Public Information Base and Consumer Welfare with Firm Level Ex-Ante Asymmetry under Quality Uncertainty

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Abstract

We develop a static model of price competition between two asymmetric firms to understand how the information base can affect the market outcome of an industry with quality uncertainty. We get two interesting finding using this simple model. Firstly, the market power of the firm with low chance of providing a high quality product always decreases with the public information base. However, the market power of a firm with high chance of providing a high quality product may increase or decrease depending up no the existing public information base. If the public information base expands from a high (low) initial level the market power of this firm increases (decreases) with the public information base. Secondly, the consumer welfare may increase or decrease depending up no the existing public information base. If the public information base expands from a high (low) initial level the then consumer welfare decreases (increases) with the public information base. Specifically, here we find a new channel namely gap in the market power of existing firms in the industry, through which the public information base which is created form consumers may harm them.

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1 Introduction

In the recent world, the information available in public domain (e.g. customer reviews, youtube contents, research article etc.) play a very important role in determining the consumption decision taken by consumers. Specifically for industry like hotels (Ye et al.[45]), movies (Duan et al.[16]), restaurants (Wu et al.[44]), and, more generally, across 600 product categories (Liu et al.[31]), the customer review can impact the sales and revenue of a firm. One can view the customer reviews as bank of information available in the public domain about the product of different firms or information base. More generally, a consumer before taking their consumption decision uses some search engine (google, youtube, amazon, facebook etc.) and collect a sub-set of information from the publicly available information base. Out of these sub-set of information it receives some noisy information as well about products of different firms in the same industry or product group, which affects her consumption decision. Moreover, several industry reports suggest that the percentage of consumers who write a post-purchase review ranges between 3% and 10% (see Zhou [46]; Weise [43]). Recent academic studies report a similar range. For instance, in Anderson and Simester [4], only 1.5% of the customers of a private label apparel firm wrote an online review, while in Brandes and Dover[9] and Brandes, Godes, and Mayzlin[10] respectively 11% and 18.6% of hotel customers posted an online review. Therefore, the bank of customer review or information base is itself a sample taken from some unknown population and the sub-set of the information base out of which consumers take their consumption decision is a sub-sample taken from the bank of customer review. Hence, the size of the bank of customer review will actually determine the efficacy of the search engine that consumer uses and hence the noise received by the customers. In this study our objective is to examine how the size of customer review affects the market power of different firms in the industry, and consequently the consumer welfare. More precisely, are more information better for the consumers?

We develop a static model to understand how the information base can affect the market outcome of an industry with quality uncertainty. The quality uncertainty in the literature of industrial organization is generally modeled using, the seminal work of Akerlof [1]. However, in Akerlof [1] we have asymmetric incomplete information about the product quality (seller are more informed than the buyers.). This may lead to price signaling in the market but we deviate from Akerlof [1] by considering the case where there is uncertainty about the quality on both sides of the market. This two-sided quality uncertainty may be relevant particularly the experience good industry. A good is called an experienced good if its quality is realized ex-post. For example, we may receive unsatisfactory service from a very reputed hotel relative to a less reputed one. Similarly, there is no guarantee that a more qualified doctor will always provide more satisfactory medical service than a less qualified one. Hence, there may exist uncertainty about service quality on part of service provider as well, which is unrelated to the reputation, qualification etc. but is purely an outcome realization of good state or bad state. Similar uncertainty may exist in quality of goods produced. Motivated by such uncertainties, we develop a duopoly model with quality uncertainty on both side of the market. The consumer in our model, on the other hand, is assumed to receive different signals from the nature (randomly) about the probability with which a firm will provide a good quality product.

We get two interesting finding using this simple model.

Firstly, the market power of the firm with low chance of providing high quality product always decreases with the public information base. However the market power of the firm with high chance of providing high quality product may increase or decrease depending upno the public information base. If the initial information base is high (low) enough then market power increases (decreases) with the information base. There are various empirical studies that show evidence to this finding. In the context of two-sided platforms, Resnick [37] show that eBay sellers with a positive reputation can sell their products for higher prices relative to sellers without or with a negative reputation. Using data from eBay coin auctions, Lucking-Reiley [**?**] find that negative feedback has a statistically significant effect on price, while the effect is not significant for positive feedback. Houser and Wooders [23] show that a 10% increase in positive feedback on eBay increases the winning price by about 0.17%, while a 10% increase in neutral or negative feedback will decrease the price by 0.24%.

Secondly, the consumer welfare may decrease depending up no the existing public information base. If the initial size of the public information base (or customer reviews) is high (low) enough then consumer welfare decreases (increases) with the information base. Theses results contrast the finding of Ananthakrishnan et al.[5] and Hollenbeck [22].

To the best of our knowledge there is no study that examines how the market outcome will change with the size of the public information available in the public domain. We explain our model in section 2 followed by determination of the demand in section 3. The market equilibrium is described in section 4. Section 5 discusses our main results. We conclude in section 6 with discussions.

1.1 Literature Review

Our study is mostly related to two streams of literature of the economic theory: information structure and quality uncertainty. Let us start with information structure. The literature on information structure is spread over various sub-sets of literature on the economic theory. In the non-oligopoly structure, for example, various studies relates information structure with equilibrium outcome such as Bar-Isaac et al. [6] with labour market, Gieczewski [20] with screening, Kartik and Zhong [27] and Pavan and Tirole [34] with lemon market. Finally, our result is very similar to Janssen and Roy[25] with lemon market. However, the channel through which our result works is different. Moreover, our study is mainly focused with oligopoly theory.[1] The literature where oligopoly theory and information structure intersect can be sub-categorized into different sub-sets. First sub-set that discuss about the consumer learning and information acquisition. The

[1]There are also studies that relates information structure with monopoly market outcomes such as Koh and Li [29] on social learning, Mensch and Ravid [33] on monopolistic screening and Roesler and Szentes [39] on buyer optimal learning.

important contribution toward this direction are Albrecht and Whitmeyer [2] with comparison shopping; Biglaiser et al. [11] with product differentiation perception; Dogon and Hu [15] with consumer search; [40] with consumer control in the two sided market. Second, type of literature focuses on information design problem by third party like information provider. Contribution towards this line of analysis are Elliott and Li [17] with the market segmentation issue and Carnehl et al. [14] with product differentiation; Li and Zhao [30] with monopoly pricing. However, our study is related to the literature of the information structure which relates the public information to the oligopoly theory. There are few recent studies towards this line among them study of Armstrong and Zhou [3] is closely related to our study. Like Feinberg and Skrzypacz [18],[2] uncertainty via noisy signal about an uncertain event (quality of the product) provide an interesting foundation our model which is not present in Armstrong and Zhou [3]. The main difference between the Armstrong and Zhou [3] and our study is that in our paper the firms are ex-ante asymmetric. We establish that the consumers optimum information is proportionately increases with the ex-ante difference between the firms. Further, the competition softening effect of information is also affects differently for different firm. The information base always increases the competition for the firs with low ex-ante probability. On the other hand the affect on the firm with high ex-ante probability is ambiguous. The study by Bergemann et al. [7] and Bergemann et al. [8] also discuss effects of information structure on the market outcomes like consumer surplus, producer surplus and total surplus. However, Bergemann et al. [7] discuss only about the alignments of these market outcome and is silent about how it changes with the information structure. [3]

Our paper is also related to a parallel literature that examines impact of price competition under quality uncertainty. For example, Gabszewicz and Resende [19] considered a two-dimensional framework with consumers having preferences for both vertical and horizontal differentiation of products to analyze how uncertainty about product quality affects equilibrium outcomes. Reme [36], on the other hand, showed that quality uncertainty has non-monotonic effects on firms' price

[2]Other studies that talks about higher order uncertainty are Kets [28], Rong [38] and Weinstein and Yildiz [42]

[3]There are other studies with information structure and oligopoly theory such that Canidio and Gall [13] with information generation, Bergemann et al. [8] with monopoly market setting with price discrimination, Shi and Zhang [41] with market segmentation and Hwang et al. [24] with competitive advertising.

setting behavior. Interestingly, quality uncertainty can force a high quality provider to be aggressive to the point where its price in the first period is below that of a low quality provider. Further, when quality uncertainty is sufficiently high, product compatibility may be used as a means of softening price competition. Grilo and Wauthy [21] modelled situations where consumers take explicit account of the risk associated with buying such products while making their consumption decisions. The consumers are heterogeneous with respect to attitude towards risk, and this significantly affects price competition.

Earlier, Bouckaert and Degryse [12] analysed the market for professional services characterized by the coexistence of ”experts” and ”non-experts” whose main difference lies in the probability that they will provide the adequate service. Krishna and Winston (1998) offered a similar perspective of product quality: the quality of a product depends on the probability that it will generate the satisfaction that a consumer is expected to enjoy. Thus, at the time of buying, the consumer faces an intrinsic uncertainty.

Our analysis, however, is different from these in the sense that not only we focus on the consumers receiving signals from the information set available in the public domain, noisy though and different from each other, regarding the probability of a producer supplying a high (or a low) quality; but also the uncertainty that prevails on part of the producers themselves depending on realization of a good or a bad state.

2 The Model

Consider an industry of experience good with two firms viz. Firm 1 and Firm 2. Let us denote the price and quality of Firm i respectively by p_i and s_i, $i = 1, 2$. The quality that Firm i produces can be either high s^H or low s^L with $s^H > s^L$: $s_i \in \{s^H, s^L\}$ for all $i = 1, 2$. However, the quality that any Firm i produces depends on a random event, the realized production state of the Firm i which can be either good or bad. If the production state of Firm i is good then it produces s^H; and s^L otherwise. The probability that the good state occurs for Firm i and it produces s^H is $\bar{\lambda}_i$.

The probability differs across the firms and without any loss of generality, we assume Firm 1 has a larger chance of having a good production state than Firm 2, that is, $\bar{\lambda}_1 > \bar{\lambda}_2$. Thus the firms are assumed to be ex-ante asymmetric in terms of the realization of the good (or bad) production states.

