To license or to cross-hold? An Analysis of Partial Passive Ownership and Alternate Forms of Technology Licensing

Rajit Biswas*
Centre For Development Studies,
Prasanth Nagar, Medical College P.O., Ulloor,
Thiruvananthapuram 695011, Kerala, India

Aishwarya Desai†
Madras School of Economics
Gandhi Mandapam Road, Chennai-600025, Tamil Nadu, India

Neelanjan Sen‡
Madras School of Economics
Gandhi Mandapam Road, Chennai-600025, Tamil Nadu, India §

December 5, 2025

Abstract

This paper develops a model of technology transfer and cross-holdings in an oligopolistic market with three firms. The efficient firm can own cross-holding in an inefficient firm's profit share and can also license its technology to that firm. If the cost difference is lower, technology will always be licensed via a fixed fee and no cross-holding will be chosen, while for a higher cost difference, a two-part tariff licensing will be chosen, and cross-holdings may be held. Consumer surplus and welfare always increase, not only when the firms undertake only technology licensing, but also in the presence of both cross-holding and technology licensing. We also extend the model by incorporating horizontal product differentiation and show that cross-holding and two-part licensing is the grand optimal choice. With relatively high cost differences, cross-holding along with licensing via a

*email address: advaita.mannanam@gmail.com; rajit@cds.edu

†Corresponding author, email address: desaaishwarya636@gmail.com; phd24aishwarya@mse.ac.in

‡email address: neelu.sen@gmail.com; neelanjan@mse.ac.in

§The authors thank Uday Bhanu Sinha, for the valuable suggestions and comments, and Arghya Dutta for his help in some computation. The usual disclaimer applies.

two-part tariff always leads to an increase in consumer surplus and total welfare.

JEL classification: L24, L13, D43

Keywords: Technology Licensing, Cross-holding, Fixed-fee, Royalty

No conflict of interest statement: The authors declare that there is no conflict of interest.
Funding statement: No funding is received from any sources.
Data Availability Statement: Data sharing not applicable –the article describes entirely theoretical research
CRediT authorship contribution statement: **Rajit Biswas, Aishwarya Desai & Neelanjan Sen**: Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization

1 Introduction

Firms may form strategic alliances amongst themselves to foster cooperation on various fronts. Caloghirou et al. (2003) discusses the importance and prevalence of these collaborations in detail. One particular mode of such inter-firm cooperation is partially passive holdings (henceforth PPOs). This refers to a situation in which a firm owns non-controlling minority shares of its rival's profits (generally less than 50 per cent). We construct a model where firms may decide to come up with PPO in another firm, while the transfer of technology is also possible either via fixed fee or royalty or both. Firms compete in terms of quantity in the output market. When the goods are homogeneous, we show that if technology is transferable only through fixed-fee licensing, then with a moderate cost difference, a scheme that involves both PPO and licensing is preferred over only licensing. On the other hand, for a very high or a sufficiently low-cost difference, PPO will never be chosen. If technology can be transferred through two-part tariff licensing, then allowing PPO is either sub-optimal (for lower marginal costs) or firms are indifferent to it (for relatively higher cost). We also extend the model by incorporating horizontal product differentiation where technology can be licensed via fixed-fee along with cross-holding. We observe that with high cost differences and very low product differentiation, it is optimal for the licensor to neither share technology via fixed fee nor cross-hold. Cross holding and fixed-fee licensing is optimal, when the cost difference is relatively low irrespective of the degree of product differentiation. Interestingly, in the presence of horizontal product differentiation, cross holding and two-part licensing will always happen and it is the grand optimal choice.

There has been substantial literature on the effect of cross-holding on knowledge transfer and market competition among firms. Malueg (1992) and Gilo et al. (2006) consider a certain level of cross-holding and study tacit collusion. Gomes-Casseres et al. (2006) argues that knowledge flow among allied firms will be much higher than among non-allied firms. Foros et al. (2011) and Ghosh and Morita (2017) are important contributions as they endogenize the level of cross-ownership, though based on different mechanisms. In a differentiated oligopoly model with three firms, Foros et al. (2011) compare the profitability of a merger between two firms in which one firm fully acquires another, with the profitability from cross holding. They show that joint profit can be higher in the latter case, because it may result in the reduction of competition with respect to an outside competitor. In Ghosh and Morita (2017), cross holdings have two distinct and opposing impacts on the joint profit. It can facilitate knowledge transfer among the firms thereby increasing the joint profits, but also causes the competitors to behave aggressively which may reduce joint profits. This trade-off determines the level of PPO in their model. Additionally, there have been contributions which have focused on the relationship between technology transfer and cross-holding. Chen et al. (2010) considers a fixed level of cross-ownership and shows that royalty licensing (fixed fee) is preferred by the patentee of a process innovation when the degree of cross-ownership is high (low). Leonardas et al. (2021) shows that technology licensing and PPO can be complimentary channels of technology transfer. In a Cournot duopoly model, where technology licensing can occur through fixed fees, they show that PPOs can promote licensing. Specifically, if the cost difference is in the intermediate range, then the firm with the superior technology will license its technology to the other firm, provided it owns a larger share in it. PPO can thus lead to higher consumer surplus and social welfare.

Gomes-Casseres et al. (2006) studies interfirm alliances as a mechanism for sharing technological knowledge, where alliances take different forms, from equity joint ventures to nonequity contractual arrangements. They argue that knowledge flows can be between alliance partners or it may flow between pairs of non-allied firms. Ono et al. (2004) mentions that at the end of the 1990s, several foreign automobile manufacturers became the largest shareholders in several Japanese automobile manufacturers and thereby entered into a partial ownership arrangement. They also show that partial ownership arrangement between these automakers facilitated the transfer of technology and managerial skills, either unilaterally or bilaterally, thereby creating cost-saving synergies. Moreover, there are also numerous examples where firms have only entered into technology licensing agreements in the absence of any partial ownership arrangement (See Vishwasrao (2007)). Brousseau et al. (2007) provides evidence on the contractual governance of technology licensing agreements using an international sample of licensing contracts and explore how contracts are designed. Their findings are based on a database of 213 licensing contracts from a survey of 160 American, Japanese, and European firms, working mainly in chemical, equipment, and service industries. It includes information on licensing conditions, payment formulas, safeguards and governance structure, where 60 percent of their database agreements are sole licenses. The other agreements included are either in a cooperative R&D project (18 percent) or in a wider alliance (9 percent) or partnership (12 percent) via equity relationships. They thereby take into account technology licensing agreements among subsidiaries of the same firm or between a subsidiary and its parent company via the "equity links" to take into account the existence of an equity relationship between the licensor and licensee. In this paper, we study the possibilities of technology licensing from an efficient firm to an inefficient firm, where the licensor may have a partial ownership arrangement and thereby own a share in the licensee's profits. We show that the final equilibrium in our model may explain why sole licenses in the absence of any equity links are sometimes dominant, where firms only have licensing contracts in the absence of any partial ownership arrangements between them (see Brousseau et al. (2007)).

Our model relates to Ghosh and Morita (2017) and Leonardas et al. (2021). We assume a three-firm Cournot model, where the efficient firm (firm 1) can have cross holdings in an inefficient firm (firm 2). Initially as in Ghosh and Morita (2017) and Leonardas et al. (2021), we assume that the firms produce homogeneous goods, however later in the paper, we also consider horizontal product differentiation. The efficient firm can also license its technology to the inefficient firm either through a fixed fee or through a two-part tariff. Contrary to Ghosh and Morita (2017) and Leonardas et al. (2021), Firm 1 and Firm 2 make two distinct decisions to maximize their joint profit, namely the optimal level of PPO and whether technology should be licensed, and if so then the optimal scheme for it (fixed fee or royalty). That is, PPO and technology licensing both are instruments for joint profit maximisation in our model. The presence of the third firm helps us to capture the potential negative impact of PPO on joint profit maximisation through a more aggressive behaviour of an outside competitor to Firms 1 and 2.[1] This feature of our model is akin to the modelling strategy of Ghosh and Morita (2017). However, in their model firms do not consider the optimal choice of licensing instrument, the decision to license and the optimal level of PPO simultaneously for the maximization of joint profit. They do discuss knowledge transfer through technology licensing vis-a-vis PPO from the perspective of a welfare-maximizing authority, but firms themselves do not consider the possibility of employing both technology licensing and PPO to maximize joint profits. These departures lead us to a set of significant and interesting results. If the products are homogeneous and we allow firms to license technology only through fixed fee[2] then when the cost difference is low, firms would never go for PPO, but technology licensing will be employed. If the marginal cost is of intermediate value, they would adopt both, and if the marginal cost is relatively higher there would be neither technology transfer nor PPO. We also argue that if two-part tariff licensing of technology can be used by firms then *PPO's may never be chosen by the firms*. Moreover, contrary to Ghosh and Morita (2017), free licensing is never adopted by the firms in the final equilibrium. If the cost difference is low the firms would decide only to license technology via fixed fee (without cross-holding), and if it is high firms would decide to license their technology via two-part tariff and there may or may not be cross-holding (as under two-part tariff licensing PPO has no impact on joint profit for higher cost difference).

Frazzani et al. (2020) says "... whether and in which circumstances common ownership is beneficial or deleterious for competition, innovation, and, ultimately, citizen welfare is still an open debate. .... In addition, there is still ongoing debate as to whether common ownership drives commonly owned firm's managements to compete less or to coordinate future business strategy with one another in a collusive fashion." There are some works that have studied the effects of common ownership on innovation (López and Vives, 2019; Antón et al., 2023; Shelegia and Spiegel, 2017), showing that common shareholding can also be welfare-improving. However, empirical evidence for efficiency gains arising from common shareholding is still lacking (Schmalz, 2017).[3]

Moreover, in the literature, there are also very few theoretical papers that have studied the combined effect of cross-holding and technology licensing opportunities on consumer surplus and welfare. Hence, given the growing interest in PPO by the policymakers and the anti-trust authorities, we explore the effect of cross-holding and technology licensing opportunities on consumer

[1]This effect is absent in Leonardas et al. (2021.) We think that it is better to avoid duopoly models when the firms are involved in deciding on a merger or the level of PPO. This is because in the duopoly model mergers and cross-holding are generally always possible

[2]See Leonardas et al. (2021) for why this may be the case.

[3]Rosati et al. (2020) says "Policymakers, and for this purpose those worried about consumer welfare, need to adjust their policy objectives to capture the new possible anti-competitive effects resulting from these changes. For these reasons, it is crucial to choose an appropriate measure of competition. In order to establish a causal relationship between common shareholding and competition, the options explored include measures of market power or market concentration, as well as considering directly the effect on prices."

surplus and welfare as in Ghosh and Morita (2017) and Leonardos et al. (2021). We show that when the goods are homogeneous, and if the cost difference is low, then firms would decide only to license technology via fixed-fee (but no cross-holding) and post-licensing consumer surplus and welfare will increase. On the other hand, if the cost difference is high, firms would decide to license their technology via two-part tariff (there may or may not be cross-holding), then both consumer surplus and welfare will increase. Therefore, in our model consumer surplus and welfare always increase, not only when the firms only select technology licensing as a tool, but also when they have agreements that involve cross-holding as well as technology licensing. However, our result is in contrast to Ghosh and Morita (2017), where the endogenously determined level of PPO can increase consumer surplus and/or total surplus only when the level of PPO is relatively large. In Leonardos et al. (2021), where there are only two firms, consumer surplus and social welfare decrease in the level of cross-holding (PPO) when the cost difference is small and licensing via fixed-fee takes place. However, they also show that PPOs have a positive impact on consumer surplus and social welfare whenever they induce technology licensing. Contrarily, in our model, we show that irrespective of the level of PPOs, the consumer surplus and welfare always increase. This is because PPOs are always accompanied by licensing which results not only in higher industry outputs but also in industry profits.

The present model is also extended by incorporating horizontal product differentiation. In the context of fixed-fee licensing and cross-holding, we observe that when cost differences are high and product differentiation is very low, it is optimal for the licensor to neither share technology through a fixed fee nor engage in cross-holding. This result is similar to the one obtained when the goods are homogeneous. Interestingly, we find that cross-holding and fixed-fee licensing become optimal when the cost difference is relatively small, irrespective of the degree of product differentiation. The paper also discusses two-part tariff licensing and cross-holding within the same framework of horizontal product differentiation. It is shown that cross-holding and two-part licensing will always occur and represent the grand optimal choice. Furthermore, when the cost difference is low, the outside firm will produce after cross-holding and two-part licensing, whereas if the cost difference is high, it will not produce.

Antitrust authorities increasingly recognize that partial cross-holdings or common ownership among competing firms can have significant implications for market competition. Both the U.S. Department of Justice (DOJ) and the Federal Trade Commission (FTC)[4] explicitly note in their Merger Guidelines that even non-controlling interests in rival firms may raise competitive concerns by softening rivalry or facilitating coordination (U.S. DOJ & FTC, 2023).[5] Empirical and theoretical research has reinforced these apprehensions. Hariskos et al. (2022) finds that increasing symmetric passive cross-ownership in a duopoly experiment leads to higher prices and more collusive outcomes. Similarly, He and Huang (2017) show that firms sharing blockholders within the same industry exhibit greater market share growth and coordination, suggesting that common ownership can influence strategic behaviour in product markets. Lopez and Vives (2019) develop a model indicating that while cross-holdings may improve welfare under specific R&D spillover conditions, they generally raise consumer surplus concerns in highly concentrated markets. From a legal and

[4] See https://news.bloomberglaw.com/antitrust/insight-ftcs-inquiry-into-common-ownership-benefits-market
[5] See https://www.justice.gov/atr/merger-guidelines/applying-merger-guidelines/guideline-11

policy perspective, Hemphill and Kahan (2020) argue that common ownership may operate as a subtle mechanism of anticompetitive coordination. Collectively, these findings underscore that competition authorities treat cross-holdings seriously, though not all such arrangements are inherently harmful. The welfare effects depend on the nature of the ownership, the degree of control, and the strategic context. These are the precisely the dimensions explored in the paper. Our analysis shows and strengthens the view that policy makers need to take a very nuanced view when they deal with the possible consequences of cross holding in the presence of various forms of technology licensing schemes. This is because the negative impacts of cross-holding could be mitigated by positive impacts of technology licensing and hence, as shown in this paper, consumer surplus and welfare may actually increase in the presence of product differentiation, if the cost difference is relatively high among the firms in the pre-licensing stage. On the other hand, when the goods are homogeneous, consumer surplus and welfare will always increase.

The structure of the paper is as follows. Section 2 sets up the basic model and considers fixed fee licensing, section 3 considers two-part tariff licensing, section 4 deals with welfare analysis. In section 5 and section 6 we extend the paper by introducing product differentiation and section 7 concludes the model.

2 Model with fixed fee licensing

We consider a market characterized by Cournot competition among three firms. All these firms produce a homogeneous good and the output produced by firm i is given by q_i. The firms face a linear demand given by $p = a - q_i - \sum q_j$, where $i \neq j$; $i, j = 1, 2, 3$ and p is the price. We assume that firm 1 owns a share in firm 2 of level x. This entitles firm 1 a share in firm 2's profit in proportion to x. We assume $x \in [0, \frac{1}{2}]$, that is firm 1 cannot own more than 50 per cent of profit shares of firm 2.[6] As common in the literature we will call this kind of cross holdings as partial passive ownership holdings or PPO for brevity's sake. If $x > 0.5$ then firm 1 will have a "controlling share" of firm 2 and it requires a different analysis. Thus, in the present analysis as in Ghosh and Morita (2017), Lopez and Vives (2019), Leonardos et al. (2021) etc. we assume that $x \leq 0.5$. Finally, we assume firm 2 and 3 to have identical per unit costs given by a positive constant $c(> 0)$ whereas firm 1, which is the efficient firm can produce at a relatively lower per unit cost c_1. For simplicity we assume $c_1 = 0$.

We consider a two-stage game. In stage 1, based on joint profit maximization[7], the level of PPO (x) is determined and also a decision is taken if technology is licensed from the efficient firm (firm 1) to the inefficient firm (firm 2). As in Fosfuri and Roca (2004)[8], we assume that firm 1 cannot reach firm 3 through licensing contracts. Adaptation of an efficient technology by one firm generally causes negative externalities on the other (non-buying) firms in the market. Thus, in the context of licensing agreements, licensing of technology from an efficient firm to all other inefficient firms in the market is less observed. It may create confusion among the licensees and also increases the

[6]See Ghosh and Morita (2017) and Leonardos et al. (2021).

[7]Total profit of firm 1 and firm 2.

[8]They say "...firm 3 cannot license in firm 1's cost-reducing innovation because of compatibility reasons that would raise adoption costs or because of transaction costs that would make the deal unfeasible."

transaction cost of the licensor significantly.[9] Getting approvals from the antitrust authorities may also become complex and sometime impossible in this kind of situations. Thus, we avoid licensing of technology to all inefficient firms in the market. Hence, throughout the paper we assume that technology can be licensed to only one firm (say firm 2). For a similar reason cross-holding many firms is also avoided in the present model. This approach is also followed in the literature by Ghosh and Morita (2017) and Leonardas et al. (2021). In Stage 2, firms produce and compete in terms of output.

