Irreversibility, Lumpy Investment, and the Dynamics of Public Capital and Sovereign Debt

Preliminary draft, comments welcome. *

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Abstract

We build a model of a small developing economy where public capital accumulation faces two investment frictions: capital irreversibility and lumpy investment induced by fixed costs. When the capital stock of the government is low, the government issues defaultable debt primarily to invest, whereas at high capital levels, it borrows mainly for consumption smoothing. We show that for the same debt-to-GDP ratio, the low capital economy is less likely to default, it borrows and invests more, and faces lower spreads. We also show that irreversible capital and lumpiness both individually and together, lower default risk. When compared with a model with convex adjustment costs and reversible investment, our model displays lower spreads, higher borrowing and more investment. We provide evidence from emerging economy data that spreads are lower in periods with high levels of public investment, which is also a feature of our model simulations.

JEL classification: F34, F41, E22, G15, H63.

Keywords: Sovereign default, sovereign spreads, public capital, investment frictions.

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1 Introduction

Developing economy governments use new borrowing (net of debt repayment) on international markets to fund spending on public consumption (bureaucrats, healthcare, education), private consumption (through transfers for example) as well as public investment (roads, dams). The sovereign default literature, building on the quantitative models of Arellano (2008) and Aguiar and Gopinath (2006) has focused on the first two types of spending while largely ignoring public investment.[1] Yet, the IMF notes that ... " Developing countries disproportionately use public borrowing to finance infrastructure projects versus developed economies... " (International Monetary Fund, 2022). In this paper, we extend sovereign default models to allow the government to accumulate public capital, which is an input in local production. This extension seems appropriate because developing economies often lack the domestic resources to undertake large investment projects without substantially reducing consumption levels in the economy. As a result, they often wish to bring in foreign resources through international borrowing. In the presence of default risk, the sovereign default literature has shown that increased indebtedness raises borrowing costs, so governments have to trade off these additional costs against higher future flows of output. While the typical sovereign default model is focused on borrowing to smooth public and private consumption, our model allows the developing economy to borrow to finance the transition to the optimal level of public capital. We show that allowing governments to fund public capital accumulation through international debt has large consequences for debt capacity, borrowing costs, default risk and welfare. The optimal use of fiscal capacity (including borrowing) between consumption and investment is shown to change over the transition as capital is accumulated.

Our Benchmark model incorporates two key features of public infrastructure investment that play an important quantitative role in reducing default risk relative to a model without these features. These features are capital irreversibility and lumpy investment and constitute the main departure of our model from the literature.[2] Irreversibility here refers to the idea that capital, once installed, cannot be converted back into consumption goods. Early work on this idea in the context of a firm can be found in Dixit and Pindyck (1994); Abel, Dixit, Eberly, and Pindyck (1996); and Abel and Eberly (1999) for example. While irreversibility can be partial or complete, in our model, it is captured by the assumption that investment

[1]While almost all models in this literature contemplate private consumption smoothing, some examples with public consumption are Cuadra, Sanchez, and Sapriza (2010) and Arellano and Bai (2017). Public spending is commonly found in models that incorporate a political element, such as Scholl (2017) and Cotoc, Johri, and Sosa-Padilla (2025) for example.

[2]As noted in the World Bank's Public Investment Management Reference Guide, "*... special features of public investment... [are] large and "lumpy" projects. Capital investment projects tend to be large scale and irreversible*" (Kim, Fallov, & Groom, 2020).

in public capital can never be negative and the capital stock of the government can only fall if depreciated assets are not replaced. Lumpiness refers to the idea that investment occurs in infrequent but large spikes as opposed to a more continual and gradual process. Public capital projects, especially infrastructure projects frequently involve substantial fixed costs, which create nonlinearities in investment decisions as discussed in Caballero and Engel (1999) in the context of firms.[3] A key aspect of these models is that agents do not adjust their capital stock continuously, creating a "zone of inaction" where capital is allowed to depreciate. Only when the capital stock hits a lower-bound trigger does the agent find it optimal to pay the fixed cost and make a large, discrete investment. In our model, the government faces a fixed cost if it chooses to increase the capital stock but does not if it invests only to cover depreciation. As a result, there is a region of the state space of the model where the government chooses to grow the public capital stock and a zone where it does not. Investment episodes are associated with low debt and capital and with higher productivity levels. We show that both irreversibility and lumpiness individually lower default risk relative to a sovereign default model with a more typical capital accumulation process with convex adjustment costs, which we call the Standard economy. As a result, the Standard economy borrows less than the Benchmark economy in order to curtail borrowing costs.

In order to understand these results, it is useful to consider how the level of debt and public capital interact with the incentives to default versus repay debt. For a given level of debt, additional public capital raises future output. This has two opposing effects: first, this increases repayment capacity and second, it raises consumption levels in a default induced autarky situation. The former lowers default risk, while the latter raises it. Irreversibility interacts with these two economic forces by limiting the government's ability to consumption smooth through liquidation of existing public capital. The government has a strong incentive to "eat its capital" in states when the marginal utility of consumption is high. These typically happen just before and during defaults when debt burdens are high and productivity is low or both. Lowering the capital stock in these situations not only reduces future ability to repay debt, it raises the value of default by limiting how much consumption falls. Recognizing this dynamic, lenders will assign positive probability to lower capital stocks in the future and higher risk of default when capital is reversible. Capital irreversibility thus effectively acts as a mechanism that prevents the liquidation of capital and lowers the likelihood of default.

Lumpiness also interacts with the forces discussed above. Since the government has to pay a fixed cost whenever it want to increase the public capital stock, there are many areas of the state space (productivity, public capital, and debt) where it would choose not

[3] See also Thomas (2002) and Khan and Thomas (2008).

to invest whereas a government not facing the fixed cost would invest. Especially when the capital stock is relatively low, this results in much more borrowing in the non-lumpy economy, which obviously raises default risk, ceteris paribus. Overall, the impact of the extra debt overwhelms the impact of the extra productive capacity which is more muted due to the opposing effects discussed above. As such, the presence of investment fixed costs in our Benchmark model discipline government borrowing for investment purposes to states in which the return on additional capital is relatively high and this discipline reduces borrowing costs.

In order to study the quantitative importance of these features of investment we calibrate the Benchmark model to Mexican data and compare the results to a model with reversible capital and no fixed costs of investment called the Standard model. Both models feature a small open economy government that issues long-term debt and accumulates a stock of public capital. The Benchmark model is subject to the non convex adjustment costs discussed above while the Standard model features quadratic adjustment costs. The government can default on its debt obligations and this creates a spread between the bond yield and the risk-free rate. The two model economies are calibrated such that the mean public capital share of output and investment share of output are the same in the model simulations. The results suggest that ignoring these key aspects of infrastructure development considerably overstates emerging economy default risk. The combination of higher spreads and lower debt lead to lower welfare in economies without irreversibility and lumpiness.

Initial conditions matter in the Benchmark model. When we calibrate the model to an initial level of public capital that is below its mature, optimal level, we observe a stochastic transition which captures an important feature of developing economy data which is the gradual building up of public infrastructure like ports, bridges, dams and highways. The model transition displays : i). considerable investment close to the low public capital initial state and ii). a more mature economy near the end of the transition where it mainly replaces depreciated capital. Consistent with the discussions in the IMF quote above, when the levels of capital are low, the economy borrows more than when capital is high and primarily uses external funds to invest in public capital. The mature economy primarily uses external funds for consumption smoothing except when productivity and the return on capital is high. When expressing indebtedness relative to repayment capacity (debt-to-GDP ratio), the economy at the beginning of the transition faces spreads that are smaller than those at the end of the transition. This reflects several opposing forces at play in the model. The less mature economy has a need to invest using international borrowing and thus would like to avoid exclusion from debt markets, despite a relatively low capacity to repay using current output. The more mature economy has a greater repayment ability but also a greater comfort with autarky due to its higher domestic production.

The highly state-contingent nature of key government decisions : how much to invest, borrow and whether to default result in an interesting pattern. For any given level of capital, there are combinations of debt and productivity that are consistent with growing the capital stock (ie., investment is greater than depreciated capital). The economy never chooses to default during these investment spikes which occur at lower debt levels than those that trigger default, ceteris paribus. Lenders in the model recognize this pattern and offer lower spreads during these investment episodes, a pattern that is consistent with the data.

In order to quantify the degree of lumpiness in public investment in emerging economies, we follow the firm investment literature (e.g., Cooper & Haltiwanger, 2006; Khan & Thomas, 2008). We identify investment spikes using the distribution of the gross general government investment-to-capital ratio (I/K) for all country-year observations in our emerging economies dataset, which covers 43 countries over 42 years (details in the Appendix). The mean ratio of I/K is approximately 0.076, with a standard deviation of 0.031. We define a *public investment spike* as any observation where $I/K > 0.10$, roughly one standard deviation above the mean. Using this threshold, about 18% of country-year observations qualify as spikes. When considering the appropriateness of this admittedly ad-hoc threshold, keep in mind that some investment must occur each year simply for the upkeep of depreciated public capital.

Table 1: Share of Positive Investment Spikes under Different Cutoff Thresholds

Cutoff Threshold (I/K)	0.075	0.100	0.125	0.150
Positive Spikes (%)	36.99	18.58	7.26	3.25

Notes: Each column reports the share (%) of observations where the public investment-to-capital ratio (I/K) exceeds the specified cutoff. Such instances are classified as *positive investment spikes*. The reported values represent the fraction of spikes among all country-year observations in emerging economies, multiplied by 100.

In order to get a sense of the robustness of the frequency of spikes to variation in the threshold, Table 1 reports the frequency of spikes as the threshold is raised and lowered. The frequency declines sharply as the cutoff increases: from 37% at 0.075 to just 3% at 0.15. The data reveal that a significant fraction of investment occurs in large, infrequent bursts providing support for the presence of lumpy investment costs. The second central feature of the Benchmark model is capital irreversibility which is captured by the assumption that gross investment can never be negative. This finding is confirmed in our emerging economy dataset. Indeed gross investment is almost always sufficient to cover depreciation of capital, meaning the public capital stock does not usually decrease.[4]

We validate our model results empirically by showing that sovereign spreads are lower in periods classified as displaying investment spikes as well as being negatively correlated with the ratio of public investment to GDP. All these results come from empirical models that include typical correlates in spread regressions (see section 3.4).

1.1 Related Literature

Our work builds on the quantitative sovereign default models of Arellano (2008) and Aguiar and Gopinath (2006) with one period debt and the contributions of Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009) that introduce long-term debt to these models. All these models assume the small open economy has an exogenous and random endowment process for the consumption good and international debt is used to smooth private consumption. Cuadra et al. (2010) explore the role of fiscal policy by adding public spending and endogenous production to a model with one period debt but production does not take capital as an input in production. Mendoza and Yue (2012) also allow endogenous production and like us, incorporate a fixed stock of private capital in the production function for calibration purposes. Unlike us the production technology also uses intermediate good imports which act as a mechanism to create recessions in default episodes.[5] Like the vast majority of sovereign default models, none of the above mentioned studies allow the accumulation of physical capital.

