Violence as a signal in electoral democracy

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Abstract

In electoral democracies around the world, majoritarian ethnic parties often mobilize political support by engaging in anti-minority violence. While such behavior is common, it is not immediately clear why this strategy appeals to rational voters. In this paper, we offer an explanation for why rational voters from the majority (minority) group may vote for a party that organizes anti-minority (anti-majority) violence. Our model suggests that by engaging in such violence, parties send a costly signal to voters, indicating that if elected, they will prioritize the provision of majority(minority)-specific public goods. This strategy is particularly effective in environments where electoral promises lack credibility. We find that in a two-party electoral competition, signals of both majority and minority bias cannot coexist in equilibrium. Using the *universal divinity* criterion, we show that minority-biased signals are sent only when the majority and minority groups are of comparable size, while majoritarian signals are sent, at high signaling costs, only when the majority group is significantly larger. Additionally, we find that the likelihood of sending extreme signals increases when parties prioritize electoral victory over implementing their ideal policy positions.

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Introduction

Ideally, elections are fought on the promise of public goods. In reality, however – specially in less developed countries – strategies such as bribing (Mitra and Mitra, 2017), clientelism (Breeding, 2011; Bardhan and Mookherjee, 2012; Veenendaal and Corbett, 2020), voter intimidation (Frye et al., 2019) and mafia involvement (De Feo and De Luca, 2017) are often employed as strategies for winning elections. Besides these strategies, campaigns laden with anti-minority rhetoric and the role of news and social media in amplifying such sentiments have been widely used to explain the rise of Donald Trump in the U.S. and several anti-immigrant parties in Europe (Hooghe and Dassonneville, 2018; Enns and Jardina, 2021; Boomgaarden and Vliegenthart, 2007; Müller and Schwarz, 2023). The scenario is even more stark in the context of less developed countries where anti-minority campaign rhetoric often culminates in episodes of ethnic violence (Kongkirati, 2016; Ezeibe, 2021; Wilkinson, 2006).

While there is ample evidence on the use of ethnic violence in manipulating election results, theoretically there is no clear mechanism that explains this phenomenon. In rational choice models, voters maximize their expected utilities under different candidates or parties and vote accordingly (Downs, 1957). While in these models voter behavior is influenced by allocation of public goods, bureaucratic efficiency and macroeconomic policies, more recent literature has attempted to broaden them to also include identity in the individual voter's preferences, thus making the ethnic, religious or racial identity of the candidate or the party an important factor in influencing electoral outcomes (Kramer, 1971; Stigler, 1973; Fair, 1996; Glaeser, 2005; Fearon, 1999). As ethnic groups become electorally viable, ethnic identity becomes politically salient in catapulting the rise of identity politics and of political parties based on ethnic identities (Posner, 2004). Studies analyzing the political logic of violence find that violence or polarizing anti-minority events take place in the most competitive seats and this is confirmed by higher electoral dividends for the party with the strongest majority identity in the subsequent election for constituencies worst affected by such events (Wilkinson, 2006; Dhattiwala and Biggs, 2012).

Why might voters respond to such electoral strategies? Sen (2007) in his seminal work on identity argues that people have multiple identities which become salient at different times given the environment and context in which they live. In India, as such, voters often engage in group based voting along the lines of gender, religion, caste or ethnicity. Much of the literature in social science, therefore, has argued that communal riots effect electoral outcomes through the salience of religious identities (Brass, 2011; Wilkinson, 2006; Varshney, 2003). Riots are, thus, instrumental in making the religious identity of voters salient, thereby consolidating vote for the party that best reflects that religious identity. In this paper, we try to theorize why use of violence can be a rational election strategy.

Specifically, why does ethnic violence against one group (say, minorities) induce votes from the other group (say, majority)? One explanation could be that the majority derives utility from violence towards minority (or, vice versa) and in doing so attracts votes. We, in this paper, offer a different theoretical explanation.

It is already known from Alesina et al. (1999) that in ethnically diverse constituencies, group identity matters for social coordination which determines the type and the size of public good because preference for different types of public goods varies across ethnic groups. Hence, to rally support, political parties representing ethnic interests must promise ethnicity specific public goods along with more general public goods. In the presence of incomplete information, parties through their promises of ethnicity specific public goods can signal their type i.e, their ethnic bias in favour of (or against) a community (or communities). However, in an institutional setting where electoral promises are not credible, voters cannot distinguish between cheap-talking and more honest intentions of promise keeping. Therefore, to signal its intention of keeping its promise of providing ethnicity specific good upon winning election, a party in our model, would take costly political actions. We show that parties transmit such costly signals i.e, undertake costly political actions only when the constituency is sufficiently polarized or if the size of the majority community is sufficiently large.

In our model, policies (or public goods) play a central role in mobilizing votes. In this framework, there can be two types of public goods – neutral public goods (e.g. hospitals) and ethnicity specific public goods (e.g. religious establishment such as temple and mosque). Voters care about both types of public goods. Political parties on the other hand, care about two things. First, they get utility from being in power. Second, they have a bliss point i.e, an ideal amount of ethnicity specific public good. Therefore, if the actual ethnic public good deviates from the party's ideal, they get a disutility. In our model, parties send costly signals to signal this ideal position which we refer to as the 'ideology' of the party. Such signals in reality take the form of ethnic riots, hate speech and other forms of violence and vandalism. More specific examples of such signals garnering electoral support would be the Hindu nationalist Bharatiya Janata Party's (then Jana Sangh) cow protection movement in 1960s, riots and their temple building campaign in the 1990s which increased their vote share in the subsequent national and state elections (after 1990s) or the onset of electoral gains for Shiv Sena in the 1980s following their militant activism and vandalism of South Indian and Gujarati business establishments, in favour of Marathi causes like language and employment for the *Marathi manoos* (Iyer and Shrivastava, 2018; Johari, 2015; Roychowdhury, 2018).

The voters are same across ethnic groups regarding their preference for neutral public good, but they differ in terms of their preference for group-specific public goods. In our model, there is no commitment device (as in Callander and Wilkie (2007)) -i.e, parties do not have to stick to their announced platforms. Hence, a party which cares about the ethnic minority, can send anti-minority signal before election and implement pro-minority policy upon winning. However, what disciplines the party is the assumption that the cost of signaling increases the further the signal is, from the party's ideal policy which is similar to Banks (1990) where the costs to the candidates increase if the distance between the announced and the true position increases. In other words, sending a pro-minority signal will be costly for a majoritarian party and vice-versa.

The sequence of the game is as follows: parties observe their ideal policy privately and then send signals. All voters observe the signals and update their beliefs regarding each party's ideal policy position. Based on their updated beliefs about each party's ideal position, voters vote sincerely, i.e. each voter votes for the party which is expected to give her higher payoff given the signals. The winner is decided using a simple majority rule and the winner implements her ideal policy.

Our paper is related to a wide variety of models in the voting literature. In its essence, our model is closely linked with the costly signaling model of Banks (1990) where in a two candidate electoral competition, voters are uncertain about the policy that the elected candidate would implement because of the distance between the candidate's announced position and his true position. Our paper is also related to a bunch of papers which model political activism as a signaling mechanism (Lohmann, 1993, 1994, 1995).

The paper is organized as follows: in the next section (Section 2) we present the model, Section 3 characterizes the equilibrium behavior of the political parties and voters and in Section 4 we conclude. Proofs of the results are relegated to the appendix following the concluding section.

Model

We set up a model of political competition with two political parties - 1 and 2 - and electorate distributed between two groups - A and B. The size of the electorate is normalized to 1. Group A is the majority group with size α and Group B is the minority group with size $1-\alpha$ where $\alpha > \frac{1}{2}$.

We assume that there is a fixed budget (again normalized to 1) to be spent on two types of public goods. There is a neutral public good the benefit of which accrues to all voters. However, spending can be diverted to some group-specific public good which only benefits the voters belonging to that particular group. In our model $q \in [-1, 1]$ denotes the policy regarding group-specific public good that is being implemented. Specifically, any $q > 0$ indicates that the amount $|q|$ is being spent on a public good that is beneficial to majority (Group A), while $q < 0$ indicates that the amount $|q|$ is being spent on a public good that is beneficial to minority (Group B). Whenever $|q|$ is spent on a group-specific public good, the rest of the amount of the public good budget, i.e. $1- |q|$ is spent on neutral public good. The policy variable q is chosen by the winner of the political competition.

Political Parties

We assume that political party i, $i = 1, 2$ has an ideal group-specific public good policy q_i which is private information. We assume that q_i is distributed uniformly over $[-1, 1]$ and the distribution is common knowledge. Party i's payoff when policy q is implemented is given by $-(q - q_i)^2$. However, a party also suffers an additional loss $l \geq 0$ from losing the election. Notice that the party that wins the election is always going to implement its ideal policy. Hence, the winning payoff for a party is 0. Specifically, the pay off for party i is given by

$$W_i(q, q_i) = \begin{cases} 0 & \text{if } i \text{ wins} \\ -\left[l + (q - q_i)^2\right] & \text{if } i \text{ loses and policy } q \text{ is implemented} \end{cases} \tag{1}$$

In our specification, higher the value of l, more a party cares about winning the election relative to its ideological position.

Before voting takes place, party i can send a signal $s_i \in S_i$ to inform voters about its ideological position. We assume that the signal is publicly observable and S_i is the set of signals available to party i. In our analysis, we restrict S_i to a finite set of three elements $\{-1, 0, 1\}$. Signals are costly and the cost of sending signal $s_i \in S_i$ for i with type q_i is

$$C(s_i) = \begin{cases} c(1 + q_i) & \text{if } s_i = -1 \\ 0 & \text{if } s_i = 0 \\ c(1 - q_i) & \text{if } s_i = 1 \end{cases} \tag{2}$$

where $c > 0$. In our specification, the signal 0 can be interpreted as no signal. However, signal 1 and signal -1 are costly signals and the cost of sending signal 1 (alternatively -1) falls as q_i rises (alternatively falls)[1].

[1]We assume that conflicts are costly only for the party and not the community members the party is trying to appease. But in reality, an anti-minority riot may be costly for majority voters as well. In our model, there is no such cost. We assume that any conflict that a party organizes against ethnic group A does not affect ethnic group B. This is consistent with the nature of ethnic violence in an electoral democracy like India. Here, ethnic riots are low-grade localised conflicts where the number of casualty is much lower than the large scale ethnic conflicts we see in a non-democratic set up. For example, compared to the Rwandan genocide which claimed 500,000 to 1 million lives, the number of casualties in religious riots in India is much lower. A case in point would be the Godhra riot of 2002, one of the biggest episodes of ethnic violence in independent India, in which around 1000 people were killed. Similarly, in the anti-Sikh riot of 1984, around 3000 people lost their lives. But for most of the riots that took place in independent India, the number of casualties is much lower.

Voters

The preferences of the voters in the two groups differ with respect to the group-specific public good, but not with respect to the neutral public good. We also assume that all members belonging to the same group do not value the group-specific public good similarly. A voter's preference over group-specific public good vis-a-vis the neutral public good is captured by the preference parameter $\gamma \in [0, 2]$. The preference parameter γ is voter-specific and captures the voter's relative valuation of her group-specific public good vis-a-vis the neutral public good. More specifically, if a voter with the preference parameter γ belongs to group A, her willingness to substitute the neutral public good for A-specific public good is given by γ. Hence, there are differences, both within group and between the group in voters' preferences. There is no envy in our model, i.e. a majority voter does not get any disutility from the minority voter getting its group-specific public good and vice versa. We have already mentioned that a policy $q > 0$ (alternatively $q < 0$) implies that the amount $|q|$ will be spent on A-specific (alternatively B-specific) public good. Specifically, the payoff to a Group A voter of type γ from policy q is

$$u_A(q, \gamma) = \begin{cases} 1 - |q| + \gamma |q| & \text{if } q \geq 0 \\ 1 - |q| & \text{if } q < 0 \end{cases} \tag{3}$$

Similarly, the payoff to a Group B voter of type γ from policy q is

$$u_B(q, \gamma) = \begin{cases} 1 - |q| + \gamma |q| & \text{if } q \leq 0 \\ 1 - |q| & \text{if } q > 0 \end{cases} \tag{4}$$

We assume that γ is uniformly distributed in $[0, 2]$. Notice that voters of type $\gamma \in [0, 1)$ value neutral public good more than the group-specific one while voters of type $\gamma \in (1, 2]$ value group-specific public good more.

