Public provision of healthcare and basic research: What are the joint effects on economic growth and welfare?

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Abstract

We propose a generalized R&D-based economic growth model that incorporates endogenous human capital accumulation in terms of education and health and the public provision of healthcare and basic science. The government taxes households to pay for healthcare personnel and basic researchers. These employees are not anymore available for applied research and for final goods production. Thus, important tradeoffs emerge for economic growth and welfare with respect to government spending policies. While increasing public spending on health and basic research may decrease economic growth and welfare in the short run, we show that they foster economic growth in the medium run and tend to raise long-run welfare when compared to actual levels of spending in Organisation for Economic Co-operation and Development (OECD) countries. In addition, since public funding for healthcare tends to be rather high in most rich countries, the overall public spending shortfall is lower than previous research has shown. Our results highlight the importance of understanding the tradeoffs involved when deciding adequate public funding for healthcare and basic research.

Keywords: R&D-Based Growth, Basic Research, Public Healthcare, Children's Health, Education, Fertility, Intertemporal Tradeoffs.

JEL Codes: H41; J24; O31; O32; O41.

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1 Introduction

How much should governments invest in different areas such as healthcare and basic research to foster economic growth and improve welfare? To answer this question, we propose a generalized research and development (R&D) based model of economic growth in which the government funds basic research, which is an important input in applied R&D, and healthcare expenditures, which improve individual productivity. We use the model i) to analyze the dependence of economic growth on government investments in basic science and in healthcare, ii) for a calibration to real-world data to highlight the welfare effects of different government policies, and iii) to address the tradeoffs that governments face when deciding to invest in healthcare and in basic science.

Most frameworks that have previously been used to analyze the growth effects of healthcare and of basic science were built around different strands of the literature that disregard at least one important dimension in this context. The effects of health investments are often analyzed in models of exogenous economic growth (Solow, 1956; Ramsey, 1928; Cass, 1965; Koopmans, 1965; Diamond, 1965). Of course, in such models and with diminishing returns to physical capital in the production process, there are only limited repercussions of health on economic growth. A few contributions rely on R&D-based endogenous and semi-endogenous growth models of the Romer (1990) and Jones (1995) types to analyze the effects of health investments (see, for example, Kuhn and Prettner, 2016; Baldanzi et al., 2019, 2021). The typical finding in this literature is that health investments can raise economic growth and welfare.[1] However, these frameworks are silent on government expenditures on basic science.

Another strand of the literature is concerned with the effects of government-funded basic research (see, for example, Gersbach et al., 2013; Gersbach and Schneider, 2015; Gersbach et al., 2018, 2023; Prettner and Werner, 2016; Akcigit et al., 2020; Huang et al., 2023).[2] These frameworks show the importance of basic scientific knowledge as an input in applied R&D, which is the main engine of long-run economic growth. The typical finding here is that basic science investments foster economic growth and that the observed levels of investment in rich countries tend to be much lower than the levels that would maximize welfare. However, these frameworks are silent on the effects of government expenditures on health.

The discussion so far implies that the only tradeoff analyzed in the discussed literature

[1]For the importance of health investments in generating human capital and for the long-run consequences of government health investments, see, for example, Weil (2007), Prettner et al. (2013), Schneider and Winkler (2021), and Kuhn et al. (2023). For empirical evidence, see Weil (2014), Madsen (2016), Bucci et al. (2021), and Bloom et al. (2024).

[2]For empirical evidence, see Czarnitzki and Thorwarth (2012), Toole (2012), Minniti and Venturini (2017a,b), Coad et al. (2021), and Mulligan et al. (2022).

is whether or not an economy should raise taxes to increase spending on healthcare *or* on basic science. The finding that these types of spending are welfare increasing is perhaps not surprising when there are no other types of spending to consider for the government. By contrast, governments usually face tradeoffs when planning expenditures in different areas that may all be growth-promoting and welfare-enhancing.

In this contribution, we are interested how the results of the two separate strands of literature change once we allow for both types of government expenditures, on health and on basic science. To this end, we propose a generalized R&D-based growth model that includes the quality-quantity tradeoff between fertility and human capital accumulation (cf. Prettner et al., 2013; Strulik et al., 2013; Prettner, 2014). We augment this model to allow for the fact that parents care not only for the education of their children, but also for their health. In so doing, we follow Baldanzi et al. (2021) who, however, abstract from public spending on basic research and on health. To close this gap, we assume that the government seeks to improve people's health by employing healthcare personnel. This, in turn, enhances labor productivity in the sectors that employ workers, e.g., by reducing production losses caused by sick employees. Finally, we include basic research as a necessary input in the production of applied R&D, which, in turn, determines economic growth (Prettner and Werner, 2016).

Government investments in healthcare and in basic science both have productivity-enhancing effects in the medium and in the long run, that is, from time $t+1$ onwards. However, a rise in the number of healthcare workers and in the number of basic scientists implies that fewer workers are available for final goods production and for applied R&D in the short run, that is, in period t when taxes are raised (for this intertemporal tradeoff see also Prettner and Werner, 2016; Gersbach et al., 2018; Gersbach and Komarov, 2020). Overall, our model therefore captures a rich set of tradeoffs in terms of government spending, along with the quality-quantity tradeoff between fertility on the one hand and human capital accumulation in terms of education and health on the other. To our knowledge, our model is the first to generalize the previous R&D-based growth literature to account for all of the following important dimensions: endogenous fertility, endogenous private health investments, endogenous education investments, and public funding for healthcare and for basic science.

Using our framework, we find that the welfare-increasing level of government expenditures on health and basic science is higher than the actual levels in Organisation for Economic Co-operation and Development (OECD) countries. However, in the short run (in period t), investments in health and basic science lead to declines in welfare. The reasons for the decrease of welfare in the short run are that i) taxes have to be increased to fund basic research and healthcare and this reduces consumption and, thus, welfare; ii) additional healthcare personnel and additional basic researchers are attracted from the other sectors in the economy, in particular, from final goods production, which reduces aggregate output. This short-run versus long-run tradeoff in combination with the fact that governments typically aim for getting re-elected in the short run could be an explanation for the under-provision of healthcare and basic research from a long-run perspective (see also Prettner and Werner, 2016; Gersbach et al., 2018; Gersbach and Komarov, 2020; Chen et al., 2021). In addition, we show that basic research expenditures are more effective in raising welfare than healthcare spending and that the optimal spending levels implied by the model do not exceed actual spending levels by a similarly great margin as in earlier research (Prettner and Werner, 2016).

The policy implications of our findings are that i) raising government expenditures on basic science and healthcare is worthwhile in the long run; ii) however, the presence of more domains on which the government can spend its funds productively implies that more care is needed to design and evaluate the corresponding policies.

Our article is structured as follows. In Section 2, we develop the generalized R&D-based economic growth model with endogenous fertility, endogenous education, endogenous health, and government expenditures on healthcare and on basic science. In Section 3, we present our analytical results that hold along a balanced growth path. In Section 4, we calibrate the model and solve it numerically for obtaining the transitional dynamics and the welfare effects of government spending. Section 5 is devoted to sensitivity analyses and robustness checks, while we conclude in Section 6.

2 The model

2.1 Consumption side

We consider an economy with three overlapping generations: children, adults, and retirees. Adults decide upon the consumption level c_t, savings for retirement s_t, the number of children n_t, and investments in each of their children in terms of education e_t, and health m_t. The time adults do not spend on raising their children, educating them, and caring for their health is supplied on the labor market. Retirees consume their entire savings carried over from adulthood. Finally, children do not make any economic decisions; instead, they are fed, educated, and cared for by their parents. Following Prettner and Werner (2016) and Baldanzi et al. (2021), the preferences of a single-parent household are captured by the utility function

$$u_t = \ln c_t + \beta \ln [(R_{t+1} - 1)s_t] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t, \tag{1}$$

where $\beta \in (0,1)$ represents the inter-generational discount factor, R_{t+1} represents the gross interest rate on assets between generation t and $t+1$, and $\xi \in (0,1)$, $\theta \in (0,1)$, and $\sigma \in (0,1)$ are utility weights on the number of children, children's education, and children's health, respectively. This type of utility function is often used in the literature (cf. Strulik et al., 2013; Prettner and Werner, 2016; Baldanzi et al., 2021) and is based on the "warm-glow motive of giving" (see Andreoni, 1989). It is a special case of the utility formulation used in Galor and Weil (2000), and Galor (2005, 2011), which leads to the same tradeoffs in terms of child quantity (the number of children) and child quality (here education and health of each child). To simplify the exposition, we assume exogenous mortality and to rule out solutions in which parents do not have any children at all, we impose the parameter restriction $\xi > \theta + \sigma$. These are very important assumptions because they prevent the nonsensical solutions in which parents would want to invest in children's education and health, while they do not have any children at all in the very first place.

For the sake of clarity, we assume that the cost of raising children, educating them, and providing them with a basic health level requires time of their parents. However, it can be shown that the model is isomorphic to a more complicated framework in which education and healthcare for the children are bought by parents on the market. Overall, the budget constraint of the household reads

$$(1-\tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t) w_t h_t = c_t + s_t, \tag{2}$$

where $\tau \in (0,1)$ represents the income tax rate; $\psi > 0$, $\eta > 0$, and $\chi > 0$ denote opportunity costs in terms of time for child-rearing, education per child, and health investment per child, respectively; w_t is the wage rate; and h_t represents effective labor (the human capital of the household as a composite of its education and health). Solving the optimization problem for the choices of consumption, savings, fertility, education, and children's health yields (see Appendix A for the derivation)

$$\begin{aligned} c_t &= \frac{(1-\tau)w_t h_t}{1+\beta+\xi}, \qquad s_t = \frac{\beta(1-\tau)w_t h_t}{1+\beta+\xi}, \qquad n_t = \frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}, \\ e_t &= \frac{\theta\psi}{\eta(\xi-\theta-\sigma)}, \qquad m_t = \frac{\sigma\psi}{\chi(\xi-\theta-\sigma)}. \end{aligned} \tag{3}$$

The population size at time $t+1$ is determined via the fertility rate n_t as

$$L_{t+1} = n_t L_t = \frac{\xi - \theta - \sigma}{\psi(1+\beta+\xi)} L_t. \tag{4}$$

We assume that the individual human capital level of the next generation depends positively on (i) education effort by the parents, e_t; (ii) parents' productivity in education, A_E; (iii) healthcare effort by parents for their children, m_t[3]; (iv) parents' productivity in healthcare for their children, A_M; and (v) the level of parental human capital h_t in the following way:

$$h_{t+1} = (A_E e_t h_t)^{\nu} (A_M m_t h_t)^{1-\nu} = \left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{\xi - \theta - \sigma} h_t. \tag{5}$$

Equations (4) and (5) capture the tradeoffs between child quantity and quality that are summarized in the following proposition.

Proposition 1. *An increase in the desire for a large family (ξ) raises fertility and population growth but reduces human capital accumulation. Increases in the desire for having better educated or healthier children (increases in θ and σ) raise human capital accumulation but reduce fertility and population growth.*

Proof. See Appendix B. □

Proposition 1 shows that there is an inverse relationship between population growth and individual human capital accumulation, which results from the quality-quantity tradeoff in the optimization problem of the household.

H_t, the aggregate human capital stock of the economy, is the product of individual human capital (h_t) and the total population size (L_t). Therefore, the human capital stock available for production, basic and applied research, and healthcare ($\tilde{H}_t$) is given by the aggregate human capital stock adjusted for the time parents spend raising their children, educating them, and caring for their health ($\psi n_t + \eta e_t n_t + \chi m_t n_t$) as

$$\tilde{H}_t = [1 - \psi n_t - \eta e_t n_t - \chi m_t n_t]\, H_t = \frac{1+\beta}{1+\beta+\xi} h_t L_t. \tag{6}$$

Note that aggregate human capital accumulation and population growth are again inversely related.

[3] Note that, along with the level of education, a better health condition is also an essential component of individual human capital (cf. Rivera and Currais, 2004; Baldanzi et al., 2021).

2.2 Production side

The final goods sector, the intermediate goods sector, the applied research sector, the basic research sector, and the healthcare sector constitute the production side of the economy. The first three sectors are based on the standard Romer (1990) and Jones (1995) R&D-driven growth framework. We modify the Romer-Jones structure to account for (*i*) a tax-financed basic research sector that employs scientists to discover and explain the natural laws and phenomena required for applied research (Prettner and Werner, 2016), (*ii*) public healthcare, which enhances the productivity of human capital (Kuhn and Prettner, 2016), and (*iii*) the endogenous evolution of aggregate human capital in the production process, which depends on fertility, education, and health choices of households (cf. Galor and Weil, 2000; Galor, 2005, 2011; Strulik et al., 2013). The model structure is displayed in Figure 1, where households demand goods and supply labor and capital (through savings), the government taxes household wage income to fund healthcare and basic research, and the different production sectors employ labor or the saved capital of households to produce their corresponding output. Demand and supply for all goods are equal due to the market clearing price vector that emerges endogenously by the interactions between households and firms on the market.

2.2.1 Final goods sector

The perfectly competitive final goods sector employs workers and machines to produce output Y_t according to

$$Y_t = \left(H_{t,M}^{\varepsilon_0} H_{t,Y} \right)^{1-\alpha} \int_0^{A_t} x_{t,i}^{\alpha} \, di, \tag{7}$$

where $H_{t,Y}$ and $H_{t,M}$ refer to the (embodied) human capital employed in the final good and healthcare sectors, respectively, A_t is the technological frontier, $x_{t,i}$ is the amount of the blueprint-specific machine i used in production, and α is the elasticity of output with respect to machines. Employment in the healthcare sector raises the health of workers and, thus, affects their productivity according to $H_{t,M}^{\varepsilon_0}$, where $\varepsilon_0 \geq 0$ measures the strength of the effect.[4] We

[4]Consider the following example. An individual's human capital level at the time of entry into the labor force in period t is h_t. This human capital level depends on the parents' decision (in period $t-1$) to devote time to education and healthcare when the individual was young. However, if the individual becomes ill, even though she continues to work, she may not be able to perform to her full potential. Public healthcare will assist her in regaining full productivity as soon as possible. As a result, she will be more productive than if she did not have access to public healthcare. In this context, it should be noted that public healthcare may have an impact on children's health. However, for the sake of simplicity, we are ignoring this pathway here. One worthwhile extension of the current model would be integrating this aspect and investigating its long-run implications. Note that the health impact on labor productivity is accounted for as a spillover effect. The strength of this spillover effect in the final goods production sector is captured by the parameter ε_0. A similar type of argument can be found in Schneider and Winkler (2017, 2021).

