The Unsettling Behavior of Exchange Rates Under Inflation Targeting*

Paul Beaudry[†]
University of British Columbia and NBER

Ippei Fujiwara[‡]
Keio University and CEPR

Yasuo Hirosi[§]
Keio University

Amartya Lahiri[¶]
University of British Columbia

July 10, 2025

Abstract

Over the last few decades, many central banks have adopted an inflation targeting framework and this has generally been associated with reduced inflation variability. In this paper we examine how inflation targeting has changed the behavior of exchange rates and we uncover a rather curious pattern. Using a large set of countries, we find that as countries switched to inflation targeting their currencies became tied to the price of oil, that is, under inflation targeting currencies tend to appreciate with rising oil prices while prior to inflation targeting regime they did not exhibit such a relationship. Importantly, this data pattern is observed independent of whether the country is a net oil exporter or importer. We argue that such a pattern may reflect that, under inflation targeting, the equilibrium dynamics for the nominal exchange rate becomes indeterminate when uncovered interest parity (UIP) *does not hold*. In such situations, oil prices may well act as a focal point for currency pricing decisions.

JEL Classification: E4, F4

Keywords: Inflation targeting, exchange rate, commodity currency, indeterminacy

*We would like to thank Wesley Ng and Rishab Devnani for excellent research assistance. Our thanks also to seminar participants at Columbia University, ISI-Delhi, RBI, SED conference in Mexico City, and the World Bank. Both Beaudry and Lahiri acknowledge research support from Social Sciences and Humanities Research Council of Canada. View expressed here are personal and do not reflect the views of ay organization.

[†]E-mail address: paul.beaudry@ubc.ca.

[‡]ippei.fujiwara@keio.jp

[§]yhirosi@keio.ac.jp

[¶]E-mail address: amartya.lahiri@ubc.ca.

1 Introduction

Inflation targeting has become an increasingly popular monetary policy operating regime over the past three decades. Starting from New Zealand's initial adoption in 1989, the list of inflation targeting countries has now grown to over thirty. The baseline versions of inflation targeting regimes essentially stipulate an inflation target for the central bank, the index to be used for measurement, some communication and review protocols, and a few exceptional conditions in which the central bank may deviate from its basic goal (such as maintaining financial stability). The proponents of inflation targeting regimes laud the clarity of the regime as well as its positive effect on anchoring private sector expectations.

Inflation targeting has proved to be remarkably successful in maintaining low and stable inflation in countries that adopted it. This partly explains its rising popularity over time. However, it also has some interesting ancillary policies. One of them is the fact that central banks that target inflation typically allow their exchange rates to float freely. The idea underneath is that in an environment with inflation targeting, fluctuating exchange rates would allow the system to accommodate shocks to the goods market that require a change in the real exchange rate. While being good in theory, it remains an open question of how inflation targeting actually affects exchange rate behavior. The object of this paper is to (1) document how exchange rate behavior within a country changed between pre and post inflation targeting and (2) offer an explanation to the observed pattern.

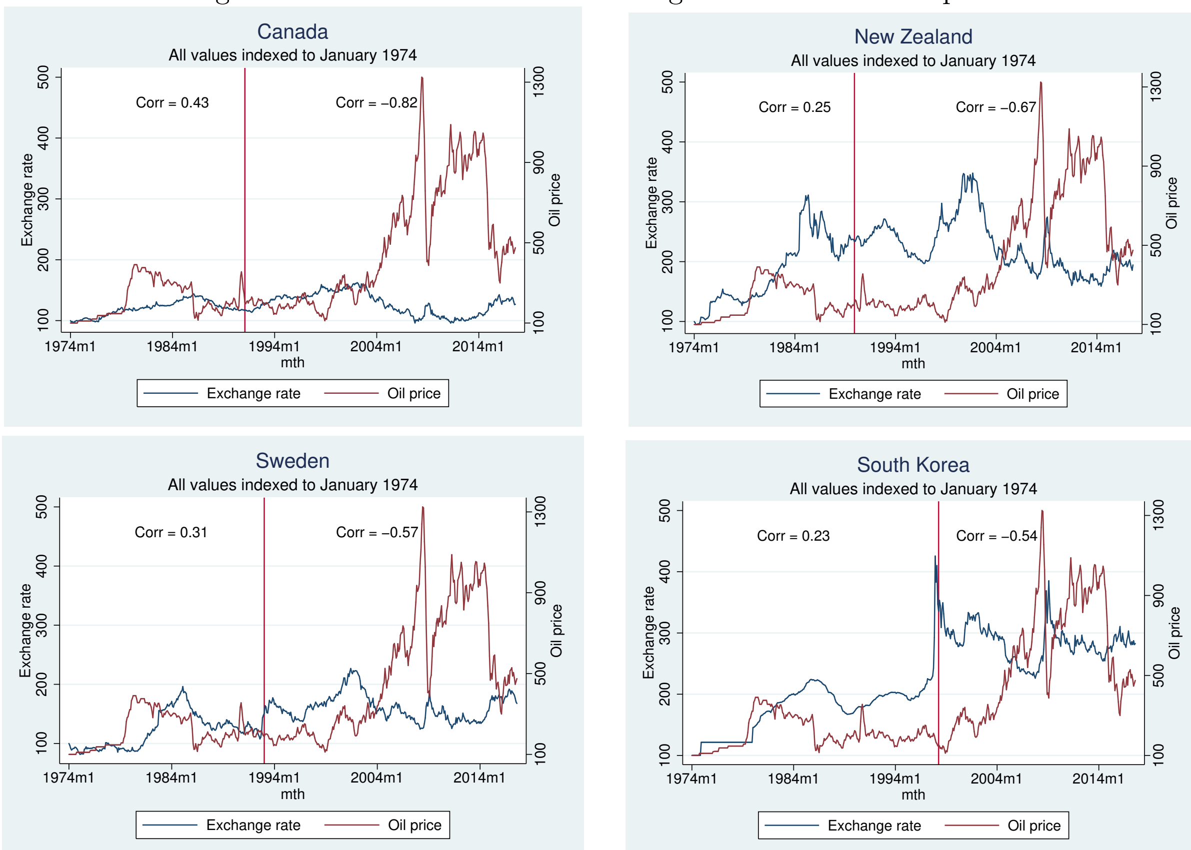
In the first part of the paper we uncover a startling data pattern: countries that adopted inflation targeting as their monetary policy framework have seen their currencies becoming systematically linked to the world oil price. Specifically, the currencies of inflation targeting countries tend to appreciate when the world oil price rises and depreciate when the oil price falls. Crucially, this relationship between the exchange rate and world oil prices emerged only after these countries switched to inflation targeting. We find that this empirical relationship is independent of whether or not the country is an oil exporter or importer, and is not driven by time or decade specific affects. This result was quite unexpected, at least to us, and calls for an explanation. In the second part of the paper we propose the elements of one potential answer. Our explanation builds on the observation that in open economies, inflation targeting when combined with the failure of interest parity tends to render the equilibrium dynamics indeterminate. We provide a two dynamic small open economy examples to demonstrate this point. Since in such a situation the exchange rate is no longer pinned down by fundamentals, there is room for exchange rate behavior to become tied to arbitrary forces. Given that many of the early adopter of inflation targeting where commodity exporters, having exchanges rate expectations under inflation targeting become tied to oil prices may have emerged as a natural focal point.

Some of the bizarre or potentially excessive fluctuations of exchanges rates under inflation targeting have recently attracted attention. For example, Canada witnessed a dramatic appreciation of its currency between 2005 and 2013 as world oil prices rose and then an even sharper depreciation since 2015 as oil prices declined. This may not be too surprising as Canada is a oil exporter. Such exchange rate movements may provide implicit exchange rate risk hedging to resource based industries whose prices and revenues are in US dollars but costs are in local currency. Canada, however, was by no means unique in witnessing this oil linked currency cycle. Thus, the Swedish Krona, for example, had a similar cycle. It appreciated from over 9 to 6.4 kronas per US dollar between March 2009 and March 2014 but then began a sharp secular depreciation that left the currency at just over 9 kronas per US dollar by March 2017. Figure 1 shows the exchange rates of four inflation targeting countries from four different continents who adopted inflation targeting at different times. The outcomes are quite similar. While the exchange rates used to typically depreciate during periods of oil price increases, the relationship flipped after they adopted inflation targeting with the correlation becoming negative, i.e., the exchange rate tended to appreciate when oil prices rose. It seems that inflation targeting converted previously non-commodity currencies into commodity currencies.

In general, one would expect the relationship between exchange rates and oil prices to be dependent on country specific factors such as whether the country imports or exports oil (or energy more generally), amongst other possible factors. Indeed, in a broader sample of twenty two inflation targeting countries, Figure 2 shows that the exchange rate and the world oil price exhibited a mix of positive and negative correlations across the sample before these countries adopted inflation targeting. The average of the country correlations during the pre-inflation targeting phase was 0.04. Intriguingly, the correlation turns negative for most of the countries after they adopted inflation targeting with the average of the country correlations becoming -0.57.

The evidence presented above raises two key questions. First, does the puzzling effect of inflation targeting on the relationship between exchange rates and the world oil price generalize to the full sample of inflation targeting countries? We find that it indeed does generalize. Moreover, it also generalizes beyond oil prices to world energy prices as well. Second, given that it does generalize, is the identified effect of inflation targeting actually representing the effect of a country's reliance on

Figure 1: Correlation between exchange rate and world oil prices

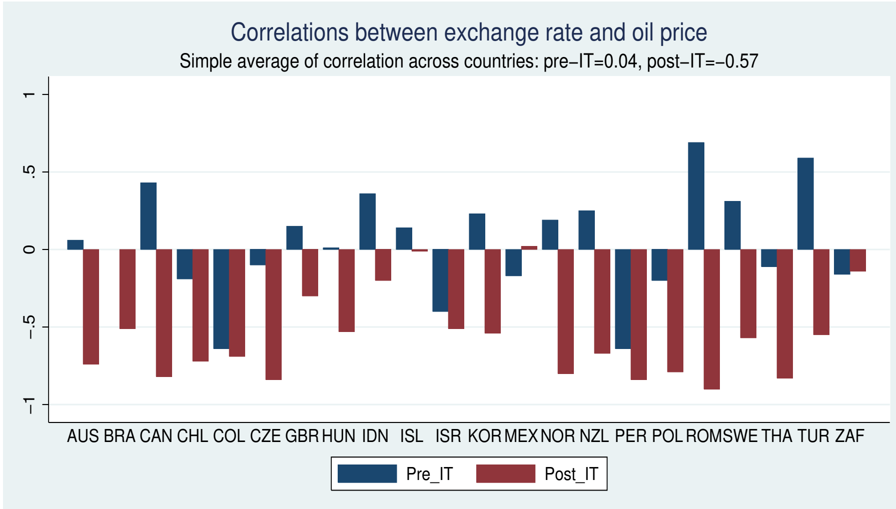


Note: The figure plots the world oil price and the nominal exchange rates of Canada, New Zealand, Sweden and South Korea over time. Exchange rates are local currency units per US dollar which are then indexed to their January 1974 value. The vertical line indicates the date when the country switched to inflation targeting. Corr indicates the correlation coefficient between the exchange rate and the world oil price. The numbers to the left of the vertical lines are the correlations before inflation targeting and the number to the right of the line is the correlation post inflation targeting.

oil exports or some time specific effects related to when most countries adopted inflation targeting? We find that the result is robust to controlling for the country's dependence on oil exports, as well as to country and time effects.

If the relationship is not being driven by factors such as oil exports or time effects, how does one explain it? We provide an explanation that is based on the interaction between a monetary policy regime of inflation targeting and the widely documented failure of uncovered interest parity. We show that the joint impact of inflation targeting and failure of uncovered interest parity renders indeterminate the equilibrium dynamics of an open economy. Intuitively, allowing the exchange to fluctuate freely in an environment with inflation targeting with no aggregate nominal anchor makes any given nominal interest rate consistent with a continuum of different levels of the domestic

Figure 2: Correlation between nominal exchange rates and world oil price



price level and the nominal exchange rate. In such environments, markets could focus on oil or commodity prices more generally to price such an asset. It is important to clarify here that our proposed explanation for the data fact unearthed here is by no means intended to be the only possible explanation. There may well be other complementary explanation as well.

Our work is focused on the interaction between monetary policy, commodity prices (generally) and its effect on the nominal exchange rate. As such it is related to at least three different bodies of research. The first is the long and distinguished work on forecasting exchange rates going back to Meese and Rogoff (1983). The bulk of this literature finds that it is hard to forecast exchange rates using structural models any better than the random walk model. A recent paper that is particularly relevant for our message is by Devereux and Smith (2017) who show that the observed contemporaneous correlation between commodity prices and exchange rates can be rationalized by incorporating the fact that changes in commodity prices impact future monetary policy which in turn, affects the current exchange rate. The second related literature is the work on uncovered interest parity. This work tends to find that uncovered interest is often violated in simple data tests, a feature that we build on in our theoretical model. A recent updated overview of this literature along with the implications for monetary policy can be found in Engel et al. (2017). The third strand of work that relates to us is the research on the macroeconomic effects of monetary policy rules, particularly the Taylor rule. An overview of this body of work can be found in Woodford (2003).

In the next section we present the basic data fact. Section 3 presents the model which we use to illustrate our explanation for the data fact. Section 4 presents some evidence of the failure of uncovered inteest parity in our sample of inflation targeting countries while the last section concludes.