Park (2017) and Gordon and Guerrón-Quintana (2018) build sovereign default models where the government accumulates capital which is an input in private production of consumption goods. In Park (2017), the government issues one-period bonds while Gordon and Guerrón-Quintana (2018) has long term debt. In the former, investment in capital requires both domestic goods and imported inputs. Like in Mendoza and Yue (2012), default triggers a loss of imports which in this context sharply raises the cost of investment. The latter paper adds convex investment adjustment costs to the accumulation process but removes imports. These studies emphasize the relationship between default incentives, debt levels and the level of the capital stock. Higher levels of capital create two opposing forces on default risk. On

[4] From a conceptual perspective, Dixit and Pindyck (1994), suggests that an investment is irreversible when it is a "sunk cost," which arises from asset specificity. Public infrastructure such as roads, ports, bridges, and dams are examples of such 'country-specific' capital; a highway cannot be dug up and moved to another economy. While ownership can be sold to private entities, the capital continues to add to the productive capacity of the economy.

[5] See also Na, Schmitt-Grohé, Uribe, and Yue (2018) for a model with an endowment stream of tradable goods and endogenous production of non-tradable goods.

the one hand repayment capacity is higher for a given level of debt while output in autarky is also higher, encouraging default. The key relationship naturally is between the size of debt relative to the capital stock. Arellano, Bai, and Mihalache (2018) build a two sector version of Gordon and Guerrón-Quintana (2018) with capital accumulation where investment uses both tradable and non-tradable goods. The model helps to explain a disproportionate decline in non-tradable activity in periods of increased default risk.

These opposing forces are also present in our model. Our model differs from these studies primarily due to the presence of lumpiness and irreversibility in the capital investment process. A more minor difference is that while the above models formally have the government accumulating capital, the models are calibrated to an economy's total private and public capital stock and total investment data whereas we specifically target public capital and public investment data. As a result, they potentially overstate the importance of capital to their quantitative results.[6] In order to understand the relative importance of non-convex adjustment costs to convex adjustment costs, we create a Standard model without irreversibility or lumpiness but with quadratic adjustment costs and find our Benchmark model implies higher borrowing and lower default risk. We also show that our Benchmark model implies higher welfare.

The rest of the paper is organized as follows. In Section 2 we introduce our model; Section 3 presents the quantitative analysis and results of the Benchmark model and its variants. Section 4 is the conclusion.

2 Theoretical Framework

This section presents a small open economy model with a representative household and a borrower government. The Benchmark model incorporates accumulation of irreversible public capital by the government with fixed costs of investment, foreign borrowing with potential for default, and household choices of consumption and labour supply. We will subsequently discuss variants of the Benchmark model.

[6] In Park (2017) the production technology is owned by the household while capital is accumulated by the government but there is no rental payment. In Gordon and Guerrón-Quintana (2018) we see a fully centralized model where the government undertakes all activities. See also Asonuma and Joo (2024) for a model that focuses on post-default restructuring of debt in the presence of public capital calibrated to only the public capital share. Finally Deng and Liu (2024) build a model with both physical and intangible capital accumulation.

2.1 Household

The model features an infinite horizon, and time is discrete, indexed by $t \in \{0, 1, \dots\}$. The representative household maximizes lifetime utility, which depends on consumption and labour supply:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t - \frac{\ell_t^{\omega}}{\omega}) \right], \tag{1}$$

where c_t denotes consumption, ℓ_t is labour, β is the discount factor and $\omega > 0$ governs the Frisch elasticity of labour supply. As is typical in the literature, We use so called GHH preferences (Greenwood, Hercowitz, and Huffman (1988)), because they ensure the marginal rate of substitution between consumption and leisure is independent of the level of consumption.

Each unit of labour supplied by the household is paid a wage rate w_t. The household also receives a lump sum transfer or pays a tax to the government T_t such that the budget constraint may be written as:

$$c_t = w_t \ell_t + T_t. \tag{2}$$

2.2 The government

The government in our model oversees the investment activities of public capital, owns the production technology, and determines the level of sovereign borrowing. We discuss the various decisions of the government in turn, starting with public capital accumulation.

2.2.1 Public Capital Accumulation and Investment

Public capital investment involves converting one unit of the consumption good into one extra unit of public capital for use in production next period. The law of motion for the public capital is given by:

$$k_{t+1} = (1 - \delta_k) k_t + I_t, \quad I_t \geq 0; \tag{3}$$

where k_t and k_{t+1} are the current and the next period's public capital stock, I_t refers to gross investment, and δ_k to the depreciation rate. There are two key frictions that make the public capital accumulation process in our model distinct. The first is that public capital investment is irreversible, $I_t \geq 0$. This implies that public capital, once created, can never be converted back into consumption goods; in other words, investment is strictly non-negative. The second key friction is that public capital investment is *lumpy*. We assume that the government must incur a fixed cost $\bar{\zeta}$ whenever it seeks to raise the capital stock above its depreciated level. This cost is paid by the government in the form of lost output:

$$\zeta_t = \begin{cases} 0, & \text{if } I_t \leq \delta_k k_t, \\ \bar{\zeta}, & \text{if } I_t > \delta_k k_t. \end{cases} \tag{4}$$

As discussed in the Introduction, the investment fixed cost forces investment, net of depreciation, to occur in discrete, non-incremental amounts, generating the lumpy dynamics often observed in large-scale public infrastructure projects. Irreversibility implies that capital will not be available in the future to serve consumption needs in states where debt repayments are high or output is low or both. Both the assumption of irreversibility and lumpy investment appear to be features of public infrastructure projects such as roads, ports, bridges, railway lines, or power plants.

2.2.2 Production of the consumption good

The government has access to a constant returns to scale production technology such that final consumption goods are produced using labour hired from households, a fixed stock of private capital and the stock of public capital:

$$y_t = A_t \ell_t^{\alpha_l} (k_t^g)^{\alpha_k} (k^p)^{1-\alpha_l-\alpha_k}; \tag{5}$$

where A_t denotes total factor productivity, k_t^g is public capital, and k^p is private capital. For computational simplicity, we follow Mendoza and Yue (2012) and hold the private capital stock fixed and also normalize it to unity[7]. The parameters α_k and α_l denote the output elasticities of public capital and labour respectively. We assume the log of A_t follows an AR(1) process:

[7] We will abstract private capital k^p in our discussion hereafter. Effectively we have assumed that any transfers or taxes charged to the household implicitly include payment for the use of private capital. Holding private capital constant simplifies the model solution by reducing the dimensionality of the state space and normalizing its value to unity effectively scales productivity measures because any true variation in private capital will be captured in total factor productivity. Later, when linking the model to the data, we will measure TFP using a modified Solow residual that captures the impact of private capital movement.

$$A_t = \rho A_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \tag{6}$$

2.2.3 Recursive Government Problem

In what follows, we present the government's problem in recursive form, expressing all decisions and value functions directly as functions of the current state variables. The government issues nonenforceable long-term bonds and is not committed to repayment. In each period, the government decides how much debt to carry, whether to default on its debt, and how much to invest in public capital. We assume there is no exclusion from the bond market in the periods after default, as in Hatchondo and Martinez (2009) and Bianchi and Sosa-Padilla (2024), but the government suffers a one-period productivity loss as the cost of default. We also assume that lenders cannot expropriate public capital to recover defaulted funds. Its value function is given by:

$$V(A, b, k) = \max_{d \in \{0,1\}} \Big\{ (1-d)\, V^{nd}(A, b, k) + d\, V^d(A, b, k) \Big\}. \tag{7}$$

where V^{nd} is the value of being in good standing, and V^d is the value of being in default. Since the government will be granted reentry to the credit market in the period after a default episode, each period the government will always be choosing between the two value functions listed above, where the investment and default/borrowing decision are being optimized simultaneously. When the government decides to repay its debt, the value function is given by:

$$V^{nd}(A, b, k^g) = \max_{b',\, k^{g\prime}} \Big\{ u(c, \ell) + \beta\, \mathbb{E}\Big[V(A', b', k^{g\prime}) \,\Big|\, A \Big] \Big\}, \tag{8}$$

subject to the following constraints:

$$w\ell + T + \kappa b + I + \zeta = q\Big[b' - (1-\delta^b) b\Big] + A\ell^{\alpha_\ell} (k^g)^{\alpha_k}, \tag{9}$$

$$k^{g\prime} = (1 - \delta_k) k^g + I, \qquad I \geq 0, \tag{10}$$

$$\zeta = \begin{cases} 0, & \text{if } I \leq \delta_k k^g, \\ \bar{\zeta}, & \text{if } I > \delta_k k^g. \end{cases} \tag{11}$$

where T denotes lump sum transfers or tax collected from the household, κ is the coupon payment per bond issued, q is the bond price offered by a risk-neutral foreign lender, and δ^b is the bond decay rate. Here b_t represents the stock of outstanding claims on the government.

Following the framework for long-duration debt pioneered by Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), when the government issues a bond in period t, it promises to pay $\kappa(1-\delta)^{j-1}$ units of the consumption good in period $t+j$, for all $j \geq 1$. Hence, the evolution of debt can be written as follows:

$$b' = (1-\delta)b + \nu,$$

where ν represents new bond issuance.

If the government defaults, it incurs a loss in the productivity $A^d \leq A$. We follow the setup in Chatterjee and Eyigungor (2012) where :

$$A^d = A - \max\left\{0,\ d_0\bar{A} + d_1 A^2\right\}. \tag{12}$$

This form of default costs suggests that the losses are increasing in output levels under repayment. Johri (2025) provides justification by showing that this form of default costs can be endogenously generated in a model with a requirement that intermediate imports face an international reserve availability requirement. The government is not allowed to trade in the bond market in the period of default as in Hatchondo and Martinez (2009), but will be granted reentry in the next period with zero debt. The value function in default is given as:

$$V^d(A, b, k^g) = \max_{k^{g'}} \left\{ u(c, \ell) + \beta\, \mathbb{E}\Big[V(A', b', k^{g'}) \,\Big|\, A \Big] \right\}, \tag{13}$$

subject to the following constraints:

$$w\ell + T + I + \zeta = A^d\, \ell^{\alpha_\ell} (k^g)^{\alpha_k}, \tag{14}$$

$$k^{g'} = (1-\delta_k)k^g + I, \qquad I \geq 0, \tag{15}$$

$$\zeta = \begin{cases} 0, & \text{if } I \leq \delta_k k^g, \\ \bar{\zeta}, & \text{if } I > \delta_k k^g. \end{cases} \tag{16}$$

The government chooses to default whenever doing so yields a higher value. Define the default set over the state space (A, b, k^g) as

$$\hat{d} = \Big\{ (A, b, k^g) \ :\ V^d(A, b, k^g) \geq V^{nd}(A, b, k^g) \Big\}. \tag{17}$$

Similarly, let $\hat{\imath}(A, b, k^g)$ denote the optimal investment policy in the state space. The investment set is then defined as

$$\hat{i} = \Big\{(A, b, k^g)\ :\ I(A, b, k^g) > \delta_k k^g\Big\}. \tag{18}$$

2.3 Lender's Problem

The bonds issued by the government are purchased by competitive risk-neutral international lenders that take the bond price as given, which satisfies the zero-profit condition:

$$q(A, b', k^{g'}) = \frac{1}{1+r^*}\mathbb{E}_t\Big(1 - \hat{d}(A', b', k^{g'})\Big)\Big[\kappa + (1-\delta_b)q\Big(A', b'', k^{g''}\Big)\Big]. \tag{19}$$

Where r^* is a constant risk-free rate, $\hat{d}$ is the policy matrix for the default decision, such that the bond price depends on the three states variable of productivity, debt and public capital level. Despite widespread evidence that developed economy interest rates affect emerging economy default risk, it is standard to assume that the risk free rate is constant in this literature, however see Johri, Khan, and Sosa-Padilla (2022) for a model with time-varying world interest rate that influences the debt and default dynamics of a small open economy.