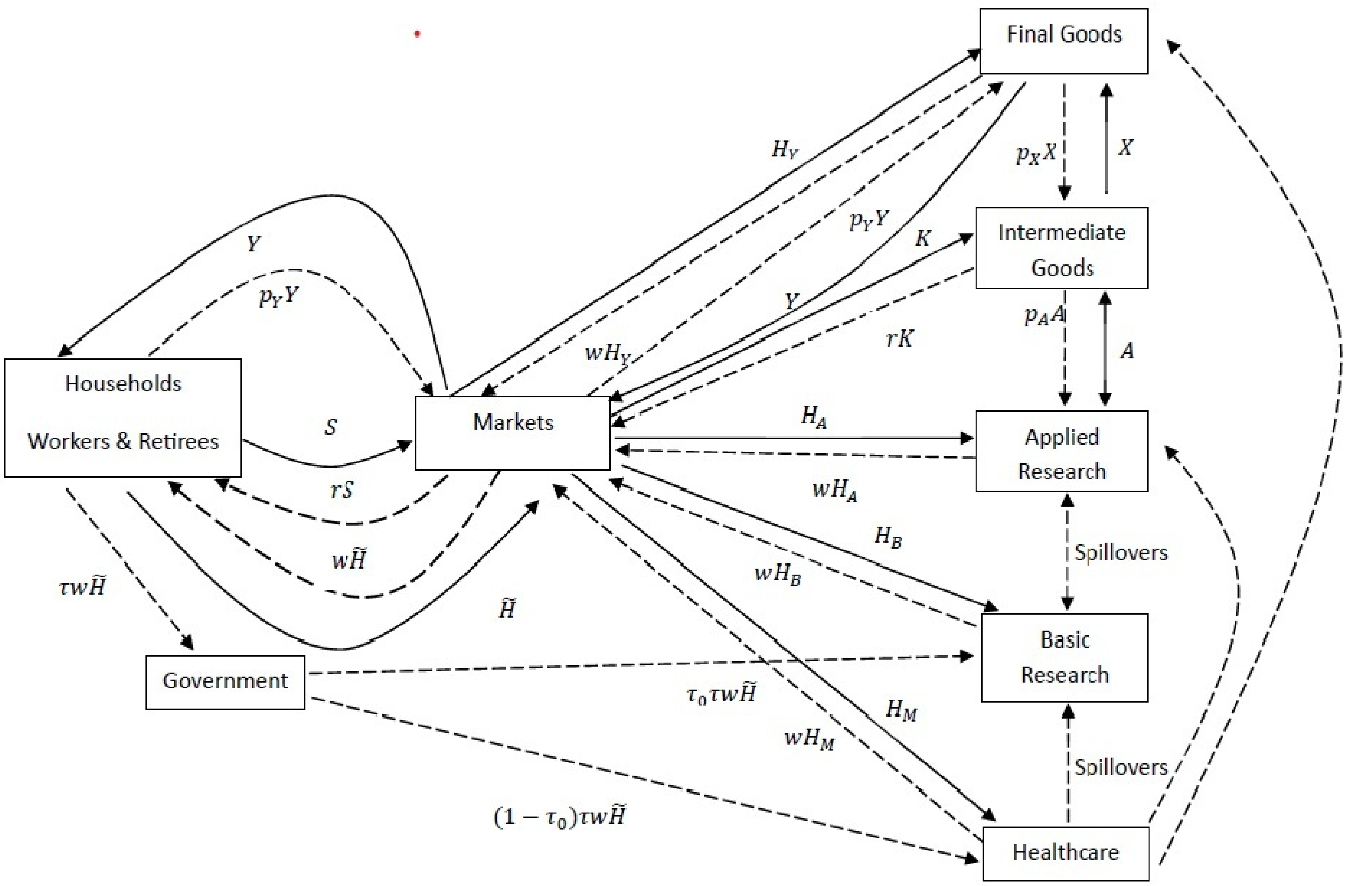


Figure 1: Overview of the structure of the model

assume that government health investments are non-zero such that $H_{t,M} > 0$. For a given technology level and health status (i.e., given A_t and $H_{t,M}$), Equation (7) exhibits constant returns to scale in $H_{t,Y}$ and $x_{t,i}$. Perfect competition implies that the wage rate $w_{t,Y}$ and the price of machines $p_{t,i}$ are given by the marginal products of workers and machines as

$$w_{t,Y} = (1-\alpha)\left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{-\alpha} H_{t,M}^{\varepsilon_0} \int_0^{A_t} x_{t,i}^{\alpha}\, di = (1-\alpha)\frac{Y_t}{H_{t,Y}}, \tag{8}$$

$$p_{t,i} = \alpha x_{t,i}^{\alpha-1} \left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha}. \tag{9}$$

We observe the standard effects of declining marginal productivity: increasing employment of workers raises the price of machines, while more intensive machine use and faster technological progress both raise the wage rate. In addition, better healthcare has a positive effect on the wage rate and, because it raises labor productivity, also on the price of machines.

2.2.2 Intermediate goods sector

Raw physical capital $k_{t,i}$ serves as variable input and one machine-specific blueprint serves as fixed input in the production of the monopolistically competitive intermediate goods sector, which produces the machines for the production of the final good. We assume full depreciation of physical capital over the course of one generation. Thus, operating profits in intermediate goods production are $\pi_{t,i} = p_{t,i}k_{t,i} - R_t k_{t,i}$ and profit maximization leads to the well-known monopolistic pricing rule for each firm

$$p_{t,i} = \frac{R_t}{\alpha}. \tag{10}$$

This pricing rule implies that intermediate sector firms charge a markup over the marginal cost of production, which leads to positive operating profits. Due to symmetry across firms, each firm employs $k_t = K_t/A_t$ units of physical capital, where K_t represents the aggregate physical capital stock. Thus, the aggregate production function can be re-written as

$$Y_t = \left(A_t H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha} K_t^{\alpha}, \tag{11}$$

where technological progress appears as labor augmenting.

2.2.3 Applied research sector

The applied research sector employs scientists with a human capital stock of $H_{t,A}$ to design new blueprints that can be patented and sold to the intermediate goods sector. In applied research, the representative firm's production function is given by

$$A_{t+1} - A_t = \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A}, \tag{12}$$

where δ_1 is the basic productivity of applied scientists, $H_{t,M}^{\varepsilon_1}$ is the effect of public healthcare on the productivity of applied scientists, B_t represents society's stock of basic knowledge, which forms the epistemic base for the stock of patented knowledge A_t (Mokyr, 2002, 2016; O'Rourke et al., 2013; Prettner and Werner, 2016; Lehmann-Hasemeyer et al., 2023), $\phi_1 \in [0,1]$ measures the extent of intertemporal knowledge spillovers in the applied research sector, and $\mu_1 \in [0,1]$ measures the extent of intersectoral knowledge spillovers from basic to applied research.[5] For a given stock of basic and applied knowledge, $\varepsilon_1 \geq 0$ measures how strongly public healthcare enhances the productivity of applied scientists. Similar to Prettner and Werner (2016), no blueprints can be developed without any basic knowledge B_t, so we assume that $B_0 > 0$ and $A_0 > 0$. Our framework nests the semi-endogenous growth model of Jones (1995) as a special case. Moreover, the Romer (1990) model could be recovered by switching off the dynamics of human capital accumulation. We summarize this in Remark 1.

Remark 1. *For $\tau = 0$, $\theta = 0$, $\sigma = 0$, $\xi > \psi(1 + \beta + \xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and $\phi_1 \in (0,1)$, our model nests the Jones (1995) framework. Furthermore, if we set $\tau = 0$, $\xi = \psi(1 + \beta + \xi)$, $\theta = 0$, $\sigma = 0$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, $\phi_1 = 1$ and remove the dynamics of human capital by assuming a constant individual human capital stock $h_t = \bar{h}$, the Romer (1990) framework would emerge from the given setup.*

Firms in the applied research sector hire human capital $H_{t,A}$ so as to maximize their profits

$$\pi_{t,A} = p_{t,A} \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A} - w_{t,A} H_{t,A} \tag{13}$$

with $p_{t,A}$ being the price of a blueprint and $w_{t,A}$ referring to the applied researchers' wage rate. This leads to the optimality condition

$$w_{t,A} = p_{t,A} \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1}. \tag{14}$$

We observe that wages of applied researchers depend positively on the price of blueprints, the productivity of applied researchers, intertemporal- and intersectoral knowledge spill-overs, and on the extent to which public healthcare raises scientists' productivity.

[5] As in Prettner and Werner (2016), given that patents are partially excludable, whereas the laws of nature, once discovered, can be exploited by scientists freely, one can expect that the spillovers from basic research to applied research are greater than the spillovers in the opposite direction.

Following Strulik et al. (2013) and Prettner and Werner (2016), we assume that patent protection lasts for one generation. Once the patent expires, the right to sell the blueprint is handed over to the government, which can either consume or invest the associated proceeds. For a blueprint, firms in the applied research sector charge the entire operating profit of an intermediate goods producer, that is,

$$p_{t,A} = \pi_{t,i} = \alpha(1-\alpha)\frac{Y_t}{A_t}. \tag{15}$$

The reason is that free entry prevails in intermediate goods production. If a firm would not be willing to pay its entire operating profit for a blueprint, another firm would always be willing to do so.

2.2.4 Basic research sector

Following Prettner and Werner (2016), the production function of basic research is given by

$$B_{t+1} - B_t = \delta_2 H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_{t,B}, \tag{16}$$

where B_t is the stock of basic knowledge, δ_2 is the productivity of basic researchers, $H_{t,B}$ is the human capital stock employed in the basic research sector, $\mu_2 \in [0,1]$ is intertemporal spillover effect within basic research, and $\phi_2 \in [0,1]$ is intersectoral spillover effect from applied research to basic research. As in final goods production and applied research, public healthcare raises the productivity of basic researchers with $\varepsilon_2 \geq 0$ measuring the strength of this effect.

A part τ_0 of the government's revenue is spent on employing scientists to discover basic knowledge. Considering aggregate labor supply as given by Equation (6), the tax revenue used for funding basic research is then given by the left-hand side of

$$\frac{\tau_0 \tau(1+\beta)}{1+\beta+\xi} w_t h_t L_t = w_t h_t L_{t,B}. \tag{17}$$

This revenue is used to cover the wage bill of scientists in the basic research sector, which is given by the right-hand side of Equation (17). It follows that the amount of human capital employed in the basic research sector is

$$H_{t,B} = L_{t,B} h_t = \frac{\tau_0 \tau(1+\beta)}{1+\beta+\xi} H_t, \quad \left(\equiv \tau_0 \tau \tilde{H}_t\right). \tag{18}$$

Using the production function (16), basic knowledge evolves according to

$$B_{t+1} - B_t = \frac{\delta_2 \tau_0 \tau (1+\beta)}{1+\beta+\xi} H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_t. \tag{19}$$

Note that the accumulation of basic knowledge rises with government spending on it as captured by the product of the overall tax rate, τ, and the share of spending devoted to basic research, τ_0.

In reality, a part of basic research is funded privately, while a part of applied research is, in turn, funded publicly. We assume the polar case of full public funding of basic research and full private funding of applied research for tractability reasons. Going more into the details of the interplay between basic and applied research and their cross-funding would definitely be a worthwhile topic for further research in the area of innovation economics. Since applied research is driving productivity and there is also a short-run versus long-run tradeoff for applied research funding, our results should not change strongly when including the possibilities of privately funded basic research and publicly funded applied research. In particular, this holds true as long as the main part of basic research is publicly funded and the main part of applied research is privately funded.

2.2.5 Healthcare sector

We assume that the government's budget is balanced so that a share $(1-\tau_0)$ of the tax revenue is spent on the wages of healthcare workers, i.e.,

$$\frac{(1-\tau_0)\tau(1+\beta)}{1+\beta+\xi} w_t h_t L_t = w_t h_t L_{t,M}.$$

Thus, the amount of human capital employed in the healthcare sector is

$$H_{t,M} = h_t L_{t,M} = \frac{(1-\tau_0)\tau(1+\beta)}{1+\beta+\xi} H_t, \quad \left(\equiv (1-\tau_0)\tau \tilde{H}_t\right). \tag{20}$$

The government aims to improve people's health by providing healthcare to them and, in doing so, affects the productivity of human capital in final goods production, applied research, and basic research as described above when discussing the production functions in the different sectors.

2.3 Market clearing and balanced growth path

The labor market clearing conditions are $\tilde{H}_t = h_t[L_{t,Y}+L_{t,A}+L_{t,B}+L_{t,M}] = H_{t,Y}+H_{t,A}+H_{t,B}+H_{t,M}$ and $w_{t,Y} = w_{t,A} = w_{t,B} = w_{t,M} = w_t$. The first of these equations states that employment in the four sectors of the economy that produce with labor adds up to total employment. The second equation states that wages in the four sectors have to be equal, otherwise workers of a sector that pays a lower wage would have an incentive to move to a sector that pays a higher wage. Using these conditions and Equations (8), (14), (15), (18), and (20) yields the demand for human capital in the final goods and applied research sectors as

$$H_{t,Y} = \frac{A_t^{1-\phi_1} B_t^{-\mu_1} H_{t,M}^{-\varepsilon_1}}{\alpha\delta_1}, \tag{21}$$

$$H_{t,A} = \tilde{H}_t - H_{t,B} - H_{t,M} - H_{t,Y}$$

$$\implies H_{t,A} = \frac{(1-\tau)(1+\beta)}{(1+\beta+\xi)} h_t L_t - \frac{A_t^{1-\phi_1} B_t^{-\mu_1}}{\alpha\delta_1} \left[\frac{(1-\tau_0)\tau(1+\beta)}{(1+\beta+\xi)} h_t L_t\right]^{-\varepsilon_1}. \tag{22}$$

The development of new blueprints in the applied research sector is then given by

$$A_{t+1} = \left(\frac{1+\beta}{1+\beta+\xi}\right)^{1+\varepsilon_1} \left[(1-\tau_0)\tau\right]^{\varepsilon_1} (1-\tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1} (h_t Lt)^{1+\varepsilon_1} - \left(\frac{1-\alpha}{\alpha}\right) A_t. \tag{23}$$

Note that the last term in this equation is negative, which implies that oscillations can occur. The reason is that, due to the timing of innovation, an increase in A_{t+1} generates an increase in the marginal value product of labor in the final goods sector in $t+1$ and, thus, a shift of employment from the other sectors to the final goods sector. This, in turn, reduces the number of scientists and slows down innovation, which dampens the growth of A from A_{t+1} to A_{t+2}. As a consequence, a shift of employment back from final goods production to the other sectors occurs. The process continues until the economy has converged to the new balanced growth path. When simulating the economy numerically, a lower ϕ_1 leads to greater oscillations because the last term in Equation (23) increases in relative importance.

Capital market clearing requires that aggregate savings are used for physical capital accumulation and purchasing new blueprints for intermediate goods production, i.e., $K_{t+1} = s_t L_t - p_{t,A}(A_{t+1} - A_t) = \frac{\beta(1-\tau)}{1+\beta+\xi} w_t h_t L_t - p_{t,A}(A_{t+1} - A_t)$. Equations (8), (11), (15), (20), (21), and (23) yield the aggregate physical capital stock of the next period as

$$K_{t+1} = \left[\frac{\beta(1-\tau)(1-\alpha)\left[(1-\tau_0)\tau(1+\beta)\right]^{\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}}{(1+\beta+\xi)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}}K_t^{\alpha}\right] \times$$
$$\left[\left(\frac{A_t^{2-\phi_1}B_t^{-\mu_1}}{\alpha\delta_1}\right)^{-\alpha} A_t\left(h_tL_t\right)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}\right]$$
$$-\alpha(1-\alpha)\frac{Y_t}{A_t}\left[\left(\frac{1+\beta}{1+\beta+\xi}\right)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}(1-\tau)\delta_1 A_t^{\phi_1}B_t^{\mu_1}(h_tLt)^{1+\varepsilon_1}-\frac{A_t}{\alpha}\right]. \tag{24}$$

Finally, with respect to the evolution of the stock of basic knowledge, Equations (19) and (20) yield

$$B_{t+1} = \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}}A_t^{\phi_2}B_t^{\mu_2}(h_tL_t)^{1+\varepsilon_2}+B_t. \tag{25}$$

Equations (4), (5), (6), (11), (19), (20), (21), (22), (23), (24), and (25) fully characterize the evolution of the economy in the short-, medium-, and long run. We obtain the analytical results for the balanced growth path next and afterwards proceed to the numerical assessment of the transitional dynamics and the welfare implications.

3 Analytical results for the long-run balanced growth path

From now on, we restrict our attention to the following parameter ranges to ensure the existence of a balanced growth path and to rule out the empirically implausible scenario of hyper-exponential growth.

Assumption 1. *The intertemporal and intersectoral knowledge spillovers are given by* $\phi_1 \in [0,1)$, $\phi_2 \in [0,1)$, $\mu_1 \in [0,1)$, *and* $\mu_2 \in [0,1)$. *Moreover, it holds that* $\phi_1+\mu_1<1$ *and* $\phi_2+\mu_2<1$.