The Game

Party i privately observes q_i and then both parties choose $s_i \in S_i$ simultaneously. All voters observe the signals (s_1, s_2) and update their beliefs about each party's ideal policy position. Based on their updated beliefs about each party's ideal position, voters vote sincerely, i.e. each voter votes for the party which is expected to give her higher payoff given the signals. If a voter is indifferent, she votes for each party with equal probability. The winner is decided using a simple majority rule and the winner implements her ideal policy.

Strategies

A strategy for party i is a mapping from her type space $[-1, 1]$ to the signal space $S_i = \{-1, 0, 1\}$, i.e.

$$s_i : [-1, 1] \to \{-1, 0, 1\}$$

A voter's strategy on the other hand depends on the group she belongs to, her preference for the neutral public good vis-a-vis the group-specific public good characterized by the parameter γ and the signals she observes. For a group j voter, $j = A, B$, the voting strategy is a mapping as defined below:

$$r_j : S_1 \times S_2 \times [0,2] \to \left\{0, \frac{1}{2}, 1\right\}$$

where $r_j(.,.,.)$ is the probability of voting for party 1.

Equilibrium Behavior

We first examine strategies which constitute a Perfect Bayesian equilibrium (PBE). Briefly, a PBE consists of signaling strategies $s_1^*(.), s_2^*(.)$ for the parties which are optimal with respect to each other as well as to the strategies $r_A^*(.,.,.), r_B^*(.,.,.)$ of the voters, where these voting strategies are optimal given the voters' beliefs, $\mu_1^*(.), \mu_2^*(.)$, regarding the true policy positions of the parties. It requires voters to form these beliefs about parties 1 and 2 for all possible signals, so that $\mu_i^*(s_i)$ is defined for all $s_i \in S_i$, $i = 1, 2$. These beliefs need to be rational in the sense that, upon observing a signal s_i, from party i, $\mu_i^*(s_i)$ is the posterior probability obtained via Bayes' Rule from the party's signaling strategy $s_i^*(.)$ and the common knowledge prior.

We look at equilibrium in symmetric cutoff[2] signaling strategies for the parties. This means that equilibrium strategies must satisfy two conditions:

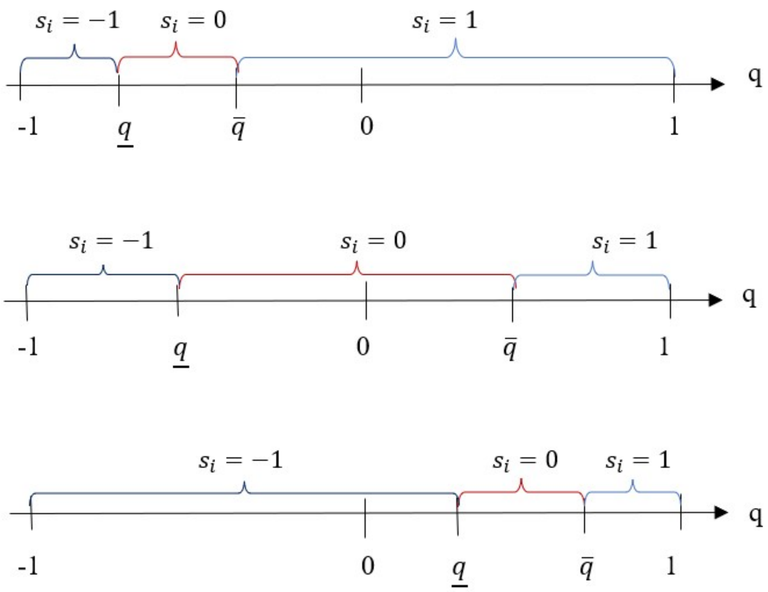
1. $q_1 = q_2 \Rightarrow s_1^*(q_1) = s_2^*(q_2)$
2. For some $\underline{q}, \bar{q} \in [-1, 1]$ with $\underline{q} \leq \bar{q}$,

$$s_i^*(q_i) = \begin{cases} -1 & \text{if } q_i \in [-1, \underline{q}) \\ 0 & \text{if } q_i \in [\underline{q}, \bar{q}] \\ 1 & \text{if } s_i \in (\bar{q}, 1] \end{cases} \tag{5}$$

Since we are looking at symmetric strategies, we can ignore the subscript in the strategy functions. We also assume that if a voter prefers the expected policy outcome under party i, she votes for party i, but if she is indifferent she votes for each party with probability $\frac{1}{2}$.

[2]These strategies are referred to as "cutoff strategies" because parties choose their optimal signal depending on whether or not their ideal q_i lies below a cutoff value (i.e, within a specific range of q).

Figure 1: Cutoff strategies for the parties: $s_i^*(q_i)$



Notes: There can be three scenarios for the values of the lower and higher cutoffs. In the above figure we have three panels for the same. The first panel represents the scenario when both the lower and higher cutoff, $\bar{q}$ and $\underline{q}$ is negative. In the second panel, $\underline{q} < 0$ and $\bar{q} > 0$ and in the third panel, $\bar{q}$ and $\underline{q}$ is positive.

Unlike the type space which is continuous, the signal space, in our model, is restricted to three signals. Therefore, a fully informative or separating equilibrium is not possible in our structure. As such, we can have a semi-pooling and/or a pooling equilibrium. Unlike the pooling equilibrium which is completely uninformative, in a semi-pooling equilibrium, some information transmission takes place and this allows voters to update their beliefs accordingly.

We now define our equilibrium in terms of strategies of the players and beliefs. We use the standard notion of Perfect Bayesian Equilibrium which in the context of our model is formally defined below.

Definition 1 *A perfect Bayesian equilibrium of the above model consists of party strategies* $s_1^*(.)\,, s_2^*(.)$*, voter strategies* $r_A^*(.,.,.)\,, r_B^*(.,.,.)$*, and beliefs* $\mu_1^*(.)\,, \mu_2^*(.)$*, such that*

1. *For all* $q_i \in [-1,1]$*,* $s^*(q_i)$ *maximizes*

$$E_{q_i \in [-1,1]}\left[W\left(q_i, s, s^*\left(q_{i'}\right), r_A^*\left(s, s^*\left(q_{i'}\right)\right), r_B^*\left(s, s^*\left(q_{i'}\right)\right)\right)\right]$$

   for $i' \neq i$.

2. *For all* $\gamma \in [0,2]$ *and for all* $(s_1, s_2) \in S_1 \times S_2$*,*

$$r_A^*\left(s_1, s_2, \gamma\right) = \begin{cases} 1 \\ \frac{1}{2} \\ 0 \end{cases} \quad \text{as } E_{q_1}\left[u_A\left(q_1, \gamma\right)\right]|_{\mu_1^*(q_1|s_1)} \gtreqless E_{q_2}\left[u_A\left(q_2, \gamma\right)\right]|_{\mu_2^*(q_2|s_2)}$$

   and

$$r_B^*\left(s_1, s_2, \gamma\right) = \begin{cases} 1 \\ \frac{1}{2} \\ 0 \end{cases} \quad \text{as } E_{q_1}\left[u_B\left(q_1, \gamma\right)\right]|_{\mu_1^*(q_1|s_1)} \gtreqless E_{q_2}\left[u_B\left(q_2, \gamma\right)\right]|_{\mu_2^*(q_2|s_2)}$$

3. *For all* $s_i \in S_i$*, if* $s_i^{*-1}(s_i) \neq \phi$*, then* $\mu_i^*(t_i|s_i)$ *is the conditional probability that* $q_i \in t_i \cap s_i^{*-1}(s_i)$ *given* $q_i \in s_i^{*-1}(s_i)$ *where* $t_i \subset [-1,1]$.

Condition 1 states that each party maximizes its expected payoff given the strategies of the other party and the voters. Condition 2 states that the voters vote for the party that gives them higher expected payoff and if indifferent chooses each party with equal probability. Condition 3 implies that voters use Bayes' Rule to update their beliefs in equilibrium after observing the signals.

Out-of-equilibrium beliefs

We restrict voters' off-equilibrium beliefs using the *universal divinity* criterion following Banks and Sobel (1987) and Banks (1990). *Universal divinity* requires that, for every out of equilibrium signal, voters form beliefs about which type of party is "most likely" to defect (i.e. the type that gets the highest possible payoff from the act of defection) and then place probability one on that type of party sending the out-of-equilibrium signal. Given our signal structure, this implies that if the out-of-equilibrium signal is $s_i = 1$ (alternatively s_i =-1), voters believe that the type "most likely" to defect is q_i= 1 (alternatively, q_i= -1) because for it the cost of signaling is zero. This follows from the differential signaling costs for signals $s_i = 1$ and $s_i =$ -1 across types. In other words, this implies that if the "most likely" type to defect does not have the incentive to defect, then no other type would either. However, if the out-of-equilibrium signal is s_i =0, every type is equally likely to send this signal since every type faces zero signaling cost.

Electoral Equilibrium

For our subsequent analysis, we concentrate on pure strategy equilibria in strategies and beliefs that satisfy Definition 1 and *universal divinity* as defined above. We call these equilibria "electoral equilibria". This is formally stated in the following definition.

Definition 2 *An electoral equilibrium of the model consists of party strategies, voter strategies, and beliefs as described in Definition 1 such that the out-of-equilibrium beliefs are restricted by universal divinity criterion.*

Voters' Optimal Strategies

Given Bayesian updating of beliefs and the proposed symmetric equilibrium strategy profile described in (5), the expected policy outcomes from party i after signals $s_i = -1$, $s_i = 0$ and $s_i = 1$ are $\frac{-1+\underline{q}}{2}$, $\frac{\underline{q}+\bar{q}}{2}$ and $\frac{\bar{q}+1}{2}$ respectively. Since voter preferences differ between the two groups with respect to q, we need to analyze the voting decision of the the voters belonging to different groups separately.

We need to analyze voter's strategies for three separate possibilities in our search for equilibrium - (i) $\bar{q} \leq 0$, (ii) $\underline{q} \geq 0$ and (iii) $\underline{q} \leq 0 \leq \bar{q}$. If both parties send the same signal, the probability of voting for either party is $\frac{1}{2}$ for all voters. However, if the signals differ, the optimal voting strategy for different members in both groups may be different. The optimal voting strategies for the voters belonging to either group when signals from the parties differ are described in Table 1 below. The derivation of these optimal strategies is relegated to the appendix.

Table 1: Equilibrium Voting Strategies

Signal Profile	$r_A^*(s_1, s_2, \gamma)$	$r_B^*(s_1, s_2, \gamma)$	Cut off
Case I: $\underline{q} \leq \overline{q} \leq 0$			
$s_1 = 0, s_2 = -1$	1 for all γ	1 for $\gamma < 1$ $\frac{1}{2}$ for $\gamma = 1$ 0 for $\gamma > 1$	
$s_1 = 1, s_2 = -1$	1 for all γ	1 for $\gamma < \gamma_1$ $\frac{1}{2}$ for $\gamma = \gamma_1$ 0 for $\gamma > \gamma_1$	$\gamma_1 = 1 - \frac{1}{\left(1-\underline{q}\right)(1-\bar{q})-\bar{q}^2}$
$s_1 = 1, s_2 = 0$	1 for $\gamma > \gamma_2$ $\frac{1}{2}$ for $\gamma = \gamma_2$ 0 for $\gamma < \gamma_2$	1 for $\gamma < \gamma_3$ $\frac{1}{2}$ for $\gamma = \gamma_3$ 0 for $\gamma > \gamma_3$	$\gamma_2 = 1 + \bar{q} + \underline{q} - \underline{q}\bar{q}$ $\gamma_3 = 1 - \frac{1}{-\bar{q}-\underline{q}+\underline{q}\bar{q}}$
Case II: $0 \leq \underline{q} \leq \overline{q}$			
$s_1 = 0, s_2 = -1$	1 for $\gamma > \gamma_4$ $\frac{1}{2}$ for $\gamma = \gamma_4$ 0 for $\gamma < \gamma_4$	1 for $\gamma < \gamma_5$ $\frac{1}{2}$ for $\gamma = \gamma_5$ 0 for $\gamma > \gamma_5$	$\gamma_4 = 1 - \frac{1}{\bar{q}+\underline{q}+\underline{q}\bar{q}}$ $\gamma_5 = 1 - (\bar{q} + \underline{q} + \underline{q}\bar{q})$
$s_1 = 1, s_2 = -1$	1 for $\gamma > \gamma_6$ $\frac{1}{2}$ for $\gamma = \gamma_6$ 0 for $\gamma < \gamma_6$	0 for all γ	$\gamma_6 = 1 - \frac{1}{\left(1+\underline{q}\right)(1+\bar{q})-\underline{q}^2}$
$s_1 = 1, s_2 = 0$	1 for $\gamma > 1$ $\frac{1}{2}$ for $\gamma = 1$ 0 for $\gamma < 1$	0 for all γ	
Case III: $\underline{q} \leq 0 \leq \overline{q}$			
$s_1 = 0, s_2 = -1$	1 for all γ	1 for $\gamma < \gamma_7$ $\frac{1}{2}$ for $\gamma = \gamma_7$ 0 for $\gamma > \gamma_7$	$\gamma_7 = 1 - \frac{\bar{q}^2}{\bar{q}-\underline{q}-\underline{q}\bar{q}}$
$s_1 = 1, s_2 = -1$	1 for $\gamma > \gamma_8$ $\frac{1}{2}$ for $\gamma = \gamma_8$ 0 for $\gamma < \gamma_8$	1 for $\gamma < \gamma_9$ $\frac{1}{2}$ for $\gamma = \gamma_9$ 0 for $\gamma > \gamma_9$	$\gamma_8 = 1 - \frac{1-\underline{q}}{1+\bar{q}}$ $\gamma_9 = 1 - \frac{1+\bar{q}}{1-\underline{q}}$
$s_1 = 1, s_2 = 0$	1 for $\gamma > \gamma_{10}$ $\frac{1}{2}$ for $\gamma = \gamma_{10}$ 0 for $\gamma < \gamma_{10}$	0 for all γ	$\gamma_{10} = 1 - \frac{\underline{q}^2}{\bar{q}-\underline{q}-\underline{q}\bar{q}}$

