Communicating Bias*

Swagata Bhattacharjee†1, Srijita Ghosh‡2, and Suraj Shekhar§3

[1]O. P. Jindal Global University

[2,3]Ashoka University

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Abstract

We consider a static cheap talk model in an environment with either one or two experts whose biases are privately known by the experts themselves. Before the experts learn the state, they send a cheap talk message about their bias to the decision maker. Subsequently, the decision maker chooses one expert to get state relevant advice from. We ask two questions - One, is there an equilibrium where the experts' bias is fully revealed? Two, is the bias revealing equilibrium welfare improving for the decision maker? We find that when there is only one expert, there is no bias revealing equilibrium. However, if there are two experts, there exists a bias revealing equilibrium, and under some conditions it gives the decision maker more utility than any equilibrium which is possible without bias revelation. This highlights a new channel through which sender competition can benefit the decision maker.

JEL codes: D82, D83

Keywords - Cheap talk, uncertain bias, multiple senders, bias revelation

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†swagata.bhattacharjee@jgu.edu.in

‡srijita.ghosh@ashoka.edu.in

§suraj.shekhar@ashoka.edu.in

1 Introduction

Consider a cheap talk model of strategic communication in which the bias of the sender(s) is unknown. Suppose the receiver can interact with multiple senders before choosing one sender to get payoff relevant advice from. For example, consider an individual who wants to consult a financial adviser. The individual may be uncertain about the risk appetites of different advisers. However, he can talk with different advisers (without disclosing his personal finances) before deciding who he will consult. In many other real-life situations, a decision maker has to decide which expert to consult, where the experts have privately known biases. For example, for legal advice a defendant has to hire one from a pool of lawyers, for policy advice companies have to hire one of several field experts etc.

Motivated by such examples, we consider a cheap talk game with $N \in \{1,2\}$ senders, where the bias of each sender is her private knowledge. It is common knowledge that the bias can be either *high* or *low* and that the bias of each sender is drawn independently from a known distribution. Before any player (sender or receiver) gets information about the payoff relevant state, we add a bias communication stage (stage 1). In this stage, the senders simultaneously send cheap talk messages about their bias type, following which the receiver selects one sender. The next stage (stage 2) is a standard cheap talk game where the chosen sender observes the true state perfectly and sends a cheap talk message to the decision maker. The decision maker chooses an action and all players get paid.

Our main result shows that an endogenous bias revelation equilibrium exists when there are two senders, but not for the one sender case. Moreover, under some conditions, a two sender bias-revelation equilibrium is preferred by the receiver over any equilibrium possible without the bias revelation stage. We also contribute to the literature on cheap talk with uncertain biases by characterizing closed form expressions for equilibria and equilibrium payoffs in a cheap talk game with quadratic loss utility functions.

In the baseline case of one sender only, note that any bias revealing equilibrium will feature a Crawford-Sobel type partition equilibrium (Crawford and Sobel (1982)) in the second stage after biases are revealed in the first stage. For any such equilibria played in stage 2, we show that the gain from deviation for the high-bias sender always exceeds the gain from not deviating for the low-bias sender. This is because the high bias sender obtains a more balanced[1] partition when she pretends to be low-type whereas it is the reverse for the low type sender. This result implies that whenever the high bias sender does not want to deviate (gains from deviation are negative), the low bias sender will want to deviate (gains from not deviating are negative), and thus we cannot have both types revealing their bias in equilibrium.

However, when two senders compete to be hired by the decision maker, there exists a bias revealing equilibrium. The two-sender case presents two new forces. First, since the decision maker hires only one expert, the senders *compete* to get hired. Second, since the probability of selection depends on every sender's bias message, and the senders are not aware of their competitors bias, there is strategic uncertainty that is not present in the one-sender case. These new forces allow us to control incentives by identifying conditions on the outside option and on the strategic uncertainty which allow the bias revealing incentive compatibility constraints to hold.

To show the welfare implications of sender competition, we analyze if and when a bias revealing equilibrium is preferred by the decision maker over any equilibrium possible without the bias revealing stage. There are two factors which affect the decision maker's payoff which help us compare bias revealing equilibria to those possible without the bias revelation stage. One, the amount of information transmitted as measured by the number of partitions possible in equilibrium. Two, the variance in payoff induced by the equilibrium as measured by the balance of the partition (more balance leads to lower variance). Since we work with the quadratic loss utility function, the decision maker prefers more information and more balance. The maximum number of partitions possible in a non-revelation equilibrium is between the maximum number of partitions possible with the high bias sender in stage 2 and the number of partitions possible with the low bias sender in stage 2[2]. Thus, compared to a non-revealing equilibrium, in a revelation equilibrium, the decision maker gains/loses in amount of information when stage 2 is the maximal partition equilibrium with low/high bias sender. Further, the variance in payoff induced by the revealing equilibrium is higher when the difference between high and low bias is more and the fraction of high bias senders in society is neither too high or too low.

[1] Li and Madarász (2008) define a partition as more balanced than another if there is a smaller increase in noise as one moves from lower messages to higher ones (for positive biases). Decision makers with concave utility functions prefer more balanced partitions.

[2] As the parameter capturing the fraction of high bias senders in society goes from zero to one, the maximum number of partitions spans the range.

We find conditions under which a bias revealing equilibrium is preferred by the decision maker in the following manner. First, with the help of a public randomization device[3], we construct a bias-revealing equilibrium where the players always play the most informative partition equilibrium in stage 2. Next, we show that fixing the level of high bias, if the lower bias is small enough, then a) the number of partitions possible when communicating with a low bias sender is strictly higher than the number of partitions possible without the bias revelation stage if the fraction of high bias senders is above a cutoff, b) the cutoff needed in the previous point becomes really small. Further, the variance induced by the bias revealing equilibrium is low when the fraction of high bias senders is low. We show that with a large fraction of low bias senders, the decision maker's benefit from the extra information (more number of partitions) obtained from the low bias sender in a revealing equilibrium is more than the decision maker's loss emanating from low information transmission with the high bias sender and higher variance in payoff[4].

We contribute to the literature on cheap talk games with uncertain sender bias. The paper closest to ours is Li and Madarász (2008) which discusses a static cheap talk game with one sender of unknown bias. They compare two regimes - one where the sender must announce her true bias before communicating about the state, and another where the sender has no possibility of revealing any information about her bias before sending state relevant information to the decision maker. They find that if the utility function of the decision maker is concave enough, then he may prefer the regime where the sender's bias is not revealed.

Our paper differs from Li and Madarász (2008) in two ways. One, we allow the senders to choose if they want to reveal their biases by adding a pre-play bias communication stage before any expert gets to observe the true state. This is different from the exogenous bias revelation regime considered in Li and Madarász (2008) for two reasons. First, since a sender's bias is her private information, in most environments, it would be very difficult to explain how an exogenous truthful bias revelation can be enforced. This assumption becomes harder to justify because we show that in the one sender case, endogenous bias revelation is not possible. Furthermore, while Li and

[3]Though it is possible to construct this equilibrium without a public randomization device for a very restricted parameter space, it is not preferred by the decision maker to the best equilibrium possible without the bias revelation stage. The reason is that such an equilibrium requires the decision maker to select the high-bias sender with probability one whenever the senders announce different biases. This reduces the decision maker's payoff in the bias-revealing equilibrium.

[4]Compared to the equilibrium without bias revelation.

Madarász (2008) considers equilibria that follow exogenous true bias announcement, they do not consider the conditions needed for the sender to endogenously reveal her bias prior to learning the state. These conditions limit the outcomes possible in equilibrium. Second, we allow for more than one sender. We show that while endogenous bias revelation is not possible with one sender, bias-revealing equilibria exist in the two sender world, and they can even be welfare improving for the decision maker.

Quement (2016) also considers a model with unknown bias. In his paper, there are two senders and the receiver gets messages from both the senders sequentially. One important difference between this setup and ours is that, in our model, the receiver can only get message from one sender and the sender has the option to reveal her bias before learning state relevant information. Further, in contrast to Quement (2016), where an increase in the number of senders reduces the receiver's payoff, we find sender competition can improve the welfare of the receiver. Antić and Persico (2020) develops a framework where the bias is endogenously determined before the cheap talk communication. This paper finds that when the endogenously determined bias is not commonly known, it involves welfare loss. In contrast, we examine a set up where bias is exogenously determined but remains a private information, and we show that bias-revealing equilibria lead to higher welfare for the receiver under certain conditions. In a repeated game scenario, Atakan et al. (2020) determines the optimal path of the stakes involved in the relationship between a sender and a receiver when the receiver is privately informed about the conflict of interest.

