the relation between θ_{MP} and ϕ for a given M, k, α. We can verify that the θ_{MP} is decreasing in $|\phi|$ with maximum value of $\theta_{MP} = \frac{M\kappa}{8\alpha}$ when $\phi = 0$. The condition $\theta_{MP} > \frac{M\kappa}{12\alpha}$ requires

$$|\phi| < \frac{M\kappa^2}{12\sqrt{3}\alpha} \tag{13}$$

We can verify that (11) has no positive real solution when condition (13) is not satisfied. When condition (13) is satisfied, there exist only one real solution of (11) that satisfies the second order condition $\theta > \frac{M\kappa}{12\alpha}$ for local maximum, and hence if this is an interior solution then this solution is a unique solution. This proves lemma 1. The profit as a function of quality for the interior solution is given by

$$\Pi_{int}(\theta) = \frac{M}{4\kappa\theta}(\kappa\theta+\phi)^2 + M\beta(\delta-\kappa) - \alpha\theta^2 \tag{88}$$

Also notice that the condition (13) results in the market coverage range between $(\frac{1}{2} - \frac{1}{2\sqrt{3}}, \frac{1}{2} + \frac{1}{2\sqrt{3}})$.

Using implicit function theorem on (11) we can show that under interior solution the monopolist's optimal choice of quality is

- increasing in market size (M) and median income (μ)
- decreasing in quality cost (α) and income inequality (σ)
- non-monotonic in advertiser's willingness to pay β and the marginal cost c with single peak at $\phi = 0$

b) Next we consider the possible corner solution when $N = M$(full market coverage) and find the set of parameter values when this solution dominates the interior solution. For full market coverage, the monopolist must set the subscription price $s = 0$ (from (1)). The profit function becomes:

$$\Pi_{zp} = M\beta\delta - Mc - \alpha\theta^2$$

Since increasing θ increases the fixed cost without additional revenue as the market is fully covered, the optimal θ for the monopolist with zero subscription price is $\theta = 0$. Therefore,

$$\Pi_{zp} = \underbrace{M\beta\delta}_{advetisement\ revenue} - \underbrace{Mc}_{variable\ cost} \tag{89}$$

Intuitively, $M\delta$ is the average income for the whole market and hence $M\beta\delta$ is the total advertisement revenue when the market is fully covered.

When $\beta = 0$, $\Pi_{int} > \Pi_{zp} = 0$. However, using envelope theorem we can show that the optimal profit with zero price increases faster with β than the optimal profit with the interior solution.

$$\frac{d\Pi_{int}}{d\beta} = M\delta - \frac{s}{\theta} < M\delta = \frac{d\Pi_{zp}}{d\beta}$$

Using intermediate value theorem, we can conclude that there exist a critical $\overline{\beta} > 0$ such that $\Pi_{zp} > \Pi_{int}$ whenever $\beta > \overline{\beta}$.

We now show that $\phi(\overline{\beta}) < \frac{M\kappa^2}{12\sqrt{3}\alpha}$ (maximum value for which we have interior solution) so that there is an internal solution at the critical point. To do so, we evaluate Π_{int} at $\phi = \frac{M\kappa^2}{12\sqrt{3}\alpha}$ when $\theta_{MP} = \frac{M\kappa}{12\alpha}$.

$$\Pi_{int}|_{\kappa\beta - c = \frac{M\kappa^2}{12\sqrt{3}\alpha}} = \frac{3\alpha}{\kappa^2}(\frac{M\kappa^2}{12\alpha} + \frac{M\kappa^2}{12\sqrt{3}\alpha})^2 + M\beta(\delta - \kappa) - \alpha(\frac{M\kappa}{12\alpha})^2 = \underbrace{M\beta\delta - Mc}_{\Pi_{zp}} + \underbrace{(3 - 2\sqrt{3})\alpha(\frac{M\kappa}{12\alpha})^2}_{<0}$$

$$\Rightarrow \Pi_{int} < \Pi_{zp} \text{ when } \phi = \frac{M\kappa^2}{12\sqrt{3}\alpha}$$

$$\Rightarrow \phi(\overline{\beta}) < \frac{M\kappa^2}{12\sqrt{3}\alpha}$$

Although the above result is sufficient for the proof of Proposition 2, we can get exact cut-off point by equating $\Pi_{int} = \Pi_{zp}$. Without showing the detail solution steps, we state that $\overline{\beta}$ is given by

$$\phi(\overline{\beta}) = \frac{M\kappa^2}{27\alpha} \Rightarrow \overline{\beta} = \phi^{-1}(\frac{M\kappa^2}{27\alpha}) \Rightarrow \overline{\beta} = \frac{c}{\kappa} + \frac{M\kappa}{27\alpha}$$

Above equation shows that $\overline{\beta}$ increases with M and decreases with α, but ambiguous with μ and σ. If c is small then $\overline{\beta}$ increases with μ and decreases with σ.

c) Now we evaluate the cutoff point for the corner solution when market is not served by the monopolist, which means $N(s, \theta) = 0$ and hence the profit is zero. We evaluate Π_{int} when $\phi = -\frac{M\kappa^2}{12\sqrt{3}\alpha}$ (lowest value for the interior solution) and the corresponding $\theta_{MP} = \frac{M\kappa}{12\alpha}$

$$\Pi_{int}|_{\phi=-\frac{M\kappa^2}{12\sqrt{3}\alpha}} = \frac{3\alpha}{k^2}(\frac{M\kappa^2}{12\alpha} - \frac{M\kappa^2}{12\sqrt{3}\alpha})^2 + M\beta(\delta - \kappa) - \alpha(\frac{M\kappa}{12\alpha})^2 = \underbrace{(3 - 2\sqrt{3})\alpha(\frac{M\kappa}{12\alpha})^2}_{<0} + M\beta(\delta - \kappa)$$