2.4 Recursive Equilibrium

Definition 1: *A recursive Markov perfect equilibrium for this economy is defined by*

1. *a set of value functions* V, V^d, *and* V^{nd}
2. *policy rules for default* $\hat{d}$, *borrowing* $\hat{b}$ *and public investment* $\hat{i}$
3. *and a bond price schedule* $\hat{q}$.

such that

(i) Given the bond price schedule $\hat{q}$, *the policy function* $\{\hat{b}, \hat{d}, \hat{i}\}$, *and the value functions* $\{V, V^d, V^{nd}\}$, *solve the government's optimization problem.*

(ii) Given the government policies, the bond price schedule satisfies eq19 and is consistent with the zero profit condition for the foreign lenders in international financial markets.

3 Quantitative Analysis

This section presents the quantitative analysis of the theoretical model. Section 3.1 outlines the computational approach, and Section 3.2 describes the calibration of the benchmark economy. Section 3.3 discusses the main results and examines alternative outcomes from variants of the Benchmark model.

3.1 Computational Approach

We solve the model's recursive problem numerically using value function iteration on a discretized state space. The state of the economy is defined by (A, b, k^g), representing the current productivity shock, the level of outstanding government debt, and the stock of public capital, respectively.

To implement the solution, we first discretize the state space. The stochastic AR(1) process for productivity is approximated as a finite-state Markov chain using the Tauchen (1986) method into a grid of 21 points. The grids for public capital, K^g, and government debt, B, are set to be 60 and 45 points, respectively. The algorithm iterates on the value functions for repayment, $V^{ND}(A, b, k^g)$, and default, $V^D(A, b, k^g)$, along with the state-contingent bond price function, $q(A, b', k^{g\prime})$. These functions are solved for simultaneously until they converge to a stationary fixed point. Convergence is achieved when the maximum absolute difference between the functions in successive iterations falls below a tolerance of 10^{-5}.

3.2 Parameterization

The values for the parameters that are used in our quantitative analysis are taken from the literature or calibrated such that the moments produced from the model simulation match those in the data. A period in the model is one year. Following Bianchi, Hatchondo, and Martinez (2018), we calibrate the Benchmark model to Mexico which is a common benchmark in the sovereign default literature.

The sovereign spread is defined as the difference between the bond yield and the risk-free rate ($spread = r_{\text{model}} - r^*$). In our long-bond setup, the model yield is computed from bond prices after solving the government's problem, $r_{\text{model}} = \frac{\kappa}{q} - \delta_b$, where κ is the coupon payment $\kappa = \frac{r^* + \delta_b}{1 + r^*}$. The spread series are obtained from Catão and Mano (2017), and cover 1973 to 2014. The data on public capital stock and public investment are drawn from the IMF Investment and Capital Stock Dataset (ICSD). Measures of public external debt are

Table 2: Model Parameters: Internal and External Calibration

Parameter	**Value**	**Target / Source**
Internally Calibrated Parameters		
Default cost (linear term) d_0	-0.56	Mean Spread (%)
Default cost (quadratic term) d_1	0.81	Mean Debt-Output Ratio (b/Y)
Capital depreciation rate δ_K	0.05	Mean Public Investment Public (I/Y)
Public capital income share α_k	0.098	Mean Public Capital Share (k^g/Y)
Investment fixed cost ζ	0.01	Maximum Public Capital Share($max(k^g/Y)$)
Externally Calibrated Parameters		
Discount factor β	0.85	Johri et al. (2022)
Risk aversion σ	2.00	Mendoza (1991)
labour supply elasticity ω	1.45	Mendoza (1991)
labour share α_l	0.64	Gordon and Guerrón-Quintana (2018)
Risk-free interest rate r^*	0.04	Standard value
Bond decay rate δ_B	0.3	Average duration
Shock persistence ρ_A	0.56	Estimated
Shock volatility σ_A	0.032	Estimated

obtained from the World Bank's World Development Indicators (WDI). We focus on the period 1973–2014 since this is the period for which spread data is available. Table 2 reports the parameter values and the target moment, or source. Table 3 reports the targeted and some non-targeted moments from the model versus data.

3.2.1 External Calibration

We set the subjective annual discount factor to $\beta = 0.85$, which corresponds to a quarterly discount rate of $\beta = 0.96$ that is commonly seen in the literature.[8] The coefficient of relative risk aversion, $\sigma = 2$, is a standard value in quantitative sovereign default models. The Frisch elasticity of labour supply is set at a conventional value of $\omega = 1.45$ (Mendoza and Yue (2012) and Park (2017)). The labour share in production, α_l, is set to 0.64, matching the average labour income share in Mexico's national accounts over the sample period. The risk-free rate, r^*, is fixed at 4% per year, which is also a standard value. The bond decay parameter, δ_b, is set to 0.30, which corresponds to a Macaulay duration of approximately three years in our model simulation which is consistent with the duration used by Bianchi et al. (2018).[9] Finally, the persistence (ρ_A) and volatility (σ_A) of the productivity shock are

[8] Similar quarterly discount factors can be found in Johri et al. (2022) ($\beta = 0.96$), Arellano (2008) ($\beta = 0.953$) and Chatterjee and Eyigungor (2012) ($\beta = 0.954$).

[9] We use the Macaulay duration for a perpetual coupon bond, which is expressed as $D = (1+r_{model})/(\delta_b + r_{model})$.

estimated by fitting a log-AR(1) process to Mexico's detrended modified Solow residual:

$$\log(A_{t+1}) = \rho_A \log(A_t) + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_A^2) \tag{20}$$

The resulting estimates are $\rho_A = 0.56$ and $\sigma_A = 0.032$.[10]

3.2.2 Internal Calibration

Our internal calibration strategy involves matching the model moments with targeted data moments. The model economy is simulated using the calculated historical adjusted Solow Residuals between 1973 and 2014 for Mexico. Because the adjusted Solow residuals from the data do not fit exactly with the discretized productivity grid, for each year in the sample, the observed productivity level is mapped to the nearest node of the discretized productivity grid so that the model transition is driven by a close approximation to the actual sequence of productivity shocks. In the appendix B we show this mapping fits the actual productivity process well. Because capital accumulation implies a transition process, the initial state matters. The initial state of debt and public capital is selected to minimize the squared distance between the model-implied capital-output and debt-output ratios and their empirical values in 1973.

Conditional on this starting point and the observed productivity sequence, the model generates a path of output, consumption, investment, debt, spreads, and default decisions by iterating on the policy functions obtained from solving the value functions. Moment calculations are restricted to repayment periods in which bond prices remain positive, and the resulting simulated statistics are then compared to their empirical counterparts for Mexico, as shown in table 3.

We calibrate five parameters internally to ensure the model replicates key features of the Mexican economy. The calibration targets are (i) an average sovereign spread of 2.96 percent; (ii) a mean public debt-to-output ratio of 21 percent; (iii) an average public capital share of 42 percent of output; (iv) a mean public investment share of 3 percent of output; and (v) a maximum public capital share of 58 percent of output. The two default cost parameters d_0 and d_1 are jointly chosen to target Mexico's average debt-to-GDP ratio and mean sovereign spread. The capital depreciation rate δ_k, the public capital income share α_k and the fixed cost of investment $\bar{\zeta}$ are jointly chosen to match the average public capital-to-GDP ratio,

[10] The modified Solow residual is constructed to capture movements in total factor productivity as well as unmodeled or unmeasured factors like private capital, which is held constant in our model. It is defined as $SR_t = Y_t/(K_t^g)^{\alpha_k}$, where both GDP ($Y_t$) and the public capital stock (K_t^g) are measured in detrended, per capita levels.

Table 3: Targeted and Non-Targeted Moments: Benchmark Model

Moment	**Data**	**Benchmark model**
Targeted Moments		
Mean Spread (%)	2.96	2.85
Mean Debt-Output Ratio (b/Y)	0.21	0.21
Mean Public Capital Share (k^g/Y)	0.42	0.42
Mean Public Investment Share (I/Y)	0.03	0.03
Maximum Public Capital Share ($\max(k^g/Y)$)	0.58	0.57
Non-Targeted Moments		
Std. of Consumption / Std. of Output	1.03	1.11
Std. of Public Capital / Std. of Output	1.88	1.38

the average public investment-to-GDP ratio, and the maximum public capital-to-GDP ratio seen in Mexican data in this time period. Since the fixed cost of investment affects the level of these variables in model simulations, we are choosing the maximum level of capital to output ratio in the data as one of the moments to tie down the fixed cost $\bar{\zeta}$. In order to generate these moments, we feed the estimated innovations to the adjusted Solow Residual series over this time period into the model. As shown in table 3, our Benchmark model matches the data moments reasonably well.

3.3 Model Variants and Comparative Analysis

This section presents the results from the quantitative model in three steps. We begin by examining the implications of our Benchmark model; we then decompose the contribution of each friction by comparing the Benchmark model to alternative variants that selectively remove capital irreversibility and investment lumpiness. Finally, we contrast the Benchmark specification with the commonly used Standard model (reversible and smooth investment) to quantify the overall importance of both frictions taken together.

3.3.1 Benchmark Model

In this section we present the key implications of the Benchmark model, highlighting how the combination of investment irreversibility and lumpiness shapes borrowing, default, and the allocation of resources to public investment relative to transfers.

Investment and Borrowing In the model, the government receives two sources of funds once any debt repayment has been made. These are domestic output and external borrowed funds. These funds have two uses: private consumption and public investment. An important question in the context of our model is whether borrowed resources are primarily used for consumption smoothing (as is the case in most sovereign default models) or for investment. If the external resources are primarily used for consumption, then it would be inconsistent with the observations of the IMF discussed in the Introduction and there would be little reason to believe that default risk should differ in this model. We can infer useful information from the ratio of investment-to-net bond revenue. We define net bond revenue (NBR) as the funds raised from new debt issuance net of debt rollover and coupon payments. This ratio therefore captures the share of incoming external resources that are directed toward investment rather than consumption.[11]

$$\text{NBR} = q \cdot [b' - (1 - \delta_b)b] - \kappa b.$$

Note that NBR will be positive when debt is zero and fall as debt rises, eventually becoming negative as bond prices continue to fall. As such, the ratio only provides interesting information when debt levels are low. In Figure 1, we plot the investment-to-net bond revenue ratio against all productivity states given a fixed public capital level and zero debt so that NBR is positive even at low productivity levels. The figure displays two curves for two different capital levels that are roughly four standard deviations below and above the mean in the simulation, providing a representative comparison between a low-capital economy and a high-capital economy. The blue line displays the situation of an economy that has a strong desire to accumulate public capital, while the red line indicates a much higher capital level where further investment is only justified at very high productivity states. As expected, Figure 1 shows that the fraction of net bond revenue used for investment is increasing with productivity. For the low public capital economy (blue circles), this ratio displays a minimum of 73 percent and a maximum above 100 percent. This indicates that most external funds are used to finance public investment and that sometimes all incoming external resources are used for investment. When public capital is much higher (red triangles), the ratio is much lower at all productivity levels, as the return to public capital accumulation is correspondingly lower. This reflects the idea that the economy has achieved a mature level of public capital. We also note that both plots display flat regions where the ratio does not respond to productivity increases. These are more pronounced at higher capital levels (red line). As the public capital stock reaches the non-stochastic steady state level, the usage of external funds pivots towards consumption.

[11] Obviously resources are fungible once inside the government's budget constraint.