The second strand of literature we connect to is the one on cheap talk models with multiple senders. Li (2010) considers a model with multiple senders and privately known bias with three different protocols of learning - sequential, simultaneous, and hierarchical. This paper finds that competition is welfare improving and that simultaneous learning is the most efficient of all three. In contrast, we introduce sender competition in a different way: only one sender is hired after the pre-play bias communication stage, so the senders compete to get hired, without knowing the state. We state conditions under which this can be welfare improving. Li et al. (2016) also considers a model of cheap talk with multiple senders, where each sender gets a private signal about their own project. In contrast, in our model, there is only one payoff relevant state; and once hired, the sender learns the state perfectly. Schmidbauer (2017) considers a multi-period version of Li et al. (2016) and shows more competition harms the information transmission and reduces payoffs. In this paper, we document a positive effect of sender competition through a unique channel. Broadly speaking, our paper also relates to multi dimensional cheap talk literature, since there are two dimensions (sender bias and state of the world) which are privately observed by the sender. Multi-dimensionality of the state of the world in itself generates an additional benefit of sender competition, as documented in Battaglini (2002). However, we find another channel through which competition helps. Chakraborty and Harbaugh (2007), Chakraborty and Harbaugh (2010) consider multi-dimenisonal cheap talk environments and show how using comparative ranking we can get full information revelation. In our paper, the two dimensions are independent of each other, and hence can not be comparatively ranked. Our focus is not on full information revelation across both dimensions, we are interested in examining the bias-revealing equilibria.

The rest of the paper is organized as follows: section 2 describes the model. In section 3, we start with a baseline case of one sender and show that there does not exist a bias revealing equilibrium in the one sender model (subsection 3.1). Subsequently, in section 3.2 we show that such an equilibrium does exist in the two-sender model. Next, we show that a bias-revealing equilibrium may give the decision maker more utility than any equilibrium possible without bias revelation.

2 Model

Primitives

We consider a one-shot strategic communication game with one decision maker (he) and $N \in \{1,2\}$ experts (she) $(S_1,...,S_n)$. The state of the world θ is commonly known to be uniformly distributed on the unit interval $[0,1]$. The decision maker can hire exactly one of the experts to get state-relevant advice from. If an expert is hired, she learns the state perfectly and can send a cheap talk message (m) to the decision maker. Following this message, the decision maker takes an action $y(m)$. All experts are biased in that their preferences do not align perfectly with the decision maker's preferences. An expert S_i's bias b_i is her private information, but it is common knowledge that biases are drawn IID from the distribution:

$$b_i = \begin{cases} b_h & \textit{with probability } p_h \in (0,1) \\ b_l & \textit{with probability } 1-p_h \end{cases}$$

where $|b_l| < |b_h|$. We assume that biases are low enough ($|b_i| \leq \frac{1}{4}\ \forall\ i$) so that there exists at least one informative cheap talk equilibrium when the hired expert's bias is known.

Since biases are unknown, we consider a two-stage game. In the first stage, the players play a cheap talk game which could reveal information about the type of the expert, i.e., her bias.[5] Following this, in the second stage, the hired expert and the decision maker play a standard cheap talk game where the expert sends a message to the DM about the payoff relevant state.

To contrast our results with those of the nondisclosure world in Li and Madarász (2008), we will often consider a strategic communication game where the experts do not have the option to reveal their types in the first stage. This game will not have the first stage game. We will call this the *LM* game. The analysis for such a game would follow the non-disclosure environment analysis presented in Li and Madarász (2008).

Timing

The timing of the game is as follows. At the beginning of the game in stage 1, each expert S_i privately learns her own bias $b_i \in \{b_h, b_l\}$ and then simultaneously sends a costless message $m_b^i \in \mathscr{M}_b$ to the DM that potentially conveys information about their own bias. Without loss of generality, we focus on direct mechanisms, so, $\mathscr{M}_b = \{b_h, b_l\}$. The decision maker then chooses to hire one expert as her advisor according to a hiring rule $h : \mathscr{M}_b \longrightarrow \Delta\{S_1, ...S_N\}$, where the hiring rule depends on the observed message vector sent by the N experts, and $h_i(m_b)$ denotes the probability of hiring expert S_i if the message vector is $m_b = (m_b^1, ...m_b^N)$. Note that, if $N = 1$, the DM hires the expert for sure.

In stage 2, the hired expert i learns the true state θ perfectly and sends another message $m_\theta^i \in \mathscr{M}$ to the DM possibly conveying some information about the state. Focusing on direct mechanisms, We assume $\mathscr{M} = [0,1]$. Upon observing m_θ^i, the DM takes an action $y\left(m_\theta^i\right) \in [0,1]$. Note that stage 2 looks like Crawford and Sobel (1982) if the bias of the chosen expert is fully revealed in stage 1. If not, then stage 2 looks like the non-disclosure world of Li and Madarász (2008). The expert who is not hired gets her outside option (described under 'Payoffs' below).

[5] Note that an expert learns the state only after being hired. Therefore, in stage 1, the only information that can be conveyed is about the experts' bias.

Beliefs

Let $P_b^i \in [0,1]$ denote the posterior belief that the DM forms about the hired expert i being high bias (b_h) type at the end of stage 1 upon receiving bias-relevant information from all the experts. Let $P \in \Delta(\theta)$ be the posterior belief about true state θ that the DM forms in stage 2 upon receiving state-relevant information from the hired expert.

Payoffs

If the true state is θ, the hired expert is expert i, and the decision maker takes the action y, then the payoffs are as follows:

$$
\begin{aligned}
U_{DM}(\theta, y) &= -(y-\theta)^2 \\
U_i(\theta, y, b_i) &= -(y-\theta-b_i)^2 \\
U_{j \neq i} &= -A_{b_j}
\end{aligned}
$$

where the expert who is not selected (for $N=2$) gets a reservation payoff of $-A_b$ for $b \in \{b_l, b_h\}$. We assume that $A_l = A, A_h = A+c$, so $c \geq 0$ captures the idea that the high bias sender's outside option is allowed to be worse than the low bias sender's outside option. To make sure that experts always want to get hired, we assume their reservation payoff is weakly worse than the lowest possible equilibrium payoff obtained by any expert from being hired, that is - worse than the payoff of from a babbling equilibrium in the Crawford-Sobel world.

Strategies

We will consider only pure strategies for the experts. A pure strategy for an expert S_i with bias b_i consists of two functions. The first function $\mu_{bi} : \{b_l, b_h\} \to \mathscr{M}_b$ determines the bias message sent by the expert in stage 1. This depends upon the expert's own true bias. The second function $\mu_{\theta i} : \{b_l, b_h\} \times P_b^i \times \theta \to \mathscr{M} = [0,1]$ determines the message ($m_\theta^i \in [0,1]$) about the state $\theta \in [0,1]$ sent by the expert i in stage 2 if hired. This message depends upon the chosen expert's true bias in $\{b_l, b_h\}$, the decision maker's posterior belief (P_b^i) that the chosen expert is type b_h, and the true state θ observed by the expert.

The DM's strategy also consists of two functions. One, a hiring function $h : \mathscr{M}_b{}^N \to \Delta\{S_1, ...S_N\}$ which determines which expert to hire as a function of the stage 1 vector of messages. Two, an action function $y : P_b^i \times \mathscr{M} \to [0,1]$ which determines the action taken as a function of the belief about the chosen expert and the message sent by the chosen expert in stage 2.

Perfect Bayesian Equilibrium

A Perfect Bayesian equilibrium consists of a profile of strategies for the decision maker and all experts, and belief vectors P, P_b such that: given the strategies of all players, the beliefs are derived using Bayes' rule whenever possible. The decision maker's hiring rule h and the action function y, maximize his ex-ante expected utility given his belief P, P_b and the strategy of all experts. We can write the DM's problem as,

$$\max_{y,h} EU_{DM}(P(P_b(m_b), m_\theta^i), y(P_b(m_b), m_\theta^{h(m_b)}))$$

For each type of expert, their strategy should maximize their expected payoff given the strategies of all the other experts and the decision maker. We can write expert i's problem as

$$\max_{m_b^i, m_\theta^i} EU_i(P(P_b(m_b), m_\theta^i), y(P_b(m_b), m_\theta^{h(m_b)}), b_i)$$

Notation

For ease of exposition, let us denote $CS_{b_{j'}}^j(k)$ as the payoff for a b_j type sender in a k partition equilibrium when the sender's bias is thought to be $b_{j'}$ with probability 1. Thus, when senders are truth-telling, from Crawford and Sobel (1982), we get:

$$CS_{b_h}^h(m) = -\frac{1}{12m^2} - \frac{b_h^2(m^2+2)}{3}$$
$$CS_{b_l}^l(n) = -\frac{1}{12n^2} - \frac{b_l^2(n^2+2)}{3}$$

In the next section, we will find expressions for the payoffs $CS_{b_l}^h(n)$ and $CS_{b_h}^l(m)$ and how they impact equilibrium. An n partition cheap talk equilibrium between a sender of known type b and the receiver will be denoted as 'n partition CS b equilibrium' (since this will be exactly the same as a n partition equilibrium in Crawford and Sobel (1982)).