If we assume $\beta = 0$ and $c = \frac{M\kappa^2}{12\sqrt{3}\alpha}$ then $\Pi_{int} < 0$. Since Π_{int} is monotonically increasing in β and is greater than 0 when $\beta = \overline{\beta}$, there exist $0 < \underline{\beta} < \overline{\beta}$ such that $Pi_{int}|_{\underline{\beta}} = 0$. If $\beta < \underline{\beta}$ then the monopolist will not serve the market. We had assumed $c = \frac{M\kappa^2}{12\sqrt{3}\alpha}$. If we decrease c then $\underline{\beta}$ also decreases, and will continue to be positive only if c is sufficiently high. Specifically, the following equation characterizes the relationship:

$$\underline{\beta} = \max(0, \phi^{-1}(-\frac{M\kappa^2}{27\alpha}))$$

The above equation implies $\underline{\beta}$ decreases with M, mu and increases with c, α and σ.

A.3 Solution of Duopoly Market

Let's consider that at-most two firms, $k \in \{1, 2\}$, can enter the market. The firms choose the non-negative subscription price, quality level, and the advertisement price, $(s_k, \theta_k, p_k) \in \mathbb{R}^3_+$.

Equilibrium Type B

Suppose Firm 1 (leader) decides to enter and chooses θ_1 in the stage 1, and the Firm 2 (follower) decides to enter in the second stage with the positive quality $\theta_2 > 0$. If we assume $\theta_1 > \theta_2$ the profit function of two firms are given by the equation (24)-(25) as it was shown in the section 6.

$$\Pi_1 = \underbrace{M\frac{1-\gamma}{\kappa\theta_1(4-\gamma)^2}(2\kappa\theta_1+\phi)^2}_{R_1} + M\beta(\delta-\kappa) - \alpha\theta_1^2 \tag{24}$$

$$\Pi_2 = \underbrace{M\frac{1-\gamma}{\kappa\theta_2(4-\gamma)^2}(\kappa\theta_2+2\phi)^2}_{R_2} - \alpha\theta_2^2 \tag{25}$$

Define the revenue expression R_1 and R_2 as shown above. We can verify that R_1 is a concave function in θ_1 and R_2 is a concave function in θ_2 if $\gamma \leq \frac{4}{7}$. Given θ_1, the optimal choice of θ_2 by Firm 2 is given by the first order condition[43]:

$$\frac{\partial R_2}{\partial \theta_2} - 2\alpha\theta_2 = 0 \Rightarrow \frac{M\kappa}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})\big[4-7\gamma-\frac{2\phi}{\kappa\theta_2}(2\gamma^2-3\gamma+4)\big] = 2\alpha\theta_2 \tag{31}$$

Lemma 2: Firm 1 will set the quality so that the Firm 2 enters with lower quality i.e. $\theta_2 < \theta_1$.
Proof: From equations (24) to (25), $\Pi_1 > \Pi_2$ for all $\theta_1 > \theta_2$. This implies that the high-quality firm has an advantage, and Firm 1 will choose a quality level that positions it in the high-quality space. Due to the concavity of the profit functions, for any given choice of θ_1, the profit function of Firm 2 will exhibit two strictly concave segments: one for $\theta_2 < \theta_1$ and another for $\theta_2 > \theta_1$. Each segment has a unique local maximum, as shown on the left side of Figure A.3. The left maximum point ($\theta_2 < \theta_1$) continuously increases with θ_1 due to relaxed competition, while the right maximum point ($\theta_2 > \theta_1$) continuously decreases with θ_1 due to increased competition. Therefore, there exist a θ_1^* such that Firm 2 will enter as a high-quality firm if $\theta_1 > \theta_1^*$. The right side of Figure A.3 shows that Firm 1 has higher profit when it preempts as a high quality firm, thus Firm 1 will choose $\theta_1 > \theta_1^*$, and Firm 2 will enter as a low-quality firm.

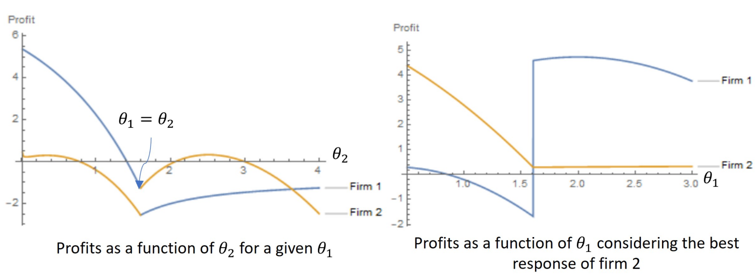

Figure 9:

If Firm 1 sets its quality to the monopolistic level, then the right maximum on the left side of Figure A.3 is negative. This effectively blocks new entry as a high quality player. □

[43] we later verify the necessary second order condition and check for uniqueness

From the reaction function (31) of Firm 2 we calculate $\frac{\partial\theta_2}{\partial\theta_1}$ using implicit function theorem[44].

$$\frac{\partial\theta_2}{\partial\theta_1} = -\frac{\frac{\partial^2 R_2}{\partial\theta_1\partial\theta_2}}{\frac{\partial^2 R_2}{\partial\theta_2^2} - 2\alpha}$$