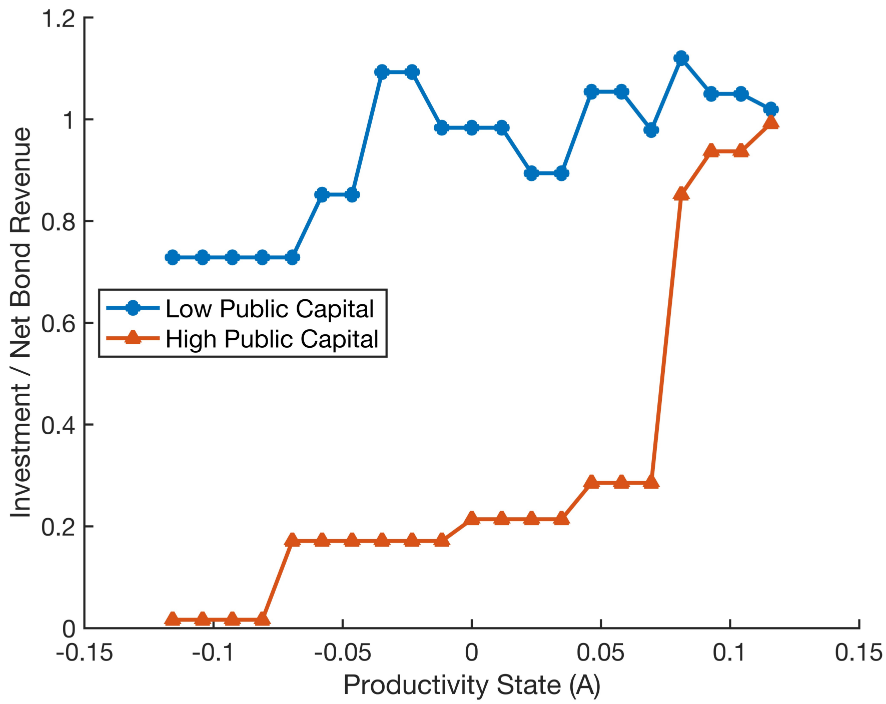


Figure 1: Investment-to-Net Revenue Ratio across Productivity States

Borrowing Behaviour Having seen how NBR is allocated by the government, we turn attention to how new borrowing varies with capital levels. Figure 2 presents the government's borrowing policy functions, conditional on mean productivity and two capital levels as in the previous figure. When the accumulated debt is at a relatively low level, the low-capital economy (blue) borrows more than the high-capital economy (red). This pattern reverses as debt rises. There are a number of forces at play that explain this reversal. The first is the diminishing return to public capital accumulation; a higher capital economy is less incentivized to invest and therefore has a lower demand on external funds. The second is the marginal utility of the household. Postponing consumption in order to invest is expensive when capital and output are low, and this encourages borrowing to pay for investment and vice versa. Finally, as indebtedness rises relative to output (repayment capacity), borrowing costs increase. The same debt level signifies higher debt-to-output ratios and higher borrowing costs when capital is low than when it is high, as can be seen in the right panel of the figure. This third economic force dominates the first two at higher debt levels and this helps to explain why the high-capital economy can sustain more debt and indeed chooses to do so.

Sovereign Spreads. The borrowing patterns discussed above already suggest that comparing default risk across different levels of capital purely in terms of the level of debt is

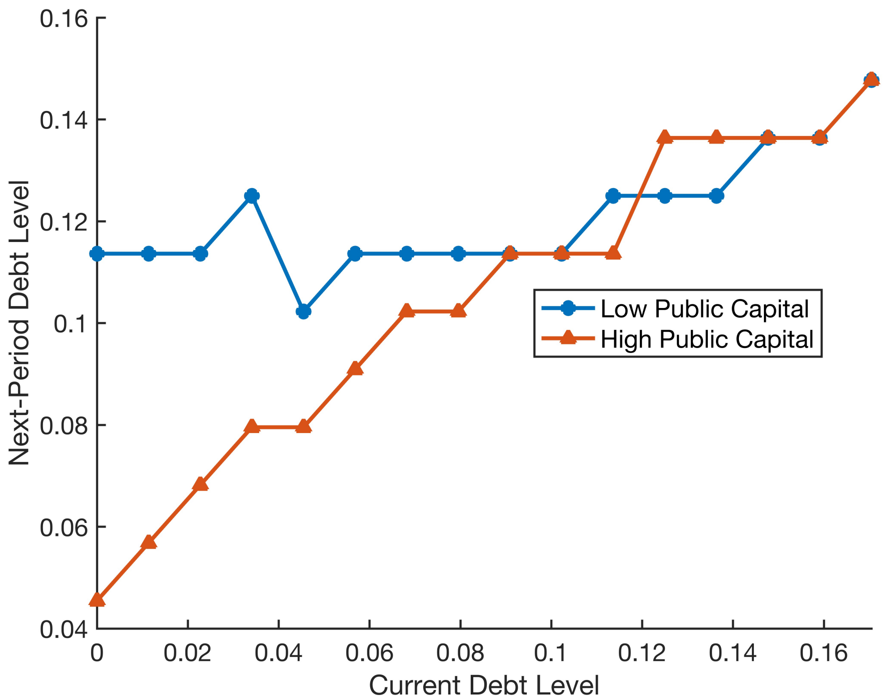


Figure 2: Borrowing Policy: b' vs. b

misleading. Low- and high-capital economies face very different output levels, and therefore the same nominal debt imposes different repayment burdens. A high-capital economy produces more, so a given amount of debt represents a smaller debt-to-output ratio and thus lower marginal default incentives. We can adjust for this scale effect by comparing spreads of economies with different levels of public capital by using the debt-to-output ratio.

Figure 3 plots the sovereign spread against the debt-to-output ratio for two different capital levels. Spreads are lower for the very low capital state (blue circles) compared to a moderately higher capital state (red squares). This appears to occur because at low capital levels the government will repay in order to remain able to borrow funds needed for investment. The comparison is deliberately restricted to low-capital regions because of this reason. While not shown, in the mature economy (around the non-stochastic steady-state capital level), differences in capital cease to meaningfully affect default risk. This is related to the fact that at high capital levels, most borrowed funds are no longer used for investment.

Keep in mind that the same debt-to-output ratio implies a higher debt level for the high-capital economy. On the one hand more capital implies greater repayment capacity while on the other hand, higher output in default makes autarky less less costly. These competing and complicated interactions between capital levels and debt levels are consistent with the

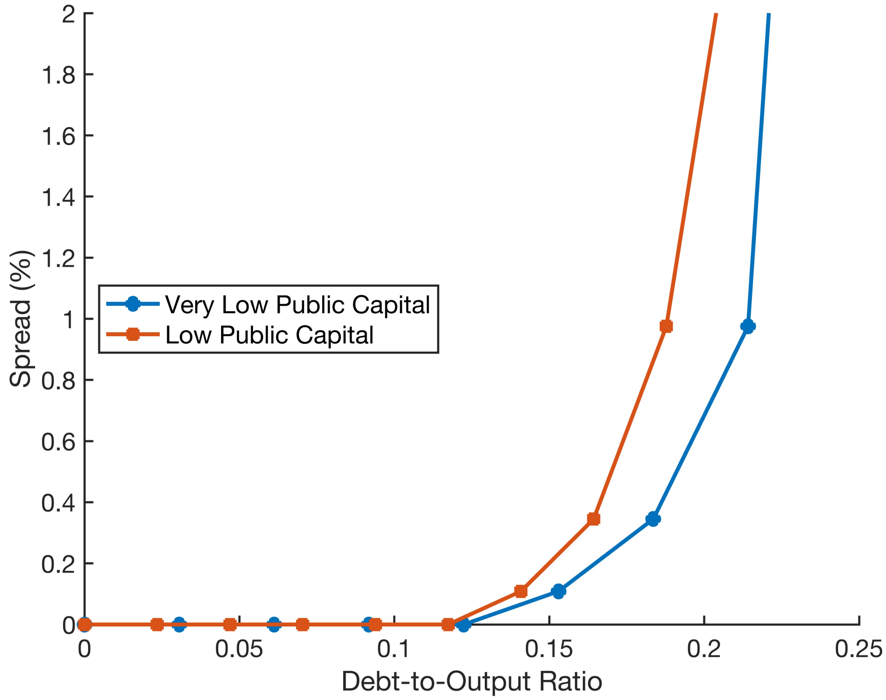
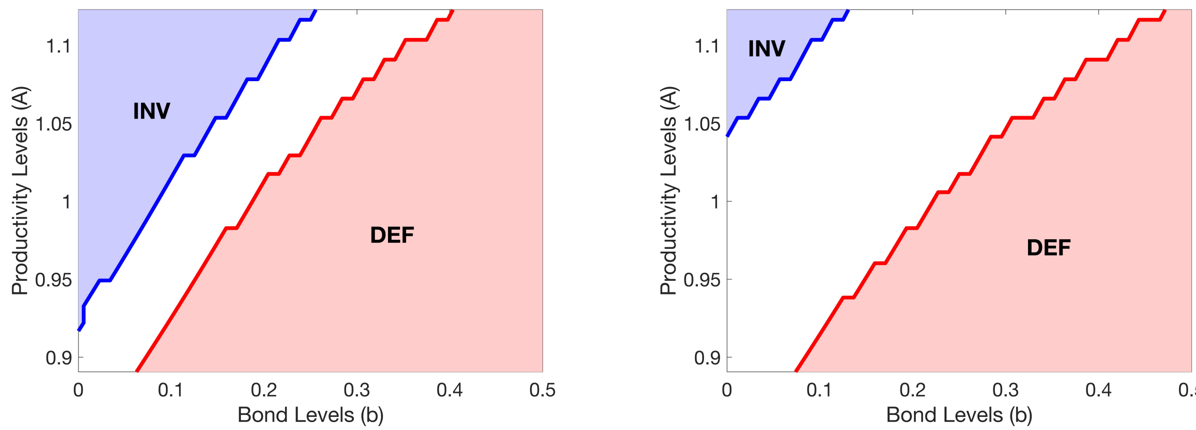


Figure 3: Benchmark Model Sovereign Spreads

discussions in Gordon and Guerrón-Quintana (2018).[12]

Default and Investment Decisions. A central implication of the Benchmark model is the *mutual exclusivity* between default and positive investment, net of depreciation ($\hat{i}$). For convenience we will subsequently simply refer to ($\hat{i}$) as investment from now onwards. Figure 4 maps the government's optimal default ($\hat{d}$) and investment ($\hat{i}$) decisions holding public capital constant at two different levels. Productivity levels are displayed on the vertical axis, while debt levels are on the horizontal axis. The red contour delineates the default region, while the blue contour defines the investment region. As shown in the figure, investment occurs primarily in the upper-left portion of the state space when productivity is high and debt is low. Defaults, however, cluster on the right-hand side when debt is high and/or productivity is low. The intermediate white region between these two boundaries represents states where the government neither adds to the public capital stock nor defaults. When capital is low (left panel), this white region is smaller than when capital is high (right panel). As the capital level increases, the investment region shrinks and will eventually disappear. To understand the role of debt, fixing any point on the productivity axis and moving right-

[12] We show in the appendix C that our benchmark model produces patterns consistent with Gordon and Guerrón-Quintana (2018).



(a) Low Capital State (b) High Capital State

Figure 4: Default Regions by Capital Level in the Benchmark model

ward along a horizontal line shows that as debt accumulates and net revenue from new borrowing (NBR) declines, a larger share of available resources must be devoted to debt rollover and coupon payments, and the optimal policy shifts sequentially from investment to no investment, and finally to default as the debt burden becomes too large.