2 The Empirical Relationship

Our interest is in systematically teasing out the relationship between oil prices and the exchange rates of countries that have chosen inflation targeting as their preferred monetary policy regime. In order to uncover this relationship we examine monthly data for sample of twenty seven countries which chose to adopt inflation targeting at some point. We examine this relationship using monthly data between January 1974 and August 2017.

2.1 Data

The list of countries along with the date on which these countries switched to inflation targeting is taken from Hammond (2012). Table 3 gives the names of the countries along with the year and month in which they adopted the inflation targeting regime. The oil price series we use is the spot rate of WTI crude taken from the FRED database of FRB St. Louis. In some of our specifications, we control for the economic dependence on oil of the countries in our sample. For this we use net exports of oil as a share of GDP. Our oil trade data comes from the United Nations COMTRADE database.

Since the primary aspect of our exercise is to determine the behavior of exchange rates, we need to select countries that actually allow their exchange rates to fluctuate in response to market pressures. We use the updated version of Reinhart and Rogoff (2004) to classify countries into flexible exchange rate regimes. Specifically, we use their fine classifications 11-14 as indicators of flexible exchange rate regimes. Clearly, countries could have periods where they are classified as flexible and other periods where they are not. For our empirical analysis we only consider years for which a country had a flexible exchange rate. Our monthly exchange rate data comes from International Financial Statistics of the IMF. Details regarding the data are provided in the Appendix.

2.2 Empirical results

We examine the empirical relationship between exchange rates and oil prices in two different ways. We first study the individual country level impulse responses of exchange rates to oil price shocks identified from a three variable vector autoregression model. We then study the relationship between exchange rates and oil prices using panel regressions that control for various possible confounding factors including oil exporter status, country fixed effects and time effects.

2.2.1 Vector AutoRegressions (VAR)

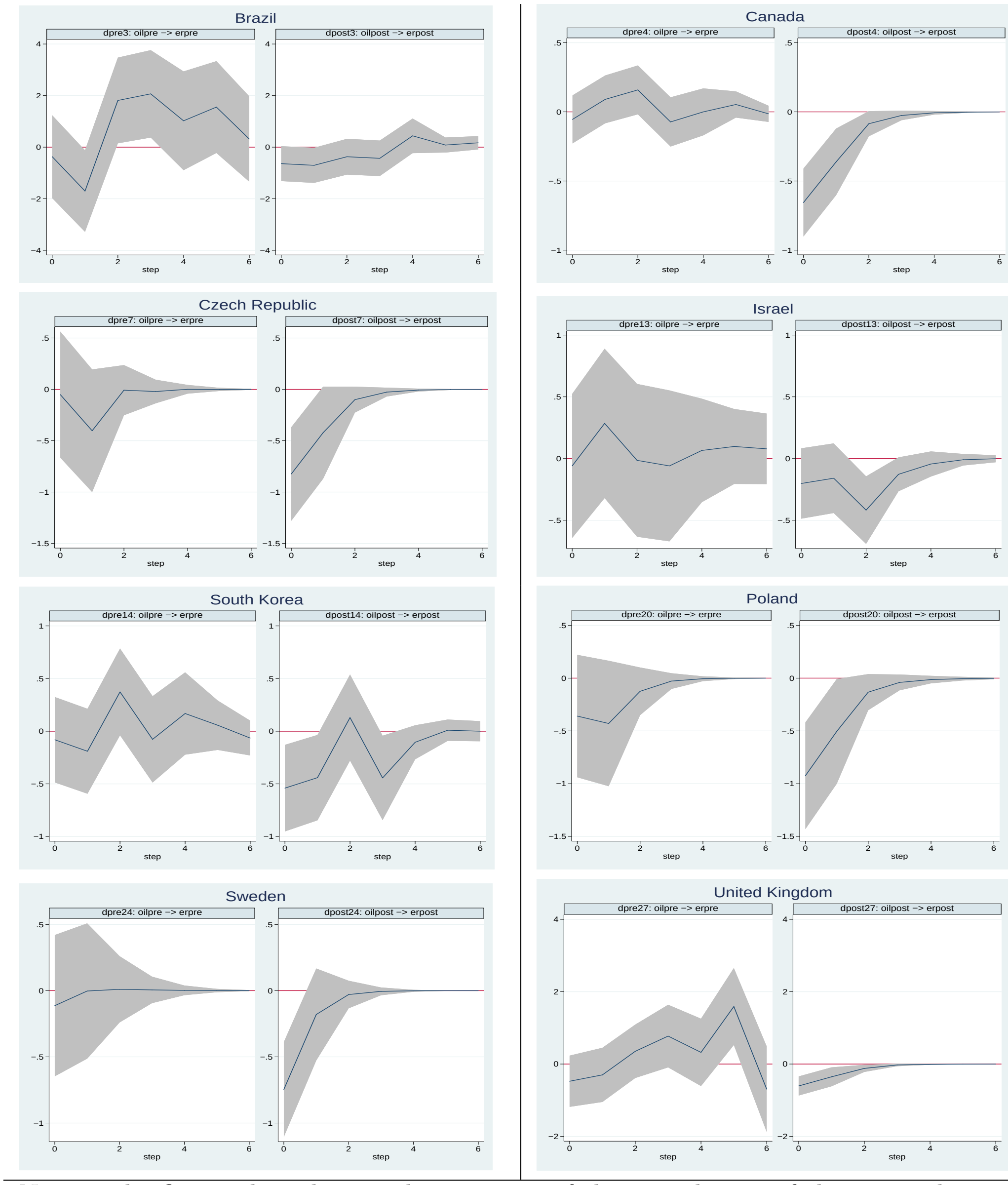
We start by estimating a three-equation VAR model for each country in our sample. Specifically, the model includes the exchange rate, the consumer price index and the world oil price. We estimate the model in growth rates rather than levels since the levels of prices are often non-stationary. For each country we estimate the VARs separately for the pre-inflation targeting and post-inflation targeting periods. Our identification scheme involves assuming that the exchange rate is most endogenous while world oil prices are most exogenous. There were insufficient observations to estimate the pre and post inflation targeting priods separately for Armenia, Australia, Chile, Colombia, Ghana, Guatemala, Indonesia, New Zealand, Peru, Philippines, Romania and Serbia. This left us with estimated models for fifteen countries. The impulse responses of the exchange rate to world oil price shocks from the estimated VAR models are shown in the figures below.

Figure 3 shows the impulse response of the exchange rate to a one unit shock to the world oil price. The shaded areas depict the 95 percent confidence intervals. The striking feature of the Figure is the sharp contrast between the pre and post IT periods n the impact effect of an oil shock on exchange rates. While in the pre-IT period the effect of an oil shock on the exchange rate is insignificantly different from zero, in the post-IT period the impact effect is significantly negative in seven out of the eight countries.

Figure 4 shows the corresponding impulse responses of exchange rates to oil price shocks in the countries that were late adopters of inflation targeting. Just as in the early adopters, in this group too the impulse responses are strikingly different in the pre and post inflation targeting periods. In six out of the seven countries, the exchange rate appreciates significantly on impact of an oil price shock during the post inflation targeting period whereas during the pre inflation targeting phase the exchange rate response was statistically insignifcantly different from zero.

We find these results quite striking both in terms of the contrast between the pre and post infla-

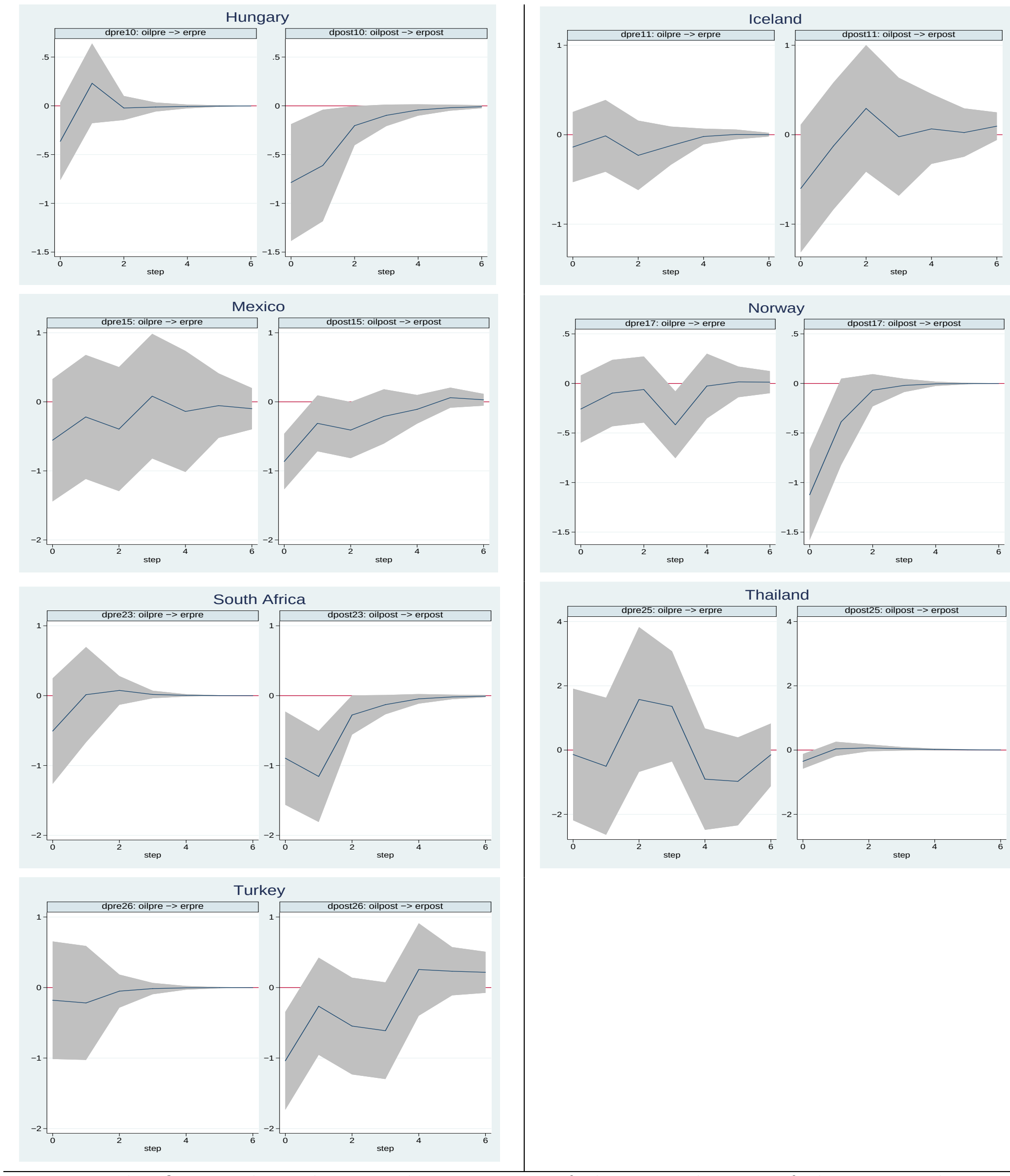
Figure 3: Exchange rate response to oil price shock in early IT adopters



Notes: The figure plots the impulse response of the growth rate of the nominal exchange rate to a one unit shock to the growth rate of the world oil price from a three variable VAR model including the exchange rate, consumer price index and the world oil price (all in growth rates). The Choleski ordering for identification is oil price, CPI, exchange rate. The shaded regions depict 95 percent confidence intervals around the impulse response functions.

tion targeting periods but also for their similarity across different groups of countries that adopted inflation targeting at very different times and which have very different country characteristics.

Figure 4: Exchange rate response to oil price shock in late IT adopters



Notes: The figure plots the impulse response of the growth rate of the nominal exchange rate to a one unit shock to the growth rate of the world oil price from a three variable VAR model including the exchange rate, consumer price index and the world oil price (all in growth rates). The Choleski ordering for identification is oil price, CPI, exchange rate. The shaded regions depict 95 percent confidence intervals around the impulse response functions.

2.2.2 Panel Regressions

The VAR results, while instructive and interesting, leave open some important issues. Thus, the results could be driven by the dependence of these countries on oil. Specifically, oil price shocks could impact countries differently depending on whether they are oil importers or exporters. One might also wonder if time effects related to when these countries adopted inflation targeting may be driving some of the results.

To test the general effect of inflation targeting on the relationship between the exchange rate and world oil prices while controlling for different confounding factors, we estimate the following baseline panel regression:

$$E_{it} = \alpha + \beta_1 Oil\_price_t + \beta_2 Oil\_price_t * IT_{it} + \varepsilon_{it}$$

where Oil_price_t is the world oil price (in dollars) at date t and IT_{it} is a dummy variable that takes value one if country i at date t is an inflation targeter, and zero otherwise. In running the regressions we also add in country fixed effects as well as quinquennial dummies to control for time effects that might also be important in driving the relationship between oil prices and exchange rates. The coefficient of interest for us is β_2 in this regression.

The column labeled (1) in Table 4 presents our baseline results. The coefficient on the interaction term is negative and significant at the 1 percent level indicating that relative to non-inflation targeting countries, exchange rates of inflation targeting countries tend to appreciate when the world oil price rises. Indeed, this appreciation of the currency in inflation targeting countries is not just relative to non-inflation targeting countries but also in absolute terms (the sum of the coefficients on the oil price and the oil price and inflation targeting dummy interaction term is negative). Importantly, the effect of oil prices on the exchange rates of non-inflation countries is insignificant. Clearly, the simple correlations and visual impressions conveyed by Figures 1 and 2 generalize to more formal econometric methods where one includes controls for other variables as well as country and time effects.