To make the analysis tractable, we first focus on licensing via a fixed fee, and subsequently on licensing via a two part tariff (in the later section). We first focus on the joint profit that firm 1 and firm 2 earn (π). Next, we find the subgame perfect equilibrium as the firms will always select the case that gives the maximum joint profit, by choosing the appropriate value of PPO (x) and fixed fee (F). The firms also use a side payment, i.e. a monetary transfer (T) from firm 1 to firm 2 in exchange for the cross-holding in Firm 2.

2.1 Joint profit of Firm 1 and Firm 2 as determined by PPO and licensing pattern.

We first present, the various cases firms 1 and 2 are faced with in Table 1.[10] Thereafter, we determine the outputs and profits of the firms in each of the cases. Post-licensing, both in the presence as well as in the absence of cross-holding, the technology may become "drastic" such that firm 3 may not be able to compete in the market (see Arrow (1962)) and thus not produce any output. Otherwise, we can call it "non-drastic". Moreover, it is to be noted that whether the technology is drastic or non-drastic is not only determined by the cost-reduction after licensing but also by the level of cross-holding (x).

Table 1

Sl.No.	Case	Description
1	NCNL	No cross-holding & No licensing
2	F	Licensing via fixed fee and good 3 is produced
3	F0	Licensing via fixed fee, but good 3 is not produced
4	C	Only Cross-holding
5	CF	Cross-holding & fixed fee licensing and good 3 is produced
6	CF0	Cross-holding & fixed fee licensing, but good 3 is not produced

[9]Fosfuri and Roca (2004) argues "For instance, Union Carbide has a long tradition in polypropylene licensing. BP is both a major player in the polyethylene market and a leading licensor of polyethylene technology. Second, rarely is a firm able to license its process technology to the whole industry. Some competitors might not be able to incorporate the cost-reducing innovation in their production process because of incompatibility or simply prohibitive costs. For instance, metallocene technology, based on new single site catalysts, has been adopted only in a relatively small share of the worldwide production of polyolefin."

[10]The notations given in the table will be used henceforth in the paper.

2.1.1 Case 1: No cross-holding & No licensing (NCNL)

The joint profit and output of firms 1 and 2 in the case of $NCNL$ are

$$\pi^{NCNL} = \pi_1^{NCNL} + \pi_2^{NCNL} = \frac{(a+2c)^2}{16} + \frac{(a-2c)^2}{16}, \tag{1}$$

where $\pi_1^{NCNL} = \frac{(a+2c)^2}{16}$ and $\pi_2^{NCNL} = \frac{(a-2c)^2}{16}$ represents the profit of firm 1 and that of firm 2 when there is no cross-holding and no technology licensing (NCNL). Firm 3 earns a profit $\pi_3^{NCNL} = \frac{(a-2c)^2}{16}$. The outputs produced are $q_1^{NCNL} = \frac{a+2c}{4}$ and $q_2^{NCNL} = q_3^{NCNL} = \frac{a-2c}{4}$.

Throughout the paper we assume that $a > 2c$ such that $\pi_i^{NCNL} > 0$ and $q_i^{NCNL} > 0;\ i = 1, 2, 3.$

2.1.2 Case 2: Fixed-fee licensing and good 3 is produced (F)

Firm 1 decides to license its technology to firm 2, via fixed fee licensing (F) but x is set to zero, i.e. no cross-holding, while the third firm continues to produce. Thus, the profit functions are as follows

$$\pi_1^F = pq_1 + F, \quad \pi_2^F = pq_2 - F \quad and \quad \pi_3^F = (p-c)q_3. \tag{2}$$

Using equation (2), we have the optimal outputs as

$$q_1^F = \frac{(a+c)}{4}, \ q_2^F = \frac{(a+c)}{4} \quad \text{and} \quad q_3^F = \frac{a-3c}{4}. \tag{3}$$

In this case, the joint profit of firm 1 and firm 2 is

$$\pi^F = \pi_1^F + \pi_2^F = \frac{2(a+c)^2}{16}, \tag{4}$$

where $\pi_1^F = \frac{(a+c)^2}{16} + F$, $\pi_2^F = \frac{(a+c)^2}{16} - F$ and $\pi_3^F = \frac{(a-3c)^2}{16}$.

Since this is derived assuming that firm 3 continues to produce its output post-licensing, the corresponding parametric restriction is $a > 3c$.

2.1.3 Case 3: Fixed-fee licensing, but good 3 is not produced (F0)

Consider the previous case (F), but the third firm does not produce post-licensing. In this case, using equation (2), given $q_3^{F0} = 0$, the outputs of firm 1 and firm 2 are

$$q_1^{F0} = \frac{a}{3} \quad \text{and} \quad q_2^{F0} = \frac{a}{3} \tag{5}$$

and the joint profit is

$$\pi^{F0} = \pi_1^{F0} + \pi_2^{F0} = \frac{2a^2}{9} \tag{6}$$

where $\pi_1^{F0} = \frac{a^2}{9} + F$, $\pi_2^{F0} = \frac{a^2}{9} - F$ and $\pi_3^{F0} = 0$.

Since this is derived assuming that firm 3 does not produce post-licensing, the corresponding parametric restriction is $a \leq 3c$.[11]

[11] Otherwise, it will lead to the previous case (Case F).

2.1.4 Case 4: Only cross-holding (C)

Firms 1 and 2 do not go for technology licensing, rather only cross-holding is chosen. The profit functions of the three firms without any technology licensing would be given by

$$\pi_1^C = pq_1 + x(p-c)q_2 - T \tag{7}$$

$$\pi_2^C = (1-x)(p-c)q_2 + T \quad \text{and} \tag{8}$$

$$\pi_3^C = (p-c)q_3. \tag{9}$$

Profit maximization entails the following optimal values of quantities for the three firms respectively $q_1^C = \frac{a(1-x)+(2+x)c}{4-x}$, $q_2^C = \frac{a-2c}{4-x}$ and $q_3^C = \frac{a-2c}{4-x}$. The joint profit of firm 1 and firm 2 is given by

$$\pi^C = \pi_1^C + \pi_2^C = \frac{[a+c(2-x)][a(1-x)+(2+x)c]}{(4-x)^2} + \frac{(a-2c)^2}{(4-x)^2} \tag{10}$$

where $\pi_1^C = \frac{\left[a+c(2-x)\right]\left[a(1-x)+(2+x)c\right]}{(4-x)^2} + \frac{x(a-2c)^2}{(4-x)^2} - T$, $\pi_2^C = (1-x)\frac{(a-2c)^2}{(4-x)^2} + T$ and $\pi_3^C = \frac{(a-2c)^2}{(4-x)^2}$.

It is to be noted that after cross-holding firm 1 reduces its output $q_1^C < q_1^{NCNL}$, whereas firm 2 and firm 3 increase their output $q_2^C > q_2^{NCNL}$ and $q_3^C > q_3^{NCNL}$.

2.1.5 Case 5: Cross-holding & fixed fee licensing, good 3 is produced (CF)

Firms 1 and 2 agree on cross holding as well as technology licensing via fixed fee. The contract involves the level of ownership (x) of firm 1 in firm 2, the money transfer (T), and the fixed fee to be levied for technology transfer (F). Assuming firm 3 continues to produce, then the profit functions of the three firms would be

$$\pi_1 = pq_1 + x(pq_2 - F) + F - T \tag{11}$$

$$\pi_2 = (1-x)(pq_2 - F) + T \quad \text{and} \tag{12}$$

$$\pi_3 = (p-c)q_3. \tag{13}$$

Maximizing these profit functions, we obtain the optimum outputs as

$$q_1^{CF} = \frac{(a+c)(1-x)}{4-x}, \; q_2^{CF} = \frac{(a+c)}{(4-x)} \;\; and \;\; q_3^{CF} = \frac{a-c(3-x)}{4-x}. \tag{14}$$

It is to be noted that $q_3^{CF} > 0$ only if $\frac{a}{3-x} > c$, which means that the initial unit cost of firm 2 and firm 3 (c) should be significantly less. The joint profit of firm 1 and firm 2 is given by

$$\pi^{CF} = \pi_1^{CF} + \pi_2^{CF} = \frac{(a+c)^2(2-x)}{(4-x)^2} \tag{15}$$

where $\pi_1^{CF} = \frac{(a+c)\left[(a+c)(1-x)\right]}{(4-x)^2} + x\left[\frac{(a+c)^2}{(4-x)^2} - F\right] + F - T$, $\pi_2^{CF} = (1-x)\left[\frac{(a+c)^2}{(4-x)^2} - F\right] + T$ and $\pi_3^{CF} = \frac{\left[a-(3-x)c\right]^2}{(4-x)^2}$.

It is to be noted that π^{CF} decreases in x. Since the quantities and profits are derived assuming that firm 3 continues to produce good 3 post-licensing, the corresponding parametric restriction is $a > c(3-x)$.

2.1.6 Case 6: Cross-holding & fixed fee licensing, but good 3 is not produced (CF0)

This case corresponds to the previous case, i.e. case 5, the only exception being firm 3 does not produce. Proceeding as in the previous case, given $q_3^{CF0} = 0$, the outputs of firm 1 and firm 2 are

$$q_1^{CF0} = \frac{a(1-x)}{3-x} \quad \text{and} \quad q_2^{CF0} = \frac{a}{(3-x)} \tag{16}$$

and the joint profit of firm 1 and firm 2 is given by

$$\pi^{CF0} = \pi_1^{CF0} + \pi_2^{CF0} = \frac{a^2(2-x)}{(3-x)^2} \tag{17}$$

which is increasing in x, where $\pi_1^{CF0} = \frac{a^2(1-x)}{(3-x)^2} + x\left[\frac{a^2}{(3-x)^2} - F\right] + F - T$, $\pi_2^{CF0} = (1-x)\left[\frac{a^2}{(3-x)^2} - F\right] + T$ and $\pi_3^{CF0} = 0$.

Since this is derived assuming that firm 3 does not continue to produce its output post-licensing, the corresponding parametric restriction is $a \leq c(3-x)$.

2.2 Profit comparison and determination of optimal outcome

It is straightforward to check that $\pi^C < \pi^{NCNL}$ (See Ghosh and Morita (2017)). Thus C is always dominated by NCNL, and so only cross-holding (C) can be ruled out and need not be considered in the further analysis. The intuition for this is straightforward. As x increases, the degree of competition between firms 1 and 2 reduces (See Ghosh and Morita (2017)) and thus the total output produced by firm 1 and firm 2 reduces. This effect works in the direction of increasing the joint profit of firm 1 and firm 2. On the other hand, weaker competition between firms 1 and 2 induces firm 3 to produce more as the quantities are strategic substitutes, and this effect works in the direction of decreasing the joint profit of firm 1 and firm 2. The latter effect dominates the former effect for all $x \in (0, \frac{1}{2}]$. Therefore, $\pi^C < \pi^{NCNL}$, i.e. only cross-holding (C) is inferior to no cross-holding and no-licensing ($NCNL$).

Moreover, comparing only fixed-fee licensing (F or $F0$) with no cross-holding and no-licensing ($NCNL$), we observe the following:
i) when $c < \frac{a}{3}$, F dominates $NCNL$ as $\pi^F > \pi^{NCNL}$,[12]
ii) while if $\frac{a}{3} \leq c < \frac{a\sqrt{7}}{6}$, $F0$ dominates $NCNL$ as $\pi^{F0} > \pi^{NCNL}$ [13] and
iii) if $\frac{a\sqrt{7}}{6} \leq c < \frac{a}{2}$, then $NCNL$ dominates $F0$ as $\pi^{F0} \leq \pi^{NCNL}$.

[12] Consider the inequality $\frac{2(a+c)^2}{16} > \frac{(a+2c)^2}{16} + \frac{(a-2c)^2}{16}$. This is equivalent to $c < \frac{a}{3}$.
[13] Consider the inequality $\frac{2a^2}{9} > \frac{(a+2c)^2}{16} + \frac{(a-2c)^2}{16}$ which reduces to $c < \frac{a\sqrt{7}}{6}$.

We know from the literature (see Marjit (1990) and Wang (1998)) that higher cost difference reduces the chances of technology licensing. We observe a similar situation here. If the cost difference is high ($\frac{a\sqrt{7}}{6} \leq c < \frac{a}{2}$) technology licensing does not occur. If the cost difference is slightly lesser, but still high enough ($\frac{a}{3} \leq c < \frac{a\sqrt{7}}{6}$), technology licensing does take place but the inefficient firm 3 (which continues to use the inefficient technology) cannot compete and stops its production. For a still lower cost difference ($c < \frac{a}{3}$) technology licensing will indeed take place always and firm 3 continues to produce output in the market.

We can also make the following observations before coming to the first proposition. In case firms choose strategies, such that they are in a situation akin to Case 5 i.e. CF, they would choose a minimal value of x as from equation (15), π^{CF} is decreasing in x. Since here firm 3 produces (positive) outputs or $a > c(3-x)$, this is possible with $a > 3c$ or $a \leq 3c$. This is because if x increases, given firm 1 licenses its technology to firm 2, the degree of competition between firms 1 and 2 reduces. As stated before, this effect positively affects the joint profit of firm 1 and firm 2. However, less competition between firms 1 and 2 induces firm 3 to produce more output as the quantities are strategic substitutes, and this negatively affects the joint profit of firm 1 and firm 2. The latter effect dominates the former effect given $a > c(3-x)$ or $x > \frac{3c-a}{c}$, thus π^{CF} decreases in x.

Thus, we have the following.
I) If $c < \frac{a}{3}$, x will be set at 0 (as π^{CF} decreases in x) and the corresponding joint profit will be $\pi^{CF} = \pi^{F} = \frac{2(a+c)^2}{16}$ (as $x = 0$).

II) On the other hand, if $c \geq \frac{a}{3}$ we argue as follows.
a) For $\frac{a}{3} \leq c < \frac{a}{2.5}$ the firms will set $x(>0)$ as low as possible (as π^{CF} decreases in x), and it will be marginally greater than $\frac{3c-a}{c}(<0.5)$, given firm 3 produce outputs, and the corresponding joint profit π^{CF} will be marginally lower than $(a-c)c$.

b) Otherwise, for $\frac{a}{2.5} \leq c < \frac{a}{2}$, we observe that $0.5 \leq \frac{3c-a}{c}$, thus firms will set $x = 0.5$ (as high as possible given firm 3 produce outputs) as it has been assumed before that $x \in (0, \frac{1}{2}]$, but this contradicts $a > c(3-x)$. Thus, for $\frac{a}{2.5} \leq c < \frac{a}{2}$ Case CF is not possible.

If firms choose strategies, such that they are in a situation akin to Case $CF0$, firms would choose a value of x which will be maximal as from equation (17), π^{CF0} is increasing in x. For Case $CF0$, since firm 3 does not produce (positive) outputs or $a \leq c(3-x)$ i.e. $x \leq \frac{3c-a}{c}$, this is possible only when $a \leq 3c$. This is because if x increases, given firm 1 licenses its technology to firm 2, the degree of competition between firms 1 and 2 reduces. This helps in increasing the joint profit of firm 1 and firm 2. However, here the latter effect (via the increase in the output of firm 3) is completely absent as firm 3 can't compete, thus π^{CF0} increases in x.

It is straight forward to check that
i) if $\frac{a}{3} \leq c < \frac{a}{2.5}$, $x = \frac{3c-a}{c}(<0.5)$ (as π^{CF0} increases in x given good 3 is not produced), and the corresponding joint profits will be $\pi^{CF0} = (a-c)c$ and
ii) if $\frac{a}{2.5} \leq c < \frac{a}{2}$, $x = 0.5$, the corresponding joint profit will be $\pi^{CF0} = \frac{a^2(1.5)}{(2.5)^2}$.

The above analysis leads us to the following Proposition 1 that relates the endogenous choices made by the firms regarding cross-holding and technology licensing in terms of cost.

Proposition 1 *When licensing can be carried out only through a fixed fee, then if*

a) $c < \frac{a}{3}$*, then optimal level of PPO* $x^* = 0$ *and the joint profit is* $\pi^F = \frac{2(a+c)^2}{16}$*. Firms would decide only to license technology but there will be no PPO (Case F).*

b) $\frac{a}{3} \leq c < \frac{a}{2.5}$*, then optimal level of cross holding* $x^* = \frac{3c-a}{c}(< 0.5)$ *and the joint profit is* $\pi^{CF0} = (a-c)c$*. Firms would decide to license their technology and there would be PPO (Case CF0).*

c) $\frac{a}{2.5} \leq c < \frac{a\sqrt{23}}{10}$*, then optimal level of cross holding* $x^* = 0.5$ *and the joint profit is* $\pi^{CF0} = \frac{1.5a^2}{(2.5)^2}$*. Firms would decide to license their technology and there would be PPO (Case CF0).*

d) $\frac{a\sqrt{23}}{10} \leq c < \frac{a}{2}$*, then optimal level of cross holding* $x^* = 0$ *and the joint profit is* $\pi^{NCNL} = \frac{(a+2c)^2}{16} + \frac{(a-2c)^2}{16}$*. In this case firms would neither license their technology nor will there be PPO (Case NCNL).*

Proof. See Appendix A ■

Moreover, the joint profit of firm 1 and firm 2 is independent (See equations (4), (6), (15) and (17)) of the level of fixed-fee (F) and the amount of money to be transferred from firm 1 to firm 2 (T) for acquiring a stake of x in firm 2. Once the joint profit is maximized and the respective schemes (licensing and/or cross-holding or none) are chosen, firm 1 and firm 2 can bargain about the distribution of the total profit using F and T as discussed in Appendix B.