This pattern that investment ceases *before* defaults occur emerges from the model's dynamic relationship between debt levels and investment decisions. Recall that earlier we showed public investment constitutes a substantial share of NBR when debt is low. As debt rises and NBR shrinks, an increasing portion of domestic output must be allocated to debt service and coupon payments, causing a reduction in funds available for investment purposes. It also raises the marginal utility of consumption as domestic resources become scarcer. In this intermediate range of debt levels, the government can still afford to repay but finds further investment infeasible, since the marginal cost of borrowing for investment through higher spreads and reduced fiscal capacity exceeds its expected return. Hence, investment stops while repayment continues. Only when debt accumulates further and debt service absorbs a large enough share of available resources does the value of repayment surpass that of exclusion, making default the optimal choice. Lenders note that the optimal policy implies a mutual exclusivity between investing and defaulting and offer lower spreads during investment periods. Table 4 shows that in the model simulations, the average spread is always lower during an investment episode than when only depreciated capital is being replaced. This pattern is also consistent with our investment spike data. As discussed above, most investment in the model occurs with the help of borrowed funds, and this creates an incentive to repay debt. This dependency on external funding thus lowers the default risk perceived by the lender.

Table 4: Average Sovereign Spread Conditional on Investment: Simulation vs Data

Condition	**Simulation**	**Data**
$I > \delta K$	1.1182	1.6457
$I \leq \delta K$	3.6298	3.1358

We will now decompose the individual impact of lumpiness from that of irreversibility in the Benchmark model results presented above. For this purpose, we introduce a series of model variants in which these frictions are selectively removed. Specifically, we consider two model variants: Rev–Lumpy and Irrev–No-Lumpy. Rev-Lumpy allows public capital to be freely converted back into consumption goods while maintaining the Benchmark model fixed investment cost level. The Irrev–No-Lumpy model maintains irreversibility of capital but turns off the fixed investment cost. In these model variants, we keep all other parameters at the same values as in the Benchmark model. Later we will introduce the Standard model similar to Gordon and Guerrón-Quintana (2018) with reversible capital, no fixed investment cost but with quadratic adjustment costs in capital accumulation. The next section begins by examining the role of capital irreversibility before turning to the effects of investment lumpiness.

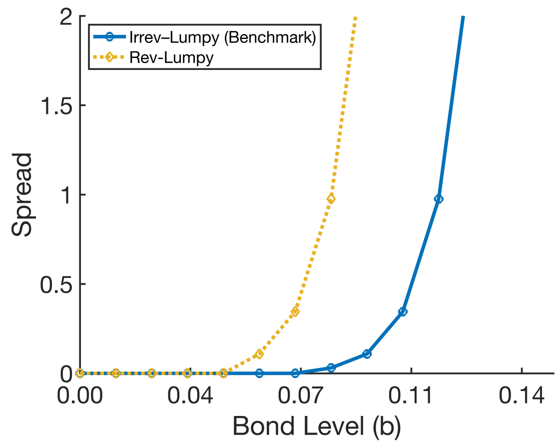
Table 5: Key Simulated Moments Across Model Variants

Model	**Mean Spread**	**Mean B**	**Mean K**	**Mean I**	**Mean Y**	**Mean C**
Irrev–Lumpy (Benchmark)	2.852	0.116	0.230	0.014	0.543	0.513
Irrev–No-Lumpy	3.639	0.124	0.264	0.017	0.556	0.534
Rev–Lumpy	4.045	0.087	0.201	0.012	0.531	0.504

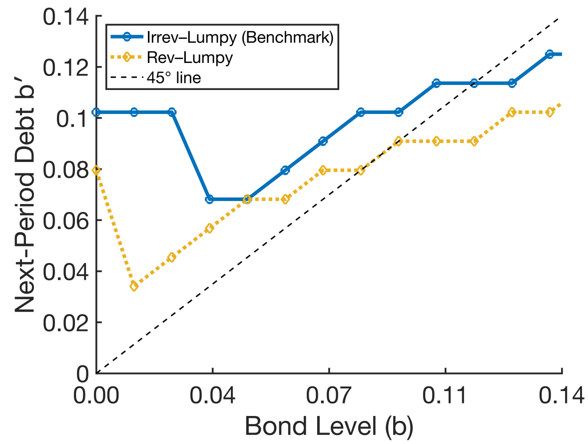
3.3.2 Effects of Irreversibility

The impact of capital irreversibility becomes clear when we compare the Benchmark economy to its reversible-capital counterpart (Rev–Lumpy). Comparing rows 1 and 3 in Table 5, we see that removing irreversibility leads to substantially higher average spreads and lower levels of debt, capital, investment, output, and consumption. This indicates that irreversibility alone plays a large role in the Benchmark model results. The lower sovereign spreads allow governments to borrow more cheaply, which in turn supports greater investment and capital accumulation.

These differences in simulated moments are directly reflected in the underlying policy func-



(a) Controls for lumpiness



(b) Controls for lumpiness

Figure 5: Irreversibility and Lumpiness: Effects on Sovereign Spread and Borrowing Policy

tions. Figure 5a shows that, for any given bond level, the Benchmark model yields consistently lower spreads than the reversible-lumpy model. Figure 5b further shows that the Benchmark economy usually chooses higher next-period borrowing, demonstrating stronger repayment capacity. As debt rises, both models increase borrowing, though at some point the rise in the spread incentivizes lowering debt (lines below the 45 degree line). We find the same quantitative effects of irreversibility in the frictionless investment environment (Irrev–No-Lumpy vs. Rev–No-Lumpy); the corresponding figures are reported in the appendix D.

To understand why irreversibility influences the probability of default, it is important to realize that in irreversible capital models, accumulated capital is not available for consumption smoothing in the future. In contrast, when capital is reversible, the government can "eat its capital" in states where the marginal utility of consumption is high. These bad times occur when productivity is low and debt repayments are high or both. Anticipating this behaviour, foreign lenders assign a positive probability to future capital depletion and, consequently, to lower expected future output. Since default risk is inversely related to expected output for a given debt level, lenders demand higher spreads from economies with reversible capital than from those in the irreversible models, holding other factors constant. This mechanism is illustrated in Figure 6, which plots the next period capital against debt level for the two models. The figure shows that, (i) the benchmark model often has a higher desired capital level for the next period, and (ii) as debt accumulates and debt services become more costly, the government chooses to liquidate or "consume" its capital stock whenever reversibility allows. This pattern emerges regardless of whether investment is lumpy or smooth, leading to lower

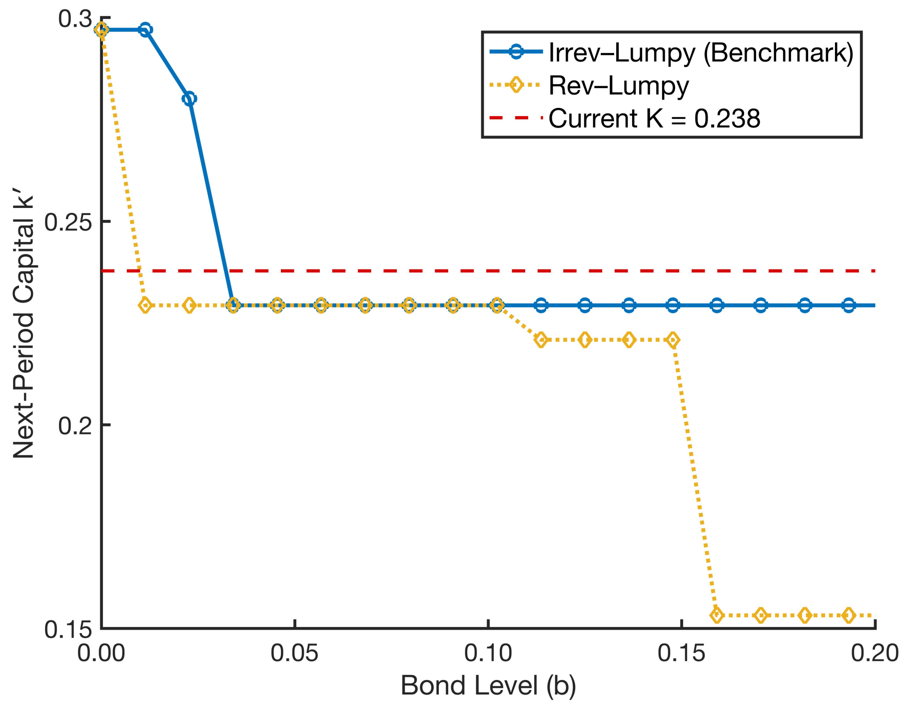


Figure 6: Capital Policy Rules: Benchmark vs Rev-Lumpy

future capital and output. Consequently, economies with reversible capital face weaker debt sustainability because lenders anticipate this behaviour and demand higher spreads ex-ante.

This mechanism of irreversibility operates alongside the standard channels emphasized in sovereign default models with capital. Greater capital accumulation increases a country's productive capacity, thereby expanding its resources and repayment ability, which lowers default risk (consumption-smoothing channel). However, higher capital also raises potential autarky consumption, which can increase the incentive to default (autarky channel). As a result, models that do not account for irreversibility may overstate default risk, particularly at higher levels of capital. As shown in the appendix, the reduction in the default region from having irreversibility is much more prominent when at a higher level of capital.

Having established the role of irreversibility in our results, we now turn to the effects of investment lumpiness. While irreversibility limits the government's ability to disinvest, lumpiness introduces discrete costs to expanding public capital, shaping both the timing and intensity of investment episodes. The next section explores how this investment friction influences sovereign spreads and borrowing dynamics.