An immediate concern regarding our baseline results is that they may be picking up the fact that a number of inflation targeting countries may be oil exporters and the estimates may just be indicating that the exchange rate tends to appreciate in oil exporting countries when the world oil price rises. Since oil prices are denominated in dollars, higher dollar earnings of oil exporting countries likely increase the demand for their own currency and cause an appreciation. Column (2) of the Table adds four additional regressors: an oil exporter dummy which takes the value one in every period in which the country is a net exporter of oil and zero otherwise; an interaction between the oil net exporter dummy and the inflation targeting dummy; an interaction between the oil exporter dummy and oil prices; and a triple interaction between the oil exporter dummy, the inflation targeting dummy and the oil price. The regression coefficient on the triple interaction gives the differential effect of oil prices on the exchange rate in inflation targeting regimes that are net oil exporters (relative to oil importers).

The results in Column (2) of the table show that the baseline results are robust to controlling for the net oil exporter status of the country. While higher oil prices do tend to appreciate the currency more in inflation targeting countries that are net oil exporters (relative to oil importing inflation targeters), the exchange rate appreciating effect of higher oil prices in inflation targeting countries becomes even stronger after controlling for the oil exporter status of the countries in the sample.

Oil prices tend to be very correlated with commodity prices and energy price indices in general. An immediate check for the robustness of our results is to examine if the results carry over to broader measures of world energy prices. Columns (3) and (4) of Table 4 re-run the regressions in Columns (1) and (2) but with the world energy price index instead. The results are very similar to those obtained from the oil price regressions. Higher energy prices cause significant exchange rate appreciations in inflation targeting countries both relative to non-inflation targeting countries and in absolute terms. Contrarily, the exchange rates of non-inflation targeting countries are statistically unaffected by changes in world energy prices. Column (4) of the Table shows that these results are robust to controlling for the energy net exporter status of the countries in our sample. Interestingly, there are no statistically differential effects of energy prices on the exchange rate of energy exporting inflation targeters relative to energy importing inflation targeting countries.

A potentially confounding issue with running regressions of nominal exchange rates on other prices such as world oil prices is the possible non-stationarity of the two series. To alleviate concerns regarding spurious inference from regressions of levels on levels of non-stationary time series, we also ran the specification with both exchange rates and oil prices in growth rates rather than levels. The results are reported in Table 5. The main result to note is that the coefficients on the interaction term between oil prices and the inflation targeting dummy remain negative and highly significant as do the coefficients on the interaction term between the growth rate of energy prices and the inflation targeting dummy. We conclude from these regression that our inference regarding inflation targeting converting currencies into commodity currencies is not spuriously driven by the non-stationarity of exchange rates and oil prices. The relationship emerges even in stationary specifications where these variables enter in growth rates rather than levels.

The set of countries that have adopted inflation targeting can be broadly broken into to groups – the early adopters who adopted before 2000 and the later adopters who waited till after 2000. Table 3 gives the list of the countries along with the date on which they adopted the regime. The early adopters tended to be more developed than the later adopters. One might wonder whether our results are being driven by specific characteristics of countries that belong to one of these two groups rather than revealing anything about inflation targeting itself. To check this we ran the baseline regressions on the early and late adopters separately. Table 6 reports the results for regressions both in levels of exchange rates and oil prices as well as in growth rates of the two variables. Our baseline results for the entire sample clearly hold within each sub-sample of early and late adopters. In both sets of countries, the coefficient on the interaction term between oil prices and the inflation targeting dummy in the level regression is negative and significant at the 1 percent level. The corresponding coefficient in the growth rate regressions are also negative for both groups. The only specification in which the oil price and IT dummy interaction term is insignificant is for the late adopter sub-sample. This is due to the greater imprecision of the estimate due to the somewhat smaller sample size of the late-adopter group.

2.2.3 Is it the USA?

Since all exchange rates are bilateral, one might logically wonder whether developments in the USA are driving the results rather than developments specific to the countries in our inflation targeting sample. To address this, we add the US Federal Funds rate to the list of regressors in our panel regressions. Tables 7 and 8 below show that the results are unaffected.

In summary, the regression results in Tables 4 and 5 confirm the simple negative correlations that summarized the relationship between oil prices and exchange rates in inflation targeting countries. This relationship is robust to including controls for net oil exporter status, country effects, time effects, split samples as well broader measures of energy prices. The fact is puzzling. Why does inflation targeting tend to convert currencies into commodity currencies?

3 An Explanation

Consider a small open representative household economy that produces a homogenous traded good and a continuum of non-traded goods. The household receives a constant endowment y^T in every period while non-traded goods are produced using the labor of household members.

The economy faces competitive world product markets where it takes the world price of the homogenous traded good as given. We normalize the world price of the good in terms of the foreign currency (USD) to one. Hence, the domestic price of the traded good is the nominal exchange rate E. E denotes the number of local currency units per unit of the foreign currency.

3.1 Households

The representative agent maximizes lifetime utility

$$V = \int_{t=0}^{\infty} e^{-\rho t} \left[\gamma \ln c_t^T + (1-\gamma) \ln c_t^N + \ln z_t - \frac{n_t^{1+\psi}}{1+\psi} \right] dt \tag{3.1}$$

where $z = \frac{M}{P}$ denotes real money balances in terms of the overall price index $P = \left(P^T\right)^\gamma \left(P^N\right)^{1-\gamma}$ and n denotes labor supply. In the following it will be convenient to also define real money balances in terms of the traded good:

$$m = \frac{M}{E}.$$

Note that using the price index gives the relationship between m and z as

$$z_t = e_t^{1-\gamma} m_t.$$

The agent's flow budget constraint is

$$\dot{b}_t = r b_t + w_t n_t + y_t^T - \frac{c_t^N}{e_t} - c_t^T - \dot{m}_t - \epsilon_t m_t + \tau_t + \Pi_t$$

where a dot over a variable denotes its time derivative, $w = \frac{W}{E}$ denotes the real wage, $e = \frac{E}{P}$ denotes the real exchange rate and $\epsilon = \frac{\dot{E}}{E}$ is the rate of depreciation of the domestic currency. b denotes foreign bonds denominated in terms of the traded good and r is the interest rate on foreign bonds which we assume to be constant. τ are lump-sum transfers received by the household from the government while Π denotes dividends received by households from firms.

Note that the budget constraint has been written in terms of the traded good which we use as the numeraire good throughout. This is with no loss of generality. Letting $a = b + m$ denote total household assets, we can rewrite the household flow budget constraint as

$$\dot{a}_t = r a_t + w_t n_t + y_t^T - \frac{c_t^N}{e_t} - c_t^T - (r + \epsilon_t) m_t + \tau_t + \Pi_t \tag{3.2}$$

The optimality conditions for the household's lifetime utility maximizing problem are:

$$
\begin{aligned}
\frac{\gamma}{c_t^T} &= \lambda_t \\
\frac{c_t^N}{c_t^T} &= \left(\frac{1-\gamma}{\gamma}\right) e_t \\
z_t &= \frac{c_t^T e_t^{1-\gamma}}{\gamma(r+\epsilon_t)} \\
n_t^{\psi} &= \gamma \frac{w_t e_t^{\gamma}}{c_t^T} \\
\dot{\lambda}_t &= (\rho - r)\lambda_t
\end{aligned}
$$

λ above is the co-state variable in the Hamiltonian for the household's problem. In the following we shall maintain the stationarity assumption $\rho = r$. This eliminates any secular movements in international bond holdings but does introduce a unit root into the dynamic structure of the model. This assumption implies that λ is constant along all perfect foresight paths which also implies that the consumption of tradables, c_t^T, is also constant over time.

3.2 Firms

There are two types of firms in the economy: final goods firms and intermediate goods firms.

3.2.1 Final goods firms

Final goods firms combine a continuum of intermediate goods of measure one to produce a non-traded final good using the technology

$$
y_t^n = \left[\int_{i=0}^{1} y_{it}^{\frac{\xi-1}{\xi}} di\right]^{\frac{\xi}{\xi-1}}
$$

These firms are perfectly competitive in all markets that they operate in. They maximize flow profits given by

$$
\Pi_t^n = P_t^n y_t^n - \int_{i=0}^{1} P_{it} y_{it} di
$$

The final goods firms optimality problem yields the demand function for intermediate goods:

$$
y_{it} = \left(\frac{P_{it}}{P_t^n}\right)^{-\xi} y_t^n
$$

This problem also gives the non-traded price index:

$$P_t^N = \left(\int P_{it}^{1-\xi} di \right)^{\frac{1}{1-\xi}} \tag{3.3}$$

3.2.2 Intermediate goods firms

Intermediate firms produce using the technology

$$y_{it} = A_t n_{it}$$

where A denotes aggregate labor productivity and n_i is labor employed by firm i. These firms are monopolistically competitive. They set prices to maximize profits. However, prices are sticky in that firms that change prices have to pay a quadratic price adjustment cost:

$$\Theta_t = \frac{\theta}{2} \left(\frac{\dot{P}_{it}}{P_{it}} \right)^2 P_t^n y_t^n$$

This formulation of Rotemberg price setting follows Kaplan et al. (2018). Here, the price P_{it} is the state variable for the intermediate firm since it is predetermined. The firm chooses $\dot{P}_{it}$ at every date t.

Letting η denote the co-state variable associated with the state variable P_{it} and ι_t denote the nominal interest rate, the optimality conditions associated with this problem are:

$$\begin{aligned} \eta_{it} &= \theta \frac{\dot{P}_{it}}{P_{it}} \frac{P_t^n}{P_{it}} y_t^n \\ \dot{\eta}_{it} &= \iota_t \eta_{it} + y_{it}^n \left[\left\{ \left(1 - \frac{W_t}{A_t P_{it}} \right) \xi - 1 \right\} \left(\frac{P_{it}}{P_t^n} \right)^{-\xi} - \theta \left(\frac{\dot{P}_{it}}{P_{it}} \right)^2 \right] \end{aligned}$$

The first condition says that at an optimum firm i should equate the marginal value of an existing price, η_{it}, with the marginal cost of changing it. The second condition gives the evolution equation of the shadow value of the price of the firm.

3.3 Government

The government in this economy comprises a fiscal and a monetary agent. The fiscal agent makes transfers to the private sector while the monetary authority chooses monetary policy and exchange rate policy. Using R to denote foreign bonds held by the government, we can write the consolidated government's flow budget constraint as:

$$\dot{R}_t = rR_t + \dot{m}_t + \epsilon_t m_t - \tau_t \tag{3.4}$$

We assume that the monetary authority adjusts the policy rate to implement its desired monetary policy while allowing the exchange rate to float freely with no intervention in foreign exchange markets. Formally, monetary policy is given by the interest rate rule

$$\iota_t = \bar{i} + \phi_\pi \pi_t + \phi_x \ln x_t \tag{3.5}$$

where $x_t = \frac{Y_t^n}{\bar{Y}^n}$ denotes the output gap between actual output of the non-traded good Y^n and the flexible price non-traded output $\bar{Y}^n$.

3.4 Equilibrium

We start with the aggregate flow resource constraint for the economy which we can derive by combining the household's flow budget constraint with the firm profit function and the government's flow budget constraint which gives

$$\dot{f}_t = rf_t + \bar{y}^T - c_t^T \tag{3.6}$$

where $f = b + R$ denotes net country foreign assets. This equation gives the country's current account at every date. From the household optimality conditions we know that c^T is constant over time. Using this in the current account equation and integrating forward subject to the standard transversality condition $\lim_{t\to\infty} e^{-rt} f_t = 0$ gives the equilibrium constant level of tradable consumption to be

$$\bar{c}^T = rf_0 + \bar{y}^T \tag{3.7}$$

Since all firms face the same problem, we impose symmetry across firms which implies that

$$P_{it} = P_t = P_t^n$$

where the last equality follows from the non-tradable price given by equation (3.3). In the following we shall use π to denote the non-tradable inflation rate so that

$$\frac{\dot{P}_t^n}{P_t^n} = \pi_t$$

Combining the non-traded firms' optimality conditions after imposing the symmetry condition on them gives the evolution equation for non-traded inflation:

$$\dot{\pi}_t = \left(\iota_t - \pi_t - \frac{\dot{y}_t^n}{y_t^n}\right)\pi_t + \frac{\xi-1}{\theta}\left(1 - \frac{\xi}{\xi-1}\frac{W_t}{A_t P_t^n}\right)$$

From the household optimality conditions we have $\frac{\dot{c}_t^n}{c_t^n} = \frac{\dot{e}_t}{e_t}$ while the market clearing condition for non-traded goods is $c^n = y^n$. Combining these and substituting them in the $\dot{\pi}_t$ equation derived above gives the equilibrium dynamics of non-traded inflation in the economy as

$$\dot{\pi}_t = \left(\iota_t - \pi_t - \frac{\dot{e}_t}{e_t}\right)\pi_t + \left(\frac{\xi-1}{\theta}\right)\left(1 - \frac{\xi}{\xi-1}\frac{W_t}{A_t P_t^n}\right) \tag{3.8}$$

In the appendix we solve for the flexible price equilibrium and show that:

$$\bar{n}_t = \left[(1-\gamma)\bar{e}_t^{\gamma}\left(\frac{\xi-1}{\xi}\right)\right]^{\frac{1}{1+\psi}} \tag{3.9}$$

$$\bar{y}_t^n = A_t\left[(1-\gamma)\bar{e}_t^{\gamma}\left(\frac{\xi-1}{\xi}\right)\right]^{\frac{1}{1+\psi}} \tag{3.10}$$

$$\bar{P}_t^n = \frac{\xi}{\xi-1}\frac{\bar{W}_t}{A_t} \tag{3.11}$$

$$\bar{e}_t = \left[\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{A_t}{rf_0 + \bar{y}^T}\right)\right]^{\frac{1+\psi}{1+\psi-\gamma}}\left[\left(\frac{\xi-1}{\xi}\right)(1-\gamma)\right]^{\frac{1}{1+\psi-\gamma}} \tag{3.12}$$