3 Analysis

We first study our benchmark case of one sender. We explore if truthful bias revelation is possible in equilibrium in stage 1. We show that this is not possible in the one sender world. However, we show in later sections that truthful bias revelation is possible in the two sender world. Moreover, under some conditions, the bias revealing equilibrium can give the decision maker a higher utility than any feasible bias hiding equilibrium. This analysis demonstrates a new channel through which competition and strategic uncertainty amongst senders can improve the welfare of the decision maker. In the two following subsections we show the role of competition and strategic uncertainty on the incentive compatibility constraints of the different types.

3.1 One sender world

Is it possible that the experts reveal their biases perfectly in stage 1 in some equilibria? We start our analysis by exploring the possibility of such bias revealing equilibria existing when there is only one sender. This will serve as our baseline case. Before going further, we note that it is obvious that if the receiver plays a babbling equilibrium in stage 2 irrespective of the messages received in stage 1, then both types of senders will have no incentive to deviate from truth telling in the bias revealing stage. However, since this equilibrium is uninteresting[6], we will henceforth only consider those equilibria in stage 2 where the receiver is playing a non babbling cheap talk equilibrium after at least one of the stage 1 messages.

Our main result in this subsection is that with one sender, there does not exist a bias revealing equilibrium in pure strategies. This is not intuitive at first glance. In stage 2 of the game, the receiver can promise different cheap talk equilibria as a reward to the sender for revealing her bias in stage 1. For example, the DM may compensate the higher bias sender with a finer partition equilibria[7] for revealing her bias as compared to the equilibrium following the lower bias revelation.[8] However,

[6]Though the bias is revealed in equilibrium, this is not payoff relevant for the decision maker.

[7]To the extent allowed by the size of the bias.

[8]The reader may wonder if the lower bias sender will want to deviate and lie about her type to benefit from the finer partition offered to the higher bias sender in such an equilibrium. The lower bias sender's incentive compatibility

we show that no matter which equilibria are offered for revealing their true bias types in stage 2, either the low bias sender or the high bias sender (or both) will want to deviate from a bias revealing equilibrium in stage 1. This is in contrast to the two sender case (section 3.2) where we show that there exist equilibria where the senders truthfully reveal their bias in equilibrium. Proposition 1 shows our main result for this section.

Proposition 1. *When there is only one sender ($n = 1$), there is no bias revealing equilibrium in pure strategies.*

Proof. The proof is presented in the appendix. In the proof we also derive expressions for $CS^h_{b_l}(n)$ and $CS^l_{b_h}(m)$. We use these expressions in the rest of the analysis. □

The intuition behind the non existence of a bias revealing equilibrium in the one sender case is as follows. Suppose that there is a bias revealing equilibrium where the high bias message results in a m partition equilibrium and the low bias message results in a n partition equilibrium. If $m \leq n$, it is easy to show that the high bias sender would deviate and pretend to be low bias since she will be able to obtain higher actions in equilibrium, whereas the low bias sender would want to announce her type honestly. That is, the gain from deviation is non-negative (i.e. $CS^h_{b_l}(n) - CS^h_{b_h}(n) \geq 0$) for the high bias sender and the gains from not deviating is non-negative (i.e. $CS^l_{b_l}(n) - CS^l_{b_h}(n) \geq 0$) for the low bias sender. Now consider any $m > n$ and the difference between these two differences i.e.:

$$[CS^h_{b_l}(n) - CS^h_{b_h}(m)] - [CS^l_{b_l}(n) - CS^l_{b_h}(m)]$$
$$= (b_h - b_l)^2(2 - \frac{1}{n} - \frac{1}{m}) + \frac{(b_h^2 - b_l^2)}{3}(m^2 - n^2) \tag{1}$$

This expression is clearly positive since $m > n$, $b_h > b_l$ and at least one of m, n is greater than 1 (else it would be a babbling equilibrium and we have stated before - we are interested in only informative equilibria). Thus, as we increase m to incentivize the high bias sender to not deviate, before the gains from deviating becomes negative for the high bias sender, the gains from not deviating becomes negative for the low bias sender. Therefore, we cannot incentivize both types of senders to reveal

constraint will be satisfied if the partition points of the high bias sender are unbalanced (many small partitions at the lower levels of the state with larger partitions at the higher levels) enough. This is because of the risk averse utility function of the senders.

their bias truthfully in any equilibrium. The intuition for why the deviation gains are higher for the high bias sender is that the high bias sender obtains a more balanced partition when she lies and pretends to be low bias whereas it is the other way around for the low bias sender.

So, what kind of equilibria exist when there is only one sender? From our result, we know that any equilibrium will be one where there is uncertainty about the sender's bias in stage 2. In these environments, 'conflict hiding'[9] equilibria may exist. One example is as follows:

Example 1. *Suppose $b_l = \frac{1}{6}, b_h = \frac{1}{5}$ and $p_h = \frac{3}{5}$. The following strategy profile constitutes a perfect Bayesian Equilibrium.*

Stage 1

$\mu_b(x) = b_l \ \forall \ x \in \{b_l, b_h\}$

$h(b) = 1 \ \forall b \in \{b_l, b_h\}$

Stage 2

Sender Strategy:

$\mu_{sb_l} = m_1 \ if \ \theta \in [0, 0.146]$

$\mu_{sb_l} = m_2 \ if \ \theta \in (0.146, 1]$

$\mu_{sb_h} = m_1 \ if \ \theta \in [0, 0.113]$

$\mu_{sb_h} = m_2 \ if \ \theta \in (0.113, 1]$

Decision maker strategy:

If DM observes b_l in stage 1 and m_1 in stage 2 $= 0.0631$

If DM observes b_l in stage 1 and m_2 in stage 2 $= 0.5631$

If DM observes b_l in stage 1 and $m \neq m_1, m_2$ in stage 2 $= 0.0631$ *If DM observes b_h in stage 1* $= 0.5$

[9] See Li and Madarasz 2008

$Beliefs$

$P_b = p_h \; \forall$ *messages in stage* 1

$P = p(b = b_h/m_1)U[0,0.146] + p(b_l/m_1)U[0,0.113]$ *; if DM observes b_l in stage 1 and m_1 in stage 2*

$P = p(b_h/m_2)U[0.146,1] + p(b_l/m_2)U[0.113,1]$ *; if DM observes b_l in stage 1 and m_2 in stage 2*

$P = p(b_h/m_1)U[0,0.146] + p(b_l/m_1)U[0,0.113]$ *; if DM observes b_l in stage 1 and $m \neq m_1, m_2$ in stage 2*

$P = U[0,1]$ *; if DM observes b_h in stage 1* (2)

The decision maker's expected payoff in this equilibrium is -0.056.

3.2 Two sender world

Now, we consider the environment with two experts. After stage 1, the decision maker hires one of the experts based on their messages about their bias. In stage 2, this hired expert gets to see the true state and sends cheap talk message about the state. We ask two questions of this environment. One, does there exist a bias revealing equilibrium? The two sender environment is notably different from the one sender world in two ways - the introduction of sender competition (only one expert is hired while the other gets her reservation payoff) and strategic uncertainty (arising because the hiring decision depends upon the vector of bias announcements). Can these forces generate a bias revealing equilibrium when the one sender case could not? The second question of interest is whether the bias revealing equilibrium can give the decision maker a higher payoff than any equilibrium that arises in an environment without the bias messaging stage.

First, we show that in contrast to the one-sender case, truthful bias revelation in pure strategies is possible in an informative equilibrium in the two-sender world. We provide conditions under which such an equilibrium exists. Later, we find conditions under which we are able to demonstrate an example where there exists a bias revelation equilibrium which is strictly preferred by the decision maker to any equilibrium that can be achieved without the bias revealing stage.

3.2.1 General bias revealing equilibrium

Let n (respectively, m) be the highest partition cheap talk equilibrium possible when the sender's bias is known to be b_l (respectively, b_h). Since $b_h > b_l$, we know $n \geq m$. Let the payoff from not getting selected for the low bias sender be $-A_l = -A$, where $-A = -\frac{1}{12} - {b_l}^2$ is the babbling payoff for low type sender when there is no uncertainty about her type. If the high bias type sender does not get hired, she gets the payoff: $-A_h = -(A+c)$.

