Personalised Incentives: Segregation in a platform with networks[*]

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Abstract

An online platform often lacks complete information about user-level preferences over networked goods. Can the platform instead utilise information about user-level social networks to maximise profit? We develop a binary choice framework where an online platform generates ad-revenue from a free basic plan (B) and subscription revenue from a fee-based exclusive plan (E). Generic content is costlessly provided on B, but the platform endogenously provides costly personalised incentives on E by leveraging user-level network information. In equilibrium, influential users receive greater incentives, which attract other susceptible users to join E. Comparative statics show how changes in ad-revenue, subscription fee, and users' bias towards B or E impact platform profit monotonically or non-monotonically, depending on how the change interacts with user-level network statistics like the average influence of users, or the distribution of influence across users. We further analyse whether an increase in strength of network and platform competition increases profit.

Keywords: Networked goods, influence, susceptibility, direct network, incentives.

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1 Introduction

Recent advances in digital infrastructure have enabled online platforms to observe *who influences whom* and *who matters*. Platforms access user-level data about user-influences and other user-level characteristics, including the users' position within social networks.[1] Typically, platforms incentivise users with higher influences over their peers to modify their preferences over the adoption of a particular product. This triggers a cascading effect through the network spillover effects and induces other *susceptible* users to adopt such products.[2] Prior research suggests that firms utilise user-level network information to target adopters to accelerate diffusion (Galeotti and Goyal, 2009, Bollinger and Gillingham, 2012), lobbyists channelise resources to influence key legislators to adopt a particular policy using the legislators' network (Battaglini and Patacchini, 2018), or politicians mobilise spending on voters with high social outreach measures to amplify outreach (Lever Guzmán, 2010). These studies collectively highlight that users who occupy 'key' positions in relevant networks exert proportional effects on others' choices. Influence, when measurable, becomes a salient measure of targeting and allocating resources. Based on this foundation, we develop a theoretical framework where an online platform credibly promises 'personalised incentives' to users in a network in a way such that it induces every user to self-select into their preferred plan out of two plans – basic or exclusive.

Platforms like $\mathbb{X}$ (formerly *Twitter* – for social media), *PlayStation* (for gaming), *Strava* (for fitness), offer users several options from which users can self-select into their preferred plan. One key characteristic of these platforms is that they provide at least one other plan besides their basic plan from which the users can self-select based on preferences. $\mathbb{X}$ offers additional benefits – like reduced or no ads, blue tick, ad-revenue sharing, reply prioritisation, etc. – to their Premium and Premium+ plans. Similarly, Moodle provides customised benefits, storage, security, and third-party app integrations to its premium users. eToro offers tiered services to users based on user characteristics, and the perks and services increase over tiers.

[1]Platforms like Twitter allow third parties to access user data through their APIs. These APIs assist third-party brokers in acquiring information about user activities, social connections, interactions, and engagement behaviour, among other things. For example, Facebook Graph API, Twitter API, and Instagram API allow developers to gather information about users' social networks. Additionally, third-party analytics firms like Kred, and PeerIndex provide *social influence* and *outreach* scores to users based on their engagement – shares, tweets, replies, mentions, friends, followers – in social media platforms.

[2]Susceptible users are those who are influenced both directly and indirectly by the actions of relatively more influential agents. In summary, susceptibility is a user-level network statistic. The notion of *influence* and *susceptibility* is suitably defined for our purpose in detail in the model section.

We study an online platform which offers two plans: a free, (advertisement) ad interrupted basic plan (B), and a subscription fee-based, ad-free exclusive plan (E). Plan B provides the uniform fixed content to all users but includes exogenously determined personalised advertisements visible only to the individual user. These personalised ads can be beneficial or detrimental to the user, which may alter their preferences towards B. However, whether an ad is beneficial or detrimental to a particular user is private information to that user. On the other hand, plan E provide users with *personalised incentives*. These incentives include in-app features tailored to individual preferences, specialised perks like data security, gifts, cash, ad-free content, in-app tokens, extra-time, early bird offers, reward coupons, etc. Henceforth, we refer to such a set of benefits as *incentives*.

The sources of revenue of the platform at the user-level consist of a fixed ad-revenue from B, and the fixed subscription fee from E. The platform incurs an endogenous cost by credibly promising personalised incentives to users upon their joining plan E. Based on these incentives, among other things, users self-select into B or E, which segments the entire market. The sole objective of the platform is to maximise profit. Here, we introduce a framework where segregation emerges not from differences in preferences or user type, but from an endogenous mechanism built on incentivising users based on influence. We depart from standard models of type-specific screening by firms and offer a rationale for optimal segregation without discriminatory pricing or access to any private information.

Another important feature of the aforementioned platforms is the notion of collaboration or co-action among its users within the basic and exclusive plans. The collaborative component is where users interact with others in the same plan and derive additional utility from every other user they are influenced by in their network (network externality). Hence, user choices are primarily governed by the subscription fee of E, bias towards the preferred plan, promised incentives from choosing plan E, private ad benefit from B, and the benefit from the network externality.

In this article, we complement the rich literature on price discrimination with network externalities. Although some platforms have access to information about user-level network, gaining complete knowledge of user preferences is seldom possible. We examine the problem of an imperfectly informed platform that cannot observe user preferences directly and allocates incentives based on observable network influence to induce self-selection among users such that it maximises platform profit. On the contrary, (Candogan et al., 2012, Fainmesser and Galeotti, 2016, 2020) studies how firms strategically target and tailor prices to subsidise influential users and seek rent from the susceptible ones. In multi-sided platforms, the profit from one side subsidises the other, which induces cross-side network externalities, (Rochet and Tirole, 2003, Armstrong, 2006). However, these papers, along with Weyl (2010) and Tan and Zhou (2021), primarily model network externalities using price as the control variable.[3] Realistically, platforms keep their prices fixed and differentiate plans to segregate users. It is difficult for platforms to continually adjust and optimise their subscription fees to maximise profit. To address this, we characterise the conditions under which the optimal strategy of the platform is to generate costly incentives and attract more users on E, as opposed to relying on ad-revenue from B, without incurring any cost on incentives.

The description of our model is as follows. In the *initial stage*, the platform credibly promises incentives to each user conditional on subscribing to E. Then, each user realises an exogenous private advertisement shock – positive or negative – from advertisements on B. The platform only learns that the shock is uniform over a non-degenerate support. Users are biased either towards B or E (being biased away from B is equivalent to being biased towards E). Such bias helps get a more robust understanding of whether the externalities generated by influential users can be high enough to lure buyers to join E when users are biased towards B.

In the *final stage*, users simultaneously choose between staying on B or subscribing to E. Revenue is generated from two sources – *ad-revenues* from B and *subscription revenues* from E – where per-user revenue from E is strictly higher than B. The per-user rate of ad-revenue, as well as subscription fee, is fixed.[4] The cost of the platform in promising personalised incentives to users is increasing and convex. In general, incentives are costly to produce and provide greater utility than uniform content. The existence of a cost element is the rationale that the direct content-utility from E must exceed that of B. Per-user network utility on the E strictly exceeds that on B. Since the benefit from the collaborative feature of exclusive content exceeds that of the B. The collaborative benefits are derived from networks and captured through directed links between any pair of representative users i and j. A directed link from user i to user j implies i is directly *susceptible* to j, or equivalently, j directly *influences* i. This overall pattern of who influences whom is given by the adjacency matrix, which helps determine the influence of each agent.

We find that the optimal investment in personalised incentives to any user is directly

[3]See Rysman (2009) for the summary of the economics of two-sided markets.
[4]We assume this for analytical convenience.

proportional to the influence the user has in their network. Here, the comparative statics establish how variations in the subscription fee, ad-revenue, and other parameters like the bias, network strength impact the optimal profit of the platform. In the process, we find that the aforementioned factors rely on aggregate user influence and higher-order moments of user influence. These parameter shifts dictate whether platform profit increases or decreases in equilibrium.[5]

Our framework allows us to concentrate on trade-offs associated with market coverage between plans B and E. Even though E generates greater per-user revenue, does it motivate the platform to provide costly incentives on E? *Proposition* 2 shows that a rise in ad-revenue increases equilibrium profit in proportion to the potential user share on B. In essence, rising ad-revenue demotivates the platform to invest in costly incentives on E. Users comprehend such under-provision, thereby preferring B. *Proposition* 4 shows the marginal effect of ad-revenue is non-monotonic to a rise in subscription fee (U-shaped around a threshold fee). Below the threshold, a rise in fee means better personalised incentives, thereby leading susceptible users to conform with their influential counterparts to join E. However, a fee rise above the threshold makes the network spillover on E less salient to users, thereby impeding them from joining E.

Proposition 3 shows that a rising subscription fee has a non-monotonic (inverse-U shaped around a threshold) effect on the optimal profit. Initially, an increase in the fee builds a strong motive to invest and provides incentives to attract influential users on E, which eventually lures the more susceptible ones. However, a further rise beyond the threshold makes susceptible users reluctant to follow their influential counterparts to join E, given incentives are only proportional to their low influence. A subsequent corollary on the net marginal effect of subscription fee and ad-revenue on profit shows that when private shocks have sufficiently large variance, the marginal effect of subscription fee strictly exceeds that of ad-revenue. This effect is also directly proportional to the aggregate influence. Intuitively, if B exposes users to extreme shocks, the presence of more influential users exerts enough externality to lure users to pay the subscription fee to join E, thereby escaping from such shocks.

Furthermore, we identify the conditions to measure the impact of preference bias on the optimal profit. If all users are uniformly biased toward the basic plan, any further increase in this bias will reduce the equilibrium profit (*Proposition 5a*). This is because a

[5] If user utility from personalised incentives is linear, then variance of user-influence determines some of the comparative statics. If the user-utility is strictly concave, then higher-order moments are relevant.

stronger bias for B makes users more reluctant to adopt plan E, thereby lowering per-user revenue, which is relatively greater on E. Our analysis shows the gravitation towards B rises as the variance of ad shocks becomes less severe, and users' influence in the network grows stronger. On the other hand, if a user group is biased towards plan E while the other group is equally biased toward plan B, a rise in bias hurts equilibrium profit if the users in B are relatively more influential than users on E (*Proposition 5b*). However, if the opposite holds, then a rise in bias strictly increases platform profit. This is because the users on plan E being more influential than those on B are more likely to attract a greater number of susceptible users towards E.

We also find that an increase in strength of the user-network, through adding new links, can lower platform's optimal profit if the subscription fee on E is below a threshold (*Proposition 6*). An increase in network strength increases overall influence, which leads more users to join E due to network spillover, thereby imposing a greater cost of providing personalised incentives. Unless the subscription fee exceeds the critical value, high cost of providing incentives will always dominate the revenue generated.

Finally, we explore if the provision of incentives increases under a duopoly setup – an established (or dominant) platform and another relatively new (or emerging) one.

2 Related Literature

Our work lies at the confluence of industrial organisation with user-level networks under uncertainty. In the ambit of Industrial Organisation (IO), we focus on the conditions and comparative statics for providing optimal incentives, keeping prices fixed and accounting for user-level networks. In platform economics, two key strands of literature are closely related to our line of work – first, where the platform endogenously determines the optimal subscription fee, and second, where the platform endogenously determines the critical mass of users required to be profitable.

The seminal work of Mussa and Rosen (1978) demonstrates how a monopolist optimally selects both quality and price. The notion of quality captures the hedonic attributes of the product. This can be construed as an incentive in our model. Nonetheless, our premise is different. In a slightly different hierarchical setting, Winter (2004) highlights the importance of personalised incentives in an asymmetric environment where agents are differently rewarded based on their efforts. They show that in the presence of some externalities, higher incentives are attributable to higher ranks in an organisational setting. Katz and Shapiro (1986) models the adoption of products (technologies) in a dynamic setting with network externalities and provides useful insights about pricing in such a setting. Similarly, Farrell and Saloner (1985) focuses on the inertia of the adoption of technologies. These contributions highlight the implications of network externalities – both indirect and direct – on the adoption of technologies.

In the niche topic of optimal pricing with externalities, platforms incentivise a portion of the agents to improve coordination, (Bernstein and Winter, 2012, Candogan et al., 2012, Bloch and Quérou, 2013, Fainmesser and Galeotti, 2016). It turns out that the optimal pricing is a function of the Bonacich Centrality of the agents. This result is a feature that arises in the seminal work of Ballester et al. (2006). A significant portion of the literature on pricing in networks deals with deterministic networks, where the firm and the users have complete information about the network structure and use the linear-quadratic utility function of users. Notable exceptions are Bloch and Quérou (2013), which studies a model with personalised prices and private valuation of the product, and Fainmesser and Galeotti (2016), which considers a random network with users having limited information about networks. If firms – monopolist or competing – have prior information about the user-level network, they can strategically target and tailor prices to subsidise influential ones and seek rent from the susceptible ones (Fainmesser and Galeotti, 2016, 2020). Similarly, Candogan et al. (2012) characterises that a monopolist offers the product at a lower price or for free to an influential agent, say i, and bolsters participation from other agents who are influenced by i.