Optimal choice for Firm 1 given the response function of Firm 2 is given by the first order condition:

$$\frac{d\Pi_1}{d\theta_1} = \frac{\partial\Pi_1}{\partial\theta_1} + \frac{\partial\Pi_1}{\partial\theta_2}\frac{\partial\theta_2}{\partial\theta_1} = 0 \Rightarrow (\frac{\partial R_1}{\partial\theta_1} - 2\alpha\theta_1)(\frac{\partial^2 R_2}{\partial\theta_2^2} - 2\alpha) - \frac{\partial R_1}{\partial\theta_2}\frac{\partial^2 R_2}{\partial\theta_1\partial\theta_2} = 0$$

Multiplying both side by $\gamma^2\theta_1 \neq 0$ and replacing $2\alpha\theta_2 = \frac{\partial R_2}{\partial\theta_2}$ we get

$$(\gamma\frac{\partial R_1}{\partial\theta_1} - \frac{\partial R_2}{\partial\theta_2})(\frac{\partial^2 R_2}{\partial\theta_2^2} - \frac{\partial R_2}{\partial\theta_2}) - \gamma^2\theta_1\frac{\partial^2 R_2}{\partial\theta_1\partial\theta_2}\frac{\partial R_1}{\partial\theta_2} = 0$$

We evaluate the left side of the above equation and replace $\frac{\phi}{\kappa\theta_1} \equiv x$ to further simplify:

$$\begin{aligned}
x^4(-3072 + 5376\gamma - 5312\gamma^2 + 3920\gamma^3 - 3232\gamma^4 + 1968\gamma^5 - 852\gamma^6 + 168\gamma^7 - 8\gamma^8) \\
+ x^3\gamma^2(-2048 + 1664\gamma - 1952\gamma^2 + 896\gamma^3 - 480\gamma^4 - 288\gamma^5 + 124\gamma^6 - 4\gamma^7) \\
+ x^2\gamma^2(1024 - 5632\gamma + 7488\gamma^2 - 6400\gamma^3 + 3112\gamma^4 - 1908\gamma^5 + 327\gamma^6 + 18\gamma^7 \\
+ x\gamma^4(256 - 928\gamma + 1048\gamma^2 - 984\gamma^3 - 184\gamma^4 + 108\gamma^5 \\
+ \gamma^4(-64 + 432\gamma - 644\gamma^2 + 675\gamma^3 - 556\gamma^4 + 112\gamma^5) = 0
\end{aligned} \tag{90}$$

Even though θ_1 and θ_2 may be non-monotonic in ϕ, we claim that the ratios $\frac{\phi}{\kappa\theta_1(\phi)}$ and $\frac{\phi}{\kappa\theta_2(\phi)}$ are continuously increasing in ϕ, that is elasticity of θ_1 and θ_2 with respect to ϕ is less than 1.

Claim: $\frac{\phi}{\kappa\theta_1(\phi)}$ *and* $\frac{\phi}{\kappa\theta_2(\phi)}$ *is continuously increasing in* ϕ *when first order condition has a solution.*

Proof: Let's assume that the (90) has a solution for a given range of ϕ. Continuity is derived from the implicit function theorem and the fact that both θ_1 and θ_2 are positive in the Type B equilibrium. Now, suppose the ratio $\frac{\phi}{\kappa\theta_1}$ is decreasing in ϕ. An increase in ϕ would imply decrease in $\frac{\phi}{\kappa\theta_1}$ and increase in θ_1. As θ_1 increases, θ_2 also increases, but by a proportionately smaller amount because $\frac{d\theta_2}{d\theta_1} < 1$, thus lowering γ. This results in lowering of all components on the left-hand side (LHS) of the equation below (first order condition), while the right-hand side (RHS) increases. Hence, this cannot be the solution of the first order condition.

$$\frac{M\kappa}{(4-\gamma)^3}(1 + \frac{2\phi}{\kappa\theta_2})\big[4 - 7\gamma - \frac{2\phi}{\kappa\theta_2}(2\gamma^2 - 3\gamma + 4)\big] + \frac{\partial\Pi_1}{\partial\theta_2}\frac{\partial\theta_2}{\partial\theta_1} = 2\alpha\theta_2$$

Therefore, $\frac{\phi}{\kappa\theta_1}$ is increasing in ϕ. We can use the same logic on (31) to show that $\frac{\phi}{\kappa\theta_2}$ is increasing in ϕ. □

[44]Note that second order condition $\frac{\partial^2 R_2}{\partial\theta_2^2} - 2\alpha < 0$ ensures that implicit function theorem can be applied.

First, we find the solution of (90) at $\phi = 0$ i.e. $x = 0$.

Lemma 3: If $\phi = 0$ then there exist a unique solution such that two firms choose quality in the ratio $\gamma = 0.195064$, which is a constant for all (μ, σ^2, α). The market share of the high quality firm is twice that of the low quality firm. The high-quality firm sets its quality lower than the monopoly level but achieves higher market coverage due to a lower price-to-quality ratio.

Proof: Replacing $x = 0$ simplifies (90) to $112\gamma^5 - 556\gamma^4 + 675\gamma^3 - 644\gamma^2 + 432\gamma - 64 = 0$ which has a unique solution $\gamma = 0.195064$ that satisfies second order condition and is constant irrespective of μ, σ, α. The corresponding θ_1 and θ_2 can be derived from (31).

$$\theta_1 = \rho\frac{M\kappa}{\alpha} \text{ and } \theta_2 = \rho\gamma\frac{M\kappa}{\alpha} \text{ where}$$

$$\gamma = 0.195064, \rho = \frac{4-7\gamma}{2\gamma(4-\gamma)^3} = 0.1226$$

$\theta_2 = 0.1225\frac{M\kappa}{\alpha}$ is lower than $\theta_{MP} = \frac{M\kappa}{8\alpha}$. The corresponding market shares of the two players using (26) and (27) yields

$$\frac{N_1}{M} = \frac{2}{4-\gamma} = 52.56\%, \frac{N_2}{M} = \frac{1}{4-\gamma} = 26.28\%$$

□

Now we characterize the solution of (90) for $\phi < 0$ and $\phi > 0$.