The growth rates of the stocks of blueprints and of basic knowledge are then given by

$$g_{t,A} \equiv \frac{A_{t+1}-A_t}{A_t} = \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1A_t^{\phi_1-1}B_t^{\mu_1}(h_tLt)^{1+\varepsilon_1}-\frac{1}{\alpha}, \tag{26}$$

$$g_{t,B} \equiv \frac{B_{t+1}-B_t}{B_t} = \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}}B_t^{\mu_2-1}A_t^{\phi_2}(h_tL_t)^{1+\varepsilon_2}. \tag{27}$$

The balanced growth factors (henceforth BGFs) of individual human capital, population size, and aggregate human capital amount to[6]

$$\tilde{h} \equiv \frac{h_{t+1}}{h_t} = \left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{\xi - \theta - \sigma}, \tag{28}$$

$$\tilde{L} \equiv \frac{L_{t+1}}{L_t} = n_t = \frac{\xi - \theta - \sigma}{\psi(1+\beta+\xi)}, \tag{29}$$

$$\Omega \equiv \tilde{h}\tilde{L} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{1+\beta+\xi}. \tag{30}$$

From now on, we assume that

$$\psi \in \left(\frac{\xi - \theta - \sigma}{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}, \frac{\xi - \theta - \sigma}{1+\beta+\xi} \right),$$

which ensures that individual human capital and the population will both grow over time. As a result, $\Omega = \tilde{h}\tilde{L} > 1$ holds unambiguously. The following proposition contains the main results for the long-run balanced growth path.

Proposition 2.

(i) The BGFs of A, B, K, and Y are given by

$$\tilde{A} \equiv \frac{A_{t+1}}{A_t} = \Omega^{\Psi_1}, \qquad \tilde{B} \equiv \left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\Psi_2},$$
$$\tilde{K} \equiv \left(\frac{K_{t+1}}{K_t}\right) = \Omega^{\Psi_3 + \varepsilon_0} = \tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t}\right),$$

where

$$\Psi_1 = \frac{(1+\varepsilon_1)(1-\mu_2) + (1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2) - \phi_2\mu_1} > 0,$$
$$\Psi_2 = \frac{(1+\varepsilon_2)(1-\phi_1) + (1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2) - \phi_2\mu_1} > 0,$$
$$\Psi_3 = \frac{(1-\mu_2)(2-\phi_1+\varepsilon_1) + \mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2) - \phi_2\mu_1} > 0.$$

(ii) The BGFs increase with aggregate human capital accumulation (Ω), with the knowledge spillovers μ_1, μ_2, ϕ_1, ϕ_2, and with the strength of the effect that healthcare has in enhancing the productivity of workers employed in the applied research sector (ε_1) and in the basic research sector (ε_2). The BGF of GDP also increases with the strength of the

[6] Note that $R_t = \alpha p_t = \alpha^2 Y_t / K_t$. Along the balanced growth path, Y_t and K_t are growing at the same rate such that R_t must be constant, i.e., $R_{t+1} = R_t = R$, for all t.

effect that healthcare has in enhancing the productivity of workers employed in the final goods sector (ε_0)*.*

(iii) *The BGFs are independent of the tax rates,* τ*,* τ_0*, and* $(1-\tau_0)$*.*

(iv) *The BGF of individual human capital* ($\tilde{h}$) *increases with the utility weight of children's education* (θ) *and health* (σ)*, and decreases with the utility weight of the number of children* (ξ)*.*

(v) *The BGF of the population* ($\tilde{L}$) *decreases with the utility weight of children's education* (θ) *and health* (σ)*, and increases with the utility weight of the number of children* (ξ)*.*

(vi) *The BGF of aggregate human capital* (Ω) *increases with the utility weight of children's education* (θ) *and health* (σ)*, and decreases with the utility weight of the number of children* (ξ)*.*

(vii) *The BGF of per capita GDP is given by*

$$\tilde{y} = \frac{\tilde{Y}}{\tilde{L}} = \left[\frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}}{1+\beta+\xi}\right]^{\Psi_3+\varepsilon_0}\left[\frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}\right]^{-1}.$$

The per capita GDP growth factor increases with the utility weight of children's education (θ) *and health* (σ)*, and decreases with the utility weight of the number of children* (ξ)*. It also increases with the knowledge spillovers* μ_1*,* μ_2*,* ϕ_1*,* ϕ_2*, and the strength of the effect that healthcare has on the productivity of workers employed in the final goods sector* (ε_0)*, the applied research sector* (ε_1)*, and the basic research sector* (ε_2)*.*

Proof. See Appendix C. □

The growth models of Prettner and Werner (2016) and Baldanzi et al. (2021) are nested as special cases within our generalized R&D-based growth model, which we summarize in Remark 2.

Remark 2. *For* $\varepsilon_0=\varepsilon_1=\varepsilon_2=0$*, and* $\nu=1$*, our model nests the Prettner and Werner (2016) framework, while for* $\varepsilon_0=\varepsilon_1=\varepsilon_2=\mu_1=\mu_2=\phi_2=0$ *our model nests the Baldanzi et al. (2021) framework as special cases.*

One implication of Proposition 2 is that human capital accumulation is a primary factor for determining long-run economic growth and, thus, welfare. A second implication is that, although aggregate human capital accumulation is increasing with the desire for educated and healthy children, it is decreasing with population growth. We summarize these insights in Remark 3.

Remark 3. *The ultimate driving force of economic growth is the accumulation of human capital. This result is in line with standard endogenous economic growth models. However, in our case, there is a negative association between population growth and human capital accumulation such that faster population growth can even reduce economic growth.*

Furthermore, the growth effect of human capital accumulation is mediated through the basic research and applied research sectors — because in these sectors new technologies are developed — and through the health sector, which improves productivity of workers. Higher intertemporal and intersectoral knowledge spillovers and a stronger effect of healthcare in enhancing the productivity of workers employed in the applied research, basic research, and final goods sectors lead to a rise in balanced growth rates. We summarize these insights in Remark 4.

Remark 4. *While economic growth is ultimately driven by the accumulation of human capital, the effect is mediated by basic research and applied research — because in these sectors, new technologies are developed — and healthcare — because better health raises the productivity of workers in all sectors.*

The effects of θ and σ on per capita GDP growth that emerge from our model are higher than in Baldanzi et al. (2021). In Baldanzi et al.'s model, the per capita GDP growth factor is influenced only by ϕ_1. By contrast, in our model, the per capita GDP growth factor is additionally influenced by intertemporal and intersectoral knowledge spillovers such as ϕ_2, μ_1, and μ_2. Another reason for the difference between Baldanzi et al.'s findings and ours is that we incorporate the impact of public healthcare in enhancing the productivity of workers in various sectors (through the spillover parameters ε_0, ε_1, and ε_2).

The impact of θ on per capita GDP growth in our model is, in turn, greater than that of Prettner and Werner (2016), particularly when (*i*) $\nu = 1$ and (*ii*) $A_E\theta = A_M\sigma$. The inclusion of the effect that healthcare raises the productivity of workers and the corresponding spillover terms ε_0, ε_1, and ε_2 play a crucial role in explaining this difference. We would also like to highlight that, unlike in Prettner and Werner (2016), who do not consider health and healthcare investments, a rise in parental health investments for children (through a rise in σ) increases the per capita GDP growth factor in our model.

4 The transitional dynamics

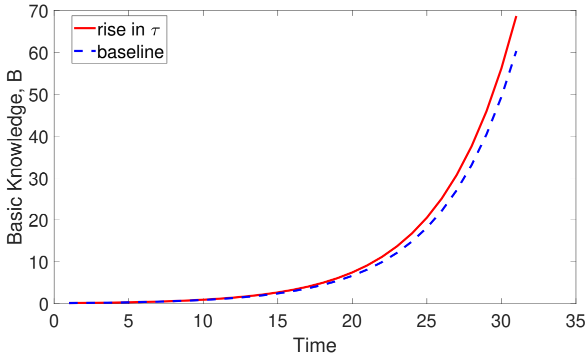
We now simulate the dynamic system represented by Equations (4), (5), (6), (11), (19), (20), (21), (22), (23), (24), and (25) to illustrate our analytical results numerically and to examine the economy's behavior during the transition. We use 20 years as the length of one generation, which corresponds to the duration of patent protection. We consider data of the 30 OECD countries such as Austria, Belgium, Chile, Czechia, Denmark, Estonia, France, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, and United States. This is because basic research data for the rest of the OECD are not available. Table 1 summarizes the parameter values that are either taken from the literature (cf. Auerbach and Kotlikoff, 1987; Jones, 1995; Acemoglu, 2008; Prettner and Werner, 2014, 2016; Baldanzi et al., 2021) or otherwise adjusted so that the model's predictions are consistent with the population growth rate and the economic growth rate of OECD countries from 2000 to 2019, with the data taken from OECD (2023). We are considering data until 2019 because the COVID-19 pandemic disrupted economic activity in the years afterwards. Data on the fraction of GDP that OECD countries spent on basic research and the fraction of GDP spent on publicly-funded healthcare between 2000 and 2019 are taken directly from OECD (2023) and we also get the population growth rate and per capita GDP from this source. The simulated value of the population growth rate is 12.96% over 20 years, which is a reasonable approximation of the inter-generational population growth rate of 12.94% for the countries considered. Similarly, the simulated GDP growth rate is 54.16% over 20 years. This, too, is reasonably close to the inter-generational GDP growth rate of 53.44% for the countries considered.

In line with Prettner and Werner (2016), the discount factor β is computed based on a yearly discount rate of 1.9% and we choose the parameter value $\alpha = 1/3$ as it is common practice (cf. Jones, 1995; Acemoglu, 2008). In addition, we calculate basic research as a percentage of GDP in 2019 for the mentioned OECD countries as 0.4523% and public health expenditure as a percentage of GDP as 9.9%. Thus, the required tax rate to fund basic research (i.e., $\tau\tau_0$) is $0.004523\times(3/2)=0.0067845$ (because $\alpha = 1/3$) and the required tax rate for funding public health expenditure (i.e., $\tau(1-\tau_0)$) is $0.0990\times(3/2)=0.1485$. As a consequence, the values of τ and τ_0 are 0.1552845 and 0.044, respectively.

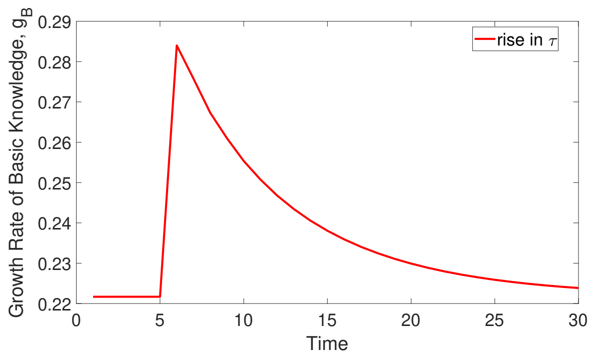
The effect of an increase in total government expenditure (basic research and health expenditures together) on the growth rates of basic knowledge, technology, per capita GDP, and the physical capital stock (or aggregate GDP) are shown in Figures 2b, 2d, 2f, and 2h. We assume that the economy initially moves along the balanced growth path. After the fifth period in the

Parameters	Values	Sources
β	0.6892	Auerbach and Kotlikoff (1987), Grossmann et al. (2013a,b), Prettner and Werner (2016)
α	1/3	Jones (1995), Acemoglu (2008), Prettner and Werner (2016)
ψ	0.052	Baldanzi et al. (2021)
τ	0.1552845	OECD(2023a,b)
τ_0	0.044	OECD(2023a,b)
η	0.139	Prettner and Werner (2016)
χ	0.139	Authors (similar to Prettner and Werner (2016))
ξ	0.8491	Baldanzi et al. (2021)
θ	0.4	Baldanzi et al. (2021)
σ	0.3	Baldanzi et al. (2021)
A_E	1.18	Authors
A_M	1.128	Authors
ϕ_1	0.4	Prettner and Werner (2016), Baldanzi et al. (2021)
ϕ_2	0.05	Prettner and Werner (2016)
μ_1	0.3	Authors (close to Prettner and Werner (2016))
μ_2	0.3	Prettner and Werner (2016)
ε_0	0.001	Authors
ε_1	0.001	Authors
ε_2	0.001	Authors
δ_1	1	Prettner and Werner (2016)
δ_2	1	Prettner and Werner (2014)
ν	0.5	Baldanzi et al. (2021)

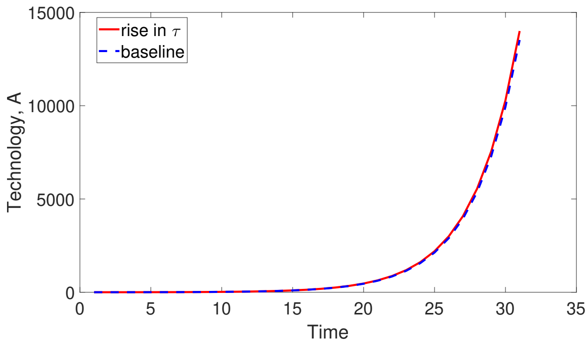
Table 1: Parameter values for the numerical analysis