A portion of the theoretical literature highlights the pricing and welfare implications of a two/multi-sided market under monopoly as well as competition (Hagiu, 2006, Weyl, 2010, Jullien, 2011, Hagiu and Spulber, 2013, Fainmesser and Galeotti, 2020, Belleflamme and Peitz, 2024).[6] Research on platforms has focused on network effects and the relevant welfare implications under such conditions (Rochet and Tirole, 2003, Anderson and Coate, 2005, Armstrong, 2006, Weyl, 2010, Jullien et al., 2021). The importance of network externalities has been highlighted in the literature on pricing networked goods. Jeon et al. (2022) analyses second-degree price discrimination in two-sided platforms through a principal-agent framework where the consumers' side interacts with the advertisers. Furthermore,

[6] Gowrisankaran and Stavins (2004) provides empirical evidence of network externalities in new technology adoption.

information about the social network among users aids advertisers in targeting users by their preferences over a product (Eliaz and Spiegler, 2020). On a similar note, Tan and Zhou (2021) highlights that for single-homing, an increase in competition may lead to an increase in platform profits and prices and a reduction in welfare. This motivates us to assume single-homing in the baseline model which is later 'relaxed' to our convenience. However, the implications of welfare and efficiency are slightly different in a monopoly than under competition. To complement the literature, we abstract away from the welfare aspect and focus on microfoundations of network externalities.

Furthermore, we complement the literature on *Freemium* models, which introduces the notion of segmenting users into basic and exclusive. Some Freemium models encounter a trade-off in generating revenues between increasing the number of premium users for the subscription-based revenues against increasing the number of basic users for the ad revenues (Zennyo, 2020). In this ambit, Sato (2019) proposes a binary pricing scheme where the basic plan includes full advertisements while the premium plan is devoid of any ads.

We deviate from the above literature on the grounds of the choice variable — providing personalised incentives to any potential user on the exclusive plan. Unlike the standard IO literature on platforms, which isolates subscription-based or ad-based monetisation, our framework integrates both and studies the trade-off faced by the platform and how their interaction shapes platform behaviour. Our model is a functional workhorse model for several types of platforms. In terms of provision of contents by platform, at one extreme, we have only ad-revenue platform, and on the other, we have a subscription-based based one.[7] Realistically, it seems complex to provide exclusively personalised incentives, but some platforms use coarse group or cohort-based targeting based on the influence of groups.[8]

[7] Based on the nature of the problem, one can attach appropriate weights to fit the required purpose.

[8] Strava, a fitness app, allows clubs or groups to form communities around specific interests. These clubs serve as platforms for sharing content, organising group challenges, effective targeting etc. The feature can be construed as an incentive in our model.
Steemit is a blockchain-powered social media platform which provides personalised rewards to users based on content generated, curated etc.(see (Li and Palanisamy, 2019))

3 Model

3.1 Basic framework

Consider a model with $n \geq 2$ users on an online platform. Initially, all users are enrolled into the basic plan (B) provided by the platform. The basic plan is free to use and provides *generic features* and restricted access to the users. Later, the users get the option to switch and subscribe to the exclusive plan (E). The exclusive plan provides users with *incentives* – which include tailored content, personalised perks, full access, and other benefits. A user enjoys the exclusive plan for a fixed fee of $c > 0$. Effectively, each user has to either stay with plan B or switch to plan E. In other words, each user chooses plan $f \in \{B, E\}$. The actual valuation of the incentives to user i is denoted by $k_i \geq 0$. In particular, the platform credibly promises incentives to each potential user who subscribes to plan H. The vector of incentives to n potential users is given by $\mathbf{k} = (k_1, k_2, \cdots, k_n)$. On the contrary, the basic plan is free to use for all users.

Typically, online platforms purchase consumer data from an external data repository, third-party brokers or access it from a parent or sister company with social connections data. We assume that the platform has complete information about the interpersonal relations among the users. This assumption keeps our optimisation problem tractable. The interpersonal relations between users are captured by a *directed network* and is formally represented via the $n \times n$ adjacency matrix $\mathbb{G} = [g_{ij}]_{n\times n}$ where $g_{ij} = \{0, 1\}$.[9] A directed link from user i to j implies that user i values choosing the same plan as j. In our model, the directed link from i to j refers to two equivalent relations – user i is directly susceptible to j is the same as user j has direct influence over i. Note, g_{ij} is 1 if users values choosing the same plan as the user they are influenced by, and 0 otherwise.[10] We assume, $g_{ii} = 0$, i.e., we rule out self-loops for all users. In our analysis, g_{ij} need not be the same as g_{ji}.[11]

3.2 Objective of users

Each user cares about the following: (a) utility from incentives in the exclusive plan, (b) users get additional disutility from the subscription fee for the exclusive plan, (c) a user

[9] Our analysis is valid for $g_{ij} \in [0, 1]$ where g_{ij} is how strongly user i is directly influenced by user j.

[10] Note, user i can be indirectly susceptible to j, if not directly susceptible, only if there exists a walk in $\mathbb{G}$ from i to j.

[11] Since the network $\mathbb{G}$ is directed, user i is susceptible to j's actions, but the converse need not be true.

may have a personal preferential bias towards or away from the exclusive plan, (d) the network utility derived from choosing a similar plan as the users who influence them, and (e) the private shock they receive from choosing the basic plan.[12]

The overall utility of user i from choosing plan $f \in \{B, E\}$ is given by[13]

$$U_i(f) \;=\; \begin{cases} b_i \;+\; \gamma\delta \cdot \sum\limits_{j \in N} g_{ij} \cdot v_j(B) \;+\; \epsilon_i & \text{if} \;\; f = B \\ \\ u(k_i) \;-\; c \;+\; \delta \cdot \sum\limits_{j \in N} g_{ij} \cdot v_j(E) & \text{if} \;\; f = E \end{cases}$$

If user i chooses the basic plan, the first term refers to the personal (dis)utility of the user from their predisposition towards or away from the basic plan. In other words, $b_i \in [-b, b]$ is the preferential bias of a user towards or away from the basic plan. The information about b_i is common knowledge.[14] $b_i = 0$ refers that user i has no predisposition towards any plan. If the user chooses the exclusive plan, the first term $u(k_i)$ is the utility of the user from the incentives promised by the platform for choosing the exclusive plan. The incentive-utility function $u(k_i)$ is strictly increasing, twice continuously differentiable, and strictly concave in k_i, with $u'(k_i) \to \infty$ as $k_i \to 0$.

The second term captures the overall network-utility derived by each user from co-aligning their actions with the influential users in their network. Any representative user cares about the actions of the members they are susceptible to in their network. If user j chooses the exclusive plan, and i is susceptible to j's action, then i derives a strictly positive spillover of δ from aligning their choice with their influencers. $v_j(f)$ is a binary indicator variable which takes the value of 1 if user j' chooses plan f, and zero otherwise. Similarly, if user j chooses the basic plan, and i is susceptible to j's action, then i derives the network benefit of δ from aligning their choice with their influencers but the network benefit per user is moderated by a discount factor $\gamma \in (0, 1]$. This implies that the network spillover effect from co-action is not as pronounced under the basic plan as the exclusive one. Therefore, being in the network strictly enhances the network spillover effects.

[12]The utility of one user from the basic plan is normalised to 0. Thus, the cost of producing the contents in the basic plan is costless.

[13]Each user derives a fixed utility of $\bar{u} \geq 0$ from being in the platform, regardless of the plan. Both plans of the platform are considered as 'good' in nature. Since $\bar{u}$ will be present in both $U_i(E)$ and $U_i(B)$, we rule out its dependence on the overall utility function.

[14]Typically, platforms require users to fill out a survey questionnaire. In some cases, the survey reveals to the platform the magnitude of bias towards the basic or the exclusive plan.

If user i subscribes to the basic plan, they are vulnerable to a private preference shock of ϵ_i which is uniformly distributed in the interval of $[-\frac{1}{2e}, \frac{1}{2e}]$ where the density $e > 0$, and the mean is zero. This shock may come in the form of ad-content which generates revenue for the platform and can either benefit or hurt the basic user. The advertisement shocks are independently and identically distributed across users, and their distribution is assumed to be common knowledge.[15] On the other hand, if user i chooses the executive plan, they are required to subscribe and pay the fixed fee of c.

3.3 Objective of the platform

The objective of the platform is to maximize the expected profit subject to a resource constraint. The platform generates *ad revenue* from streaming advertisements to basic users and *subscription revenue* from the exclusive users. It is costly for the platform to promise and provide incentives to its exclusive users. Let $\mathbf{K}$ denote the set of all feasible incentive vectors $\mathbf{k} = (k_1, k_2, \cdots, k_n)$ such that each $k_i \geq 0$. Formally speaking, the platform announces a feasible incentive vector $\mathbf{k} \in \mathbf{K}$ that maximises the profit of the platform.

For any distribution of the private information shock, the ex-ante probability that user j chooses the exclusive plan is the expected value of $v_j(H)$. The ex-ante probability of choosing the exclusive plan p_j is derived from $E[v_j(H)]$.[16] Note that p_j is derived after the vector of incentives is announced by the platform but before the information shocks are observed by the user.

The expected profit of the platform is

$$\begin{aligned} \Pi(\mathbf{k}) &= \sum_{j=1}^{n} c \cdot p_j(\mathbf{k}) + \sum_{j=1}^{n} (1 - p_j(\mathbf{k})) \cdot r - \sum_{j=1}^{n} \frac{\alpha}{2} \cdot k_j^2 \\ &= c \cdot \sum_{j=1}^{n} p_j(\mathbf{k}) + r \cdot \sum_{j=1}^{n} (1 - p_j(\mathbf{k})) - \sum_{j=1}^{n} \frac{\alpha}{2} \cdot k_j^2 \end{aligned} \tag{1}$$

where r is the ad-revenue earned from each basic user. The first part of the expression is the expected revenue of the platform from the premium users, and the second part is the expected revenue from the basic users. The final term is the cost of preparation and

[15] In any platform, ads are randomly shown to all basic customers. Some ads are beneficial to a user, and some ads cause hindrances.

[16] We restrict our attention to the pure incentive strategies of the platform.

delivery of the promised incentives to users, where α is the cost of incentives.[17]

4 Equilibrium

The equilibrium $(\mathbf{k}^*, \mathbf{p}^*(\mathbf{k}^*))$ is an ordered pair where $\mathbf{k}^*$ is the optimal incentive vector announced by the platform, and $\mathbf{p}^*(\mathbf{k}^*)$ is the optimal ex-ante probability of a user subscribing to the exclusive plan. The timing of the game indicates that it is a two-stage game, which we solve via backward induction.

Firstly, the platform chooses the incentive vector $\mathbf{k}$ such that it maximises the profit of the platform. Then the users observe their private shock, which affects their utility. The shock introduces sufficient uncertainty about the user's utility to the platform. Due to the private shock to the users, the platform cannot perfectly predict the exact level of incentives required by the users that induce them to choose the desired plan. Later, each user chooses their preferred plan. Our analysis is from an ex-ante perspective; hence, to determine the equilibrium, we analyse from an expected sense.

The expected utility of user i conditional on ϵ_i for $f \in \{B, E\}$ is as follows

$$EU_i(f) \ = \ \begin{cases} b_i \ + \ \gamma\delta \cdot \sum_{j \in N} g_{ij} \cdot (1 - p_j) \ + \ \epsilon_i & \text{if} \quad f = B \\ \\ u(k_i) \ + \ \delta \cdot \sum_{j \in N} g_{ij} \cdot p_j \ - \ c & \text{if} \quad f = E \end{cases} \tag{2}$$

For any user, the expected value of aligning the choice of the exclusive plan with those who influence them is represented by p_j. Conditional on ϵ_i, a user subscribes to the exclusive plan if their expected utility from the exclusive plan is at least as high as that of the basic plan, i.e. if

$$\begin{aligned} EU_i(E|\epsilon_i) \ &\geq \ EU_i(B|\epsilon_i) \\ \epsilon_i \ &\leq \ u(k_i) \ - \ b_i \ + \ \delta(1+\gamma)\cdot[\sum_{j \in N} g_{ij} \cdot p_j(\mathbf{k}) - (\frac{\gamma}{1+\gamma}) \sum_{j \in N} g_{ij}] \ - \ c \end{aligned} \tag{3}$$

[17]The upper bound on resources that can be spent on incentivizing is K such that $\frac{\alpha}{2} \sum_j k_j^2 \leq K$ and $K \leq \bar{K} < \infty$ where $\bar{K}$ is sufficiently high. Additionally, the cost of producing generic content is normalized to zero.