The above result stated in Proposition 1 is depicted in Figure. In this figure, we measure the cost of firm 2 and firm 3 (c) on the horizontal axis and plot the optimal level of PPO (x^*), as well as the joint profit of firm 1 and firm 2 for different situations on the vertical axis. First, we plot π^F, the joint profit when technology is licensed to firm 2 in the absence of PPO i.e. $x^* = 0$ when $c < \frac{a}{3}$. This is represented by the red curve. It is to be noted that if x increases (above zero) and for relatively small c , when firm 1 licenses its technology to firm 2, the degree of competition between firms 1 and 2 reduces. This thereby positively affects the joint profit of firm 1 and firm 2. However, firm 3 reacts by producing more output and this effect negatively affects the joint profit of firm 1 and firm 2. If c is small enough, the latter effect dominates the former effect, and thus firm 1 and firm 2 will only license in this context and will set $x^* = 0$.

Next, we plot π^{CF0}, the joint profit when technology is licensed to firm 2 (good 3 is not produced) in the presence of PPO, i.e. $x^* = \frac{3c-a}{c}(< 0.5)$ when $\frac{a}{3} \leq c < \frac{a}{2.5}$. This is represented by the two green curves. This case corresponds to a high cost difference i.e $\frac{a}{3} \leq c \leq \frac{a}{2.5}$. If firm 1 licenses to firm 2, and firm 3 does not produce then the joint profit becomes π^{F0} in the absence of cross-holding. On the other hand, in the presence of both PPO and licensing, the firms will set x as high as possible (given good 3 is not produced), which gives a joint profit of π^{CF0}. Moreover, here $\pi^{CF0} > \pi^{F0}$, thus the firms will prefer a combination of cross-holding and licensing ($CF0$) rather

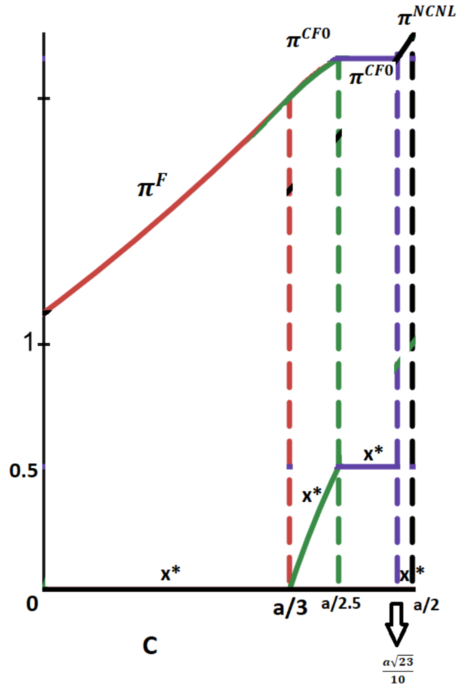

Figure 1: Fixed-fee licensing & Cross-holding

than only licensing ($F0$). As c is relatively high, higher than the first case, so that firm 3 can't compete post-licensing, cross-holding ($x^* = \frac{3c-a}{c}(> 0)$) always results in higher profits as the firms can reduce the level of competition by producing lower outputs (in comparison to only licensing). Thus, when $\frac{a}{3} \leq c \leq \frac{a}{2.5}$ the strategy that involves both PPO (x^*) and licensing is preferred over only licensing.

We also plot π^{CF0}, the joint profit when technology is licensed to firm 2 (but good 3 is not produced) in the presence of PPO, but $x^* = 0.5$ when $\frac{a}{2.5} \leq c < \frac{a\sqrt{23}}{10}$. This is represented by the two purple curves. For a similar reasoning as above, in this case too, cross-holding and fixed-fee licensing jointly is preferred over $F0$ or $NCNL$.

Finally, we plot π^{NCNL}, the joint profit when technology is not licensed to firm 2 in the absence of cross-holding, i.e. $x^* = 0$ when $\frac{a\sqrt{23}}{10} \leq c < \frac{a}{2}$. This is represented by the two black curves. In this case as the cost difference is too high cross-holding and licensing jointly is not preferred over $NCNL$. Thus, licensing is not preferred due to the high-cost difference. Even though in this case post-licensing firm 3 will not produce, firm 2 produces a larger output and this results in lower joint profits of firm 1 and firm 2.

3 Model with two-part tariff licensing

In this section, we explore the possibility of a two-part tariff licensing (a combination of royalty and fixed-fee), when the efficient firm transfers technology to the inefficient firm.[14] The structure of the game is identical to that of the previous section (see Section 2), except that in Stage 1 of the game, firms can choose to transfer technology via a two-part tariff along with cross-holding. In the Table 2[15] we present all cases available to firms 1 & 2.[16]

Table 2

No.	Case	Description
1	NCNL	No cross-holding & No licensing
2	RF	Licensing via Royalty & Fixed Fee, good 3 is produced
3	RF0	Licensing via Royalty & Fixed Fee, good 3 is not produced
4	C	Only Cross-holding
5	CRF	Cross-holding & Two-part tariff licensing, good 3 is produced
6	CRF0	Cross-holding & Two-part tariff licensing, good 3 is not produced

3.1 Joint profit of Firm 1 and Firm 2 as determined by PPO and licensing pattern.

As discussed before in Section 2.1, here we avoid stating the profits for only cross-holding (C) and no-cross holding and no-licensing (NCNL). Let us now consider the other cases of Table 2.

3.1.1 Licensing via Royalty & Fixed Fee, good 3 is produced(RF)

Suppose firm 1 charges a royalty rate $r \in [0, c]$ and a fixed fee F when it transfers technology to firm 2. To start the analysis, let us ignore PPO (i.e. set $x = 0$). It is straightforward to check that the outputs of the firms are

$$q_1^{RF} = \frac{a+c+r}{4},\ q_2^{RF} = \frac{a+c-3r}{4}\ \ and\ \ q_3^{RF} = \frac{a-3c+r}{4} \tag{18}$$

and the joint profit of the firms would be

$$\pi^{RF} = \pi_1^{RF} + \pi_2^{RF} = \frac{(a+c+r)^2}{16} + \frac{(a+c-3r)^2}{16} + \frac{r(a+c-3r)}{4} \tag{19}$$

if good 3 is produced, where $\pi_1^{RF} = \frac{(a+c+r)^2}{16} + \frac{r(a+c-3r)}{4} + F$, $\pi_2^{RF} = \frac{(a+c-3r)^2}{16} - F$ and $\pi_3^{RF} = \frac{(a+r-3c)^2}{16}$. This situation arises only when $3c < a + r$.

[14]It has been shown by Rostoker (1984) that royalties (39 percent) and a combination of royalties and fees (49 percent) are largely more common than upfront fees (13 percent) in corporate-licensing transactions. Vishwasrao (2007) also shows the prevalence of royalty licensing contracts. It shows that licensing contracts are more likely to use royalties when sales are relatively high. Vishwasrao (2007) also found that firms are more likely to use output based payments (e.g. royalties) to control the sale and diffusion of R&D or brand intensive know-how to unaffiliated firms. Neubig and Wunsch-Vincent (2017) argues That “Still today, high-income countries make up for the bulk, precisely close to 99 percent of RLF (Royalties and license fees) receipts – almost unchanged from ten to fifteen years earlier – and for 84 percent of royalty payments – a decline from 91 percent in 1999.”

[15]The notations given in the table will be used henceforth in the paper

[16]If optimal $r = 0$, then we will drop term R in this context. It will then be treated as fixed-fee licensing as mentioned in Table 1.

3.1.2 Licensing via Royalty & Fixed Fee, good 3 is not produced(RF0)

On the other hand, when $3c \geq a + r$, and good 3 is not produced, the outputs of firm 1 and firm 2 are

$$q_1^{RF0} = \frac{a+r}{3} \quad and \quad q_2^{RF0} = \frac{(a-2r)}{3}, \tag{20}$$

then joint profit is

$$\pi^{RF0} = \pi_1^{RF0} + \pi_2^{RF0} = \frac{1}{9}\big[(a+r)^2 + 3r(a-2r) + (a-2r)^2\big] \tag{21}$$

and it is increasing in the royalty rate (r), as long as $a + r \leq 3c$,[17] where $\pi_1^{RF0} = \frac{(a+r)^2}{9} + r\frac{a-2r}{3} + F$, $\pi_2^{RF0} = \frac{(a-2r)^2}{9} - F$ and $\pi_3^{RF0} = 0$.

3.1.3 Cross-holding & Two-part tariff licensing, good 3 is produced(CRF)

Firms, in our model, of course, have the option to choose two-part tariff licensing along with $x > 0$. Suppose firms decide to choose $x > 0$. Then the outputs of the three firms are

$$q_1^{CRF} = \frac{(a+c)(1-x) + r(1+2x)}{4-x}, q_2^{CRF} = \frac{a+c-3r}{4-x} \quad and \quad q_3^{CRF} = \frac{a - c(3-x) + r(1-x)}{4-x} \tag{22}$$

and the joint profit if good 3 is produced would be given by

$$\pi^{CRF} = \pi_1^{CRF} + \pi_2^{CRF} = \frac{[(a+c) + r(1-x)][(a+c)(1-x) + r(1+2x)]}{(4-x)^2} + \frac{(a+c-3r)^2}{(4-x)^2} + r\frac{(a+c-3r)}{4-x}, \tag{23}$$

where $\pi_1^{CRF} = \frac{\big[(a+c)+r(1-x)\big]\big[(a+c)(1-x)+r(1+2x)\big]}{(4-x)^2} + x\big[\frac{(a+c-3r)^2}{(4-x)^2} - F\big] + r\frac{a+c-3r}{4-x} + F - T$, $\pi_2^{CRF} = (1-x)\big[\frac{(a+c-3r)^2}{(4-x)^2} - F\big] + T$ and $\pi_3^{CRF} = \frac{\big[a-(3-x)c+r(1-x)\big]^2}{(4-x)^2}$. This case arises when $a + r(1-x) > c(3-x)$ such that $q_3^{CRF} > 0$.

3.1.4 Cross-holding & Two-part tariff licensing, good 3 is not produced(CRF0)

On the other hand, when good 3 is not produced i.e. $a + r(1-x) \leq c(3-x)$, the outputs of firm 1 and firm 2 are

$$q_1^{CRF0} = \frac{a(1-x) + r(1+x)}{3-x} \quad and \quad q_2^{CRF0} = \frac{(a-2r)}{(3-x)} \tag{24}$$

and the joint profit is

$$\pi^{CRF0} = \pi_1^{CRF0} + \pi_2^{CRF0} = \frac{\big[a + r(1-x)\big]\big[a(1-x) + r(1+x)\big]}{(3-x)^2} + \frac{(a-2r)^2}{(3-x)^2} + r\frac{a-2r}{3-x}, \tag{25}$$

where $\pi_1^{CRF0} = \frac{\big[a+r(1-x)\big]\big[a(1-x)+r(1+x)\big]}{(3-x)^2} + x\big[\frac{(a-2r)^2}{(3-x)^2} - F\big] + r\frac{a-2r}{3-x} + F - T$, $\pi_2^{CRF0} = (1-x)\big[\frac{(a-2r)^2}{(3-x)^2} - F\big] + T$ and $\pi_3^{CRF0} = 0$. Moreover, $\frac{\partial \pi^{CRF0}}{\partial r} > 0$.

[17] Derivative of the expression of joint profit turns out to be $a - 2r$ which is positive for $\frac{a}{3} \leq c < \frac{a}{2}$.

3.2 Profit comparison and determination of optimal outcome

3.2.1 No cross-holding

Let us first consider, the case when firms ignore cross-holding i.e. $x = 0$.

For RF, when cost is relatively low, i.e. $c < \frac{a}{3}$, then to entail joint profit maximization firms would set $r = 0$. The joint profit in this case would be the same as in equation (4), i.e. under fixed-fee licensing π^F, moreover $\pi^F > \pi^{NCNL}$ (see Proposition 1). This idea is similar to Fosfuri and Roca (2004), as they discuss technology transfer with three firms but no PPO. They show that for the low-cost difference, the fixed fee is better than two-part tariff licensing. In Fosfuri and Roca (2004) fixed fee dominates royalty because licensing through a fixed fee generates a negative pecuniary externality on the third incumbent, and the licensor has an incentive to make the licensee more aggressive in the product market because in so doing it will steal the market share from the other competitor. This is also the same in our model for $c < \frac{a}{3}$, and if good 3 is produced post-licensing, then to entail joint profit maximization firms would set $r = 0$.

On the other hand, for $\frac{a}{3} \leq c < \frac{a}{2}$, firms would set the optimal royalty rate $r^* = 3c - a$ $(< c)$ as π^{RF0} increases in royalty rate. The joint profit in this case would be given by $\pi^{RF0} = c(a - c)$. At a high-cost difference ($\frac{a}{3} \leq c < \frac{a}{2}$) since the market share is already too high for the efficient firm, it does not have much incentive for stealing market share from the third competitor (which is an inefficient firm). Instead, it focuses on royalty $r^* = 3c - a$ (> 0) to make the licensee less competitive and increase the joint profit of firm 1 and firm 2; and $\pi^{RF0} = c(a - c)$ is higher than π^{F0} and π^{NCNL}.

The above discussion leads us to the following Lemma 1.

Lemma 1 *If $x = 0$,*
a) for $c < \frac{a}{3}$, only fixed fee licensing (F) is better than two-part tariff (RF) licensing and NCNL; and

b) for $\frac{a}{3} \leq c < \frac{a}{2}$, the optimal royalty rate is $r^* = 3c - a$, good 3 is not produced and two-part tariff licensing (RF0) is better than only fixed-fee licensing (F0) and NCNL.

3.2.2 Possibility of cross-holding

Let us assume the other case where $x \geq 0$.

If good 3 is produced or $a + r(1 - x) > c(3 - x)$, then using expression (23) we get $\frac{\partial \pi^{CRF}}{\partial r} < 0$. Thus, when good 3 is produced, firms would choose fixed fee licensing and $r = 0$ for the same reason as stated before. The joint profit in this case would be the same as in equation (4), π^{CF} given $a > c(3 - x)$. Moreover, in such situations, optimal $x = 0$ as stated in Lemma 8. Thus, if $c < \frac{a}{3}$, optimal $x = 0$, and firms choose only fixed-fee licensing.

Lemma 2 *If $c < \frac{a}{3}$, optimal $x = 0$, then only fixed-fee licensing (F) is the grand optimal.*

If good 3 is not produced or $a + r(1 - x) \leq c(3 - x)$, then using equation (25), we observe that the optimal royalty rate should be set as high as possible i.e. $r^* = \frac{c(3-x)-a}{1-x}$ $(< c)$ and

$$\pi^{CRF0} = c(a - c) \tag{26}$$

which is independent of the level of x. Hence, x can be set as zero also. Thus, we can conclude that, given $x = 0$, if $\frac{a}{3} \leq c < \frac{a}{2}$, firms would set the optimal royalty rate $r^* = 3c - a(< c)$. The joint profit in this case would be given by $\pi^{R0} = c(a - c)$ as stated before. Hence, we observe that the purpose of maximizing the joint profit can be ensured with and without PPO under royalty licensing. As $r^* = \frac{c(3-x)-a}{1-x}(< c)$ for $\frac{a}{3-x} \leq c$, we observe that $r^*(> 0)$ is decreasing in x. Thus, if the firms want to increase x, it will decrease r^* and vice-versa. This is because, as in this context firm 3 will not compete, thus the purpose of maximizing the joint profit can be done with and without cross-holding, as in this context a higher royalty rate r^* solves the problem. This idea is similar to Wang (1998) where in the presence of two firms, the low-cost firm licenses the technology to the high-cost firm by setting the royalty rate as high as possible, as by doing so the inter-firm competition is minimized. Moreover, for the same reason as in Wang (1998), in our model too two-part tariff licensing is superior to fixed-fee.

Lemma 3 *If $c \geq \frac{a}{3}$, then two-part tariff licensing (CRF0 or RF0) is the grand optimal both with and without cross-holding such that $0 < r^* = \frac{c(3-x^*)-a}{1-x^*} < c$.*

It is to be noted that as $r^* < c$, firm 1 can charge a fixed-fee too. As mentioned in Section 2.1, here also we assume that the firms determine the level of royalty rate (r), fixed-fee (F) and the monetary transfer (T) from firm 1 to firm 2, while deciding the contract that involves cross-holding (x^*), using Nash-bargaining. As r, F and T do not have direct implications on the joint profits and thus on the main results of the paper as stated in Proposition 2, we discuss the determination of r, F and T in Appendix C.