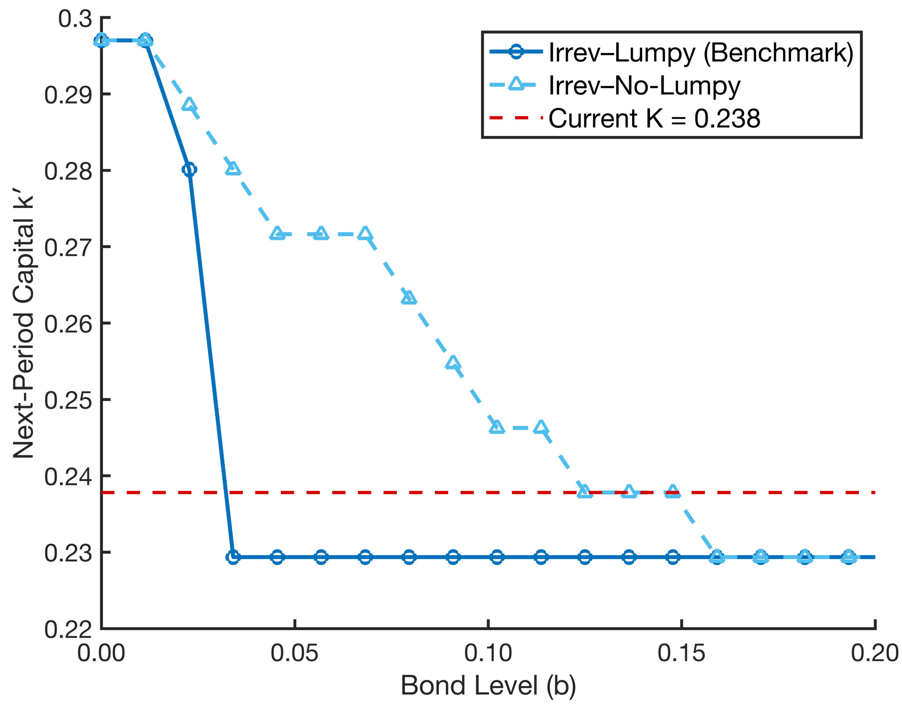
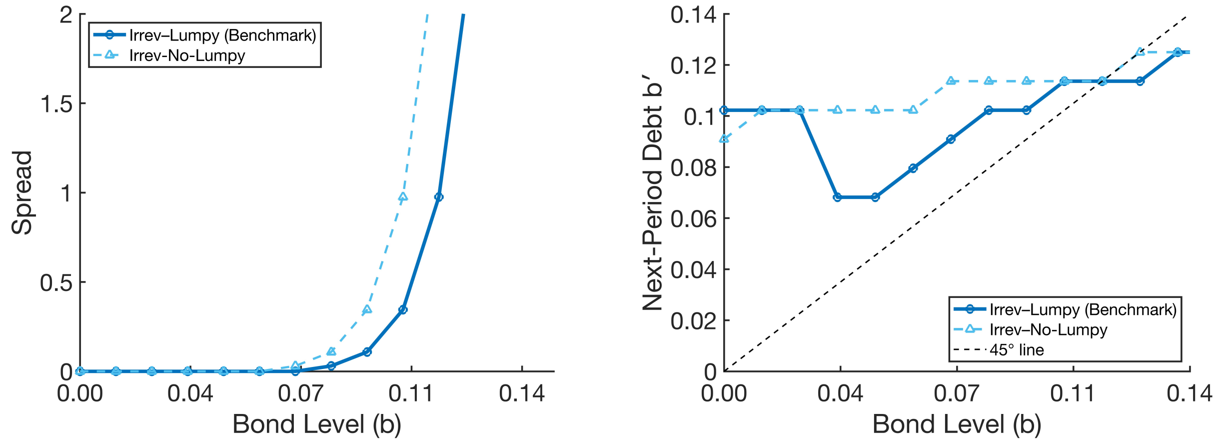


Figure 7: Capital Policy Rules: Benchmark vs Irrev-No-Lumpy

3.3.3 Effects of Lumpiness

Comparing rows 1 and 2 in Table 5 shows the impact of reducing the fixed cost of investment to zero (Irrev-No-Lumpy) in the presence of capital irreversibility. The absence of fixed costs to invest naturally results in higher average levels of public capital and investment, but also higher average debt and sovereign spreads.

To understand these results, we first examine the investment decision in Figure 7. This figure plots the policy function for next-period capital (k') against current debt (b) for both models holding productivity and capital stock at its mean value in the simulation. Without the fixed cost to create a "zone of inaction," the government in the Irrev-No-Lumpy model (light blue diamonds) chooses a higher level of next-period capital compared to the Benchmark model (dark blue circles) at almost all debt levels, but especially when low. The current capital level is illustrated in the horizontal dashed line. Both models choose to increase their capital stock at very low debt levels, but the Benchmark model allows depreciation to lower the capital stock at higher levels. In the absence of fixed costs, the government finds this productivity level sufficient to invest but the optimal amount to invest falls as debt rises. As explained above, this raises the marginal utility of consumption and lowers the overall value of investment. Eventually both models find it optimal to let depreciation lower the capital



(a) Controls for Irreversibility

(b) Controls for Irreversibility

Figure 8: Effect of Lumpiness

stock but this happens sooner in the Benchmark model.

This higher investment demand has a direct impact on the government's financial position, as shown in Figure 8. To finance the higher level of capital shown in the previous figure, the government must rely on the international credit markets more heavily. Panel 8b confirms this, showing that the Irrev-No-Lumpy model exhibits a higher borrowing function (b') when it sets a higher (k') than the benchmark model. The extra borrowing is reflected directly in higher spreads as shown in Panel 8a.

3.3.4 Capital and Sovereign Risk

At various points in the discussion so far, we have emphasized that additional capital has two opposing effects on the default/repay decision. These economic forces have been discussed in Gordon and Guerrón-Quintana (2018) and Park (2017) previously and operate to a lesser extent in our model since the share of public capital in production is lower compared to their calibration to the total capital stock. To summarize, for a given level of current debt, more capital implies more production in the future. This extra output cuts both ways: on the one hand, repayment capacity is higher, on the other hand, autarky induced by default does not reduce consumption as much. These two opposing influences on default risk interact with our two investment frictions: lumpiness and irreversibility. At low levels of debt, the latter dominates while at higher levels of debt (when default risk is high), the former dominates.

Comparing the Benchmark to the Irrev-No-Lumpy model, we see that removing the fixed cost leads to higher capital and output and higher spreads. Here, the absence of the fixed cost incentivizes the government to invest more. To finance this higher investment, the government accumulates significantly more debt. Therefore the higher capital stock and increases future output is accompanied by higher debt. The overall lowering of spreads is dominated by the increase of the debt burden required to build it. Thus, the Benchmark is safer because the fixed cost acts as a constraint on investment-driven borrowing in periods when the overall benefit of investment is relatively low.

When comparing the Benchmark to the Rev-Lumpy model, we see that the Benchmark model has higher capital and output, yet lower spreads. In the Reversible model, the government holds less capital on average because it can easily disinvest in periods when the marginal utility of consumption is high. In particular, this ability to convert capital into consumption is very tempting during or close to a default event, making financial autarky less painful while also reducing repayment capacity. The Benchmark economy is safer because the irreversibility constraint shuts down this consumption-smoothing channel.

Having analyzed these frictions in isolation, we now turn to the Standard model (Rev-No-Lumpy), which removes both lumpiness and irreversibility while introducing the typical quadratic investment costs often found in the literature.

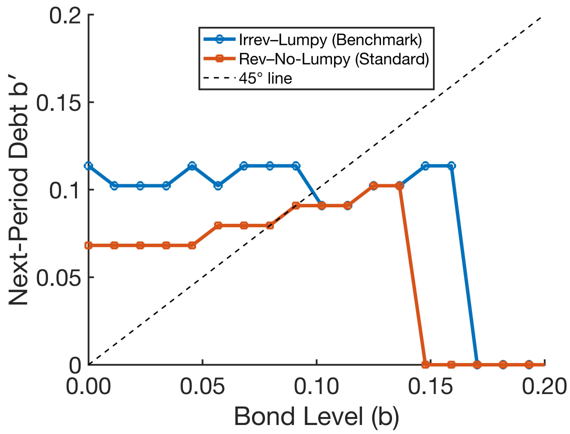
3.3.5 Standard Model

In order to compare our Benchmark model to a sovereign default model with a more standard formulation of capital accumulation, we develop a comparable variant called the Standard model. The Standard model removes both lumpiness and irreversibility while keeping all other parameters the same as in the Benchmark model. Following Gordon and Guerrón-Quintana (2018) quadratic capital adjustment costs limit the level of capital accumulated and investment volatility in the Standard model.[13]

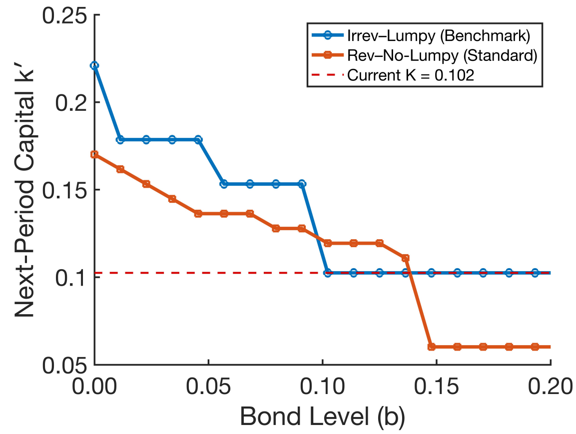
Borrowing, Investment and Default Risk Figures 9 and 10 report three key policy rules as functions of current debt: the government's next-period debt choice b', next-period public capital k' choice, and the chosen sovereign spread at two different levels of capital: (i) a low-capital state corresponding to roughly two standard deviations below the simulated mean, and (ii) the mean-capital state. We examine these two capital levels because investment incentives are concentrated in the lower part of the capital distribution. When capital is near or above its steady-state region, the marginal return to public investment is too low for the government to actively accumulate capital. Moreover, comparing models

[13] The capital adjustment cost is set to match the mean public capital-to-output ratio.

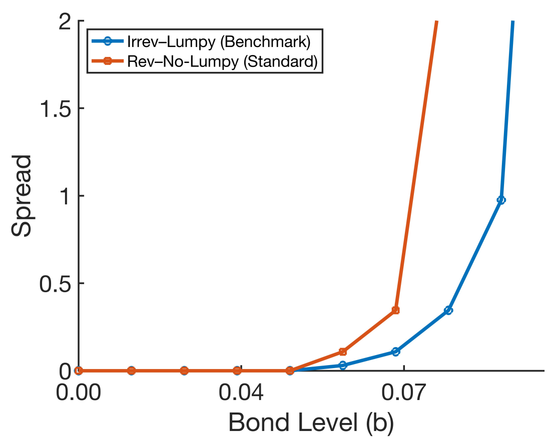
across various levels of capital is more informative about how investment frictions influence debt and default decisions along the transition from a low-capital economy to a high-capital economy.



(a) Debt Policy (Low k)



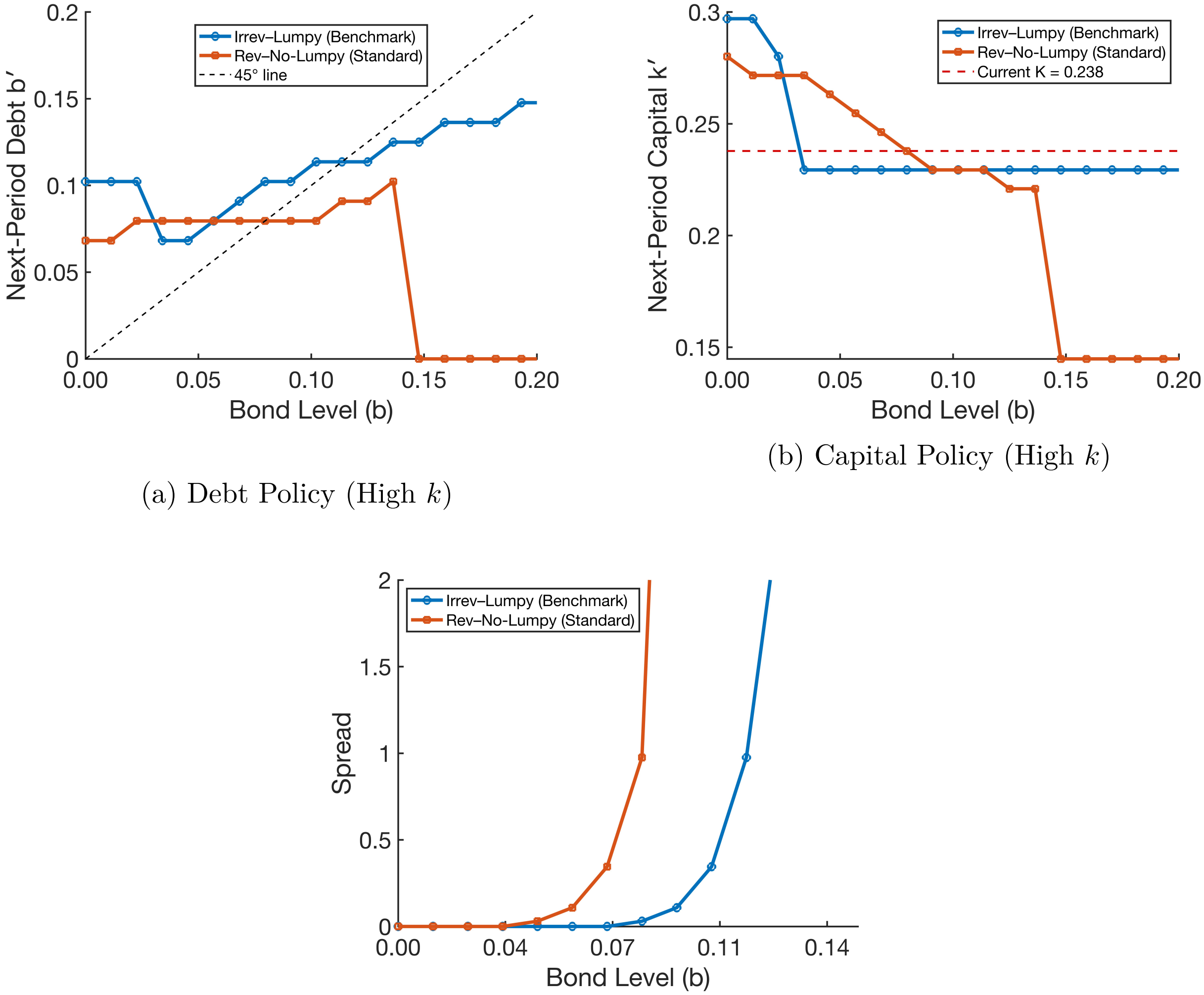
(b) Capital Policy (Low k)