Equilibrium Behavior of the Parties

We denote by $v_i(s_1, s_2, \alpha)$ party i's vote share from any signal profile (s_1, s_2). From the previous discussion we know that Party 1's vote share for any signal profile (s_1, s_2) is

$$v_1(s_1, s_2, \alpha) = \alpha \int r_A^*(s_1, s_2, \gamma)\, d\gamma + (1-\alpha) \int r_B^*(s_1, s_2, \gamma)\, d\gamma$$

In Table 2 in the Appendix, we present party 1's vote-share and expected payoff for different signal profiles. If both parties send the same signal, the vote share and win probability for both of them is $\frac{1}{2}$. Given simple majority voting, party 1 wins the election with probability 1 iff $v_1(s_1, s_2, \alpha) > \frac{1}{2}$. If $v_1(s_1, s_2, \alpha) = \frac{1}{2}$, party 1 wins with probability $\frac{1}{2}$. Notice that if party 1 wins, its payoff is 0 (since it always implements q_1 on winning). Hence, each party chooses the signal so as to minimize the expected loss.

We first show that if party 2 adopts the signaling strategy described in (5), for $\underline{q} \leq 0$, party 1's sequentially rational strategy would never include $s_1 = -1$ for any realization of $q_1 \in [-1, 1]$. On the other hand, for $\underline{q} > 0$, party 1's sequentially rational strategy would never include $s_1 = 1$ for any $q_1 \in [-1, 1]$. Thus, we cannot have an equilibrium in monotone symmetric cutoff strategies in which all three signals are being sent by some types. This is summarized in the following lemma.

Lemma 1 *No symmetric electoral equilibrium in monotone cutoff strategies exists in which every $s_i \in \{-1, 0, 1\}$ is chosen by some $q_i \in [-1, 1]$.*

Proof. Proof of the lemma is in the Appendix. ■

The last lemma rules out any monotone cutoff strategy equilibrium with three intervals. We now examine whether a perfect Bayesian equilibrium in monotone cutoff strategies exists with two intervals. We once again have three possible cases:

1. $\underline{q} = -1$, which implies in equilibrium party i sends $s_i = 0$ if $q_i \in [-1, \bar{q}]$ and $s_i = 1$ if $q_i \in (\bar{q}, 1]$ for some $\bar{q} \in (-1, 1)$.
2. $\bar{q} = 1$, which implies in equilibrium party i sends $s_i = -1$ if $q_i \in [-1, \underline{q})$ and $s_i = 0$ if $q_i \in \left[\underline{q}, 1\right]$ for some $\underline{q} \in (-1, 1)$.
3. $\underline{q} = \bar{q} \in (-1, 1)$, which implies in equilibrium party i sends $s_i = -1$ if $q_i \in [-1, \underline{q})$ and $s_i = 1$ if $q_i \in (\underline{q}, 1]$.

We first argue that we cannot have an equilibrium with $\underline{q} = \bar{q}$ such that only the signals -1 and $+1$ are sent. As shown in the proof of Lemma 1, if $\underline{q} = \bar{q} < 0$, the best response of party i never includes $s_i = -1$ as $s_i = 0$ leads to lower expected loss given the strategy of the other player. Similarly, if $\underline{q} = \bar{q} \geq 0$, the best response of party i never includes $s_i = 1$.

Notice that if $\bar{q} = 1$, we cannot have an equilibrium with $\underline{q} < 0$ either. This is because if $\underline{q} < 0$, then for all $q_i \in [-1, 1]$, $s_i = 0$ leads to lower expected loss relative to $s_i = -1$ as argued in the proof of Lemma 1.

We next characterize the conditions under which a symmetric Perfect Bayesian equilibrium in cutoff strategies with two intervals may exist.

First we take up the case with $\bar{q} = 1$. From our discussion in the last paragraph we already know that we cannot have such an equilibrium with

$$s_i(q_i) = \begin{cases} -1 & \text{for } q_i \in [-1, \underline{q}) \\ 0 & \text{for } q_i \in [\underline{q}, 1] \end{cases} \tag{6}$$

for $\underline{q} < 0$. We show that there exists a symmetric equilibrium in cutoff strategies described in (6) under certain restrictions on the parameters of the model. We then examine the case of $\underline{q} = -1$. We show that a cutoff equilibrium with symmetric strategies described as

$$s_i(q_i) = \begin{cases} 0 & \text{for } q_j \in [-1, \bar{q}] \\ 1 & \text{for } q_j \in (\bar{q}, 1] \end{cases} \tag{7}$$

exists with $\bar{q} < 0$. Finally, we show that a symmetric equilibrium with the same cutoff strategy in (7) with $\bar{q} \geq 0$ exists for different values of the parameters.

All these are stated in the following lemma.

Lemma 2 *1. Suppose $l \leq \frac{2}{3}$. Then, for all $c > \frac{l}{4} + \frac{1}{3}$ there exists $\alpha_2(l,c) > \frac{1}{2}$ such that for all $\alpha \geq \alpha_2(l,c)$, we have a cutoff equilibrium with*

$$s_i^*(q_i) = \begin{cases} 0 & \text{for } q_i \in [-1, \bar{q}_2(l,c)] \\ 1 & \text{for } q_i \in (\bar{q}_2(l,c), 1] \end{cases}$$

for some $\bar{q}_2(l,c) \in [0,1)$.

2. Suppose $l > \frac{2}{3}$. Then for $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$, there exists $\alpha_1(l,c) > \frac{1}{2}$ such that for all $\frac{1}{2} < \alpha \leq \alpha_1(l,c)$, we have a cutoff equilibrium with

$$s_i^*(q_i) = \begin{cases} -1 & \text{for } q_i \in [-1, \underline{q}(l,c)) \\ 0 & \text{for } q_i \in [\underline{q}(l,c), 1] \end{cases}$$

for some $\underline{q}(l,c) \in [0,1)$. For all values of $\alpha > \frac{1}{2}$, there is another cutoff equilibrium with

$$s_i^*(q_i) = \begin{cases} 0 & \text{for } q_i \in [-1, \bar{q}_1(l,c)] \\ 1 & \text{for } q_i \in (\bar{q}_1(l,c), 1] \end{cases}$$

for some $\bar{q}_1(l,c) \in (-1,0)$. Lastly, for $c > \frac{l}{6} + \frac{1}{2}$ there exists $\alpha_2(l,c) > \frac{1}{2}$ such that for all $\alpha \geq \alpha_2(l,c)$, we have another {0,1} equilibrium with

$$s_i^*(q_i) = \begin{cases} 0 & \text{for } q_i \in [-1, \bar{q}_2(l,c)] \\ 1 & \text{for } q_i \in (\bar{q}_2(l,c), 1] \end{cases}$$

for some $\bar{q}_2(l,c) \in [0,1)$.

Proof. The proof is in the Appendix ■

The second part of Lemma 2 shows when parties attach high value to loss from election and the signaling cost is middling - we have two cutoff equilibria: one where besides the neutral signal, only signal indicative of minority specific public good preference is transmitted and one where besides the neutral signal, only signal indicative of majority specific public good preference is. However, both these results differ in terms of their parametric specification: the first one holds only when α lies below a certain threshold value and the second one holds for all value of α. The first result is counter-intuitive, since the majority never prefers the public good budget being spent on minority specific public good. This explains why the result holds only when the size of the minority is sufficiently large, i.e, the size of the majority is below a critical level $\alpha_1(l, c)$. However, with higher signaling costs, the only cutoff equilibrium that may exist is the one where along with the neutral signal, the signal indicative of majority specific public good preference is transmitted if and only if the size of the majority is sufficiently large.

Semi-pooling equilibria under universal divinity

In this section, we analyze which of our semi-pooling equilibria survives under universal divinity. Taking the first of our proposed equilibrium $\{-1, 0\}$, the only signal that a deviating party can send is that of s_i=1. Under universal divinity, voters on seeing the s_i= 1 attach probability 1 to the deviating party to be of type q_i=1. Under such a belief, the minority would never vote for the deviating party. To win, the party would then need to rely on the majority voters. We show that under universal divinity, the majority votes received are not enough to win (for the deviating party). Therefore, no party has incentive to deviate from the proposed equilibria $\{-1, 0\}$. Similarly, we also show that for our two majoritarian semi-pooling equilibria $\{0, 1\}$ with $\bar{q} < 0$ $[\bar{q}_1(l, c)]$ and the one with $\bar{q} > 0$ $[\bar{q}_2(l, c)]$, any deviation to signal s_i= -1 would be sub-optimal for a party as the majority would never would vote it. This is described in the following proposition.

Proposition 1 *All equilibria described in Lemma 2 surive under universal divinity.*

Proof. Proof is in the Appendix. ■

Next, we look at how the parameters l and c affect the cutoffs described in Proposition 1 in the relevant range. We establish the properties of the cutoffs in the Lemma 3.

Lemma 3 *1. For $l > \frac{2}{3}$ and $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$, $\underline{q}(l, c)$ is strictly increasing in l and strictly decreasing in c.*

2. For $l > \frac{2}{3}$ and $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$, $\bar{q}_1(l, c)$ is strictly decreasing in l, but strictly increasing in c.

3. If $c > \max\left\{\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right\}$, $\bar{q}_2(l, c)$ is strictly decreasing in l, but strictly increasing in c.

Proof. Proof is in the Appendix. ■

We now look at the ex-ante probability of observing extreme signals in different equilibria. We first consider the equilibrium with

$$s_i^*(q_i) = \begin{cases} -1 & \text{for } q_i \in [-1, \underline{q}(l, c)) \\ 0 & \text{for } q_i \in [\underline{q}(l, c), 1] \end{cases}$$

This equilibrium exists when $l > \frac{2}{3}$ and $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$. Moreover, we also need $\alpha \leq \alpha_1 (l, c)$. Suppose these conditions are met. Then, the probability that at least one party will be sending signal -1 is

$$1 - \left(\frac{1 - \underline{q}(l, c)}{2}\right)^2$$

As l increases $\underline{q}(l, c)$ increases. This will not affect the requirement $\alpha \leq \alpha_1 (l, c)$ since $\alpha_1 (l, c)$ is strictly increasing in l. Thus the probability of observing the signal indicative of minority specific public good preference will increase.