Now using Lemmas 1-3 we conclude the following:

Proposition 2 *a) If $c < \frac{a}{3}$, then optimal level of cross holding $x^* = 0$ and the joint profit is $\pi^F = \frac{2(a+c)^2}{16}$. Firms would decide only to license technology via fixed-fee but no cross-holding (F).*

b) If $\frac{a}{3} \leq c < \frac{a}{2}$, then optimal level of cross holding x^* can be anything in $[0, 0.5]$ and the joint profit is $\pi^{CRF0} = \pi^{RF0} = (a - c)c$. Firms would decide to license their technology via two-part tariff ($0 < r^* = \frac{c(3-x^*)-a}{1-x^*} < c$ & $F > 0$) and there may be cross-holding (CRF0 or RF0).

In our work, the outside firm (firm 3) may free-ride and cause negative externalities on the firms that are involved in determining the level of cross-holding (this is akin to Ghosh and Morita (2017)). If the level of PPO (x) increases the degree of competition between firms 1 and 2 reduces. This effect works in the direction of increasing the joint profit of firm 1 and firm 2. However, weaker competition between firms 1 and 2 induces firm 3 to produce more as the quantities are strategic substitutes under quantity competition, and this effect works in the direction of decreasing the joint profit of firm 1 and firm 2. The latter effect dominates the former and hence (only) PPO is never possible. For a similar reason for $c < \frac{a}{3}$, both cross-holding and licensing, CF and CRF, are never optimal, given the third firm is active in the market. Hence, firm 1 and firm 2, would decide only to license technology via fixed fee (F) when the cost difference is low $c < \frac{a}{3}$, as royalty also dampens the joint profit for a similar economic reason. Thus, it is similar to a standard patent licensing problem. Fixed fee dominates two-part tariff because licensing through a fixed fee generates a negative pecuniary externality on the third incumbent thus to entail joint profit maximization firms would set $r = 0$ (see Fosfuri and Roca (2004)). On the other hand, the firms would decide to license their technology via two-part tariff and there may be cross-holding when the cost difference is high $\frac{a}{3} \leq c < \frac{a}{2}$. In this situation, both $CRF0$ and $RF0$ are optimal.

Let us consider the possibility of free technology transfer (as in Ghosh and Morita (2017)) for $c < \frac{a}{3}$, where firms would decide only to license technology via fixed-fee but no cross-holding (See Proposition 2a).[18] Suppose firm 1 transfers its technology for free to firm 2, i.e. $F = 0$. Here, the outputs produced by the firms will be the same as discussed under fixed-fee licensing. Moreover, in this context after licensing firm 1 will lose as given $x^* = 0$ (See Proposition 2a), the profit of firm 1 will always reduce as it will now face more competition from firm 2, which gets the better technology from firm 1 for free. Thus, free licensing is never possible.

Leonardos et al. (2021) mentions that technology transfer could be done with positive partial ownership ($x^* > 0$) and zero fixed-fee ($F = 0$). However, this is not possible in our model, due to the presence of the third firm. In our model, for $c < \frac{a}{3}$ post-licensing irrespective of the value of F, the joint profit of firm 1 and firm 2, reduces if x increases. As x increases, the competition between firm 1 and 2 is relaxed and their joint output $(q_1 + q_2)$ falls w.r.t no cross-holding ($x = 0$). This effect results in increasing their joint profit. However, quantities being strategic substitutes, firm 3 increases its output (q_3), thereby reducing the impact of joint output reduction by firms 1 & 2 on their profit. Given the goods are perfect substitute the second effect dominates the first effect and thus $x > 0$ is not optimal. Moreover, if $\frac{a}{3} \leq c < \frac{a}{2}$ such that post-licensing firm 3 can't compete, $x > 0$ and $F = 0$ is also not possible. As then the firms would decide to license their technology via two-part tariff where r^* (where $0 < r^* = \frac{c(3-x^*)-a}{1-x^*} < c$) and there may be cross-holding (see Proposition 2b).

Proposition 3 *Free licensing of technology from a firm that produces at a lower unit cost to another firm that produces at a higher unit cost is never possible, even if the efficient firm can cross-hold the inefficient firm.*

4 Welfare analysis

In this section, we discuss the implications of the cross-holding and licensing on the consumer surplus (CS), industry profit (Π_I) and welfare ($W = CS + \Pi_I$). Here, we discuss the main results and the entire calculations are relegated to Appendix D. We observe in Appendix D that post-licensing (Case F) as well as when both cross-holding and licensig takes place (Cases CF0 and CRF0) industry output always increases. The final results as discussed in Appendix D is stated below in the following proposition.

Proposition 4 *a) If $c < \frac{a}{3}$, firms would decide only to license technology via fixed-fee (but no cross-holding) and post-licensing consumer surplus and welfare will increase.*

[18] It is to be noted that (See Proposition 2) if $\frac{a}{3} \leq c < \frac{a}{2}$, then optimal level of cross holding x^* can be anything in $[0, 0.5]$ and the firms would decide to license their technology via royalty ($r^* = \frac{c(3-x^*)-a}{1-x^*} (> 0)$). Thus, free-licensing is not possible.

b) If $\frac{a}{3} \leq c < \frac{a}{2}$, then firms would decide to license their technology via two-part tariff (there may be cross-holding) and post-licensing consumer surplus and welfare will increase.

5 Product differentiation: Model with fixed fee licensing

In this section we explore the possibilities of fixed-fee licensing and/or cross-holding when the goods are horizontally differentiated.[19]

Consider a market characterized by Cournot competition among three firms. All three firms produce a differentiated good and the output produced by firm i is given by q_i. The preference of the consumer is represented by the following utility function, as in Singh and Vives (1984),

$$U(q_1, q_2, q_3, I) = aq_1 + aq_2 + aq_3 - \frac{1}{2}[q_1^2 + q_2^2 + q_3^2 + 2\gamma q_1 q_2 + 2\gamma q_1 q_3 + 2\gamma q_2 q_3] + I, \qquad (27)$$

where $\gamma \in (0, 1)$ is an inverse measure of horizontal product differentiation, q_i represents the quantity consumed by the individual, P_i is the price charged by firm i and I represents the income spent on the composite good, whose price is normalized to one. The consumer maximizes the utility subject to the budget constraint $\sum p_i q_i + I \leq M$, where M is the income. Thus the inverse demand function for good i is given by $P_i = a - q_i - \gamma q_j - \gamma q_k$, $i = 1, 2, 3$, $i \neq j \neq k$.

Firm 1 can own a share in firm 2, i.e. a share (x) of the firm 2's profit is transferred to firm 1 and $x \in [0, \frac{1}{2}]$ as discussed before in Section 2. Finally, as before, we assume firm 2 and 3 to have identical per unit costs given by a positive constant c $(a > c > 0)$ and firm 1, which is the efficient firm can produce at a relatively lower per unit cost c_1. For simplicity let us assume $c_1 = 0$. The game proceeds in the same manner as presented in section 2.

5.1 Joint profit of Firm 1 and Firm 2 as determined by PPO and licensing pattern.

We start our analysis, by presenting the joint profits and output of the firms as per the cases presented in Table 1 in Section 2.1.

5.1.1 Case 1: No cross-holding & No licensing (NCNL)

The joint profit of firms 1 and 2 when there is no cross-holding or no technology licensing is

$$\pi^{NCNL} = \pi_1^{NCNL} + \pi_2^{NCNL} = \left[\frac{a(2-\gamma) + 2\gamma c}{4 + 2\gamma(1-\gamma)}\right]^2 + \left[\frac{a(2-\gamma) - 2c}{4 + 2\gamma(1-\gamma)}\right]^2, \qquad (28)$$

where $\pi_1^{NCNL} = \left[\frac{a(2-\gamma)+2\gamma c}{4+2\gamma(1-\gamma)}\right]^2$ and $\pi_2^{NCNL} = \left[\frac{a(2-\gamma)-2c}{4+2\gamma(1-\gamma)}\right]^2$ are the profits of firm 1 and firm 2 respectively. Firm 3 earns a profit $\pi_3^{NCNL} = \left[\frac{a(2-\gamma)-2c}{4+2\gamma(1-\gamma)}\right]^2$. The outputs produced are $q_1^{NCNL} = \frac{a(2-\gamma)+2\gamma c}{4+2\gamma(1-\gamma)}$ and $q_2^{NCNL} = q_3^{NCNL} = \frac{a(2-\gamma)-2c}{4+2\gamma(1-\gamma)}$. Throughout the paper we assume that $a(2-\gamma) > 2c$ such that $q_2^{NCNL} = q_3^{NCNL} > 0$ & $\pi_2^{NCNL} = \pi_3^{NCNL} > 0$.

[19] We thank the anonymous referee for this suggestion.

5.1.2 Case 2: Fixed-fee licensing and good 3 is produced (F)

If firm 1 decides to license its technology via a fixed fee, and x is set to zero, i.e. no cross-holding, while the third firm continues to produce. Then, we have

$$q_1^F = \frac{a(2-\gamma)+\gamma c}{4+2\gamma(1-\gamma)} = q_2^F \quad \text{and} \quad q_3^F = \frac{a(2-\gamma)-c(2+\gamma)}{4+2\gamma(1-\gamma)}. \tag{29}$$

In this case, the joint profit of firm 1 and firm 2 is

$$\pi^F = \pi_1^F + \pi_2^F = 2\left[\frac{a(2-\gamma)+\gamma c}{4+2\gamma(1-\gamma)}\right]^2, \tag{30}$$

where $\pi_1^F = \left[\frac{a(2-\gamma)+\gamma c}{4+2\gamma(1-\gamma)}\right]^2 + F$, $\pi_2^F = \left[\frac{a(2-\gamma)+\gamma c}{4+2\gamma(1-\gamma)}\right]^2 - F$ and $\pi_3^F = \left[\frac{a(2-\gamma)-c(2+\gamma)}{4+2\gamma(1-\gamma)}\right]^2$. The corresponding parametric restriction to this case is $a(2-\gamma) > c(2+\gamma)$ such that $\pi_3^F > 0$.

5.1.3 Case 3: Fixed-fee licensing, good 3 is not produced (F0)

Consider the previous case (Case 2), but firm 3 does not produce post-licensing. Then the outputs of firm 1 and firm 2 are

$$q_1^{F0} = \frac{a}{2+\gamma} = q_2^{F0} \tag{31}$$

and the joint profit of firm 1 and firm 2 is

$$\pi^{F0} = \pi_1^{F0} + \pi_2^{F0} = 2\left[\frac{a}{2+\gamma}\right]^2, \tag{32}$$

where $\pi_1^{F0} = \left[\frac{a}{2+\gamma}\right]^2 + F$, $\pi_2^{F0} = \left[\frac{a}{2+\gamma}\right]^2 - F$ and $\pi_3^{F0} = 0$. Since this is derived assuming that firm 3 does not continue to produce post-licensing, the corresponding parametric restriction is $a(2-\gamma) \leq c(2+\gamma)$.[20]

5.1.4 Case 4: Only cross-holding (C)

Suppose firms 1 and 2 do not go for technology licensing, but only cross-holding is selected (Case C). Then the profit functions of the three firms without any technology licensing would be given by

$$\pi_1 = p_1 q_1 + x(p_2 - c)q_2 - T \tag{33}$$

$$\pi_2 = (1-x)(p_2 - c)q_2 + T \quad \text{and} \tag{34}$$

$$\pi_3 = (p_3 - c)q_3. \tag{35}$$

Profit maximization entails the following optimal values of quantities for the three firms respectively $q_1^C = \frac{a[2-\gamma(1+x)]+\gamma(2+x)c}{2(2+\gamma)-\gamma^2(2+x)}$, $q_2^C = q_3^C = \frac{a(2-\gamma)-2c}{2(2+\gamma)-\gamma^2(2+x)}(>0)$. The joint profit of firm 1 and firm 2 is given

[20] The parametric restriction implying $q_3^{F0} = 0$ is not exogenous, but is endogenously determined from $q_3^F > 0$.

by

$$\pi^C = \pi_1^C + \pi_2^C = \frac{\Big[a[2-\gamma+\gamma x(1-\gamma)] + c\gamma(2-x)\Big]\Big[a[2-\gamma(1+x)] + \gamma(2+x)c\Big]}{\big[2(2+\gamma) - \gamma^2(2+x)\big]^2} + \left[\frac{a(2-\gamma)-2c}{2(2+\gamma)-\gamma^2(2+x)}\right]^2 \tag{36}$$

where $\pi_1^C = \frac{\big[a[2-\gamma+\gamma x(1-\gamma)]+c\gamma(2-x)\big]\big[a[2-\gamma(1+x)]+\gamma(2+x)c\big]}{\big[2(2+\gamma)-\gamma^2(2+x)\big]^2} + x\big[\frac{a(2-\gamma)-2c}{2(2+\gamma)-\gamma^2(2+x)}\big]^2 - T$,
$\pi_2^C = (1-x)\big[\frac{a(2-\gamma)-2c}{2(2+\gamma)-\gamma^2(2+x)}\big]^2 + T$ and $\pi_3^C = \big[\frac{a(2-\gamma)-2c}{2(2+\gamma)-\gamma^2(2+x)}\big]^2$.

5.1.5 Case 5: Cross-holding & fixed fee licensing, good 3 is produced (CF)

Suppose now that in stage 1 firms 1 and 2 agrees on cross holding and fixed-fee licensing and assume post-licensing firm 3 is able to compete. The profit functions of the three firms would be,

$$\pi_1 = p_1 q_1 + x(p_2 q_2 - F) + F - T \tag{37}$$

$$\pi_2 = (1-x)(p_2 q_2 - F) + T \quad \text{and} \tag{38}$$

$$\pi_3 = (p_3 - c)q_3. \tag{39}$$

Maximizing these profit functions, we obtain the optimum output for the three firms 1, 2, and 3 respectively as

$$q_1^{CF} = \frac{[2-\gamma(1+x)][a(2-\gamma)+c\gamma]}{(2-\gamma)[2(2+\gamma)-\gamma^2(2+x)]},\ q_2^{CF} = \frac{(2-\gamma)[a(2-\gamma)+c\gamma]}{(2-\gamma)[2(2+\gamma)-\gamma^2(2+x)]} \ \textit{and} \ q_3^{CF} = \frac{a(2-\gamma)^2 - c[4-\gamma^2(1+x)]}{(2-\gamma)[2(2+\gamma)-\gamma^2(2+x)]}(>0). \tag{40}$$

It is to be noted that $q_3^{CF} > 0$ only if $a(2-\gamma)^2 > c[4-\gamma^2(1+x)]$, which means that the initial unit cost of firm 2 and firm 3 (c) should be significantly less. The joint profit of firm 1 and firm 2 in this context is given by

$$\pi^{CF} = \pi_1^{CF} + \pi_2^{CF} = \frac{\Big([2-\gamma+x\gamma(1-\gamma)]\big[2-\gamma(1+x)\big] + (2-\gamma)^2\Big)\big[a(2-\gamma)+c\gamma\big]^2}{(2-\gamma)^2\big[2(2+\gamma)-\gamma^2(2+x)\big]^2} \tag{41}$$

and is decreasing (increasing) in x when γ is high (low), where
$\pi_1^{CF} = \frac{[2-\gamma+x\gamma(1-\gamma)][2-\gamma(1+x)]\big[a(2-\gamma)+c\gamma\big]^2}{(2-\gamma)^2[2(2+\gamma)-\gamma^2(2+x)]^2} + x\left[\left(\frac{(2-\gamma)[a(2-\gamma)+c\gamma]}{(2-\gamma)[2(2+\gamma)-\gamma^2(2+x)]}\right)^2 - F\right] + F - T$,
$\pi_2^{CF} = (1-x)\left[\left(\frac{(2-\gamma)[a(2-\gamma)+c\gamma]}{(2-\gamma)[2(2+\gamma)-\gamma^2(2+x)]}\right)^2 - F\right] + T$ and $\pi_3^{CF} = \left[\frac{a(2-\gamma)^2-c[4-\gamma^2(1+x)]}{(2-\gamma)[2(2+\gamma)-\gamma^2(2+x)]}\right]^2$.
The parametric restriction here is $a(2-\gamma) > \frac{c[4-\gamma^2(1+x)]}{(2-\gamma)}$.