Suppose there is a set of experience population E. Each individual $e \in E$ carries an information about the Firm i's quality if it had the prior experience of consuming the Firm i's product. Therefore, each $e \in E$ is characterized by an information vector $u^e = (u_1^e, u_2^e)$ where $u_i^e \in \{NA, 0, 1\}$ for all $i = 1, 2$. If individual $e \in E$ had no prior experience of consuming the Firm i's product then $u_i^e = NA$. If individual $e \in E$ had prior experience of consuming good quality the Firm i's product then $u_i^e = 1$. Otherwise $u_i^e = 0$. Define the set of individuals having prior experience of consuming Firm i's product is $E_i = \{e \in E \mid u_i^e \neq NA\}$ and the set of individual having prior experience of consuming high quality of Firm i's product is $E_i^h = \{e \in E \mid u_i^e = 1\}$. Therefore, proportion of experienced buyers who purchased from Firm i and had the experience of having good quality is $\lambda_i^E = |E_i^h|/|E_i|$ where for any set S, $|S|$ denotes the cardinality of the set S. We assume for any $i \in \{1, 2\}$, E_i is sufficiently large set. Therefore, using law of large numbers we can conclude that $\lambda_i^E \to \bar{\lambda}_i$ for every $i \in \{1, 2\}$. Moreover, there is a non-empty set of experienced individual $E^p \subset E$ with $E^p \neq \emptyset$, who truly reveals their experience vector in the public domain.

Suppose there is a unit mass of new buyer who can purchase the product. The buyers do not know the true vale $(\bar{\lambda}_1, \bar{\lambda}_2)$. However, any buyer b can collect information about the experienced quality of both firms' products by enumerating the following two sets of individuals.

(i) The set E^p which is available in the public domain.

(ii) The set of experienced individual who are connected to the buyer b, $E^b \subset E$ which is assign by the nature randomly to buyer b. We assume for every buyer b we have fixed $|E^b \cap E_i|$.[4]

Therefore, any buyer b can only enumerate a subset set $(E^p \cup E^b)$ of E to predict $(\bar{\lambda}_1, \bar{\lambda}_2)$.

Assumption 1 Any consumer b is naive enough in predicting $(\bar{\lambda}_1, \bar{\lambda}_2)$ always assume $(E^p \cap E^b) = \emptyset$.

[4]In the discussion section we will discuss how relaxing this assumption do not change our result qualitatively.

The Assumption 1 implies that the consumers have ignorance biase to enumerate the available data points and take the information available in her personal domain and public domain are separate.

Hence, the predicted value $(\lambda_1^b, \lambda_2^b)$ of $(\bar{\lambda}_1, \bar{\lambda}_2)$ for the buyer b can be erroneous based on its true values. In other words nature send a noise signal to any buyer b about the probability that Firm i will provide good quality is λ_i^b which include two components, its true value $\bar{\lambda}_i$ and sampling error ϵ_i^b. That is,

$$\lambda_i^b = \bar{\lambda}_i + \epsilon_i^b.$$

Further, given E^b varies with b therefore ϵ_i^b also varies across b. Also, random assignment of E^b by the nature, makes ϵ_i^b random as well. Ignoring the superscript b we can in generally write each individual in the unit muss received a noise signal from the nature about the probability that Firm i will provide good quality is λ_i which include two components, its true value $\bar{\lambda}_i$ and sampling error ϵ_i.

$$\lambda_i = \bar{\lambda}_i + \epsilon_i \;\; \forall \;\; i \in \{1, 2\}.$$

The noise vector (ϵ_1, ϵ_2) independently and jointly distributed with joint probability density function $g(\epsilon_1, \epsilon_2)$ over the interval $[-\bar{\epsilon}_1, \bar{\epsilon}_1] \times [-\bar{\epsilon}_2, \bar{\epsilon}_2]$ and symmetric about (0,0) where $\bar{\epsilon}_i$ is the maximum noise length for ϵ_i. We assume maximum noise length is same for both the noises that is $\bar{\epsilon}_i = \bar{\epsilon}$ for all $i \in \{1, 2\}$. Further, $\sigma_i^2 = var(\epsilon_i)$ which decreases with number of sample size. Note, if $|E^p|$ increases then for each individual the sample size increases hence σ decreases.

Definition 1 An increase in consumer review base $|E^p|$ will be called increase in information in the public domain if and only if $|E^p|$ reduces the mean preserving spread of the noises. Therefore, an increase in $|E^p|$ leads to increase in information if and only if $\bar{\epsilon}$ decreases with an increase in $|E^p|$.

Assumption 2 We assume $\bar{\epsilon} = \sigma(\sigma_1, \sigma_2)$ with $(\partial\sigma/\partial\sigma_i) > 0$ for all $i = 1, 2$.

Therefore, using definition 1, the assumption 2 implies an increase in $|E^p|$ always implies an increase in information. Hence, we call increases in $|E^p|$ as increase in the public information base. Finally, $g(\epsilon_1, \epsilon_2)$ is the common knowledge. The consumers are heterogeneous in terms of signal they received (λ_1, λ_2) from nature. Therefore, unit mass of consumer distribute over $[\bar{\lambda}_1 - \sigma, \bar{\lambda}_1 + \sigma] \times [\bar{\lambda}_2 - \sigma, \bar{\lambda}_2 + \sigma]$ with probability density function specified by the

$$f(\lambda_1, \lambda_2) = g(\lambda_1 - \bar{\lambda}_1, \lambda_2 - \bar{\lambda}_2)$$

as shown in the Figure 1. Finally, we need Assumption 3 along with Assumption 2 to structure

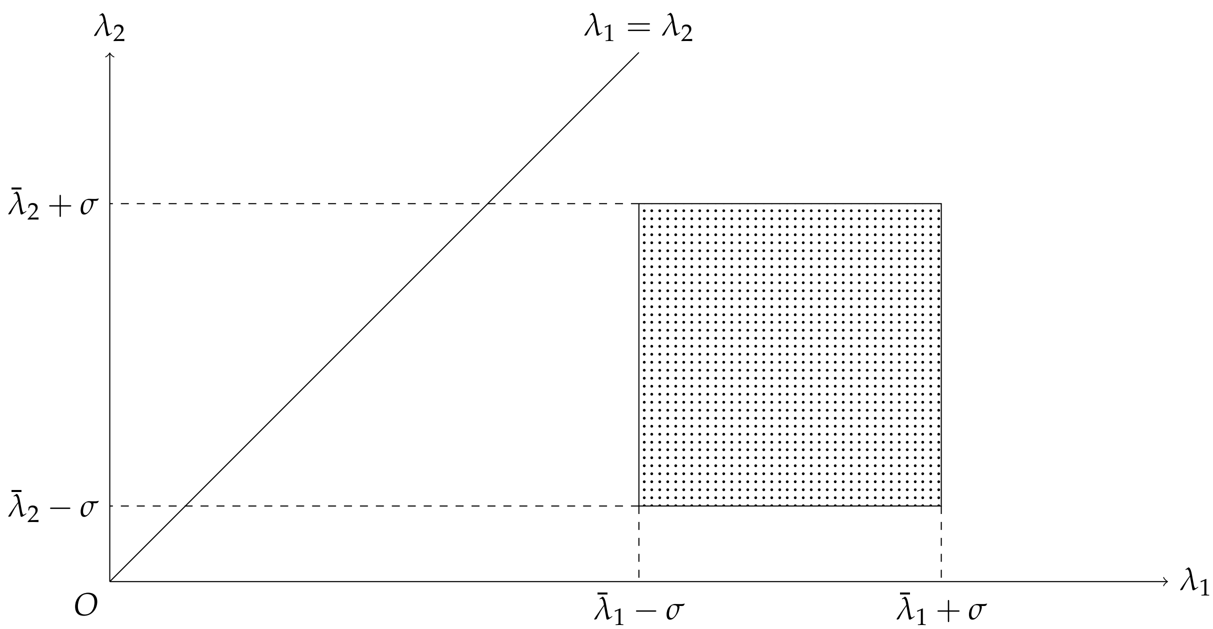

Figure 1: The Consumers Distribution

the parameter space.

Assumption 3 We assume $0 \leq \sigma \leq Min\{\bar{\lambda}^d/2, (1 - \bar{\lambda}^d)/2\}$.

Assumption 3 implies two properties together.

Firstly, we assume that there is no consumer receiving signal such that the chance of providing high quality by Firm 2 is larger than Firm 1. In other word, nature never provides signal in contrast to the true ranking of $\bar{\lambda}_1$ and $\bar{\lambda}_2$. Notionally, we have $\bar{\lambda}_1 - \sigma > \bar{\lambda}_2 + \sigma$ implies

$$\bar{\lambda}^d = \bar{\lambda}_1 - \bar{\lambda}_2 > 2\sigma. \tag{1}$$

Secondly, we assume each consumer received legitimated signal, that is, $[\bar{\lambda}_i - \sigma, \bar{\lambda}_i + \sigma] \subseteq [0,1]$. Given, $\bar{\lambda}_1 - \sigma > \bar{\lambda}_2 + \sigma$ we have for $i = 1,2$, $[\bar{\lambda}_i - \sigma, \bar{\lambda}_i + \sigma] \subseteq [0,1]$ if only if $\bar{\lambda}_2 - \sigma \geq 0$ implies $\bar{\lambda}_2 \geq \sigma$ and $\bar{\lambda}_1 + \sigma \leq 1$ implies $\bar{\lambda}_1 \leq 1 - \sigma$, combining we have

$$\bar{\lambda}^d < 1 - 2\sigma. \tag{2}$$

Hence, conditions (1) and (2) together give us Assumption 3. In particular, Assumption 3 determines the parameter space of our model which is given by the shaded region in the Figure 2.