Consider the following strategy profile:

Stage 1

$\mu_{ib}(x) = x \; \forall \; x \in \{b_l, b_h\} \; and \; i \in \{1,2\}$

$h(b_l, b_l) = (\frac{1}{2}, \frac{1}{2})$

$h(b_l, b_h) = (0, 1)$

$h(b_h, b_l) = (1, 0)$

$h(b_h, b_h) = (\frac{1}{2}, \frac{1}{2})$

Stage 2

If senders reports (b_l, b_l) in stage 1: Play a n partition CS b_l equilibrium with chosen expert

If senders reports (b_l, b_h) in stage 1: Play a babbling equilibrium with chosen expert

If senders reports (b_h, b_l) in stage 1: Play a babbling equilibrium with chosen expert

If senders reports (b_h, b_h) in stage 1: Play a m partition CS b_h equilibrium with chosen expert

Deviation by chosen expert in stage 2: take the lowest equilibrium action in n partition CS b_l equilibrium

Deviation by decision maker in hiring in stage 1: Play a babbling equilibrium with chosen expert

(3)

The following proposition shows that this strategy profile constitutes a PBE, so bias revelation is supported in equilibrium.

Proposition 2. *There exists a p' such that if $p_h \in [0, p']$, then the above strategies are part of an informative Perfect Bayesian equilibrium.*

Proof. The proof is presented in the appendix. □

The existence of this bias revealing equilibrium depends crucially on both strategic uncertainty amongst the experts and sender competition. In particular, suppose there was no strategic uncertainty because the senders knew each other's bias. Consider the same conditions as outlined in this proposition and one low bias expert and one high bias expert. The above strategies would not constitute an equilibrium because the low bias sender would never be selected giving her a low payoff (her outside option). The low bias sender would deviate and announce her type as high. Sender competition is modelled here by our assumption that only one sender is selected whereas the other sender gets her outside option. As the proof indicates, by manipulating the outside option, we can incentivize experts to reveal their bias accurately. For example, here when p_h is low, the high bias sender faces a tradeoff between most likely getting a babbling payoff (if bias is revealed truthfully) or lying about her bias and getting a payoff which mixes between hiding in CS b_l n partition equilibrium and getting her outside option with probability half. If the outside option is low enough, the former is preferred.

3.2.2 Welfare

In this subsection, we want to determine conditions under which a revealing equilibrium is payoff superior to any equilibrium possible without the bias revealing stage. First, we notice that while the equilibrium illustrated in proposition 2 allows for endogenous bias revelation, it fares poorly on receiver welfare owing to the babbling equilibrium (which gives the worst equilibrium payoff) being played when the senders' messages don't match in stage 1. There is a trade-off between bias revelation and decision maker's payoff maximization: in stage 2, if the most informative equilibrium is selected, that will enhance the utility of the decision maker, but in stage 1, the incentive constraints will not be satisfied, especially for the high bias type sender, if that most informative equilibrium is played later. The issue is that if we pick either type with probability one after mixed messages (when the senders' reports about bias do not match), we are unable to generate incentives for truth-telling for both types. On the other hand, if we let the receiver use a non-deterministic hiring strategy after mixed messages and this is followed by the most informative equilibria then the receiver will choose to deviate from the non-deterministic hiring strategy and will always pick the low-type sender after mixed messages in any bias revealing equilibrium.

To mitigate this we allow the receiver to use a public randomization device to commit to mixed strategies as a response to bias announcements. Any deviations by the receiver will now be observable and can be punished with a babbling equilibrium.

First, we consider the one sender case again and show that allowing the receiver the ability to commit to mixing on hiring does not alter the result there. The following result shows there cannot be a bias revealing equilibrium in the one sender case even after allowing for mixed strategy in hiring by the receiver.

Proposition 3. *There does not exist any informative bias revealing equilibrium with one sender.*

Proof. In the appendix. □

Next, we turn to the two sender case and show an example where under some conditions, not only does a bias-revealing equilibrium exist, but it gives the decision maker a higher utility than any equilibrium that can be achieved without the bias-revealing stage 1.

First we will find the payoff maximizing bias revealing equilibrium. Then, we will construct an example in which this equilibrium does better than the best possible equilibrium when there is no bias revealing stage.

Consider the following bias revealing strategy profile:

Stage 1

$\mu_{ib}(x) = x \; \forall \; x \in \{b_l, b_h\}$ *and* $i \in \{1,2\}$

$h(b_l, b_l) = (\frac{1}{2}, \frac{1}{2})$

$h(b_l, b_h) = (1-v, v)$ $, v \in [0,1]$

$h(b_h, b_l) = (v, 1-v)$

$h(b_h, b_h) = (\frac{1}{2}, \frac{1}{2})$

Stage 2

If senders reports (b_l, b_l) in stage 1: Play a n partition CS b_l equilibrium with chosen expert

If senders reports (b_l, b_h) or (b_h, b_l) in stage 1: If b_l chosen - play a n partition CS b_l equilibrium, If b_h chosen - play a m partition CS b_h equilibrium

If senders reports (b_h, b_h) in stage 1: Play a m partition CS b_h equilibrium with chosen expert

Deviation by chosen expert in stage 2: take the lowest equilibrium action in n partition CS b_l equilibrium

Deviation by decision maker in hiring in stage 1: Play a babbling equilibrium with chosen expert

(4)

Proposition 4. *There exists a* v_1 *such that* $\forall v \in [\frac{1}{2}, v_1]$*, there exists a* c_3 *such that for all* $c > c_3$*, there exists an interval* $I \subset [0,1]$ *such that if* $p_h \in I$ *then the strategies described in 4 constitute an equilibrium.*

Proof. IC for b_h:

$$p_h > \frac{\frac{1}{2}(CS^h_{b_l}(n) - CS^h_{b_h}(m)) + (\frac{1}{2} - v)(CS^h_{b_h}(m) + A + c)}{(v - \frac{1}{2})(CS^h_{b_l}(n) - CS^h_{b_h}(m))} = p_{h_0} \tag{5}$$

We need that the RHS is less than 1 to get a feasible region for p_h.

Consider $v > \frac{1}{2}$. Then, the denominator is positive, since $CS^h_{b_l}(n) \geq CS^h_{b_h}(m)$. In the numerator, since $CS^h_{b_l}(n) \geq CS^h_{b_h}(m)$, and $CS^h_{b_h}(m) > -(A+c)$, we have the first expression to be positive while the second expression to be negative.

Then, for every $v_1 > \frac{1}{2}, \exists c_1(v) = \frac{(v+1)\left(CS^h_{b_l}(n) - CS^h_{b_h}(m)\right)}{\left(v-\frac{1}{2}\right)} - CS^h_{b_h}(m) - A$. Thus, $\forall c > c_1(v), RHS < 1$.

IC for b_l:

$$p_h < \frac{\frac{1}{2}(CS^l_{b_l}(n) - CS^l_{b_h}(m)) + (\frac{1}{2} - v)(CS^l_{b_h}(m) + A)}{(v - \frac{1}{2})(CS^l_{b_l}(n) - CS^l_{b_h}(m))} = p_{h_1} \tag{6}$$

We need the RHS is greater than zero to get a feasible region for p_h. Again consider $v > \frac{1}{2}$. Then, the denominator is positive. Furthermore, the first expression of the numerator is positive while the second expression is negative. At $v = \frac{1}{2}, RHS > 0$, and RHS is continuous and decreasing in v, so there must exist a $v_1 = \frac{1}{2}\left(CS^l_{b_l}(n) + A\right) > \frac{1}{2}$, such that RHS=0. So, for all $\frac{1}{2} < v < v_1, RHS > 0$.

To satisfy both the ICs simultaneously, we need:

$$\frac{\frac{1}{2}(CS^h_{b_l}(n) - CS^h_{b_h}(m)) + (\frac{1}{2} - v)(CS^h_{b_h}(m) + A + c)}{(v - \frac{1}{2})(CS^h_{b_l}(n) - CS^h_{b_h}(m))} < \frac{\frac{1}{2}(CS^l_{b_l}(n) - CS^l_{b_h}(m)) + (\frac{1}{2} - v)(CS^l_{b_h}(m) + A)}{(v - \frac{1}{2})(CS^l_{b_l}(n) - CS^l_{b_h}(m))}$$

Choose a v'' such that $\frac{1}{2} < v'' < v_1$. For each v in this range, we can always choose c high enough to make the above inequality hold. Suppose it holds when $c > c_2\left(v''\right)$. Now for this v'' $(> \frac{1}{2})$, we can find $c_1\left(v''\right)$ which makes the IC for b_h feasible.