We use equation 4 to determine the ex-ante probability of users. The probability that user i subscribes to the exclusive plan is given by

$$p_i(\mathbf{k}) \;=\; \frac{1}{2} \;+\; e \cdot \Big(u(k_i) \;-\; b_i \;-\; c \;+\; \delta(1+\gamma) \cdot \big[\sum_{j \in N} g_{ij} \cdot p_j(\mathbf{k}) - (\frac{\gamma}{1+\gamma}) \sum_{j \in N} g_{ij} \big] \Big) \tag{4}$$

To ensure that the solution to equation 5 lies in the interior and is unique, we introduce the two following assumptions:

Assumption 1 (Interiority). Let $\bar{p}$ and $\underline{p}$ denote the highest and lowest probabilities of a user choosing the exclusive plan. The maximum probability of i for choosing H is

$$p_i^{\max} \;=\; \min\{1, \bar{p}\},$$

where $\bar{p}$ lies in the interval $[0, 1]$. The value of $\bar{p}$ is given by

$$\bar{p} \;=\; \frac{1}{2} \;+\; e\Big(u(\sqrt{\frac{2K}{\alpha}}) + b - c\Big) \;+\; \frac{\beta(n-1)}{1+\gamma}$$

where $\beta \;=\; e\delta(1+\gamma) \;\in\; (0, 1)$. The above expression captures the situation where (i) the platform invests all its resources on incentivizing the user, (ii) the user is preferentially biased towards the exclusive plan where $\min\{b_i\} \;=\; -b$, (iii) the user is susceptible to the action of every other user, i.e. $g_{ij} = 1$ for all $j \neq i$, and (iv) the probability of all other users will certainly subscribe to the exclusive plan, i.e. $p_j = 1$ for all $j \neq i$.

Analogously, the minimum probability of i for choosing H is

$$p_i^{\min} \;=\; \max\{0, \underline{p}\},$$

where $\underline{p}$ lies in the interval $[0, 1]$. The value of $\underline{p}$ is given by

$$\underline{p} \;=\; \frac{1}{2} - eb \;-\; ec \;-\; \frac{\beta(n-1)}{1+\gamma}$$

The previous expression entails the scenario where (i) the platform does not incentivize the user, i.e. $u(0) \;=\; 0$, (ii) the user is preferentially biased towards the basic plan where $\max\{b_i\} \;=\; b$, (iii) the user is susceptible to the action of every other user, i.e. $g_{ij} \;=\; 1$ for all $j \neq i$, and (iv) the probability of all other users will certainly subscribe to the basic plan, i.e.

$p_j = 0$ for all $j \neq i$. Note that Assumption 1 ensures that the solutions are interior.

The vector of probabilities of users choosing H is given by $\mathbf{p} = (p_1, \cdots, p_n)$. The probability function p_i takes the vector of announced incentives $\mathbf{k}$ as input and maps to $[0,1]$.[18] Hence, the vector of probabilities is a mapping from the domain $[0,1]^n$ to itself. Using equation 5, we generate the system of linear equations of users' probabilities of choosing the exclusive plan, which is represented as:

$$\begin{pmatrix} p_1(\mathbf{k}) \\ \vdots \\ p_n(\mathbf{k}) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + e\Big(u(k_1) - b_1 - c + \delta(1+\gamma)\cdot[\sum_{j\in N} g_{1j}\cdot p_j(\mathbf{k}) - (\frac{\gamma}{1+\gamma})\sum_{j\in N} g_{1j}]\Big) \\ \vdots \\ \frac{1}{2} + e\Big(u(k_n) - b_n - c + \delta(1+\gamma)\cdot[\sum_{j\in N} g_{nj}\cdot p_j(\mathbf{k}) - (\frac{\gamma}{1+\gamma})\sum_{j\in N} g_{nj}]\Big) \end{pmatrix} \tag{5}$$

The vector of probabilities of all users adopting the exclusive plan turns out to be

$$\mathbf{p}(\mathbf{k}) = \frac{1}{2}\cdot\mathbf{1} + e\cdot\mathbf{u}(\mathbf{k}) - e\cdot\mathbf{b} - ec\cdot\mathbf{1} + \beta\cdot\mathbb{G}\cdot\mathbf{p}(\mathbf{k}) - e\delta\gamma\cdot\mathbb{G}\cdot\mathbf{1} \tag{6a}$$

where $\beta = e\delta(1+\gamma) \in (0,1)$, $\mathbf{1}$ is the $n \times 1$ vector of ones, $\mathbf{u}(\mathbf{k}) = (u(k_1), \cdots, u(k_n))$ is the vector of incentives derived by users, $\mathbf{b} = (b_1, \cdots, b_n)$ is the vector that accounts for predispositions towards or away from the basic plan. Solving equation 6a, we get

$$\mathbf{p}^*(\mathbf{k}) = \Big(\frac{1-\gamma}{2(1+\gamma)}\Big)\cdot(\mathbb{I}-\beta\mathbb{G})^{-1}\cdot\mathbf{1} + (\frac{\gamma}{1+\gamma})\cdot\mathbf{1} + e\cdot(\mathbb{I}-\beta\mathbb{G})^{-1}\cdot[\mathbf{u}(\mathbf{k}) - \mathbf{b} - c\cdot\mathbf{1}] \tag{6}$$

where $(\mathbb{I} - \beta\mathbb{G}^\top)^{-1} = \mathbb{X}(\beta,\mathbb{G}) = [x_{ij}]$ is a $n \times n$ matrix of inverses where x_{ij} is the total number of discounted walks from j to i. The following assumption of invertibility ensures that the solution is unique.

Assumption 2 (Invertibility). For any feasible network $\mathbb{G}$, the matrix $(\mathbb{I} - \beta\mathbb{G}^\top)$ is invertible, if β is sufficiently small. The largest eigenvalue of matrix $\mathbb{G}$ is $\zeta(\mathbb{G})$ such that $e\delta(1+\gamma)\cdot\zeta(\mathbb{G}) < 1$. This condition is sufficient to ensure that $(\mathbb{I} - \beta\mathbb{G}^\top)$ is invertible which is similar to Ballester et al. (2006). In the paper, we also deal with changes in the network structure; hence, we assume a slightly tighter sufficiency condition $e\delta(1+\gamma)\cdot n < 1$. Now,

[18] We utilise Brouwer's fixed point theorem to ensure that the solution exists.

we define the notion of influence of users.

Influence. We define the vector of *Influence* of the users for any $\mathbb{G}$ by

$$\boldsymbol{\mathcal{I}}(\beta, \mathbb{G}) \;=\; (\mathbb{I} - \beta \mathbb{G}^{\top})^{-1} \cdot \mathbf{1} \;=\; \sum_{z=0}^{+\infty} \beta^{z} \cdot (\mathbb{G}^{\top})^{z} \cdot \mathbf{1} \tag{7}$$

where z-th power of $\mathbb{G}$ tracks all the walks of length z in $\mathbb{G}$ between j and i. To reiterate, each entry x_{ij} measures the susceptibility of user j on i. In other words, x_{ij} measures the influence of user i on j. x_{ij} measures the total number of discounted walks of all possible lengths between j and i.

The influence of any user i on others who are susceptible to user i's action is $I_i(\beta, \mathbb{G}) = \sum_{j=1}^{n} x_{ij}(\beta, \mathbb{G})$ where $x_{ij}(\beta, \mathbb{G}) \geq 1$ is the row sum which is interpreted as the influence of user i to on any j.[19] We refer to the aggregate of the influences of all users in the system as $\mathcal{I}_{agg}$ where $\mathcal{I}_{agg} = \sum_{i} I_i$. In a directed network, $\mathbb{G}$ may not be the same as $\mathbb{G}^{\top}$, hence the influence measure of a user i on others may not be analogous to their susceptibility measure of i towards others.[20] Note, the influence measure of any user appeals to the structure of $\mathbb{G}$ and is independent of any other primitives.

4.1 First-order conditions

The vector of probabilities of users opting for the exclusive plan is given by $\mathbf{p} = (p_1, \cdots, p_n)$. Differentiating with respect to incentive yields

$$J_i[\mathbf{p}^{*\top}] \;=\; e \cdot J_i[\mathbf{u}^{\top}] \cdot \mathbb{X} \qquad [\because \; \frac{\partial k_j}{\partial k_i} = 0, \; \frac{\partial u(k_j)}{\partial k_i} = 0 \;\forall\; j \neq i]$$

$$J_i[\mathbf{p}^{*\top}] \cdot \mathbf{1} \;=\; \sum_{j} \frac{\partial p_j}{\partial k_i} \;=\; e \cdot \sum_{j} \frac{\partial u_i}{\partial k_i} \cdot x_{ij} \;=\; e \cdot u_i' I_i$$

[19]Similarly, we can construct the susceptibility of any user i from the column sum. The susceptibility of any user i is $S_i(\beta, \mathbb{G}) = \sum_{i=1}^{n} x_{ij}(\beta, \mathbb{G})$. The susceptibility vector is

$$(\mathbb{I} - \beta \mathbb{G})^{-1} \cdot \mathbf{1} \;=\; [\, x_{ij} \,]_{n \times n}^{\top} \cdot \mathbf{1} \;=\; (S_1, S_2, \cdots, S_n)^{\top} \;=\; \boldsymbol{\mathcal{S}}(\delta, \theta, \mathbb{G})$$

[20]If the network $\mathbb{G}$ is undirected, then $\mathbb{G} = \mathbb{G}^{\top}$, then $I_i = S_i$. The influence of any i is the same as their susceptibility. In social network literature, for any undirected network, the measure of the influence of user i can be interpreted as the unweighted Katz-Bonacich centrality of i in $\mathbb{G}$.

where $J_i = \frac{\partial}{\partial k_i}[\cdot]$ is the first order differentiation with respect to k_i.[21] Note that, $J_i[\mathbf{u}^\top] = (0, \cdots, \frac{\partial u(k_i)}{\partial k_i}, \cdots, 0)$, and $J_i[\mathbf{k}^\top] = (0, \cdots, \underbrace{1}_{\text{i-th term}}, \cdots, 0)$.

The vector expression of the expected profit (in equation 2) of the platform for an arbitrary level of incentive $\mathbf{k}$ is

$$\Pi(\mathbf{k}) \ = \ rn \ + \ (c-r) \cdot (\mathbf{p}^*)^\top \cdot \mathbf{1} \ - \ \frac{\alpha}{2} \cdot \mathbf{k}^\top \cdot \mathbf{k}$$

Differentiating the constrained profit function of the platform with respect to incentives is given by

$$\begin{aligned} \Pi'(\mathbf{k}) \ &= \ (c-r) \cdot J_i[\mathbf{p}^{*\top}] \cdot \mathbf{1} \ - \ \frac{\alpha}{2} \cdot (J_i[\mathbf{k}^\top] \cdot \mathbf{k} \ + \ \mathbf{k}^\top \cdot J_i[\mathbf{k}]) \ = \ 0 \\ \implies \alpha k_i \ &= \ e(c-r) \cdot u_i' \cdot I_i \quad \implies \ \frac{k_i}{u'(k_i)} \ = \ e\frac{(c-r)}{\alpha} \cdot I_i \end{aligned} \tag{8}$$

In the absence of any networks, i.e. when $\mathbb{G} = [\mathbf{0}]$, the influence of any user is 1 and the above expression boils down to $\frac{k_i}{u'(k_i)} = e\frac{(c-r)}{\alpha}$. The solution vector is feasible if $c > r$, and $\alpha, c, r, e, a > 0$. The equilibrium value of the profit of the platform is

$$\Pi(\mathbf{k}^*) \ = \ rn \ + \ (c-r) \cdot \Big(\frac{\mathcal{I}_{agg}}{2(1+\gamma)} \ - \ ce \cdot \mathcal{I}_{agg} \ + \ (\frac{\gamma}{1+\gamma})(n - \frac{\mathcal{I}_{agg}}{2}) \ + \ e \cdot [\mathbf{u}(\mathbf{k}^*)^\top - \mathbf{b}^\top] \cdot \boldsymbol{\mathcal{I}} \Big) \ - \ \frac{\alpha}{2} \cdot \sum_{j=1}^{n} k_j^{*2}$$

In summary, Assumption 1 ensures that the solution lies in the interior, and Assumption 2 guarantees that the influence score of each user is positive and well-defined. In particular, we obtain the following proposition using Assumption 1 and Assumption 2.

Proposition 1. *There exists a unique a pure strategy equilibrium* $(\mathbf{k}^*, \mathbf{p}^*(\mathbf{k}^*))$.