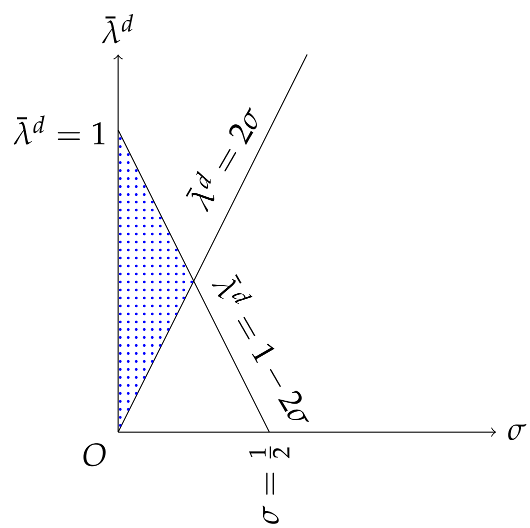

Figure 2: The Parameter Space

The sequence of event occur as follows,

- **Stage-0:** Given any E, nature assign any subset of E randomly to each customer. Then each customer combine the information in her subset with the information available in the public domain E^p, received signal (λ_1, λ_2). Therefore, each consumer signal is there private information. However, nature reveals the distribution of subset assign over the customers to the firms and E^p is available in the public domain. Hence, distribution of (λ_1, λ_2), that is $f(\lambda_1, \lambda_2)$, is common knowledge.

- **Stage-1:** Each firm charges its optimum the price.
- **Stage-2:** Consumers decides from which firms to purchase.

For simplicity of our analysis we assume that the noises (ϵ_1, ϵ_2) independently and identically follow the uniform distribution over $[-\sigma, \sigma]^2$.[5] Therefore, the joint probability density function of (ϵ_1, ϵ_2) is

$$g(\epsilon_1, \epsilon_2) = \frac{1}{4\sigma^2} \; \forall \; (\epsilon_1, \epsilon_2) \in [-\sigma, \sigma]^2. \tag{3}$$

Moreover, using the joint density function of (ϵ_1, ϵ_2) in equation (3), one can write the joint probability density function of (λ_1, λ_2) as

$$f(\lambda_1, \lambda_2) = \frac{1}{4\sigma^2} \; \forall \; (\lambda_1, \lambda_2) \in [\bar{\lambda}_1 - \sigma, \bar{\lambda}_1 + \sigma] \times [\bar{\lambda}_2 - \sigma, \bar{\lambda}_2 + \sigma]. \tag{4}$$

3 The Demand Functions

Each individual of the unit mass either consumes one unit of the commodity or do not consume anything. If she consumes from Firm i and received quality s_i then she receives surplus $(s_i - p_i)$, $i = 1, 2$.[6] If she does not consume then "net utility" is zero. Therefore, each consumers have three options:

(i) Purchase from Firm 1 ($d^c = 1$). If any consumer receives signal (λ_1, λ_2) from the search-engine then purchasing from Firm 1 implies that the consumer will get good quality (s^H) with probability λ_1 and will get bad quality (s^L) with probability $1 - \lambda_1$. Hence, the expected utility of the consumer is

$$EU(p_1, p_2; \lambda_1, \lambda_2, d^c = 1) = \lambda_1(s^H - p_1) + (1 - \lambda_1)(s^L - p_1) = \lambda_1 s^d + s^L - p_1$$

[5] In the discussion section we will discuss the channel through which information may harm consumers will actually become stronger by relaxing the assumption of uniform distribution.

[6] We assume linear gross utility from consuming quality s_i for analytical convenience. However, it is inconsequential because there is no quality choice by Firm i.

where $s^d = s^H - s^L > 0$.

(ii) Purchase from Firm 2 ($d^c = 2$). If any consumer receives signal (λ_1, λ_2) from the search-engine then purchasing from Firm 2 implies that the consumer will get good quality (s^H) with probability λ_2 and will get bad quality (s^L) with probability $1 - \lambda_2$. Hence, the expected utility of the consumer is

$$EU(p_1, p_2; \lambda_1, \lambda_2, d^c = 2) = \lambda_2(s^H - p_2) + (1 - \lambda_2)(s^L - p_2) = \lambda_2 s^d + s^L - p_2.$$

(iii) It can abstain from purchasing ($d^c = 0$). The reservation utility of each consumer is zero. Hence, in this case

$$EU(p_1, p_2; \lambda_1, \lambda_2, d^c = 0) = 0.$$

Therefore, combining all three possibilities we have

$$EU(p_1, p_2; \lambda_1, \lambda_2, d^c) = \begin{cases} \lambda_1 s^d + s^L - p_1 & \text{for } d^c = 1 \\ \lambda_2 s^d + s^L - p_2 & \text{for } d^c = 2 \\ 0 & \text{for } d^c = 0 \end{cases} \tag{5}$$

Assumption 4 We assume $s^L \geq \max\{p_1, p_2\}$.

The Assumption 4 implies full market coverage, since even when the bad state realizes all consumers get strictly non-negative net utility (or, surplus) regardless of which Firm they buy from.[7] Subsequently, we will relax these assumption and allow for "partial" market coverage to check robustness of our results. However, in these section we assume the full market coverage. The consumer choose, $d^c = 1$ over $d^c = 2$ if and only if it is incentive compatible:

$$\lambda_1 s^d + s^L - p_1 \geq \lambda_2 s^d + s^L - p_2 \Rightarrow \lambda_1 \geq \lambda_2 + \frac{p_1 - p_2}{s^d}.$$

[7]Observe that in Assumption 5 we take more stringent condition than what we requires for full coverage. However, we will relax this assumption when we extend our result to the partial coverage.

Therefore, given any (p_1, p_2) and quality difference s^d any consumer who received signal (λ_1, λ_2) such that

$$\lambda_1 \geq \lambda_2 + \frac{p_1 - p_2}{s^d} \tag{6}$$

will consume from Firm 1. Hence the market segmentation condition is

$$\lambda_1 = \lambda_2 + \frac{p_1 - p_2}{s^d}. \tag{7}$$

Since, the unit mass of population is uniformly distributed over the $[\lambda_1 - \sigma, \lambda_1 + \sigma] \times [\lambda_2 - \sigma, \lambda_2 + \sigma]$ with joint density function given by the condition (4), hence the following properties of the condition (7) are important to derive different possibilities of demand faced by Firm 1 and 2.

(i) If $p_1 - p_2 = (\bar{\lambda}^d + 2\sigma)s^d$ then locus of condition (7) satisfies the point $(\bar{\lambda}_1 + \sigma, \bar{\lambda}_2 - \sigma)$. This market segmentation line is represented by A_1B_1 line in Figure 3.

(ii) If $p_1 - p_2 = \bar{\lambda}^d s^d$ then locus of condition (7) satisfies the points $(\bar{\lambda}_1 - \sigma, \bar{\lambda}_2 - \sigma)$ and $(\bar{\lambda}_1 + \sigma, \bar{\lambda}_2 + \sigma)$simultaneously. This market segmentation line is represented by A_2B_2 line in Figure 3.

(iii) If $p_1 - p_2 = (\bar{\lambda}^d - 2\sigma)s^d$ then locus of condition (7) satisfies the points $(\bar{\lambda}_1 - \sigma, \bar{\lambda}_2 + \sigma)$. This market segmentation line is represented by A_3B_3 line in Figure 3.

Therefore, based on the value of (p_1, p_2) which in tern determines the position of the line (7), we have the following possibilities of demand faced by the firms.

Case-I: If $p_1 - p_2 \geq (\bar{\lambda}^d + 2\sigma)s^d$ then the locus of (7) always lies on or below A_1B_1. Hence, for every (λ_1, λ_2), condition (6) never holds. Hence, every consumer purchases from Firm 2. Therefore, for all $p_1 - p_2 \geq (\bar{\lambda}^d + 2\sigma)s^d$, we have $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = 1$.

Case-II: If $(\bar{\lambda}^d + 2\sigma)s^d > p_1 - p_2 > \bar{\lambda}^d s^d$ then the locus of (7) always passes between the line A_1B_1 and A_2B_2, such as A_4B_4 line in Figure (4). Therefore, the consumer who has received the signal to the right of the line A_4B_4 in the box, that is, the red shaded region in Figure 4,

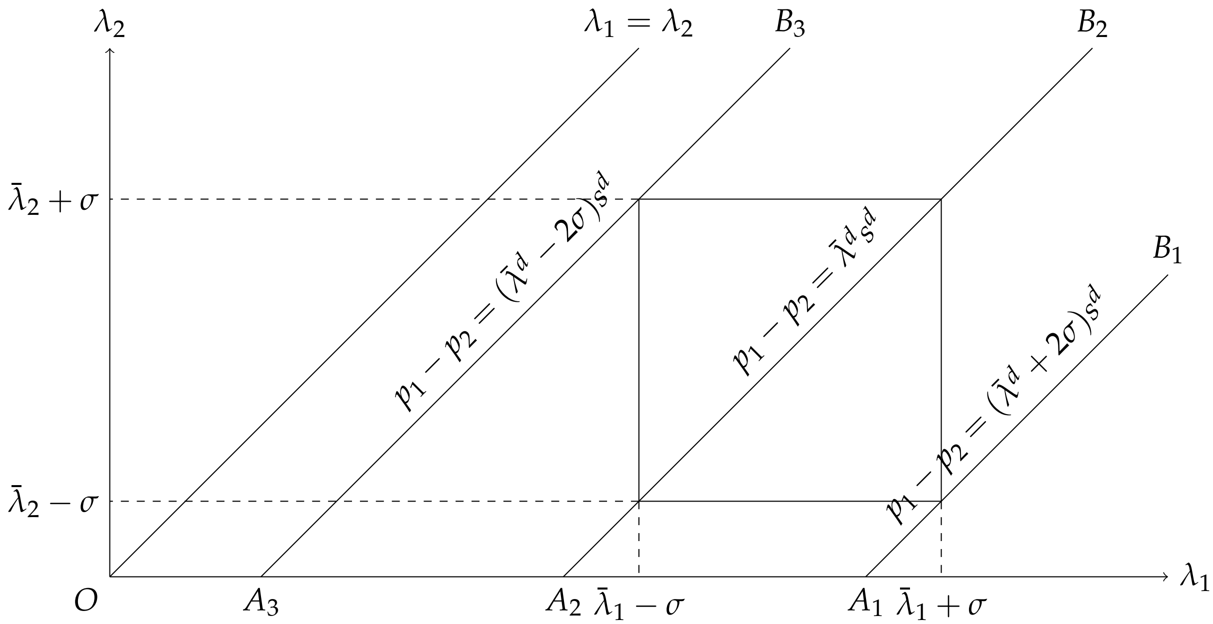

Figure 3: Different Possibilities

will purchase from Firm 1. Similarly, any consumer who has received the signal in blue shaded region will purchase from Firm 2. Therefore, for all $(\bar{\lambda}^d + 2\sigma)s^d > p_1 - p_2 > \bar{\lambda}^d s^d$ we have $D_1(p_1, p_2) = \frac{1}{8\sigma^2}\left(\bar{\lambda}_1 + \sigma - \bar{\lambda}_2 + \sigma - \frac{p_1 - p_2}{s^d}\right)^2 = \frac{\left[(\bar{\lambda}^d + 2\sigma)s^d - p_1 + p_2\right]^2}{8\sigma^2 (s^d)^2}$ and $D_2(p_1, p_2) = 1 - \frac{\left[(\bar{\lambda}^d + 2\sigma)s^d - p_1 + p_2\right]^2}{8\sigma^2 (s^d)^2}$.