5.1.6 Case 6: Cross-holding & fixed fee licensing, good 3 is not produced (CF0)

Suppose now that in stage 1 firms 1 and 2 agree on cross holding and fixed-fee licensing and assume post-licensing firm 3 is not able to compete. Proceeding as in the previous case, given $q_3^{CF0} = 0$, the outputs of firm 1 and firm 2 are

$$q_1^{CF0} = \frac{a[2-\gamma(1+x)]}{4-\gamma^2(1+x)} \quad \text{and} \quad q_2^{CF0} = \frac{a(2-\gamma)}{4-\gamma^2(1+x)} \tag{42}$$

and the joint profit of firm 1 and firm 2 is given by

$$\pi^{CF0} = \pi_1^{CF0} + \pi_2^{CF0} = \frac{a^2\left[2-\gamma+\gamma x(1-\gamma)\right]\left[2-\gamma(1+x)\right]}{\left[4-\gamma^2(1+x)\right]^2} + \left[\frac{a(2-\gamma)}{4-\gamma^2(1+x)}\right]^2 \tag{43}$$

which is increasing in x, where $\pi_1^{CF0} = \frac{\left(a[2-\gamma+\gamma x(1-\gamma)]\right)\left(a[2-\gamma(1+x)]\right)}{\left[4-\gamma^2(1+x)\right]^2} + x\left[\left(\frac{a(2-\gamma)}{4-\gamma^2(1+x)}\right)^2 - F\right] + F - T$, $\pi_2^{CF0} = (1-x)\left[\left(\frac{a(2-\gamma)}{4-\gamma^2(1+x)}\right)^2 - F\right] + T$ and $\pi_3^{CF0} = 0$. Since this is derived assuming that firm 3 does not continue to produce its output post-licensing, the corresponding parametric restriction is $a(2-\gamma) \leq \frac{c[4-\gamma^2(1+x)]}{(2-\gamma)}$.

5.2 Profit comparison and determination of optimal outcome

In this section, we compare the profits across the afore-mentioned cases. This comparison provides us with the optimal choice for firms 1 & 2 that maximizes their joint profit. The sequence of the analysis is as follows: First we compare the cases NCNL and C, followed by the comparison of F or F0 and CF or CF0; and finally we discuss the optimal choice.[21]

For better presentation and comprehension of the findings, we make use of diagrams that help us in deriving feasible ranges for parameters for each of the cases. We measure c in the x-axis and γ in the y-axis, and without loss of any generality we set the demand parameter $a = 1$. We also restrict to the zone such that $a(2-\gamma) > 2c$ or $\pi_2^{NCNL} = \pi_3^{NCNL} > 0$. Thus, the zone situated on the right hand side of the downward sloping black line in Figures 2, 3 and 4 is ignored.[22]

5.2.1 NCNL vs C

With reference to Figure 2, the zone marked as $NCNL$ represents the region where $\pi^{NCNL} > \pi^C$ and thus $NCNL$ dominates C. In this region for any given level of cost (c), the product differentiation is low (γ is high). As x increases, the competition between firm 1 and 2 is relaxed[23] and their joint output ($q_1 + q_2$) falls w.r.t no cross-holding ($x = 0$). This effect (we call it the positive effect) results in increasing their joint profit. However, quantities being strategic substitutes, firm 3 increases its output (q_3), thereby reducing the impact of joint output reduction by firms 1 & 2

[21]It is to be noted that the distinction between F or FO and CF or CFO is not exogenous or arbitrary but is determined from the optimal choice of variables i.e. F & x.

[22]In the ignored region all three firms cannot compete under NCNL.

[23]This has been well established in the literature

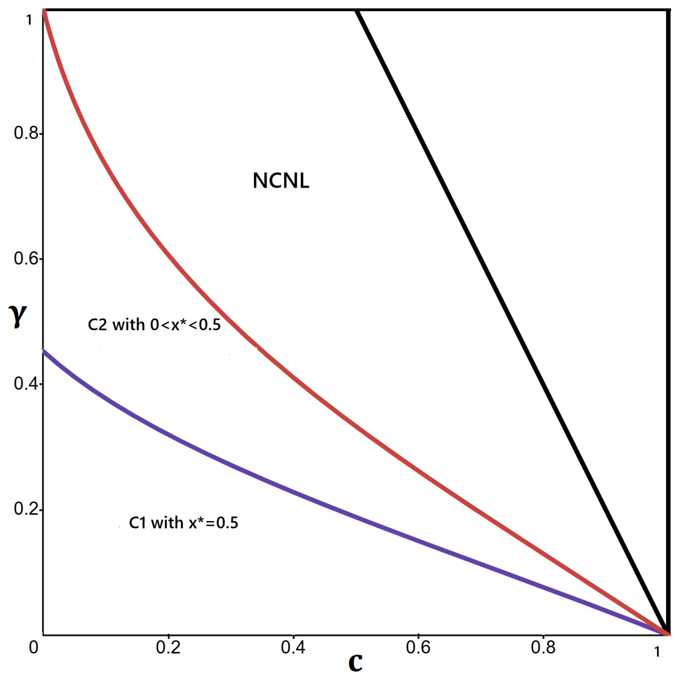

Figure 2: NCNL & C Comparison

on their profit (we call it the negative effect). Since goods are close substitutes (γ is high) in zone $NCNL$ of Figure 2, the business stealing effect by firm 3 from firms 1 & 2 is stronger and reduces the joint profit of firm 1 and 2, thereby rendering cross-holding as sub-optimal w.r.t No cross-holding. Since in this zone the negative effect outweighs the positive effect therefore, $\pi^C < \pi^{NCNL}$, thus only cross-holding (C) is inferior to no cross-holding and no-licensing ($NCNL$).

Cross-holding (C) dominates ($NCNL$) in two zones as shown in Figure 2. In zone $C1$ of Figure 2, the maximum PPO holding i.e. $x = 0.5$ maximizes the joint profit of firm 1 & 2.[24] In zone $C2$ of Figure 2

$$x^* = \frac{2\big[a(2-\gamma)-2c\big]\big[a(2-\gamma)(1-\gamma)-2c(1+\gamma^2)\big]}{2\big[2(2+\gamma)-2\gamma^2\big](a-c)\big[a(1-\gamma)-c\big]+a\gamma^2\big[a(2-\gamma)+2\gamma c\big]} \in (0, 0.5) \tag{44}$$

maximizes the joint profit of firm 1 & 2 under case C.

In the former zone $C1$ of Figure 2, for all ranges of c, product differentiation is extremely high (γ is very low). High product differentiation entails increased market power for producer of each brand, thereby weakening the business stealing (negative) effect imposed by firm 3. Also, due to high product differentiation positive effect of reduced joint output by firms 1 & 2 dominates. Thus in this zone, despite increase in output by firm 3, joint profit of firm 1 and 2 with cross- holding alone (C) is greater than the joint profit under NCNL. Thus firm 1 has an incentive to take the highest possible PPO (x) from firm 2. Hence, C with $x^* = 0.5$ dominates $NCNL$ in zone $C1$ of Figure 2.

[24]The 0.5 comes from the assumption that $x \in [0, 0.5]$.

In the zone $C2$ of Figure 2, the product differentiation is moderate (γ neither very high, nor very low), thus the business stealing effect (negative effect) becomes relatively stronger w.r.t to zone $C1$, whereas the positive effect of reduced joint output weakens. This creates an incentive for firm 1 to hold just as much PPO in firm 2 that will offset the negative business stealing effect by firm 3 at the margin. The case C with $x \in (0, 0.5)$ as stated in equation (44) at all ranges of product differentiation in this zone is high enough to ensure for firm 1 substantial share in profits of firm 2, yet not high enough to severely relax the competition between firm 1 and 2 and resultant stronger retaliation by firm 3. Hence, in zone $C2$ of Figure 2, C with $x \in (0, 0.5)$ as stated in equation (44), dominates $NCNL$.

Lemma 4 *With reference to Figure 2, $NCNL$ dominates C in zone $NCNL$ where γ is very high. Otherwise, in zone $C2$, $x \in (0, 0.5)$ (as in equation (44)), as well as in zone $C1$ with $x = 0.5$; C dominates $NCNL$.*

5.2.2 F or F0 vs CF or CF0

We now compare the joint profit of firm 1 & 2 where firm 1 engages only in technology sharing with a fixed fee in exchange and no PPO (Case F *or* $F0$) with the other scenario wherein firm 1 engages in both i.e. fixed fee licensing and cross-holding (Case CF *or* $CF0$).

Maximizing the joint profit of firm 1 and 2, i.e. π^{CF} and π^{CF0}, w.r.t x we find that optimal $x > 0$ for all values of c and γ. This suggests that when $\gamma \in (0, 1)$ so that the products are differentiated, if firm 1 is licensing via fixed fee, then it is always better to cross-hold than to only license via fixed-fee. With fixed fee licensing, the licensor and licensee become at par in terms of costs, intensifying the competition between them. Thus, to safeguard itself from intense competition, it is better for the licensor to also cross-hold in the licensee firm. This can be seen as a cushion for the licensor against the fall in the joint profit due to intense competition. With this we eliminate only Fixed fee licensing (F *or* $F0$) as reasonable option for firm 1, since it is strictly dominated by Cross-holding along with Fixed fee licensing (CF *or* $CF0$).

Lemma 5 *Cross-holding along with Fixed fee licensing (CF or $CF0$) always dominates only Fixed fee licensing (F or $F0$).*

Thus, we now restrict our analysis to CF *or* $CF0$. First we discuss Case CF represented by zone $C1F$ and $C2F$ in Figure 3, wherein despite cross-holding and licensing, firm 3 continues to compete in the market ($q_3^{CF} > 0$). Subsequently, we discuss the Case $CF0$ represented by zone $C1F0$ and $C2F0$ in Figure 3, where after cross-holding and licensing, firm 3 can't compete in the market ($q_3^{CF} = 0$).

In zone $C1F$ of Figure 3, the optimal cross-holding (x^*) is maximum i.e. 0.5.[25] This zone is characterized by high product differentiation (γ is low), indicating low business stealing by firm 3 (negative effect) and higher positive effect emerging from joint output reduction by firms 1 & 2. Since, the positive effect of fall in joint output outweighs the negative effect of business stealing, it provides the scope for firm 1 to increase its cross-holding (x) in firm 2 to maximum possible level

[25] This comes from the joint profit maximization exercise and also from assumption $x \in [0, \frac{1}{2}]$.

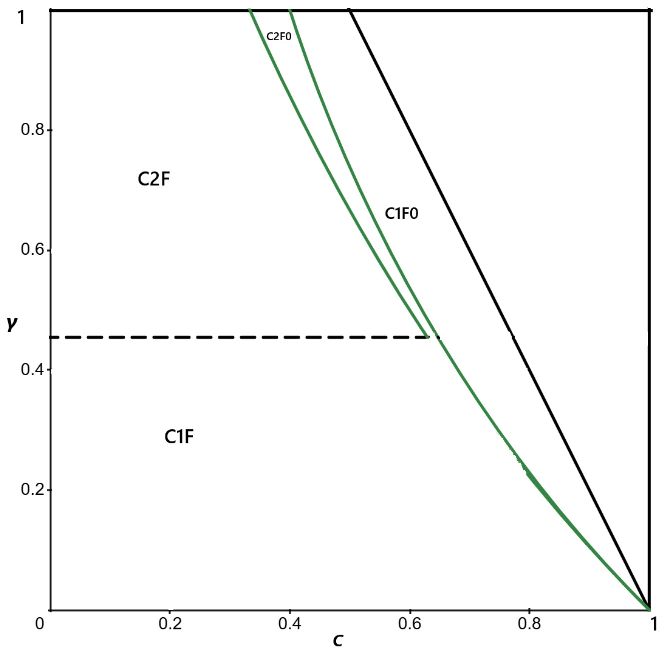

Figure 3: CF and CF0 Comparison

of 0.5.[26]

In zone $C2F$ of Figure 3, product differentiation is less, thus business stealing (negative) effect is relatively high, but not too high to offset the positive effect of joint output reduction on the total profit of firm 1 & 2. Thus, to reduce this negative effect, firm 1 keeps cross-holding to a level that is not severely anti-competitive. This way it prevents a stronger retaliation by firm 3, in terms of business stealing. Therefore, although firm 1 cross-holds but in this case

$$x = \frac{2(1-\gamma)(2-\gamma)}{4-3\gamma^2} \in (0, 0.5) \tag{45}$$

maximizes the joint profit of firm 1 & 2 under case CF.

As in $C1F$, the zone $C1F0$ of Figure 3 is characterized by highest level of cross-holding i.e. $x = 0.5$. In this zone, for any range of γ and given $x = 0.5$, c is high. Due to high cost, firm 3 is unable to compete against two efficient firms 1 & 2, i.e. $q_3 > 0$. Since, there is no presence of firm 3, firm 1 & 2 maximize their joint profits and firm 1 takes highest possible cross-holding ($x = 0.5$).

Lastly, we have the zone $C2F0$ in Figure 3, where

$$x = \frac{(4-\gamma^2)c - a(2-\gamma)^2}{c\gamma} \in (0, 0.5) \tag{46}$$

maximizes the joint profit of firm 1 & 2 under $CF0$. Here, costs of firm 3 are relatively lesser than zone $C1F0$ in Figure 3. Therefore, under $C2F0$, optimal x is lesser than 0.5, so that competition is high and firm 3 is forced to leave the market.

[26]We observe that $(q_1 + q_2)$ is a decreasing function of x and q_3 increases in x.

Lemma 6 *Using Figure 3, we observe that*

i) in zone $C1F$ where $x = 0.5$ and in zone $C2F$ where $x \in (0, 0.5)$ as in equation (45); CF dominates only fixed-fee licensing, however,

ii) in zone $C1F0$ where $x = 0.5$ and in zone $C2F0$ where $x \in (0, 0.5)$ as in equation (46); $CF0$ dominates only fixed-fee licensing.

5.2.3 Final results

In this section we discuss the final results using Lemmas 4 - 6. In the Figure 4 we sketch out the various zones and derive the optimal choices in the respective zones using Lemmas 4 - 6, where Figure 4 is derived by superimposing Figure 2 on Figure 3.

Before, arriving at the optimal choices that maximizes the joint profit of firm 1 and firm 2, we describe the zones of Figure 4 as follows:

Zone $A1$: In this zone firm 1 & 2 can either chose C with $x = 0.5$ or CF with $x = 0.5$.

Zone $A2$: Firm 1 can either chose CF with $x = 0.5$ or C with $x \in (0, 0.5)$ as stated in equation (44).

Zone $A3$: Firm 1 can chose between CF with $x = 0.5$ and $NCNL$.

Zone $A4$: Firm 1 has an option of CF with $x \in (0, 0.5)$ as as stated in equation (45) or C with $x \in (0, 0.5)$ as stated in equation (44).

Zone $A5$: Firm 1 can chose between CF with $x \in (0, 0.5)$ as stated in equation (45) and $NCNL$.

Zone $A6$: Firm 1 can chose between $CF0$ with $x \in (0, 0.5)$ as stated in equation (46) and $NCNL$.

Zone $A7$ **and** $A8$: Firm 1 can chose between $CF0$ with $x = 0.5$ or $NCNL$.

After comparing the joint profit across different zones we observe the following results. In zones $A1$, $A2$ & $A3$ of Figure 4, CF with $x^* = 0.5$ is the optimal choice. As discussed before, due to the high differentiation of products in these zones, firm 1 has an incentive to take the highest possible cross-holding. Moreover, due to higher product differentiation, it is also optimal to license the technology to firm 2, as the competition in the market does not increase too much in these zones post-licensing. Post-licensing firm 3 competes in these zones ($q_3^{CF} > 0$). However, in zones $A4$ & $A5$ of Figure 4, CF with $x^* \in (0, 0.5)$ as in equation (45) is the optimal choice. Due to relatively less product differentiation, the cross holding (x^*) is lesser than 0.5, thus, these zones experience intense competition among firms.

In zone $A6$ of Figure 4, since firm 3 does not operate due to high costs this results in relaxing the competition between firm 1 & 2. However, to prevent any business stealing by firm 3 (so that

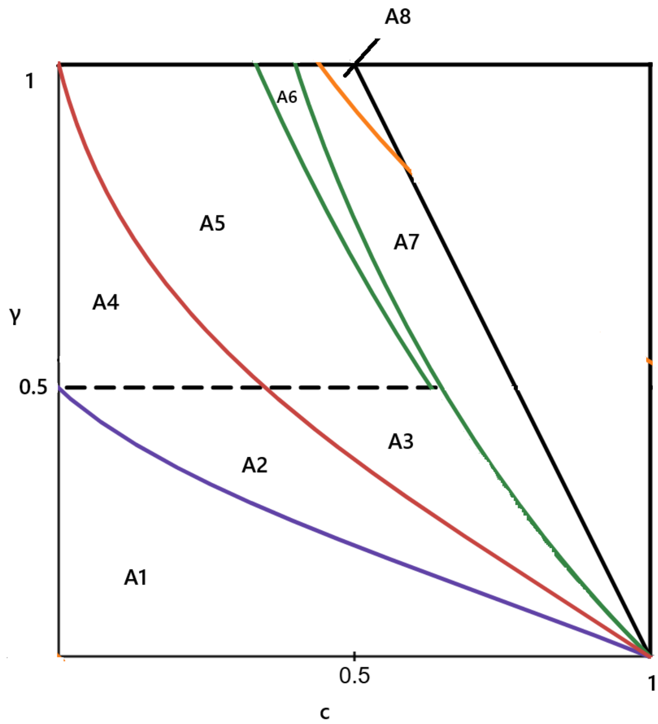

Figure 4: Final Comparison

$q_3^{CF0} = 0$), as goods are close substitutes, firm 1 adopts $CF0$ with $x^* \in (0, 0.5)$. In zone $A7$ of Figure 4 however, it is optimal for firm 1 to adopt $CF0$ with $x^* = 0.5$. This is because, costs being higher than in zone $A6$, firm 3 leaves the market even when $x^* = 0.5$, making it a duopoly thus giving leverage to firm 1 to take highest possible cross-holding.