Then, for $\frac{1}{2} < v'' < v_1$, we can find a $c_3 = max\left\{c_2\left(v''\right), c_1\left(v''\right)\right\}$ such that for all $c > c_3$, our bias revealing strategy profile described in 4 is an equilibrium.

□

Proposition 5. *Given b_h, there exists $\bar{b}$ such that if $b_l < \bar{b}$, there exists an interval $G \subset I$ such that if $p_h \in G$, the bias-revealing equilibrium described in 4 gives the decision maker higher utility compared to any equilibrium that can exist without the bias revealing stage. That is, the bias revealing equilibrium payoff for the DM is better than (a) any conflict hiding (CH) equilibrium, and (b) any equilibrium without the bias revealing stage where endogenously a range of bias may be revealed.*

Proof. The result has two parts. Part a states that in any equilibrium without the bias revealing stage,if there is no endogenous bias revelation, that is, if we restrict our attention to conflict hiding (CH) equilibria, then the bias revelation equilibrium in 4 is better for the DM. However, in LM, there is another kind of equilibrium where endogenous bias revelation is possible even without the bias revealing stage (call this CR equilibrium). Part b of the proposition states that even compared to those endogenous bias revealing CR equilibria, the bias revealing equilibrium is better for the DM.

Proof of Proposition 5 a: First, let us consider the CH equilibrium with no endogenous bias revelation.

Let us fix a b_h such that it admits a maximum of m partitions when the bias is known (i.e. in the Crawford-Sobel world). Let n be the maximum number of partitions admissible for b_l in a CS world. Suppose the highest number of partitions possible in a conflict hiding (CH) equilibrium without the bias revealing stage is $k \geq 2$.

Then, we know from proposition 4, that there exists a v_1 such that given any $v \in [\frac{1}{2}, v_1]$, there exist a c_3 such that if $c > c_3$ the bias revealing strategies described in 4 constitute an equilibrium. Let us now focus on the closed-form expression of the difference between the payoffs from this bias-revealing equilibrium and a k partition CH equilibrium as derived in section (????). This difference can be given as a quadratic expression:

$$\Delta(p_h) = Ap_h^2 + 2Bp_h + C$$

$$where$$

$$A = (CS^h_{b_h}(m) - CS^l_{b_l}(n))(1-2v) + (k-1)(b_h - b_l)^2(\frac{k+1}{3} - \frac{1}{k})$$

$$2B = (CS^h_{b_h}(m) - CS^l_{b_l}(n))v + (k-1)(b_h - b_l)(\frac{(k+1)b_l}{3} + \frac{b_h - b_l}{2k})$$

$$C = CS^l_{b_l}(n) - CS^l_{b_l}(k)$$

We need to show that $\Delta > 0$ for a range of p_h. The roadmap of this proof is as follows. First, we note that for the existence of an equilibrium, we need $p_h \in [p_{h_0}, p_{h_1}]$, as given in 5 and 6. Now, with a very low b_l, we can choose c high enough such that p_{h_0} goes to zero, so for a low p_h we know that equilibrium exists. For very low p_h, whenever $C > 0$, we get $\Delta > 0$, so next we find conditions for which $C > 0$. This gives us another lower bound on p_h: $p_h > z$. For $b_l \to 0$, we can show that $z \to 0$, and there exists a threshold of v, $v^* > \frac{1}{2}$ such that at $p_h = z$, the bias revealing equilibrium exists and $\Delta > 0$. Hence, for all $p_h \in [z, p_{h_1}]$, bias revealing equilibrium is better than the CH equilibria.

First of all, to ensure the existence of this bias-revealing equilibrium, we need $p_h \in I =$

$[p_{h_0}, p_{h_1}]$ (given by 5 and 6 in Proposition 4. Since the function is quadratic in p_h, if $C > 0$, then for a very low p_h,the difference in the payoff in the bias-revealing equilibrium and that in the conflict-hiding equilibrium is positive. $C > 0$ requires that the maximum number of partitions in the conflict hiding equilibrium is strictly less than the maximum number of partitions possible when the bias of the sender is known to be b_l [10] i.e. $n > k$. To ensure that no n partition conflict hiding equilibrium exists, we impose the condition that whenever there is an n partition conflict hiding equilibrium, the lowest equilibrium action is negative, so this can never happen. This requires

$$p_h > z = \frac{\frac{1}{2n(n-1)} - b_l}{b_h - b_l} \tag{7}$$

So, for the existence of a revealing equilibrium where $n > k$, we need

$$p_h > max(z, p_{h_0}) \tag{8}$$

where p_{h_0} is given in 5.

Now, let us pick a $v \in [\frac{1}{2}, v_1]$. From Proposition 4, we know that there exists a c_3 such that $\forall c > c_3$, 5 holds, so let us pick a $c = c^{'} > c_3$ which makes $p_{h_0} \to 0$. Thus, $max(z, p_{h_0}) = z$, so we just need to show that at $p_h = z, \Delta > 0$. For this to be an equilibrium, we also need IC for b_l to hold. Using 6, we need:

$$z < p_{h_1} = \frac{\frac{1}{2}(CS^l_{b_l}(n) - CS^l_{b_h}(m)) + \frac{1}{2}(CS^l_{b_h}(m) + A) - v(CS^l_{b_h}(m) + A)}{(v - \frac{1}{2})(CS^l_{b_l}(n) - CS^l_{b_h}(m))} \tag{9}$$

Let $b_l \to 0$. Then, $n \to \infty$, hence $z \to 0$ from above, hence satisfies 9.

At $p_h = z$,

$$Ap_h^2 + 2Bp_h + C > 0 \ \Rightarrow v < \overline{v} \tag{10}$$

$$where\ \overline{v} = \frac{z^2 CS^h_{b_h}(m) + (1-z)^2 CS^l_{b_l}(n) - CS^z_{b_z}(k) + \frac{(k-1)(b_h - b_l)^2 z(1-z)}{k}}{2z(1-z)(CS^l_{b_l}(n) - CS^h_{b_h}(m))}$$

$$and\ b_z = zb_h + (1-z)b_l$$

[10] We will prove in a corollary that when the number of partitions in the conflict hiding equilibrium equals the number of partitions in the most informative equilibrium when the bias is known to be b_l, then the conflict hiding equilibrium gives the decision maker higher utility than the bias revealing equilibrium. Further, this condition also implies that $m < n$.

As $b_l \to 0$, 10 holds because $\overline{v}$ goes to ∞. For 9 to hold, we get the condition:

$$v < \frac{1}{2}(\frac{A}{CS^l_{b_h}(m) + A}) \tag{11}$$

Thus, we can find a $v^*(> \frac{1}{2})$ such that 9 holds if $\frac{1}{2}(\frac{A}{CS^l_{b_h}(m)+A}) > \frac{1}{2}$. This is true because the highest possible payoff for the decision maker at any state is zero and therefore $CS^l_{b_h}(m) < 0$.

Thus, there exists a cutoff $\bar{b}$ such that if $b_l < \bar{b}$ then there exists a set G of the form $[z, p_{h_1}]$, a $v^* > \frac{1}{2}$ and a c^* such that the bias revealing strategies in 4 constitute an equilibrium and they give the decision maker a higher payoff than any payoff possible in a conflict hiding equilibrium without the bias revelation stage. This concludes the first part of the proof.

Proof of Proposition 5 b

Let us now turn our attention to the type of LM equilibria where there is endogenous bias revelation even without any explicit bias revelation stage (CR equilibria).

$$U'^r_{R,[d,1]}(q) = -\frac{(1-d)^3}{12q^2} - (1-d)\frac{q^2-1}{3}b^2 - (1-d)\frac{q-1}{q}p(1-p)d^2 \tag{12}$$

By similar calculations the maximum payoff from a q partition b_l equilibrium in $[0,d]$ is given by,

$$U'^r_{R,[0,d]}(q) = -\frac{d^3}{12q^2} - db_l^2\frac{(q^2-1)}{3} \tag{13}$$

If the maximum number of partitions in the conflict hiding equilibrium is given by $k < n$, where n denotes the max number of partitions in a CS b_l equilibrium, then the max number of partitions in conflict revealing equilibrium will allow only $k-1$ partitions on $[d,1]$. Thus there can be at most $n-k+1$ partition in $[0,d]$ for CS b_l. Thus the payoff from such an endogenous CR equilibrium would be

$$\begin{aligned} U'^r_R(n) = &-\frac{d^3}{12(n-k+1)^2} - db^2\frac{((n-k+1)^2-1)}{3} \\ &-\frac{(1-d)^3}{12(k-1)^2} - (1-d)\frac{(k-1)^2-1}{3}b_l^2 - (1-d)\frac{k-2}{k-1}p(1-p)d^2 \end{aligned} \tag{14}$$

□

4 Conclusion

In this paper, we consider a model of strategic information transmission where the experts have uncertain biases and may choose to disclose them before communicating state-relevant information. We build on the framework developed in Li and Madarász (2008) and make bias revelation an endogenous choice. We find that if there is only one sender, full revelation of bias is not possible in equilibrium. With two senders, we identify conditions for a bias-revealing equilibrium to exist. Moreover, we find that if the players have access to a public randomization device, then the decision maker could be better off with a bias-revealing equilibrium compared to the best equilibrium possible with no bias revelation.