Case-III: If $p_1 - p_2 = \bar{\lambda}^d s^d$ then locus of condition (7) will coincide with the line A_2B_2. Therefore, half of the consumer received the signal that satisfy condition (6). Hence, we have $D_1(p_1, p_2) = 1/2$ and $D_2(p_1, p_2) = 1/2$.

Case-IV: If $\bar{\lambda}^d s^d > p_1 - p_2 > (\bar{\lambda}^d - 2\sigma)s^d$ the the locus the the condition (7) will passes between the line A_2B_2 and A_3B_3 as shown in Figure 5 by line A_5B_5. Therefore, for all $s^d\bar{\lambda}^d > p_1 - p_2 > (\bar{\lambda}^d - 2\sigma)s^d$ we have $D_1(p_1, p_2) = 1 - \frac{\left[p_1 - p_2 - (\bar{\lambda}^d - 2\sigma)s^d\right]^2}{8\sigma^2 (s^d)^2}$ and $D_2(p_1, p_2) = \frac{\left[p_1 - p_2 - (\bar{\lambda}^d - 2\sigma)s^d\right]^2}{8\sigma^2 (s^d)^2}$.

Case-V: If $p_1 - p_2 < (\bar{\lambda}^d - 2\sigma)s^d$ then every customer received signal that satisfy condition (6) and we will have $D_1(p_1, p_2) = 1$ and $D_2(p_1, p_2) = 0$.

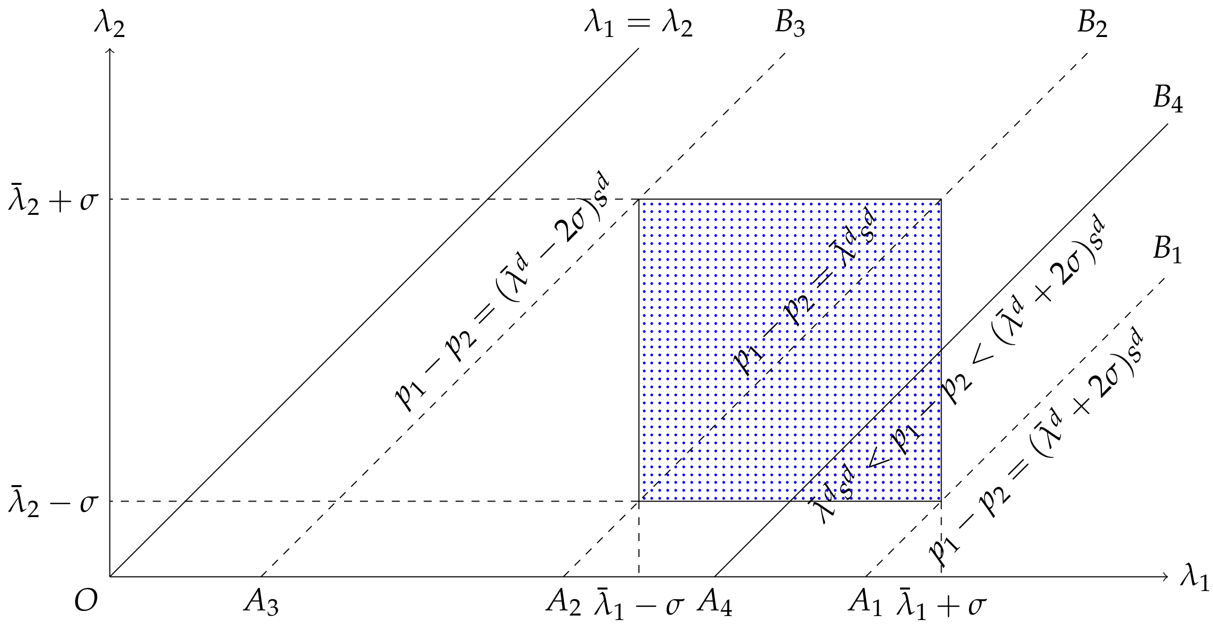

Figure 4: Case-II

Hence, combining all the five cases we have

$$D_1(p_1, p_2) = \begin{cases} 1 & \forall\ p_1 \leq p_2 + (\bar{\lambda}^d - 2\sigma)s^d \\ 1 - \frac{\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)\right]^2}{8\sigma^2(s^d)^2} & \forall\ p_2 + s^d(\bar{\lambda}^d - 2\sigma) < p_1 < p_2 + \bar{\lambda}^d s^d \\ \frac{1}{2} & \forall\ p_1 = p_2 + \bar{\lambda}^d s^d \\ \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]^2}{8\sigma^2(s^d)^2} & \forall\ p_2 + s^d\bar{\lambda}^d < p_1 < p_2 + (\bar{\lambda}^d + 2\sigma)s^d \\ 0 & \forall\ p_1 \geq p_2 + (\bar{\lambda}^d + 2\sigma)s^d \end{cases} \tag{8}$$

and

$$D_2(p_1, p_2) = \begin{cases} 0 & \forall\ p_2 \geq p_1 - (\bar{\lambda}^d - 2\sigma)s^d \\ \frac{\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)\right]^2}{8\sigma^2(s^d)^2} & \forall\ p_1 - s^d(\bar{\lambda}^d - 2\sigma) > p_2 > p_1 - \bar{\lambda}^d s^d \\ \frac{1}{2} & \forall\ p_2 = p_1 - \bar{\lambda}^d s^d \\ 1 - \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]^2}{8\sigma^2(s^d)^2} & \forall\ p_1 - s^d\bar{\lambda}^d > p_2 > p_1 - (\bar{\lambda}^d + 2\sigma)s^d \\ 1 & \forall\ p_2 \leq p_1 - (\bar{\lambda}^d + 2\sigma)s^d \end{cases} \tag{9}$$