Lemma 4: There exist $\phi(\underline{\beta}) < \phi_{il} < 0$ and $0 < \phi_{ir} < \phi(\overline{\beta})$ such that an interior solution exist iff $\phi \in (\phi_{il}, \phi_{ir})$ and this solution is unique for a given ϕ.

Steps to prove lemma 4: It is easy to verify that γ is a continuous function of x using implicit function theorem on (90). We have already established that $\frac{\phi}{\kappa\theta_2}$ is increasing in ϕ. So when ϕ decreases below 0 so that $\frac{\phi}{\kappa\theta_2}$ approaches $-\frac{1}{2}$, the lhs of (31) becomes zero which implies that (31) does not have a solution. Therefore, there exist $\phi_{il} < 0$ such that if $\phi < \phi_{il}$ no interior solution exist. Similarly on the right side, $\frac{s_2}{\theta_2}$ decreases as ϕ increases (see (29)). When $\frac{s_2}{\theta_2} \to 0$, $\frac{\phi}{\kappa\theta_2} \to \frac{1-\gamma}{2+\gamma}$, and the profit function (25) of the low quality firm becomes:

$$\Pi_2 = \frac{M}{2+\gamma}\phi - \alpha\theta_2^2 \Rightarrow \frac{d\Pi_2}{d\theta_2} < 0$$

Hence the optimal quality is zero, which means that there is no interior solution. Therefore, there exist $\phi_{ir} > 0$ such that no interior solution exist.

For uniqueness and the cutoff points, we need to find the solution of equation (92) which is a complex equation and does not have a close form solutions. However, we already have found a solution $\phi = 0$ (lemma 3). We now change x numerically with small precision on both directions, and find roots of the polynomial of γ that satisfy the necessary second order condition. Then we we use (31) to find θ_1 and θ_2. We find that no solution exist if $\phi \geq \frac{M\kappa^2}{255\alpha}$ and $\phi \leq -\frac{M\kappa^2}{187\alpha}$, and there is a unique solution when the solution exists. This proves that $= \frac{-M\kappa^2}{27\alpha} = \phi(\underline{\beta}) < \phi_{il} = -\frac{M\kappa^2}{187\alpha}$ and $\phi_{ir} = \frac{M\kappa^2}{255\alpha} < \phi\overline{\beta} = \frac{M\kappa^2}{27\alpha}$. Please note that the cutoffs are not precise due to numerical method but within the reasonable precision limit of $0.005\frac{M\kappa^2}{\alpha}$. □

Lemma 6: γ decreases continuously with ϕ when $\phi > 0$ and the high-quality firm sets lower quality than the monopolist for all ϕ.

Proof: Using implicit function theorem on (92) we evaluate that γ is a continuously decreasing function of x when $x > 0$. We have already established that x is a continuously increasing function of ϕ, which implies that γ is decreasing in ϕ. $\theta_2(\phi) < \theta_{MP}(\phi)$ for all ϕ is verified through the solution.

□

Lemma 5: There exist a critical $\phi_0 \in (\phi_{il}, 0)$ such that for all $\phi \in (\phi(\underline{\beta}), \phi_0)$ only one firm enters the market (type A equilibrium).

Proof: Proof follows from the IVT and the following statements:

a) Low quality firm makes a negative profit when $\phi = \phi_{il} + \epsilon = -\frac{M\kappa^2}{188\alpha}$ [45]

b) Profit function of the low quality firm, Π_2, is continuously increasing in ϕ when $\phi \in (\phi_{il}, \phi_{ir})$(see Lemma 8)

c) Low quality firm makes positive profit when $\phi = 0$ (shown above).

d) A monopolist will enter the market if $\phi > -\frac{M\kappa^2}{27\alpha}$

□

Using (31) and (92) we can show that this cutoff point is $\phi_1 \approx -\frac{M\kappa^2}{242\alpha}$. Therefore, when $\frac{-M\kappa^2}{27\alpha} \leq \phi \leq -\frac{M\kappa^2}{242\alpha}$ only one firm (monopolist) exist and further entry is blockaded.

Lemma 8: Profit of both firms increases continuously with ϕ under Type B equilibrium.

Proof: We evaluate

$$\frac{\partial \Pi_1}{\partial \theta_2} = \frac{\partial \Pi_1}{\partial \gamma}\frac{\partial \gamma}{\partial \theta_2} = -\frac{M(2+\gamma)}{\kappa\theta_1(4-\gamma)^3}(2\kappa\theta_1+\phi)^2(\frac{1}{\theta_1}) = -\frac{4Mk(2+\gamma)}{(4-\gamma)^3}(1+\frac{\phi}{2\kappa\theta_1})^2$$

$$\frac{\partial \Pi_2}{\partial \theta_1} = \frac{\partial \Pi_2}{\partial \gamma}\frac{\partial \gamma}{\partial \theta_1} = -\frac{M(2+\gamma)}{\kappa\theta_2(4-\gamma)^3}(\kappa\theta_2+2\phi)^2(-\frac{\theta_2}{\theta_1^2}) = \frac{Mk\gamma^2(2+\gamma)}{(4-\gamma)^3}(1+\frac{2\phi}{\kappa\theta_2})^2$$

$$\frac{\partial \Pi_1}{\partial \phi} = \frac{4M(1-\gamma)}{(4-\gamma)^2}(1+\frac{\phi}{2\kappa\theta_1}) > 0$$