In the other equilibria with

$$s_i^* (q_i) = \begin{cases} 0 & \text{for } q_i \in [-1, \bar{q}(l, c)] \\ 1 & \text{for } q_i \in (\bar{q}(l, c), 1] \end{cases}$$

the probability of observing the extreme signal is

$$1 - \left(\frac{1 + \bar{q}}{2}\right)^2$$

Since the cutoffs $\bar{q}_1 (l, c)$ and $\bar{q}_2 (l, c)$ are strictly decreasing in l, the above probability is also increasing in l. The above discussion is summarized in the following proposition.

Proposition 2 *Whenever a cutoff equilibrium exists, the probability that an extreme signal is sent by one of the parties increases as the parties care relatively more about losing the election and lower is the signaling cost.*

As parties care more about electoral loss, the cutoffs $\underline{q}(l, c)$ increases and that of $\bar{q}_1 (l, c)$ and $\bar{q}_2 (l, c)$ decreases. This implies that the range of values of q_i, the party's ideal position, for which an extreme signal indicating minority or majority bias is sent, increases as electoral loss matters more. The opposite happens when the cost of signaling goes up.

We finally look at what happens to equilibrium if α changes. Suppose $l > \frac{2}{3}$ and $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$ and $\alpha \leq \alpha_1 (l, c)$. In this case both types of equilibrium with

$$s_i^* (q_i) = \begin{cases} -1 & \text{for } q_i \in [-1, \underline{q}(l, c)) \\ 0 & \text{for } q_i \in [\underline{q}(l, c), 1] \end{cases}$$

and

$$s_i^* (q_i) = \begin{cases} 0 & \text{for } q_i \in [-1, \bar{q}(l, c)] \\ 1 & \text{for } q_i \in (\bar{q}(l, c), 1] \end{cases}$$

are possible. As α increases beyond $\alpha_1 (l, c)$, the equilibrium with

$$s_i^* (q_i) = \begin{cases} -1 & \text{for } q_i \in [-1, \underline{q}(l, c)) \\ 0 & \text{for } q_i \in [\underline{q}(l, c), 1] \end{cases}$$

gets knocked out, but the $\{0,1\}$ cutoff equilibrium with $\bar{q} < 0$ is sustained. As α increases further, the only equilibrium that survives is the $\{0,1\}$

equilibrium with $\bar{q} > 0$ if and only if $\alpha \geq \alpha_2(l, c)$. If $l < \frac{2}{3}$, then for all $c > \frac{l}{4} + \frac{1}{3}$, the equilibrium with

$$s_i^*(q_i) = \begin{cases} 0 & \text{for } q_i \in [-1, \bar{q}(l, c)] \\ 1 & \text{for } q_i \in (\bar{q}(l, c), 1] \end{cases}$$

exists if and only if $\alpha \geq \alpha_2(l, c)$. Thus, the chance of parties sending the extreme signal indicative of majority specific public good preference is higher when the size of the majority is large.

When parties attach high value to electoral loss, for middling signaling cost, an equilibrium with the parties sending the signal indicative of preference for minority specific public good preference, as mentioned before is counter-intuitive and exists only when the sizes of the majority and minority are relatively close. As parties care more about electoral loss, the range of values for which an extreme signal is sent increases. By doing so a party, given its ideal position, ensures that minority voters preferring group-specific public good and those preferring neutral public good also vote for the party. However, to win a party needs simple majority, which implies that because of the higher range of values the cutoff now takes, the majority voters who prefer neutral public good expect the party's ideal position to be closer to 0, also vote for the party. The equilibrium with signal indicative of preference for majority specific public good preference exists irrespective of the relative sizes of the two groups. When the size of the minority is not large enough, the equilibrium where an extreme signal indicating minority bias is sent gets knocked out, because sending such costly signals leads to higher expected loss for the party, hence not an optimal choice. Therefore, when the size of the majority and minority groups are not close, to win, a party must woo the majority voters and send costly signals to that end– indicating majority bias. Hence, the only extreme signal that would be sent in equilibrium besides the neutral signal would be the one representing majority bias. Lastly, when signaling cost is high, sending an extreme signal, indicating majority bias is optimal for a party if and only if the size of the majority is sufficiently large, consolidating whose support justifies the huge costs incurred.

Pooling Equilibria

Given that our signal space is restricted to three signals, the three candidates for pooling equilibria are $\{-1, -1\}$, $\{0, 0\}$ and $\{1, 1\}$. To see, if these can be sustained as pooling equilibria, we need to define our off-equilibrium beliefs. Here, again we use the criterion of *universal divinity.*

Our first candidate for pooling equilibria is $\{1, 1\}$. Any deviating party can send either a signal s_i= -1 or s_i= 0. Checking if a party has incentive to deviate to either one of them is enough to knock this candidate pooling equilibrium out. Suppose a party deviates to signal

0, then under universal divinity the voters believe that all parties are equally likely to deviate to signal s_i=0. This is because except for type q_i=1, deviating to 0 allows all parties to save on the signaling cost. However, since all parties are equally likely to deviate to 0, the winning probability is same for all parties, since in expected terms a party's ideal q_i can be anywhere between -1 and 1. We show that because of the positive cost of sending an extreme signal in our model, under universal divinity, a party would have incentive to deviate to signal s_i=0. This knocks out the $\{1, 1\}$ equilibrium.

The second candidate for pooling equilibria $\{-1, -1\}$ is even more difficult to sustain. A deviating party would send either a signal s_i=1 or s_i=0. We show that following the same line of argument as before, a party would have incentive to deviate to signal s_i=0 (instead of deviating to signal s_i=1) , thereby also knocking this equilibrium out.

Our last candidate for pooling equilibria is $\{0, 0\}$. A deviating party can send either signal s_i=-1 or s_i= 1. Again, it is obvious why a party would not deviate to signal -1, as under universal divinity no majority voter would vote for it, thus making deviation sub-optimal. Therefore, we check if a party has incentive to deviate to signal 1. What makes this case particularly interesting is that the majority voter has a kinked payoff function, hence his utility from the expected position of the party in the two ranges $[-1, 0)$ and $(0, 1]$ is different. We show that under universal divinity, this equilibrium will not survive if $\alpha \geq \frac{3}{4}$.

Proposition 3 *Under universal divinity, the only pooling equilibrium that survives is the* $\{0, 0\}$ *equilibrium if and only if* $\alpha < \frac{3}{4}$

Given that under universal divinity, upon defection to s_i=1 voters believe a party to be of the extreme majoritarian type, i.e, q_i=1, this type actually has incentive to deviate (since the cost of signaling is zero for it, same as that of sending the zero signal) if and only if the share of the majority in the population is large enough.

Characterization of semi-pooling and pooling equilibria

Characterization of equilibria is important to understand which kind of equilibrium exists under different parametric configurations. In the following pictures, we show, for different values of l, the parametric zones in the (c, α) space where different types of equilibria are possible. We plot two pictures for the cases $l \leq \frac{2}{3}$ and $l > \frac{2}{3}$. From Proposition 3, we know that a pooling equilibrium with {0,0} signal exists if and only if $\alpha < \frac{3}{4}$ in both cases. This is shown in the figure by the PE zone to the left of the $\alpha = \frac{3}{4}$ (green) line. Therefore, we have a PE zone for all values of c as long as α is below a critical value.

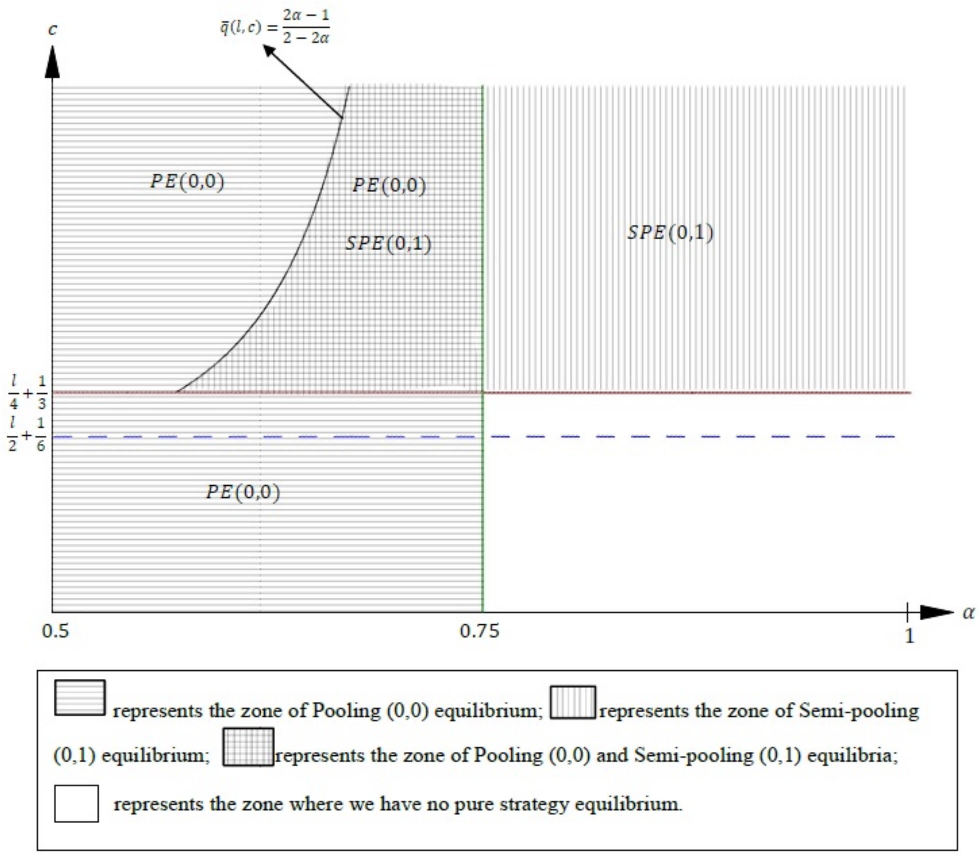
To identify the parameter ranges in the said space where the semi-pooling equilibria may arise, we need to distinguish between two cases $l \leq \frac{2}{3}$ and $l > \frac{2}{3}$.

Case I: $l \leq \frac{2}{3}$

The only semi-pooling equilibrium we have in this case is the {0,1} equilibrium which is valid only if $c > \frac{l}{4} + \frac{1}{3}$ and $\alpha \geq \alpha_2(l,c)$[3]. Figure 2 shows that the SPE(0,1) zone lies to the right of the black curve and above the $c = \frac{l}{4} + \frac{1}{3}$ (red) line.

Therefore, two kinds of equilibria can exist in the zone above the $c = \frac{l}{4} + \frac{1}{3}$ line and in between the black curve and the green line under universal divinity: PE(0,0) and SPE(0,1)

Figure 2: Characterization of equilibria for $l \leq \frac{2}{3}$



Case II: $l > \frac{2}{3}$

From Lemma 1 we know that for the SPE with {-1,0} to exist, $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$ and $\alpha \leq \alpha_1(l,c)$[4]. Figure 3 shows that the SPE(-1,0)

