## A characterization of the $\alpha_{min}$ -egalitarian rule

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December 8, 2024

## Abstract

This paper provides a characterization of the  $\alpha_{\min}$ -egalitarian rule, first introduced by Giménez-Gómez and Peris (2014), which offers a novel compromise between proportionality and egalitarianism in claims problems. The rule is implemented via a two-step process: an initial egalitarian distribution up to the smallest claim, followed by a proportional allocation of the remaining estate among the residual claims. We characterize this rule using a modified version of the "no-advantageous reallocation" axiom and a "lower-bound" axiom reflecting the principle of sustainability. Additionally, we introduce and characterize the  $\beta_{\min}$ -egalitarian rule, which serves as the dual counterpart to the  $\alpha_{\min}$ -egalitarian rule.

The problem of dividing a limited estate among claimants when the total claims exceed the available estate is known in the literature as a *claims problem*. In this paper, we provide an axiomatic characterization of the  $\alpha_{\min}$ -egalitarian rule (Giménez-Gómez and Peris, 2014), which guarantees each claimant the smallest claim that is *sustainable* (Herrero and Villar, 2002). Any remaining estate is then distributed proportionally. If the smallest claim is not sustainable, the rule divides the estate equally among all claimants. Our characterization of the  $\alpha_{\min}$ -egalitarian rule employs the axioms of *no advantageous reallocation* (Moulin, 1985) and *lower bounds* (Moulin, 2002).

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In bankruptcy problems, a court divides a bankrupt firm's net worth among creditors (O'Neill, 1982; Aumann and Maschler, 1985). Two key rules commonly used are the *proportional rule*, which divides the estate based on contributions, and the *equal division rule*, which allocates it equally among creditors. The proportional rule often leaves little for the lowest claimants when claims are highly unequal, while the equal division rule may violate *claim-boundedness*, a principle ensuring claimants receive no more than their claims. The  $\alpha_{\min}$ -egalitarian rule resolves this tension by prioritizing the lowest claimants while maintaining *claim-boundedness*. Other studies addressing compromises between these rules include Dominguez and Thomson (2006) and Thomson (2015a,b). For a comprehensive survey of claims problems, see Moulin (2002) and Thomson (2019).

We also discuss several practical applications of the  $\alpha_{\min}$ -egalitarian rule. For instance, in the division of fishing or milk quotas, the rule ensures each producer receives a minimum allocation necessary for industry viability. In food rationing contexts, it guarantees a baseline caloric intake for all beneficiaries. Similarly, for drinking water or medical supplies, the rule secures a minimum supply for each region, with the remaining resources distributed proportionally. Another example is the global limitation of  $CO_2$  emissions—a "carbon bankruptcy" problem (Giménez-Gómez et al., 2016)—where the carbon budget represents the total endowment and countries act as creditors.

The  $\alpha_{\min}$ -egalitarian rule, introduced by Giménez-Gómez and Peris (2014), modifies standard division rules in allocation problems to ensure that no agent receives more than their claim while guaranteeing a minimum viable amount for each agent. This rule strikes a balance between egalitarian and proportional principles by prioritizing an egalitarian distribution that respects claim-boundedness. Instead of providing an axiomatic framework, the original authors define the rule equivalently as a convex combination of the proportional and equal award principles. Although widely applied across various allocation problems, the  $\alpha_{\min}$ -egalitarian rule had not been formally characterized axiomatically until now. This study fills that gap by offering the first axiomatic characterization of the rule, elucidating its theoretical foundations while highlighting its practical significance.

**JEL code**: C71, D63, D81

Keywords: claims problem, proportionality, egalitarianism, sustainability.

## References

R. J. Aumann and M. Maschler. Game theoretic analysis of a bankruptcy problem from the talmud. *Journal of economic theory*, 36(2):195–213, 1985.

- D. Dominguez and W. Thomson. A new solution to the problem of adjudicating conflicting claims. *Economic Theory*, 28:283–307, 2006.
- J.-M. Giménez-Gómez and J. E. Peris. A proportional approach to claims problems with a guaranteed minimum. *European Journal of Operational Research*, 232(1):109–116, 2014.
- J.-M. Giménez-Gómez, J. Teixidó-Figueras, and C. Vilella. The global carbon budget: a conflicting claims problem. *Climatic change*, 136:693–703, 2016.
- C. Herrero and A. Villar. Sustainability in bankruptcy problems. Top, 10(2):261–273, 2002.
- H. Moulin. Egalitarianism and utilitarianism in quasi-linear bargaining. *Econometrica: Journal* of the Econometric Society, pages 49–67, 1985.
- H. Moulin. Axiomatic cost and surplus sharing. Handbook of Social Choice and Welfare, 1: 289–357, 2002.
- B. O'Neill. A problem of rights arbitration from the talmud. Mathematical Social Sciences, 2 (4):345–371, 1982.
- W. Thomson. For claims problems, compromising between the proportional and constrained equal awards rules. *Economic Theory*, 60:495–520, 2015a.
- W. Thomson. For claims problems, another compromise between the proportional and constrained equal awards rules. 2015b.
- W. Thomson. How to divide when there isn't enough. Cambridge University Press, 2019.