[45] Corresponding $\gamma = 0.158679, \theta_2 = 0.01896\frac{M\kappa}{\alpha}$; Plugging these values in profit function (34) we get negative profit

$$\frac{\partial \Pi_2}{\partial \phi} = \frac{4M(1-\gamma)}{(4-\gamma)^2}(1+\frac{2\phi}{\kappa\theta_2}) > 0$$

Using envelope theorem,

$$\frac{d\Pi_1(\theta_1,\theta_2,\phi)}{d\phi} = \underbrace{\frac{\partial \Pi_1}{\partial \theta_2}\frac{\partial \theta_2}{\partial \phi}}_{\text{strategic effect}} + \underbrace{\frac{\partial \Pi_1}{\partial \phi}}_{\text{direct effect}} > 0$$

$$\frac{d\Pi_2(\theta_1,\theta_2,\phi)}{d\phi} = \underbrace{\frac{\partial \Pi_2}{\partial \theta_1}\frac{\partial \theta_1}{\partial \phi}}_{\text{strategic effect}} + \underbrace{\frac{\partial \Pi_1}{\partial \phi}}_{\text{direct effect}} > 0$$

Direct effect is always positive. It can be easily shown that for all values of γ strategic effect either reinforces the direct effect or is much smaller than the direct effect because $|\kappa\frac{\partial\theta_1}{\partial\phi}| < 1$ and $|\kappa\frac{\partial\theta_2}{\partial\phi}| < 1$. We have already shown that when $\phi \geq 0$, there is a positive strategic effect because γ decreases with ϕ (Lemma 6). γ increases with ϕ at small value of ϕ when $\frac{\phi}{\kappa\theta_2} = -\frac{1}{2}$ when the strategic effect is small.

Type C Equilibriurm

Lemma 9: Suppose Firm 2 chooses $\theta_2 = 0$ and $\phi \in [0, \frac{M\kappa^2}{12\sqrt{3}\alpha})$. Then, the best response of Firm 1 is $\theta_1 = \theta_{MP}$, and corresponding subscription prices are $s_1 = s_{MP}, s_2 = 0$.

Proof: If Firm 2 chooses $\theta_2 = 0$, then to maintain positive demand, it must also set $s_2 = 0$ in the second stage for all $\theta_1 \in \mathbb{R}+$. $\phi \geq 0$ ensures that Firm 2 will have non-negative profit even without subscription revenue. With $s_2 = 0$ and $\theta_2 = 0$, the consumers of Firm 2 receive zero utility. Hence, the demand for Firm 1 is comprised of all consumers who gain positive utility from subscribing to its product, i.e., $v_iY_i\theta_1 - s_1 > 0$, which yields the same demand curve as that of a monopolist, as described in Equation (1). Therefore, the optimal response for Firm 1 is (s_{MP}, θ_{MP}), as detailed in Lemma 1, provided the necessary condition $|\phi| < \frac{M\kappa^2}{12\sqrt{3}\alpha}$ for the monopolist's interior solution is satisfied (see (13)).

Note that $\theta_1 = 0$ is never the best response for Firm 1, because if both firms set their subscription prices to zero, they would have to share the advertising revenue. However, Firm 1 can achieve higher profits by capturing the full advertising revenue if it increases its quality by an infinitesimally small amount. □

Lemma 11: There exists a critical $\phi_1 \in (0, \phi_{ir})$ such that for $\phi \in (\phi_1, \phi_2]$ there is a unique equilibrium of type C.

Proof: Let us use the subscript 'b', 'c','o' to denote endogenous parameter under type B, type C, and when Firm 2 unilaterally deviates from type B to zero quality, respectively. Profit of Firm

2 in type B is given by (25)

$$\Pi_{2b} = M\frac{1-\gamma}{\kappa\theta_{2b}(4-\gamma)^2}(\kappa\theta_{2b}+2\phi)^2 - \alpha\theta_{2b}^2 \tag{25}$$

If Firm 2 unilaterally deviates from type B to zero quality its profit is given by (91)

$$\Pi_{2o} = \frac{M}{2}(1-\frac{\phi}{\kappa\theta_{1b}})\phi \tag{91}$$

We evaluate Π_{2b} at $\phi = 0$ and at $\phi = \phi_{ir} = \frac{M\kappa^2}{255\alpha}$ using (90)

$$\Pi_{2b}|_{\phi=0} = 0.00076\frac{M^2\kappa^2}{\alpha} \text{ and } \Pi_{2b}|_{\phi=\phi_{ir}} = 0.0019\frac{M^2\kappa^2}{\alpha}$$

Similarly, Π_{2o} when $\phi = 0$ and $\phi = \phi_{ir} = \frac{M\kappa^2}{255\alpha}$ using (91)

$$\Pi_{2o}|_{\phi=0} = 0 \text{ and } \Pi_{2o}|_{\phi=\phi_{ir}} = 0.002\frac{M^2\kappa^2}{\alpha}$$

Therefore

- If $\phi = 0$ then $\Pi_{2b} > \Pi_{2o}$ and $\frac{d\Pi_{2o}}{d\phi} = \frac{M}{2} > \frac{d\Pi_{2b}}{d\phi}$. Therefore, Π_{2o} is lower than Π_{2b} but increasing faster.
- If $\phi = \phi_{ir} = \frac{M\kappa^2}{255\alpha}$ then $\Pi_{2b} < \Pi_{2o}$.
- Π_{2b} is a continuously increasing and convex function of ϕ when $\phi \in [0, \phi_{ir}]$ (from (25)), and Π_{2o} is a continuously increasing and concave function of ϕ (from (91)).