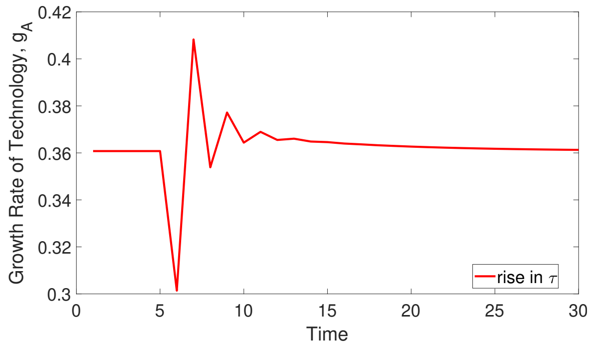
(a) Basic knowledge



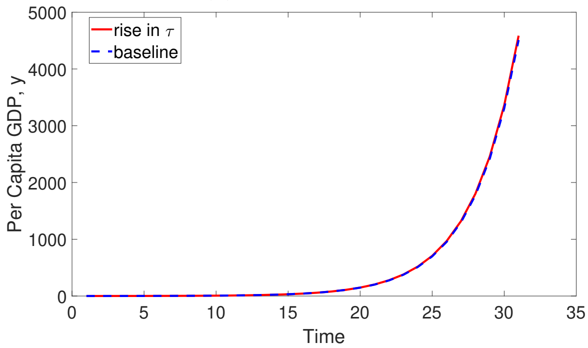
(b) Basic knowledge growth



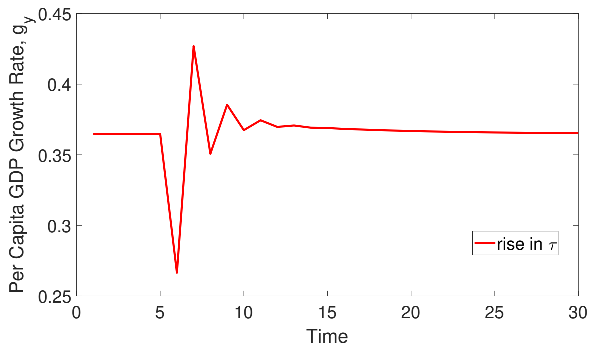
(c) Technology



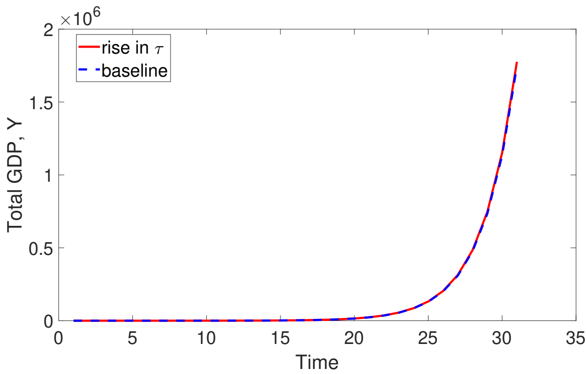
(d) Technology growth



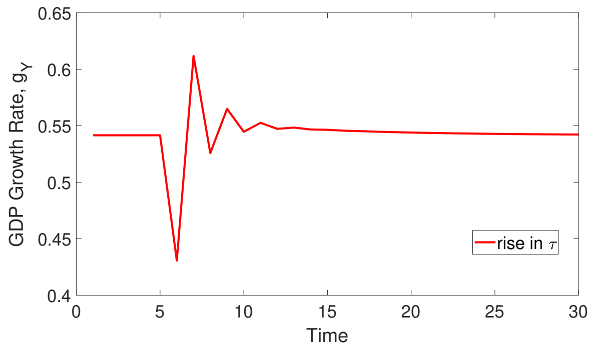
(e) GDP per capita



(f) Growth of GDP per capita



(g) GDP



(h) GDP growth

Figure 2: Effects of a rise in τ by 1 percentage point in terms of GDP

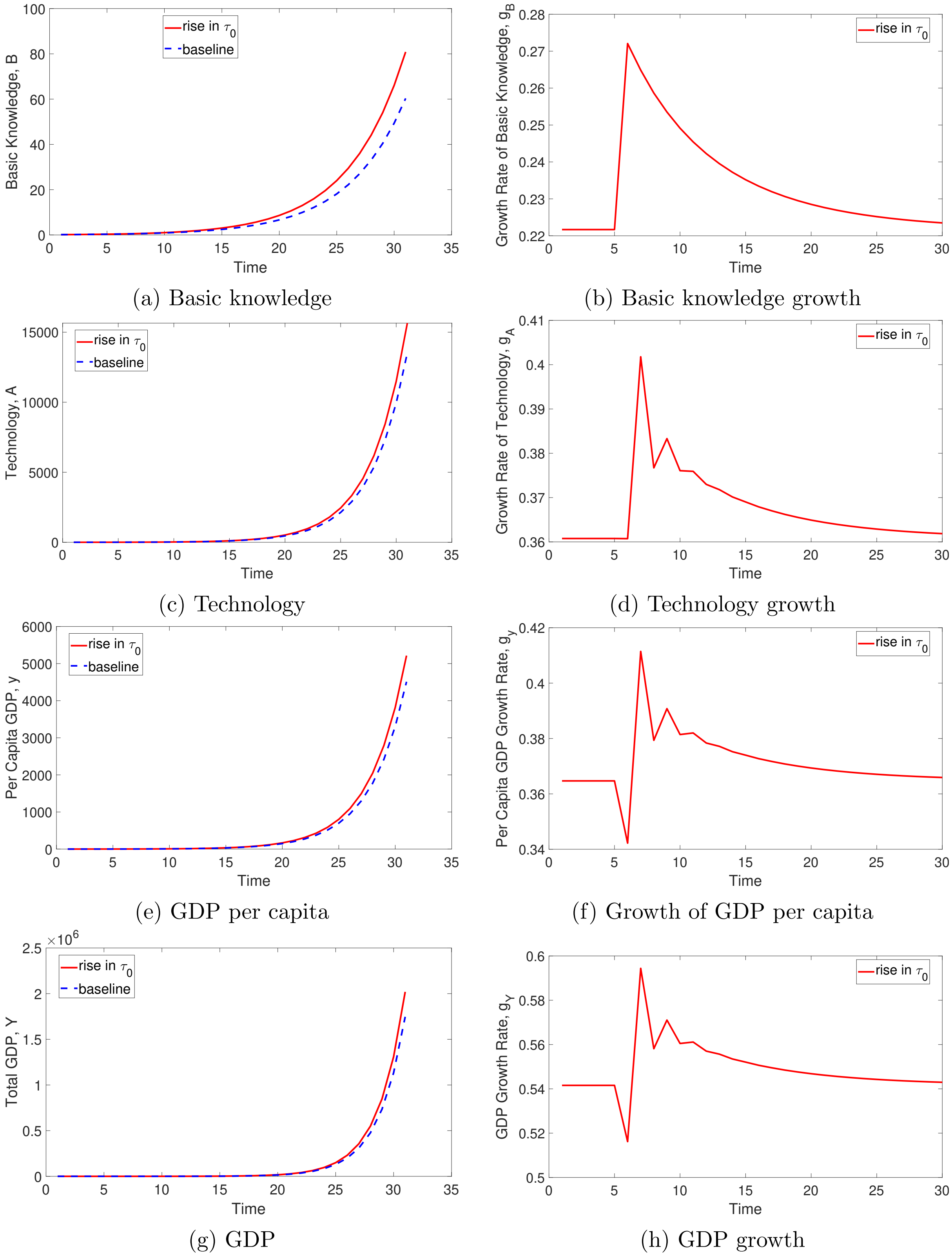
numerical analysis, we increase government expenditures by 1 percentage point in terms of GDP. Thus, τ increases from 0.1552845 to 0.1702845 ($0.01\times(3/2)+0.1552845=0.015+0.1552845=0.1702845$).

The rise in public spending has the effect of drawing labor away from applied research towards basic research. This slows down the expansion of applied research (see Figure 2d), while accelerating the evolution of basic knowledge (see Figure 2b). Because basic knowledge is a necessary input for applied research, the accumulation of new blueprints accelerates in the medium run despite a short-run slowdown. In the short- and medium run, the physical capital stock grows faster than if no policy changes were put into effect (see Figure 2h). As a result of the temporary slowdown in the accumulation of applied research, economic growth in terms of per capita GDP slows down in the short run, whereas growth in per capita GDP picks up in the medium run (see Figure 2f). However, there is no growth effect in the long run because the beneficial growth effects of increased investment in basic research gradually fade away. This is expected due to the semi-endogenous growth structure of the model (see Jones, 1995). The effects of an increase in τ on the levels of the different variables are shown in Figures 2a, 2c, 2e, and 2g. The solid (red) line illustrates an economy that experienced a rise in τ, whereas the dashed (blue) line depicts an economy that did not experience such an increase. The level of basic knowledge (Figure 2a), the number of patents (Figure 2c), aggregate output (Figure 2g), and per capita GDP (Figure 2e) are all higher in the economy with a rise in τ.

As mentioned above, the oscillating pattern emerges because of the timing in the discrete version of the R&D-based growth model. The rise of A_{t+1} raises the marginal value product of labor in the final goods sector and draws employment away from the other sectors, including applied R&D. This reduces innovation activity in period $t+1$ and, thus, the accumulation of A between periods $t+1$ and $t+2$. As a consequence, the marginal value product of labor in the final goods sector falls again as compared to the other sectors and employment shifts back. The greater ϕ_1, the smoother is the transition. However, if ϕ_1 reaches very high levels, the model would not predict the economic growth rate accurately anymore.

The effect of a change in the composition of government expenditures in favor of basic research vis-a-vis health expenditure (i.e., a rise in τ_0) on the growth rates of basic knowledge, technology, per capita GDP, and the physical capital stock (or aggregate GDP) are shown in Figures 3b, 3d, 3f, and 3h. Again, we assume that the economy initially moves along the balanced growth path and that an increase in the composition of government expenditures in favor of basic research by 1 percentage point occurs after the fifth period. Thus, τ_0 increases from 0.044 to 0.054.

The rise of τ_0 has the effect of drawing labor away from the final goods sector towards basic



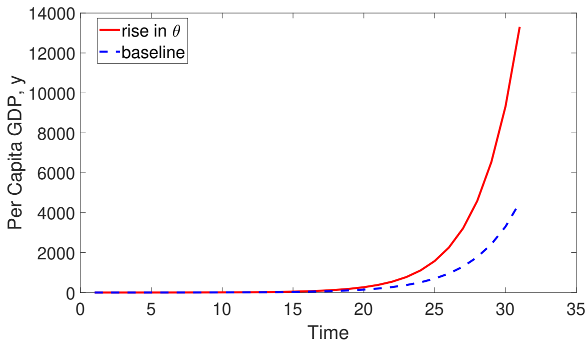
(a) Basic knowledge (b) Basic knowledge growth

(c) Technology (d) Technology growth

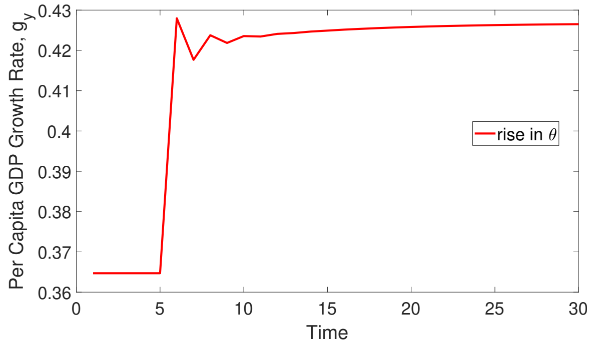
(e) GDP per capita (f) Growth of GDP per capita

(g) GDP (h) GDP growth

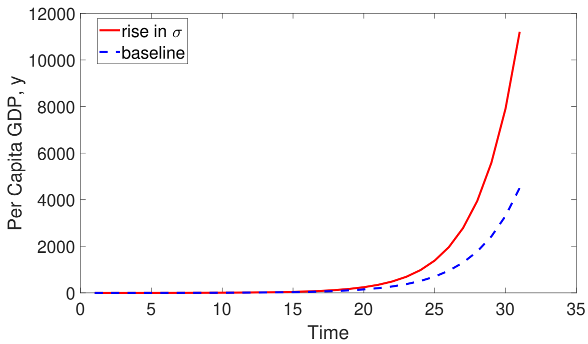
Figure 3: Effects of a rise in τ_0 by 1 percentage point



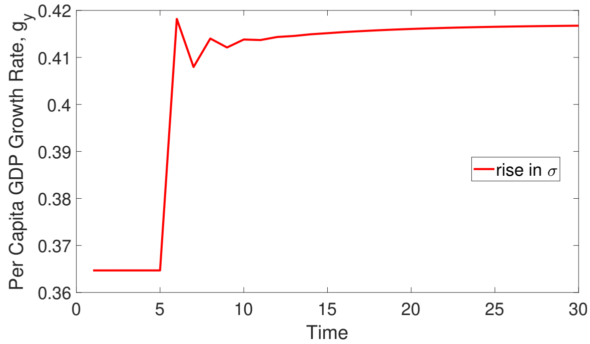
(a) A rise in θ on per capita GDP



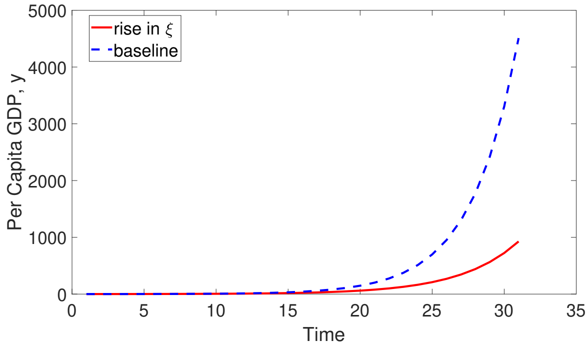
(b) A rise in θ on per capita GDP growth



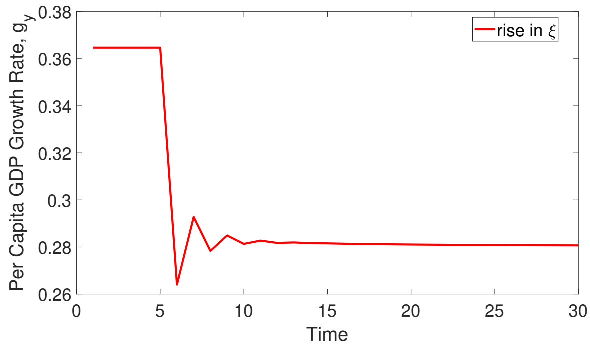
(c) A rise in σ on per capita GDP



(d) A rise in σ on per capita GDP growth

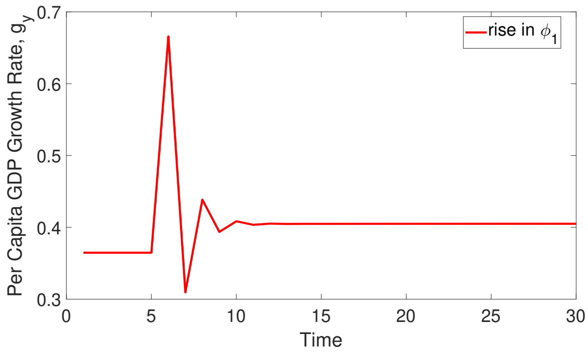


(e) A rise in ξ on per capita GDP

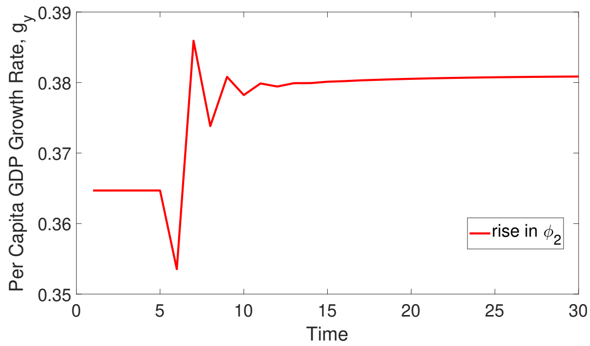


(f) A rise in ξ on per capita GDP growth

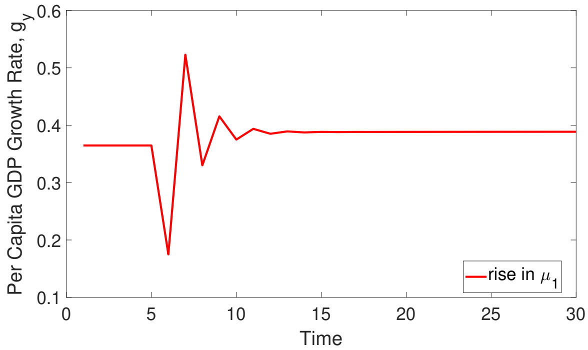
Figure 4: Effect of rise in σ, θ and ξ by 1 percent on per capita GDP and its growth



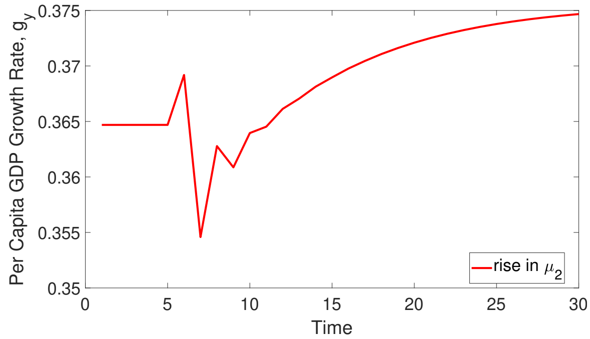
(a) A rise in ϕ_1 on per capita GDP growth



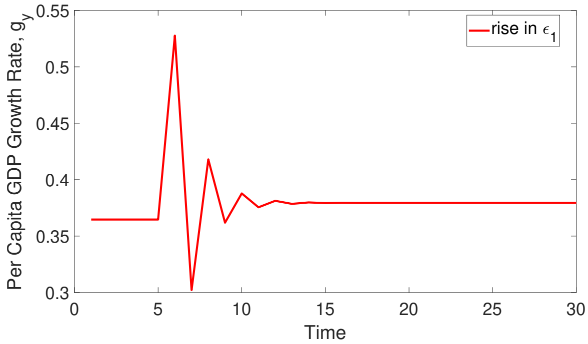
(b) A rise in ϕ_2 on per capita GDP growth