Lastly, zone $A8$ of Figure 4 is characterized by high costs (high c for firm 2 and firm 3 in the no licensing scenario) and low product differentiation. Here, the sharing of technology by firm 1 would imply both (firm 1 & 2) being at par as close competitors in the post-licensing stage. This, irrespective of the level of cross-holding, intensifies competition in the market even though only firm 1 and firm 2 compete, than in $NCNL$ where both firm 2 and firm 3 operates. Thus, sharing of technology and cross-holding $CF0$ would render less profits w.r.t. NCNL. Thus, NCNL is the optimal choice.

Lastly, we can simplify the results in the following lemma.[27]

Lemma 7 *i) CF is the optimal choice in zones $A1$, $A2$, $A3$, $A4$ & $A5$ of Figure 4.*
ii) $CF0$ is the optimal choice in zones $A6$ & $A7$ of Figure 4.
iii) $NCNL$ is the optimal choice in zone $A8$ of Figure 4.

With high product differentiation (γ very low) and low unit-cost (c) as in zones $A1$, $A2$ & $A3$ of Figure 4, it is optimal for firm 1 to share the technology with firm 2 via fixed fee licensing and also

[27]It is to be noted that within CF & $CF0$ zones, there are varying levels of x^*, depending on the value of γ and c.

to take the highest possible cross-holding CF of $x^* = 0.5$. Otherwise, when product differentiation is moderate to low and low unit-cost (c) as in zones $A4$ & $A5$ of Figure 4, the licensor firm adopts both fixed fee and cross-holding as the optimal strategy however, the cross-holding is less than 0.5. When the cost difference between firms is high (high c), firm 3 is unable to compete in the post-licensing stage, firm 1 can optimally choose $CF0$ with $x^* = 0.5$. This happens in zone $A7$ of Figure 4. With cost differences relatively lower (than in zone $A7$) and product differentiation relatively low as in zone $A6$ of Figure 4, to keep firm 3 out of the market firm 1 should adopt $CF0$ with $x^* \in (0, 0.5)$, such that sufficiently intense competition between firm 1 & 2 drives the firm 3 out of market. Interestingly, with high cost differences and very low product differentiation as in zone $A8$ of Figure 4, it is optimal for firm 1 to neither share technology via fixed fee nor cross-hold, i.e. $NCNL$.

Thus with this, we arrive at the following propositions.

Proposition 5 *i) With high cost differences and very low product differentiation, it is optimal for firm 1 to neither share technology via fixed fee nor cross-hold ($NCNL$).*

ii) Otherwise, cross holding and fixed-fee licensing ($CF/CF0$) is always optimal.

6 Product differentiation: Model with two-part tariff licensing

In this section, we explore the possibility of a two-part tariff licensing as discussed in Section 3.

6.1 Joint profit of Firm 1 and Firm 2 as determined by PPO and licensing pattern.

We start our analysis, by presenting the joint profits and output of the firms as per the cases presented in Table 2 as in Section 3.1. As discussed before in Section 5, here we avoid stating the profits for only cross-holding (C) and no-cross holding and no-licensing (NCNL).

6.1.1 Licensing via Royalty & Fixed Fee, good 3 is produced(RF)

Assuming that all three firms produce, the outputs of the firms are;

$$q_1^{RF} = \frac{a(2-\gamma)+\gamma(c+r)}{2(2+\gamma-\gamma^2)},\ q_2^{RF} = \frac{a(2-\gamma)+\gamma c - r(2+\gamma)}{2(2+\gamma-\gamma^2)}\ \ and\ \ q_3^{RF} = \frac{a(2-\gamma)-c(2+\gamma)+\gamma r}{2(2+\gamma-\gamma^2)} \tag{47}$$

and the joint profit of the firms would be

$$\pi^{RF} = \pi_1^{RF} + \pi_2^{RF} = \left[\frac{a(2-\gamma)+\gamma(c+r)}{2(2+\gamma-\gamma^2)}\right]^2 + r\left[\frac{a(2-\gamma)+\gamma c - r(2+\gamma)}{2(2+\gamma-\gamma^2)}\right] + \left[\frac{a(2-\gamma)+\gamma c - r(2+\gamma)}{2(2+\gamma-\gamma^2)}\right]^2 \tag{48}$$

where, $\pi_1^{RF} = \left[\frac{a(2-\gamma)+\gamma(c+r)}{2(2+\gamma-\gamma^2)}\right]^2 + r\left[\frac{a(2-\gamma)+\gamma c-r(2+\gamma)}{2(2+\gamma-\gamma^2)}\right] + F$, $\pi_2^{RF} = \left[\frac{a(2-\gamma)+\gamma c-r(2+\gamma)}{2(2+\gamma-\gamma^2)}\right]^2 - F$ and $\pi_3^{RF} = \left[\frac{a(2-\gamma)-c(2+\gamma)+\gamma r}{2(2+\gamma-\gamma^2)}\right]^2$. When cost is relatively low, which implies firm 3 continues to produce, i.e. $c < \frac{a(2-\gamma)+\gamma r}{2+\gamma}$, then to entail joint profit maximization firms would set r as

$$r^*_{RF} = \frac{\big[a(2-\gamma)+\gamma c\big](2\gamma-2\gamma^2)}{8+12\gamma-8\gamma^2-4\gamma^3} \quad or \quad c \text{ whichever is lower.} \tag{49}$$

6.1.2 Licensing via Royalty & Fixed Fee, good 3 is not produced(RF0)

On the other hand, when $c \geq \frac{a(2-\gamma)+\gamma r}{2+\gamma}$ and good 3 is not produced, the outputs are

$$q_1^{RF0} = \frac{a(2-\gamma)+\gamma r}{4-\gamma^2} \quad and \quad q_2^{RF0} = \frac{a(2-\gamma)-2r}{4-\gamma^2}, \tag{50}$$

and joint profit is

$$\pi^{RF0} = \pi_1^{RF0} + \pi_2^{RF0} = \left[\frac{a(2-\gamma)+\gamma r}{4-\gamma^2}\right]^2 + r\left[\frac{a(2-\gamma)-2r}{4-\gamma^2}\right] + \left[\frac{a(2-\gamma)-2r}{4-\gamma^2}\right]^2 \tag{51}$$

where $\pi_1^{RF0} = \left[\frac{a(2-\gamma)+\gamma r}{4-\gamma^2}\right]^2 + r\left[\frac{a(2-\gamma)-2r}{4-\gamma^2}\right] + F$, $\pi_2^{RF0} = \left[\frac{a(2-\gamma)-2r}{4-\gamma^2}\right]^2 - F$ and $\pi_3^{RF0} = 0$.

Maximizing the joint profit w.r.t r, results in optimal royalty to be charged as

$$r^*_{RF0} = \frac{a(2-\gamma)(2\gamma-\gamma^2)}{8-6\gamma^2} \quad or \quad \frac{c(2+\gamma)-a(2-\gamma)}{\gamma}. \tag{52}$$

6.1.3 Cross-holding & Two-part tariff licensing, good 3 is produced(CRF)

Furthermore, firms also have an option to choose two-part licensing along with $x > 0$. Then the outputs of the three firms are

$$q_1^{CRF} = \frac{(2-\gamma)\big[a(2-\gamma)+\gamma(c+r)\big] - \gamma x\big[a(2-\gamma)-2r+\gamma c\big]}{(2-\gamma)\big[2(2+\gamma-\gamma^2)-\gamma^2 x\big]},$$
$$q_2^{CRF} = \frac{a(2-\gamma)^2+\gamma c(2-\gamma)-r(2+\gamma)(2-\gamma)}{(2-\gamma)\big[2(2+\gamma-\gamma^2)-\gamma^2 x\big]} \quad and$$
$$q_3^{CRF} = \frac{a(2-\gamma)^2+\gamma r(2-\gamma)-c(2+\gamma)(2-\gamma)+\gamma^2 x(c-r)}{(2-\gamma)\big[2(2+\gamma-\gamma^2)-\gamma^2 x\big]} \tag{53}$$

and the joint profit if good 3 is produced would be given by

$$\pi^{CRF} = \pi_1^{CRF} + \pi_2^{CRF} = P_1^{CRF} q_1^{CRF} + \big(q_2^{CRF}\big)^2 + r q_2^{CRF} \tag{54}$$

where $\pi_1^{CRF} = P_1^{CRF} q_1^{CRF} + x\big[(q_2^{CRF})^2 - F\big] + r q_2^{CRF} + F - T$, $\pi_2^{CRF} = (1-x)\big[(q_2^{CRF})^2 - F\big] + T$ and $\pi_3^{CRF} = [q_3^{CRF}]^2$. Moreover, $P_1^{CRF} = \frac{a(2-\gamma)^2+\gamma(c+r)(2-\gamma)+\gamma x\Big((1-\gamma)\big(a(2-\gamma)+\gamma c\big)-r(2-\gamma^2)\Big)}{(2-\gamma)\Big(2(2-\gamma)(1+\gamma)-\gamma^2 x\Big)}$. If, good 3 is produced then the royalty rate should satisfy the condition such that $\frac{a(2-\gamma)^2+\gamma r(2-\gamma)-\gamma^2 xr}{(2+\gamma)(2-\gamma)-\gamma^2 x} > c$.

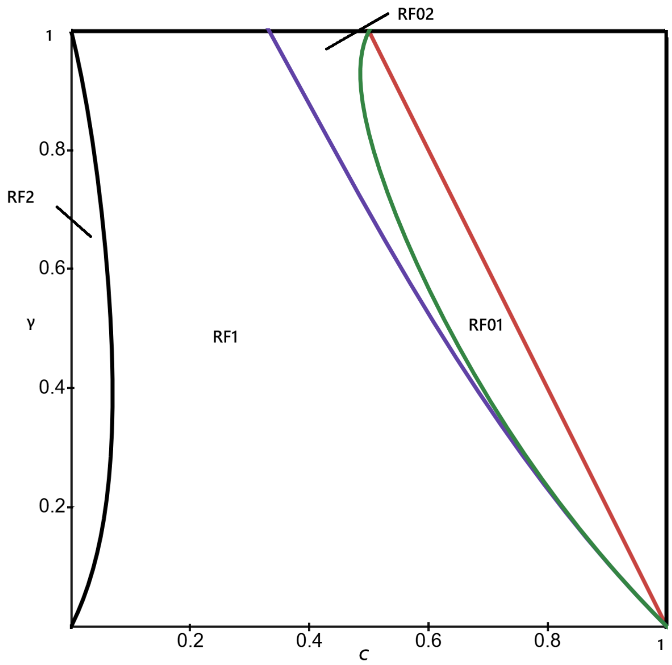

Figure 5: RF & RF0 Comparison

6.1.4 Cross-holding & Two-part tariff licensing, good 3 is not produced(CRF0)

If firm 3 doesn't compete (${q_3}^{CRF0} = 0$) such that $\frac{a(2-\gamma)^2+\gamma r(2-\gamma)-\gamma^2 xr}{(2+\gamma)(2-\gamma)-\gamma^2 x} \leq c$, then the outputs of firm 1 and firm 2 are

$$q_1^{CRF0} = \frac{a(2-\gamma) - x\gamma(a-r) + r\gamma}{4-\gamma^2(1+x)} \quad and \quad q_2^{CRF0} = \frac{a(2-\gamma) - 2r}{4-\gamma^2(1+x)} \tag{55}$$

and the joint profit is

$$\pi^{CRF0} = \pi_1^{CRF0} + \pi_2^{CRF0} = P_1^{CRF0} q_1^{CRF0} + \left(q_2^{CRF0}\right)^2 + rq_2^{CRF0} \tag{56}$$

where $\pi_1^{CRF0} = P_1^{CRF0} q_1^{CRF0} + x\left[(q_2^{CRF0})^2 - F\right] + rq_2^{CRF0} + F - T$ and $\pi_2^{CRF0} = (1-x)\left[(q_2^{CRF0})^2 - F\right] + T$ and ${P_1}^{CRF0} = \frac{a(2-\gamma)+x\gamma(a-r-a\gamma)+r\gamma}{4-\gamma^2(1+x)}$.

6.2 Profit comparison and determination of optimal outcome

6.2.1 Preliminary analysis

Using Figure 5, we identify regions feasible for RF & $RF0$.

The RF region is divided in $RF1$ and $RF2$. In the former i.e. $RF1$, the optimal royalty is $r^*_{RF} = \frac{[a(2-\gamma)+\gamma c](2\gamma-2\gamma^2)}{8+12\gamma-8\gamma^2-4\gamma^3}$ ($< c$). However, in the region $RF2$, due to the low cost of firm 2, the licensor is implicitly faced with a upper royalty limit, and thus $r^* = c$.

The region $RF0$, is also divided into $RF01$ and $RF02$. This division is on the basis of royalty that can keep firm 3 out of market. In $RF01$, the optimal royalty charged is $r^*_{RF01} = \frac{a(2-\gamma)(2\gamma-\gamma^2)}{8-6\gamma^2}$. However, in the region $RF02$, in order to keep the firm 3 out of market, the licensor firm has to increase the extent of competition in the market. Since, the royalty (r^*) is implicitly the effective cost for firm 2, lowering it means firm 3 facing intense competition and thus leaving the market. Thus, lowering the royalty further to $r^*_{RF02} = \frac{c(2+\gamma)-a(2-\gamma)}{\gamma}$, satisfies the condition $c \geq \frac{a(2-\gamma)+\gamma r}{2+\gamma}$.[28]

6.2.2 Final analysis: optimal outcome

As solving this problem is quite challenging, we have taken different values of c and γ and have found the optimal values of r^* and x^* numerically. Based on these values of r^* and x^*, we arrive at Figure 6, Figure 7 and Figure 8, where we show different cases (CRF and CRF0).

i) Figure 6, shows the feasible region of CRF where $r^* \geq 0$ as well as $x^* \geq 0$.

ii) Figure 7, shows the case CRF where $r^* = c$ and $x^* > 0$.

iii) Figure 8, demonstrates the case $CRF0$, i.e. region with $r^* \geq 0$ as well as $x^* \geq 0$.

iv) Figure 9, demonstrates the case $CRF0$, i.e. region with $r^* \geq 0$ and $x^* = 0.5$.

The pattern of these zones as depicted in Figures 6 - 9, are similar to what has been observed in Figure 5, which depicts RF and RF0.

We observe that for cases CRF and $CRF0$, $r^* > 0$ and $x^* > 0$. This implies that $RF/RF0$ as well as $CF/CF0$ are dominated by $CRF/CRF0$. Thus, $RF/RF0$ as well as $CF/CF0$ can be ignored as it is not optimal. It is to be noted that if $r^* < c$, firm 1 can charge a fixed-fee too. Moreover, if $r^* = c$, then also firm 1 can levy a fixed-fee on firm 2, as in the presence of cross-holding ($x^* > 0$), the output of firm 2 as well as the profit from selling good 2 as a whole increase.

This leaves us with two possible strategies $CRF/CRF0$ and $NCNL$.[29] We further observe that $\pi^{NCNL} < \pi^{CRF}$ as well as $\pi^{NCNL} < \pi^{CRF0}$. Therefore, CRF or $CRF0$ emerges as the (grand) optimal choice for joint profit maximization. Thus, we have the following proposition.

Proposition 6 *Cross holding and two-part licensing (CRF or $CF0$) will always happen,*

i) If the cost difference is low, then good 3 will be produced post cross-holding and two-part licensing (CRF).

ii) If the cost difference is high, then good 3 will not be produced post cross-holding and two-part licensing (CRF0).

[28]Satisfying this condition implies $q_3 = 0$ as well as $r^*_{RF01} \geq r^*_{RF02}$.

[29]See zone $A8$ of Figure 4, where $NCNL$ is the optimal strategy and also see Proposition 5.