In equilibrium, the expected profit of the platform is

$$\Pi(\mathbf{k}^*) \ = \ c \cdot \mathbb{P}(\mathbf{k}^*) \ + \ r \cdot (n - \mathbb{P}(\mathbf{k}^*)) \ - \ \frac{\alpha}{2} \cdot \sum_{j=1}^{n} k_j^{*\,2} \tag{9}$$

where $\mathbb{P}(\mathbf{k}^*) = \sum_{j=1}^{n} p_j(\mathbf{k}^*)$. In equilibrium, the probability of adopting the exclusive plan is increasing in the utility from the incentives and decreasing in the subscription fee. Consequently, the optimal profit of the platform is increasing in the probability of adopting

[21] We suppress the dependence of the probability vector $\mathbf{p}(\mathbf{k})$ on the quality $\mathbf{k}$. We refer to $\mathbf{p}(\mathbf{k})$ as $\mathbf{p}$ for notational convenience.

the exclusive plan and decreasing in the probability of the basic plan. In the next section, we discuss the impact of changes in the parameters used in the model on the equilibrium profit.

Social connections are important not only in the context of problems in political economics, but also in problems in the ambit of IO. In political economics, interest groups allocate resources among legislators to gain their vote on the interest group's preferred agenda. The optimal allocation of resources (money) is proportional to the *Katz-Bonacich centrality measure*, among other factors like voting rule and preferences of legislators Battaglini and Patacchini (2018), Das Chaudhury et al. (2023). Lever Guzmán (2010) also posits how potential voters take cues from their social environment before they cast their vote. They solve the optimal resource allocation strategy to be the one where influential voters, carrying a higher value of eigen-vector centrality, are bestowed with higher resources. In our model, the optimal investment in the allocation of incentives based on the relative influence of the user is somewhat similar.

5 Comparative statics

Here, we study the impact of changes in the elements of the tuple $\Omega = (\alpha, c, r, e, \mathbb{G})$ and their interactions on the objective of the platform. We show how the platform's equilibrium profit responds to changes in parameters across different network architectures. For the rest of the analysis, we assume $c > r$. This implies that the return per user from the exclusive plan is always larger than the return per user from the basic plan. Otherwise, the platform is always better off providing no exclusive incentives.

5.1 Ad-returns

Ceteris paribus, any increase in the per-user ad-returns (r) benefits the platform from higher revenue from the basic plan. An increase in r strengthens the platform's resources to invest in providing incentives to its exclusive users. On the other hand, the increase in r weakens the platform's incentive to invest in the exclusive plan as the basic plan has become relatively more rewarding. We are interested in the net effect on the equilibrium profit due to an increase in ad-returns per user. Suppose that assumptions 1 and 2 hold. Fix the tuple $(\alpha, c, e, \mathbb{G})$, we want to see the impact of a change in r on the equilibrium profit in any given network $\mathbb{G}$.

Proposition 2. *An increase in per-user ad-returns from the basic plan (weakly) increases the equilibrium profit of the platform in proportion to the expected share of basic users* $(n - \mathbb{P}(\mathbf{k}^*))$.

Simply speaking, an increase in r makes the basic plan appear more attractive to the platform. The platform generates higher revenue per user from the exclusive plan than the basic, i.e. $c > r$. However, providing incentives to exclusive users is costly. One direct effect is that an increase in revenue from r weakens the incentive of the platform towards providing costly incentives on the exclusive plan. From the perspective of some users, staying on the basic is more reasonable as the cost of being susceptible and switching to exclusive is more expensive. The possibility of a reduction in incentives on the exclusive plan impedes some users from joining E.

The first-order condition of the equilibrium profit with respect to r gives the following:

$$J_r[\Pi(\mathbf{k}^*)] = \underbrace{(n - \mathbb{P}(\mathbf{k}^*))}_{\text{Share of Basic users}} + \underbrace{(c - r) \cdot J_r[\mathbb{P}(\mathbf{k}^*)]}_{\text{Marginal Impact on revenue from switching}} - \underbrace{\frac{\alpha}{2} \cdot (J_r[\mathbf{k}^{*\top} \cdot \mathbf{k}^*])}_{\text{Change on investment in incentives}}$$

The first term in the above expression is the additional revenue generated due to an increase in r which is the share of basic users. The second term is the impact of the rise in r on the probability of choosing the exclusive plan. An increase in r reduces the additional benefit from acquiring exclusive users which leads to a reduction in investment on incentives. Additionally, $(c-r)$ is the incremental return per user in the exclusive plan compared to the basic plan. The third term indicates the reduction in the cost of investment on incentives due to the increase in r. Using the envelope theorem, the second and the third terms cancel out, i.e. $J_r[\Pi(\mathbf{k}^*)] = (n - \mathbb{P}(\mathbf{k}^*))$. In other words, an increase in r increases the optimal profit by the share of basic users. This implies that an increase in the per-user return from ad revenue reduces the probability of adopting the exclusive plan. The proof is in the appendix A.1.

Furthermore, the rate of change of equilibrium profit is increasing in r, $J_{rr}[\Pi(\mathbf{k}^*)] > 0$. As a consequence, the adoption of exclusive plan goes down. This causes the subscription revenue to go down. Additionally, the cost of providing incentives goes down, since as $r \to c$, then $k_i^* \to 0$. Hence, increasing r would erode all exclusive users.

5.2 Subscription fees

Ceteris paribus, any increase in the per-user subscription fee (c) benefits the platform from higher revenue from the exclusive plan. An increase in subscription fee generates a stronger incentive for the platform towards investing in costly investments in order to provide users with more exclusive content upon joining E. However, investment in incentives is costly. The question is, what should be the optimal level of incentives such that influential users are able to attract relatively susceptible users on plan E, due to an increase in c.

The first-order condition of the equilibrium profit with respect to c gives the following:

$$J_c[\Pi(\mathbf{k}^*)] \;=\; \underbrace{\mathbb{P}(\mathbf{k}^*)}_{\text{Share of Exclusive users}} \;+\; \underbrace{(c-r)\cdot J_c[\mathbb{P}(\mathbf{k}^*)]}_{\text{Marginal Impact on revenue from switching}} \;-\; \underbrace{\frac{\alpha}{2}\cdot\left(J_c[\mathbf{k}^{*\top}\cdot\mathbf{k}^*]\right)}_{\text{Change on investment in incentives}}$$

The first term in the above expression is the additional profit due to an increase in c which is the share of exclusive users. The second term is the impact of the rise in c on the probability of choosing the exclusive plan. The third term indicates the cost of investment on incentives due to an increase in c. The aggregate of the three terms produces the following

$$J_c[\Pi(\mathbf{k}^*)] \;=\; \mathbb{P}(\mathbf{k}^*) \;-\; e(c-r)\cdot\mathcal{I}_{agg}.$$

This result follows from the envelope theorem. The impact of an increase in the subscription fee on equilibrium profit is determined by two key factors – (i) the first term is the *direct adoption effect* generated by additional revenue from existing exclusive users, and (ii) the second term is the *amplified spillover effect* from the difference in revenue generated per user from exclusive to basic amplified the aggregate network influence. J_c is the net effect of the revenue gain due to an increase in c on the adoption of E and the revenue loss due to some users being discouraged towards E modified by the network spillover effect. Suppose that assumptions 1 and 2 hold. Fix the tuple $(\alpha, r, e, \mathbb{G})$, we want to see the impact of a change in c on the equilibrium profit in any given network $\mathbb{G}$.

Proposition 3. *In equilibrium, the impact of c on equilibrium profit is non-monotonic. There exists a critical level of subscription fee $\bar{c}$ such that*

(a) *for any $c < \bar{c}$, the equilibrium profit strictly increases with an increase in c.*

(b) *for any $c \geq \bar{c}$, the equilibrium profit does not increase with an increase in c.*

Part (a) of the proposition demonstrates that there exists a critical subscription fee $\bar{c}$ which is determined by the net of direct adoption effect, supersedes the amplified spillover effect. In other words, $\bar{c}$ is obtained by the difference between the adoption effect and the amplified spillover effect. An increase in c increases the optimal profit if $J_c[\Pi(\mathbf{k}^*)] > 0$. If c is sufficiently low, the share of adoption of exclusive users is high. In this case, if c increases, the gap between c and r widens, the amplified spillover effect grows and negatively impacts J_c. In relatively denser networks, the spillover effects amplify the effect of adoption (or lack thereof) of the exclusive plan. However, the adoption effect may still be higher, leading to an overall positive effect on profit.

Part (b) of the proposition demonstrates that an increase in c decreases the optimal profit if $J_c[\Pi(\mathbf{k}^*)] \leq 0$. This happens if the amplified spillover effect supersedes the adoption effect. If c is sufficiently high, the share of adoption of exclusive users is not too high and will decrease. In this case, if c increases, the gap between c and r widens, the amplified spillover effect grows and negatively impacts J_c. In this case, an increase in c hurts the platform. The proof is available in the appendix A.2.

Remember, as c rises, the investment in incentives goes up. As a consequence, the adoption of the basic plan goes down. This causes the subscription revenue to go up. On the other hand, the cost of providing incentives goes up as well. The rate of change of equilibrium profit is increasing in c if $J_{cc}[\Pi(\mathbf{k}^*)] > 0$. Note that the impact of an increase in c has two conflicting impacts on the probability of the adoption of the exclusive plan. Firstly, due to the increase in subscription fees, the investment in incentives goes up, which improves the level of incentives provided to the potential users. On the other hand, the increase in c reduces the adoption of the exclusive plan, but the crowding out slows down compared to the rise in utility which increases the adoption. The impact of the rate of change of c on the amplified spillover effect is proportional to the aggregate influence of the network. Conversely, if the effect of the rate of change of c on the amplified spillover effect supersedes the effect on the adoption of the exclusive plan, then $J_{cc}[\Pi(\mathbf{k}^*)] \leq 0$.

Net marginal effect of subscription fees and ad-returns per user. Note, r is monotonically increasing in platform profit in proportion to the potential share of basic users. However, the effect of c remains non-monotonic. Given market is always covered, it is natural to examine the relative magnitudes of the above two effects, which also explains how users, amidst rising c or r, gravitate towards E or B, and how the underlying model parameters and network connections determine the strength of such gravitation. From the previous two subsections,

$$J_c[\Pi(\mathbf{k}^*)] - J_r[\Pi(\mathbf{k}^*)] \; = \; 2\mathbb{P}(\mathbf{k}^*) \; - \; n \; - \; e(c-r) \cdot \mathcal{I}_{agg}$$

Clearly, the net effect is determined by the parameters b, γ, c, r, and $\mathcal{I}_{agg}$. In addition, e, which determines the variance of the shock a user faces on plan B, has a non-deterministic effect. If e is sufficiently small, which implies extreme shocks, the net marginal effect is primarily driven by the first term, or the above effect reduces to

$$J_c[\Pi(\mathbf{k}^*)] - J_r[\Pi(\mathbf{k}^*)] \to \frac{(1-\gamma)}{(1+\gamma)} \cdot \Big(\mathcal{I}_{agg} - n\Big)$$

The above implies that for any discount factor γ, if users on plan B experience shocks with relatively high variance, the net marginal effect of c over r on platform profit rises monotonically with the aggregate influence of users.

Corollary: *For any discount factor γ, if users on plan B experience shocks with relatively high variance, then users are more likely to subscribe to E. Their propensity to do so monotonically rises with the presence of more influential users in their network.*

In other words, when say, more number of influential users are in the network, users are strongly drawn towards plan E to avoid any such shocks by paying a subscription fee. As aggregate influence increases, users pay the price to avoid shocks.

5.3 Interaction between ad-returns and subscription fees

In this subsection, we examine the interaction of both parameters on the equilibrium profit. Suppose assumptions 1 and 2 hold and $u(k_i)$ is twice differentiable.

Proposition 4. *The impact of any change in the subscription fee on the rate of change of equilibrium profit with respect to change in ad revenue is symmetric in cross-effects. There exists a critical level of subscription fee $\tilde{c}$ such that*

(a) *for any $c > \tilde{c}$, the rate of change of equilibrium profit with respect to changes in ad returns increases with an increase in subscription fee.*

(b) *for any $c \leq \tilde{c}$, the rate of change of equilibrium profit with respect to changes in ad returns* *decreases with an increase in subscription fee.*

Proposition $4a$ entails that if the subscription fee is beyond the critical threshold $\tilde{c}$, then any increase in c amplifies the impact of a change in r on equilibrium profit. The exclusive plan appears less attractive due to the higher fee. We know, an increase in r increases the adoption of the basic plan. Now, if c is sufficiently high and increases, then an increase in c amplifies the adoption of the basic plan which increases the profit of the platform, but this effect is moderated by the spillover effect. The relative strength of these two effects determines the effect of c on J_r. The spillover effect is salient where users feel that they could not be rewarded enough on plan E. Here, higher incentives are not strong enough to override the cost of a higher c. The rate of adoption of the basic plan – as well as profits – due to a rise in basic users is amplified by the rise in sufficiently high subscription fees. If the subscription fee is beyond the threshold $\tilde{c}$, an increase in c reinforces some users to stay with the basic plan which furthers the adoption of B, i.e. $J_{rc} > 0$.