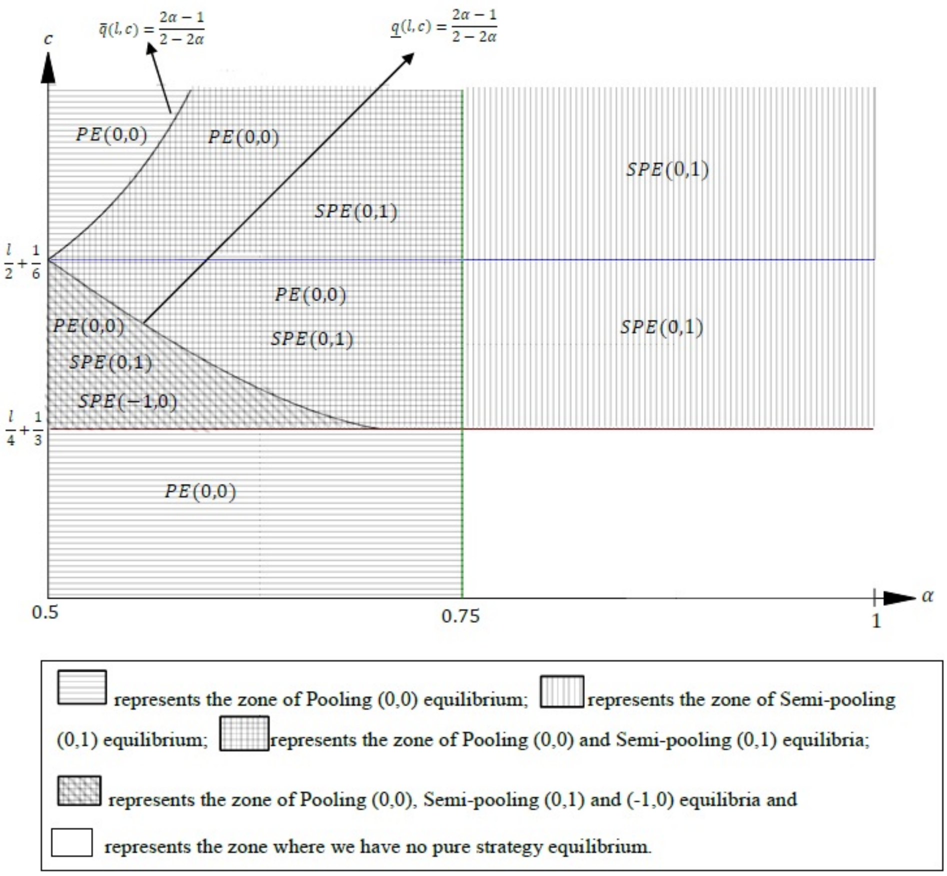
[3]This critical value of α, $\alpha_2(l,c)$ is derived from the prerequisite $\bar{q}(l,c) < \frac{2\alpha-1}{2(1-\alpha)}$
[4]This critical value of α, $\alpha_1(l,c)$ is derived from the prerequisite $\underline{q}(l,c) > \frac{2\alpha-1}{2(1-\alpha)}$

zone lies below the black curve and between the $c = \frac{l}{2} + \frac{1}{6}$ (red) and $c = \frac{l}{4} + \frac{1}{3}$ (blue) lines. It is easy to verify that at the lower bound of c, $c = \frac{l}{4} + \frac{1}{3}$, $\alpha_1(l, c) < \frac{3}{4}$ for all $l < \frac{8}{3}$. However, for $l \geq \frac{8}{3}$, $\alpha_1(l, c) = \frac{3}{4}$ at $c = \frac{l}{4} + \frac{1}{3}$.

The second equilibrium under this category is the {0,1} equilibrium with $\bar{q} < 0$. It holds for all values of α, $\alpha > \frac{1}{2}$ as long as $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$. Thus we have the zone for the SPE(0,1) with $\bar{q} < 0$ lying between the $c = \frac{l}{2} + \frac{1}{6}$ (blue) and $c = \frac{l}{4} + \frac{1}{3}$ (red) lines for all values of $\alpha > \frac{1}{2}$. Another zone for SPE(0,1) for $\bar{q} > 0$ will lie to the right of the $\bar{q}(l, c) = \frac{2\alpha - 1}{2 - 2\alpha}$ curve and above the blue line.

Thus, under universal divinity, three kinds of equilibria can exist in the triangular area below the downward sloping black curve and in between the red and blue lines : PE(0,0), SPE(-1,0) and SPE(0,1) with $\bar{q} < 0$. Again, two kinds of equilibria exist in the area enclosed between the upward and downward sloping black curves and the green line: PE(0,0), SPE(0,1) with $\bar{q} < 0$ and SPE(0,1) with $\bar{q} > 0$

Figure 3: Characterization of equilibria for $l > \frac{2}{3}$



From the above discussion we can draw the following conclusions regarding equilibrium violence:

1. For $\alpha > \frac{3}{4}$ and a given l, as the cost of signaling crosses a certain threshold i.e, $c > \frac{l}{4} + \frac{1}{3}$, we always have an SPE with extreme majoritarian signal.
2. For a given c, at low values of l and α (i.e, the size of the two communities is very close), we do not have an SPE with majoritarian violence for any c.
3. For a given c, as l increases and reaches a threshold such that c falls below $\frac{l}{4} + \frac{1}{3}$, we no longer have an SPE with violence (majoritarian or minority-specific) for any value of α.
4. The range of values c, given any l, for which we have an SPE with minority-specific violence is less than the range for which we have an SPE with majoritarian violence.
5. As l increases, that is parties are more office-motivated, given any c, the chance of sending an extreme (majoritarian and minority-specific) signal increases.

Conclusion

In different electoral democracies around the world we have seen rhetoric against ethnic minority being used as an election strategy by parties representing ethnic majorities. Manifestation of such strategies range from hate speech to different forms of violence. The target groups of such violence are context specific – in some cases they are immigrants while in some others, they are the religious minorities. In the recent past, it has been acknowledged that such strategies have been successful in mobilizing electoral support in different countries. Nevertheless, it is not very clear why a rational, utility maximizing ethnic majority voter would vote for a party that incites violence against the minority community. This is so because such a strategy does not directly contribute to the consumption bundle of a typical member of the majority community. We reckon that such acts of violence work as signals to the voters; they signal the ethnic bias of the party and its intent to provide ethnicity specific public good if it wins the election. In many countries such as India, electoral promises are considered to be mere cheap talk which does not entail any commitment from the parties. In such a set up, costly signal such as violence works as a credible signal of a party's intent.

We have motivated our paper by suggesting that violence is a costly signal that is sent by ethnic parties to signal their ideal position. However, our theory is general enough to accommodate signals sent by parties which care about any group of people and not necessarily ethnic groups. One such example would be the use of costly signals by left ideological parties which care about class identities. If there are many parties which compete for votes of poor people and voters do not know the ideological position of the parties, the parties will resort to costly activities to signal their ideological positions. Such signals may include violence, vandalism and other costly protest activities.

We show that in equilibrium, besides the neutral signal only one other extreme signal indicating majority or minority bias is sent. We also show that the signal representing minority bias is sent only when the size of the majority and minority groups are sufficiently close. The chance of an extreme signal being sent increases as the parties care relatively more about electoral loss (than their ideal policy position being implemented) and decreases with the cost of signaling. In the presence of high cost of signaling with less-office motivated parties, a semi-pooling equilibria exists, in which an extreme signal indicating majority bias is sent if and only if the size of the majority group is sufficiently large. Restricting our off-equilibrium belief with the criterion of *university divinity*, we find that all of our three candidates of semi-pooling equilibria can be sustained. However, the only pooling equilibrium that can be sustained is $\{0,0\}$, where both parties do not send any signal, only if the size of the majority is smaller that a critical value.

Our paper is mainly a contribution to the theories of positive political economy that analyze behavior of voters, candidates and parties. There is no direct policy implication of our theory. But we argue that ethnic parties engage in costly violence to signal their position on ethnic bias. They need to signal their ethnic bias using violence because there is no credibility behind one's promise to build or provide ethnicity-specific public good. Given this mechanism, our theory implies that if there are institutional arrangements such that parties implement their electoral promises, there should be no reason to send costly signals such as ethnic violence. Consequently, ethnic violence around elections would go down.

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Appendix

Tables

Table 2: Party 1's Vote Share and Expected Loss for Different Signal Profiles

Signal Profile	Vote share of Party 1	Expected loss of party 1	
Case I: $\underline{q} \leq \bar{q} \leq 0$			
$s_1 = 0, s_2 = -1$	$\frac{1+\alpha}{2}$	0	
$s_1 = 1, s_2 = -1$	$\alpha + (1-\alpha)\max\left\{0, \frac{\gamma_1}{2}\right\}$	0	
$s_1 = 1, s_2 = 0$	$\alpha \min\left\{1, 1 - \frac{\gamma_2}{2}\right\}$ $+(1-\alpha)\max\left\{0, \frac{\gamma_3}{2}\right\}$	0 $\frac{1}{2}\left[l + \int_{\underline{q}}^{\bar{q}} (q-q_1)^2 f\left(q\vert_{\underline{q} \leq q_2 \leq \bar{q}}\right) dq\right]$ $\left[l + \int_{\underline{q}}^{\bar{q}} (q-q_1)^2 f\left(q\vert_{\underline{q} \leq q_2 \leq \bar{q}}\right) dq\right]$	if $\gamma_2 < \frac{2\alpha-1}{\alpha}$ if $\gamma_2 = \frac{2\alpha-1}{\alpha}$ if $\gamma_2 > \frac{2\alpha-1}{\alpha}$
Case II: $0 \leq \underline{q} \leq \bar{q}$			
$s_1 = 0, s_2 = -1$	$\alpha \min\left\{1, 1 - \frac{\gamma_4}{2}\right\}$ $+(1-\alpha)\max\left\{0, \frac{\gamma_5}{2}\right\}$	0 $\frac{1}{2}\left[l + \int_{-1}^{\underline{q}} (q-q_1)^2 f\left(q\vert_{q_2 < \underline{q}}\right) dq\right]$ $\left[l + \int_{-1}^{\underline{q}} (q-q_1)^2 f\left(q\vert_{q_2 < \underline{q}}\right) dq\right]$	if $\gamma_4 < \frac{2\alpha-1}{\alpha}$ if $\gamma_4 = \frac{2\alpha-1}{\alpha}$ if $\gamma_4 > \frac{2\alpha-1}{\alpha}$
$s_1 = 1, s_2 = -1$	$\alpha \min\left\{1, 1 - \frac{\gamma_6}{2}\right\}$	0 $\frac{1}{2}\left[l + \int_{-1}^{\underline{q}} (q-q_1)^2 f\left(q\vert_{q_2 < \underline{q}}\right) dq\right]$ $\left[l + \int_{-1}^{\underline{q}} (q-q_1)^2 f\left(q\vert_{q_2 < \underline{q}}\right) dq\right]$	if $\gamma_6 < \frac{2\alpha-1}{\alpha}$ if $\gamma_6 = \frac{2\alpha-1}{\alpha}$ if $\gamma_6 > \frac{2\alpha-1}{\alpha}$
$s_1 = 1, s_2 = 0$	$\frac{\alpha}{2}$	$\left[l + \int_{\underline{q}}^{\bar{q}} (q-q_1)^2 f\left(q\vert_{\underline{q} \leq q_2 \leq \bar{q}}\right) dq\right]$	
Case III: $\underline{q} \leq 0 \leq \bar{q}$			
$s_1 = 0, s_2 = -1$	$\alpha + (1-\alpha)\max\left\{0, \frac{\gamma_7}{2}\right\}$	0	
$s_1 = 1, s_2 = -1$	$\alpha \min\left\{1, 1 - \frac{\gamma_8}{2}\right\}$ $+(1-\alpha)\max\left\{0, \frac{\gamma_9}{2}\right\}$	0 $\frac{1}{2}\left[l + \int_{-1}^{\underline{q}} (q-q_1)^2 f\left(q\vert_{q_2 < \underline{q}}\right) dq\right]$ $\left[l + \int_{-1}^{\underline{q}} (q-q_1)^2 f\left(q\vert_{q_2 < \underline{q}}\right) dq\right]$	if $\gamma_8 < \frac{2\alpha-1}{\alpha}$ if $\gamma_8 = \frac{2\alpha-1}{\alpha}$ if $\gamma_8 > \frac{2\alpha-1}{\alpha}$
$s_1 = 1, s_2 = 0$	$\alpha \min\left\{1, 1 - \frac{\gamma_{10}}{2}\right\}$	0 $\frac{1}{2}\left[l + \int_{\underline{q}}^{\bar{q}} (q-q_1)^2 f\left(q\vert_{\underline{q} \leq q_2 \leq \bar{q}}\right) dq\right]$ $\left[l + \int_{\underline{q}}^{\bar{q}} (q-q_1)^2 f\left(q\vert_{\underline{q} \leq q_2 \leq \bar{q}}\right) dq\right]$	if $\gamma_{10} < \frac{2\alpha-1}{\alpha}$ if $\gamma_{10} = \frac{2\alpha-1}{\alpha}$ if $\gamma_{10} > \frac{2\alpha-1}{\alpha}$

Derivation of optimal voting strategies

We need to analyze voter's strategies for three separate possiblities in our search for equilibrium - (i) $\bar{q} \leq 0$, (ii) $\underline{q} \geq 0$ and (iii) $\underline{q} \leq 0 \leq \bar{q}$. We take up each case subsequently.