(c) Spreads (Low k)

Figure 9: Benchmark vs Standard Model at Low Capital Level

The discussion about the individual impact of lumpiness and irreversibility suggests that we should expect irreversibility to play a larger role in any differences between the Benchmark and Standard models. The patterns from the debt and spread policy rules show that for both capital levels, the Standard model consistently exhibits higher sovereign spreads than the Benchmark model while the debt-policy functions show that the Benchmark economy borrows as much as, and frequently more than, the Standard model at almost all debt levels prior to default. Moreover, as capital increases, this spread gap widens. The Benchmark



(a) Debt Policy (High k)

(b) Capital Policy (High k)

(c) Spreads (High k)

Figure 10: Benchmark vs Standard Model at High Capital Level

economy's spread schedule shifts further to the right at higher capital, while the Standard model's schedule moves very little, indicating that the risk-reducing effect of capital accumulation is largely nullified by reversibility. The Benchmark economy's borrowing capacity also increases substantially, while that of the Standard model rises only modestly in the low-debt region.

The capital-policy functions show minor but still noticeable effects from having the fixed investment cost versus the capital adjustment costs. In the Benchmark economy, investment occurs in concentrated episodes due to the fixed cost, leading the government to invest more intensely than in the Standard model at low debt but to also stop investing at debt level where the Standard model still displays an increase in the capital stock. As before, the horizontal dashed line indicates the current capital stock. In the Standard model, the quadratic capital adjustment cost ensures that the government adjusts capital more smoothly and gradually. The impact of reversibility, while somewhat constrained by the adjustment cost can be seen at debt levels consistent with default. These can be found when $b' = 0$ in the Standard model. At these levels of debt, the capital policy k' shows a sharp drop below current capital levels, suggesting a substantial liquidation of public capital in default for consumption smoothing purposes.

The differences between the two models become more prominent at higher capital levels. When the economy has accumulated more capital, both models reduce their incentive to invest further, but the Benchmark model retains its higher borrowing capacity and lower spreads. In contrast, the Standard model begins disinvesting at much lower debt levels. The ability to reverse capital therefore alters the intertemporal trade-offs faced by the government: in the Standard model, capital becomes a substitute for debt in smoothing consumption, leading to early disinvestment, limited borrowing, and the early onset of high spreads. Figure 11 illustrates this by showing the default regions of both models at the low and high capital. In the figure we can see that the default boundary of the Benchmark model (blue) lies strictly to the right of that of the Standard model (red), implying a strictly smaller default region. Moreover, the reduction in the default region at higher capital is far more prominent under irreversibility: the Benchmark model boundary shifts substantially to the right, while the Standard model boundary moves little and in places overlaps almost exactly with its low-capital counterpart.

Implications of Investment Frictions A key takeaway from comparing the Benchmark and Standard models is that standard models with reversible capital and convex rather than non-convex adjustment costs may substantially overstate default risk if irreversibility and lumpiness are important in the real world. In models where governments can freely disinvest, capital provides an additional consumption-smoothing channel, weakening the repayment capacity and incentive, making economies appear riskier than they truly are. Our results demonstrate that modeling capital using standard, reversible accumulation rules yields lower borrowing and higher default risks than when these frictions are included.

These changes in debt, default and investment dynamics and the resulting lower borrowing costs influence welfare in the two economies. Figure 12 compares the value functions of the Benchmark and Standard models at the mean productivity and debt levels. Across the entire capital dimension, the Benchmark model yields uniformly higher lifetime utility, with an average welfare gain of 0.43% relative to the Standard economy.

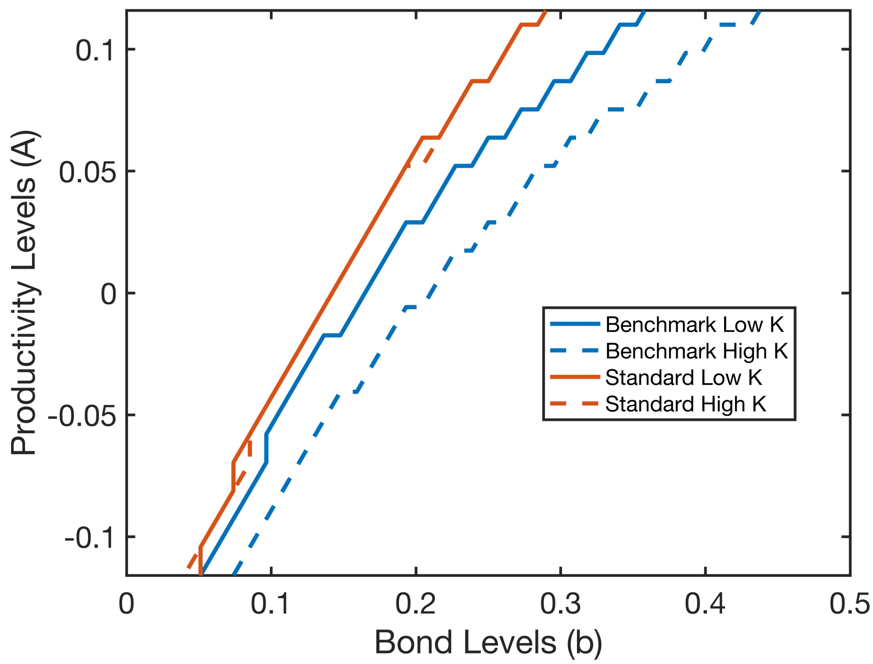


Figure 11: Default region boundaries under the Benchmark and Standard models.

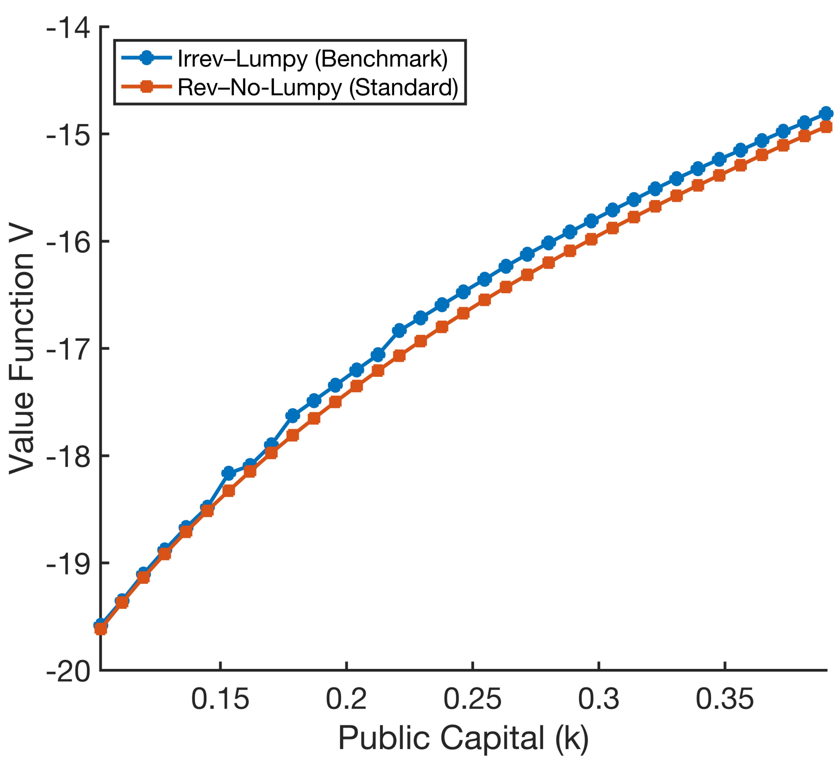


Figure 12: Value Functions:$V_{Benchmark}$ versus $V_{Standard}$

3.4 Empirical Validation

To assess whether the results associated with the Benchmark model are reflected in sovereign borrowing data, we examine the relationship between sovereign spreads and a country's debt and public investment. The analysis uses a panel of emerging economies covering the period 1973–2014. [14] Following Catão and Mano (2017), we specify the sovereign spread as a function of observable macroeconomic fundamentals and country fixed effects:

$$s_{i,t} = \alpha_i + F_{i,t}\beta + \epsilon_{i,t}$$

where $s_{i,t}$ denotes the sovereign spread for the country i in year t, α_i is the country fixed effect, $F_{i,t}$ is a vector of macroeconomic fundamentals and $\epsilon_{i,t}$ represent errors. We adopt the baseline set of fundamentals used in Catão and Mano (2017) and extend it by including the public investment-to-GDP ratio, and measures of investment spikes. These additions allow us to directly test the empirical relevance of our model's key mechanisms, specifically, how variations in public capital and investment affect sovereign spreads across emerging economies.

Sovereign Spreads, Debt, and Public Capital. Table 6 reports the results of the panel regressions. Column (1) reproduces the baseline specification from Catão and Mano (2017). Consistent with their findings, most of the well-established correlates of country risk, such as debt-to-GDP, GDP growth, international reserves, and market volatility, are highly significant and display the expected signs.

Column (2) and (3) introduce a dummy variable capturing investment "spikes" (defined as i/k $>$ 0.10 in (2) and i/k $>$ 0.08, in (3)). Column (4) replaces the investment spike dummy variables with the continuous public investment-to-GDP ratio. All three investment variables are negative and statistically significant, indicating that periods of higher public investment are associated with lower sovereign spreads. This finding is consistent with the model mechanism in which productive public investment reduces default risk.[15]

[14] We restrict the sample to the set of emerging economies identified in Catão and Mano (2017) for consistency.

[15] We do not claim that these results imply causality since in both the models and the empirical work, it is clear that spreads, debt, capital, investment and output are all simultaneously determined in equilibrium.