Therefore, Π_{2o} must cross Π_{2b} from below, and there is exactly one cutoff point $\phi_1 \in (0, \phi_{ir})$ such that for $\phi < \phi_1$, we have $\Pi_{2o} < \Pi_{2b}$. This implies that if $< \phi_0 < \phi < \phi_1$ there is no Type C equilibrium, and we get a unique type B equilibrium.

If $\phi > \phi_1$, then Firm 2 has a profitable deviation to zero quality. Should it choose zero quality, Firm 1 will set the monopolistic level of price and quality, θ_{MP} (Lemma 9), which is greater than θ_{1b} (Lemma 6). This relaxes competition and reduces the market coverage of Firm 1, thereby increasing the profit of Firm 2 in both scenarios: a) when Firm 2 acts according to the reaction function of Type B (31), and b) if it maintains zero quality under Type C (note: $\Pi_{2c} > \Pi_{2o}$ because $\theta_{MP} > \theta_{1b}$). Except for a small interval when ϕ is closer to ϕ_1, the latter profit (b) is greater, and thus a Type C equilibrium is sustained because both Firm 1 and Firm 2 are acting optimally after deviation. Therefore, we have a unique Type C equilibrium when $\phi \in (\phi_1, \phi_2)$, with the exception noted below.

□

Exception: In the small interval where ϕ is very close to ϕ_1, the profit under scenario a) is greater, and thus the Type C equilibrium with $\theta_1 = \theta_{MP}$ is not sustained. In this case, Firm 2 will increase its quality, making θ_{MP} suboptimal for Firm 1. However due to lower competition, Firm 1 profits more when Firm 2 opts for zero quality. Consequently, Firm 1 preempts by setting a quality level lower than θ_{MP} but greater than θ_{1d} to make Firm 2 indifferent between zero and positive quality levels under Type B. For simplicity, we categorize this scenario also as a Type C equilibrium, despite Firm 1's quality being lower than θ_{MP}.

Type D Equilibriurm

Lemma 12: There exist a unique type D equilibrium if $\phi > \phi_2$

Proof: If $\phi > \phi_2$, Firm 2 will respond with $\theta_2 = \theta_{MP+}$ when Firm 1 sets $\theta_1 = \theta_{MP}$, which will result in Firm 1 losing all demand and will have negative profit. This is because the price of both firms will become zero, but Firm 2 has better quality and hence is preferred by consumers. Anticipating this, Firm 1, who is a first mover, will increase quality and set $\theta_1 = \theta_c$ (see definition 5) when Firm 2 is indifferent between choosing zero quality and θ_{c+}. If Firm 1 chooses $\theta_1 = \theta_c$, the best response for Firm 2 is $\theta_2 = 0$.

It is not profitable for Firm 2 to set $\theta \in (0, \theta_c)$ because $\phi_2 > \phi_{ir}$, which means no interior solution exists (Lemma 4). This is evident from the reaction function of Firm 2 (Equation (31)). As ϕ increases and approaches ϕ_{ir}, the ratio $\frac{\phi}{\kappa\theta_2}$, which increases with ϕ, approaches $\frac{1}{2}$, and the last factor on the LHS becomes negative for all γ. Therefore, for $\phi > \phi_{ir}$, $\frac{d\Pi_2}{d\theta_2} < 0$, and the optimal quality for Firm 2 is zero.

$$\frac{M\kappa}{(4-\gamma)^3}(1 + \frac{2\phi}{\kappa\theta_2})\left[4 - 7\gamma - \frac{2\phi}{\kappa\theta_2}(2\gamma^2 - 3\gamma + 4)\right] = 2\alpha\theta_2 \tag{31}$$

It is also not profitable for Firm 2 to set $\theta_2 > \theta_c$ because we have shown in Lemma 2 that if Firm 1 sets quality greater than or equal to monopoly, then entry of Firm 2 as a high quality player is not profitable under Type B equilibrium. Since there are no profitable deviations for Firm 2 from $\theta_2 = 0$, a unique Type D equilibrium exist. □

A.4 Solution of Simultaneous Duopoly Market

Reaction functions of Firm 1 and Firm 2 at stage 1:

$$\frac{4M\kappa}{(4-\gamma)^3}(1 + \frac{\phi}{2\kappa\theta_1})\left[(2\gamma^2 - 3\gamma + 4) - \frac{\phi}{2\kappa\theta_1}(4 - 7\gamma)\right] = 2\alpha\theta_1 \tag{30}$$

$$\frac{M\kappa}{(4-\gamma)^3}(1 + \frac{2\phi}{\kappa\theta_2})\left[4 - 7\gamma - \frac{2\phi}{\kappa\theta_2}(2\gamma^2 - 3\gamma + 4)\right] = 2\alpha\theta_2 \tag{31}$$

Necessary condition for the Type B equilibrium is that the above two equation has a solution. Assuming that the above reaction function has a solution, we can infer (using implicit function theorem on (30)-(31)) that θ_1 and θ_2 are non-monotonic in ϕ. However, the ratios $\frac{\phi}{\kappa\theta_1(\phi)}$ and $\frac{\phi}{\kappa\theta_2(\phi)}$ are continuously increasing in ϕ. In other words, elasticity of θ_1 and θ_2 with respect to ϕ is less than 1[46].