Case I: $\underline{q} \leq \bar{q} \leq 0$

Given the equilibrum strategies of the parties, consider the case when party i sends the signal $s_i = -1$. If party i wins, the expected payoff of a Group A voter with preference parameter γ is

$$E_{q_i}\left[u_A\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=-1)} = 1 + \frac{-1+\underline{q}}{2}$$

since $\frac{-1+\underline{q}}{2} < 0$. Similarly, the payoff for the same voter after $s_i = 0$ and $s_i = 1$ are

$$E_{q_i}\left[u_A\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=0)} = 1 + \frac{\underline{q}+\bar{q}}{2}$$

and

$$E_{q_i}\left[u_A\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=1)} = 1 - \frac{\bar{q}^2+1-\gamma}{2\left(1-\bar{q}\right)}$$

respectively. The derivation of the last expression is little bit more involved than the other two. $s_i = 1$ implies that $q_i \in (\bar{q}, 1]$ with $\bar{q} \leq 0$. However, the payoff function of the voters have a kink at $q = 0$ and hence the expected payoff for group A voters after observing $s_i = 1$ is

$$\frac{0-\bar{q}}{1-\bar{q}}\left[1+\frac{\bar{q}}{2}\right] + \frac{1-0}{1-\bar{q}}\left[1-\left(1-\gamma\right)\frac{1}{2}\right]$$

which gives us the above expression.

In case of Group B voters, the expected payoffs are

$$E_{q_i}\left[u_B\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=-1)} = 1 + \left(1-\gamma\right)\frac{-1+\underline{q}}{2}$$

$$E_{q_i}\left[u_B\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=0)} = 1 + \left(1-\gamma\right)\frac{\underline{q}+\bar{q}}{2}$$

and

$$E_{q_i}\left[u_B\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=1)} = 1 - \frac{\left(1-\gamma\right)\bar{q}^2+1}{2\left(1-\bar{q}\right)}$$

We can now write down the optimal voting strategy for any voter in either group. We examine the voting strategies for all possible signal profiles in what follows:

Consider the signal profile $s_1 = 0, s_2 = -1$. Since $\frac{\underline{q}+\bar{q}}{2} > \frac{-1+\underline{q}}{2}$,

$$r_A^*\left(s_1, s_2, \gamma\right) = 1$$

for all γ. In this case,

$$r_B^*\left(s_1, s_2, \gamma\right) = \begin{cases} 1 & \text{if } \gamma < 1 \\ \frac{1}{2} & \text{if } \gamma = 1 \\ 0 & \text{if } \gamma > 1 \end{cases}$$

For signal profile $s_1 = 1, s_2 = -1$, it is easily verifiable that

$$r_A^*(s_1, s_2, \gamma) = 1$$

for all γ while

$$r_B^*(s_1, s_2, \gamma) = \begin{cases} 1 & \text{if } \gamma < \gamma_1 \\ \frac{1}{2} & \text{if } \gamma = \gamma_1 \\ 0 & \text{if } \gamma > \gamma_1 \end{cases}$$

where $\gamma_1 = 1 - \frac{1}{(1-\underline{q})(1-\bar{q})-\bar{q}^2} \in (0,1)$.

Finally, for signal profile $s_1 = 1, s_2 = 0$,

$$r_A^*(s_1, s_2, \gamma) = \begin{cases} 1 & \text{if } \gamma > \gamma_2 \\ \frac{1}{2} & \text{if } \gamma = \gamma_2 \\ 0 & \text{if } \gamma < \gamma_2 \end{cases}$$

where $\gamma_2 = 1 + \bar{q} + \underline{q} - \underline{q}\bar{q} < 1$. Notice that if $\gamma_2 \leq 0$, $r_A^*(s_1, s_2, \gamma) = 1$ for all $\gamma \in [0, 2]$. In this case,

$$r_B^*(s_1, s_2, \gamma) = \begin{cases} 1 & \text{if } \gamma < \gamma_3 \\ \frac{1}{2} & \text{if } \gamma = \gamma_3 \\ 0 & \text{if } \gamma > \gamma_3 \end{cases}$$

where $\gamma_3 = 1 - \frac{1}{-\bar{q}-\underline{q}+\underline{q}\bar{q}} < 1$. Once again, if $\gamma_3 \leq 0$, $r_B^*(s_1, s_2, \gamma) = 0$ for all $\gamma \in [0, 2]$. Notice that $\gamma_2 > 0$ iff $\gamma_3 < 0$.

Case II: $0 \leq \underline{q} \leq \bar{q}$

Given the equilibrum strategies of the parties, consider the case when party i sends the signal $s_i = -1$. If party i wins, the expected payoff of a Group A voter with preference parameter γ is

$$E_{q_i}[u_A(q_i, \gamma)]\,|_{\mu_i^*(q_i|s_i=-1)} = 1 - \frac{1 + (1-\gamma)\underline{q}^2}{2\left(1 + \underline{q}\right)}$$

Similarly, the payoff for the same voter after $s_i = 0$ and $s_i = 1$ are

$$E_{q_i}[u_A(q_i, \gamma)]\,|_{\mu_i^*(q_i|s_i=0)} = 1 - (1-\gamma)\frac{\underline{q} + \bar{q}}{2}$$

and

$$E_{q_i}[u_A(q_i, \gamma)]\,|_{\mu_i^*(q_i|s_i=1)} = 1 - (1-\gamma)\frac{1 + \bar{q}}{2}$$

respectively.

In case of Group B voters, the expected payoffs are

$$E_{q_i}[u_B(q_i, \gamma)]\,|_{\mu_i^*(q_i|s_i=-1)} = 1 - \frac{(1-\gamma) + \underline{q}^2}{2\left(1 + \underline{q}\right)}$$

$$E_{q_i}[u_B(q_i, \gamma)]\,|_{\mu_i^*(q_i|s_i=0)} = 1 - \frac{\underline{q} + \bar{q}}{2}$$

and

$$E_{q_i}[u_B(q_i, \gamma)]\,|_{\mu_i^*(q_i|s_i=1)} = 1 - \frac{1 + \bar{q}}{2}$$

Once again, if the signals of the parties match, the probability of voting for either party is $\frac{1}{2}$. As in the earlier case, we can derive voting strategied for different signal profiles.

For signals $s_1 = 0, s_2 = -1$,

$$r_A^*(s_1, s_2, \gamma) = \begin{cases} 1 & \text{if } \gamma > \gamma_4 \\ \frac{1}{2} & \text{if } \gamma = \gamma_4 \\ 0 & \text{if } \gamma < \gamma_4 \end{cases}$$

where $\gamma_4 = 1 - \frac{1}{\bar{q} + \underline{q} + \underline{q}\bar{q}} < 1$. If $\gamma_4 < 0$, $r_A^*(s_1, s_2, \gamma) = 1$ for all $\gamma \in [0, 2]$. For group B voters,

$$r_B^*(s_1, s_2, \gamma) = \begin{cases} 1 & \text{if } \gamma < \gamma_5 \\ \frac{1}{2} & \text{if } \gamma = \gamma_5 \\ 0 & \text{if } \gamma > \gamma_5 \end{cases}$$

where $\gamma_5 = 1 - \left(\bar{q} + \underline{q} + \underline{q}\bar{q}\right) < 1$. If $\gamma_5 < 0$, $r_B^*(s_1, s_2, \gamma) = 0$ for all $\gamma \in [0, 2]$. Also that $\gamma_4 > 0$ iff $\gamma_5 < 0$.

In case $s_1 = 1, s_2 = -1$,

$$r_A^*(s_1, s_2, \gamma) = \begin{cases} 1 & \text{if } \gamma > \gamma_6 \\ \frac{1}{2} & \text{if } \gamma = \gamma_6 \\ 0 & \text{if } \gamma < \gamma_6 \end{cases}$$

where $\gamma_6 = 1 - \frac{1}{(1+\bar{q})\left(1+\underline{q}\right) - \underline{q}^2} < 1$. If $\gamma_6 < 0$, $r_A^*(s_1, s_2, \gamma) = 1$ for all $\gamma \in [0, 2]$. In this case $r_B^*(s_1, s_2, \gamma) = 0$.

Finally, for $s_1 = 1, s_2 = 0$,

$$r_A^*(s_1, s_2, \gamma) = \begin{cases} 0 & \text{if } \gamma < 1 \\ \frac{1}{2} & \text{if } \gamma = 1 \\ 1 & \text{if } \gamma > 1 \end{cases}$$

while $r_B^*(s_1, s_2, \gamma) = 0$.

Case III: $\underline{q} \leq 0 \leq \bar{q}$

In this case,

$$E_{q_i}\left[u_A(q_i, \gamma)\right]|_{\mu_i^*(q_i|s_i=-1)} = 1 + \frac{-1 + \underline{q}}{2}$$

$$E_{q_i}\left[u_A(q_i, \gamma)\right]|_{\mu_i^*(q_i|s_i=0)} = 1 - \frac{\underline{q}^2 + (1-\gamma)\bar{q}^2}{2\left(\bar{q} - \underline{q}\right)}$$

and

$$E_{q_i}\left[u_A(q_i, \gamma)\right]|_{\mu_i^*(q_i|s_i=1)} = 1 - (1-\gamma)\frac{1 + \bar{q}}{2}$$

while

$$E_{q_i}\left[u_B(q_i, \gamma)\right]|_{\mu_i^*(q_i|s_i=-1)} = 1 - (1-\gamma)\frac{1 - \underline{q}}{2}$$

$$E_{q_i}\left[u_B(q_i, \gamma)\right]|_{\mu_i^*(q_i|s_i=0)} = 1 - \frac{(1-\gamma)\underline{q}^2 + \bar{q}^2}{2\left(\bar{q} - \underline{q}\right)}$$

and

$$E_{q_i}\left[u_B\left(q_i,\gamma\right)\right]|_{\mu_i^*(q_i|s_i=1)}=1-\frac{1+\bar{q}}{2}$$

For signals $s_1=0,s_2=-1$, $r_A^*\left(s_1,s_2,\gamma\right)=1$ for all γ while

$$r_B^*\left(s_1,s_2,\gamma\right)=\begin{cases}1 & \text{if } \gamma<\gamma_7\\ \frac{1}{2} & \text{if } \gamma=\gamma_7\\ 0 & \text{if } \gamma>\gamma_7\end{cases}$$

where $\gamma_7=1-\frac{\bar{q}^2}{\bar{q}-\underline{q}-\underline{q}\bar{q}}\in(0,1)$.

In case $s_1=1,s_2=-1$,

$$r_A^*\left(s_1,s_2,\gamma\right)=\begin{cases}1 & \text{if } \gamma>\gamma_8\\ \frac{1}{2} & \text{if } \gamma=\gamma_8\\ 0 & \text{if } \gamma<\gamma_8\end{cases}$$

and

$$r_B^*\left(s_1,s_2,\gamma\right)=\begin{cases}1 & \text{if } \gamma<\gamma_9\\ \frac{1}{2} & \text{if } \gamma=\gamma_9\\ 0 & \text{if } \gamma>\gamma_9\end{cases}$$

where $\gamma_8=1-\frac{1-\underline{q}}{1+\bar{q}}$ and $\gamma_9=1-\frac{1+\bar{q}}{1-\underline{q}}$ respectively.

Finally, for $s_1=1,s_2=0$,

$$r_A^*\left(s_1,s_2,\gamma\right)=\begin{cases}1 & \text{if } \gamma>\gamma_{10}\\ \frac{1}{2} & \text{if } \gamma=\gamma_{10}\\ 0 & \text{if } \gamma<\gamma_{10}\end{cases}$$

where $\gamma_{10}=1-\frac{\underline{q}^2}{\bar{q}-\underline{q}-\underline{q}\bar{q}}$ while $r_B^*\left(s_1,s_2,\gamma\right)=0$ for all γ.

Proof of Lemma 1

Suppose party 2 adopts the strategy described in (5). We show that in each of the cases $\underline{q}\le\bar{q}\le 0$, $0\le\underline{q}\le\bar{q}$ and $\underline{q}\le 0\le\bar{q}$, one of the signals is never sent by party 1 for any $q_1\in[-1,1]$.