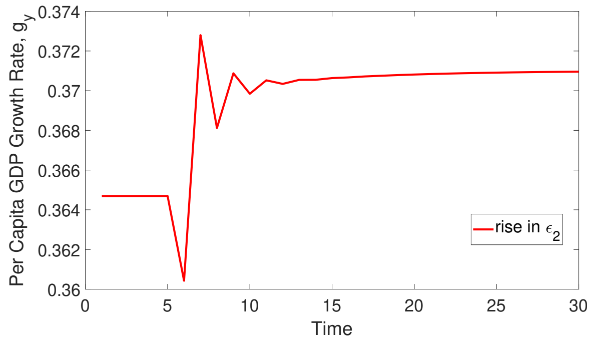
(c) A rise in μ_1 on per capita GDP growth



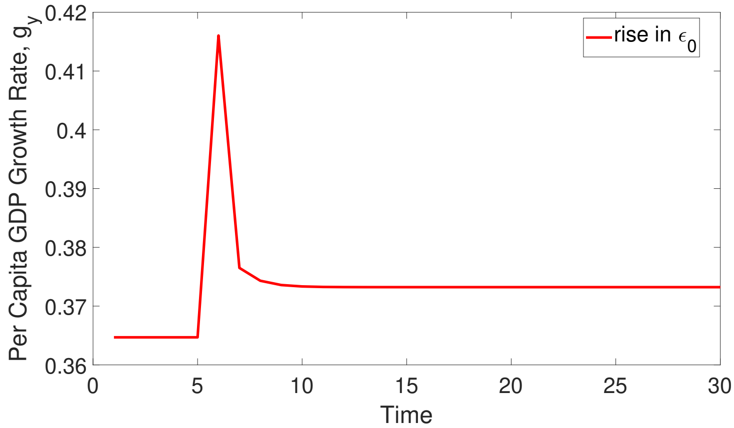
(d) A rise in μ_2 on per capita GDP growth



(e) A rise in ε_1 on per capita GDP



(f) A rise in ε_2 on per capita GDP growth



(g) A rise in ε_0 on per capita GDP growth

Figure 5: Effect of rises in ϕ_1, ϕ_2, μ_1, μ_2, ε_1, ε_2 and ε_0 by 5 percentage points on per capita GDP growth

research. This slows down the expansion of final goods (see Figure 3h), while accelerating the evolution of basic knowledge (see Figure 3b). Because basic knowledge is a necessary input for applied research, the accumulation of new blueprints accelerates in the short as well as in the medium run. In the short and medium run, the physical capital stock grows faster than if no policy changes were enacted (see Figure 3h). Economic growth in terms of per capita GDP slows down in the short run, whereas growth in per capita GDP picks up in the medium run (see Figure 3f). Again, there is no growth effect in the long run.

The effects of an increase in τ_0 on the levels of the different variables are shown in Figures 3a, 3c, 3e, and 3g. The solid (red) line illustrates an economy that experienced a rise in τ_0, whereas the dashed (blue) line depicts an economy that did not experience such an increase. The level of basic knowledge (see Figure 3a), the number of patents (Figure 3c), aggregate output and the capital stock (Figure 3g), and per capita GDP (Figure 3e) are all higher in the economy that witnessed a change in the composition of public expenditure in favor of basic research.

Figures 4b and 4a display the impact of an increase in the value of the weight of children's education in the parental utility function (θ) by 1% after the fifth period on the per capita GDP growth rate and on the level of per capita GDP. We observe that, after the increase in θ, the economy exhibits a higher growth rate as compared to no change. Similarly, an increase in the value of the weight of children's health in the parental utility function (σ) leads to a rise in the per capita GDP growth rate (see Figure 4d). By contrast, an increase in the desire for a large family (ξ) leads to a fall in the growth rate and the level of per capita GDP as shown in Figures 4f and 4e . These are exactly the effects we stated in Proposition 2 *part (vii)* and they are also consistent with the empirical evidence (Brander and Dowrick, 1994; Ahituv, 2001; Li and Zhang, 2007; Cohen and Soto, 2007; Hanushek and Woessmann, 2012; Herzer et al., 2012).

Figure 5 illustrates the impact of increases in the value of intertemporal and intersectoral knowledge spillovers ϕ_1, ϕ_2, μ_1, μ_2, and the strength of the effect of healthcare in enhancing the productivity of workers employed in the final goods sector (ε_0), applied research sector (ε_1), and the basic research sector (ε_2) by 5 percentage points on the per capita GDP growth rate. The results confirm the analytical findings expressed in Proposition 2 *part (vii)*.

Next, we focus on the welfare effect of the policy change. Our objective is to determine whether a tax policy change leads to an improvement in welfare. If it does, we also aim to identify the optimal tax rate that maximizes welfare over a specific time period. Individuals in our analysis value both present and future consumption, the number of children they have, as well as the education level and the health status of each child (see Equation (1)). As a result, we assess the effects of changes in basic research and healthcare expenditures on utility levels over

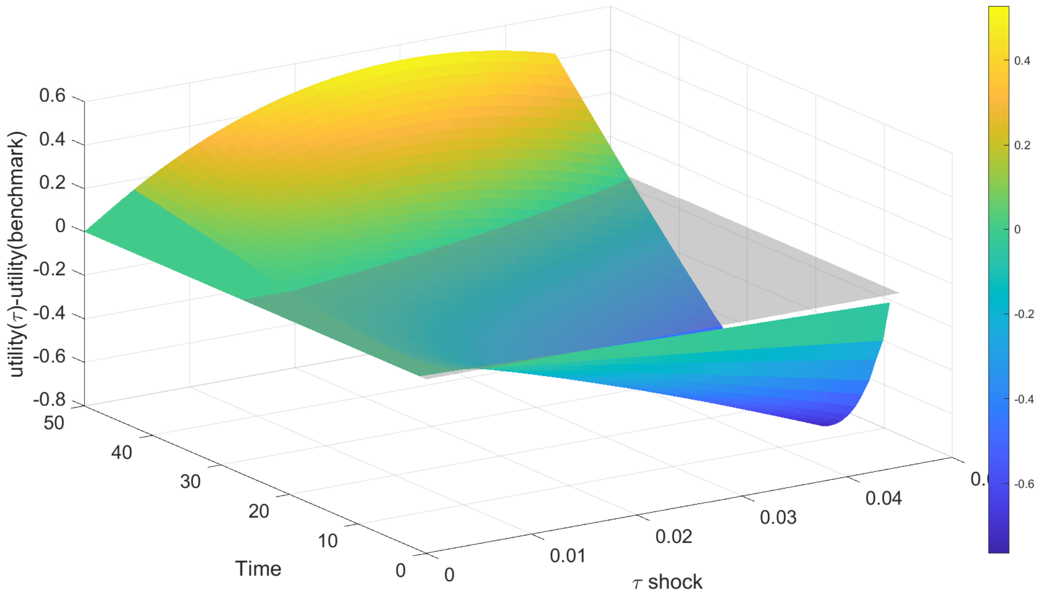


Figure 6: Changes in lifetime utility for changes in τ over different time horizons (x-axis) and different changes in τ (y-axis)

Note: The figure displays the difference in aggregate utility levels between inhabitants of an economy that changes its public policies related to basic research and healthcare and inhabitants of an economy without such a change. The time horizon is displayed on the x-axis, while the change in τ is displayed on the y-axis. If the difference is positive, the inhabitants of the economy with the corresponding change in τ are better off in the relevant time period (because the social welfare is higher). The shaded plane corresponds to the case in which inhabitants of both economies are equally well off, that is, the difference equals zero.

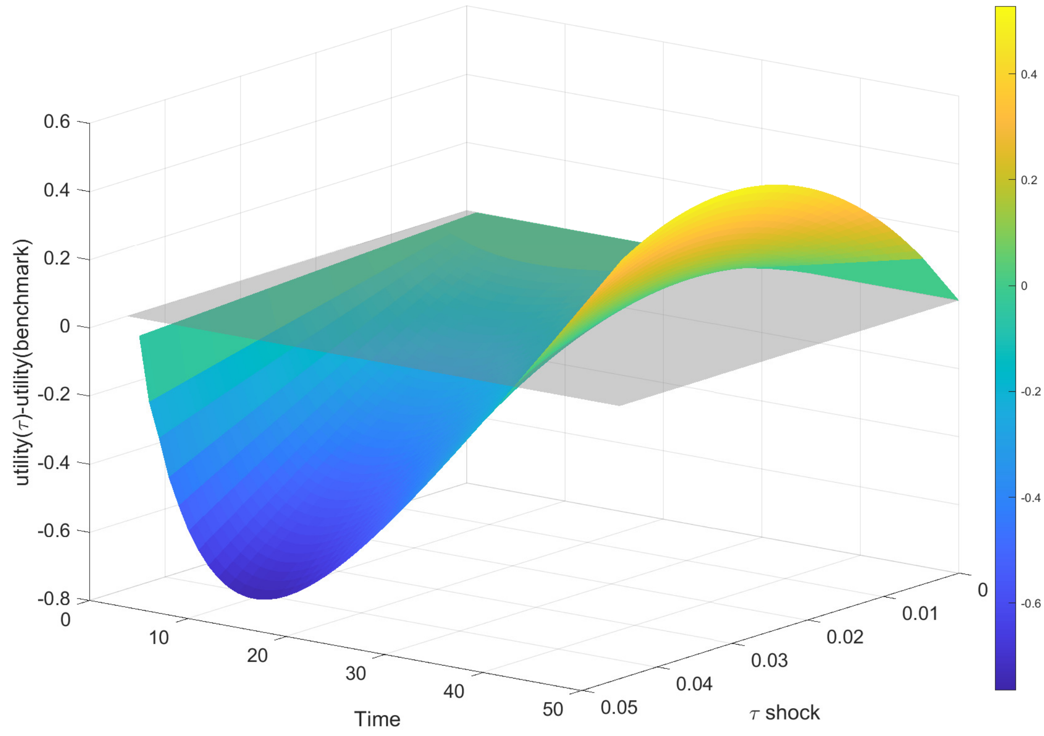


Figure 7: Figure 6 from a different angle

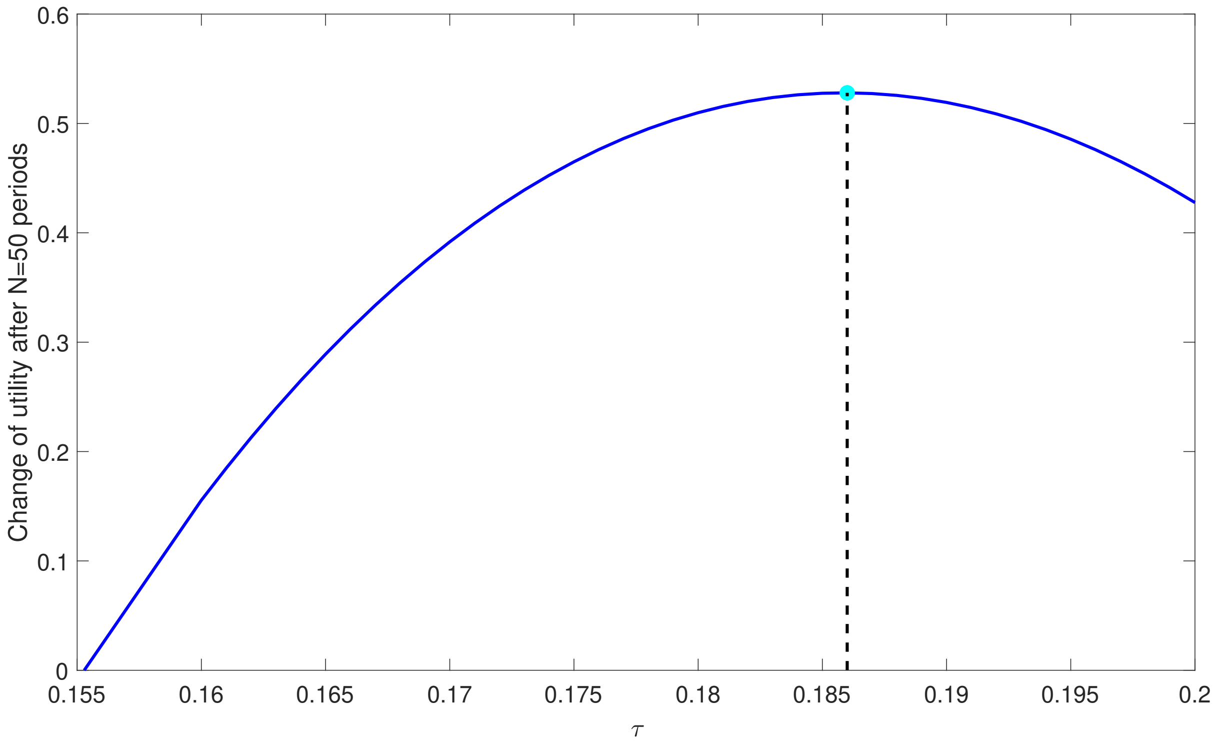


Figure 8: Changes in lifetime utility for changes in τ for the time horizon $N = 50$

different time horizons, and various changes in these public expenditures (τ). The difference in the overall utility level between the inhabitants of an economy that implements changes in its basic research and healthcare policies and the inhabitants of an economy that does not make such changes (i.e., $\Delta\tau = 0$) is depicted in Figure 6 (and also in Figure 7 from a different angle). Aggregate utility is calculated by summing up the average lifetime utilities until time horizon N according to

$$U_N = \sum_{j=1}^{N} \lambda^{j-1} u_j(c_j, c_{j+1}, e, n, m), \tag{31}$$

where $\lambda = 1/(1+\rho^s)$ represents the social discount factor with ρ^s being the social discount rate and $u_j(c_j, c_{j+1}, e, n, m)$ represents the value of life divided by the consumption level in our case of a logarithmic utility function (cf. Hall and Jones, 2007).[7]

We follow the standard Millian type of welfare measure approach, where the average individual determines social well-being. Note, however, that the positive welfare effect of fertility and, thus, population growth is still accounted for because n shows up in Equation (31) as one of the determinants of individual utility. If we used the Benthamite type of welfare measure instead, Equation (31) would still apply but the social discount factor would change to $\lambda = 1/(1+\rho^s - n)$. In addition, this social discount factor would now be different between the baseline scenario and the scenario with the parameter change. There would be an additional beneficial effect of population growth because it increases the population size over time and, thus, aggregate utility (as compared to average utility in the Millian approach). The effect would be that all policy measures that reduce fertility get an additional "punishment" in terms of utility and, thus, they would become less worthwhile from a welfare perspective. In our case, it would mean that optimal expenditures for health and basic research decrease.

In Figure 6, the time horizon is represented on the X-axis. Initially (prior to $N = 0$), the economy progresses along a balanced growth path, and at $N = 0$, it encounters a policy change (τ), the magnitude of which is depicted on the Y-axis. The corresponding change in overall utility, compared to the benchmark scenario without a policy change, is shown on the Z-axis. The graph illustrates that an increase in government expenditure on basic research and public healthcare initially leads to a decline in welfare in the immediate years following the impact. This is because of the increase in taxes that reduce consumption and the fact that healthcare and basic research both attract workers from the other sectors such as final goods production, which decreases in the short run (in period t). However, increases in government expenditures on basic research and public healthcare enhance economic growth from period $t+1$

[7] The value of life is given by $u_j(c_j, c_{j+1}, e, n, m)/u_j'(c_j, c_{j+1}, e, n, m) = c_j \cdot u_j(c_j, c_{j+1}, e, n, m)$, which is the Millian utility function multiplied by the consumption level in the case of logarithmic utility.