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A Appendix

Proof of proposition 1

Proof. We will prove this result by contradiction. Suppose there exists a bias revealing equilibrium in pure strategies. WLOG, let the equilibrium be the following:

Stage 1:

$\mu_b(x) = x \; \forall \; x \in \{b_l, b_h\}$

$h(b) = 1 \; \forall \; b \in \{b_l, b_h\}$

Stage 2:

If sender reports type b_l in stage 1: Play an n partition CS b_l equilibrium

If sender reports type b_h in stage 1: Play an m partition CS b_h equilibrium

If the decision maker arrives at an off equilibrium node, she takes the lowest equilibrium action in n partition CS b_l equilibrium

First, let us consider the incentives of the high bias sender. If she plays according to the strategies proposed above, her expected payoff is:

$$\frac{-1}{12m^2} - \frac{{b_h}^2(m^2+2)}{3} \tag{15}$$

Clearly there is no reason to deviate in stage 2 of the game if she reveals her type truthfully in stage

1 (since stage 2 play is an equilibrium, there is no incentive to deviate). If she deviates in stage 1 and reports her type to be b_l, then in stage 2 she can exploit the n partition CS b_l equilibrium to her advantage. In particular, while she does not have the incentives to deviate from the equilibrium messages (else the decision maker plays the action $\frac{1}{2}$), she will change the interval of the state space on which the messages are reported (a la Li and Madarasz's conflict hiding equilibrium). In an n partition CS b_l equilibrium, the equilibrium actions are given by

$$y_i = \frac{2i-1}{2n} + b_l(2i^2 + (1+n)(1-2i))$$

where $i = 1, 2, ..., n$. Now, in equilibrium, the high bias expert will not deviate from the messages the low bias expert was meant to send in equilibrium (else the dm takes the action half). However, the high bias expert does not have to choose the same partition function as the low bias expert. In fact, she will choose cut off points on the state space to maximize her own payoff from the equilibrium messages. In particular, in equilibrium, she will choose points $a_1, ..., a_{n-1}$ such that $a_i + b_h = \frac{y_i + y_{i+1}}{2}$. When the state is between a_i and a_{i+1}, the sender will send the message so that action y_i will be played in response. The expected payoff to the high bias expert from deviating is therefore given by:

$$CS^h_{b_l}(n) = \int_0^{a_1} -(y_1 - \theta - b_h)^2 \, d\theta + \int_{a_1}^{a_2} -(y_2 - \theta - b_h)^2 \, d\theta + \ldots \int_{a_{n-1}}^{1} -(y_n - \theta - b_h)^2 \, d\theta$$

Substituting the expressions for y_i and a_i and simplifying, we get that the expected payoff to the high bias expert from deviating is

$$CS^h_{b_l}(n) = \frac{-1}{12n^2} + {b_l}^2(\frac{4}{3} - \frac{1}{n} - \frac{n^2}{3}) + b_l b_h \frac{2(1-n)}{n} - \frac{{b_h}^2}{n} \tag{16}$$

Comparing 15 and 16, we get that the high bias sender will not deviate if:

$${b_l}^2(4 - \frac{3}{n} - n^2) + {b_h}^2(m^2 + 2 - \frac{3}{n}) - b_l b_h \frac{6(n-1)}{n} \;\leq\; \frac{1}{4}(\frac{1}{n^2} - \frac{1}{m^2}) \tag{17}$$

Inequality 17 captures the incentive compatibility constraint of the high bias expert for the prescribed strategies to constitute an equilibrium.

Now, let us consider the incentives of the low bias sender. Doing the same analysis as before, we can show that the low bias expert will not deviate from the prescribed strategies if:

$$-{b_h}^2(4-\frac{3}{m}-m^2)-{b_l}^2(n^2+2-\frac{3}{m})+b_lb_h\frac{6(m-1)}{m} \;\geq\; \frac{1}{4}(\frac{1}{n^2}-\frac{1}{m^2}) \tag{18}$$

Looking at the bias revealing incentives of the two types of senders jointly, we see that 17 and 18 can simultaneously hold only if:

$$-{b_h}^2(4-\frac{3}{m}-m^2)-{b_l}^2(n^2+2-\frac{3}{m})+b_lb_h\frac{6(m-1)}{m} \geq {b_l}^2(4-\frac{3}{n}-n^2)+{b_h}^2(m^2+2-\frac{3}{n})-b_lb_h\frac{6(n-1)}{n} \tag{19}$$

$$\iff (b_h-b_l)^2(2-\frac{1}{n}-\frac{1}{m}) \leq 0 \tag{20}$$

This inequality cannot hold unless $b_h = b_l$ or $n = m = 1$ (only babbling equilibrium is played). Since we have assumed that $b_l < b_h$ and we are only looking for non trivial equilibria in stage two, we conclude that the proposed strategies do not constitute an equilibrium since at least one type of sender will have incentives to deviate from truth telling in period 1. □

Proof of proposition 2

Proof. For a high bias sender:

$$\text{Payoff from announcing } b_h = p_h[\frac{1}{2}(-A-c)+\frac{CS^h_{b_h}(m)}{2}]+(1-p_h)(CS^h_{b_h}(1)) \tag{21}$$

$$\text{Payoff from announcing } b_l = p_h(-A-c)+(1-p_h)[\frac{1}{2}(-A-c)+\frac{CS^h_{b_l}(n)}{2}] \tag{22}$$

Thus, to satisfy this IC, we must have:

$$p_h \geq \frac{\frac{CS^h_{b_l}(n)}{2}-CS^h_{b_h}(1)-\frac{A+c}{2}}{\frac{CS^h_{b_l}(n)}{2}-CS^h_{b_h}(1)+\frac{1}{2}CS^h_{b_h}(m)}$$

Note that for a feasible solution to exist, we need

$$\frac{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) - \frac{A+c}{2}}{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) + \frac{1}{2}CS^h_{b_h}(m)} \leq 1$$

Since $\frac{1}{2}CS^h_{b_h}(m) > -\frac{A+c}{2}$[11], this is always satisfied.

Next, consider the incentives for a low bias sender:

$$\text{Payoff from announcing } b_h = p_h[\frac{1}{2}(-A) + \frac{CS^l_{b_h}(m)}{2}] + (1-p_h)CS^l_{b_l}(1) \tag{23}$$

$$\text{Payoff from announcing } b_l = p_h(-A) + (1-p_h)[\frac{1}{2}(-A) + \frac{CS^l_{b_l}(n)}{2}] \tag{24}$$

This incentive constraint is satisfied if:

$$p_h \leq \frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}} \tag{25}$$

Note that for a feasible solution to exist, we need

$$\frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}} \geq 0$$

Now, the numerator

$$\frac{CS^l_{b_l}(n) - CS^l_{b_l}(1)}{2} \geq 0$$

since $-A = CS^l_{b_l}(1)$, and $CS^l_{b_l}(n) \geq CS^l_{b_l}(1)$ since $n \geq 1$. The denominator can be rewritten as:

$$\begin{aligned} & \frac{CS^l_{b_l}(n) - CS^l_{b_l}(1)}{2} + \frac{CS^l_{b_h}(m) - CS^l_{b_l}(1)}{2} \\ \geq \; & CS^l_{b_h}(m) - CS^l_{b_l}(1) \quad \left(\sin ce\ n \geq m \Rightarrow CS^l_{b_l}(n) \geq CS^l_{b_h}(m)\right) \end{aligned}$$

This holds if ${b_h}^2 < \frac{1}{4m^2}$. We know this is true because $b_h < \frac{1}{2m(m-1)}$ (to guarantee an m partition equilibrium in the CS world for the sender's bias equal to b_h).

[11] As the payoff from being hired is always designed to be greater than not being hired.