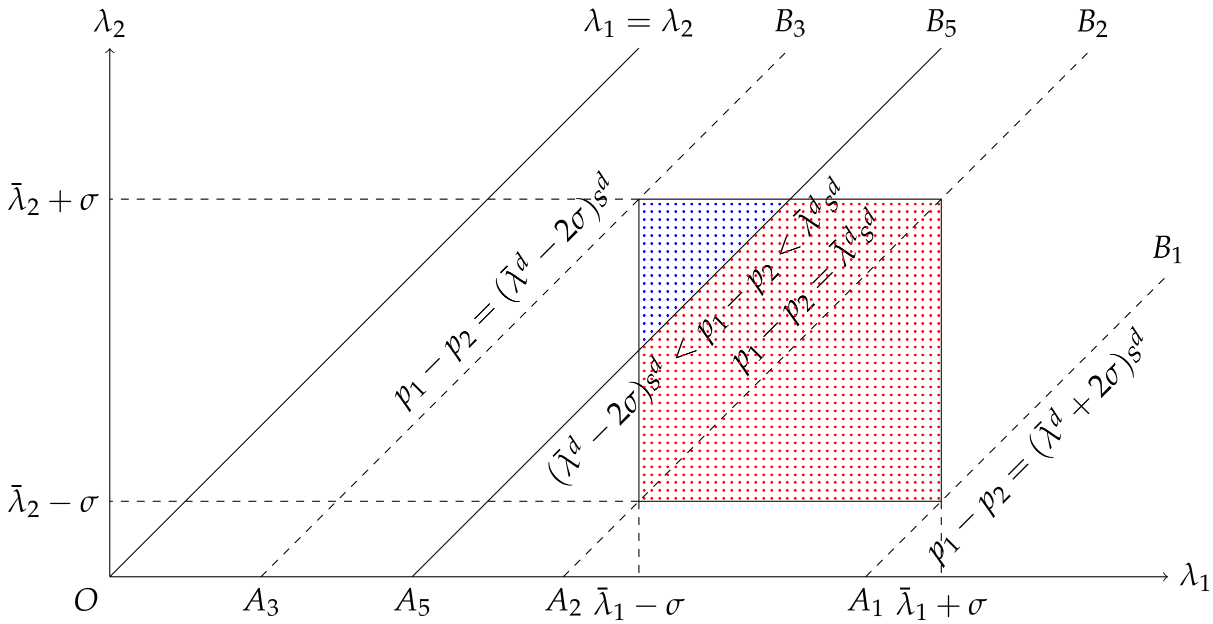

Figure 5: Case-IV

We will explain the demand function using the Figure 6. In this figure we consider the $p_1 - p_2$ plane. Suppose that $s^L = \overline{OL} = \overline{OL'}$ and $s^H = \overline{OH} = \overline{OH'}$. Moreover, the line OA, A_1B_1, A_2B_2 and A_3B_3 respectively represent the locus of $p_1 = p_2$, $p_1 = p_2 + s^d(\bar{\lambda}^d - 2\sigma)$, $p_1 = p_2 + s^d\bar{\lambda}^d$ and $p_1 = p_2 + s^d(\bar{\lambda}^d + 2\sigma)$. Since, we consider $(p_1, p_2) \in [0, s^L]^2$, therefore we focus in the box $OLCL'$. Since, in the gray shaded area $OA_1C_1CL'O$ we have $p_1 \leq p_2 + s^d(\bar{\lambda}^d - 2\sigma)$ therefore $D_1(p_1, p_2) = 1$ and $D_2(p_1, p_2) = 0$. In the red shaded area $A_1A_2C_2C_1A_1$ we have $p_2 + s^d(\bar{\lambda}^d - 2\sigma) < p_1 < p_2 + s^d\bar{\lambda}^d$ implies $D_1(p_1, p_2) = 1 - [p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)]^2/8\sigma^2(s^d)^2$ and $D_2(p_1, p_2) = [p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)]^2/8\sigma^2(s^d)^2$. On the line A_2B_2 we have $p_1 = p_2 + s^d\bar{\lambda}^d$ implies $D_1(p_1, p_2) = D_2(p_1, p_2) = 1/2$. In the green shaded region $A_2C_2C_3A_3A_2$ we have $p_2 + s^d\bar{\lambda}^d < p_1 < p_2 + s^d(\bar{\lambda}^d + 2\sigma)$ implies $D_1(p_1, p_2) = [s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2]^2/8\sigma^2(s^d)^2$ and $D_2(p_1, p_2) = 1 - [s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2]^2/8\sigma^2(s^d)^2$. Finally, in the region $A_3C_3LA_3$ we have $p_1 \geq p_2 + s^d(\bar{\lambda}^d + 2\sigma)$ implies $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = 1$. Moreover, note that the demand functions are continuous in (p_1, p_2).

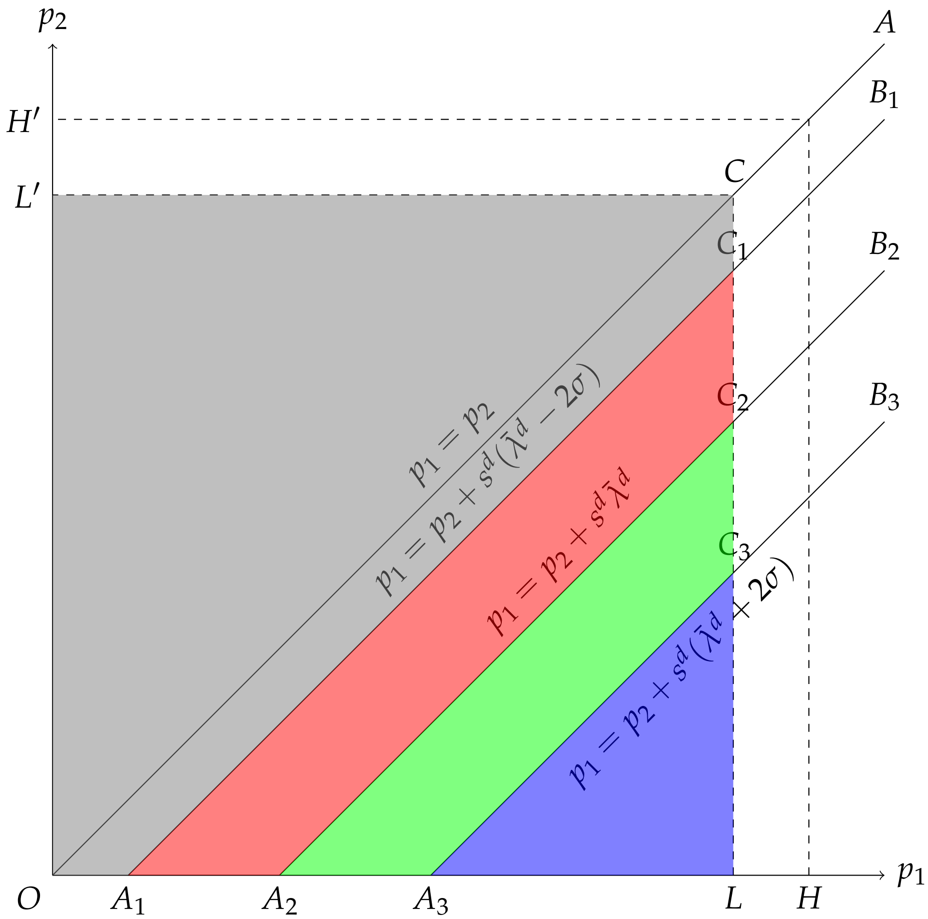

Figure 6: Domain of the demand function

Therefore, given (p_1, p_2) the consumer's welfare is

$$CS(p_1, p_2) = \left[\iint_{\{(\lambda_1,\lambda_2)|d^c=1\}} EU(p_1, p_2; \lambda_1, \lambda_2, d^c = 1) + \iint_{\{(\lambda_1,\lambda_2)|d^c=2\}} EU(p_1, p_2; \lambda_1, \lambda_2, d^c = 2) \right] f(\lambda_1, \lambda_2) d\lambda_2 d\lambda_1. \quad (10)$$

4 The Market Equilibrium

In this section initially we ignore our Assumption 4 and focus on the non-negative prices, $(p_1, p_2) \in \mathbb{R}^2_+$. We assume price competition among the two firms. Given our focus on how information availability influence market equilibrium, the production costs are normalized to zero. Given the price of the Firm 2 (p_2), Firm 1 maximizes his profit:

$$\pi_1(p_1,p_2) = p_1 D_1(p_1,p_2) = \begin{cases} p_1 & \forall\ p_1 \le p_2 + (\bar{\lambda}^d - 2\sigma)s^d \\ p_1\left[1 - \frac{\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)\right]^2)}{8\sigma^2 (s^d)^2}\right] & \forall\ p_2 + s^d(\bar{\lambda}^d - 2\sigma) < p_1 < p_2 + \bar{\lambda}^d s^d \\ \frac{p_1}{2} & \forall\ p_1 = p_2 + \bar{\lambda}^d s^d \\ \frac{p_1\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]^2}{8\sigma^2 (s^d)^2} & \forall\ p_2 + s^d\bar{\lambda}^d < p_1 < p_2 + (\bar{\lambda}^d + 2\sigma)s^d \\ 0 & \forall\ p_1 \ge p_2 + (\bar{\lambda}^d + 2\sigma)s^d. \end{cases} \quad (11)$$

Similarly, given the price of Firm 1 (p_1), Firm 2 maximizes his profit

$$\pi_2(p_1,p_2) = p_2 D_2(p_1,p_2) = \begin{cases} 0 & \forall\ p_2 \ge p_1 - (\bar{\lambda}^d - 2\sigma)s^d \\ \frac{p_2\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)\right]^2)}{8\sigma^2 (s^d)^2} & \forall\ p_1 - s^d(\bar{\lambda}^d - 2\sigma) > p_2 > p_1 - \bar{\lambda}^d s^d \\ \frac{p_2}{2} & \forall\ p_2 = p_1 - \bar{\lambda}^d s^d \\ p_2\left[1 - \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]^2}{8\sigma^2 (s^d)^2}\right] & \forall\ p_1 - s^d\bar{\lambda}^d > p_2 > p_1 - (\bar{\lambda}^d + 2\sigma)s^d \\ p_2 & \forall\ p_2 \le p_1 - (\bar{\lambda}^d + 2\sigma)s^d. \end{cases} \quad (12)$$

Note that the domain of the profit function is also explained by Figure 6.

4.1 Reaction Function of Firm 1

If we ignore the Assumption 4 then from Figure 6 it is clear that for every $p_2 \ge 0$ as p_1 increases every piece of the domain of $\pi(p_1,p_2)$ function as given by equation (11) will be reached. Therefore, we will derive the reaction function of Firm 1 by enumerating the following properties of $\pi_1(p_1,p_2)$ function given by the equation (11). Given any $p_2 \ge 0$,

(i) $\pi_1(p_1,p_2)$ is continuous in p_1.

(ii) $\pi_1(p_1,p_2)$ is increasing in p_1 for all $p_1 \in (0, p_2 + (\bar{\lambda}^d - 2\sigma)s^d)$.

(iii) For all $p_1 \in (p_2 + s^d(\bar{\lambda}^d - 2\sigma), p_2 + s^d\bar{\lambda}^d)$ we have

$$\frac{\partial \pi_1}{\partial p_1} = 1 - \frac{[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)][3p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)]}{8\sigma^2(s^d)^2}.$$

Therefore

$$\lim_{p_1 \to [p_2 + s^d(\bar{\lambda}^d - 2\sigma)]^+} \frac{\partial \pi_1}{\partial p_1} = 1 > 0 \text{ and } \lim_{p_1 \to [p_2 + s^d\bar{\lambda}^d]^-} \frac{\partial \pi_1}{\partial p_1} = -\frac{p_2 + (\bar{\lambda}^d - \sigma)s^d}{4\sigma s^d} < 0$$

implies that there exist at least one $\hat{p}_1(p_2)$ such that $\partial \pi_1(\hat{p}_1(p_2), p_2)/\partial p_1 = 0$. Further, we have

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = -\frac{3\left[p_1 - \frac{2}{3}(p_2 + s^d(\bar{\lambda}^d - 2\sigma))\right]}{4\sigma^2(s^d)^2} < 0 \text{ and } \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{2\left[p_1 - \frac{1}{2}(p_2 + s^d(\bar{\lambda}^d - 2\sigma))\right]}{4\sigma^2(s^d)^2} > 0$$

implies in $p_1 - p_2$ plane the locus of $\partial \pi_1/\partial p_1 = 0$ is monotonically increasing. Hence, for any p_2 we have an unique $\hat{p}_1(p_2)$ such that $\partial \pi_1(\hat{p}_1(p_2), p_2)/\partial p_1 = 0$. Finally, since $\partial^2 \pi_1/\partial p_1^2 < 0$, therefore $\hat{p}(p_2)$ always maximizes $\pi_1(p_1, p_2)$ in the domain $(p_2 + s^d(\bar{\lambda}^d - 2\sigma), p_2 + s^d\bar{\lambda}^d)$. In particular, we have:

$$\hat{p}_1(p_2) = \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{p_2 + (\bar{\lambda}^d - 2\sigma)s^d\right\}^2} + 2\left\{p_2 + (\bar{\lambda}^d - 2\sigma)s^d\right\}}{3}$$