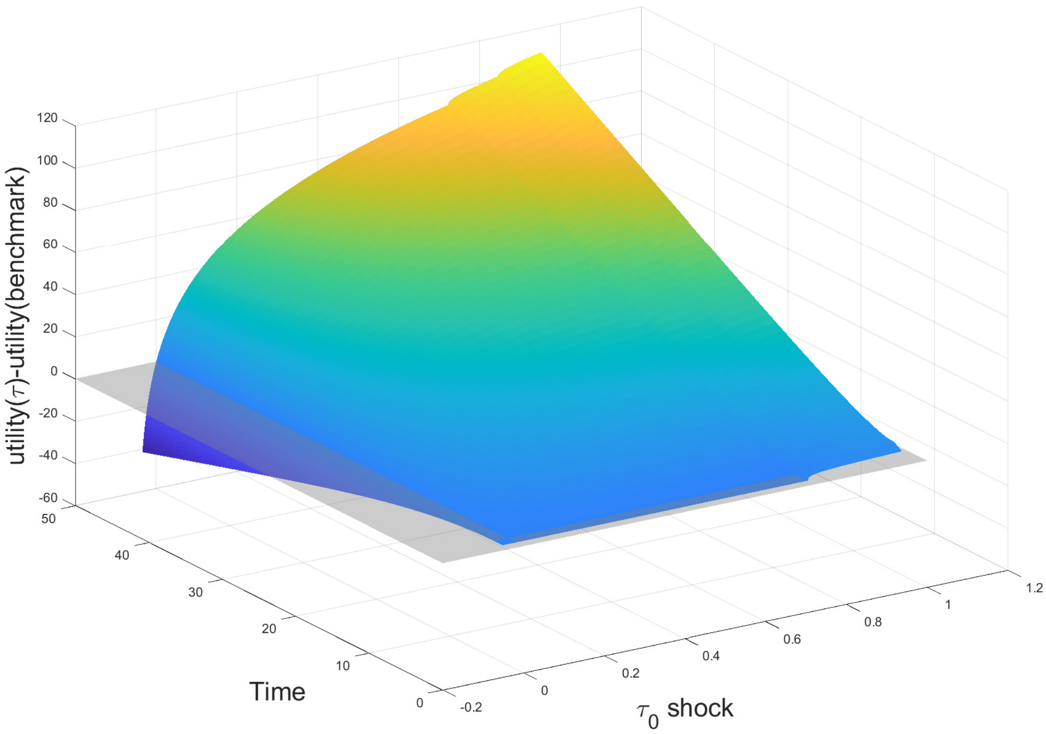


Figure 9: Changes in lifetime utility for changes in τ_0 for the optimal level of τ over different time horizons (x-axis) and different changes in τ_0 (y-axis)

Note: The figure displays the difference in aggregate utility levels between inhabitants of an economy that changes its public policies related to basic research and healthcare and inhabitants of an economy without such a change. The time horizon is displayed on the x-axis, while the change in τ_0 is displayed on the y-axis. If the difference is positive, the inhabitants of the economy with the corresponding change in τ_0 are better off in the relevant time period (because the social welfare is higher). The shaded plane corresponds to the case in which inhabitants of both economies are equally well off, that is, the difference equals zero. Note that for the optimal size of the budget (i.e., for the optimal level of τ), a rise in τ_0 represents a higher level of investment by the government in basic research at the cost of healthcare.

onwards, which raises long-run welfare levels (cf. Prettner and Werner, 2016; Gersbach et al., 2018; Gersbach and Komarov, 2020). After 50 generations, the welfare levels show a positive response to a slight increase in public healthcare and basic research expenditures. However, this positive response turns negative once a certain level of public spending is reached. This suggests that for each increase in τ and for each time horizon, an optimal rate of public spending (i.e., τ_{optimal}) exists that maximizes welfare. According to our results, maximum welfare after 50 generations will be achieved by raising τ to approximately 18.6%, which corresponds to 12.4% of GDP (see Figure 8). This level of public spending on both basic research and healthcare surpasses to a substantial degree the current expenditure levels in the specified OECD countries corresponding to approximately 10.35% of GDP in 2019.

Finally, we examine the effect that a change in the composition of government expenditures

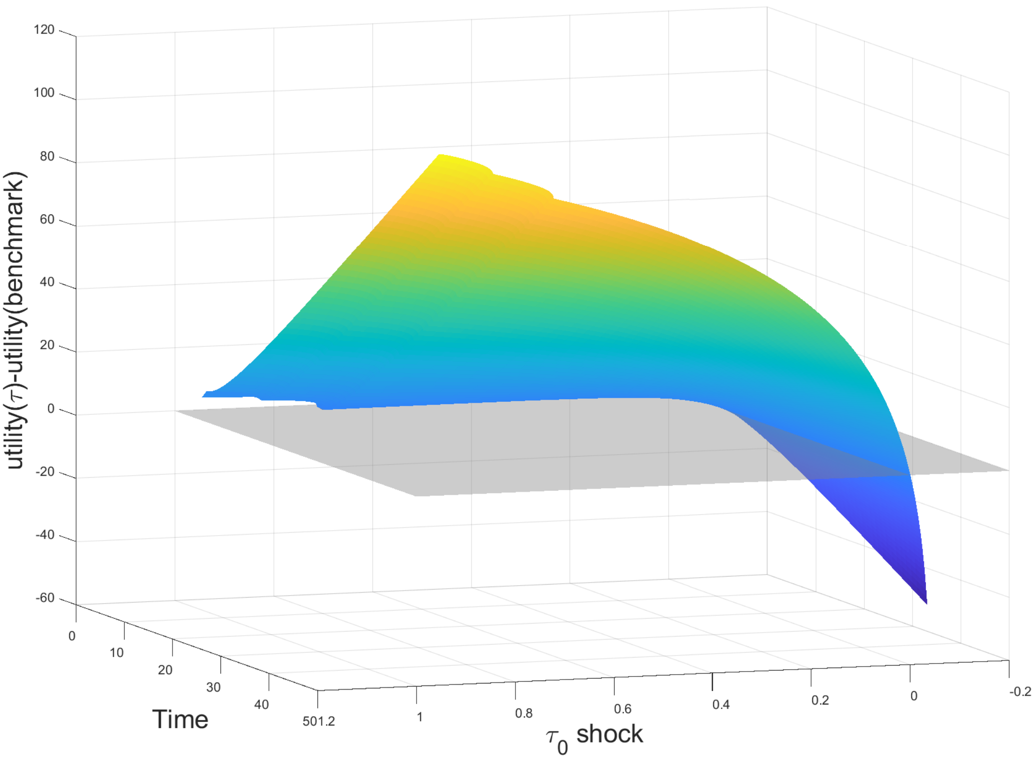


Figure 10: Figure 9 from a different angle

has on welfare. Figure 9 shows that when the government selects the optimal size of the budget (i.e., when τ equals 18.6%, which corresponds to 12.4% of GDP), welfare increases if the level of investment by the government in basic research rises at the cost of government healthcare expenditure. As Figure 11 suggests, there is no interior level of τ_0 for which welfare would be maximized. Of course, before we can jump to any policy conclusions on this aspect, it needs to be cautioned that basic research enhances economic growth directly in our model, while healthcare has only an indirect effect through its impact on enhancing the productivity of workers in different sectors. Moreover, we have not considered longevity increases related to healthcare investments and the associated effects on welfare. Prior research has shown that this effect typically dwarfs the welfare effect of healthcare that is due to rising economic growth (Kuhn and Prettner, 2016). Therefore, incorporating endogenous longevity is an important aspect for further research. However, doing so will increase the complexity of the model considerably.

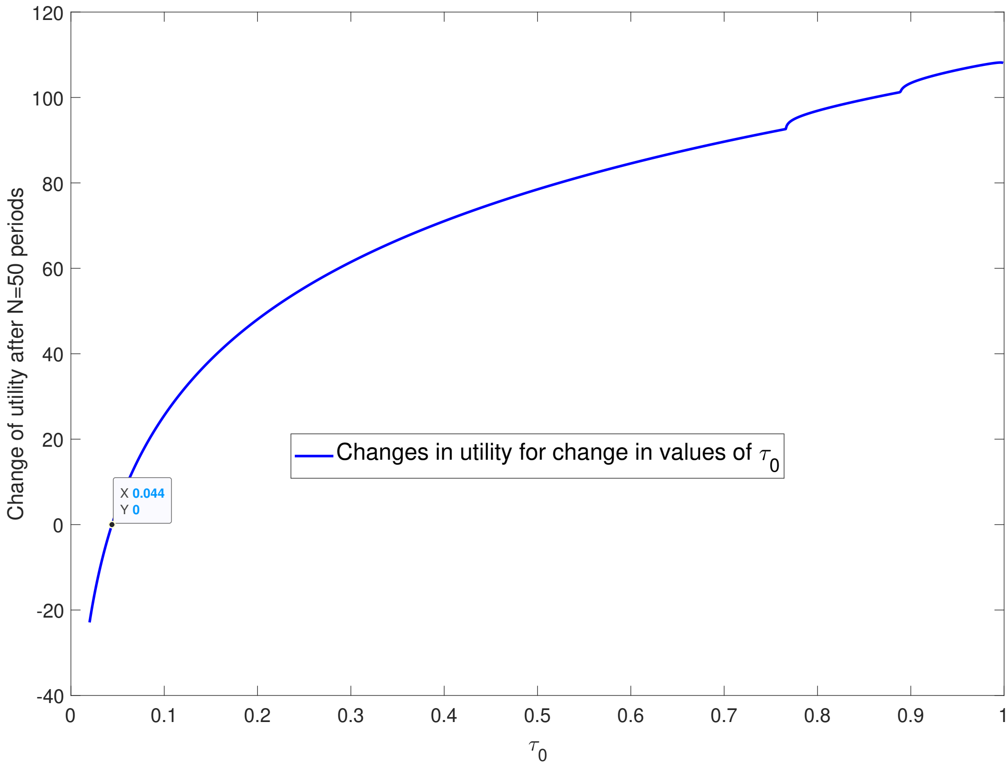


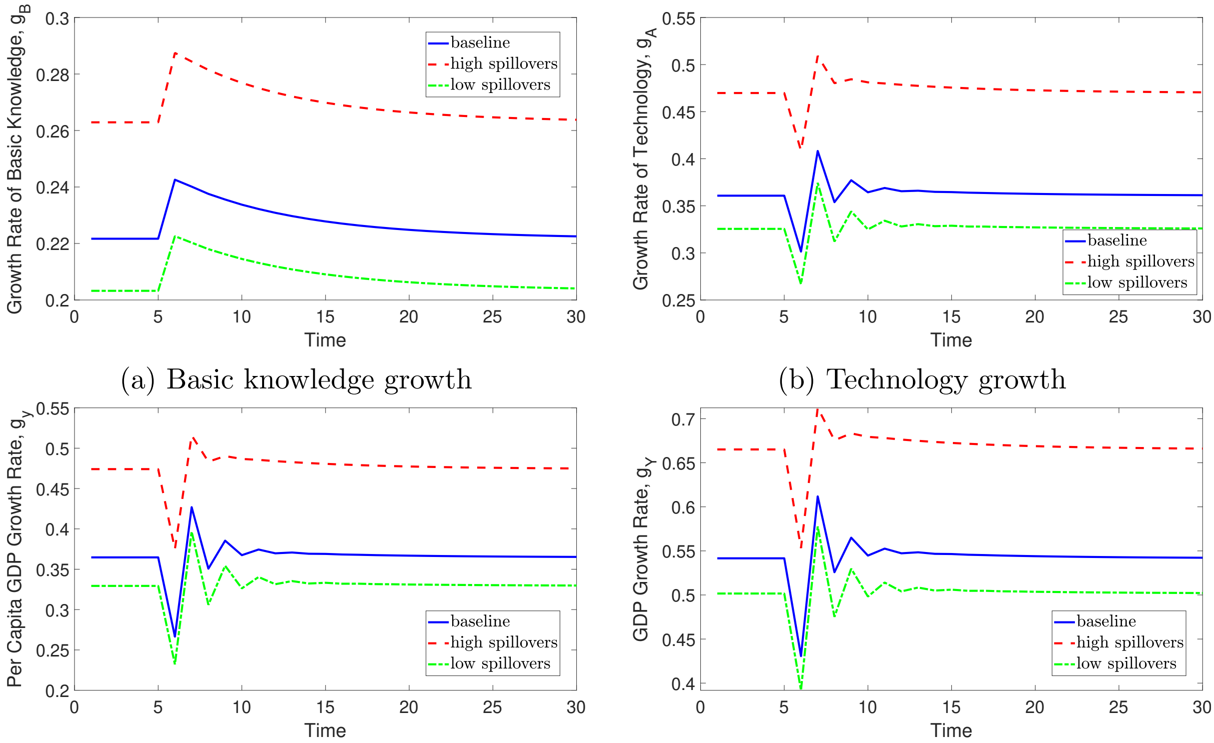
Figure 11: Changes in lifetime utility for changes in τ_0 for the time horizon $N = 50$ while τ is at the optimal level

5 Sensitivity analysis

5.1 Sensitivity analysis with respect to parameter changes

As previously indicated, the parameter settings for the illustrative simulation were chosen to represent either observed data from OECD countries from 2000 to 2019 or common sense derived from empirical findings, as they are frequently used in other research. However, as stated in Prettner and Werner (2016), measuring intertemporal and intersectoral knowledge spillovers is exceptionally difficult. Most research calibrates the relevant parameters so that the model's projected growth series match the observed ones, which is the approach that we followed here as well. In this section, our purpose is to demonstrate that our qualitative findings are fairly robust to changes in the spillover parameters ϕ_1, ϕ_2, μ_1, and μ_2. In doing so, we simulate growth in basic knowledge, applied knowledge, GDP, and per capita GDP in three alternative scenarios: (i) a low spillover scenario with $\phi_1 = 0.38$, $\phi_2 = 0.03$, $\mu_1 = 0.27$, and $\mu_2 = 0.28$; (ii) a high spillover scenario with $\phi_1 = 0.47$, $\phi_2 = 0.07$, $\mu_1 = 0.34$, and $\mu_2 = 0.35$; and (iii) an intermediate spillover scenario corresponding to our baseline specification with $\phi_1 = 0.40$, $\phi_2 = 0.05$, $\mu_1 = 0.30$, and $\mu_2 = 0.30$. Figure 12 depicts the growth effects, with the red dashed line indicating the high spillover scenario, the blue solid line representing the intermediate spillover scenario, and the green dash-dotted line reflecting the low spillover scenario. In summary, boosting public spending on basic research and healthcare (an increase in τ) results in a medium-run rise in the growth rates of basic knowledge, applied knowledge, per capita GDP, and aggregate GDP (or physical capital), a short-run slowdown in the growth rates of these variables, and no long-run growth effects. Thus, our findings related to growth rates remain valid across all of these specifications.

In addition, we examine how changes in spillovers affect the sensitivity of welfare. We observe an interior optimal level of public expenditure on basic research and healthcare (τ) for each generation irrespective of whether the intersectoral and intertemporal knowledge spillovers are low, high, or at the intermediate level. Nevertheless, the optimal public expenditures on basic research and healthcare (τ) are sensitive to changes in the intersectoral and intertemporal knowledge spillovers. Figure 13 exhibits the effects of increases in aggregate utility for low spillovers (green dash-dotted line), intermediate spillovers (blue solid line), and high spillovers (red dashed line) up to generation $N = 50$. The peak of the change in utility concerning a change in τ increases with an increase in spillovers. In particular, for the low spillover scenario, the peak occurs at $\tau = 0.162$, which corresponds to 10.8% of GDP, and for the high spillover case, the maximum occurs at $\tau = 0.231$, which corresponds to 15.4% of GDP. Nonetheless, the



(a) Basic knowledge growth

(b) Technology growth

(c) Growth of GDP per capita

(d) GDP growth

Figure 12: Sensitivity check with respect to intertemporal and intersectoral knowledge spillovers

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

optimal public expenditure on basic research and healthcare (τ_{optimal}) from the point of view of future generations is still higher in all three scenarios than the levels presently observed in the above-specified OECD countries.