To solve the above equations, we further simplify it by defining $x \equiv \frac{\phi}{\kappa\theta_1}$ and eliminating M and α from the above two equations we get:

$$x^2(16 - 12\gamma + 8\gamma^2 - 4\gamma^3 + 7\gamma^4) + x\gamma^2(2 + \gamma + 2\gamma^2 + \gamma^3) + \gamma^2(-4 + 23\gamma - 12\gamma^2 + 8\gamma^3) = 0 \quad (92)$$

In place of (90) for sequential entry we have (92) as FOC for the simultaneous entry. We solve this in a similar manner starting with $x = 0$ and then changing x on both positive and negative side by a small value, identifying γ that satisfies the second order condition, and then using (31) to calculate value of θ_1 and θ_2. We do this until (92) has no solution, which will provide the cutoff points ϕ_{il} and ϕ_{ir}.

First, we find the solution of (92) at the $\phi = 0$ i.e. $x = 0$. Replacing $x = 0$ simplifies (92) to $8\gamma^3 - 12\gamma^2 + 23\gamma - 4 = 0$ which has a unique solution $\gamma = 0.194031$ and is constant irrespective of μ, σ, α. We verify that the necessary second order condition is also satisfied for this solution. The corresponding θ_1 and θ_2 can be derived from (30) and (31).

$$\theta_1 = \rho\frac{M\kappa}{\alpha} \text{ and } \theta_2 = \rho\gamma\frac{M\kappa}{\alpha} \text{ where}$$

$$\gamma = 0.1904, \rho = \frac{4 - 7\gamma}{2\gamma(4-\gamma)^3} = \frac{2(2\gamma^2 - 3\gamma + 4)}{(4-\gamma)^3} = 0.1267$$

The corresponding market shares of the two players using (26) and (27) yields

$$\frac{N_1}{M} = \frac{2}{4-\gamma} = 52.4\%, \frac{N_2}{M} = \frac{1}{4-\gamma} = 26.2\%$$

The following table compares the solution for simultaneous and sequential entry model for $\phi = 0$.

[46] Suppose that when ϕ increases θ_1 (resp. θ_2) increases proportionately by larger amount so that $\frac{\phi}{\theta_1}$ (resp. $\frac{\phi}{\theta_2}$) decreases then we can easily see that the LHS (marginal revenue) of (30) (resp. (31)) decreases as long as $\gamma < \frac{4}{7}$ while the RHS (marginal cost) increases, which is a contradiction. In equilibrium, $\gamma < \frac{4}{7}$ is satisfied in our model

	Sequential Entry	Simultaneous Entry
$\gamma \equiv \frac{\theta_2}{\theta_1}$	0.1951	0.1904
θ_1	$0.1226\frac{M\kappa}{\alpha} < \theta_{MP}$	$0.1265\frac{M\kappa}{\alpha} > \theta_{MP}$
$(\frac{N_1}{M}, \frac{N_2}{M})$	$(52.6\%, 26.3\%)$	$(52.4\%, 26.2\%)$
$(\frac{s_1}{\theta_1}, \frac{s_2}{\theta_2})$	$(0.4231\kappa, 0.2116\kappa)$	$(0.4254\kappa, 0.2127\kappa)$
(Π_1, Π_2)	$(12.235\frac{M^2\kappa^2}{1000\alpha}, 0.758\frac{M^2\kappa^2}{1000\alpha})$	$(12.22\frac{M^2\kappa^2}{1000\alpha}, 0.764\frac{M^2\kappa^2}{1000\alpha})$

Table 1: Difference between simultaneous entry and sequential entry when $\phi = 0$

Table 1 clearly establishes the properties of Lemma 13. This can be verified for all ϕ in (ϕ_{il}, ϕ_{ir}).

Next we evaluate the cut-off points ϕ_0 and ϕ_1 for the simultaneous entry model by identifying the lowest ϕ at which the profit of Firm 2 is positive, and the lowest ϕ at which Firm 2 has profitable deviation to $\theta_2 = 0$. Table 2 compares the cut-off points of sequential and simultaneous entry model, establishing the properties of Lemma 14.

	Simultaneous Entry	Sequential Entry
Interior solution interval (ϕ_{il}, ϕ_{ir})	$-(\frac{M\kappa^2}{185\alpha}, \frac{M\kappa^2}{240\alpha})$	$-(\frac{M\kappa^2}{187\alpha}, \frac{M\kappa^2}{255\alpha})$
ϕ_0	$-\frac{M\kappa^2}{243\alpha}$	$-\frac{M\kappa^2}{242\alpha}$
ϕ_1	$\frac{M\kappa^2}{297\alpha}$	$\frac{M\kappa^2}{302\alpha}$

Table 2: Cut-off points comparison between simultaneous entry and sequential entry

Note: There exist ϵ, however small, such that there does not exist any equilibrium when $\phi = [\phi_1, \phi_1 + \epsilon)$.

$\phi = \phi_1$ is the lowest ϕ for which Firm 2 has a profitable deviation to $\theta_2 = 0$. If it deviates at $\phi = \phi_1$, then Firm 1 will set the monopolistic quality (see Lemma 9), which is lower than the one before the deviation (see Lemma 13). This reduces the profit of Firm 2, prompting it to switch back to positive quality. Thus, in the small right-side neighborhood of ϕ_1, the two firms cycle between Type B and Type C configurations without reaching any equilibrium.

Lemma 15: There does not exist any pure strategy equilibrium if $\phi > \phi_2$.