Consider $\underline{q}\le\bar{q}\le 0$. Suppose party 1 sends $s_1=-1$. Then, from Table 2, the expected loss of party 1 (ignoring the signaling cost) is given by

$$\begin{aligned}&EL_1\left(s_1=-1,s_2\left(q_2\right),q_1\right)\\ =\;&\Pr\left[q_2<\underline{q}\right].\frac{1}{2}\phi\left(-1,\underline{q};q_1\right)\\ &+\Pr\left[\underline{q}\le q_2\le\bar{q}\right].\phi\left(\underline{q},\bar{q};q_1\right)\\ &+\Pr\left[\bar{q}<q_2\right]\phi\left(\bar{q},1;q_1\right)\end{aligned}$$

If party 1 sends $s_1=0$, then party 1's expected loss is

$$\begin{aligned}&EL_1\left(s_1=0,s_2\left(q_2\right),q_1\right)\\ =\;&\Pr\left[q_2<\underline{q}\right].0\\ &+\Pr\left[\underline{q}\le q_2\le\bar{q}\right]\frac{1}{2}\phi\left(\underline{q},\bar{q};q_1\right)\\ &+\Pr\left[\bar{q}<q_2\right]\phi^{(1)}\end{aligned}$$

where

$$\phi^{(1)} = \begin{cases} 0 & \text{if } \gamma_2 > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi(\bar{q},1;q_1) & \text{if } \gamma_2 = \frac{2\alpha-1}{\alpha} \\ \phi(\bar{q},1;q_1) & \text{if } \gamma_2 < \frac{2\alpha-1}{\alpha} \end{cases}$$

Whatever the value of γ_2, it is easy to see $EL_1(s_1 = 0, s_2(q_2), q_1) < EL_1(s_1 = -1, s_2(q_2), q_1)$ for all $q_1 \in [-1,1]$. Hence, party 1 never sends $s_1 = -1$ in equilibrium.

Now consider $\underline{q} \leq 0 \leq \bar{q}$. In this case,

$$\begin{aligned} & EL_1(s_1 = -1, s_2(q_2), q_1) \\ = \; & \Pr\left[q_2 < \underline{q}\right] . \frac{1}{2}\phi\left(-1, \underline{q}; q_1\right) \\ & + \Pr\left[\underline{q} \leq q_2 \leq \bar{q}\right] . \phi\left(\underline{q}, \bar{q}; q_1\right) \\ & + \Pr\left[\bar{q} < q_2\right] \phi^{(2)}(l, \bar{q}) \end{aligned}$$

where

$$\phi^{(2)} = \begin{cases} 0 & \text{if } \gamma_8 > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi(\bar{q},1;q_1) & \text{if } \gamma_8 = \frac{2\alpha-1}{\alpha} \\ \phi(\bar{q},1;q_1) & \text{if } \gamma_8 < \frac{2\alpha-1}{\alpha} \end{cases}$$

Similarly,

$$\begin{aligned} & EL_1(s_1 = 0, s_2(q_2), q_1) \\ = \; & \Pr\left[q_2 < \underline{q}\right] .0 \\ & + \Pr\left[\underline{q} \leq q_2 \leq \bar{q}\right] \frac{1}{2}\phi\left(\underline{q}, \bar{q}; q_1\right) \\ & + \Pr\left[\bar{q} < q_2\right] \phi^{(3)} \end{aligned}$$

where

$$\phi^{(3)} = \begin{cases} 0 & \text{if } \gamma_{10} > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi(\bar{q},1;q_1) & \text{if } \gamma_{10} = \frac{2\alpha-1}{\alpha} \\ \phi(\bar{q},1;q_1) & \text{if } \gamma_{10} < \frac{2\alpha-1}{\alpha} \end{cases}$$

It is easy to verify that $\gamma_{10} \geq \gamma_8$ for any $\underline{q} \leq 0 \leq \bar{q}$. Hence, $EL_1(s_1 = 0, s_2(q_2), q_1) < EL_1(s_1 = -1, s_2(q_2), q_1)$ for all $q_1 \in [-1,1]$. Hence, party 1 would never choose $s_1 = -1$.

Finally, consider $0 \leq \underline{q} \leq \bar{q}$. In this case,

$$\begin{aligned} & EL_1(s_1 = 0, s_2(q_2), q_1) \\ = \; & \Pr\left[q_2 < \underline{q}\right] \phi^{(4)} \\ & + \Pr\left[\underline{q} \leq q_2 \leq \bar{q}\right] . \frac{1}{2}\phi\left(\underline{q}, \bar{q}; q_1\right) \\ & + \Pr\left[\bar{q} < q_2\right] .0 \end{aligned}$$

where

$$\phi^{(4)} = \begin{cases} 0 & \text{if } \gamma_4 < \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi\left(-1,\underline{q};q_1\right) & \text{if } \gamma_4 = \frac{2\alpha-1}{\alpha} \\ \phi\left(-1,\underline{q};q_1\right) & \text{if } \gamma_4 > \frac{2\alpha-1}{\alpha} \end{cases}$$

Similarly,

$$\begin{aligned} & EL_1\left(s_1=1, s_2\left(q_2\right), q_1\right) \\ = & \operatorname{Pr}\left[q_2<\underline{q}\right] \phi^{(5)} \\ & +\operatorname{Pr}\left[\underline{q} \leq q_2 \leq \bar{q}\right] . \phi\left(\underline{q}, \bar{q} ; q_1\right) \\ & +\operatorname{Pr}\left[\bar{q}<q_2\right] \frac{1}{2} \phi\left(\bar{q}, 1 ; q_1\right) \end{aligned}$$

where

$$\phi^{(5)}=\begin{cases} 0 & \text{if } \gamma_6<\frac{2 \alpha-1}{\alpha} \\ \frac{1}{2} \phi\left(-1, \underline{q} ; q_1\right) & \text{if } \gamma_6=\frac{2 \alpha-1}{\alpha} \\ \phi\left(-1, \underline{q} ; q_1\right) & \text{if } \gamma_6>\frac{2 \alpha-1}{\alpha} \end{cases}$$

Once again it is easy to verify that $\gamma_4<\gamma_6$ for all $0 \leq \underline{q} \leq \bar{q}$. Hence, $EL_1\left(s_1=0, s_2\left(q_2\right), q_1\right)<EL_1\left(s_1=1, s_2\left(q_2\right), q_1\right)$ for all $q_1 \in[-1,1]$. Hence, party 1 would never choose $s_1=1$.

Proof of Lemma 2

For existence of such a semi-pooling equilibrium, i.e, {-1,0}, it must be the case that given that player j plays a strategy as described in (6), player 1's expected loss from sending signal $s_i=-1$ is less than its expected loss from sending signal $s_i=0$ if and only if $q_i \in[-1, \underline{q})$ for some $\underline{q} \geq 0$. Party i's expected loss from $s_i=-1$, if $\underline{q} \geq 0$, is

$$\begin{aligned} & EL_i\left(s_i=-1, s_j\left(q_j\right), q_i\right)+c\left(1+q_i\right) \\ = & \operatorname{Pr}\left[q_j<\underline{q}\right] . \frac{1}{2} \phi\left(-1, \underline{q} ; q_i\right) \\ & +\operatorname{Pr}\left[\underline{q} \leq q_j\right] . \phi_i^{-1}+c\left(1+q_i\right) \end{aligned}$$

where

$$\phi_i^{-1}=\begin{cases} 0 & \text{if } \gamma_4>\frac{2 \alpha-1}{\alpha} \\ \frac{1}{2} \phi\left(\underline{q}, 1 ; q_i\right) & \text{if } \gamma_4=\frac{2 \alpha-1}{\alpha} \\ \phi\left(\underline{q}, 1 ; q_i\right) & \text{if } \gamma_4<\frac{2 \alpha-1}{\alpha} \end{cases}$$

Similarly,

$$\begin{aligned} & EL_i\left(s_i=0, s_j\left(q_j\right), q_i\right) \\ = & \operatorname{Pr}\left[q_j<\underline{q}\right] . \phi_i^0 \\ & +\operatorname{Pr}\left[\underline{q} \leq q_j\right] . \frac{1}{2} \phi\left(\underline{q}, 1 ; q_i\right) \end{aligned}$$

where

$$\phi_i^0=\begin{cases} \phi\left(-1, \underline{q} ; q_i\right) & \text{if } \gamma_4>\frac{2 \alpha-1}{\alpha} \\ \frac{1}{2} \phi\left(-1, \underline{q} ; q_i\right) & \text{if } \gamma_4=\frac{2 \alpha-1}{\alpha} \\ 0 & \text{if } \gamma_4<\frac{2 \alpha-1}{\alpha} \end{cases}$$

If $\gamma_4 \leq \frac{2 \alpha-1}{\alpha}$,

$$EL_i\left(s_i=0, s_j\left(q_j\right), q_i\right)<EL_i\left(s_i=-1, s_j\left(q_j\right), q_i\right)+c\left(1+q_i\right)$$

for all q_i. So we cannot have an equilibrium where $s_i = -1$ is sent by some types. However, for $\gamma_4 > \frac{2\alpha-1}{\alpha}$,

$$EL_i\left(s_i = 0, s_j\left(q_j\right), q_i\right) \leq EL_i\left(s_i = -1, s_j\left(q_j\right), q_i\right) + c\left(1 + q_i\right)$$

if and only if

$$\begin{aligned} & \Pr\left[q_j < \underline{q}\right].\phi\left(-1, \underline{q}; q_i\right) + \Pr\left[\underline{q} \leq q_j\right].\frac{1}{2}\phi\left(\underline{q}, 1; q_i\right) \\ \leq \; & \Pr\left[q_j < \underline{q}\right].\frac{1}{2}\phi\left(-1, \underline{q}; q_i\right) + c\left(1 + q_i\right) \end{aligned}$$

After some manipulation, the above inequality simplifies to

$$h\left(l, c, q_i\right) = l + \frac{1}{3} - 2c\left(1 + q_i\right) + q_i^2 \leq 0$$

For existence of our proposed equilibrium, we need this inequality to hold if and only if $q_i \in \left[\underline{q}, 1\right]$ for some $\underline{q} \geq 0$. Given the convexity of $h\left(.\right)$ in q_i and the fact that $h\left(l, c, -1\right) > 0$, the equilibrium exists if and only if $h\left(l, c, 1\right) < 0$ and $h\left(l, c, 0\right) \geq 0$, i.e.

$$c > \frac{l}{4} + \frac{1}{3}$$

and

$$c \leq \frac{l}{2} + \frac{1}{6}$$

A prerequisite for both conditions to be met simultaneously is $l > \frac{2}{3}$. If $l > \frac{2}{3}$ and $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$, then equilibrium $\underline{q}$ can be solved from $h\left(l, c, \underline{q}\left(l, c\right)\right) = 0$. However, the equilibrium is valid only if $\gamma_4 > \frac{2\alpha-1}{\alpha}$ at equilibrium $\underline{q}$. But in this case,

$$\gamma_4 = 1 - \frac{1}{1 + 2\underline{q}\left(l, c\right)} > \frac{2\alpha - 1}{\alpha}$$

$$\Leftrightarrow \underline{q}\left(l, c\right) > \frac{2\alpha - 1}{2\left(1 - \alpha\right)}$$

$$\Leftrightarrow \alpha < \alpha_1\left(l, c\right)$$

For existence of a {0,1} equilibrium with $\bar{q} < 0$ and $\underline{q} = -1$, it must be the case that given that player j plays the strategy in (7), player i's expected loss from sending signal $s_i = 0$ is less than its expected loss from sending signal $s_i = 1$ if and only if $q_i \in [-1, \bar{q}]$ for some $\bar{q} < 0$. Party i's expected loss from $s_i = 1$, if $\bar{q} < 0$, is

$$\begin{aligned} & EL_i\left(s_i = 1, s_j\left(q_j\right), q_i\right) + c\left(1 - q_i\right) \\ = \; & \Pr\left[q_j \leq \bar{q}\right].\phi_i^1 \\ & + \Pr\left[\bar{q} < q_j\right].\frac{1}{2}\phi\left(\bar{q}, 1; q_1\right) + c\left(1 - q_i\right) \end{aligned}$$

where

$$\phi_i^1 = \begin{cases} \phi\left(-1, \bar{q}; q_1\right) & \text{if } \gamma_2 > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi\left(-1, \bar{q}; q_1\right) & \text{if } \gamma_2 = \frac{2\alpha-1}{\alpha} \\ 0 & \text{if } \gamma_2 < \frac{2\alpha-1}{\alpha} \end{cases}$$

Similarly, party i's expected loss from $s_i = 0$ is

$$\begin{aligned} & EL_i\left(s_i=0, s_j\left(q_j\right), q_i\right) \\ = \; & \Pr\left[q_j \leq \bar{q}\right].\frac{1}{2}\phi\left(-1, \bar{q}; q_1\right) \\ & + \Pr\left[\bar{q} < q_j\right].\phi_i^0 \end{aligned}$$

where in this case

$$\phi_i^0 = \begin{cases} 0 & \text{if } \gamma_2 > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi\left(\bar{q}, 1; q_1\right) & \text{if } \gamma_2 = \frac{2\alpha-1}{\alpha} \\ \phi\left(\bar{q}, 1; q_1\right) & \text{if } \gamma_2 < \frac{2\alpha-1}{\alpha} \end{cases}$$