In the Taylor rule equation (B.40) we have defined the output gap as $x = \frac{y^n}{\bar{y}^n}$. Using the solutions for y^n and $\bar{y}^n$ along with equation (3.12), we get

$$x_t = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{rf_0 + \bar{y}^T}{A_t}\right)\left[\frac{\xi}{(\xi-1)(1-\gamma)\bar{e}^{\gamma}}\right]^{\frac{1}{1+\psi}} e_t = \frac{e_t}{\bar{e}_t} \tag{3.13}$$

Next, from equation (3.11) we get

$$\left(\frac{\xi}{\xi-1}\right)\frac{W_t}{A_t P_t^n} = \left(\frac{W_t/P_t^n}{\bar{W}_t/\bar{P}_t^n}\right)$$

Using this in equation (3.8) and rearranging gives

$$\dot{\pi}_t = \left(\iota_t - \pi_t - \frac{\dot{e}_t}{e_t}\right)\pi_t - \left(\frac{\xi - 1}{\theta}\right)\left(\frac{W_t/P_t^n}{\bar{W}_t\bar{P}_t^n} - 1\right)$$

We can use the market clearing for the non-traded good along with the first order condition for optimal labor supply, $\frac{c^n}{1-\gamma}n^{\psi} = \frac{W}{P^n}$, to get

$$\frac{W_t/P_t^n}{\bar{W}_t\bar{P}_t^n} = x_t^{1+\psi}\left(\frac{\bar{e}_t}{e_t}\right)^{\gamma} = x_t^{1+\psi-\gamma}$$

where we have used the fact that $x = \frac{e}{\bar{e}}$.

Substituting this into the $\dot{\pi}$ equation above gives the equilibrium evolution of non-tradable inflation:

$$\dot{\pi}_t = \left(\iota_t - \pi_t - \frac{\dot{e}_t}{e_t}\right)\pi_t + \left(\frac{\xi - 1}{\theta}\right)\left(x_t^{1+\psi-\gamma} - 1\right)$$

The output gap in the model is $x = \frac{e}{\bar{e}}$. Since $e = \frac{E}{P^n}$ by definition, $\frac{\dot{e}_t}{e_t} = \frac{\dot{E}_t}{E_t} - \pi_t$. Hence, the evolution of the output gap is given by

$$\frac{\dot{x}_t}{x_t} = \frac{\dot{E}_t}{E_t} - \pi_t - \left(\frac{1+\psi}{1+\psi-1}\right)g$$

To close the model we need a description of the evolution of the nominal exchange rate, E. As we noted above, we are assuming that the central bank allows the exchange rate to float freely without interference through foreign exchange market interventions. We assume that capital flows in the economy induce a modified interest parity condition

$$\frac{\dot{E}_t}{E_t} = \phi(\iota_t - r) \tag{3.14}$$

Equation (3.14) reduces to the standard uncovered interest parity (UIP) condition when $\phi = 1$. All $\phi < 1$ are typically classified as failures of UIP. The empirical evidence on the forward premium anomaly actually finds $\phi < 0$ for a large set of bilateral currency pairs in developed economies.

From the Taylor rule expression for monetary policy, equation (B.40) above, we have

$$\iota_t = \bar{\iota} + \phi_\pi \pi_t + \phi_x \ln x_t$$

The evolution of non-traded inflation and the output gap under sticky prices are given by:

$$\dot{\pi}_t = \left[\bar{\iota} + \frac{\phi}{1-\phi} r + \phi_\pi \pi_t + \phi_x \ln x_t\right](1-\phi)\pi_t - \left(\frac{\xi-1}{\theta}\right)\left(x_t^{1+\psi-\gamma} - 1\right) \tag{3.15}$$

$$\dot{x}_t = \left[\phi\left[\bar{\iota} - r - \left(\frac{1+\psi}{1+\psi-1}\right)\frac{g}{\phi}\right] + (\phi\phi_\pi - 1)\pi_t + \phi\phi_x \ln x_t\right] x_t \tag{3.16}$$

This is a two-equation system that describes the entire equilibrium dynamics of the economy. Once the time paths of x and π are determined, all the other variables (including the real exchange rate) can be determined recursively. Both x and π are jump variables as they are not predetermined. Consequently, stability requires two unstable (positive) roots.

3.4.1 Stability

To analyze the stability properties of this economy, we start by noting that in steady state $\dot{x}_t = \dot{\pi}_t = 0$. we study the Jacobian matrix associated with equations (3.15) and (3.16) around the steady state.

In the rest of the analysis we shall use hats over variables to denote their steady state values. Moreover, we shall also impose the conditions

$$\begin{aligned}\bar{\iota} &= r + \left(\frac{1+\psi}{1+\psi-1}\right)\frac{g}{\phi} \\ \phi_x &= 0\end{aligned}$$

The first assumption implies that steady state inflation is zero, i.e., $\hat{\pi} = 0$, and steady state output gap is one, $\hat{x} = 1$. Hence, output is at the flexible price level in steady state. The second assumption says that the monetary authority only focuses on inflation and does not react to the output gap. This is an analytical simplification which facilitates closed form solutions.

The steady state value of the real exchange rate is

$$\hat{e} = \bar{e}\left[\left(\frac{\xi}{\xi-1}\right)\left\{(1-\phi)\bar{\iota} + \frac{\phi(1-\phi_\pi)}{1-\phi\phi_\pi} r\right\} + 1\right]^{\frac{1}{1+\psi-\gamma}}$$

The determinant of the Jacobian matrix evaluated around the steady state is

$$(\phi\phi_\pi - 1)\left(\frac{\xi-1}{\theta}\right)(1+\psi-\gamma)$$

The sign of the determinant clearly depends on the sign of $\phi\phi_\pi - 1$. Thus, $\phi\phi_\pi < 1$ implies that the determinant is negative so that one of the two roots is negative. On the other hand, $\phi\phi_\pi > 1$ implies that both roots are positive and the system is a source. In that event, the economy must jump to its steady state immediately without any transition dynamics.

When $\phi\phi_\pi < 1$ the equilibrium dynamics around the steady state are indeterminate. In this event, any initial configuration for e and π converges asymptotically to the unique steady state of the economy. This case opens the door to sunspot equilibria where a non-fundamental shock can drive the dynamics of the economy.

There are two interesting cases worth discussing.

1. $\phi < 0$: In this case the equilibrium is indeterminate independent of the value of the ϕ_π. ϕ_π could always satisfy the Taylor principle by exceeding unity and yet the system would exhibit equilibrium indeterminacy. Clearly, this case would apply if empirical UIP regressions show a negative coefficient on the differential between home and foreign interest rates.

2. $\phi \in [0, 1)$: In this case, the equilibrium will be determinate ***if and only if***

$$\phi_\pi > \frac{1}{\phi} > 1$$

We collect these results inthe following proposition:

Proposition 3.1. *A small open economy characterized by equations (B.40), (3.14), (3.15) and (3.16) is characterized by a unique steady state. The perfect foresight dynamic equilibrium path to this unique steady state is unique if* $\phi_\pi > \frac{1}{\phi} > 1$ *and indeterminate if* $\phi\phi_\pi < 1$.

Proposition 3.1 implies that satisfying the Taylor principle is no longer enough to guarantee a determinate equilibrium in a small open economy when UIP fails. When UIP fails strongly so that $\phi < 0$, the equilibrium is indeterminate. When UIP fails but fails mildly so that $1 > \phi > 0$, the monetary authority has to be more aggressive than the baseline Taylor principle in responding to inflation in order to guarantee determinacy of equilibrium. More generally, the standard Taylor principle is enough for stability only when UIP holds, i.e., $\phi = 1$.

In the context of our focus on exchange rates, these results suggest that one rationalization of our empirical findings is that inflation targeting along with the failure of interest parity renders the equilibrium dynamics of open economies indeterminate. The indeterminacy implies that the path of the exchange rate cannot be pinned down uniquely by the initial conditions. In these conditions, oil prices may well serve as a way for markets to price the currencies of these countries.

3.5 Policy Persistence

Our formulation of monetary policy had the central bank freely varying the policy interest rate in response to inflation and the output gap. In practice however, there is often significant persistence in the policy rate. In effect, central banks vary the policy rate relative to its previous level. In continuous time, this implies that the central bank policy rate is a state variable. Consequently, monetary policy under such policy rigidity can be represented as

$$\dot{\iota}_t = \phi_\pi \pi_t + \phi_x \ln x_t$$

The equilibrium system can now be represented as a three equation system in π, x, ι:

$$\begin{aligned}
\dot{\pi}_t &= \left[(1-\phi)\iota_t + \phi r\right]\pi_t - \left(\frac{\xi-1}{\theta}\right)\left(x_t^{1+\psi-\gamma} - 1\right) && (3.17) \\
\dot{x}_t &= \left[\phi(\iota_t - r) - \pi_t - \left(\frac{1+\psi}{1+\psi-1}\right) g\right] x_t && (3.18) \\
\dot{\iota}_t &= \phi_\pi \pi_t + \phi_x \ln x_t && (3.19)
\end{aligned}$$

This differential equation system is the same as the one that we derived above except for the fact that the policy rate ι is now a state variable and consequently its evolution equation enters the equilibrium system independently.

The steady state for this economy is characterized by $\dot{\pi}_t = \dot{x}_t = \dot{\iota}_t = 0$. We assume that the steady state value of the policy rate is $\hat{\iota} = r + \left(\frac{1+\psi}{1+\psi-1}\right)\frac{g}{\phi}$. This implies that the steady state values of inflation and the output gap are, as before, $\hat{\pi} = 0$ and $\hat{x} = 1$.

Using $\mathcal{J}$ to denote the Jacobian matrix, it is easy to check that

$$\begin{aligned}
Trace(\mathcal{J}) &= (1-\phi)\hat{\iota} + \phi r \\
Determinant(\mathcal{J}) &= -\phi\left[\phi_x\left\{(1-\phi)\hat{\iota} + \phi r\right\} + \phi_\pi(1+\psi-\gamma)\left(\frac{\xi-1}{\theta}\right)\hat{x}^{1+\psi-\gamma} + \phi_x\left(\frac{1-\phi}{\phi}\right)\hat{\pi}\right]
\end{aligned}$$

The positive trace implies that at least one of the three roots of the system is positive.

3.5.1 Inflation Targeting: $\phi_x = 0$

The first case of interest is when $\phi_x = 0$. This case corresponds to a central bank that is a strict inflation targeter. In this case the determinant of the Jacobian matrix reduces to

$$Determinant(\mathcal{J}) = -\phi\phi_\pi(1+\psi-\gamma)\left(\frac{\xi-1}{\theta}\right)\hat{x}^{1+\psi-\gamma}$$

The sign of the determinant depends on the sign of ϕ_π. For $\phi_\pi < 0$ the determinant is positive which indicates that the system is characterized by two negative roots and one positive root. With only one predetermined variable ι and two jump variables, this implies that the convergence to the steady state of the system is indeterminate.

Contrarily, $\phi_\pi > 0$ implies that the determinant is negative indicating that system has two positive roots. With two jump variables and two positive roots, the equilibrium dynamics evolve along a unique saddle path in this case rendering the equilibrium determinate.

Collecting results, under interest rate persistence, the stability of the system depends on the sign of ϕ_π. If UIP fails strongly with $\phi_\pi < 0$ then the equilibrium dynamics become indeterminate. More specifically, there exist a continuum of initial values for π and x that are all equilibria.

3.5.2 No Inflation Targeting: $\phi_\pi = 0$

This is the polar opposite case where the monetary authority does not target inflation at all and instead responds only to the output gap. The determinant of the Jacobian matrix in this case reduces to

$$Determinant(\mathcal{J}) = -\phi\phi_x\left[(1-\phi)\hat{\imath} + \phi r + \left(\frac{1-\phi}{\phi}\right)\hat{\pi}\right] < 0$$

Since the determinant is unambiguously negative, the system has one negative root and two positive roots. Consequently, the equilibrium dynamics are unique and characterized by saddle path stability. This is independent of whether or not UIP holds or fails.

This result is important because it indicates that when UIP fails, a monetary policy **switch** to inflation targeting can render the equilibrium indeterminate where previously it would have been uniquely determined.

To summarize, under inflation targeting regimes, equilibrium dynamics are uniquely determined when uncovered interest parity holds but can become indeterminate when it fails. Equilibrium indeterminacy of the real exchange rate implies that the path of nominal exchange rate is also indeterminate which opens the door to non-fundamental equilibria such as sunspots and focal points for exchange rate pricing such as oil price movements.

4 Uncovered Interest Parity?

The key condition that determines whether or not the model outlined above exhibits equilibrium indeterminacy under inflation targeting regimes is whether or not uncovered interest parity. While there is a lot of work on testing uncovered interest parity and voluminous evidence that it often fails in the data, for our proposed explanation to pass a preliminary data test, we need to show that uncovered interest parity also fails in the sample of inflation targeting countries that we have studied. To check this we run the following panel regression for our sample of countries:

$$\ln E_{t+1}^{i} - \ln E_{t}^{i} = \alpha_i + \beta \left(R_t^i - R_t^{US} \right) + \eta_{t+1}^{i}$$

where E^i is the nominal exchange rate in country i (local currency units per unit of the US dollar) while R^i is the nominal interest rate in country i. We run this regression for the flexible exchange rate periods for each inflation targeting country in our sample during the period January 1974 to August 2016. We use both one month and three month interest rate spreads to check for robustness of the results. Table 1 below reports the results.