	θ_{Low} 0.38	$\theta_{\text{Intermediate}}$ 0.4	θ_{High} 0.417	σ_{Low} 0.28	$\sigma_{\text{Intermediate}}$ 0.3	σ_{High} 0.37
τ_{optimal}	0.178	0.186	0.189	0.174	0.186	0.190

Table 2: Sensitivity of the long run optimal public expenditure rate on basic research and healthcare (i.e., τ) with respect to the preferences of households

Similarly, we examine the sensitivity of welfare to changes in the weight of children's education (θ) and health (σ) in the parental utility function. We set higher and lower bounds of these values according to Table 2 in which intermediate values correspond to our benchmark specification. We observe an interior welfare-maximizing level of τ for each generation irrespective of whether θ and σ are low, high, or at the intermediate level. However, the optimal value of τ is again sensitive to changes in the high and low values of θ and σ. Figure 14 exhibits the effects of increases in aggregate utility for a low value of θ (green dash-dotted line), an intermediate

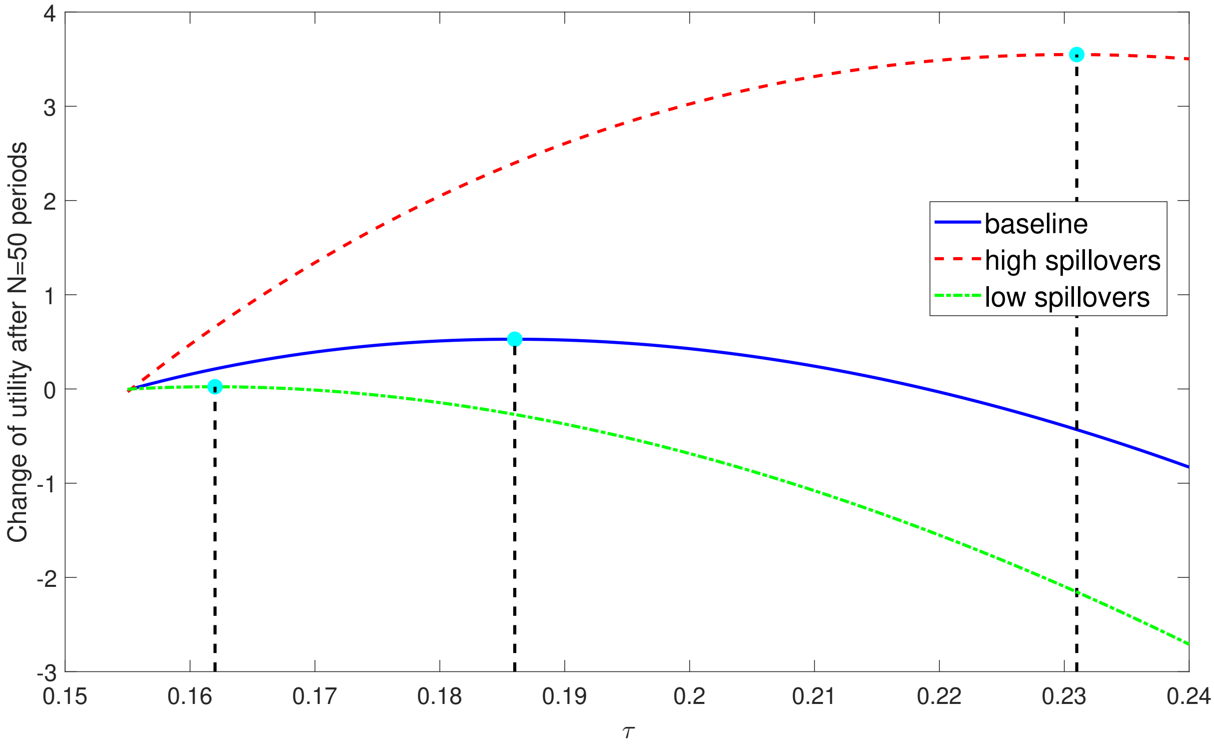


Figure 13: Changes in lifetime utility for changes in τ for the time horizon $N = 50$ and changing ϕ_1, ϕ_2, μ_1, and μ_2

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

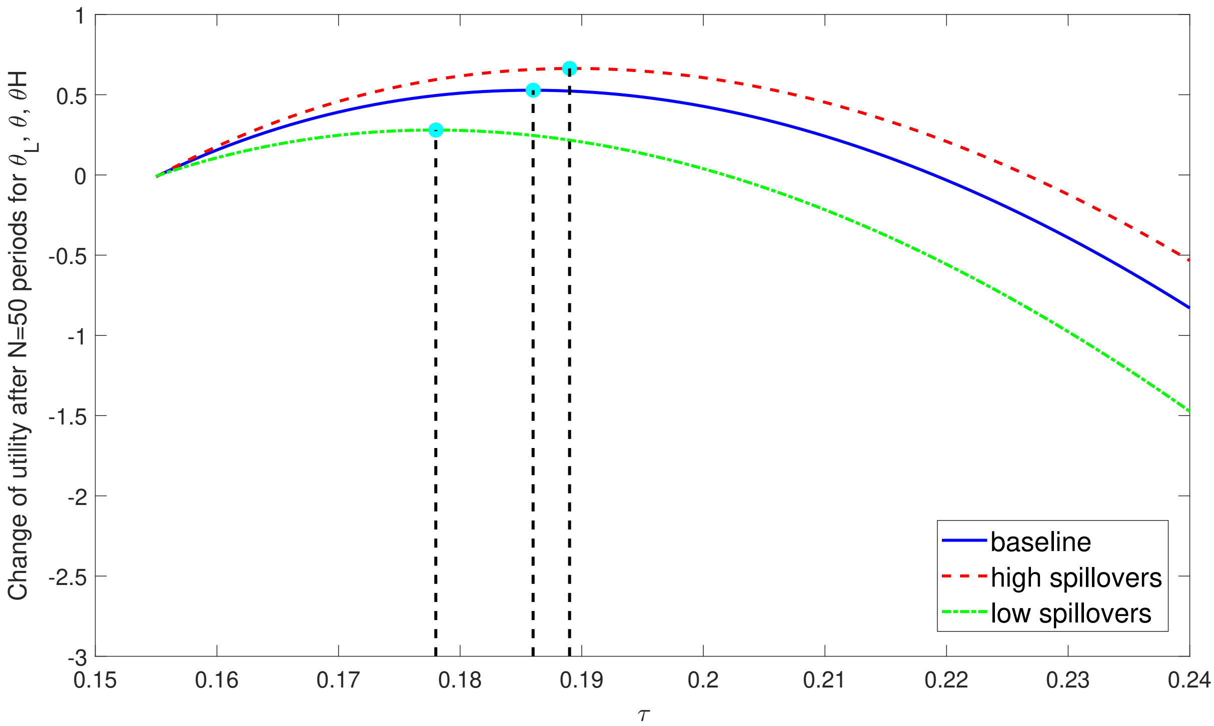


Figure 14: Changes in lifetime utility for changes in τ for the time horizon $N = 50$ and changing θ

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

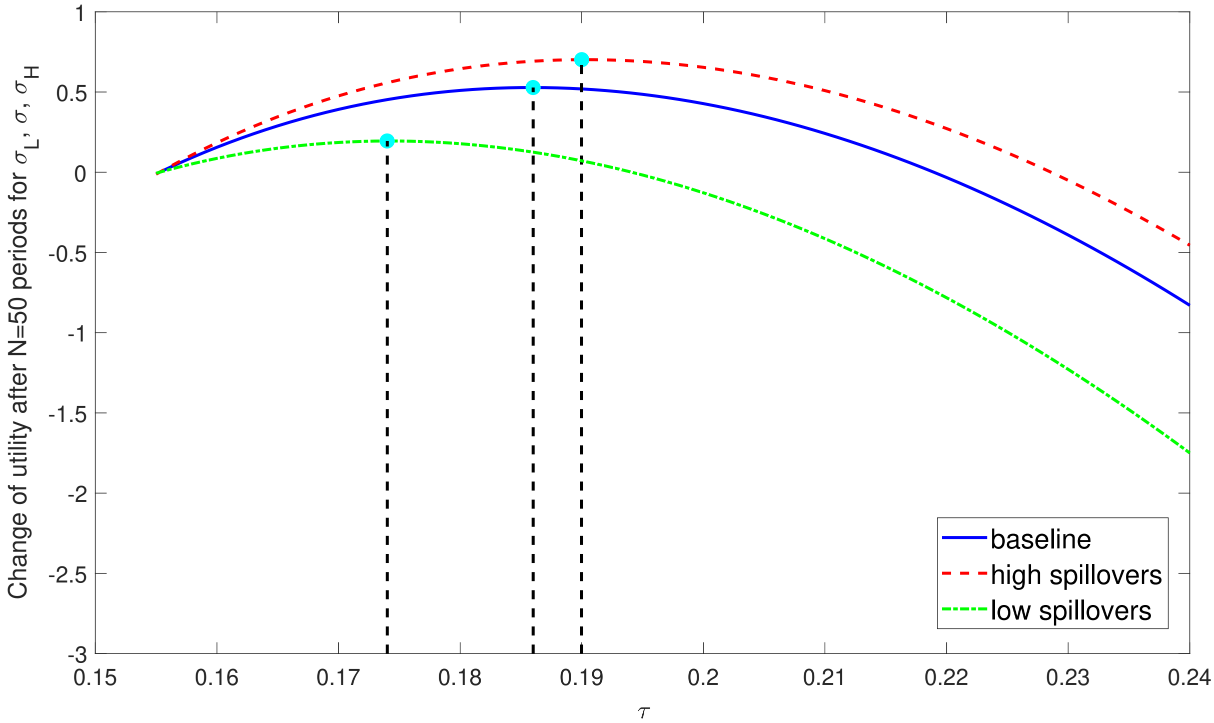


Figure 15: Changes in lifetime utility for changes in τ for the time horizon $N = 50$ and changing σ

Note: The solid (blue) line refers to the intermediate spillover scenario, the dashed (red) line refers to the high spillover scenario, and the dash-dotted (green) line refers to the low spillover scenario.

value of θ (blue solid line), and a high value of θ (red dashed line) up to generation $N = 50$.

As the value of θ increases, so does the peak of the change in utility due to a change in τ. Specifically, for the low value of θ ($\theta_{\text{Low}} = 0.38$), the peak occurs at $\tau = 0.178$, which corresponds to 11.87% of GDP, and for the high value of θ ($\theta_{\text{High}} = 0.417$), the peak occurs at $\tau = 0.189$, which corresponds to 12.6% of GDP. Figure 15 exhibits the effects of increases in aggregate utility for a low value of σ (green dash-dotted line), an intermediate value of σ (blue solid line), and a high value of σ (red dashed line) up to generation $N = 50$. As is evident from Figure 15, the change in utility due to a change in τ rises as the value of σ grows. For instance, for the low value of σ ($\sigma_{\text{Low}} = 0.28$), the peak occurs at $\tau = 0.174$, which corresponds to 11.6% of GDP, while for the high value of σ ($\sigma_{\text{High}} = 0.317$), the peak occurs at $\tau = 0.190$, which corresponds to 12.67% of GDP.

5.2 Effects of relaxing central assumptions

We had to apply a number of assumptions and modeling choices to keep the model tractable and accessible for the reader. In this subsection, we discuss some central assumptions and how the results would change if we relax or modify them. First of all, we abstracted from endogenous life expectancy depending on private and public healthcare. Including these important aspects would complicate the model substantially but is definitely a worthwhile avenue for further research. Endogenizing life expectancy would raise the welfare effects of health spending as prior literature has shown (cf. Hall and Jones, 2007; Kuhn and Prettner, 2016; Chen et al., 2021; Schneider and Winkler, 2021) and, thus, strengthen the result that current spending levels of the government are too low from a welfare maximizing perspective. With respect to the result that basic research spending promotes welfare by more than healthcare spending, we expect that this result would weaken and, likely, even be overturned.

Another crucial point is the choice of the social welfare function. While our Millian approach includes population growth as a positive component of the value of life and of welfare, it could be argued that a social planner should put even more weight on the number of individuals and, thus, attain the perspective of the Benthamite approach. Doing so would reduce the desirability of spending on healthcare and basic research because they reduce fertility in our model.

Finally, as far as relaxing the assumption of full public funding of basic research versus full private funding of applied research, we would not expect large changes. The reasons are that i) still the largest part of basic research is funded publicly, whereas the opposite holds true for applied research. This means that our assumption is arguably not too far from reality; ii) since basic and applied research are both growth-promoting and come with an investment phase so that welfare would be reduced in the short run, our qualitative findings should be robust to changes in this assumption.

6 Conclusion

We present a highly general R&D-based endogenous growth model emphasizing the role of patentable applied research, publicly-funded basic research, and publicly-funded healthcare. In so doing, we nest two recent contributions by Prettner and Werner (2016) and Baldanzi et al. (2021) as special cases. We use the model to assess the growth and welfare effects of public basic research and public healthcare investments and to illustrate the role of healthcare for enhancing productivity in various sectors of the economy.

Overall, we show that the basic insights of the previous literature remain intact from a qualitative perspective even in a situation in which different growth and welfare promoting areas are in competition regarding public funding. We show that an increase in publicly funded basic research and health expenditures is still welfare improving as compared with the observable spending levels in the OECD. However, as compared with the results of Prettner and Werner

(2016), the quantitative findings are altered to the extent that the welfare-maximizing level of expenditures does not anymore differ from the actual expenditure levels by such a wide margin.

The two main economic policy implications that emanate from our research are that i) government expenditures on basic science and healthcare are worthwhile and still these areas are under-funded from a welfare-maximizing perspective; ii) in the presence of more domains on which the government can spend its funds productively, it is even more important to consider thorough cost-benefit analyses to make sound policy decisions and to understand the various tradeoffs involved.

Interesting avenues for future research include i) endogenizing life expectancy depending on public and private healthcare in a realistic way and assessing the extent to which the welfare implications for the split between health expenditures and basic research expenditures changes; ii) extending the proposed model to the context of a developing country that is far from the technological frontier and imitates innovations that were made in developed countries. Presumably, investments in basic research would not be similarly important in such a setting and health expenditures would become more prominent; iii) designing a model in which governments have even more scope for enacting different policies such as childcare subsidies, education subsidies, research subsidies (see, e.g., Minniti and Venturini, 2017a,b), etc., and using the model to analyze the extent to which different policies contribute to increase welfare; iv) modeling more explicitly the interactions between private and public healthcare and the complementarities and tradeoffs that they imply; v) modeling in more detail the interaction between basic and applied research and acknowledging that a part of applied research is carried out publicly funded and a part of basic research is carried out privately funded. While we do not expect that this leads to strong changes in our qualitative and quantitative findings on the growth effects of public healthcare and public basic research policies, doing so may lead to additional insights that are valuable for innovation research.