(iv) For any $p_1 \in (p_2 + s^d\bar{\lambda}^d, p_2 + s^d(\bar{\lambda}^d + 2\sigma))$ we have

$$\frac{\partial \pi_1}{\partial p_1} = \frac{[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2][s^d(\bar{\lambda}^d + 2\sigma) - 3p_1 + p_2]}{8\sigma^2(s^d)^2}.$$

Given, $p_1 < s^d(\bar{\lambda}^d + 2\sigma) + p_2$ implies $s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2 > 0$, therefore, $\partial \pi_1/\partial p_1 \gtreqless 0$ if and only if $s^d(\bar{\lambda}^d + 2\sigma) - 3p_1 + p_2 \gtreqless 0$ implies $p_1 \lesseqgtr \frac{s^d(\bar{\lambda}^d + 2\sigma) + p_2}{3}$. Further, we have

$$p_2 + s^d\bar{\lambda}^d - \frac{s^d(\bar{\lambda}^d + 2\sigma) + p_2}{3} = \frac{3p_2 + 3s^d\bar{\lambda}^d - s^d(\bar{\lambda}^d + 2\sigma) - p_2}{3} = \frac{2\left[p_2 + s^d(\bar{\lambda}^d - \sigma)\right]}{3} > 0.$$

Therefore, given any $p_2 \geq 0$ for all $p_1 \in (p_2 + s^d \bar{\lambda}^d, p_2 + s^d(\bar{\lambda}^d + 2\sigma))$ we have $\partial \pi_1 / \partial p_1 < 0$.

Combining the properties (i) to (iv), for any $p_2 \geq 0$ the graph of $\pi_(p_1, p_2)$ against p_1 is shown in the Figure

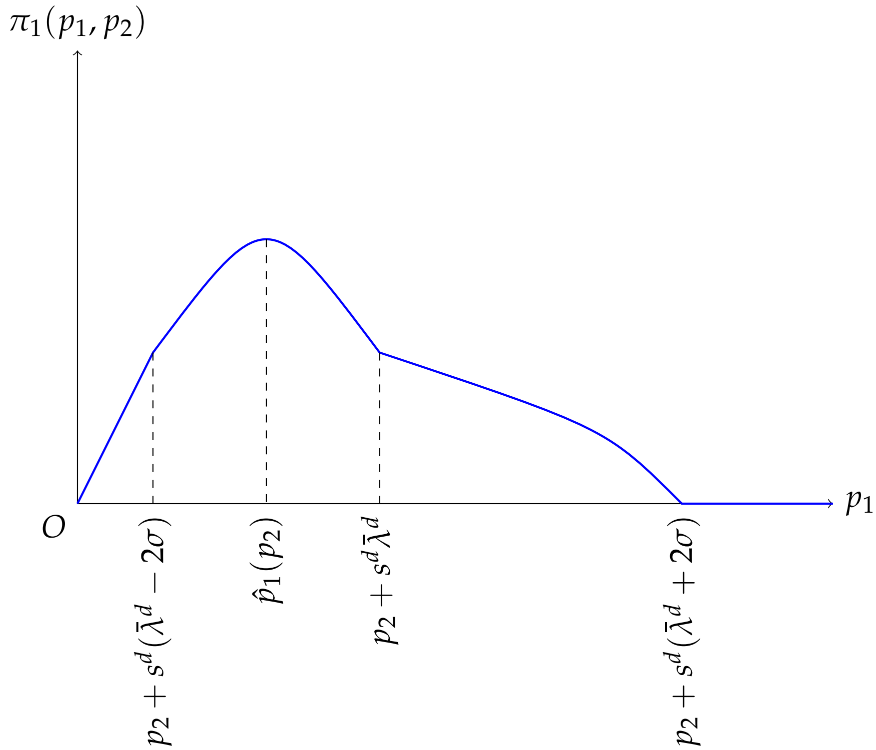

Figure 7: The $\pi_(p_1, p_2)$ for $p_1 \geq 0$.

Hence, given $p_2 \geq 0$ the reaction equation of Firm 1 is

$$\hat{p}_1(p_2) = \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{p_2 - (\bar{\lambda}^d + 2\sigma)s^d\right\}^2} + 2\left\{p_2 - (\bar{\lambda}^d - 2\sigma)\right\}}{3}. \tag{13}$$

Now, by Assumption 4 (the case of full market coverage), we restrict our-self in $(p_1, p_2) \in [0, s^L]^2$. Therefore, $\hat{p}_1(p_2)$ be the reaction equation of the Firm 1 if and only if $\hat{p}_1(p_2) \leq s^L$ implies $p_2 \leq \hat{p}_2 = 2s^L - s^d(\bar{\lambda}^d - 2\sigma) + \sqrt{(s^L)^2 + 8(s^d)^2\sigma^2}$. Finally for $p_2 > \hat{p}_2$ we have $p_1 = s^L$ be the reaction of the Firm 1. Therefore, the reaction function of the Firm 1 is

$$R_1(p_2) = \begin{cases} \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{p_2 - (\bar{\lambda}^d + 2\sigma)s^d\right\}^2} + 2\left\{p_2 - (\bar{\lambda}^d - 2\sigma)\right\}}{3} & \forall\ p_2 \leq \hat{p}_2 \\ s^L & \forall\ p_2 > \hat{p}_2. \end{cases} \tag{14}$$

The reaction function of the Firm 1 is represented by the blue curve $R_1R_1'R_1''$ in Figure 8.

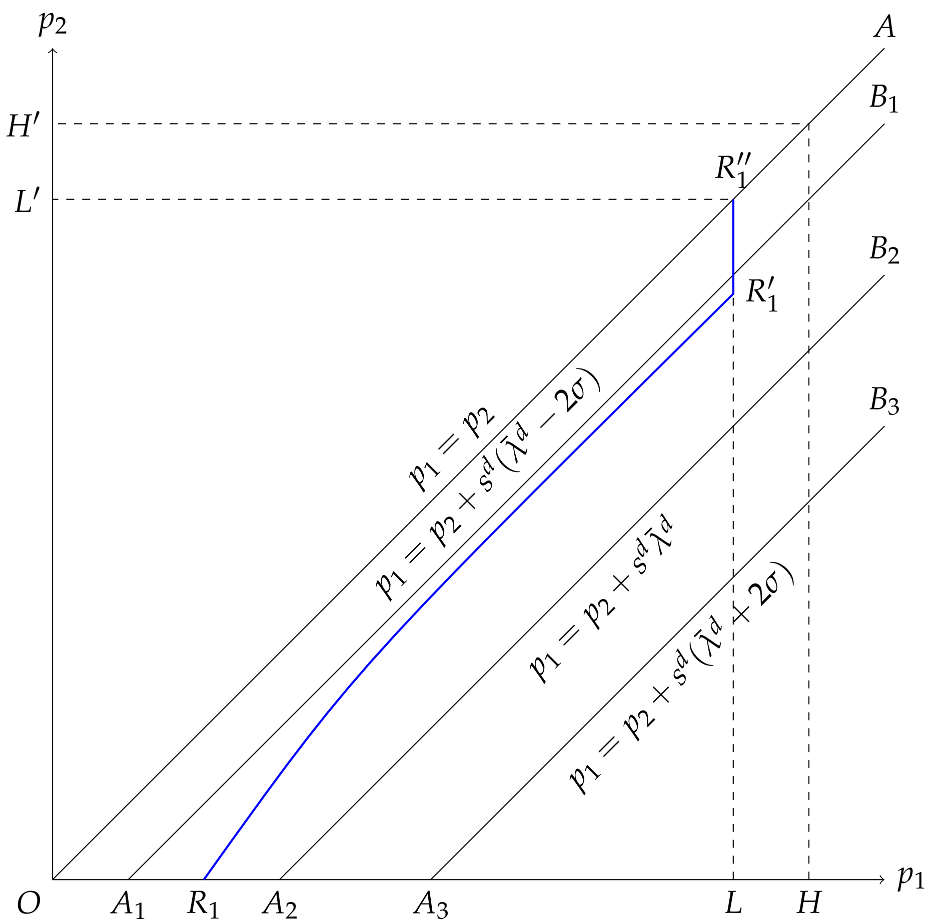

Figure 8: Reaction function of Firm 1

4.2 Reaction Function of Firm 2

If we ignore Assumption 4 then from Figure 6 it is clear that the domain of $\pi_2(p_1, p_2)$ function as specified by equation (11) will depend on the value of p_1. Therefore, we will derive the reaction function of Firm 2 by enumerating the following cases.

(i) Given any $p_1 \leq (\bar{\lambda}^d - 2\sigma)s^d$ we always have $p_2 \geq 0 \geq p_1 - (\bar{\lambda}^d - 2\sigma)s^d$ implies $\pi_2(p_1, p_2) = 0$. Hence any $p_2 \in [0, \infty)$ is the best response of Firm 2.

(ii) Given any $(\bar{\lambda}^d - 2\sigma)s^d < p_1 < \bar{\lambda}^d s^d$ we have

$$\pi_2(p_1, p_2) = \begin{cases} \frac{p_2\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma\right]^2)}{8\sigma^2(s^d)^2} & \forall\, 0 < p_2 < p_1 - (\bar{\lambda}^d - 2\sigma)s^d \\ 0 & \forall\, p_2 \geq p_1 - (\bar{\lambda}^d - 2\sigma)s^d. \end{cases}$$

Moreover, we have for any $0 < p_2 < p_1 - (\bar{\lambda}^d - 2\sigma)s^d$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma)\right]\left[p_1 - 3p_2 - s^d(\bar{\lambda}^d - 2\sigma)\right]}{8\sigma^2(s^d)^2}.$$

Therefore we have

$$\frac{\partial \pi_2}{\partial p_2} \gtreqless 0 \;\forall\, p_2 \lesseqgtr \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma)) < (p_1 - s^d(\bar{\lambda}^d - 2\sigma)).$$

Further, $\pi_2(p_1, p_2)$ is continuous. Hence, $p_2 = \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma))$ be the unique maximizer of $\pi_2(p_1, p_2)$.