Part (b) of Proposition 4 demonstrates that if the subscription fee is below the critical threshold $\tilde{c}$, then any increase in c dampens the impact of a change in r on equilibrium profit. If the subscription fee is below $\tilde{c}$, an increase in c reduces the adoption of the basic plan. This dampens J_r. This implies, although an increase in r increases the adoption of B, if c is reasonably small, any increase in c any induce higher investment in incentives and adoption of E, which dampens the adoption of B. Hence, if the subscription fee is below the threshold $\tilde{c}$, an increase in c reinforces some users to stay with the exclusive plan, which dampens the adoption of B, i.e. $J_{rc} \leq 0$.

5.4 Preference bias

We now examine the impact of preference bias of the users on the equilibrium profit of the platform.

Proposition 5. *Let* $\mathbf{k}^*$ *be the optimal level of incentives that maximises the expected equilibrium profit of the platform. The following holds:*

(a) *Suppose all users are uniformly biased towards the basic plan such that* $b_i = b > 0$ *for all users, then any increase in preferential bias disadvantages the platform.*

(b) *Suppose some users are uniformly biased towards the basic plan and the remaining users are uniformly equally biased towards the exclusive plan such that* $b_i = \{-b, b\}$ *and* $b > 0$*, then any increase in the preferential bias disadvantages the platform if and only if the basic users are relatively* *more influential than the exclusive users.*

In Part (a) of Proposition 5, we consider the case where all users are uniformly biased towards the basic plan, i.e. $b_i = b$ for all i. An increase in b strictly decreases the equilibrium platform profits. In other words, $\frac{\partial \Pi(\mathbf{k}^*)}{\partial b} = -e(c-r) \cdot \mathcal{I}_{agg} < 0$. This implies that a well-connected influential network amplifies the negative impact of the increased preferential bias and cascades through influential users, further reducing the adoption of the exclusive. Since the per-user subscription revenue from the exclusive plan always exceeds the per-user ad-revenue return from basic, i.e. $c > r$, an increase in preferential bias decreases $\Pi(\mathbf{k}^*)$. The platform loses subscription revenue and gains lower return ad-revenue from users who switched to the basic plan.

The negative effect is worsened if e is higher, which implies that the variance of the shock on plan B is moderate. Here, higher aggregate influence will further magnify the network spillover effect. Any increase in uniform bias towards B lowers the probability of users to adopt E. Note that the characterisation of k^* is independent of b and increases with individual influence, e and $(c-r)$. In this case, influence, $(c-r)$, and e are double-edged swords. If b increases, users are more likely to join plan B, which provides the platform with r per user, yet the platform invests in incentives in proportion to their influence due to uncertainty. Furthermore, as the gap between c and r increases, it further hurts the platform.

Part (b) of the proposition highlights the impact of an increase in preferential bias on the platform on the equilibrium profit and the share of adoption of the exclusive plan. Here, n_B are uniformly biased towards the basic plan, and the remaining $n_E = (n - n_B)$ users have equal and opposite bias towards the exclusive plan. In other words, let $Q \subseteq N$ be the set of users that have a fixed uniform preferential bias towards the basic plan, i.e. $b_i = b$ for all $i \in Q$, and the remaining $N \setminus \{Q\}$ users have an equal opposite bias towards the exclusive plan i.e. $b_{i'} = -b$ for all $i' \in N \setminus \{Q\}$.[22] We show the importance of the relative influence of the users in determining whether the increasing magnitude of preferential bias disadvantages the platform. An increase in b decreases the expected probability of users adopting the exclusive plan in proportion to the differences in the influence of the users in each plan. In other words, $\frac{\partial \mathbb{P}(\mathbf{k}^*)}{\partial b} = -e \cdot (\mathcal{I}_B - \mathcal{I}_E) < 0$ if $\mathcal{I}_B > \mathcal{I}_E$ where $\mathcal{I}_B$ is the aggregate influence of the basic plan users and $\mathcal{I}_E$ is the aggregate influence of the users of

[22]Note, equally opposed bias towards the exclusive plan is synonymous to those users being biased away from the basic plan.

the exclusive plan. This implies that the platform loses subscription revenue and gains low return ad-revenue from users who switched to the basic plan if the basic users are relatively more influential. Moreover, any increase in the density e always increases the uncertainty which further amplifies the network spillover effect regardless of the sign.

On the other hand, if the aggregate influence of the exclusive users exceeds that of the basic users, i.e. $\mathcal{I}_E > \mathcal{I}_B$, then the presence of bias towards the basic plan or any increase in the magnitude of such bias is beneficial for the platform.

Furthermore, the platform can overcome numerical disadvantages in the exclusive plan under certain conditions and benefit from the increase in the magnitude of the bias. Consider the case where $\mathbb{G} = [\mathbf{0}]_{n\times n}$ and the platform has relatively more basic users than exclusive users. In the absence of any network, if $n_B > n_E$, then any increase in the magnitude of the bias disadvantages the platform, i.e. $\frac{\partial \Pi(\mathbf{k}^*)}{\partial b} = -e(c-r)\cdot(n_B - n_E) < 0$. Now suppose $n_B > n_E$, but $\mathcal{I}_B \leq \mathcal{I}_E$. This happens when the users in the exclusive plan are very well-connected compared to the basic users. In this case, an increase in bias amplifies the adoption of the exclusive plan and boosts profit.

5.5 Network strength

Consider two networks $\mathbb{G} = [g_{ij}]$ and $\mathbb{G}^+ = [g^+_{ij}]$. We define $\mathbb{G}^+$ is *stronger* than network $\mathbb{G}$ if $g^+_{ij} \geq g_{ij}$, with at least one strict inequality. In other words, we say that one network is stronger than another if at least one user becomes directly influenced by another to whom they were not initially influenced by. Take any incomplete network $\mathbb{G}$ where user i is not directly susceptible to k. $\mathbb{G}^+$ is a stronger directed network where user i is directly susceptible to m in $\mathbb{G}^+$ but not in $\mathbb{G}$, i.e. $\mathbb{G}^+ = \mathbb{G} + \{\vec{im}\}$. In $\mathbb{G}$, the link g_{im} is zero. Whereas, $g_{im} = 1$ in the network $\mathbb{G}^+$.

Lemma 1. *An increase in the strength of the network (weakly) increases the influence of all users. The increase in the influence of any user j increases with the following:*
(a) *the aggregate prior influence of agent i in $\mathbb{G}$,*
(b) *the influence of agent i on m in $\mathbb{G}$,*
(c) *the influence of agent j on m in $\mathbb{G}$.*[23]

Lemma 1 highlights the effect of an increase in the strength of the network govern the

[23]The proof of the Lemma is due to Das Chaudhury et al. (2023).

influence and susceptibility of users. In network $\mathbb{G}^+$, user i becomes directly susceptible to user m. We show that the change in the influence of any user j is

$$\Delta I_j \equiv I_j^+ - I_j = (\frac{\beta \cdot I_i}{1 - \beta \cdot x_{im}}) \cdot x_{jm}$$

The proof is available in the appendix A.5. We know, $\beta > 0$, and $x_{ij} \geq 0$ for all i. In the appendix, we show $(1 - \beta \cdot x_{im})$ is strictly positive. Any increase in the strength of the network alters the relative influence of users, but never decreases the absolute influence of any user.

The stronger network $\mathbb{G}^+$ benefits the platform if the equilibrium profit generated under $\mathbb{G}^+$ weakly exceeds the equilibrium profit under $\mathbb{G}$. The optimal vectors of incentives for $\mathbb{G}$ and $\mathbb{G}^+$ are $\mathbf{k}^* = (k_1^*, \cdots, k_n^*)$ and $\mathbf{k}^{+*} = (k_1^{+*}, \cdots, k_n^{+*})$ respectively. Any increase in the strength of the network benefits the platform if $\Pi(\mathbf{k}^{+*}|\mathbb{G}^+) \geq \Pi(\mathbf{k}^*|\mathbb{G})$. For $\mathbb{G}$ and $\mathbb{G}^+$, the following must hold in equilibrium

$$\frac{k_i^*}{u'(k_i^*)} = (\frac{e(c-r)}{\alpha}) \cdot I_i \quad \text{and} \quad \frac{k_i^{+*}}{u'(k_i^{+*})} = (\frac{e(c-r)}{\alpha}) \cdot I_i^+$$
$$\implies \frac{k_i^{+*}}{u'(k_i^{+*})} \geq \frac{k_i^*}{u'(k_i^*)} \qquad [\because I_i^+ \geq I_i]$$

The above expression holds for user utility functions with diminishing marginal utility. By monotonicity, if $k_i^{+*} \geq k_i^*$, then $u(k_i^{+*}) \geq u(k_i^*)$ which implies $u'(k_i^{+*}) \leq u'(k_i^*)$. The optimal profit of the platform under $\mathbb{G}$ and $\mathbb{G}^+$ is given by

$$\Pi(\mathbf{k}^{+*}|\mathbb{G}^+) = rn + (c-r) \cdot \mathbb{P}(\mathbf{k}^{+*}) - \frac{\alpha}{2} \cdot \mathbb{C}(\mathbf{k}^{+*})$$
$$\Pi(\mathbf{k}^*|\mathbb{G}) = rn + (c-r) \cdot \mathbb{P}(\mathbf{k}^*) - \frac{\alpha}{2} \cdot \mathbb{C}(\mathbf{k}^*)$$

where $\mathbb{P}(\mathbf{k}^{+*})$ and $\mathbb{P}(\mathbf{k}^*)$ are the optimal expected share of exclusive users. Additionally, $\mathbb{C}(\mathbf{k}^{+*}) = \sum_{j=1}^{n} (k_j^{+*})^2$ and $\mathbb{C}(\mathbf{k}^*) = \sum_{j=1}^{n} (k_j^*)^2$ are the optimal investment on incentives. Suppose assumptions 1 and 2 hold. Fix the tuple (α, r, c, e, b) and use Lemma 1. The following proposition demonstrates the impact of a stronger network on the equilibrium profit of the platform.

Proposition 6. *A stronger network benefits the platform if the gain from an increase in the expected share of exclusive users exceeds increased investment in incentives. There exists a critical level of subscription fee $\widehat{c}$ such that*

(a) *for any $c > \widehat{c}$, the stronger network benefits the platform.*

(b) *for any $c \leq \widehat{c}$, stronger network disadvantages the platform.*

In this proposition, we highlight the conditions under which a stronger network benefits the platform. A stronger network benefits the platform if $\Delta\Pi^* \geq 0$ where $\Delta\Pi^* = \Pi(\mathbf{k}^{+*}) - \Pi(\mathbf{k}^*)$. A stronger network increases the influence of all users (Lemma 1). In simple terms, this happens when the gains to the platform from users' adoption of the exclusive plan exceed the additional cost of investment in incentives. If the subscription fee is beyond a critical threshold $\widehat{c}$, any increase in the strength of the network benefits the platform by increasing its equilibrium profit. If the subscription fee is beyond $\widehat{c}$, the subscription fees can amplify the revenue gain from increased exclusive adoption, but it also increases the cost of incentives. The increase in influence amplifies both these effects.

6 Alternative application

In this section, we provide some alternative applications of our analysis. Here, we allow for the possibility of competition between two platforms, which allows us to understand the magnitude of the difference in personalised incentives. Any representative user i can subscribe to one exclusive plan based on their relative expected utility from the plan.[24] Moreover, competition helps us understand whether personalised incentives improve or decline in the presence of competition.

Consider two competing platforms - (a) Dominant (P_D), and (b) Emerging (P_M). Each platform P_y has one basic (B_y) and one exclusive plan (E_y) where $y = \{D, M\}$. User i always prefers exclusive over basic for any platform, i.e. $\mathbb{E}U_i(E_y) > \mathbb{E}U_i(B_{y'})$ for all $y, y' \in \{D, M\}$.[25] This ensures that a user chooses exactly one exclusive plan subscription $E_y \in \{E_D, E_M\}$ from the two platforms.[26]

The two competing platforms are such that they vary across their brand value, and one is relatively more dominant than the other. For instance, one platform is a subsidiary of an established big-tech player.[27] The users choose between subscribing to the exclusive

[24] We rule out multihoming. We are interested in the micro-foundations of the choice of platforms based on the network spillover and other associated factors.

[25] This implies that a user prefers the exclusive of any platform over the basic of another platform, i.e. $\mathbb{E}U_i(E_y) > \mathbb{E}U_i(B_{y'})$ for all $y, y' \in \{D, M\}$.

[26] Note that in this application, a user always subscribes for only one exclusive plan. Both exclusive plans offer similar content, so it is only reasonable to select only one. Under such circumstances, which platform will the user choose? Remember that the user enjoys the basic content of alternative platforms for free.