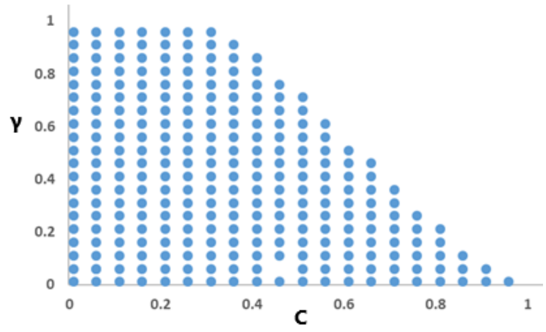

Figure 6: CRF

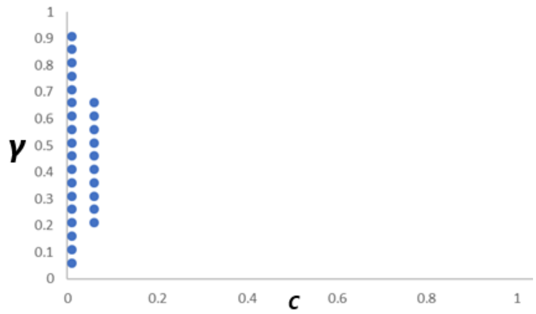

Figure 7: CRF with $r^* = c$

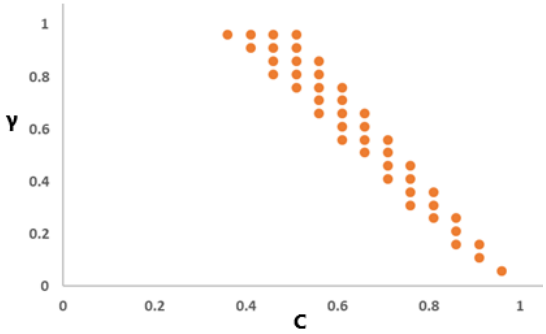

Figure 8: CRF0

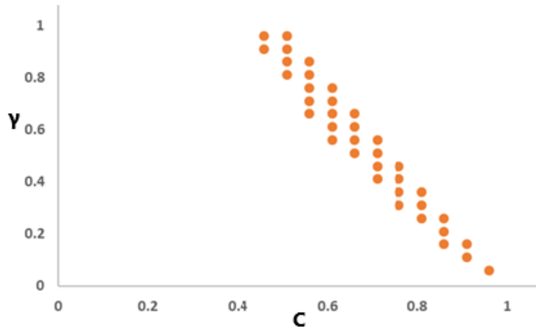

Figure 9: CRF0 with $x^* = 0.5$

6.3 Welfare Analysis

In this section, we discuss the impact of cross-holding and licensing on consumer surplus, industry profit and welfare as discussed in Section 4. Using equation (27), consumer surplus is given by

$$CS = \frac{1}{2}[(q_1)^2 + (q_2)^2 + (q_3)^2 + 2\gamma q_1 q_2 + 2\gamma q_1 q_3 + 2\gamma q_2 q_3]. \tag{57}$$

The industry profit is given by $\Pi_I = \sum_i \pi_i$[30] whereas welfare is the sum of consumer surplus and industry profit i.e., $W = CS + IP$. We now present the welfare analysis with respect to two cases i.e. CRF & $CRFO$ vis-a-vis $NCNL$. We restrict to these two cases (CRF & $CRFO$) because these are the grand optimal outcomes as discussed before (see Proposition 6).

With reference to Figure 10 & 11, the highlighted regions represent the regions where $CS^{CRF} > CS^{NCNL}$ and $CS^{NCNL} > CS^{CRF}$ respectively. From Figure 11, we see that at lower levels of cost differences (c is low), for any level of product differentiation, consumer surplus is higher in $NCNL$ than in CRF. This zone largely coincides with zone wherein $r^* = c$ (see Figure 7). This implies that as optimal royalty rate (r^*) approaches its upper limit (as it approaches c), the gains accruing to firm 2 due to technology licensing tend to be minimal or negligible. This results in dominance of anti-competitive effects of cross-holding over pro-competitive effects of technology licensing, thereby reducing consumer surplus in case of CRF. As this zone is marked with low cost differences, firms

[30]Here i denotes the number of active firms in the market.

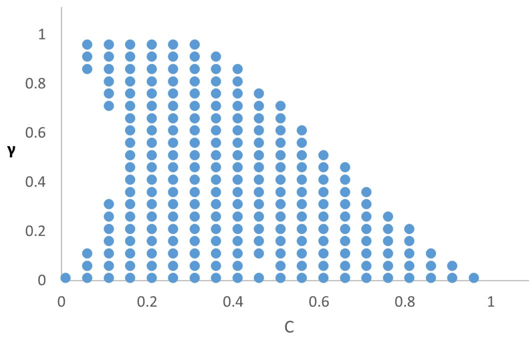

Figure 10: $CS^{CRF} > CS^{NCNL}$

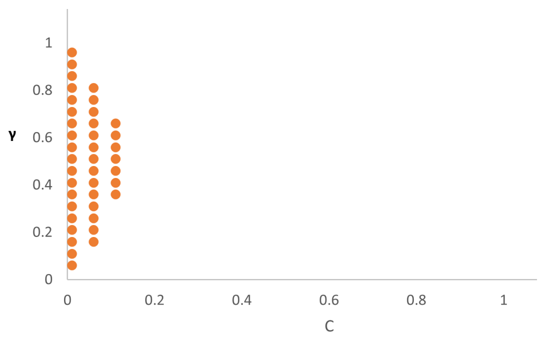

Figure 11: $CS^{NCNL} > CS^{CRF}$

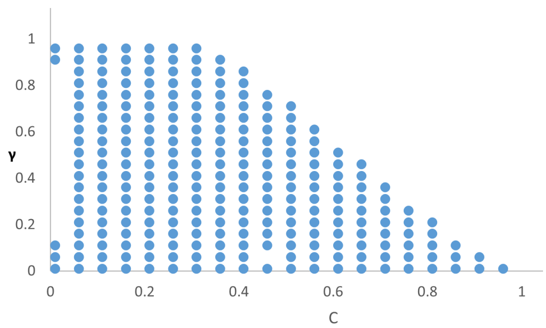

Figure 12: $W^{CRF} > W^{NCNL}$

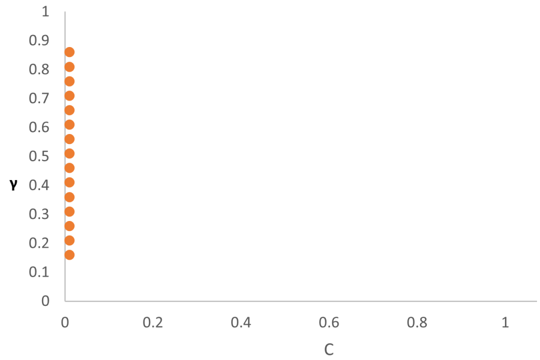

Figure 13: $W^{NCNL} > W^{CRF}$

are at par competing intensively with each other in $NCNL$. Thereby making the benchmark case $NCNL$ yield greater consumer surplus in this zone than in CRF.

On the other hand, with relatively higher cost differences, as seen from shaded regions of Figure 10, at all levels of product differentiation, CRF yields higher consumer surplus. This zone[31] is marked with $r^* > 0$, but r^* much lesser than c. Here, the pro-competitive effects of technology licensing dominate the anti-competitive effects of cross-holding, leading to a higher consumer surplus in CRF. Additionally, with high cost differences, competition is softened creating incentives for firm 1 to share its technology as well as to cross-hold in firm 2. The interplay of technology licensing and cross-holding along with product differentiation creates a surplus for consumers through greater output.

With respect to welfare, we observe a similar pattern as that of consumer surplus (see Figure 12 and Figure 13). Despite industry profits always being higher in CRF than in $NCNL$, we see a shrink in the zone representing $W^{NCNL} > W^{CRF}$ with respect to zone representing $CS^{NCNL} > CS^{CRF}$ (See Figures 11 & 13). This shrink is attributed to the fall in consumer surplus which dominates the increase in industry profits.

It is also to be noted that consumer surplus as well as industry profits are always higher in

[31] Also, see Figure 6.

$CRF0$ than in $NCNL$. Thus, welfare in $CRF0$ is always greater than in $NCNL$.

Proposition 7 *At lower levels of cost differences,* $NCNL$ *yields higher consumer surplus as well as total welfare than* $CRF/CRF0$*. Otherwise, cross-holing along with licensing via a two-part tariff always leads to an increase in consumer surplus and total welfare.*

7 Conclusion

We construct a model with three firms, and one of them is relatively efficient. The efficient firm can have cross holdings in any one of the inefficient firms and also license its technology. Both these instruments are adopted to maximize joint profit but post production they compete in terms of output. It is shown that the cost difference plays a crucial role in determining the pattern of licensing (if at all) and the level of PPO. When the goods are homogeneous, for lower cost differences firms would choose fixed fee licensing with no PPO and for higher costs they will license technology through royalty but would be indifferent to PPO. Thus, we may expect PPO to exist only if the option of royalty (or two-part tariff) licensing is not available, or firms may prefer a fixed fee because observing the output of the licensee may be difficult. We also show that free knowledge transfer is never going to take place.

We also study the impact of technology licensing and PPO on the consumer surplus and welfare. It is shown that the consumer surplus and welfare always increase, not only when the firms only select technology licensing as a tool, but also when they have agreements that involve cross-holding as well as technology licensing.

The present model is also extended by incorporating horizontal product differentiation. In the context of fixed-fee licensing and cross-holding we observe that with high cost differences and very low product differentiation, it is optimal for the licensor to neither share technology via fixed fee nor cross-hold. This is similar to what we observe when the goods are homogeneous. Interestingly, we observe that cross holding and fixed-fee licensing is optimal, when the cost difference is relatively less irrespective of the degree of product differentiation. We also discuss about the two-part tariff licensing and cross-holding in the same model of horizontal product differentiation. We show that cross holding and two-part licensing will always happen and it is the grand optimal choice. Moreover, if the cost difference is low, then good 3 will be produced post cross-holding and two-part licensing. On the other hand, if the cost difference is high, then good 3 will not be produced post cross-holding and two-part licensing. We observe that with low cost differences among firms, technology licensing via a two part tariff along with cross-holding adversely affects the consumer surplus and total welfare. However, with high cost differences, it increases the consumer surplus and total welfare.

The present paper offers new insights into the literature on cross-holding and technology licensing. However, we believe that it can be further extended by incorporating price competition within the same framework, as well as by introducing vertical product differentiation.

Appendix

A Proof of Proposition 1

We observe the following Lemma from the discussion in the main text:

Lemma 8 *a) For $c < \frac{a}{3}$, optimal $x = 0$, and firms choose only licensing.*
b) For $\frac{a}{3} \leq c < \frac{a}{2.5}$, $x = \frac{3c-a}{c} (< 0.5)$ maximizes π^{CF0}, and $\pi^{CF0} = (a-c)c > \pi^{F0} = \frac{2a^2}{9}$.[32]
c) For $\frac{a}{2.5} \leq c < \frac{a\sqrt{7}}{6}$, $x = 0.5$ maximizes π^{CF0}, and $\pi^{CF0} = \frac{a^2(1.5)}{(2.5)^2} > \pi^{F0} = \frac{2a^2}{9}$.
d) For $\frac{a\sqrt{7}}{6} \leq c < \frac{a\sqrt{23}}{10}$, $x = 0.5$ maximizes π^{CF0}, and $\pi^{CF0} = \frac{a^2 1.5}{(2.5)^2} > \pi^{NCNL} = \frac{(a+2c)^2}{16} + \frac{(a-2c)^2}{16}$.
e) For $\frac{a\sqrt{23}}{10} \leq c < \frac{a}{2}$, $x = 0$ maximizes π^{NCNL}, as $\pi^{CF0} = \frac{a^2 1.5}{(2.5)^2} \leq \pi^{NCNL} = \frac{(a+2c)^2}{16} + \frac{(a-2c)^2}{16}$.

Proof of Proposition 1.

Proof. a) When $c < \frac{a}{3}$, from equations (4) and (1), joint profit is higher for Case F than Case $NCNL$. Considering Case CF, and since π^{CF} is decreasing in x, we conclude that the joint profit of Case CF for $x^* = 0$ is equal to that of Case F. Finally, since $c < \frac{a}{3}$, Case $F0$ does not arise. This proves a of Proposition 1.

b) If $\frac{a}{3} \leq c < \frac{a}{2.5}$, then from Lemma 1, $\pi^{CF0} = (a-c)c > \pi^{F0}$. Also, $\pi^{F0} > \pi^{NCNL}$. Cases $NCNL$ and $F0$ are thus ruled out, while case F is not applicable as here we are considering $\frac{a}{3} \leq c$. This proves b of Proposition 1.

c) When $\frac{a}{2.5} \leq c < \frac{a\sqrt{7}}{6}$, Case CF is not possible as discussed before. For Case $CF0$, profits are $\frac{1.5a^2}{(2.5)^2}$. From Lemma 1(c), Case $CF0$ dominates Case $F0$. Also Case $F0$ dominates Case $NCNL$. Case F is ruled out as it doesn't satisfy the parametric restriction under consideration. Thus, Case $CF0$ will be the equilibrium when $c \in [\frac{a}{2.5}, \frac{a\sqrt{7}}{6})$.

For $\frac{a\sqrt{7}}{6} \leq c < \frac{a\sqrt{23}}{10}$, the proof is similar to above, the only difference being here Case $NCNL$ dominates Case $F0$, but then from Lemma 1(d) Case $CF0$ dominates Case $NCNL$. Thus, Case $CF0$ will be the equilibrium when $c \in [\frac{a\sqrt{7}}{6}, \frac{a\sqrt{23}}{10})$.

This proves c of Proposition 1.

d) When $\frac{a\sqrt{23}}{10} \leq c < \frac{a}{2}$, Case $NCNL$ dominates Case $F0$ and Case F is ruled out as it doesn't satisfy the parametric restriction under consideration. From Lemma 1(e), Case $NCNL$ dominates Case $CF0$. This proves d of Proposition 1. ■

[32] Here x is very close but greater than $\frac{3c-a}{c}$ and π^{CF} is very close but lower than $(a-c)c$ as discussed before.

B Appendix : Determination of Fixed-fee and Money transfer for cross-holding

In the main text (See Proposition 1 as well as Lemma 8) we argue about the possibilities of fixed-fee licensing as well as about cross-holding and thereby determine x^* (the level of cross-holding). In doing so we maximize the joint profit of firm 1 and firm 2, which is independent of the level of fixed-fee (F) and the amount of money transfer from firm 1 to firm 2 (T) for acquiring a stake of x^* in firm 2. Thus, here in the appendix, we determine the fixed-fee F^* and the amount of money to be transferred T^* respectively in the following sections, as doing so in the main paper looks a little bit clumsy. Once the joint profit is maximized firm 1 and firm 2 can bargain about the distribution of the total profit using F and T. Further, to keep this analysis more general we have used Nash-bargaining to determine F^* and T^*, where we denote the bargaining power of firm 1 and firm 2 by $\alpha(>0)$ and $\beta(>0)$ respectively such that $\alpha+\beta=1$.

B.1 For Proposition 1a

We observe that if $c<\frac{a}{3}$, then as discussed in Proposition 1a only fixed-fee licensing will happen and then the profits of the firms (See Case F of Section 2.1) will be

$$\pi_1^F=\frac{(a+c)^2}{16}+F \quad and \quad \pi_2^F=\frac{(a+c)^2}{16}-F. \tag{58}$$

Moreover, $\pi_1^{NCNL}=\frac{(a+2c)^2}{16}$ and $\pi_2^{NCNL}=\frac{(a-2c)^2}{16}$. Thus, using Nash-bargaining, we can determine the fixed-fee F^* by solving

$$\max_F \ \left[\pi_1^F-\pi_1^{NCNL}\right]^{\alpha}\left[\pi_2^F-\pi_2^{NCNL}\right]^{\beta} \tag{59}$$

given $\pi_1^F-\pi_1^{NCNL}>0$ and $\pi_2^F-\pi_2^{NCNL}>0$. Moreover, after solving equation (59) we get

$$F^*=\alpha\left[\frac{(a+c)^2}{16}-\frac{(a-2c)^2}{16}\right]-\beta\left[\frac{(a+c)^2}{16}-\frac{(a+2c)^2}{16}\right]. \tag{60}$$

For example, let $\alpha=\beta=0.5$, then $F^*=\frac{(a+2c)^2}{32}-\frac{(a-2c)^2}{32}(>0)$, hence $\pi_1^F=\frac{(a+c)^2}{16}+\frac{(a+2c)^2}{32}-\frac{(a-2c)^2}{32}>\pi_1^{NCNL}$ and $\pi_2^F=\frac{(a+c)^2}{16}-\frac{(a+2c)^2}{32}+\frac{(a-2c)^2}{32}>\pi_2^{NCNL}$.

B.2 For Proposition 1b

Let us now assume that $\frac{a}{3}\leq c<\frac{2a}{5}$. Then according to Proposition 1b, both cross-holding and fixed-fee licensing will occur (CF0) and $x^*=\frac{3c-a}{c}(<0.5)$. Then the profits of the firms will be (See Case CF0 of Section 2.1 and Lemma 8)

$$\pi_1^{CF0}=\frac{a^2(1-x^*)}{(3-x^*)^2}+x^*\left[\frac{a^2}{(3-x^*)^2}-F\right]+F-T=c^2+\frac{(a-2c)F}{c}-T \tag{61}$$

and

$$\pi_2^{CF0}=(1-x^*)\left[\frac{a^2}{(3-x^*)^2}-F\right]+T=c(a-2c)-\frac{(a-2c)F}{c}+T. \tag{62}$$

Thus, using Nash-bargaining, we can determine the fixed-fee F^* and the monetary transfer T^* by solving

$$\max_{F,T} \left[\pi_1^{CF0} - \pi_1^{NCNL}\right]^\alpha \left[\pi_2^{CF0} - \pi_2^{NCNL}\right]^\beta \tag{63}$$

given $\pi_1^{CF0} - \pi_1^{NCNL} > 0$ and $\pi_2^{CF0} - \pi_2^{NCNL} > 0$. Moreover, after solving equation (63) we get

$$\alpha\left[c(a-2c) - \frac{(a-2c)F^*}{c} + T^* - \frac{(a-2c)^2}{16}\right] = \beta\left[c^2 + \frac{(a-2c)F^*}{c} - T^* - \frac{(a+2c)^2}{16}\right] \tag{64}$$

or

$$\frac{(a-2c)F^*}{c} - T^* = \alpha\left[c(a-2c) - \frac{(a-2c)^2}{16}\right] - \beta\left[c^2 - \frac{(a+2c)^2}{16}\right], \tag{65}$$

where $\frac{(a-2c)F^*}{c} - T^*$ is the net payment from firm 2 to firm 1.