Table 1: Uncovered interest parity tests

	One-month	Three-month
Interest rate spread	0.463***	-1.102*
	(0.0185)	(0.616)
Observations	7114	683
R-squared	0.081	0.005
Number of countries	22	16
Country fixed effects	Yes	Yes

Notes: 1. The table reports the coefficient on the interest rate differential in a regression of the nominal exchange rate depreciation on interest rate differential with the USA. Standard errors are reported in parenthesis. 2. The one month spread is between money market rates while the three month spreads are between T-bill rates. 2. ***p<0.01, **p<0.05, *p<0.1

The coefficient on the one-month interest rate spread is significantly different from both zero and one, the latter indicating that interest parity fails to hold in the panel. Correspondingly, the coefficient on the three-month interest differential is significantly less than zero indicating a more extreme version of failure of uncovered interest parity.[1]

We view this evidence as supportive of the key condition underlying equilibrium indeterminacy result highlighted by our model.

5 Role of Sunspots: Evidence from Structural Estimation for Canada

We have argued that inflation-targeting monetary policy rule together with the failure of the UIP condition can give rise to equilibrium indeterminacy in a small open-economy model. Under indeterminacy, sunspots — non-fundamental disturbances to agents' expectations — can affect economic fluctuations. In this section, we empirically examine the role of sunspot shocks in explaining exchange rate fluctuations and how they are related to oil price movements by estimating an extended version of the model considered in Section 3 for the Canadian economy in a stochastic setting.

5.1 The Extended Model

For a better fit to the aggregate data, we extend the model to incorporate habit persistence in consumption preferences, price indexation to past inflation, and monetary policy smoothing. Moreover, to ensure the stationarity of the small-open economy model with incomplete asset markets, the model features an endogenous discount factor, *à la* Uzawa preferences, following Schmitt-Grohé and Uribe (2003).

For estimation, the model is embedded with five structural shocks about tradable-good technology, non-tradable-good technology, monetary policy, US inflation, and the US interest rate, all of which except for the monetary policy shock follow first-order autoregressive (AR(1)) processes. In addition to these structural shocks, we allow for sunspot shocks to affect equilibrium dynamics under indeterminacy.

The full description of the extended model is provided in Appendix B.

5.2 Estimation strategy

As shown in Section 3, equilibrium can be indeterminate when the modified UIP condition exhibits a negative relationship between exchange rate depreciation and interest rate differentials. Because

[1] We also ran these interest parity regressions individually for each country for both the one-month and three-month horizons. The average of the one-month interest spread coefficient was 0.60 while the average for the three-month interest rate spread was -1.16.

this indeterminacy property carries over to the extended model, we estimate the model by allowing for both determinacy and indeterminacy of equilibrium with full-information Bayesian methods developed by Lubik and Schorfheide (2004). While Lubik and Schorfheide (2004) conduct model estimation separately for each region, we estimate the model for both the determinacy and indeterminacy regions in one step by adopting a SMC algorithm, as implemented by Hirose et al. (2020, 2023). For a detailed description of the estimation procedure, see Fujiwara and Hirose (2024), who use the same approach to estimate a small open-economy model with a modified UIP condition similar to that employed in the present paper.

The data used in the model estimation are seven quarterly time series on $\Delta \log Y_t$, $\log \pi_t$, $\log i_t$,$\Delta \log S_t$, $\hat{\pi}_t^*$, and $\hat{i}_t^*$. The first four series are constructed from Canadian data: real GDP per capita, the GDP deflator, the Bank of Canada's policy (overnight) rate, and the Canadian to US dollar exchange rate. The other series are proxied by the US data: the US GDP deflator inflation rate and the federal funds rate. In the model, foreign variables are treated as exogenous shocks that follow stationary AR(1) processes. Thus, these exogenous processes are directly identified from these two time series. Not considering differences in the trend components and steady states between the US and Canada, we demean all the data before estimation. The sample period is from 1991:Q1 to 2019:Q4 to consider the period since the Bank of Canada introduced inflation-targeting policy explicitly. We exclude the COVID-19 pandemic period from the sample.

To avoid possible identification issues, we fix several parameters in the model. The parameter on Uzawa preferences is set at $\kappa = 0.001$ to minimize its effect on equilibrium dynamics. The steady-state subjective discount factor and the elasticity of substitution among differentiated goods are fixed at $\beta = 0.99$ and $\theta = 8$, respectively, following Justiniano and Preston (2010), who estimate a small open-economy version of the standard New Keynesian model for Canada. The steady-state growth rate is set at $g = 0.307/100$, based on the sample average of the per capita real GDP growth rate in Canada. The ratio of tradable goods to aggregate consumption is fixed at $\gamma = 0.5$, according to the estimate of Stockman and Tesar (1995).

All other parameters in the model are estimated, and their prior distributions are presented in the Online Appendix A.1.1.

5.3 Estimation results

In Appendix A.1.1, the last two columns of Table 9 present the posterior estimates of parameters in the extended model. A notable finding here is that the coefficient ψ on interest rate differentials

Table 2: Variance decomposition

	Output growth	Inflation	Interest rate	Exchange rate
Sunspot	0.287	0.359	0.326	0.790
Technology (tradable)	0.646	0.001	0.003	0.000
Technology (non-tradable)	0.030	0.551	0.608	0.011
Monetary policy	0.003	0.047	0.008	0.097
US inflation	0.005	0.006	0.002	0.026
US interest rate	0.030	0.037	0.053	0.077

Notes: The table shows the forecast error variance decomposition of output growth, inflation, the interest rate, and exchange rate depreciation at an infinite horizon, given the posterior mean estimates of parameters.

in the modified UIP condition is estimated to be negative, leading to indeterminacy, whereas the normal prior with mean zero, *ex ante*, allows for either positive or negative values. This is consistent with the single-equation estimation result in Section 4 as well as our conjectures in the preceding sections.

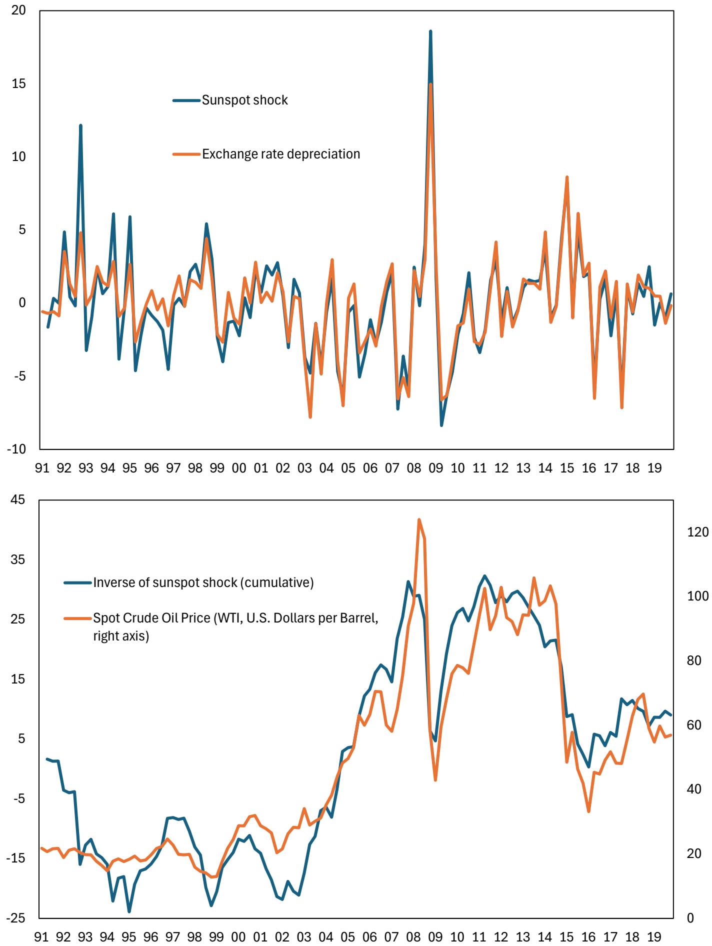
As the estimated model exhibits equilibrium indeterminacy, sunspot shocks can affect macroeconomic dynamics including exchange rate fluctuations. To quantify the contribution of sunspots to aggregate variables, Table 2 shows the forecast error variance decompositions of output growth, inflation, the interest rate, and exchange rate depreciation at an infinite horizon, given the posterior mean estimates of the parameters in the model. Markedly, the sunspot shock is the primary driver of exchange rate fluctuations; that is, 79 percent of exchange rate variability is explained by the sunspot shock.

Indeed, as shown in the upper panel of Figure 5, the smoothed estimate of the sunspot shock series, computed based on the posterior mean estimates of parameters, closely corresponds to the data on exchange rate depreciation, with a correlation coefficient of 0.927. In our framework, the sunspot shock is specified as a non-fundamental belief on expectations. This result demonstrate that exchange rate fluctuations are mostly driven by such a belief shock.

Even more striking, the lower panel of Figure 5 indicates that the cumulative sum of the inverse sunspot series nearly coincides with the oil price, yielding a correlation coefficient of 0.935. This finding suggests that sunspots—representing non-fundamental beliefs—and oil prices are linked in some manner.

As shown in Figure 6, we cannot detect such coincidences among these sequences, when we compute the smoothed estimate of the sunpot shock for the period before the Bank of Canada adopted inflation-targeting policy in 1991. A correlation coefficient between the sunspot sequences

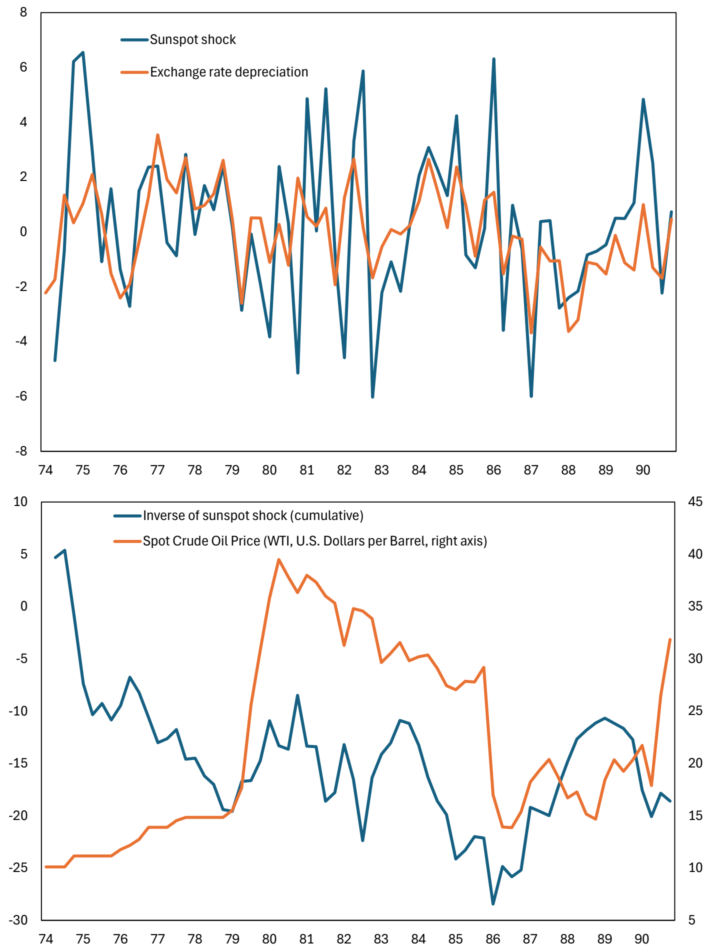
Figure 5: Smoothed estimate of sunspot shock and the data on exchange rate depreciation and oil price: 1991–2019



Notes: The upper panel plots the Kalman smoothed estimate of the sunspot shock, given the posterior mean estimates of parameters, and the data on depreciation in the Canada-US exchange rate. The lower panel shows the cumulative sum of the inverse of the smoothed estimate of the sunspot shock and the data on the spot price of the WTI crude oil.

and exchange rate depreciation and that between the cumulative sum of their inverse and the oil price are 0.530 and -0.258, respectively. Therefore, the connection of sunspots to exchange rates and oil prices are pronounced after the introduction of inflation targeting, as is consistent with our arguments in the preceding sections.

Figure 6: Smoothed estimate of sunspot shock and the data on exchange rate depreciation and oil price: 1974–1990



Notes: The upper panel plots the Kalman smoothed estimate of the sunspot shock, given the posterior mean estimates of parameters, and the data on depreciation in the Canada-US exchange rate. The lower panel shows the cumulative sum of the inverse of the smoothed estimate of the sunspot shock and the data on the spot price of the WTI crude oil.

6 Conclusions

Over the past three decades inflation targeting as a monetary policy paradigm has gained increasing popularity and central banking acceptance. In this paper we have documented a previously little known consequence of inflation targeting: the currencies of countries that adopt this regime tend to get systematically linked to the world oil prices specifically, and world energy prices more generally. Somewhat startlingly, this link is very systematic: higher world oil prices tend to appreciate the currency of an inflation targeting country. This systematic relationship in inflation targeting countries emerges independent of whether or not they are oil exporters or importers and other country characteristics.