[27] One can incorporate the notion of dominance by assuming that the network-spillover effect of the exclusive plan in the emerging platform is a discounted version of the exclusive plan of the dominant platform.

plan of either P_D or P_M based on the value of the ex-ante expected utility. For simplicity, we suppress the impact of choosing the basic plan of P_y on choosing the exclusive plan P_y where $y \in \{D, M\}$. The expected utility of any user from the dominant platform's exclusive plan is deterministic. The advertisement shock ϵ_i can be represented as positive or negative news about the emerging platform, which may alter the user's preferences.[28]

The overall utility of user i from choosing platform P_y is given by

$$\mathbb{E}U_i(P_y|\epsilon_i) \;=\; \begin{cases} u(k_{Di}) \;+\; \delta \cdot \sum\limits_{j \in N} g_{ij} \cdot p_j \;-\; c_D & \text{if } \; P_y = P_D \\ u(k_{Mi}) \;+\; \delta \cdot \sum\limits_{j \in N} g_{ij} \cdot (1 - p_j) \;-\; c_M \;+\; b_i \;+\; \epsilon_i & \text{if } \; P_y = P_M \end{cases}$$

where $p_j(\mathbf{k_D}, \mathbf{k_M})$ is the probability of a user choosing the exclusive plan provided by the dominant platform. Alternatively, $p_j(\mathbf{k_D}, \mathbf{k_M})$ can be interpreted as the probability of a user choosing the basic plan provided by the emerging platform. Furthermore, $(1 - p_j(\mathbf{k_D}, \mathbf{k_M})$ is the probability of a user choosing the basic plan provided by the dominant platform. Additionally, $(1 - p_j(\mathbf{k_D}, \mathbf{k_M}))$ can be interpreted as the probability that user j chooses the exclusive plan of the emerging platform.

For simplicity, $\gamma = 1$. Additionally, the users have a predisposed bias $b_i \in [-b, b]$ towards or away from the dominant platform. Conditional on the private preference shock, a user chooses H_D if

$$\mathbb{E}U_i(P_D|\epsilon_i) \;\geq\; \mathbb{E}U_i(P_M|\epsilon_i)$$

$$\epsilon_i \;\leq\; u(k_{Di}) - u(k_{Mi}) \;-\; (c_D - c_M) \;-\; b_i \;+\; 2\delta \cdot [\sum_{j \in N} g_{ij} \cdot p_j - \frac{1}{2} \sum_{j \in N} g_{ij}]$$

Subscription fees. The following proposition demonstrates the impact of subscription fees on the investment of incentives of the competing platforms.

Proposition 7. *If the subscription fee of the dominant platform P_D exceeds that of the emerging platform P_M, P_D's equilibrium investment on incentives always exceeds that of P_M.*

In this proposition, we show that if the dominant platform P_D charges a relatively higher subscription fee, then their investment per user on providing incentives for the exclusive plan is also higher. Relatively higher subscription fee generates more revenue for the exclusive plan which consequently leads to higher investment in providing incentives. The

[28]Here, the information about the network $\mathbb{G}$ is the same for both platforms.

first order condition entails

$$\frac{k_{Di}^*}{u'(k_{Di}^*)} = \left(\frac{e(c_D - r)}{\alpha}\right) \cdot I_i \quad \text{and} \quad \frac{k_{Mi}^*}{u'(k_{Mi}^*)} = \left(\frac{e(c_M - r)}{\alpha}\right) \cdot I_i$$

Formally, if $c_D \geq c_M$, then $k_{Di}^* \geq k_{Mi}^*$. This implies $\mathbf{k_D^*} \geq \mathbf{k_M^*}$. Interestingly, the difference in investment on incentives is completely governed by the gap between r and c_D or c_M. If the subscription fee for the exclusive plan is equal in both the dominant and the emerging platform, i.e. $c_D = c_M = c$, then the equilibrium investment on investments is symmetric.

Consider the scenario where the subscription fee from the dominant platform exceeds that from the emerging platform, i.e. $c_D \geq c_M$. If the ad-revenue per user across the two platforms is not identical where $r_D \geq r_M$ such that $(c_D - r_D) \leq (c_M - r_M)$, then in equilibrium the emerging platform invests relatively more on incentives. Suppose the difference in subscription fee and ad-revenue per-user revenue from the emerging platform exceeds that of the dominant platform. In that case, the marginal revenue per user is larger for the emerging platform is larger which implies relatively more investment in incentives.

Cost of Investments. Consider both platforms with identical subscription fees. Here, the profits of platforms are distinguishable based on the per-unit cost of the incentives. The emerging platform has a higher per-unit cost of premium content than the dominant platform, i.e. $\alpha_M \geq \alpha_D$. In other words, if the per unit cost of providing incentives of the emerging platform P_M exceeds that of the dominant platform P_D, then in equilibrium, P_D always provides higher incentives than P_M. Fix the tuple $\Omega = (\alpha_M, \alpha_D, \delta, c, r)$ such that $c_M = c_D = c > r$ and $\alpha_M \geq \alpha_D$. Formally, the equilibrium investment on incentives by the two platforms

$$\frac{k_{Di}^*}{u'(k_{Di}^*)} = \left(\frac{e(c - r)}{\alpha_D}\right) \cdot I_i \quad \text{and} \quad \frac{k_{Mi}^*}{u'(k_{Mi}^*)} = \left(\frac{e(c - r)}{\alpha_M}\right) \cdot I_i$$

where $\alpha_M \geq \alpha_D$. This implies $k_{Di}^* \geq k_{Mi}^*$. This provides an explanation for why some dominant platforms sometimes provide higher incentives to influential users.

Ad-returns. Now, consider both platforms with identical subscription fees. Here, the profits of platforms are distinguishable based on the per-user revenue from ad-returns. The emerging platform has higher per-user ad revenue than the dominant platform. In other words, if the per-user revenue from the emerging platform P_M exceeds that of the dominant platform P_D, then in equilibrium, P_D always provides higher incentives than P_M. Fix the tuple $\Omega = (\alpha, r_D, r_M, \delta, c)$ such that $r_M \geq r_D$. Mathematically, the equilibrium investment on incentives by the two platforms

$$\frac{k_{Di}^*}{u'(k_{Di}^*)} = \left(\frac{e(c - r_D)}{\alpha}\right) \cdot I_i \quad \text{and} \quad \frac{k_{Mi}^*}{u'(k_{Mi}^*)} = \left(\frac{e(c - r_M)}{\alpha}\right) \cdot I_i$$

where $r_M \geq r_D$. This implies $k_{Di}^* \geq k_{Mi}^*$. This happens because increased revenue from the basic plan reduces the impetus to invest in incentives.

Preference Bias. Two competing platforms with equal subscription fees can lead to unequal profit for the platforms, which is due to the preferential bias b_i, for any i. In equilibrium, the premium contents provided by the platform are equal. If both platforms have identical subscription fees, the optimal content is identical. Fix the tuple $\Omega = (\alpha, \delta, c_D, c_E, r)$ such that $c_D = c_E = c > r$.

Proposition 8. *If the two platforms have identical subscription fee, the following must hold:*
(a) *If all users are uniformly biased towards the emerging platform, i.e. $b_i = b$ where $b > 0$, then the profit of the emerging platform always exceeds the profit of the dominant platform.*
(b) *If some users are uniformly biased towards the dominant platform and the remaining users have symmetrically opposite bias towards the emerging platform such that $b_i = \{-b, b\}$ and $b > 0$, then profit of the emerging platform always exceeds the profit of the dominant platform if the exclusive users in the emerging platform are relatively more influential $\mathcal{I}_M \geq \mathcal{I}_D$.*

If both platforms have identical subscription fees, the competition between the two platforms implies equal investment in incentives. In this scenario, the profit of the emerging platform exceeds the profit of the dominant platform if the expected share of the exclusive users in the dominant platform does not exceed $\frac{n}{2}$. Part (a) of the proposition implies that if all users are uniformly biased towards the emerging platform, then the expected share of the exclusive users in the dominant platform does not exceed $\frac{n}{2}$. Hence, the dominant platform earns subscription revenue from (weak) minority of users and ad-revenue from the remaining users. The positive preference bias towards the emerging platform and the network spillover effect disadvantage the dominant platform.

In Part (b) of the proposition, we consider some users who are uniformly biased towards the dominant platform, and the remaining users are uniformly equally biased towards the emerging platform. If the users in the emerging platform are relatively more influential than the users in the dominant platform, i.e. $\mathcal{I}_M \geq \mathcal{I}_D$, then the network spillover effect of the exclusive users in the emerging platform advantages them over the dominant platform. For the formal proof, see the Appendix.

7 Conclusion

In this paper, we examine how an online platform leverages social networks to maximise profits amidst market coverage between two plans — a free, ad-interrupted plan B and a paid ad-free plan E. Even though plan E generates greater network-spillover to users and greater per-user revenue to the platform, than plan B, our analysis shows when profit is primarily driven through revenues from B rather than E, and vice versa.

The binary choice framework highlights a non-trivial relationship between ad-revenues and subscription fees, which can act as either complements or substitutes depending on the model primitives. The underlying mechanism determining the primary channel of profit relies not only on the absolute value of *influence* and *susceptibility* experienced by users, but also their distribution across users in their connections network. This further dictates whether the platform should incur investment cost to provide personalized incentives to attract more influential users, or eschew such costs to derive profit via ad-revenues.

Even though the baseline model is a monopoly, we provide additional insights by introducing competing platforms. One can extend our model by incorporating endogenous pricing and competition between multiple platforms. Furthermore, empirical validation using data from digital platforms can offer insights into the extent to which network-driven incentives influence user subscription behaviour. Additionally, one can incorporate dynamic network formation to explore how platforms manipulate user connections to optimise segregation.

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A Appendix

A.1 Proof of proposition 2

Proof. In equilibrium,

$$\begin{aligned} \alpha k_i^* &= e(c-r)\cdot u'(k_i^*)\cdot I_i \\ \implies \alpha \mathbf{k}^* &= e(c-r)\cdot (\mathbf{u}'(\mathbf{k}^*)\cdot \boldsymbol{\mathcal{I}}) \end{aligned}$$

where $\mathbf{u}'(\mathbf{k}^*)\cdot\boldsymbol{\mathcal{I}} = (u'(k_1^*)\cdot I_1, \cdots, u'(k_n^*)\cdot I_n)$ is a $n\times 1$ vector. The impact of a change in r on the equilibrium level of incentives is given by

$$\frac{\partial k_i^*}{\partial r} \;=\; -\frac{eu'(k_i^*)\cdot I_i}{\alpha - e(c-r)u''(k_i^*)I_i} \;<\; 0 \tag{A.1}$$

The aggregate equilibrium probability of adopting the exclusive plan is

$$\mathbb{P}(\mathbf{k}^*) \;=\; \mathbf{p}^*(\mathbf{k}^*)^\top\cdot\mathbf{1} \;=\; \Big(\frac{1-\gamma}{2(1+\gamma)}\Big)\cdot\mathcal{I}_{agg} \;+\; (\frac{n\gamma}{1+\gamma}) \;+\; e\cdot[\mathbf{u}(\mathbf{k}^*)^\top \;-\; \mathbf{b}^\top \;-\; c\cdot\mathbf{1}^\top]\cdot\boldsymbol{\mathcal{I}} \tag{A.2}$$

Hence, the impact of a change in r on the aggregate equilibrium share of adopting the exclusive plan is

$$\frac{\partial\mathbb{P}(\mathbf{k}^*)}{\partial r} \;\equiv\; J_r[\mathbb{P}(\mathbf{k}^*)] \;=\; e\cdot\sum_{i=1}^{n} u'(k_i^*)\cdot I_i\cdot\frac{\partial k_i^*}{\partial r} \;<\; 0 \tag{A.3}$$

In equilibrium, the expected profit of the platform is

$$\Pi(\mathbf{k}^*) \;=\; c\cdot\mathbb{P}(\mathbf{k}^*) \;+\; r\cdot(n-\mathbb{P}(\mathbf{k}^*)) \;-\; \frac{\alpha}{2}\cdot\sum_{j=1}^{n} k_j^{*\,2}$$

The impact of an increase in r on the equilibrium profit of the platform is given by

$$\begin{aligned}
J_r[\Pi(\mathbf{k}^*)] \; &= \; (n - \mathbb{P}(\mathbf{k}^*)) \; + \; (c-r)\cdot J_r[\mathbb{P}(\mathbf{k}^*)] \; - \; \frac{\alpha}{2}\cdot (J_r[\mathbf{k}^{*\top}\cdot \mathbf{k}^*]) \\
&= \; (n - \mathbb{P}(\mathbf{k}^*)) \; + \; e(c-r)\cdot J_r[\mathbf{u}(\mathbf{k}^*)]\cdot \boldsymbol{\mathcal{I}} \; - \; \frac{\alpha}{2}\cdot (J_r[\mathbf{k}^{*\top}\cdot \mathbf{k}^*]) \\
&= \; (n - \mathbb{P}(\mathbf{k}^*)) \; + \; \sum_i \Big[e(c-r)\cdot u'(k_i^*)\cdot I_i \; - \; \alpha\cdot k_i^* \Big]\cdot \frac{\partial k_i^*}{\partial r} \\
&= \; (n - \mathbb{P}(\mathbf{k}^*)) \; \geq \; 0 \\
&= \; (\frac{n}{1+\gamma}) \; - \; \frac{(1-\gamma)}{2(1+\gamma)}\cdot \mathcal{I}_{agg} \; + \; ec\cdot \mathcal{I}_{agg} \; - \; e\cdot [\mathbf{u}(\mathbf{k}^*)^\top \; - \; \mathbf{b}^\top]\cdot \boldsymbol{\mathcal{I}} \; \geq \; 0
\end{aligned} \tag{A.4}$$