For example, let $\alpha = \beta = 0.5$, then the net payment from firm 2 to firm 1 is $\frac{(a-2c)F^*}{c} - T^* = \frac{3c(a-2c)}{4}(> 0)$, hence $\pi_1^{CF0} = c^2 + \frac{3c(a-2c)}{4} > \pi_1^{NCNL}$ and $\pi_2^{CF0} = c(a-2c) - \frac{3c(a-2c)}{4} > \pi_2^{NCNL}$.

B.3 For Proposition 1*c*

Now assume that $\frac{a}{2.5} \leq c < \frac{a\sqrt{23}}{10}$, then from Proposition 1*c*, we argue that both cross-holding (such that $x^* = 0.5$) and fixed-fee licensing will occur, but good 3 will not be produced (CF0). Then the profits of the firms will be (See Case CF0 of Section 2.1 and Lemma 8)

$$\pi_1^{CF0} = \frac{a^2(0.5)}{(2.5)^2} + 0.5\left[\frac{a^2}{(2.5)^2} - F\right] + F - T = \frac{a^2}{(2.5)^2} + 0.5F - T \tag{66}$$

and

$$\pi_2^{CF0} = (0.5)\left[\frac{a^2}{(2.5)^2} - F\right] + T = \frac{0.5a^2}{(2.5)^2} - 0.5F + T. \tag{67}$$

Thus, using Nash-bargaining, we can determine the fixed-fee F^* and the monetary transfer T^* by solving

$$\max_{F,T} \left[\pi_1^{CF0} - \pi_1^{NCNL}\right]^\alpha \left[\pi_2^{CF0} - \pi_2^{NCNL}\right]^\beta \tag{68}$$

given $\pi_1^{CF0} - \pi_1^{NCNL} > 0$ and $\pi_2^{CF0} - \pi_2^{NCNL} > 0$. Moreover, after solving equation (68) we get

$$\alpha\left[\frac{0.5a^2}{(2.5)^2} - 0.5F^* + T^* - \frac{(a-2c)^2}{16}\right] = \beta\left[\frac{a^2}{(2.5)^2} + 0.5F^* - T^* - \frac{(a+2c)^2}{16}\right] \tag{69}$$

or

$$0.5F^* - T^* = \alpha\left[\frac{0.5a^2}{(2.5)^2} - \frac{(a-2c)^2}{16}\right] - \beta\left[\frac{a^2}{(2.5)^2} - \frac{(a+2c)^2}{16}\right], \tag{70}$$

where $0.5F^* - T^*$ is the net payment from firm 2 to firm 1.

For example, let $\alpha = \beta = 0.5$, then the net payment from firm 2 to firm 1 is $0.5F^* - T^* = \frac{1}{2}\left[\frac{ac}{2} - \frac{0.5a^2}{(2.5)^2}\right](> 0)$, hence $\pi_1^{CF0} = \frac{3a^2}{4(2.5)^2} + \frac{ac}{4} > \pi_1^{NCNL}$ and $\pi_2^{CF0} = \frac{3a^2}{4(2.5)^2} - \frac{ac}{4} > \pi_2^{NCNL}$.

C Determination of Royalty rate, Fixed-fee and Money transfer for cross-holding

Now let us consider two-part tariff licensing, which is possible only when $\frac{a}{3} \le c < \frac{a}{2}$. As mentioned in Proposition 2b we observe that if $\frac{a}{3} \le c < \frac{a}{2}$, then optimal level of cross holding x^* can be anything in $[0, 0.5]$. Then the firms would decide to license their technology via royalty ($0 < r = \frac{c(3-x^*)-a}{1-x^*} < c$) and post-licensing good 3 will not be produced. Then the profits of the firms will be (See Section 3)

$$\begin{aligned} \pi_1^{CRF0} &= \frac{\big[a + r(1-x^*)\big]\big[a(1-x^*) + r(1+x^*)\big]}{(3-x^*)^2} + x^*\left[\frac{(a-2r)^2}{(3-x^*)^2} - F\right] + r\frac{a-2r}{3-x^*} + F - T \\ &= c(a-c) - \frac{(a-2c)^2}{(1-x^*)} + (1-x^*)F - T = c(a-c) - z \end{aligned} \quad (71)$$

and

$$\pi_2^{CRF0} = (1-x^*)\left[\frac{(a-2r)^2}{(3-x^*)^2} - F\right] + T = \frac{(a-2c)^2}{(1-x^*)} - (1-x^*)F + T = z, \quad (72)$$

where $z = \frac{(a-2c)^2}{(1-x^*)} - (1-x^*)F + T$. Thus, using Nash-bargaining, we can determine the royalty rate r^*, fixed-fee F^* and the monetary transfer T^* by solving

$$\max_{r>0, F>0, T>0} \left[\pi_1^{CRF0} - \pi_1^{NCNL}\right]^\alpha \left[\pi_2^{CRF0} - \pi_2^{NCNL}\right]^\beta \quad (73)$$

given $\pi_1^{CRF0} - \pi_1^{NCNL} > 0$ and $\pi_2^{CRF0} - \pi_2^{NCNL} > 0$. Moreover, after solving equation (73) we get

$$z^* = \alpha\left[\frac{(a-2c)^2}{16}\right] + \beta\left[c(a-c) - \frac{(a+2c)^2}{16}\right]. \quad (74)$$

For example, let $\alpha = \beta = 0.5$, then $\pi_1^{CRF0} = c(a-c) - z^* = \frac{1}{2}\left[c(a-c) - \frac{(a-2c)^2}{16} + \frac{(a+2c)^2}{16}\right] > \pi_1^{NCNL}$ and $\pi_2^{CRF0} = z^* = \frac{1}{2}\left[c(a-c) + \frac{(a-2c)^2}{16} - \frac{(a+2c)^2}{16}\right] > \pi_2^{NCNL}$.

D Welfare Analysis

In the absence of cross-holding and licensing (initial situation, i.e. NCNL) the industry output is $q_I^{NCNL} = \frac{3a-2c}{4}$ (See Case NCNL of Section 2.1). The consumer surplus, industry profit and the welfare are respectively

$$CS^{NCNL} = \frac{1}{2}\left(\frac{3a-2c}{4}\right)^2, \Pi_I^{NCNL} = \frac{(3a^2 - 4ac + 12c^2)^2}{16} \ \& \ W^{NCNL} = \frac{15a^2 - 20ac + 28c^2}{36}. \quad (75)$$

If $c < \frac{a}{3}$, then as discussed in Proposition 1a only fixed-fee licensing (F) will happen and then the industry output (See Case F of Section 2.1) is $q_I^F = \frac{3a-c}{4} (> q_I^{NCNL})$. Thus, after fixed-fee licensing as the industry output increases as firm 2 uses the efficient technology, the consumer surplus will increase ($CS^F > CS^{NCNL}$). The joint output and profit of firm 1 and firm 2 increases, but the output and profit of firm 3 falls. This is because, the first effect dominates the second effect as industry-wide there is a reduction in the average cost as now firm 2 produces the good at a lower unit cost. The industry profit after fixed-fee licensing is

$$\Pi_I^F = \pi^F + \pi_3^F = \frac{(3a^2 - 2ac + 7c^2)^2}{16} \tag{76}$$

and it increases post-licensing as $\Pi_I^F > \Pi_I^{NCNL}$. Thus welfare will increase ($W^F > W^{NCNL}$) as both consumer surplus and industry profit increase.

Let us now assume that $\frac{a}{3} \leq c < \frac{2a}{5}$. Then according to Proposition 1*b*, both cross-holding and fixed-fee licensing will occur (CF0) and $x^* = \frac{3c-a}{c}(< 0.5)$. Then the industry output will be $q_I^{CF0} = a - c$ (see Case CF0 of Section 2.1 and Lemma 8). It is important to note that even though q_3^{CF0} (and π_3^{CF0}) is zero, industry output increases as $q_I^{CF0} > q_I^{NCNL}$. As discussed in Ghosh and Morita (2017), we also argue that cross-holding itself weakens competition, but it benefits the consumers and improves welfare in our model as licensing also takes place in this context. Thus, in the presence of both cross-holding and fixed-fee licensing as the industry output increases, the consumer surplus will increase ($CS^{CF0} > CS^{NCNL}$). Moreover, the industry profit under cross-holding and fixed-fee licensing is

$$\Pi_I^{CF0} = \pi^{CF0} + \pi_3^{CF0} = (a - c)c \tag{77}$$

and $\Pi_I^{CF0} > \Pi_I^{NCNL}$. Thus welfare will increase ($W^{CF0} > W^{NCNL}$) as both consumer surplus and industry profit increase.

Now assume that $\frac{a}{2.5} \leq c < \frac{a\sqrt{23}}{10}$, then from Proposition 1*c*, we argue that both cross-holding and fixed-fee licensing will occur and $x^* = 0.5$, but good 3 will not be produced (CF0). Then the industry output will be $q_I^{CF0} = \frac{3a}{5}(> q_I^{NCNL})$. As discussed before, here too, in the presence of both cross-holding and fixed-fee licensing as the industry output increases, the consumer surplus will increase ($CS^{CF0} = \frac{9a^2}{50} > CS^{NCNL}$). Moreover, the industry profit under cross-holding and fixed-fee licensing is

$$\Pi_I^{CF0} = \frac{6a^2}{25}. \tag{78}$$

We also observe that $\Pi_I^{CF0} > \Pi_I^{NCNL}$ if $\frac{a}{2.5} \leq c < 0.47936a$ (*approx*) (cost difference is less); otherwise, $\Pi_I^{CF0} < \Pi_I^{NCNL}$. Therefore, industry profit increases only if the cost difference is less. However, welfare will increase always post cross-holding and fixed-fee licensing as $W^{CF0} = \frac{21a^2}{50} > W^{NCNL}$.

Now let us consider two-part tariff licensing, which is possible only when $\frac{a}{3} \leq c < \frac{a}{2}$. As mentioned in Proposition 2*b* we observe that if $\frac{a}{3} \leq c < \frac{a}{2}$, then optimal level of cross holding x^* can be anything in $[0, 0.5]$ and the joint profit is $\pi^{CRF0} = \pi^{RF0} = (a - c)c$. Then the firms would decide to license their technology via two-part tariff and post-licensing good 3 will not be produced. Using equation (24) and substituting r^* we observe that $q_I^{CRF0} = a - c$. Here also, in the presence of both cross-holding and two-part tariff licensing as the industry output increases ($q_I^{CRF0} > q_I^{NCNL}$), the consumer surplus will increase ($CS^{CRF0} > CS^{NCNL}$). Moreover, the industry profit under cross-holding and royalty licensing is $\Pi_I^{CRF0} = (a - c)c$ (See equation (25)). We also observe that $\Pi_I^{CRF0} > \Pi_I^{NCNL}$, therefore, industry profit increases. Thus welfare will increase ($W^{CRF0} > W^{NCNL}$) as both consumer surplus and industry profit increase.

Hence, from the above discussion and we have Proposition 4 in the main text as stated below:

a) If $c < \frac{a}{3}$, firms would decide only to license technology via fixed-fee (but no cross-holding) and post-licensing consumer surplus and welfare will increase.

b) If $\frac{a}{3} \leq c < \frac{a}{2}$, then firms would decide to license their technology via royalty (there may be cross-holding) and post-licensing consumer surplus and welfare will increase.

References

[1] Anton, M., Ederer, F., Gine, M., & Schmalz, M. (2023). "Common Ownership, Competition, and Top Management Incentives", *Journal of Political Economy*, 131, 1294-1355.

[2] Arrow, K.J. (1962). "Economic Welfare and the Allocation of Resources for Invention", In: Nelson, R.R. (Ed.), The Rate and Direction of Inventive Activity. Princeton University Press.

[3] Brousseau, E., Coeurderoy, R., & Chaserant, C. (2007). "The Governance of Contracts: Empirical Evidence on Technology Licensing Agreements", *Journal of Institutional and Theoretical Economics*, 163, 205–235.

[4] Caloghirou, Y., Ioannies, S., & Vonortas, N.S. (2022). "Research Joint Ventures", *Journal of Economic Surveys*, 17, 541–570.

[5] Chen, H., Shieh, S., & Su, P. (2010). "Fees versus Royalties under Partial Cross Ownership", *Economics Bulletin*, 30, 3248-3259.

[6] Foros, O., Kind, H.J., & Shaffer, G. (2011) "Mergers and Partial Ownership" *European Economic Review*, 55, 916–926.

[7] Fosfuri, A., & Roca, E. (2004). "Optimal Licensing Strategy: Royalty or Fixed Fee?" *International Journal of Business and Economics*, 3, 13-19.

[8] Frazzani, S., Noti, K., Schinkel, M. P., Seldeslachts, J., Banal Estanol, A., Boot, N., & Angelici, C. (2020). "Barriers to Competition through Common Ownership by Institutional Investors", *Study for the Committee on Economic and Monetary Affairs, Policy Department for Economic, Scientific and Quality of Life Policies, European Parliament, Luxembourg, 2020.*

[9] Ghosh, A. & Morita, H. (2017) "Knowledge transfer and partial equity ownership" *RAND Journal of Economics*, 48, 1044–1067.

[10] Gilo, D., Moshe, Y., & Speigel, Y. (1992) "Partial Cross Ownership and Tacit Collusion." *RAND Journal of Economics*, 37, 81–99.

[11] Gomes-Casseres, B., Hagedoorn, J., & Jaffe, A.B. (1992) "Do Alliances Promote Knowledge Flows?" *Journal of Financial Economics*, 80, 5–33.

[12] Hariskos, W., Königstein, M. & Papadopoulos, K.G., (2022) "Anti-competitive effects of partial cross-ownership: Experimental evidence," *Journal of Economic Behavior & Organization*, 193, 399-409.

[13] He, J., & Huang, J. (2017). "Product Market Competition in a World of Cross-Ownership: Evidence from Institutional Blockholdings", *The Review of Financial Studies*, 30, 2674–2718.

[14] Hemphill, C.S., & Kahan, M. (2020). "The Strategies of Anticompetitive Common Ownership", *The Yale Law Journal*, 129, 1276-1599.

[15] Leonardos, S., Petrakis, E., Skartados, P., & Stamatopoulos, G. (2021). "Partial passive ownership holdings and licensing", *Economic Letter*, 204, 109910.

[16] Lopez, A.L., & Vives, X. (2019). "Overlapping ownership, R&D spillovers, and antitrust policy", *Journal of Political Econonomy*, 12, 2394–2437.

[17] Malueg, D.A. (1992) "Collusive Behavior and Partial Ownership of Rivals" *International Journal of Industrial Organization*, 10, 27–34.

[18] Marjit, S. (1990) "On a non-cooperative theory of technology transfer" *Economic Letters*, 33, 293–298.

[19] Neubig, T. S., & Wunsch-Vincent, S. (2017). "A missing link in the analysis of global value chains: Cross-border flows of intangible assets, taxation and related measurement implications", *WIPO Economic Research Working Paper No.37*

[20] Ono, H., Nakazato, T., Davis, C., & Andaley, W. (2004) "Partial Ownership Arrangements in the Japanese Automobile Industry: 1990–2000." *Journal of Applied Economics*, 12, 355–367.

[21] Rosati, N., Bomprezzi, P., Ferraresi, M., Frigo, A., & Nardo, M. (2020). "Common Shareholding in Europe, EUR 30312 EN, Publications Office of the European Union", *JRC121476, Luxembourg, 2020, ISBN 978-92-76-20876-1*, doi:10.2760/734264

[22] Rostoker, M.D. (1984). "A Survey of Corporate Licensing", *IDEA: Journal of Law Technolology*, 24, 59–92

[23] Schmalz M.C. (2017) "Common ownership and competition: facts, misconceptions and what to do about it", *Manuscript. Background paper for the 128th meeting of the OECD Competition committee.*

[24] Shelegia S., & Spiegel Y. (2017) "Bertrand competition when firms hold passive ownership stakes in one another", *Economic Letters*, 114, 136-138.

[25] Vishwasrao, S. (2007) "Royalties vs. fees. How do firms pay for foreign technology?" *International Journal of Industrial Organization*, 25, 741–759.

[26] Wang, X.H. (1998) "Fee versus royalty licensing in a Cournot duopoly model" *Economic Letters*, 60, 55–62.