We have proposed one rationalization of this data pattern based on equilibrium indeterminacy under inflation targeting. We have shown that a small open economy characterized by sticky prices but where uncovered interest parity does not hold will exhibit equilibrium indeterminacy if it pursues inflation targeting. The equilibrium indeterminacy disappears if the country ceases to target inflation. In the presence of equilibrium indeterminacy, one rationale for the link of the exchange rate to oil prices is that markets use oil prices as a focal point to select an equilibrium.

The main contribution of the paper is documenting the data fact. Our explanation for the oil price linkage of currencies of inflation targeting countries is not intended as the only possible explanation of the phenomenon. There may doubtless be other possible explanations as well. We hope to explore such other possibilities in future work.

References

Devereux, M. B. and G. W. Smith (2017). Commodity Currencies and Monetary Policy: A Simple Test. Working paper, University of British Columbia.

Engel, C., D. Lee, C. Liu, C. Liu, and S. P. Y. Wu (2017, November). The uncovered interest parity puzzle, exchange rate forecasting, and taylor rules. Working Paper 24059, National Bureau of Economic Research.

Fujiwara, I. and Y. Hirose (2024). Connecting exchange rates to fundamentals under indeterminacy. CEPR Discussion Papers 19744.

Hammond, G. (2012). *State of the art of inflation targeting.* Number 29 in Handbooks. Centre for Central Banking Studies, Bank of England.

Hirose, Y., T. Kurozumi, and W. Van Zandweghe (2020). Monetary policy and macroeconomic stability revisited. *Review of Economic Dynamics 37*, 255–274.

Hirose, Y., T. Kurozumi, and W. V. Zandweghe (2023, December). Inflation Gap Persistence, Indeterminacy, and Monetary Policy. *Review of Economic Dynamics 51*, 867–887.

Justiniano, A. and B. Preston (2010). Can structural small open-economy models account for the influence of foreign disturbances? *Journal of International Economics 81*(1), 61–74.

Kaplan, G., B. Moll, and G. L. Violante (2018, March). Monetary Policy According to HANK. *American Economic Review 108*(3), 697–743.

Lubik, T. A. and F. Schorfheide (2004, February). Testing for indeterminacy: An application to U.S. monetary policy. *American Economic Review 94*(1), 190–217.

Meese, R. A. and K. Rogoff (1983, February). Empirical exchange rate models of the seventies : Do they fit out of sample? *Journal of International Economics 14*(1-2), 3–24.

Reinhart, C. M. and K. S. Rogoff (2004, February). The modern history of exchange rate arrangements: A reinterpretation. *The Quarterly Journal of Economics 119*(1), 1–48.

Schmitt-Grohé, S. and M. Uribe (2003, October). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Smets, F. and R. Wouters (2007, June). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review 97*(3), 586–606.

Stockman, A. C. and L. L. Tesar (1995, March). Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. *American Economic Review 85*(1), 168–185.

Taylor, J. B. (1993, December). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*(1), 195–214.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

A Appendix

A.1 Tables

Table 3: Inflation Targeting Countries: Early and Late Adopters

Early adopters		Late adopters	
Country	*Date of adoption*	*Country*	*Date of adoption*
New Zealand	December 1989	South Africa	February 2000
Canada	February 1991	Thailand	May 2000
United Kingdom	October 1992	Mexico*	January 2001
Sweden	January 1993	Norway	March 2001
Australia	June 1993	Hungary	June 2001
Israel	June 1997	Peru	January 2002
Czech Republic	December 1997	Philippines	January 2002
Poland*	January 1998	Guatemala*	January 2005
South Korea	April 1998	Iceland	March 2005
Brazil	June 1999	Indonesia	July 2005
Chile	September 1999	Romania	August 2005
Colombia	October 1999	Armenia	January 2006
		Turkey	January 2006
		Serbia	September 2006
		Ghana	May 2007

Notes: 1. The table reports the list of countries that have adopted inflation targeting. The countries have been collected in two groups – early adopter who adopted prior to 2000 and late adopters who adopted from 2000 onwards. 2. * indicates that the precise date of adoption is not available. For our econometric analysis below we assume the adoption month to be January for these countries.

Table 4: Exchange rate and oil prices

VARIABLES	(1) E_{it}	(2) E_{it}	(3) E_{it}	(4) E_{it}
Oil_price_t	1.060***	1.235***		
	(0.123)	(0.126)		
IT_{it}	35.91***	39.84***	39.34***	48.03***
	(3.127)	(3.047)	(3.209)	(3.064)
$Oil\_price_t * IT_{it}$	-1.412***	-1.573***		
	(0.012)	(0.125)		
Oil_NX_{it}		-387.4***		
		(22.78)		
$Oil\_NX_{it} * IT_{it}$		-15.97		
		(26.61)		
$Oil\_NX_{it} * IT_{it} * Oil\_price_t$		-1.032**		
		(0.308)		
$Energy\_price_t$			0.894***	1.060***
			(0.098)	(0.096)
$Energy\_price_t * IT_{it}$			-1.197***	-1.368***
			(0.095)	(0.093)
$Energy\_NX_{it}$				-334.9***
				(20.47)
$Energy\_NX_{it} * IT_{it}$				-121.0***
				(22.57)
$Energy\_NX_{it} * IT_{it} * Energy\_price_t$				1.121***
				(0.194)
Observations	6553	4830	6553	6019
R-squared	0.454	0.396	0.456	0.477
Countries	20	20	20	20
Country fixed effects	YES	YES	YES	YES
Year dummies	YES	YES	YES	YES

Notes: 1. The table reports the results of regressions of the nominal exchange rate on various country, time and world characteristics. IT_{it} is a dummy for an inflation targeting country i at date t. Oil_NX_{it} denotes a dummy variable for that takes value 1 when net exports of oil by country i at date t are positive. Oil_price_t denotes the world oil price at date t (measured in US dollars). 2. All exchange rates are measured in local currency units per US dollar. 3. Standard errors of the estimates are in parenthesis. *** indicates significance at the 1 percent level, ** at the 5 percent level and * indicates significance at the 10 percent level.

Table 5: Growth Rates of Exchange Rates and Oil Price

VARIABLES	(1) E_{it}	(2) E_{it}	(3) E_{it}	(4) E_{it}
Oil_price_t	-0.0269**	-0.0312**		
	(0.0127)	(0.0129)		
IT_{it}	-0.0012	-0.0012	-0.001	-0.0070***
	(0.0026)	(0.0028)	(0.0026)	(0.0025)
$Oil\_price_t * IT_{it}$	-0.0436***	-0.0439***		
	(0.0155)	(0.0151)		
Oil_NX_{it}		-0.0797		
		(0.0447)		
$Oil\_NX_{it} * IT_{it}$		-0.0177		
		(0.0324)		
$Oil\_NX_{it} * IT_{it} * Oil\_price_t$		-0.338*		
		(0.185)		
$Energy\_price_t$			-0.0267*	0.0239*
			(0.0146)	(0.0136)
$Energy\_price_t * IT_{it}$			-0.0629***	-0.0677***
			(0.0180)	(0.0164)
$Energy\_NX_{it}$				-0.0314
				(0.0357)
$Energy\_NX_{it} * IT_{it}$				0.0365
				(0.0262)
$Energy\_NX_{it} * IT_{it} * Oil\_price_t$				-0.290*
				(0.158)
Observations	6981	5072	6981	6407
R-squared	0.037	0.041	0.058	0.065
Countries	24	23	24	23
Country fixed effects	YES	YES	YES	YES
Quinquennial time dummies	YES	YES	YES	YES

Notes: 1. The table reports the results of regressions of the growth rate of the nominal exchange rate on various country, time and world characteristics. IT_{it} is a dummy for an inflation targeting country i at date t. Oil_NX_{it} denotes a dummy variable for that takes value 1 when net exports of oil by country i at date t are positive. Oil_price_t denotes the growth rate of the world oil price at date t (measured in US dollars). 2. All exchange rates are measured in local currency units per US dollar. 3. Standard errors of the estimates are in parenthesis. *** indicates significance at the 1 percent level, ** at the 5 percent level and * indicates significance at the 10 percent level.

Table 6: Exchange rate and oil prices in early and late adopters

	Early IT adopters		Late IT adopters	
VARIABLES	E_{it}		E_{it}	
	Level	*Growth rate*	*Level*	*Growth rate*
Oil_price_t	0.566	-0.0237	5.776**	-0.0505**
	(0.468)	(0.0145)	(2.924)	(0.0228)
IT_{it}	0.298	-0.0040*	19.87	0.0039
	(13.38)	(0.0032)	(127.6)	(0.0076)
$Oil\_price_t * IT_{it}$	-1.423***	-0.0649***	-5.262*	-0.0110
	(0.4820)	(0.0170)	(3.072)	(0.0338)
Oil_NX_{it}	13.07	0.0006	9.228	-0.0077
	(8.486)	(0.0029)	(253.7)	(0.0218)
$Oil\_NX_{it} * IT_{it}$	94.95***	-0.0006	-3.251	0.0114
	(11.09)	(0.0031)	(127.63)	(0.0071)
$Oil\_NX_{it} * IT_{it} * Oil\_price_t$	-2.383***	0.0076	0.0014	-0.0564
	(0.143)	(0.0166)	(1.490)	(0.0385)
Observations	3417	3408	1679	1664
R-squared	0.153	0.048	0.201	0.056
Countries	11	11	13	12
Country fixed effects	YES	YES	YES	YES
Quinquennial time dummies	YES	YES	YES	YES

Notes: 1. The table reports the results of regressions of the nominal exchange rate on various country, time and world characteristics, separately for early late adopters of inflation targeting. IT_{it} is a dummy for an inflation targeting country i at date t. Oil_NX_{it} denotes a dummy variable for that takes value 1 when net exports of oil by country i at date t are positive. Oil_price_t denotes the world oil price at date t (measured in US dollars). 2. All exchange rates are measured in local currency units per US dollar. 3. Standard errors of the estimates are in parenthesis. *** indicates significance at the 1 percent level, ** at the 5 percent level and * indicates significance at the 10 percent level.

Table 7: Robustness: Adding US Federal Funds Rate as a control

	Level	Growth rate
VARIABLES	E_{it}	E_{it}
Oil_price_t	1.101***	-0.0300***
	(0.129)	(0.0125)
IT_{it}	26.32***	0.0141**
	(4.491)	(0.0058)
$Oil\_price_t * IT_{it}$	-1.447***	-0.0433***
	(0.127)	(0.0151)
Oil_NX_{it}	-366***	0.0709
	(22.98)	(0.0448)
$Oil\_NX_{it} * IT_{it}$	48.51***	-0.0153
	(16.62)	(0.0324)
$Oil\_NX_{it} * IT_{it} * Oil\_price_t$		-0.346*
		(0.185)
$Federal\_Funds\_Rate_{t-1}$	-1.080**	0.0003
	(0.464)	(0.0009)
$Federal\_Funds\_Rate_{t-1} * IT$	2.248***	-0.0027**
	(0.587)	0.0011)
Observations	4815	5072
R-squared	0.396	0.062
Countries	20	23
Country fixed effects	YES	YES
Year dummies	YES	YES

Notes: 1. The table reports the results of regressions of the growth rate of the nominal exchange rate on various country, time and world characteristics. IT_{it} is a dummy for an inflation targeting country i at date t. Oil_NX_{it} denotes a dummy variable for that takes value 1 when net exports of oil by country i at date t are positive. Oil_price_t denotes the growth rate of the world oil price at date t (measured in US dollars). 2. All exchange rates are measured in local currency units per US dollar. 3. Standard errors of the estimates are in parenthesis. *** indicates significance at the 1 percent level, ** at the 5 percent level and * indicates significance at the 10 percent level.

Table 8: Robustness: Adding US Federal Funds Rate as a control

	Level	Growth rate
VARIABLES	E_{it}	E_{it}
IT_{it}	38.34***	0.0051
	(4.529)	(0.0048)
$Energy\_price_t$	0.984***	-0.0227*
	(0.099)	(0.0136)
$Energy\_price_t * IT_{it}$	-0.1.296***	-0.0677***
	(0.0948)	(0.0164)
$Energy\_NX_{it}$	-336.3***	0.0246
	(20.73)	(0.0358)
$Energy\_NX_{it} * IT_{it}$	-120.7***	0.0402
	(22.51)	(0.0262)
$Energy\_NX_{it} * IT_{it} * Oil\_price_t$		-0.293*
		(0.158)
$Federal\_Funds\_Rate_{t-1}$	-0.712	0.0004
	(0.443)	(0.0008)
$Federal\_Funds\_Rate_{t-1} * IT$	1.667***	-0.0021**
	(0.583)	(0.0009)
Observations	5991	6407
R-squared	0.476	0.065
Countries	20	23
Country fixed effects	YES	YES
Year dummies	YES	YES

Notes: 1. The table reports the results of regressions of the growth rate of the nominal exchange rate on various country, time and world characteristics. IT_{it} is a dummy for an inflation targeting country i at date t. Oil_NX_{it} denotes a dummy variable for that takes value 1 when net exports of oil by country i at date t are positive. Oil_price_t denotes the growth rate of the world oil price at date t (measured in US dollars). 2. All exchange rates are measured in local currency units per US dollar. 3. Standard errors of the estimates are in parenthesis. *** indicates significance at the 1 percent level, ** at the 5 percent level and * indicates significance at the 10 percent level.

A.1.1 Prior and Posterior Distributions of Parameters

Table 9 summarizes the prior and posterior distributions of parameters in the estimated model.