The final expression in the previous equation is in terms of the primitives of the model. The impact of the increase in r on the rate of change of equilibrium profit is given by

$$\begin{aligned}
J_{rr}[\Pi(\mathbf{k}^*)] \; &= \; -J_r[\mathbb{P}(\mathbf{k}^*)] \\
&= \; \sum_{i=1}^{n} \frac{e^2 u'(k_i^*)^2 I_i}{(\alpha - e(c-r)u''(k_i^*)I_i)} \; > \; 0
\end{aligned} \tag{A.5}$$

A.2 Proof of proposition 3

Proof. The impact of a change in c on the equilibrium level of incentives is given by

$$\frac{\partial k_i^*}{\partial c} \; = \; \frac{eu'(k_i^*)\cdot I_i}{\alpha - e(c-r)u''(k_i^*)I_i} \; > \; 0 \tag{A.6}$$

Hence, the impact of a change in c on the aggregate equilibrium share of adopting the exclusive plan is

$$\frac{\partial \mathbb{P}(\mathbf{k}^*)}{\partial c} \; \equiv \; J_c[\mathbb{P}(\mathbf{k}^*)] \; = \; e\cdot \sum_{i=1}^{n} u'(k_i^*)\cdot I_i\cdot \frac{\partial k_i^*}{\partial c} - e\cdot \mathcal{I}_{agg} \tag{A.7}$$

The impact of an increase in c on the equilibrium profit of the platform is given by

$$
\begin{aligned}
J_c[\Pi(\mathbf{k}^*)] &= \mathbb{P}(\mathbf{k}^*) + (c-r)\cdot J_c[\mathbb{P}(\mathbf{k}^*)] - \frac{\alpha}{2}\cdot(J_c[\mathbf{k}^{*\top}\cdot\mathbf{k}^*]) \\
&= \mathbb{P}(\mathbf{k}^*) + \sum_i \Big[e(c-r)\cdot u'(k_i^*)\cdot I_i - \alpha\cdot k_i^*\Big]\cdot\frac{\partial k_i^*}{\partial c} - e(c-r)\cdot\mathcal{I}_{agg} \\
&= \mathbb{P}(\mathbf{k}^*) - e(c-r)\cdot\mathcal{I}_{agg} \\
&= (\frac{n\gamma}{1+\gamma}) + \frac{(1-\gamma)}{2(1+\gamma)}\cdot\mathcal{I}_{agg} - ec\cdot\mathcal{I}_{agg} + e\cdot[\mathbf{u}(\mathbf{k}^*)^\top - \mathbf{b}^\top]\cdot\boldsymbol{\mathcal{I}} - e(c-r)\cdot\mathcal{I}_{agg} \\
&= (\frac{n\gamma}{1+\gamma}) + \frac{(1-\gamma)}{2(1+\gamma)}\cdot\mathcal{I}_{agg} - 2ec\cdot\mathcal{I}_{agg} + er\cdot\mathcal{I}_{agg} + e[\mathbf{u}(\mathbf{k}^*)^\top - \mathbf{b}^\top]\cdot\boldsymbol{\mathcal{I}} \qquad \text{(A.8)}
\end{aligned}
$$

The final expression in the previous equation is in terms of the primitives of the model, and the sign will depend on the relative sizes. Using Assumption 1, we know that $\mathbb{P}(\mathbf{k}^*) \geq 0$.

(a) Hence, $J_c[\mathbb{P}(\mathbf{k}^*)] > 0$, if $\mathbb{P}(\mathbf{k}^*) > e(c-r)\mathcal{I}_{agg}$. Note that, k^* implicitly depends on the value of c. Hence, there exists a critical $\bar{c}$ such that if $c < \bar{c}$, then $\mathbb{P}(\mathbf{k}^*) > e(c-r)\mathcal{I}_{agg}$.

(b) On the other hand, $J_c[\mathbb{P}(\mathbf{k}^*)] \leq 0$, if $\mathbb{P}(\mathbf{k}^*) \leq e(c-r)\mathcal{I}_{agg}$. Note that, k^* implicitly depends on the value of c. Hence, there exists a critical $\bar{c}$ such that if $c \geq \bar{c}$, then $\mathbb{P}(\mathbf{k}^*) \leq e(c-r)\mathcal{I}_{agg}$.

The impact of the increase in c on the rate of change of equilibrium profit is given by

$$
\begin{aligned}
J_{cc}[\Pi(\mathbf{k}^*)] &= J_c[\mathbb{P}(\mathbf{k}^*) - e(c-r)\cdot\mathcal{I}_{agg}] \\
&= e\cdot\sum_{i=1}^{n} u'(k_i^*)\cdot I_i\cdot\frac{\partial k_i^*}{\partial c} - 2e\cdot\mathcal{I}_{agg} \qquad \text{(A.9)}
\end{aligned}
$$

A.3 Proof of proposition 4

Proof. The impact of a change in r on the rate of change of equilibrium profit with respect to change in c is given by

$$
\begin{aligned}
J_c[\Pi(\mathbf{k}^*)] &= \mathbb{P}(\mathbf{k}^*) - e(c-r)\cdot\mathcal{I}_{agg} \\
J_{cr}[\Pi(\mathbf{k}^*)] &= e\cdot\sum_{i=1}^{n} u'(k_i^*)\cdot I_i\cdot\frac{\partial k_i^*}{\partial r} + e\cdot\mathcal{I}_{agg} \qquad [\because \frac{\partial k_i^*}{\partial r} < 0] \qquad \text{(A.10)}
\end{aligned}
$$

Similarly, the impact of a change in c on the rate of change of equilibrium profit with respect to a change in r is given by

$$J_r[\Pi(\mathbf{k}^*)] \;=\; n - \mathbb{P}(\mathbf{k}^*)$$
$$J_{rc}[\Pi(\mathbf{k}^*)] \;=\; -e \cdot \sum_{i=1}^{n} u'(k_i^*) \cdot I_i \cdot \frac{\partial k_i^*}{\partial c} + e \cdot \mathcal{I}_{agg} \qquad [\because\; \frac{\partial k_i^*}{\partial c} \;>\; 0 \;\;\&\;\; \frac{\partial k_i^*}{\partial c} = -\frac{\partial k_i^*}{\partial r}] \quad \text{(A.11)}$$

This implies, $J_{cr}[\Pi(\mathbf{k}^*)] \;=\; J_{rc}[\Pi(\mathbf{k}^*)]$.

(a) Using Assumption 1, we know that $\mathbb{P}(\mathbf{k}^*) \;\geq\; 0$. Hence, if $J_{rc}[\Pi(\mathbf{k}^*)] \;>\; 0$, implies $\mathcal{I}_{agg} > \sum_{i=1}^{n} u'(k_i^*) \cdot I_i \cdot \frac{\partial k_i^*}{\partial c}$. There exists a critical $\tilde{c}$ such that if $c > \tilde{c}$, then $J_{rc}[\Pi(\mathbf{k}^*)] > 0$.

(b) On the other hand, if $J_{rc}[\Pi(\mathbf{k}^*)] \leq 0$, implies $\mathcal{I}_{agg} \leq \sum_{i=1}^{n} u'(k_i^*) \cdot I_i \cdot \frac{\partial k_i^*}{\partial c}$. There exists a critical $\tilde{c}$ such that if $c \leq \tilde{c}$, then $J_{rc}[\Pi(\mathbf{k}^*)] \leq 0$.

A.4 Proof of proposition 5

Proof. For part (a) of proposition 5, we assume $b_i \;=\; b$ for all i. By equation A.2, the expected share of users adopting the exclusive plan in equilibrium is given by

$$\mathbb{P}(\mathbf{k}^*) \;=\; \Big(\frac{1-\gamma}{2(1+\gamma)}\Big) \cdot \mathcal{I}_{agg} \;+\; (\frac{n\gamma}{1+\gamma}) \;+\; e \cdot [\mathbf{u}(\mathbf{k}^*)^\top \;-\; c \cdot \mathbf{1}^\top] \cdot \boldsymbol{\mathcal{I}} \;-\; eb \cdot \mathcal{I}_{agg}.$$

An increase in b decreases the expected probability of users adopting the exclusive plan in proportion to the aggregate influence. Formally, $\frac{\partial \mathbb{P}(\mathbf{k}^*)}{\partial b} \;=\; -e \cdot \mathcal{I}_{agg} < 0$.

The impact of an increase in b on the optimal profit of the platform is $\frac{\partial \Pi(\mathbf{k}^*)}{\partial b} \;=\; -e(c - r) \cdot \mathcal{I}_{agg} < 0$.

For part (b) of Proposition 5, we assume $b_i \;=\; \{-b, b\}$ for all i. Here, n_B are uniformly biased towards the basic plan, and the remaining $n_E \;=\; (n \;-\; n_B)$ users have equal and opposite bias towards the exclusive plan. In other words, n_B basic users that have a fixed uniform preferential bias towards the basic plan, i.e. $b_i \;=\; b$, and the remaining n_E users have equal opposite bias towards the exclusive plan i.e. $b_{i'} \;=\; -b$. Using equation A.2, the expected share of exclusive users adopting the exclusive plan in equilibrium is given by

$$\begin{aligned}
\mathbb{P}(\mathbf{k}^*) \;&=\; \Big(\frac{1-\gamma}{2(1+\gamma)}\Big) \cdot \mathcal{I}_{agg} \;+\; (\frac{n\gamma}{1+\gamma}) \;+\; e \cdot [\mathbf{u}(\mathbf{k}^*)^\top \;-\; c \cdot \mathbf{1}^\top] \cdot \boldsymbol{\mathcal{I}} \;-\; e \cdot \mathbf{b}^\top \cdot \boldsymbol{\mathcal{I}} \\
&=\; \Big(\frac{1-\gamma}{2(1+\gamma)}\Big) \cdot \mathcal{I}_{agg} \;+\; (\frac{n\gamma}{1+\gamma}) \;+\; e \cdot [\mathbf{u}(\mathbf{k}^*)^\top \;-\; c \cdot \mathbf{1}^\top] \cdot \boldsymbol{\mathcal{I}} \;-\; eb \cdot (\mathcal{I}_B - \mathcal{I}_E)
\end{aligned}$$

where $\mathbf{b} = b \cdot (1,\ 1,\ 1, \cdots, -1, -1, -1)$. If $\mathcal{I}_B > \mathcal{I}_E$, an increase in b decreases the expected probability of users adopting the exclusive plan in proportion to the aggregate influence. Formally if $\mathcal{I}_B > \mathcal{I}_E$, then

$$\frac{\partial \mathbb{P}(\mathbf{k}^*)}{\partial b} = -e \cdot (\mathcal{I}_B - \mathcal{I}_E) < 0.$$

The impact of an increase in b on the optimal profit of the platform is

$$\frac{\partial \Pi(\mathbf{k}^*)}{\partial b} = -e(c-r) \cdot (\mathcal{I}_B - \mathcal{I}_E) < 0 \quad \text{if} \quad \mathcal{I}_B > \mathcal{I}_E.$$

A.5 Proof of Lemma 1

Proof. Consider a pair of networks $\mathbb{G}$ and $\mathbb{G}^+$ such that the network $\mathbb{G}$ is relatively stronger than network $\mathbb{G}^+$. The stronger network $\mathbb{G}^+ = \mathbb{G} + \{\vec{im}\}$. For simplicity, consider $g_{im} = \{0, 1\}$. Any perturbation in the adjacency matrix will be of magnitude 1. The entry in the i-th row and m-th column is 1 and zero elsewhere. Define $\Delta\mathbb{G} = (\mathbb{G}^+ - \mathbb{G})$ as the perturbation of the adjacency matrix where $\Delta\mathbb{G}$ is also an $n \times n$ matrix.

$$\begin{aligned}
(\Delta\mathbb{G})^\top = (\mathbb{G}^+ - \mathbb{G})^\top &= \begin{bmatrix} 0 & & \cdots & & 0 \\ \vdots & \cdots & h_{mi} & \cdots & \vdots \\ 0 & & \cdots & & 0 \end{bmatrix} \qquad [\ \because\ g^+_{mi} - g_{mi} = h_{mi}\ ] \\
&= h_{mi} \cdot \begin{bmatrix} 0 & & \cdots & & 0 \\ \vdots & \cdots & 1_{mi} & \cdots & \vdots \\ 0 & & \cdots & & 0 \end{bmatrix} = h_{mi} \cdot \mathbb{E} \qquad [\because\ h_{mi} = 1\ ] \\
&= h_{mi} \cdot \mathbf{v} \cdot \mathbf{w} \qquad [\ \because\ \ \mathbf{v} = (0 \cdots 1_m \cdots 0)^\top \ \text{and}\ \ \mathbf{w} = (0 \cdots 1_i \cdots 0)\ ]
\end{aligned}$$

where $\mathbb{E}$ is a matrix with 1 in m-th row and i-th column and zero elsewhere. The matrix $\mathbb{E}$ is represented as a product of two vectors $\mathbf{v}$ and $\mathbf{w}$ where $\mathbf{v}$ is $n \times 1$ and has the entry 1 in the m-th row and zero elsewhere, and the vector $\mathbf{w}$ is $1 \times n$ has the entry 1 in the i-th column and zero elsewhere. If $g_{im} = [0, 1]$, then i-th row and m-th column in network $\mathbb{G}$ and $\mathbb{G}^+$ are g_{im} and g^+_{im} respectively. In that case, h_{mi} may or may not be 1.