So, there is a range of p_h for which both types' incentive constraints hold only if:

$$\frac{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) - \frac{A+c}{2}}{\frac{CS^h_{b_l}(n)}{2} - CS^h_{b_h}(1) + \frac{CS^h_{b_h}(m)}{2}} \leq \frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}}$$

For $c > (CS^h_{b_l}(n) - CS^h_{b_h}(1)) + (CS^l_{b_l}(1) - CS^h_{b_h}(1))$, the LHS is negative. We have already shown that the right-hand side is positive. Thus, the ICs can be satisfied simulteneously for a range of p_h.

Equating the IC for the low bias sender, we find

$$p' = \frac{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) - \frac{A}{2}}{\frac{CS^l_{b_l}(n)}{2} - CS^l_{b_l}(1) + \frac{CS^l_{b_h}(m)}{2}}$$

such that if $p_h \in [0, p']$ then bias revelation is an equilibrium for the strategies given in 3. □

Proof of proposition 3

Proof. Consider the following strategies. Both types of senders reveal their type. For the decision maker's strategy, we abuse notation and write the equilibrium she will play:

Decision maker strategy:

If DM observes b_l in stage $1 = (v_l)$Play j partition CS b_l eq $+ (1 - v_l)$Play k partition CS b_l eq

If DM observes b_h in stage $1 = (v_h)$Play x partition CS b_h eq $+ (1 - v_h)$Play y partition CS b_h eq

where $j, k, x, y \in \mathbb{N}$ and $v_l, v_h \in [0, 1]$. WLOG let $j \geq k, x \geq y$

Suppose such an equilibrium exists for some choice of parameters. The IC conditions for truth-telling is give by,

$$IC_{b_l} : \; v_l CS^l_{b_l}(j) + (1 - v_l) CS^l_{b_l}(k) \geq v_h CS^l_{b_h}(x) + (1 - v_h) CS^l_{b_h}(y) \tag{26}$$

$$IC_{b_h} : \; v_h CS^h_{b_h}(x) + (1 - v_h) CS^h_{b_h}(y) \geq v_l CS^h_{b_l}(j) + (1 - v_l) CS^h_{b_l}(k) \tag{27}$$

Adding, we get,

$$v_l(CS^l_{b_l}(j) - CS^h_{b_l}(j)) + (1-v_l)(CS^l_{b_l}(k) - CS^h_{b_l}(k)) \geq$$
$$v_h(CS^l_{b_h}(x) - CS^h_{b_h}(x)) + (1-v_h)(CS^l_{b_h}(y) - CS^h_{b_h}(y)) \tag{28}$$

Using equations 15, 16 and the corresponding expressions for the low bias sender, we can calculate:

$$CS^l_{b_l}(n) - CS^h_{b_l}(n) = (b_h - b_l)(b_h \frac{1}{n} + b_l(2 - \frac{1}{n})) \tag{29}$$
$$CS^l_{b_h}(m) - CS^h_{b_h}(m) = (b_h - b_l)(b_h(2 - \frac{1}{m}) + b_l \frac{1}{m}) \tag{30}$$

Plugging in the values from above in 28 we get,

$$(b_h - b_l)\left(\frac{v_l}{j} + \frac{(1-v_l)}{k} + \frac{v_h}{x} + \frac{(1-v_h)}{y} - 2\right) \geq 0 \tag{31}$$

Since $b_h \geq b_l$ this would be true if and only if

$$\frac{v_l}{j} + \frac{(1-v_l)}{k} + \frac{v_h}{x} + \frac{(1-v_h)}{y} \geq 2 \tag{32}$$

which would require $j,k,x,y \leq 1$. This would imply the equilibrium play would be babbling under all possible revelation in the first stage. However, this is not true since we were looking for an informative bias revealing equilibrium. Therefore our assumption is wrong and there does not exist any informative equilibrium in the one sender case even if we permit the receiver to mix with commitment in hiring. □

A.1 $v = 1$ revelation equilibrium

Statement: the revelation equilibrium with $v = 1$ is dominated by a non-revelation equilibrium (CH) where $k = n - 1$.

Proof. Consider the following revelation equilibrium: when both announcements are of same bias the DM chooses one at random and plays the most informative equilibrium. When the announcements differ DM choose b_h with probability 1 and plays most informative CS equilibrium (m partitions). We want to show that this equilibrium is dominated by CH equilibrium even when $k = n-1$. The payoff from revelation equilibrium is

$$U^r = (p^2 + 2p(1-p))(-\frac{1}{12m^2} - b_h^2\frac{m^2-1}{3}) + (1-p)^2(-\frac{1}{12n^2} - b_l^2\frac{n^2-1}{3})$$

The payoff from non-revelation equilibrium is

$$U^{nr} = -\frac{1}{12k^2} - b^2\frac{k^2-1}{3} - \frac{k-1}{k}p(1-p)(b_h - b_l)^2$$

......; $\square$

B General Expression for CH Equilibrium

$$\begin{aligned}
y_1 &= p_h\frac{a_1^h}{2} + (1-p_h)\frac{a_1^l}{2} \\
y_2 &= p_h\frac{a_1^h + a_2^h}{2} + (1-p_h)\frac{a_1^l + a_2^l}{2} \\
&\quad . \\
&\quad . \\
y_k &= p_h\frac{a_{k-1}^h + a_k^h}{2} + (1-p_h)\frac{a_{k-1}^l + a_k^l}{2} \\
&\quad . \\
&\quad . \\
y_n &= p_h\frac{1 + a_{n-1}^h}{2} + (1-p_h)\frac{1 + a_{n-1}^l}{2}
\end{aligned}$$

Now, since

$$a_1^h = \frac{y_1 + y_2}{2} - b_h; a_1^l = \frac{y_1 + y_2}{2} - b_l$$

we get:

$$\begin{aligned}
y_1 &= p_h \frac{\frac{y_1+y_2}{2} - b_h}{2} + (1-p_h)\frac{\frac{y_1+y_2}{2} - b_l}{2} \\
&= \frac{y_1+y_2}{4} - b/2 \\
& \quad \textit{where} \\
b &= p_h b_h + (1-p_h)\, b_l \\
&= \text{expected bias} \\
\frac{y_2}{4} &= \frac{3y_1}{4} + \frac{b}{2} \Longrightarrow y_2 = 3y_1 + 2b
\end{aligned}$$

Now,

$$\begin{aligned}
y_k &= p_h \frac{\frac{y_{k-1}+y_k}{2} - b_h + \frac{y_{k+1}+y_k}{2} - b_h}{2} + (1-p_h)\frac{\frac{y_{k-1}+y_k}{2} + \frac{y_{k+1}+y_k}{2} - 2b_l}{2} \\
&= \frac{y_{k-1}+2y_k+y_{k+1}}{4} - b \qquad (33) \\
2y_k &= y_{k-1} + y_{k+1} - 4b \\
(y_{k+1}-y_k) - (y_k - y_{k-1}) &= 4b \qquad (34)
\end{aligned}$$

Now,

$$\begin{aligned}
y_2 - y_1 &= 2y_1 + 2b \\
y_3 - y_2 &= y_2 - y_1 + 4b = 2y_1 + 2b + 4b \\
y_4 - y_3 &= y_3 - y_2 + 4b = 2y_1 + 2b + 2(4b) \\
& . \\
& . \\
y_{k+1} - y_k &= 2y_1 + 2b + (k-2)\,4b \\
y_n - y_{n-1} &= 2y_1 + 2b + (n-2)\,4b \qquad (35)
\end{aligned}$$

And:

$$\begin{aligned}
y_n &= \frac{p_h}{2}\left(\frac{y_{n-1}+y_n}{2}+1-b_h\right)+\frac{1-p_h}{2}\left(\frac{y_{n-1}+y_n}{2}+1-b_l\right) \\
&= \frac{y_{n-1}+y_n}{4}-\frac{1}{2}-\frac{b}{2} \\
Simplifying &: \\
y_n &= 1-y_1-2\left(n-1\right)b \qquad (36)
\end{aligned}$$

Another way to find the expression of y_n :

$$\begin{aligned}
y_2 &= y_1+2y_1+2b=3y_1+2b \\
y_3 &= y_2+2y_1+6b \\
&= 5y_1+8b \\
y_4 &= y_3+2y_1+10b=7y_1+18b \\
y_k &= \left(2k-1\right)y_1+2\left(k-1\right)^2 b \forall k>2 \\
y_n &= \left(2n-1\right)y_1+2\left(n-1\right)^2 b \qquad (37)
\end{aligned}$$

Equating expressions, we get:

$$\begin{aligned}
\left(2n-1\right)y_1+2\left(n-1\right)^2 b &= 1-y_1-2\left(n-1\right)b \\
2ny_1 &= 1-b\left(2n-2+2n^2+2-4n\right) \\
&= 1-2bn\left(n-1\right) \\
y_1 &= \frac{1}{2n}-b\left(n-1\right)
\end{aligned}$$