Statements and Declarations

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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Appendix

A Derivation of the optimal values for c_t, s_t, n_t, e_t, and m_t

Using (1) and (2), we set up the Lagrangian as

$$\begin{aligned}\mathcal{L} = &\ln c_t + \beta \ln\left[(R_{t+1} - 1)s_t\right] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t \\ &+ \lambda[(1-\tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t)w_t h_t - c_t - s_t].\end{aligned}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \implies \frac{1}{c_t} = \lambda, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = 0 \implies \frac{\beta}{s_t} = \lambda, \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \implies \frac{\xi}{n_t} = \lambda(1-\tau)(\psi + \eta e_t + \chi m_t)w_t h_t, \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = 0 \implies \frac{\theta}{e_t} = \lambda(1-\tau)\eta n_t w_t h_t, \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = 0 \implies \frac{\sigma}{m_t} = \lambda(1-\tau)\chi n_t w_t h_t, \tag{A.5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \implies (1-\tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t)w_t h_t - c_t - s_t = 0. \tag{A.6}$$

Dividing (A.4) by (A.5), we obtain

$$\chi m_t = \frac{\eta\sigma}{\theta} e_t. \tag{A.7}$$

Dividing (A.1) by (A.2), we obtain

$$s_t = \beta c_t. \tag{A.8}$$

Dividing (A.3) by (A.4) and using (A.7) yields

$$e_t = \frac{\theta\psi}{\eta(\xi - \theta - \sigma)}. \tag{A.9}$$

Inserting the value of e_t into (A.7), we get

$$m_t = \frac{\sigma\psi}{\chi(\xi - \theta - \sigma)}. \tag{A.10}$$

Dividing (A.3) by (A.1) and rearranging yields

$$(1-\tau)(\psi + \eta e_t + \chi m_t) n_t w_t h_t = \xi c_t. \tag{A.11}$$

Inserting (A.11) into (A.6), we get

$$(1-\tau)w_th_t-(1-\tau)(\psi+\eta e_t+\chi m_t)n_tw_th_t-c_t-s_t=0$$

$$\implies c_t=\frac{(1-\tau)w_th_t}{1+\beta+\xi}. \tag{A.12}$$

Therefore,

$$s_t=\frac{\beta(1-\tau)w_th_t}{1+\beta+\xi}. \tag{A.13}$$

Inserting the values of c_t, s_t, e_t, and m_t into (A.6) and rearranging, we obtain

$$n_t=\frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}. \tag{A.14}$$

B Proof of Proposition 1

The partial derivatives of fertility n_t with respect to ξ, θ, and σ are

$$\begin{aligned}&\frac{\partial n_t}{\partial \xi}=\frac{1+\beta+\theta+\sigma}{\psi\,(1+\beta+\xi)^2}>0, \qquad \frac{\partial n_t}{\partial \theta}=-\frac{1}{\psi\,(1+\beta+\xi)}<0,\\ &\frac{\partial n_t}{\partial \sigma}=-\frac{1}{\psi\,(1+\beta+\xi)}<0.\end{aligned} \tag{B.1}$$

The partial derivatives of individual human capital h_{t+1} with respect to ξ, θ, and σ are

$$\begin{aligned}\frac{\partial h_{t+1}}{\partial \xi}&=-\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\frac{\psi}{(\xi-\theta-\sigma)^2}h_t<0,\\ \frac{\partial h_{t+1}}{\partial \theta}&=\frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi h_t}{(\xi-\theta-\sigma)}\left[\frac{\nu A_E}{\eta}\left(A_E\frac{\theta}{\eta}\right)^{-1}+\frac{1}{(\xi-\theta-\sigma)}\right]>0,\\ \frac{\partial h_{t+1}}{\partial \sigma}&=\frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi h_t}{(\xi-\theta-\sigma)}\left[\frac{(1-\nu)A_M}{\chi}\left(A_M\frac{\sigma}{\chi}\right)^{-1}+\frac{1}{(\xi-\theta-\sigma)}\right]>0.\end{aligned} \tag{B.2}$$

C Proof of Proposition 2

(i) Growth rates of the endogenous variables have to be constant along the balanced growth path. Thus,

$$\frac{g_{t+1,A}-g_{t,A}}{g_{t,A}}=0 \implies g_{t+1,A}=g_{t,A}$$

$$\implies \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1A_{t+1}^{\phi_1-1}B_{t+1}^{\mu_1}(h_{t+1}L_{t+1})^{1+\varepsilon_1}-\frac{1}{\alpha}$$

$$=\frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1A_t^{\phi_1-1}B_t^{\mu_1}(h_tL_t)^{1+\varepsilon_1}-\frac{1}{\alpha}$$

$$\implies \left(\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t}\right)^{1+\varepsilon_1}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_1-1}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_1}=1$$

$$\Omega^{1+\varepsilon_1}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_1-1}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_1}=1. \tag{C.1}$$

$$\frac{g_{t+1,B}-g_{t,B}}{g_{t,B}} = 0 \implies g_{t,B} = g_{t-1,B}$$

$$\implies \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} B_{t+1}^{\mu_2-1} A_{t+1}^{\phi_2} (h_{t+1}L_{t+1})^{1+\varepsilon_2}$$

$$= \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} B_t^{\mu_2-1} A_t^{\phi_2} (h_t L_t)^{1+\varepsilon_2}$$

$$\implies \left(\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t}\right)^{1+\varepsilon_2}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_2}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_2-1} = 1$$

$$\left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\frac{1+\varepsilon_2}{1-\mu_2}}\left(\frac{A_{t+1}}{A_t}\right)^{\frac{\phi_2}{1-\mu_2}}. \tag{C.2}$$

Inserting Equation (C.2) into Equation (C.1), we obtain

$$\tilde{A} \equiv \left(\frac{A_{t+1}}{A_t}\right) = \Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}} = \Omega^{\Psi_1}. \tag{C.3}$$

Therefore,

$$\tilde{B} \equiv \left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}} = \Omega^{\Psi_2}. \tag{C.4}$$

Along the balanced growth path, Y_t and K_t must grow at the same rate. Thus, Equation (11) suggests

$$\tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t}\right) = \left(\frac{A_{t+1}}{A_t}\right)^{2-\phi_1}\left(\frac{B_{t+1}}{B_t}\right)^{-\mu_1}\left(\frac{H_{t+1,M}}{H_{t,M}}\right)^{\varepsilon_0-\varepsilon_1} = \left(\frac{K_{t+1}}{K_t}\right) \equiv \tilde{K} \tag{C.5}$$

$$\implies \left(\frac{Y_{t+1}}{Y_t}\right) \equiv \tilde{Y} = \tilde{K} \equiv \left(\frac{K_{t+1}}{K_t}\right) = \Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right]+\varepsilon_0} = \Omega^{\Psi_3+\varepsilon_0}. \tag{C.6}$$

(ii) First, let us focus on the signs of Ψ_1 and Ψ_2. We know from Assumption 1 that $\phi_1 \in [0,1)$, $\phi_2 \in [0,1)$, $\mu_1 \in [0,1)$, and $\mu_2 \in [0,1)$. Furthermore, $\phi_1+\mu_1 < 1$ and $\phi_2+\mu_2 < 1$.

$$\phi_1+\mu_1 < 1 \implies \mu_1 < (1-\phi_1) \tag{C.7}$$

$$\phi_2+\mu_2 < 1 \implies \phi_2 < (1-\mu_2) \tag{C.8}$$

$\phi_1 \in [0,1), \phi_2 \in [0,1), \mu_1 \in [0,1), \mu_2 \in [0,1),$ Equation (C.7) and Equation (C.8) together imply

$$\mu_1\phi_2 < (1-\phi_1)(1-\mu_2)$$

$$\implies [(1-\phi_1)(1-\mu_2)-\mu_1\phi_2] > 0$$

Note that as $(1+\varepsilon_1) > 0$, $(1-\mu_2) > 0$ and $(1+\varepsilon_2)\mu_1 \geq 0$, we get $[(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1] > 0$, and as $(1+\varepsilon_2) > 0$, $(1-\phi_1) > 0$ and $(1+\varepsilon_1)\phi_2 \geq 0$, we get $[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2] > 0$. Consequently,

$$\Psi_1 = \frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} > 0$$

$$\text{and } \Psi_2 = \frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} > 0.$$

As we assumed in the paper that $\Omega > 1$ (see page 15), therefore $\ln(\Omega) > 0$. Consequently, we get the following results.

$$\frac{\partial \tilde{A}}{\partial \mu_1} = \frac{\overbrace{(1-\mu_2)}^{>0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{A}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{B}}{\partial \mu_1} = \frac{\overbrace{\phi_2}^{\geq 0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{B}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{K}}{\partial \mu_1} = \frac{\overbrace{(1-\mu_2)}^{>0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{K}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{A}}{\partial \mu_2} = \frac{\overbrace{\mu_1}^{\geq 0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{A}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{B}}{\partial \mu_2} = \frac{\overbrace{(1-\phi_1)}^{>0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{B}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{K}}{\partial \mu_2} = \frac{\overbrace{\mu_1}^{\geq 0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{K}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{A}}{\partial \phi_1} = \frac{\overbrace{(1-\mu_2)}^{>0}\overbrace{[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{A}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{B}}{\partial \phi_1} = \frac{\overbrace{\phi_2}^{\geq 0}\overbrace{[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{B}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{K}}{\partial \phi_1} = \frac{\overbrace{(1-\mu_2)}^{>0}\overbrace{[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{K}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{A}}{\partial \phi_2} = \frac{\overbrace{\mu_1}^{\geq 0}\overbrace{[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{A}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{B}}{\partial \phi_2} = \frac{\overbrace{(1-\phi_1)}^{>0}\overbrace{[(1+\varepsilon_2)\mu_1 + (1+\varepsilon_1)(1-\mu_2)]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]^2}_{>0}} \overbrace{\tilde{B}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{K}}{\partial \phi_2} = \frac{\overbrace{\mu_1}^{\geq 0}\overbrace{[(1+\varepsilon_2)\mu_1 + (1+\varepsilon_1)(1-\mu_2)]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]^2}_{>0}} \overbrace{\tilde{K}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_0} = 0, \qquad \frac{\partial \tilde{B}}{\partial \varepsilon_0} = 0, \qquad \frac{\partial \tilde{K}}{\partial \varepsilon_0} = \tilde{K}\ln(\Omega) > 0,$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_1} = \frac{\overbrace{(1-\mu_2)}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]}_{>0}} \overbrace{\tilde{A}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{B}}{\partial \varepsilon_1} = \frac{\overbrace{\phi_2}^{\geq 0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]}_{>0}} \overbrace{\tilde{B}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{K}}{\partial \varepsilon_1} = \frac{\overbrace{(1-\mu_2)}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]}_{>0}} \overbrace{\tilde{K}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_2} = \frac{\overbrace{\mu_1}^{\geq 0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]}_{>0}} \overbrace{\tilde{A}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{B}}{\partial \varepsilon_2} = \frac{\overbrace{(1-\phi_1)}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]}_{>0}} \overbrace{\tilde{B}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{K}}{\partial \varepsilon_2} = \frac{\overbrace{\mu_1}^{\geq 0}}{\underbrace{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]}_{>0}} \overbrace{\tilde{K}\ln(\Omega)}^{>0} \geq 0.$$

(iii)

$$\frac{\partial \tilde{A}}{\partial \tau\tau_0} = \frac{\partial \tilde{B}}{\partial \tau\tau_0} = \frac{\partial \tilde{K}}{\partial \tau\tau_0} = 0,$$

$$\frac{\partial \tilde{A}}{\partial \tau(1-\tau_0)} = \frac{\partial \tilde{B}}{\partial \tau(1-\tau_0)} = \frac{\partial \tilde{K}}{\partial \tau(1-\tau_0)} = 0.$$

(iv)

$$\frac{\partial \tilde{h}}{\partial \xi} = -\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\frac{\psi}{(\xi-\theta-\sigma)^2} < 0,$$

$$\frac{\partial \tilde{h}}{\partial \theta} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \psi}{(\xi - \theta - \sigma)} \left[\frac{\nu A_E}{\eta}\left(A_E \frac{\theta}{\eta}\right)^{-1} + \frac{1}{(\xi - \theta - \sigma)}\right] > 0,$$

$$\frac{\partial \tilde{h}}{\partial \sigma} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \psi}{(\xi - \theta - \sigma)} \left[\frac{(1-\nu) A_M}{\chi}\left(A_M \frac{\sigma}{\chi}\right)^{-1} + \frac{1}{(\xi - \theta - \sigma)}\right] > 0.$$

(v)

$$\frac{\partial \tilde{L}}{\partial \xi} = \frac{1+\beta+\theta+\sigma}{\psi(1+\beta+\xi)^2} > 0; \quad \frac{\partial \tilde{L}}{\partial \theta} = \frac{-1}{\psi(1+\beta+\xi)} < 0; \quad \frac{\partial \tilde{L}}{\partial \sigma} = \frac{-1}{\psi(1+\beta+\xi)} < 0,$$

(vi)

$$\frac{\partial \Omega}{\partial \xi} = -\frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)^2} < 0,$$

$$\frac{\partial \Omega}{\partial \theta} = \frac{\frac{\nu A_E}{\eta}\left(A_E \frac{\theta}{\eta}\right)^{\nu-1} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)} > 0,$$

$$\frac{\partial \Omega}{\partial \sigma} = \frac{\frac{(1-\nu) A_M}{\chi}\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{-\nu}}{(1+\beta+\xi)} > 0.$$

(vii) Let us now focus on the sign of Ψ_3. We know that $\mu_2 \in [0,1)$, $\phi_1 \in [0,1)$, and $\phi_2 \in [0,1)$.

$$\mu_2 \in [0,1) \implies (1-\mu_2) > 0$$

$$\phi_1 \in [0,1) \implies (1-\phi_1) > 0$$

$$\phi_2 \in [0,1) \implies (1-\phi_2) > 0$$

$$(\varepsilon_1 \geq 0) \text{ and } (1-\phi_1) > 0 \implies (2-\phi_1+\varepsilon_1) > 0$$

$$(\varepsilon_2 \geq 0) \text{ and } (1-\phi_2) > 0 \implies (1+\varepsilon_2-\phi_2) > 0$$

Therefore, $[(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)] > 0$. As a result,

$$\Psi_3 = \frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\mu_1\phi_2} > 0$$

Furthermore, we assumed that $\xi > \theta+\sigma$ (see page no. 5). ε_0 is also non-negative by assumption. Consequently,

$$\frac{\partial \tilde{y}}{\partial \xi} = \overbrace{\tilde{y}}^{>0} \overbrace{\left[\frac{-1}{(1+\beta+\xi)}\right]}^{<0} \overbrace{\left[\Psi_3 + \varepsilon_0 + \frac{1+\beta+\theta+\sigma}{\xi-\theta-\sigma}\right]}^{>0} < 0,$$

$$\frac{\partial \tilde{y}}{\partial \theta} = \overbrace{\tilde{y}}^{>0} \left[\overbrace{(\Psi_3+\varepsilon_0)}^{>0}\overbrace{\left(\frac{\nu}{\theta}\right)}^{>0} + \overbrace{\left(\frac{1}{\xi-\theta-\sigma}\right)}^{>0}\right] > 0,$$

$$\frac{\partial \tilde{y}}{\partial \sigma} = \overbrace{\tilde{y}}^{>0} \left[\overbrace{(\Psi_3+\varepsilon_0)}^{>0}\overbrace{\left(\frac{1-\nu}{\sigma}\right)}^{>0} + \overbrace{\left(\frac{1}{\xi-\theta-\sigma}\right)}^{>0}\right] > 0,$$

Note that as $(1+\varepsilon_1) > 0$, $(1-\mu_2) > 0$ and $(1+\varepsilon_2)\mu_1 \geq 0$, we get $[(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1] > 0$, and as $(1+\varepsilon_2) > 0$, $(1-\phi_1) > 0$ and $(1+\varepsilon_1)\phi_2 \geq 0$, we get $[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2] > 0$. Moreover, $(1-\mu_2) > 0$, $\mu_1 \geq 0$, and $[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1] > 0$. Therefore,

$$\frac{\partial \tilde{y}}{\partial \mu_1} = \frac{\overbrace{(1-\mu_2)}^{>0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{y}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{y}}{\partial \mu_2} = \frac{\overbrace{\mu_1}^{\geq 0}\overbrace{[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{y}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{y}}{\partial \phi_1} = \frac{\overbrace{(1-\mu_2)}^{>0}\overbrace{[(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{y}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{y}}{\partial \phi_2} = \frac{\overbrace{\mu_1}^{\geq 0}\overbrace{[(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1]}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]^2}_{>0}}\overbrace{\tilde{y}\ln(\Omega)}^{>0} \geq 0,$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_0} = \tilde{y}\ln(\Omega) > 0,$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_1} = \frac{\overbrace{(1-\mu_2)}^{>0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]}_{>0}}\overbrace{\tilde{y}\ln(\Omega)}^{>0} > 0,$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_2} = \frac{\overbrace{\mu_1}^{\geq 0}}{\underbrace{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]}_{>0}}\overbrace{\tilde{y}\ln(\Omega)}^{>0} \geq 0.$$