Table 9: Prior and posterior distributions of parameters

	Prior			Posterior	
Parameter	Distribution	Mean	S.D.	Mean	90% interval
ψ	Normal	0.000	0.250	-0.446	[-0.828, -0.009]
α_π	Gamma	1.500	0.100	1.140	[1.002, 1.264]
α_y	Gamma	0.125	0.050	0.108	[0.040, 0.176]
ρ	Beta	0.500	0.100	0.920	[0.903, 0.937]
b	Beta	0.500	0.100	0.799	[0.719, 0.887]
η	Gamma	1.000	0.300	6.733	[5.663, 7.725]
ϕ	Beta	0.500	0.200	0.130	[0.019, 0.230]
χ	Gamma	70.000	10.000	60.161	[44.439, 74.026]
ρ_{aT}	Beta	0.500	0.200	0.970	[0.949, 0.990]
ρ_{aN}	Beta	0.500	0.200	0.787	[0.692, 0.878]
ρ_{π^*}	Beta	0.500	0.200	0.515	[0.375, 0.665]
ρ_{i^*}	Beta	0.500	0.200	0.923	[0.903, 0.943]
σ_{aT}	Inverse Gamma	0.250	4.000	0.432	[0.245, 0.630]
σ_{aN}	Inverse Gamma	0.250	4.000	8.688	[6.500, 10.748]
σ_i	Inverse Gamma	0.250	4.000	0.154	[0.137, 0.173]
σ_{π^*}	Inverse Gamma	0.250	4.000	0.191	[0.169, 0.212]
σ_{i^*}	Inverse Gamma	0.250	4.000	0.117	[0.103, 0.130]
σ_ζ	Inverse Gamma	0.250	4.000	3.364	[2.931, 3.783]
M_{aT}	Normal	0.000	1.000	1.832	[0.949, 2.575]
M_{aN}	Normal	0.000	1.000	-4.116	[-5.186, -3.107]
M_i	Normal	0.000	1.000	2.176	[0.629, 3.887]
M_{π^*}	Normal	0.000	1.000	-1.290	[-2.778, 0.334]
M_{i^*}	Normal	0.000	1.000	-1.315	[-3.144, 0.458]

Notes: Inverse gamma distributions are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, where ν and s are respectively set at the mean and standard deviation (S.D.) values in the table. The posterior mean and 90 percent highest posterior density intervals are calculated based on $10,000$ particles from the final importance sampling in the SMC algorithm.

A.1.2 Prior distributions

The second to fourth columns of Table 9 present the prior distributions.

For the coefficient ψ in the modified UIP condition to take either positive or negative values, we impose a normal distribution with mean zero and standard deviation 0.25. The prior mean for the coefficients on inflation and output growth in the monetary policy rule is set according to Smets and Wouters (2007). The prior mean for price adjustment cost parameter is selected at $\chi = 70$ so that the slope of the Phillips curve is 0.1 when $\theta = 8$ and $\phi = 0$. The priors for the other structural parameters for the household and firms follow from Justiniano and Preston (2010).

Regarding the shock parameters, the priors for persistence parameters in the AR(1) processes ρ_x, $x \in \{aT, aN, \pi^*, i^*\}$ are the same as those in Smets and Wouters (2007). For the shock standard deviations σ_x, $x \in \{aT, aN, i, \pi^*, i^*, \zeta\}$, we set $\nu = 0.25$ and $s = 4$ in the inverse gamma distribution of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

For the priors on the components $M_x, x \in \{aT, aN, i, \pi^*, i^*\}$ of the arbitrary matrix $\mathbf{M}$ in the indeterminacy solution, we use the standard normal distribution, following Lubik and Schorfheide (2004).

A.2 Posterior distributions

The last two columns of Table 9 display the posterior mean and 90% highest posterior density intervals for estimated parameters.

The coefficient ψ on interest rate differentials in the modified UIP condition is estimated to be negative in terms of both the posterior mean and 90% credible intervals. The monetary policy response to inflation α_π is somewhat smaller than that in the original Taylor (1993) rule, but it still satisfies the Taylor principle.

The standard deviation of the sunspot shock σ_ζ is estimated to be large. This explains the large contribution of the sunspot shock to exchange rate fluctuations shown in Table 2 of Section 5.3.

Finally, regarding the indeterminacy-related parameters, all the components (M_{aT}, M_{aN}, M_i, M_{π^*}, and M_{i^*}) in the arbitrary matrix $\mathbf{M}$ are substantially different from zero. This finding suggests that the propagation of fundamental shocks can differ dramatically under indeterminacy, compared to the case under determinacy.

A.3 Flexible price equilibrium

It is useful at this stage to illustrate the flexible price equilibrium in this economy. Note that setting $\theta = 0$ in the intermediate goods firms' problem corresponds to the flexible price case. Under $\theta = 0$ the optimality conditions for intermediate goods firms reduces to $\eta_{it} = 0$. Moreover, the optimality condition for $\dot{\eta}_{it}$ reduces to

$$\bar{P}_t^n = \frac{\xi}{\xi - 1}\frac{\bar{W}_t}{A_t} \tag{A.20}$$

Note that here and in the rest of the paper we work with the notational convention of using a bar over a variable to indicate its flexible price value. Substituting $\bar{P}^n$ into the optimality condition for household labor supply gives $A_t \bar{n}_t^{1+\psi} = A_t(1-\gamma)\bar{e}_t^{\gamma}\left(\frac{\xi-1}{\xi}\right)$ which yields

$$\bar{n}_t = \left[(1-\gamma)\bar{e}_t^{\gamma}\left(\frac{\xi-1}{\xi}\right)\right]^{\frac{1}{1+\psi}}$$

Substituting this equilibrium flexible price employment in the production function gives $\bar{y}_t^n = A_t\left[(1-\gamma)\bar{e}_t^{\gamma}\left(\frac{\xi-1}{\xi}\right)\right]^{\frac{1}{1+\psi}}$. Differentiating this with respect to time we get the growth rate of non-traded output under flexible prices:

$$\frac{\dot{\bar{y}}_t^n}{\bar{y}_t^n} = g + \frac{\gamma}{1+\psi}\frac{\dot{\bar{e}}_t}{\bar{e}_t} \tag{A.21}$$

where we are assuming that $\frac{\dot{A}_t}{A_t} = g$ so that productivity grows at a constant rate at all times.

To solve for the equilibrium real exchange rate under flexible rates recall from the household optimality conditions that $\frac{c^{\bar{n}}}{c^T} = \frac{1-\gamma}{\gamma}\bar{e}$. Combining this with the market clearing condition for the non-traded good and the flexible price employment solution derived above gives

$$\bar{e}_t = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{A_t}{rf_0+\bar{y}^T}\right)\left[\left(\frac{\xi-1}{\xi}\right)(1-\gamma)\bar{e}_t^{\gamma}\right]^{\frac{1}{1+\psi}} \tag{A.22}$$

Note that equation (A.22) can be used to solve for equilibrium real exchange rate under flexible prices at any date t:

$$\bar{e}_t = \left[\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{A_t}{rf_0+\bar{y}^T}\right)\right]^{\frac{1+\psi}{1+\psi-\gamma}}\left[\left(\frac{\xi-1}{\xi}\right)(1-\gamma)\right]^{\frac{1}{1+\psi-\gamma}}$$

The flexible price real exchange rate depreciates (appreciates) along all paths where productivity A is rising (falling). More generally,

$$\frac{\dot{\bar{e}}_t}{\bar{e}_t} = \left(\frac{1+\psi}{1+\psi-\gamma}\right)g$$

Intuitively, in any equilibrium with flexible prices, rising productivity induces an excess supply of the non-traded good. This requires a falling relative price of the non-traded good (a rising $\bar{e}$) to clear the market.

Online Appendix

B Description of the Extended Model

B.1 Household

A representative household in the home country maximizes the utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{M_t}{P_t}, h_t\right),$$

subject to contemporaneous utility:

$$u\left(C_t, \frac{M_t}{P_t}, h_t\right) := \ln\left(C_t - b\bar{C}_{t-1}\right) + \omega_m \ln\left(\frac{M_t}{P_t}\right) - \omega_h \frac{1}{1+\eta} h_t^{1+\eta},$$

and the budget constraint:

$$P_t C_t + B_t + M_t = (1 + i_{t-1}) B_{t-1} + W_t h_t + G_t + M_{t-1} + \Pi_t^T + \Pi_t^N, \tag{B.23}$$

where C_t is aggregate consumption, $b\bar{C}_{t-1}$ is an external habit taken as given by the household, M_t/P_t is a real money balance, h_t is labor supply, P_t is the aggregate price index, B_t is the international riskless bonds, i_t is the nominal interest rate, W_t is nominal wage, G_t is the net transfers from the government, and Π_t^T and Π_t^N are profits from tradable and non-tradable good firms, respectively.

From the FONCs, we obtain

$$(C_t - bC_{t-1})^{-1} = \beta \mathbb{E}_t (C_{t+1} - bC_t)^{-1} \frac{(1 + i_t)}{\pi_{t+1}}, \tag{B.24}$$

$$(C_t - bC_{t-1})^{-1} = \omega_m \frac{P_t}{M_t} + \beta \mathbb{E}_t (C_{t+1} - bC_t)^{-1} \frac{1}{\pi_{t+1}}, \tag{B.25}$$

$$\omega_h h_t^{\eta} = (C_t - bC_{t-1})^{-1} \frac{W_t}{P_t}, \tag{B.26}$$

where $\pi_t := P_t/P_{t-1}$. Note that a real money balance is separable in utility and therefore does not play any significant role in the model.

To ensure stationarity of the small open-economy model with incomplete asset markets, we impose external Uzawa preferences, so that the subjective discount factor is now endogenized after solving the optimization problem and given by

$$\beta = \beta_t := \left(1 + \frac{C_t}{C} - \omega_h \frac{1}{1+\eta} h_t^{1+\eta}\right)^{-\kappa},$$

following Schmitt-Grohé and Uribe (2003).

Aggregate consumption C_t is a composite of tradable and non-tradable goods, C_t^T and C_t^N, given by

$$C_t = \left(\frac{C_t^T}{\gamma}\right)^{\gamma} \left(\frac{C_t^N}{1-\gamma}\right)^{1-\gamma}.$$

Since the preferences allow for two-stage budgeting, the household problem can be decomposed into two sub-problems: one with intertemporal and the other with intratemporal allocations. From the optimality conditions for the total cost minimization problem, we obtain

$$C_t^T = \gamma \left(\frac{P_t^T}{P_t}\right)^{-1} C_t, \tag{B.27}$$

$$C_t^N = (1-\gamma) \left(\frac{P_t^N}{P_t}\right)^{-1} C_t, \tag{B.28}$$

$$P_t = \left(P_t^T\right)^{\gamma} \left(P_t^N\right)^{1-\gamma}, \tag{B.29}$$

where P_t^T and P_t^N are prices for tradable and non-tradable goods, respectively. The homothetic aggregator gives

$$P_t C_t = P_t^T C_t^T + P_t^N C_t^N. \tag{B.30}$$

B.2 Firms

There are three types of firms: a tradable good producer, a non-tradable good aggregator, and non-tradable intermediate good producers, each of which produces Y_t^T, Y_t^N, and $Y_t^N(i)$, respectively.

B.2.1 Tradable good producer

Under the perfect competition, the representative tradable good producer maximizes its profit:

$$\Pi_t^T := P_t^T Y_t^T - W_t h_t^T, \tag{B.31}$$

subject to the production technology:

$$Y_t^T = \exp\left(a_t^T\right) Z_t h_t^T, \tag{B.32}$$

where a_t^T and Z_t are the stationary and non-stationary components of the TFP, the latter of which grows at a constant rate:

$$\frac{Z_t}{Z_{t-1}} = 1 + g.$$

The FONC gives

$$\exp\left(a_t^T\right) Z_t = \frac{W_t}{P_t^T}. \tag{B.33}$$

The law of one price holds for the tradable goods:

$$P_t^T = S_t P_t^{T*}, \tag{B.34}$$

where S_t is the nominal exchange rate (price of foreign currency in terms of domestic currency) and P_t^{T*} is the tradable good price in terms of foreign currency.

B.2.2 Non-tradable good aggregator

The non-tradable good aggregator produces Y_t^N by aggregating the variety of intermediate goods $Y_t^N(i)$ subject to the CES technology:

$$Y_t^N = \left[\int_0^1 \left(Y_t^N(i)\right)^{1-\frac{1}{\theta}} \mathrm{d}i\right]^{\frac{\theta}{\theta-1}}.$$

The optimality conditions for the cost minimization problem are given by

$$Y_t^N(i) = \left[\frac{P_t^N(i)}{P_t^N}\right]^{-\theta} Y_t^N, \tag{B.35}$$

$$P_t^N = \int_0^1 \left[P_t^N(i)\right]^{1-\theta} \mathrm{d}i.$$

B.2.3 Non-tradable intermediate good producers

Under the monopolistic competition, each non-tradable intermediate good producer i maximizes its value:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{u_C\left(C_t, \frac{M_t}{P_t}, h_t\right)}{u_C\left(C_0, \frac{M_0}{P_0}, h_0\right)} \Pi_t^N(i),$$

subject to the profit function:

$$\Pi_t^N(i) := P_t^N(i) Y_t^N(i) - W_t h_t^N(i) - \frac{\chi}{2}\left[\frac{P_t^N(i)}{\left(\pi_{t-1}^N\right)^{\phi}\left(\pi^N\right)^{1-\phi} P_{t-1}^N(i)} - 1\right]^2 P_t^N Y_t^N, \tag{B.36}$$

the production technology:

$$Y_t^N(i) = \exp\left(a_t^N\right) Z_t h_t^N(i), \tag{B.37}$$

and the downward sloping demand curve given by (B.35), where $\pi_t^N := P_t^N / P_{t-1}^N$. Under the symmetric equilibrium, the FONCs lead to

$$\begin{aligned} 0 \quad = \quad & (1-\theta) + \theta \frac{W_t}{P_t^N \exp\left(a_t^N\right) Z_t} - \chi\left[\frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^{\phi}\left(\pi^N\right)^{1-\phi}} - 1\right] \frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^{\phi}\left(\pi^N\right)^{1-\phi}} \\ & + \chi\beta\mathbb{E}_t\left(\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}}\right)^{-1}\left[\frac{\pi_{t+1}^N}{\left(\pi_t^N\right)^{\phi}\left(\pi^N\right)^{1-\phi}} - 1\right] \frac{\pi_{t+1}^N}{\left(\pi_t^N\right)^{\phi}\left(\pi^N\right)^{1-\phi}} \frac{Y_{t+1}^N}{Y_t^N}. \end{aligned} \tag{B.38}$$

B.3 Government

The government budget constraint is given by

$$G_t + M_{t-1} + R_t = M_t + \left(1 + i_{t-1}\right) R_{t-1}, \tag{B.39}$$

where R_t is the interest bearing international reserves.