(iii) Given any $\bar{\lambda}^d s^d < p_1 < (\bar{\lambda}^d + \sigma)s^d$ we have

$$\pi_2(p_1, p_2) = \begin{cases} p_2\left[1 - \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]^2}{8\sigma^2(s^d)^2}\right] & \forall\, 0 \leq p_2 < p_1 - s^d\bar{\lambda}^d \\ \frac{p_2}{2} & \forall\, p_2 = p_1 - \bar{\lambda}^d s^d \\ \frac{p_2\left[p_1 - p_2 - s^d(\bar{\lambda}^d - 2\sigma\right]^2)}{8\sigma^2(s^d)^2} & \forall\, p_1 - \bar{\lambda}^d s^d < p_2 < p_1 - s^d(\bar{\lambda}^d - 2\sigma) \\ 0 & \forall\, p_2 \geq p_1 - s^d(\bar{\lambda}^d - 2\sigma) \end{cases} \tag{15}$$

Therefore, we have the following

(a) For all $0 \leq p_2 < p_1 - s^d\bar{\lambda}^d$ we have

$$\frac{\partial \pi_2}{\partial p_2} = 1 - \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + 3p_2\right]}{8\sigma^2(s^d)^2}$$

Further,

$$\frac{\partial^2 \pi_2}{\partial p_2^2} = \frac{3\left[p_2 - \frac{2}{3}\left\{p_1 - s^d(\bar{\lambda}^d + 2\sigma)\right\}\right]}{4\sigma^2(s^d)^2} < 0.$$

Since,

$$\lim_{p_2 \to 0^+} \frac{\partial \pi_2}{\partial p_2} = 1 - \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1\right]^2}{8\sigma^2(s^d)^2} \gtreqless 0 \Rightarrow p_1 \gtreqless (\bar{\lambda}^d - 2(\sqrt{2} - 1)\sigma)s^d. \tag{16}$$

Given, $p_1 > \bar{\lambda}^d s^d$ we have $\lim_{p_2 \to 0^+} (\partial \pi_2 / \partial p_2) > 0$. Further,

$$\lim_{p_2 \to (p_1 - s^d\bar{\lambda}^d)^-} \frac{\partial \pi_2}{\partial p_2} = \frac{s^d(\bar{\lambda}^d + \sigma) - p_1}{2\sigma s^d} > 0 \; \forall \; p_1 < s^d(\bar{\lambda}^d + \sigma). \tag{17}$$

Therefore, $\pi_2(p_1, p_2)$ is increasing for $0 \geq p_2 > p_1 - s^d\bar{\lambda}^d$.

(b) For all $p_1 - \bar{\lambda}^d s^d < p_2 < p_1 - s^d(\bar{\lambda}^d - 2\sigma)$ using (ii) and the continuity of the $\pi_2(p_1, p_2)$, we can conclude that $p_2 = \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma))$ be the unique maximizer of $\pi_2(p_1, p_2)$ if and only if $\frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma)) > p_1 - s^d\bar{\lambda}^d$ solving for p_1, we have $p_1 < s^d(\bar{\lambda}^d + \sigma)$. Hence, $p_2 = \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma))$ be the unique maximizer of $\pi_2(p_1, p_2)$.

(iv) Given any $p_1 \geq (\bar{\lambda}^d + \sigma)s^d$ using the argument of (iii) (b) one can conclude that $p_2 = \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma))$ is not a maximizer of $\pi_2(p_1, p_2)$. However, using the argument of (iii) (a) one can conclude that the best response will be obtained by solving

$$1 - \frac{\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + p_2\right]\left[s^d(\bar{\lambda}^d + 2\sigma) - p_1 + 3p_2\right]}{8\sigma^2(s^d)^2} = 0$$

for p_2. Hence, $p_2 = \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{(\bar{\lambda}^d + 2\sigma) - p_1\right\}^2} + 2\left\{(\bar{\lambda}^d + 2\sigma) - p_1\right\}^2}{3}$ is the best response of the Firm 2.

Therefore, if we ignore the Assumption 4 the reaction function of the Firm 2 is

$$r_2(p_1) = \begin{cases} [0, \infty) & \forall\ 0 \leq p_1 \leq (\bar{\lambda}^d - 2\sigma)s^d \\ \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma)) \text{ be the unique maximizer of } \pi_2(p_1, p_2) & \forall\ (\bar{\lambda}^d - 2\sigma)s^d < p_1 \leq (\bar{\lambda}^d + \sigma)s^d \\ \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{(\bar{\lambda}^d + 2\sigma) - p_1\right\}^2} + 2\left\{(\bar{\lambda}^d + 2\sigma) - p_1\right\}^2}{3} & \forall\ p_1 > (\bar{\lambda}^d + \sigma)s^d. \end{cases} \tag{18}$$

If we consider the Assumption 4 the value s^L will determine the right corner point of the reaction function of Firm 2 on $r_2(p_2)$. Since, we are interested in the interior solution therefore we ignore those complication and assume that s^L is sufficiently high enough. One upper limit of s^L is mention in the next Assumption 5.

Assumption 5 We assume $s^L \geq s^d(\bar{\lambda}^d + \sigma)$.

Given the Assumption 5 and equation (18) we can conclude the reaction function of Firm 2 by equation (19).

$$R_2(p_1) = \begin{cases} [0, \infty) & \forall\ 0 \leq p_1 \leq (\bar{\lambda}^d - 2\sigma)s^d \\ \frac{1}{3}(p_1 - s^d(\bar{\lambda}^d - 2\sigma)) \text{ be the unique maximizer of } \pi_2(p_1, p_2) & \forall\ (\bar{\lambda}^d - 2\sigma)s^d < p_1 \leq (\bar{\lambda}^d + \sigma)s^d \\ \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{(\bar{\lambda}^d + 2\sigma) - p_1\right\}^2} + 2\left\{(\bar{\lambda}^d + 2\sigma) - p_1\right\}^2}{3} & \forall\ (\bar{\lambda}^d + \sigma)s^d < p_1 \leq s^L. \end{cases} \tag{19}$$

The reaction function of Firm 2 is represented the Figure 9 by the red shaded area and the red curve $A_1 R_2 R_2'$

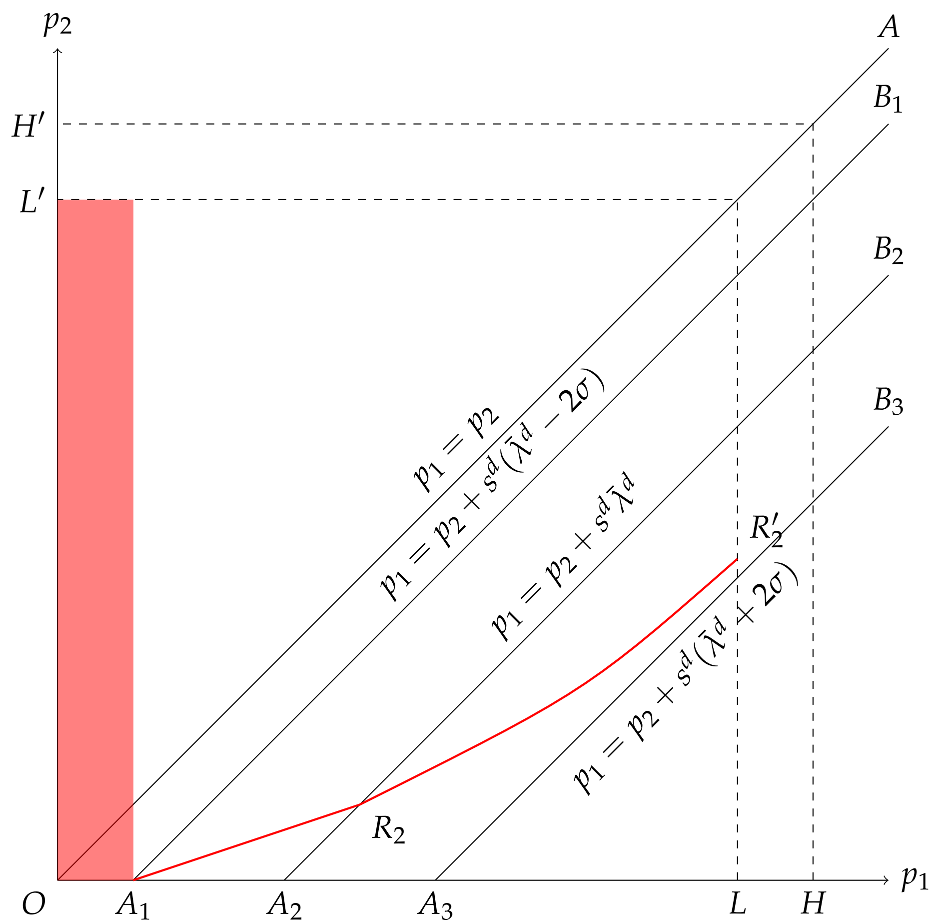

Figure 9: Reaction function of Firm 2

4.3 Equilibrium Outcome

The market equilibrium is obtained from intersection of the reaction curves. We superimpose the Figure 8 and 9 in Figure 10 together to determine the intersection. Using the Figure 10 one can conclude that there is unique intersection between the reaction curves at point E. Algebraically, unique market equilibrium (p_1^*, p_2^*) is obtained by solving the following equations

$$p_1^* = \frac{\sqrt{24(s^d)^2\sigma^2 + \left\{p_2^* - (\bar{\lambda}^d + 2\sigma)s^d\right\}^2} + 2\left\{p_2^* - (\bar{\lambda}^d - 2\sigma)\right\}}{3} \text{ and } p_2^* = \frac{1}{3}\left(p_1^* - s^d(\bar{\lambda}^d - 2\sigma)\right).$$