Table 6: Sovereign Spread Regressions — Country Fixed Effects

	Dependent variable: Sovereign Spread			
	(1)	(2)	(3)	(4)
Public Debt / GDP	2.369***	2.232***	2.074***	2.102***
	(0.444)	(0.519)	(0.542)	(0.547)
Real GDP Growth (3y MA, %)	−0.116***	−0.110***	−0.104***	−0.112***
	(0.032)	(0.033)	(0.033)	(0.036)
Reserves / GDP	−6.520***	−6.599***	−6.642***	−6.445***
	(1.710)	(1.709)	(1.703)	(1.616)
Fixed Exchange Rate	−0.356	−0.333	−0.337	−0.285
	(0.268)	(0.266)	(0.268)	(0.259)
Market Volatility	0.054***	0.052***	0.052***	0.051***
	(0.011)	(0.011)	(0.011)	(0.011)
Current Account / GDP	−2.537	−2.840	−2.607	−3.358*
	(1.595)	(1.773)	(1.763)	(1.966)
Primary Issuance	−1.225***	−1.320***	−1.325***	−1.272***
	(0.346)	(0.353)	(0.356)	(0.340)
Loan Issuance	−1.233***	−1.114***	−1.083***	−1.094***
	(0.390)	(0.384)	(0.380)	(0.359)
Investment Spike ($i/k > 0.10$)		−0.363**		
		(0.174)		
Investment Spike ($i/k > 0.08$)			−0.491***	
			(0.183)	
Public Investment / GDP				−0.125***
				(0.019)
Country FE	Yes	Yes	Yes	Yes
Observations	952	934	934	933
R^2	0.331	0.327	0.331	0.345
Adjusted R^2	0.294	0.289	0.293	0.309

Notes: Robust standard errors clustered by country in parentheses. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

4 Conclusion

We build a model of a small open developing economy in which the government accumulates public capital while borrowing long term debt on international markets. The accumulation process is subject to two frictions that are often associated with infrastructure development: irreversibility of capital and lumpy investment. Since government debt is subject to default, the model allows us to study the interaction of default incentives with the public capital accumulation process and the interplay of investment with debt dynamics.

We show that both capital irreversibility and lumpiness lower default risk, both individually and together and encourage borrowing for investment purposes. Together, these features help explain why welfare is higher in the Benchmark model than a similar model with a more standard accumulation process. We also show that economies with low levels of public capital primarily use external debt to fund investment, whereas more mature economies with higher levels of capital, primarily use debt for consumption smoothing. As a result, the borrowing costs of low capital economies are lower (for a given debt to GDP ratio) than those that are further along in the capital accumulation process.

These results suggest that if lumpy investment and irreversibility are important aspects of public capital accumulation in the real world, then standard sovereign default models with capital may overstate the default risk of developing economies. Implicitly the results also imply that what matters for debt sustainability is not simply how much governments borrow, but what they borrow for. Borrowing that finances irreversible, output-enhancing infrastructure strengthens the sovereign's repayment ability and moderates default risk, while debt used for non-productive public or private consumption smoothing does not. It also suggests that potentially lenders can charge lower interest rates for infrastructure loans as compared to bonds that finance general government spending.

A Data Details

The analysis uses an unbalanced panel of 43 countries observed from 1973 to 2014, the period for which sovereign spread data are available. Annual series for public investment and public capital stock are obtained from the IMF Investment and Capital Stock Dataset (ICSD) and converted to comparable real units. For each country–year, we compute $\frac{I}{K} = \frac{\text{General Government Investment}}{\text{General Government Capital Stock}}$. The resulting panel of I/K ratios provides the basis for identifying investment spikes following the nonconvex investment literature (e.g., Cooper and Haltiwanger 2006; Khan and Thomas 2008). Only observations corresponding to the 43 emerging economies found in the sovereign spread dataset are retained.

Countries included: Argentina, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Mexico, Panama, Peru, Uruguay, Venezuela, Jamaica, Jordan, Lebanon, Egypt, India, Indonesia, Malaysia, Pakistan, Philippines, Thailand, Gabon, Morocco, Tunisia, Bulgaria, Russia, China, Ukraine, Czech Republic, Slovakia, Estonia, Latvia, Hungary, Lithuania, Croatia, Poland, Romania, Turkey, South Africa.

B Productivity Mapping

To simulate the model using Mexico's productivity shocks (1973–2014), we map each year's adjusted Solow residual to the closest node in the discretized productivity grid. The figure below compares the observed Solow residuals with their nearest grid-mapped counterparts, showing that the discretized process closely approximates the data.

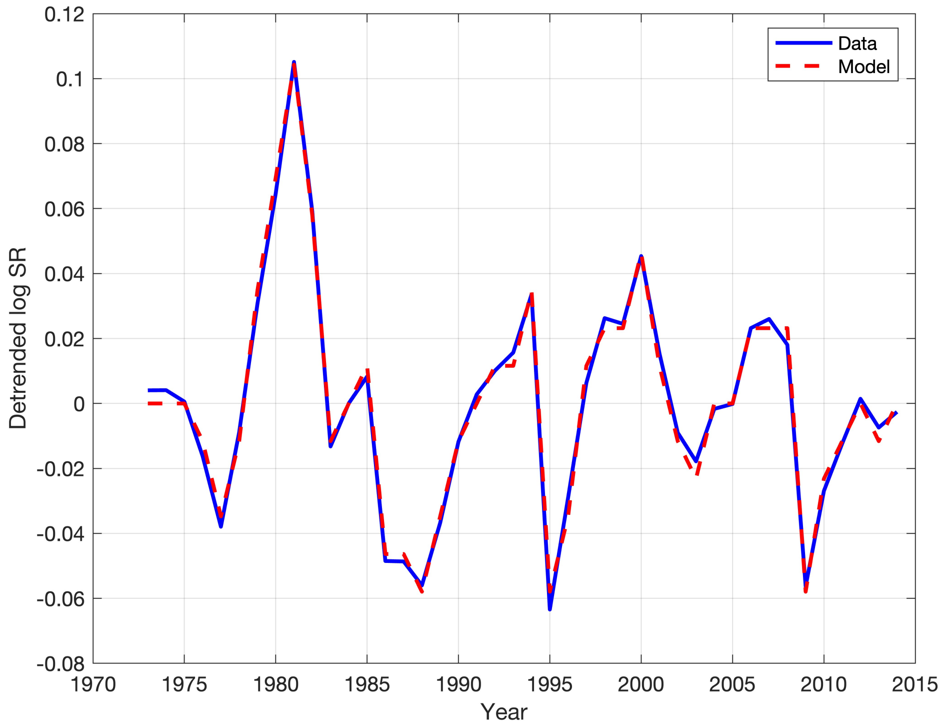


Figure 13: Observed vs. Grid-Mapped Solow Residual

C Value of Repayment

In Figure 14, we plot the spread between the value function of repayment and the value function of default. This spread measures the relative values of repayment: if $V_{nd} - V_d > 0$ then it is optimal to repay, and vice versa. When debt is low, the value of repayment is decreasing in the level of capital, as the loss from default becomes trivial as the economy grows. On the contrary, when debt level is high, the value of repayment is increasing in capital level, as the higher capital enhances the economy's ability for repayment. This figure is consistent with Figure.6 in Gordon and Guerrón-Quintana (2018).

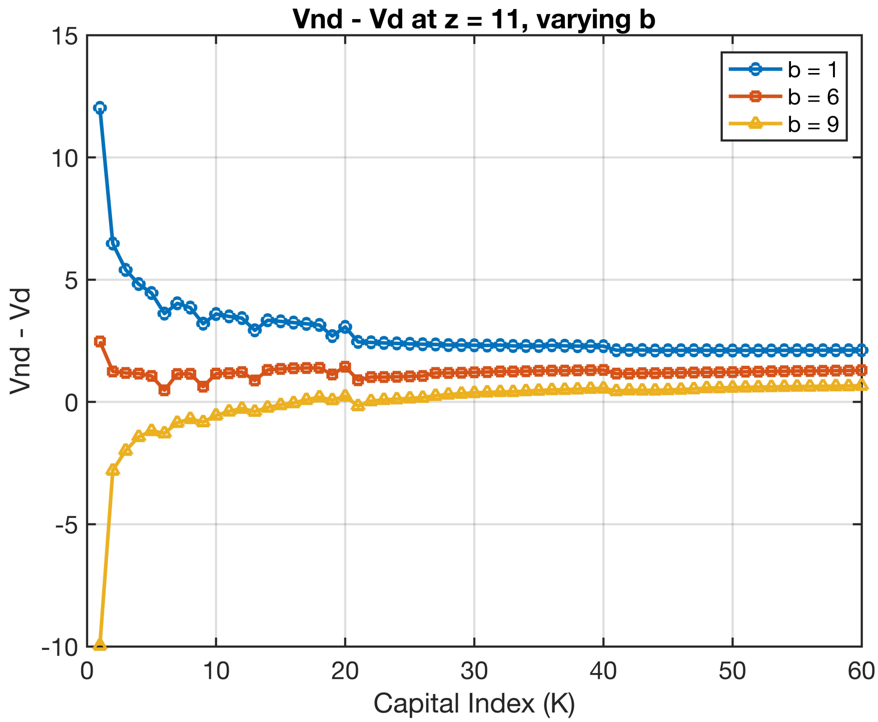
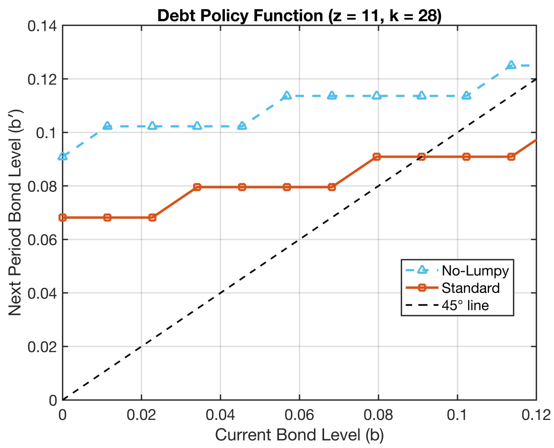


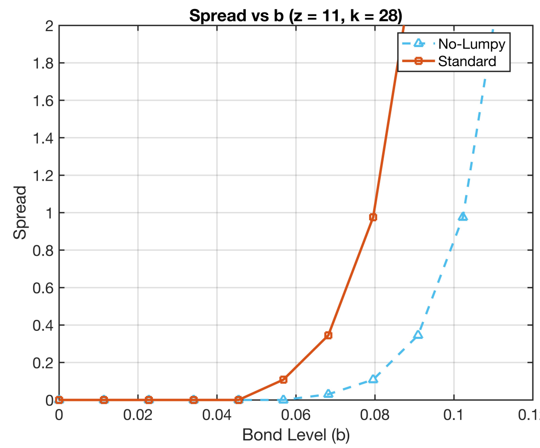
Figure 14: Value Function Spread

D Irreversibility on No-Lumpy Models

Here we focus on the Rev–No-Lumpy (Standard) model versus the Irrev–No-Lumpy model, which isolates the effect of irreversibility under smooth investment. These patterns are consistent with the observations we made on the simulated model moments and in the pattern observed from the lumpy models, which indicates that capital irreversibility alone contributes to a reduction in default risk and reflects a direct consequence of lower borrowing costs: because irreversibility reduces spreads, borrowing becomes cheaper, thereby incentivizing governments to issue more debt.



(a) Controls for no-lumpiness



(b) Controls for no-lumpiness

Figure 15: Irreversibility and Lumpiness: Effects on Sovereign Spread and Borrowing Policy

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