In network $\mathbb{G}^+$, the influence vector of users are given by

$$
\begin{aligned}
\boldsymbol{\mathcal{I}}^{+} &= (\mathbb{I} - \beta \cdot (\mathbb{G}^{+})^{\top})^{-1} \cdot \mathbf{1} \;=\; (\mathbb{I} - \beta \mathbb{G}^{\top} - \beta (\Delta \mathbb{G})^{\top})^{-1} \cdot \mathbf{1} \\
&= [\, (\mathbb{I} - \beta \mathbb{G}^{\top}) - \beta (\Delta \mathbb{G})^{\top} \,]^{-1} \cdot \mathbf{1} \;=\; [\, (\mathbb{I} - \beta \mathbb{G}^{\top}) - \beta h_{mi} \cdot \mathbf{v} \cdot \mathbf{w} \,]^{-1} \cdot \mathbf{1} \\
&= (\mathbb{I} - \beta \mathbb{G}^{\top})^{-1} \cdot \mathbf{1} \;+\; \frac{\beta \cdot h_{mi} \cdot I_i}{1 - \beta \cdot x_{im}} \cdot (x_{1m}, \;\cdots\;, x_{nm})^{\top} \qquad [\because \text{by Sherman-Morrison formula}\,] \\
&= \boldsymbol{\mathcal{I}} \;+\; (\frac{\beta \cdot h_{mi} \cdot I_i}{1 - \beta \cdot x_{im}}) \cdot (x_{1m}, \;\cdots\;, x_{nm})^{\top} \\
\Delta \boldsymbol{\mathcal{I}} &\equiv \boldsymbol{\mathcal{I}}^{+} \;-\; \boldsymbol{\mathcal{I}} \;=\; (\frac{\beta \cdot I_i}{1 - \beta \cdot x_{im}}) \cdot (x_{1m}, \;\cdots\;, x_{nm})^{\top}
\end{aligned}
$$

A.6 Proof of Proposition 6

A stronger network benefits the platform if the gain from an increase in expected share of exclusive users exceeds increased investment in incentives, i.e. $\Delta\Pi^* \geq 0$. Formally,

$$
\begin{aligned}
\Delta\Pi^* &\equiv \Pi(\mathbf{k}^{+*}) - \Pi(\mathbf{k}^{*}) \;=\; (c - r) \cdot \Delta\mathbb{P} \;-\; \frac{\alpha}{2} \cdot \Delta\mathbb{C} \\
&= (c - r) \cdot \Big[(\frac{1-\gamma}{1+\gamma}) \cdot \frac{\Delta \mathcal{I}_S}{2} \;+\; e \cdot \Big(\sum_j (u(k_j^{+*}) \cdot I_j^{+} - u(k_j^{*}) \cdot I_j) \;-\; \sum_j (b_j + c) \cdot \Delta I_j \Big) \Big] \\
&\quad - \; \frac{\alpha}{2} \cdot \sum_j ((k_j^{+*})^2 - (k_j^{*})^2)
\end{aligned}
$$

(a) Using Lemma 1, we know that $u(k_j^{+*}) \cdot I_j^{+} \geq u(k_j^{*}) \cdot I_j$. There exists a critical $\widehat{c}$ such that if $c \geq \widehat{c}$, then $(c - r) \cdot \Delta\mathbb{P} \;\geq\; \frac{\alpha}{2} \cdot \Delta\mathbb{C}$.

(b) We know that $u(k_j^{+*}) \cdot I_j^{+} \geq u(k_j^{*}) \cdot I_j$. A stronger network disadvantages the platform if $\Delta\Pi^* < 0$. There exists a critical $\widehat{c}$ such that if $c < \widehat{c}$, then $(c - r) \cdot \Delta\mathbb{P} \;<\; \frac{\alpha}{2} \cdot \Delta\mathbb{C}$.

A.7 Proof of Proposition 7

Note that $p_j(\mathbf{k_D}, \mathbf{k_M})$ is the probability of a user choosing the exclusive plan provided by the dominant platform. Alternatively, $p_j(\mathbf{k_D}, \mathbf{k_M})$ can be interpreted as the probability of a user choosing the basic plan provided by the emerging platform. Furthermore, $(1 - p_j(\mathbf{k_D}, \mathbf{k_M})$ is the probability of a user choosing the basic plan provided by the dominant platform. Additionally, $(1 - p_j(\mathbf{k_D}, \mathbf{k_M}))$ can be interpreted as the probability that user j chooses the exclusive plan of the emerging platform. The user chooses the exclusive plan of the dominant platform over the exclusive plan of the emerging platform iff

$$\mathbb{E}U_i(P_D|\epsilon_i) \;\geq\; \mathbb{E}U_i(P_M|\epsilon_i)$$
$$\epsilon_i \;\leq\; u(k_{Di}) - u(k_{Mi}) \;-\; (c_D - c_M) \;-\; b_i \;+\; 2\delta \cdot [\sum_{j\in N} g_{ij} \cdot p_j - \frac{1}{2}\sum_{j\in N} g_{ij}]$$

Hence, the ex-ante probability of a user choosing the exclusive plan provided by the dominant platform turns out to be

$$\mathbf{p} \;=\; \frac{1}{2}\cdot\mathbf{1} \;+\; e\cdot(\mathbf{u_D} - \mathbf{u_M}) \;-\; e\cdot\mathbf{b} \;-\; e(c_D - c_M)\cdot\mathbf{1} \;+\; \beta\cdot\mathbb{G}\cdot\mathbf{p} \;-\; e\delta\cdot\mathbb{G}\cdot\mathbf{1}$$
$$\mathbf{p} \;=\; \frac{1}{2}\cdot\mathbf{1} \;+\; e\cdot(\mathbb{I} - \beta\mathbb{G})^{-1}\cdot[(\mathbf{u_D} - \mathbf{u_M}) \;-\; \mathbf{b} \;-\; (c_D - c_M)\cdot\mathbf{1}] \tag{A.12}$$

where u_D and u_M are the vectors of incentives derived by exclusive users in platform D and M respectively. The probability of a user choosing the exclusive plan provided by the dominant platform with respect to the investment in incentives on their respective platforms is given by

$$\sum_j \frac{\partial p_j}{\partial k_{Di}} \;=\; J_{Di}[\mathbf{p}^\top]\cdot\mathbf{1} \;=\; \quad e\cdot J_{Di}[\mathbf{u_D}^\top]\cdot(\mathbb{I} - \beta\mathbb{G}^\top)^{-1}\cdot\mathbf{1} \;=\; e\cdot u'_{Di}I_i$$
$$\sum_j \frac{\partial p_j}{\partial k_{Mi}} \;=\; J_{Mi}[\mathbf{p}^\top]\cdot\mathbf{1} \;=\; -e\cdot J_{Mi}[\mathbf{u_M}^\top]\cdot(\mathbb{I} - \beta\mathbb{G}^\top)^{-1}\cdot\mathbf{1} \;=\; -e\cdot u'_{Mi}I_i$$

where $J_{Di} = \frac{\partial}{\partial k_{Di}}[\cdot]$ and $J_{Mi} = \frac{\partial}{\partial k_{Mi}}[\cdot]$ are the first order differentiations with respect to k_{Di} and k_{Mi} respectively. Also note that $\frac{\partial k_{yj}}{\partial k_{yi}} = 0,\ \frac{\partial u(k_{yj})}{\partial k_{yi}} = 0$ for all $j \neq i$ and $y \in \{D, M\}$.

In equilibrium, the profit of the dominant and emerging platforms are as follows:

$$\Pi(\mathbf{k_y^*}) \;=\; \begin{cases} nr \;+\; (c_D - r)\cdot(\mathbf{p})^\top\cdot\mathbf{1} \;-\; \frac{\alpha}{2}\cdot\mathbf{k_y}^{*\top}\cdot\mathbf{k_y^*} & \text{if } \mathbf{k_y^*} = \mathbf{k_D^*} \\ nc_M \;-\; (c_M - r)\cdot(\mathbf{p})^\top\cdot\mathbf{1} \;-\; \frac{\alpha}{2}\cdot\mathbf{k_y}^{*\top}\cdot\mathbf{k_y^*} & \text{if } \mathbf{k_y^*} = \mathbf{k_M^*} \end{cases}$$

Differentiating the equilibrium profit function of the platforms with respect to their respective incentives is given by

$$J_{Di}[\Pi(\mathbf{k_D}^*)] \;= 0 \;\Longrightarrow\; \alpha k^*_{Di} \;=\; e(c_D - r)\cdot u'_{Di}\cdot I_i \;\;\Longrightarrow\;\; \frac{k^*_{Di}}{u'(k^*_{Di})} \;=\; e\frac{(c_D - r)}{\alpha}\cdot I_i \tag{A.13}$$
$$J_{Mi}[\Pi(\mathbf{k_M}^*)] \;= 0 \;\Longrightarrow\; \alpha k^*_{Mi} \;= e(c_M - r)\cdot u'_{Mi}\cdot I_i \;\;\Longrightarrow\;\; \frac{k^*_{Mi}}{u'(k^*_{Mi})} \;=\; e\frac{(c_M - r)}{\alpha}\cdot I_i \tag{A.14}$$

If $c_D \geq c_M$, then $k^*_{Di} \geq k^*_{Mi}$.

A.8 Proof of Proposition 8

Consider two platforms with identical subscription fees. This implies if $c_D = c_M$, then $k^*_{Di} = k^*_{Mi} = k^*$. The equilibrium probability of the user choosing exclusive plan provided by the dominant platform turns out to be

$$\mathbf{p}^* = \frac{1}{2} \cdot \mathbf{1} - e \cdot (\mathbb{I} - \beta \mathbb{G})^{-1} \cdot \mathbf{b}$$
$$\mathbb{P}^* = \frac{n}{2} - e \cdot \mathbf{b}^\top \cdot \mathcal{I} = \frac{n}{2} - e \cdot \sum_j b_j I_j$$

In equilibrium, the profits of the dominant and emerging platforms are as follows:

$$\Pi_y(\mathbf{k}^*) = \begin{cases} nr + (c_D - r) \cdot (\mathbf{p}^*)^\top \cdot \mathbf{1} - \frac{\alpha}{2} \cdot \mathbf{k}^{*\top} \cdot \mathbf{k}^* & \text{if } P_y = P_D \\ nc_M - (c_M - r) \cdot (\mathbf{p}^*)^\top \cdot \mathbf{1} - \frac{\alpha}{2} \cdot \mathbf{k}^{*\top} \cdot \mathbf{k}^* & \text{if } P_y = P_M \end{cases}$$

The following condition entails that the profit of the emerging platform exceeds the profit of the dominant platform

$$\Pi_M(\mathbf{k}^*) \geq \Pi_D(\mathbf{k}^*)$$
$$nc_M - (c_M - r) \cdot \mathbb{P} \geq nr + (c_D - r) \cdot \mathbb{P}$$
$$\frac{n}{2} \geq \mathbb{P} = \frac{n}{2} - e \cdot \sum_j b_j I_j$$
$$- e \cdot \sum_j b_j I_j \leq 0$$

(a) If $b_i = b$, then $\mathbb{P}^* = \frac{n}{2} - eb\mathcal{I}_{agg}$. In this case, the profit of the emerging platform exceeds the profit of the dominant platform if $-eb\mathcal{I}_{agg} \leq 0$. This is always true.

(b) Now consider, $b_i = \{-b, b\}$. In this case, $\mathbb{P}^* = \frac{n}{2} - eb \cdot (\mathcal{I}_M - \mathcal{I}_D)$. Hence, the profit of the emerging platform exceeds the profit of the dominant platform if $\mathcal{I}_M \geq \mathcal{I}_D$.