Proof: We have already discussed that if two firms are infinitesimally close in quality, the firm with the lower quality will lose all demand and earn negative profit. Suppose $\phi > \phi_2$. If Firm 1 sets $\theta_1 = \theta_{MP}$, Firm 2 will respond with $\theta_2 = \theta_{MP+}$ (by definition of ϕ_2, see Definition 7), which will result in negative profits for Firm 1. Subsequently, each firm will attempt to outdo the other by marginally increasing quality until one of the firms, say Firm 1, sets $\theta_1 > \theta_c$. At this point, Firm

2 will deviate to $\theta_2 = 0$ (by definition of θ_c, see Definition 5). However, when Firm 2 sets $\theta_2 = 0$, the best response for Firm 1 is to revert to θ_{MP}. Thus, this cycle will continue without reaching an equilibrium.

Furthermore, according to Assumption 1 and Lemma 4, a Type B equilibrium does not exist when $\phi > \phi_2$. Additionally, both firms cannot have the same quality in equilibrium because this would imply that both firms have zero subscription price and merely share the advertisement revenue. Therefore, any firm can deviate by slightly increasing its quality to capture full advertisement revenue. Since $\phi > 0$, a Type A equilibrium does not exist either.

Therefore, there does not exist any pure strategy equilibrium. $\square$

A.5 Solution for Three Firms Model

Steps to prove Proposition 5

Suppose there exist an equilibrium where all three firms set positive prices. Since the higher quality firm has higher profit (see lemma 2), early entrants will choose higher quality, which means $0 < \theta_3 < \theta_2 < \theta_1$. The demand function based on indifferent consumers:

$$\begin{aligned}
N_1(s_1, s_2, s_3, \theta_1, \theta_2, \theta_3) &= M(1 - \frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)}) \\
N_2(s_1, s_2, s_3, \theta_1, \theta_2, \theta_3) &= M(\frac{s_1 - s_2}{\kappa(\theta_1 - \theta_2)} - \frac{s_2 - s_3}{\kappa(\theta_2 - \theta_3)}) \\
N_3(s_1, s_2, s_3, \theta_1, \theta_2, \theta_3) &= M(\frac{s_2 - s_3}{\kappa(\theta_2 - \theta_3)} - \frac{s_3}{\kappa\theta_3})
\end{aligned}$$

The price stage solution for subscription prices when firms simultaneously choose prices:

$$s_1 = \frac{\kappa(\theta_1 - \theta_2)(4\theta_1\theta_2 - \theta_3(3\theta_2 + \theta_1))}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))} - \frac{\theta_2(7\theta_1 - \theta_2) - \theta_3(5\theta_2 + \theta_1)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))}\phi \tag{93}$$

$$s_2 = \frac{\kappa\theta_2(\theta_1 - \theta_2)(\theta_2 - \theta_3)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))} - \frac{3\theta_2(\theta_1 - \theta_3)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))}\phi \tag{94}$$

$$s_3 = \frac{\kappa\theta_3(\theta_1 - \theta_2)(\theta_2 - \theta_3)}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))} - \frac{\theta_2(4\theta_1 - \theta_2) + 2\theta_3(\theta_1 - \theta_2) - 3\theta_3^2}{2(\theta_2(4\theta_1 - \theta_2) - \theta_3(2\theta_2 + \theta_1))}\phi \tag{95}$$

Using above equations, the corresponding profit functions on the product choice stage are given by:

$$\Pi_1 = \frac{M(\theta_1-\theta_2)}{4\kappa}\left(\frac{\kappa(4\theta_1\theta_2-\theta_3(\theta_1+3\theta_2))+c(\theta_2-\theta_3)}{\theta_2(4\theta_1-\theta_2)-\theta_3(2\theta_2+\theta_1)}\right)^2+M\beta(\delta-\kappa)-\alpha\theta_1^2 \tag{96}$$

$$\Pi_2 = \frac{M(\theta_1-\theta_2)(\theta_2-\theta_3)(\theta_3-\theta_1)}{\kappa}\left(\frac{\kappa\theta_2+c}{\theta_2(4\theta_1-\theta_2)-\theta_3(2\theta_2+\theta_1)}\right)^2-\alpha\theta_2^2 \tag{97}$$

$$\Pi_3 = \frac{M\theta_2(\theta_2-\theta_3)}{4\kappa\theta_3}\left(\frac{\kappa\theta_3(\theta_1-\theta_2)+c(4\theta_1-\theta_2-3\theta_3)}{\theta_2(4\theta_1-\theta_2)-\theta_3(2\theta_2+\theta_1)}\right)^2-\alpha\theta_3^2 \tag{98}$$

To solve this problem, we begin by calculating the reaction function of Firm 3. Firm 2 will then choose its optimal quality based on the reaction function of Firm 3, given θ_1, thereby forming Firm 2's reaction function. Subsequently, Firm 1 will choose its optimal quality based on the reaction function of Firm 2. Given the complexity of the resulting set of equations, we utilize Wolfram Mathematica for the solution[47]. Additionally, we simplify the problem by solving it for $\phi = 0$.

At $\phi = 0$, the above equations yield a unique solution where all three firms make positive profits. Similar to the duopoly case described in Lemma 3, the ratio $\frac{\theta_3}{\theta_2} = 0.198$ and $\frac{\theta_2}{\theta_1} = 0.196$ remains constant for all μ, σ, and α. These ratios are greater than the corresponding duopoly ratio, which is $\gamma = \frac{\theta_2}{\theta_1} = 0.195$.

Also, no firm will deviate to a corner solution, as setting a zero price will result in zero profit. Similarly, none of the two firms will choose the same quality, as in that case, the price will be zero in the price stage, and the firm will make zero profit. Hence, the above interior solution is a unique pure strategy equilibrium. Since the profit function is a continuous function of ϕ (see Lemma 8) and firms make positive profit at $\phi = 0$, there exists a neighborhood, however small, where the above equilibrium configuration will persist. This proves Proposition 5.

[47] Equations and codes can be provided on request.