For this to be valid, we need:

$$y_1 \geq 0 \Longleftrightarrow \frac{1}{2n}-b\left(n-1\right) \geq 0 \Longleftrightarrow b \leq \frac{1}{2n\left(n-1\right)} \qquad (38)$$

This is similar to CS So, we have:

$$
\begin{aligned}
y_k &= (2k-1)\,y_1 + 2\,(k-1)^2\,b \\
&= \frac{(2k-1)}{2n} - (2k-1)\,b\,(n-1) + 2\,(k-1)^2\,b \\
&= \frac{(2k-1)}{2n} + b\left[2k^2 - 2k + 1 - 2kn + n\right] \\
&= \frac{(2k-1)}{2n} + b\left[2k^2 - (2k-1)\,(n+1)\right] \qquad (39)
\end{aligned}
$$

Now, Receiver's payoff from this CH eqlm:

$$
U_{CH}^{DM} = p_h U_h^{DM} + (1-p_h)\,U_l^{DM} \qquad (40)
$$

where $U_h^{DM} =$ DM's payoff if the hired expert is high biased

Calculate U_h^{DM} separately:

$$
\begin{aligned}
U_h^{DM} &= \\
&= \int_0^{a_1^h} -(y_1-\theta)^2\,d\theta + \int_{a_1^h}^{a_2^h} -(y_2-\theta)^2\,d\theta + \ldots + \int_{a_{k-1}^h}^{1} -(y_n-\theta)^2\,d\theta \\
&= -\frac{1}{3}\left[\left[(y_1-\theta)^3\right]_0^{a_1^h} + \left[(y_2-\theta)^3\right]_{a_1^h}^{a_2^h} + .. + \left[(y_k-\theta)^3\right]_{a_{k-1}^h}^{1}\right] \\
&= \frac{1}{3}\left[(1-y_k)^3 - y_1^3 - \frac{1}{4}\sum_{j=1}^{k-1}\left(\left(y_j - a_j^h\right)^3 - \left(y_{j+1} - a_j^h\right)^3\right)\right] \qquad (41)
\end{aligned}
$$

Now,

$$
\begin{aligned}
& y_j - a_j^h \\
&= \frac{(2j-1)}{2k} + b\left[2j^2 - (2j-1)\,(k+1)\right] - \frac{j}{k} + b2j\,(k-j) + (1-p_h)\,d \\
&= \frac{-1}{2k} + b\,[k-2j] + b_h \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
& y_{j+1} - a_j^h \\
= \ & \frac{(2j+1)}{2k} + b\left[2(j+1)^2 - (2j+1)(k+1)\right] - \frac{j}{k} + b(2j(k-j)-1) + b_h \\
= \ & \frac{1}{2k} + b[2j-k] + b_h \qquad (43)
\end{aligned}
$$

So,

$$
\begin{aligned}
& \left(y_j - a_j^h\right)^3 - \left(y_{j+1} - a_j^h\right)^3 \\
= \ & \left(-\frac{1}{2k} - b[2j-k] + b_h\right)^3 - \left(\frac{1}{2k} + b[2j-k] + b_h\right)^3 \\
= \ & -2\left(\frac{1}{2k} + b[2j-k]\right)^3 - 6b_h^2\left(\frac{1}{2k} + b[2j-k]\right)
\end{aligned}
$$

since $(a-b)^3 - (a+b)^3 = -2b^3 - 6a^2b$

Thus,

$$
\begin{aligned}
& \frac{1}{4}\sum_{j=1}^{k-1}\left(\left(y_j - a_j^h\right)^3 - \left(y_{j+1} - a_j^h\right)^3\right) \\
= \ & \frac{1}{4}\sum_{j=1}^{k-1}\left(-2\left(\frac{1}{2k} + b[2j-k]\right)^3 - 6b_h^2\left(\frac{1}{2k} + b[2j-k]\right)\right) \\
= \ & -\left[\frac{k-1}{4k^3} + b^2(k-1)(k-2) + 3b_h^2\frac{k-1}{k}\right]
\end{aligned}
$$

Using this, we get

$$
\begin{aligned}
U_h^{DM} &= -\frac{1}{3}\left[\frac{k-1}{4k^3} + b^2(k-1)(k-2) + 3b_h^2\frac{k-1}{k} + \left(\frac{1}{2k} + b(k-1)\right)^3 + \left(\frac{1}{2k} - b(k-1)\right)^3\right] \\
&= -\frac{1}{12k^2} - \frac{b^2}{3}(k-1)\left(k+1-\frac{3}{k}\right) - b_h^2\frac{k-1}{k} \qquad (44)
\end{aligned}
$$

Similarly, we can calculate

$$
U_l^{DM} = -\frac{1}{12k^2} - \frac{b^2}{3}(k-1)\left(k+1-\frac{3}{k}\right) - b_l^2\frac{k-1}{k} \qquad (45)
$$

This gives us the final expression for DM's payoff in a conflict hiding (CH) equilibrium:

$$\begin{aligned} U_{CH}^{DM} &= p_h U_h^{DM} + (1-p_h) U_l^{DM} \\ &= -\frac{1}{12n^2} - \frac{b^2}{3}(n-1) - d^2 p_h (1-p_h) \frac{n-1}{n} \end{aligned} \tag{46}$$

Now, DM's payoff from the best revealing equilibrium is:

$$U_R^{DM} = \left(p_h^2 + 2v p_h (1-p_h)\right) u_m + \Big((1-p_h)^2 + 2(1-v) p_h (1-p_h)\Big) u_n \tag{47}$$

where u_m (respectively, u_n) is the CS equilibrium payoff for the DM when there are m (respectively n) partitions by a high bias (respectively, low bias) expert.

$$u_m = -\frac{1}{12m^2} - \frac{b_h^2}{3}\left(m^2-1\right); u_n = -\frac{1}{12n^2} - \frac{b_l^2}{3}\left(n^2-1\right)$$

Simplifying, we get the difference between these two payoffs:

$$\begin{aligned} U^D &= U_R^{DM} - U_{CH}^{DM} \\ &= p_h^2 \left((1-2v)(u_m - u_n) + \frac{k^2-1}{3} d^2 - \frac{k-1}{k} d^2 \right) \\ &\quad + p_h \left(2v (u_m - u_n) + \frac{k^2-1}{3} 2 b_l d + \frac{k-1}{k} d^2 \right) \\ &\quad + \left(u_n + \frac{1}{12k^2} + \frac{k^2-1}{3} b_l^2 \right) \end{aligned} \tag{48}$$

$$u_m = -\frac{1}{12m^2} - \frac{b_h^2(m^2-1)}{3}; \quad u_n = -\frac{1}{12n^2} - \frac{b_l^2(n^2-1)}{3}$$

Define the difference between payoff from best revelation and non-revelation (CH) equilibrium as follows:

$$\begin{aligned} U^D = U^r - U^{nr} &= (p^2 + 2vp(1-p))u_m + ((1-p)^2 + 2(1-v)p(1-p))u_n \\ &\quad + \frac{1}{12k^2} + b^2 \frac{k^2-1}{3} + \frac{k-1}{k} p(1-p)(b_h - b_l)^2 \end{aligned}$$

Rearranging we get,

$$U^D(p) = p^2((1-2v)(u_m - u_n) + \frac{k^2-1}{3}d^2 - \frac{k-1}{k}d^2)$$
$$+p(2v(u_m - u_n) + \frac{k^2-1}{3}2b_l d + \frac{k-1}{k}d^2) + (u_n + \frac{1}{12k^2} + \frac{k^2-1}{3}b_l^2)$$

Note that this is a quadratic function in p (for $d > 0$) and the following expressions are the first and the second order derivative of $U^D(p)$,

$$\frac{dU^D(p)}{dp} = 2((1-2v)(u_m - u_n) + \frac{k^2-1}{3}d^2 - \frac{k-1}{k}d^2)p + (2v(u_m - u_n) + \frac{k^2-1}{3}2b_l d + \frac{k-1}{k}d^2)$$
$$\frac{d^2U^D(p)}{dp^2} = 2((1-2v)(u_m - u_n) + \frac{k^2-1}{3}d^2 - \frac{k-1}{k}d^2)$$

To find the global extremum let us consider the FOC as follows,

$$\frac{dU^D(p)}{dp} = 0$$
$$\hat{p} = \frac{-(2v(u_m - u_n) + \frac{k^2-1}{3}2b_l d + \frac{k-1}{k}d^2)}{2((1-2v)(u_m - u_n) + \frac{k^2-1}{3}d^2 - \frac{k-1}{k}d^2)}$$