If $\gamma_2 \geq \frac{2\alpha-1}{\alpha}$,

$$EL_i\left(s_i=0, s_j\left(q_j\right), q_i\right) < EL_i\left(s_i=1, s_j\left(q_j\right), q_i\right) + c\left(1-q_i\right)$$

for all q_i. So we cannot have an equilibrium where $s_i = 1$ is sent by some types. However, for $\gamma_2 < \frac{2\alpha-1}{\alpha}$,

$$EL_i\left(s_i=0, s_j\left(q_j\right), q_i\right) \leq EL_i\left(s_i=1, s_j\left(q_j\right), q_i\right) + c\left(1-q_i\right)$$

if and only if

$$\begin{aligned} & \Pr\left[q_j \leq \bar{q}\right].\frac{1}{2}\phi\left(-1, \bar{q}; q_1\right) + \Pr\left[\bar{q} \leq q_j\right].\phi\left(\bar{q}, 1; q_1\right) \\ \leq \; & \Pr\left[\bar{q} < q_j\right].\frac{1}{2}\phi\left(\bar{q}, 1; q_1\right) + c\left(1-q_i\right) \end{aligned}$$

After some manipulation, the above inequality simplifies to

$$g\left(l, c, q_i\right) = l + \frac{1}{3} - 2c\left(1-q_i\right) + q_i^2 \leq 0$$

For existence of our proposed equilibrium, we need this inequality to hold if and only if $q_i \in [-1, \bar{q}]$ for some $\bar{q} < 0$. Given the convexity of $g(.)$ in q_i and the fact that $g(l, c, 1) > 0$, the equilibrium exists if and only if $g(l, c, -1) < 0$ and $g(l, c, 0) > 0$, i.e.

$$c > \frac{l}{4} + \frac{1}{3}$$

and

$$c < \frac{l}{2} + \frac{1}{6}$$

A prerequisite for both conditions to be met simultaneously is $l > \frac{4}{3}$. If $l > \frac{2}{3}$ and $c \in \left(\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right]$, then equilibrium $\bar{q}$ can be solved from $g\left(l, c, \bar{q}\left(l, c\right)\right) = 0$. However, the equilibrium is valid only if $\gamma_2 < \frac{2\alpha-1}{\alpha}$ at equilibrium $\bar{q}$. But in this case,

$$\gamma_2 = 2\bar{q}\left(l, c\right) < 0$$

Hence, we have this equilibrium for all values of $\alpha > \frac{1}{2}$.

For existence of a {0,1} equilibrium with $\bar{q} \geq 0$ and $\underline{q} = -1$, it must be the case that given that player j plays the strategy in (7), player i's expected loss from sending signal $s_i = 0$ is less than its expected loss from sending signal $s_i = 1$ if and only if $q_i \in [-1, \bar{q}]$ for some $\bar{q} \geq 0$. As in the earlier case, party i's expected loss from sending $s_i = 1$ is

$$\begin{aligned} & EL_i\left(s_i = 1, s_j\left(q_j\right), q_i\right) + c\left(1 - q_i\right) \\ = \; & \Pr\left[q_j \leq \bar{q}\right].\phi_i^1 \\ & + \Pr\left[\bar{q} < q_j\right].\frac{1}{2}\phi\left(\bar{q}, 1; q_1\right) + c\left(1 - q_i\right) \end{aligned}$$

except that in this case

$$\phi_i^1 = \begin{cases} \phi\left(-1, \bar{q}; q_1\right) & \text{if } \gamma_{10} > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi\left(-1, \bar{q}; q_1\right) & \text{if } \gamma_{10} = \frac{2\alpha-1}{\alpha} \\ 0 & \text{if } \gamma_{10} < \frac{2\alpha-1}{\alpha} \end{cases}$$

Similarly,

$$\begin{aligned} & EL_i\left(s_i = 0, s_j\left(q_j\right), q_i\right) \\ = \; & \Pr\left[q_j \leq \bar{q}\right].\frac{1}{2}\phi\left(-1, \bar{q}; q_1\right) \\ & + \Pr\left[\bar{q} < q_j\right].\phi_i^0 \end{aligned}$$

where

$$\phi_i^0 = \begin{cases} 0 & \text{if } \gamma_{10} > \frac{2\alpha-1}{\alpha} \\ \frac{1}{2}\phi\left(\bar{q}, 1; q_1\right) & \text{if } \gamma_{10} = \frac{2\alpha-1}{\alpha} \\ \phi\left(\bar{q}, 1; q_1\right) & \text{if } \gamma_{10} < \frac{2\alpha-1}{\alpha} \end{cases}$$

Once again, if $\gamma_{10} < \frac{2\alpha-1}{\alpha}$,

$$EL_i\left(s_i = 0, s_j\left(q_j\right), q_i\right) \leq EL_i\left(s_i = 1, s_j\left(q_j\right), q_i\right) + c\left(1 - q_i\right)$$

if and only if

$$g\left(l, c, q_i\right) = l + \frac{1}{3} - 2c\left(1 - q_i\right) + q_i^2 \leq 0$$

For existence of our proposed equilibrium, we need this inequality to hold if and only if $q_i \in [-1, \bar{q}]$ for some $\bar{q} \geq 0$. Given the convexity of $g(.)$ in q_i and the fact that $g(l, c, 1) > 0$, the equilibrium exists if and only if $g(l, c, -1) < 0$ and $g(l, c, 0) \leq 0$, i.e.

$$c > \frac{l}{4} + \frac{1}{3}$$

and

$$c \geq \frac{l}{2} + \frac{1}{6}$$

Hence, if $c > \max\left\{\frac{l}{4} + \frac{1}{3}, \frac{l}{2} + \frac{1}{6}\right\}$, then equilibrium $\bar{q}$ can be solved from $g(l, c, \bar{q}(l, c)) = 0$. However, the equilibrium is valid only if $\gamma_{10} < \frac{2\alpha-1}{\alpha}$ at equilibrium $\bar{q}$. But in this case,

$$\begin{aligned} \gamma_{10} = 1 - \frac{1}{1 + 2\bar{q}\left(l, c\right)} &< \frac{2\alpha - 1}{\alpha} \\ \Leftrightarrow \bar{q}\left(l, c\right) &< \frac{2\alpha - 1}{2\left(1 - \alpha\right)} \\ \Leftrightarrow \alpha &> \alpha_2\left(l, c\right) \end{aligned}$$

Hence, we have this equilibrium for all values of $\alpha > \alpha_2(l, c)$.
This completes the proof of Lemma 1.

Proof of Proposition 1

Equilibrium I: $\{-1, 0\}$

Expected utility of a majority voter from voting for the deviating party is:

$$E_{q_i}\left[u_A\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=1)} = \gamma$$

Expected utility of a majority voter from voting for the party following the proposed equilibrium will depend on the party's expected position. Since $\underline{q}\left(l, c\right) > 0$, the payoff function for the majority voter is kinked, depending on where the q_i of the party lies. Therefore, we have two cases:

Case 1: Voter observes s_i= -1

$$E_{q_i}\left[u_A\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=-1)} = \frac{1}{2(1+\underline{q}\left(l, c\right))} + \frac{\underline{q}\left(l, c\right)}{1+\underline{q}\left(l, c\right)}\left(1 - \frac{\underline{q}\left(l, c\right)}{2} + \frac{\gamma \underline{q}\left(l, c\right)}{2}\right)$$

Thus, a majority voter will vote for the deviating party iff,

$$\gamma > \frac{1 + 2\underline{q}\left(l, c\right) - \underline{q}^2\left(l, c\right)}{2 + 2\underline{q}\left(l, c\right) - \underline{q}^2\left(l, c\right)} = \gamma_c$$

To win the deviating party needs

$$\frac{\alpha\left(2 - \gamma_c\right)}{2} > \frac{1}{2}$$

$$\Leftrightarrow \gamma_c < \frac{2\alpha - 1}{\alpha}$$

However, the equilibrium is valid only if $\gamma_4 > \frac{2\alpha-1}{\alpha}$ at equilibrium $\underline{q}$. We have,

$$\Leftrightarrow 1 - \frac{1}{1 + 2\underline{q}\left(l, c\right)} > \frac{1 + 2\underline{q}\left(l, c\right) - \underline{q}^2\left(l, c\right)}{2 + 2\underline{q}\left(l, c\right) - \underline{q}^2\left(l, c\right)}$$

Solving this we get,

$$1 - \underline{q}^2\left(l, c\right) < 0$$

which is not possible since $\underline{q}\left(l, c\right) \in [0, 1)$

Hence, there is no incentive for a party to deviate and send s_i=1 under universal divinity.

Case 2: Voter observes s_i=0

$$E_{q_i}\left[u_A\left(q_i, \gamma\right)\right]|_{\mu_i^*(q_i|s_i=0)} = 1 - \frac{1 + \underline{q}\left(l, c\right)}{2} + \frac{\gamma\left(1 + \underline{q}\left(l, c\right)\right)}{2}$$

A majority voter will vote for deviating party iff,

$$1 - \frac{1 + \underline{q}\left(l, c\right)}{2} + \frac{\gamma\left(1 + \underline{q}\left(l, c\right)\right)}{2} < \gamma$$

$$\Leftrightarrow \gamma > 1$$

which means that half of the the majority, i.e, those majority voters whose γ>1 will vote for the deviating party. This makes the vote share $\frac{\alpha}{2}$ which is not enough to win. Therefore, no party has incentive to deviate to signal 1 when the proposed equilibrium is $\{-1, 0\}$

Equilibrium II: $\{0,1\}$

Taking our second semi-pooling equilibrium $\{0,1\}$ with no restriction on values of α, it is fairly simple to see why a party would not deviate to signal s_i= -1. Under universal divinity, any deviation to -1 would make the voters believe that the party is of type q_i=-1 with probability 1, and as such none of the majority voter would vote for the deviating party. Hence, no party has the incentive to deviate to signal s_i= -1. The same holds for the third proposed equilibrium $\{0,1\}$ with $\alpha \geq \alpha_2(l,c)$ where $\alpha_2(l,c) > \frac{1}{2}$.

Proof of Lemma 3

The first statement follows directly from $\frac{\delta}{\delta l}[h(l,c,q)] > 0$, $\frac{\delta}{\delta c}[h(l,c,q)] < 0$ and $h\left(l,c,\underline{q}(l,c)\right) = 0$ since given $h(l,c,1) < 0$, $\frac{\delta}{\delta c}[h(l,c,q)] < 0$ at $q = \underline{q}(l,c)$. Similar arguments hold for statements 2 and 3 of the Lemma as well.

Proof of Proposition 3

Pooling equilibria:$\{1,1\}$

$$EL_i(s_i = 1, s_j = 1, q_i) + c(1-q_i) > EL_i(s_i = 0, s_j = 1, q_i)$$

for all $q_i \in [-1,1)$. Hence, a party will have incentive to deviate to signal 0. This knocks out the $\{1,1\}$ equilibrium.

Pooling equilibria:$\{-1,-1\}$

$$EL_i(s_i = -1, s_j = -1, q_i) + c(1+q_i) > EL_i(s_i = 0, s_j = -1, q_i)$$

for all $q_i \in (-1,1]$.

Pooling equilibria:$\{0,0\}$

$$E_{q_i}[u_A(q_i,\gamma)]\,|_{\mu_i^*(q_i|s_i=0)} = \frac{1}{2}(1-\frac{1}{2}) + \frac{1}{2}(1-\frac{1}{2}+\frac{\gamma}{2})$$

A majority voter would vote for the deviating party iff,

$$\frac{1}{4} + \frac{\gamma+1}{4} < \gamma$$

$$\Leftrightarrow \gamma > \frac{2}{3}$$

Therefore, to win the party would need,

$$(2-\frac{2}{3})\frac{\alpha}{2} \geq \frac{1}{2}$$

$$\Leftrightarrow \alpha \geq \frac{3}{4}$$

This implies that the $\{0,0\}$ equilibrium would not survive under universal divinity if $\alpha \geq \frac{3}{4}$.