B.4 Central bank

The inflation-targeting central bank adjusts the nominal interest rate in response to deviations of inflation and output growth from their steady-state values with policy smoothing:

$$1 + i_t = \left(1 + i_{t-1}\right)^{\rho}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi}\left(\frac{Y_t/Y_{t-1}}{1+g}\right)^{\alpha_y} \exp\left(\varepsilon_{i,t}\right)\right]^{1-\rho}, \tag{B.40}$$

where $\bar{\pi}$ is the steady-state inflation and $\varepsilon_{i,t} \sim$ i.i.d. $N(0, \sigma_i^2)$ is a monetary policy shock.

B.5 Modified UIP condition and exchange rates

The modified UIP condition is given by

$$\mathbb{E}_t s_{t+1} = \left(\frac{1+i_t}{1+i_t^*}\right)^{\psi}, \tag{B.41}$$

where $s_t := S_t/S_{t-1}$ is the exchange rate depreciation.

The real exchange rate e_t is defined as

$$e_t := S_t \frac{P_t^*}{P_t} = S_t \frac{P_t^{T*}}{P_t}, \tag{B.42}$$

where we assume that $P_t^* = P_t^{T*}$.

B.6 Aggregate conditions

Aggregate conditions with respect to output, labor, and non-tradable goods are respectively given by

$$Y_t = Y_t^T + Y_t^N, \tag{B.43}$$

$$h_t = h_t^T + h_t^N, \tag{B.44}$$

$$C_t^N = Y_t^N. \tag{B.45}$$

The balance of payment identity is given by

$$B_t + R_t = (1+i_{t-1})(B_{t-1} + R_{t-1}) + P_t^T \left(Y_t^T - C_t^T\right) - \frac{\chi}{2}\left[\frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^{\phi}\left(\pi^N\right)^{1-\phi}} - 1\right]^2 P_t^N Y_t^N,$$

or

$$F_t = (1+i_{t-1}) F_{t-1} + P_t^T \left(Y_t^T - C_t^T\right) - \frac{\chi}{2}\left[\frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^{\phi}\left(\pi^N\right)^{1-\phi}} - 1\right]^2 P_t^N Y_t^N, \tag{B.46}$$

where $F_t := B_t + R_t$ is the net foreign asset.

B.7 Detrending and the system of equations

To ensure the stationarity of the system of equations, nonstationary variables are detrended by the trend component of the TFP Z_t and/or the aggregate price level P_t.

The equilibrium conditions in terms of detrended variables can be reduced to the system of 10 equations for 10 endogenous variables y_t, c_t, h_t, h_t^T, h_t^N, π_t, π_t^N, i_t, e_t, s_t:

$$\left(c_t - \frac{bc_{t-1}}{1+g}\right)^{-1} = \mathbb{E}_t \left(1 + \frac{c_t}{c} - \omega_h \frac{1}{1+\eta} h_t^{1+\eta}\right)^{-\kappa} \left(\left(1+g\right)c_{t+1} - bc_t\right)^{-1} \frac{1+i_t}{\pi_{t+1}},$$

$$\omega_h h_t^\eta = \left(c_t - \frac{b}{1+g} c_{t-1}\right)^{-1} \exp\left(a_t^T\right) e_t,$$

$$\pi_t = \left(\frac{e_t}{e_{t-1}}\right)^{\frac{1}{1-\gamma}} \pi_t^N,$$

$$\left(1-\gamma\right) c_t e_t^{\frac{\gamma}{1-\gamma}} = \exp\left(a_t^N\right) h_t^N,$$

$$\begin{aligned} 0 \quad = \quad & \left(1-\theta\right) + \theta \frac{e_t^{\frac{1}{1-\gamma}} \exp\left(a_t^T\right)}{\exp\left(a_t^N\right)} - \chi \left[\frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^\phi \left(\pi^N\right)^{1-\phi}} - 1\right] \frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^\phi \left(\pi^N\right)^{1-\phi}} \\ & + \chi \mathbb{E}_t \left(1 + \frac{c_t}{c} - \omega_h \frac{1}{1+\eta} h_t^{1+\eta}\right)^{-\kappa} \left(\frac{\left(1+g\right)c_{t+1} - bc_t}{c_t - \frac{b}{(1+g)} c_{t-1}}\right)^{-1} \left[\frac{\pi_{t+1}^N}{\left(\pi_t^N\right)^\phi \left(\pi^N\right)^{1-\phi}} - 1\right] \frac{\pi_{t+1}^N}{\left(\pi_t^N\right)^\phi \left(\pi^N\right)^{1-\phi}} \frac{\left(1+g\right)c_{t+1}}{c_t} \left(\frac{e_{t+1}}{e_t}\right)^{\frac{\gamma}{1-\gamma}}, \end{aligned}$$

$$\mathbb{E}_t s_{t+1} = \left(\frac{1+i_t}{1+i_t^*}\right)^\psi,$$

$$s_t = \frac{e_t}{e_{t-1}} \frac{\pi_t}{\pi_t^*},$$

$$h_t = h_t^T + h_t^N,$$

$$y_t = \exp\left(a_t^T\right) h_t^T + \exp\left(a_t^N\right) h_t^N,$$

$$1 + i_t = \left(1 + i_{t-1}\right)^\rho \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\alpha_y} \exp\left(\varepsilon_{i,t}\right)\right]^{1-\rho}.$$

where $y_t := Y_t/Z_t$ and $c_t := C_t/Z_t$.

Comment B.2. *Others are pinned down as follows:*

$$p_t^T = e_t,$$

$$w_t = \exp\left(a_t^T\right) e_t,$$

$$p_t^N = \left(\frac{1}{e_t}\right)^{\frac{\gamma}{1-\gamma}},$$

$$y_t^N = c_t^N,$$

$$c_t^N = (1-\gamma)\, c_t e_t^{\frac{\gamma}{1-\gamma}},$$

$$c_t^T = \gamma \frac{c_t}{e_t},$$

$$\pi_t^T = \frac{e_t}{e_{t-1}} \pi_t,$$

$$y_t^T = \exp\left(a_t^T\right) h_t^T,$$

$$m_t = \frac{\omega_m}{\left(c_t - \frac{bc_{t-1}}{\gamma}\right)^{-1} - \left(1 + \frac{c_t}{c} - \omega_h \frac{1}{1+\eta} h_t^{1+\eta}\right)^{-\kappa} (c_{t+1}\gamma - bc_t)^{-1} \frac{1}{\pi_{t+1}}},$$

$$f_t = \frac{1+i_{t-1}}{\pi_t \gamma} f_{t-1} + p_t^T \left(y_t^T - c_t^T\right) - \frac{\chi}{2} \left[\frac{\pi_t^N}{\left(\pi_{t-1}^N\right)^{\phi} \left(\pi^N\right)^{1-\phi}} - 1\right]^2 p_t^N y_t^N.$$

B.8 Steady state

We set $\beta = 0.99$ and $\kappa = 0.001$ and normalize $\omega_h = 1$. Then, the steady-state values (variables without time-subscripts) are pinned down as follows:

$$h = \left[\frac{(1+\eta)\left(2 - \beta^{-\frac{1}{\kappa}}\right)}{\omega_h}\right]^{\frac{1}{1+\eta}}.$$

$$\pi = \bar{\pi},$$

$$i = \frac{(1+g)\,\bar{\pi} - \beta}{\beta},$$

$$s = \frac{\bar{\pi}}{\pi^*},$$

$$e = \left(\frac{\theta-1}{\theta}\right)^{1-\gamma},$$

$$\pi^N = \bar{\pi},$$

$$c = \frac{\left(\frac{\theta-1}{\theta}\right)^{1-\gamma}}{\frac{1+g-b}{1+g}\omega_h} \left[\frac{\omega_h}{(1+\eta)\left(2-\beta^{-\frac{1}{\kappa}}\right)}\right]^{\frac{\eta}{1+\eta}},$$

$$h^N = \frac{\theta-1}{\theta} \frac{1-\gamma}{\frac{1+g-b}{1+g}\omega_h} \left[\frac{\omega_h}{(1+\eta)\left(2-\beta^{-\frac{1}{\kappa}}\right)}\right]^{\frac{\eta}{1+\eta}},$$

$$h^T = \left[\frac{(1+\eta)\left(2-\beta^{-\frac{1}{\kappa}}\right)}{\omega_h} \right]^{\frac{1}{1+\eta}} - \frac{\theta-1}{\theta} \frac{1-\gamma}{\frac{1+g-b}{1+g}\omega_h} \left[\frac{\omega_h}{(1+\eta)\left(2-\beta^{-\frac{1}{\kappa}}\right)} \right]^{\frac{\eta}{1+\eta}},$$

$$y = \left[\frac{(1+\eta)\left(2-\beta^{-\frac{1}{\kappa}}\right)}{\omega_h} \right]^{\frac{1}{1+\eta}}.$$

B.9 Log-linearized system of equations

Log-linearizing the detrended equilibrium conditions around the steady state yields the following linear system of equations:

$$\left(\frac{1+g+b}{1+g-b} - \psi\beta^{\frac{1}{\kappa}}\right)\hat{c}_t = \frac{b}{1+g-b}\hat{c}_{t-1} + \frac{1+g}{1+g-b}\mathbb{E}_t\hat{c}_{t+1} - \kappa(1+\eta)\left(2\beta^{\frac{1}{\kappa}}-1\right)\hat{h}_t - i_t + \mathbb{E}_t\hat{\pi}_{t+1},$$

$$\eta\hat{h}_t = -\frac{1+g}{1+g-b}\hat{c}_t + \frac{b}{1+g-b}\hat{c}_{t-1} + a_t^T + \hat{e}_t,$$

$$\hat{\pi}_t = \frac{1}{1-\gamma}\left(\hat{e}_t - \hat{e}_{t-1}\right) + \hat{\pi}_t^N,$$

$$\hat{c}_t + \frac{\gamma}{1-\gamma}\hat{e}_t = a_t^N + \hat{h}_t^N,$$

$$\hat{\pi}_t^N = \frac{\phi}{1+\phi\beta}\hat{\pi}_{t-1}^N + \frac{\beta}{1+\phi\beta}\mathbb{E}_t\hat{\pi}_{t+1}^N + \frac{\theta-1}{\chi(1+\phi\beta)}\left(\frac{1}{1-\gamma}\hat{e}_t + a_t^T - a_t^N\right),$$

$$\mathbb{E}_t\hat{s}_{t+1} = \psi\left(i_t - i_t^*\right),$$

$$\hat{s}_t = \hat{e}_t - \hat{e}_{t-1} + \hat{\pi}_t - \hat{\pi}_t^*,$$

$$\hat{h}_t = \frac{h^T}{h}\hat{h}_t^T + \frac{h^N}{h}\hat{h}_t^N,$$

$$\hat{y}_t = \frac{h^T}{h}\left(a_t^T + \hat{h}_t^T\right) + \frac{h^N}{h}\left(a_t^N + \hat{h}_t^N\right),$$

$$i_t = \rho i_{t-1} + (1-\rho)\left[\alpha_\pi\hat{\pi}_t + \alpha_y\left(\hat{y}_t - \hat{y}_{t-1}\right)\right] + \varepsilon_{i,t},$$

where hatted variables denote the percentage deviation from their steady-state values.

We treat foreign inflation $\hat{\pi}_t^*$ and the foreign nominal interest rate $\hat{i}_t^*$ as exogenous shocks. All the shocks except for the monetary policy shock $\varepsilon_{i,t}$ follow stationary AR(1) processes:

$$a_t^T = \rho_{aT}a_{t-1}^T + \varepsilon_{aT,t},$$

$$a_t^N = \rho_{aN}a_{t-1}^N + \varepsilon_{aN,t},$$

$$\hat{\pi}_t^* = \rho_{\pi^*}\hat{\pi}_{t-1}^* + \varepsilon_{\pi^*,t},$$
$$\hat{i}_t^* = \rho_{i^*}\hat{i}_{t-1}^* + \varepsilon_{i^*,t},$$

where $\varepsilon_{x,t} \sim$ i.i.d. $N(0, \sigma_x^2)$, $x \in \{aT, aN, \pi^*, i^*\}$.