The Nash equilibrium pair of prices are given as:

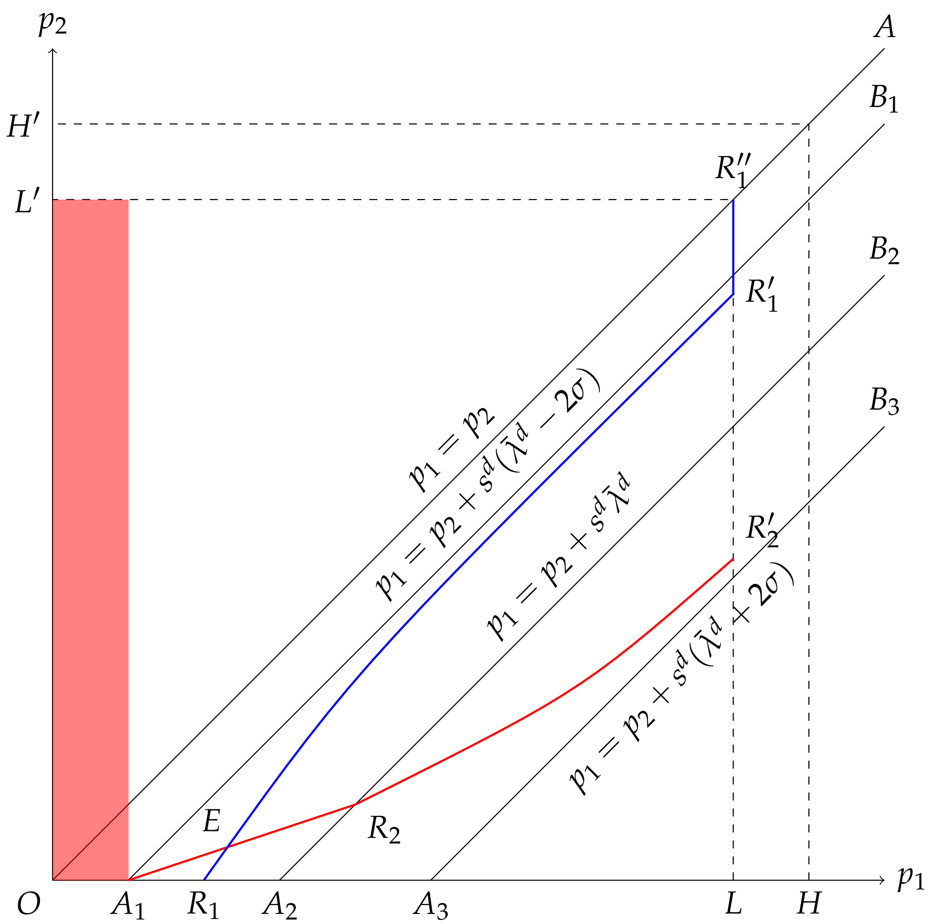

Figure 10: Market Equilibrium

$$p_1^* = \frac{s^d\left[5(\bar{\lambda}^d - 2\sigma) + 3\sqrt{(\bar{\lambda}^d - 2\sigma)^2 + 32\sigma^2}\right]}{8} = \frac{s^d\bar{\lambda}^d\left[5(1-2v) + 3\sqrt{(1-2v)^2 + 32v^2}\right]}{8} \tag{20}$$

and

$$p_2^* = \frac{s^d\left[-(\bar{\lambda}^d - 2\sigma) + \sqrt{(\bar{\lambda}^d - 2\sigma)^2 + 32\sigma^2}\right]}{8} = \frac{s^d\bar{\lambda}^d\left[-(1-2v) + \sqrt{(1-2v)^2 + 32v^2}\right]}{8} \tag{21}$$

where $v = \sigma/\bar{\lambda}^d$.Next the resulting equilibrium quantities are

$$D_1^* = D_1(p_1^*, p_2^*) = 1 - \frac{\left\{\bar{\lambda}^d - 2\sigma - \sqrt{\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2}\right\}^2}{128\sigma^2} = 1 - \frac{\left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2}{128v^2} \tag{22}$$

and

$$D_2^* = D_2(p_1^*, p_2^*) = \frac{\left\{\bar{\lambda}^d - 2\sigma - \sqrt{\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2}\right\}^2}{128\sigma^2} = \frac{\left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2}{128v^2}. \tag{23}$$

Hence, the equilibrium profit earned by the Firm 1 and Firm 2 are respectively

$$\pi_1^* = p_1^* D_1^* = \frac{s^d \bar{\lambda}^d \left[5(1-2v) + 3\sqrt{(1-2v)^2 + 32v^2}\right]}{8} \left[1 - \frac{\left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2}{128v^2}\right] \tag{24}$$

and

$$\pi_2^* = p_2^* D_2^* = \frac{s^d \bar{\lambda}^d \left[-(1-2v) + \sqrt{(1-2v)^2 + 32v^2}\right]^3}{1024v^2}. \tag{25}$$

Hence the aggregate profit is

$$\begin{aligned}\pi^* &= \pi_1^* + \pi_2^* = p_1^* D_1^* + p_2^* D_2^* = p_1^*(1 - D_2^*) + p_2^* D_2^* = p_1^* - (p_1^* - p_2^*) D_2^* \\ &= \frac{s^d \bar{\lambda}^d \left[(212v^2 - 20v + 5)\sqrt{36v^2 - 4v + 1} - (36v^2 - 4v + 1)^(3/2) + 4(1-2v)(68v^2 + 4v - 1)\right]}{512v^2}\end{aligned} \tag{26}$$

5 The comparative Static

Here in this section we will check, how the information base in the public domain will affect the market outcome. Note that given our structure the variability of consumer's signal is inversely related to the size of the information available in the public domain. Therefore, comparative static with respect to σ is nothing but the comparative static with respect to the size of information. We denote $v := \sigma/\bar{\lambda}^d$ to present our comparative static results. The next Lemma 1 discuss the comparative static results of equilibrium prices with respect to σ.

Lemma 1 (i) We always have $\partial p_2^*/\partial\sigma > 0$.

(ii) If $\bar{\lambda}^d < 1/(1+2\tilde{v}_1^d)$ then we have $\partial p_1^*/\partial\sigma \gtreqless 0$ iff $v = \sigma/\bar{\lambda}^d \gtreqless \tilde{v}_1 \approx 0.1605456870$. Otherwise we always have $\partial p_1^*/\partial\sigma < 0$.

(iii) We always have $\partial(p_1^* - p_2^*)/\partial\sigma < 0$.

(iv) We always have $\partial(D_2^*/D_1^*)/\partial\sigma > 0$.

Proof: Proof of (i): Differentiating p_2^* given by the equation (21) with respect to σ we get,

$$\frac{\partial p_2^*}{\partial\sigma} = \frac{s^d\left[\sqrt{\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2} - \left(\bar{\lambda}^d - 18\sigma\right)\right]}{4\sqrt{\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2}}. \tag{27}$$

Observe the following

(a) If $\bar{\lambda}^d \leq 18\sigma$ then we have $\partial p_2^*/\partial\sigma > 0$.

(b) If $\bar{\lambda}^d > 18\sigma$ then $\partial p_2^*/\partial\sigma > 0$ if and only if $\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2 > \left(\bar{\lambda}^d - 18\sigma\right)^2$ implies $\bar{\lambda}^d > 9\sigma$. Therefore, for every $\bar{\lambda}^d > 18\sigma > 9\sigma$ we have $\partial p_2^*/\partial\sigma > 0$.

Therefore, combining (a) and (b) we have $\partial p_2^*/\partial\sigma > 0$ for all $0 \leq \sigma \leq Min\{\bar{\lambda}^d/2, (1-\bar{\lambda}^d)/2\}$.

Proof of (ii): Differentiating p_1^* given by the equation (20) with respect to σ we get,

$$\frac{\partial p_1^*}{\partial\sigma} = -\frac{s^d\left[5\sqrt{\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2} + 3\left(\bar{\lambda}^d - 18\sigma\right)\right]}{4\sqrt{\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2}}. \tag{28}$$

Observe the following

(a) If $\bar{\lambda}^d \geq 18\sigma$ then we have $\partial p_1^*/\partial\sigma < 0$.

(b) If $\bar{\lambda}^d < 18\sigma$ then $\partial p_1^*/\partial\sigma \lesseqgtr 0$ if and only if $25\left[\left(\bar{\lambda}^d - 2\sigma\right)^2 + 32\sigma^2\right] \gtreqless 9\left(\bar{\lambda}^d - 18\sigma\right)^2$ implies $v \lesseqgtr \tilde{v}_1$ or $\sigma \lesseqgtr \tilde{v}_1\bar{\lambda}^d$.

(c) Given, $\tilde{v}_1 < 1/2$ we have $\tilde{v}_1\bar{\lambda}^d < \bar{\lambda}^d/2$. Further, $\tilde{v}_1\bar{\lambda}^d \lesseqgtr (1-\bar{\lambda}^d)/2$ implies $\bar{\lambda}^d \lesseqgtr 1/(1+2\tilde{v}_1) > 1/2$.

Combining (a), (b), and (c) we have the Lemma 1(ii).

Proof of Lemma 1(iii): We have

$$p_1^* - p_2^* = \frac{\bar{\lambda}^d s^d \left[3(1-2v) + \sqrt{(1-2v)^2 + 32v^2}\right]}{4}.$$

Differentiating with respect to σ we have

$$\frac{\partial \left(p_1^* - p_2^*\right)}{\partial \sigma} = \frac{s^d \left[(1-18v) - 3\sqrt{(1-2v)^2 + 32v^2}\right]}{2}.$$

Since,

$$(1-18v)^2 - 9\left[(1-2v)^2 + 32v^2\right] = -8 < 0$$

therefore, we have $(\partial(p_1^* - p_2^*)/\partial\sigma) < 0$.

Proof of Lemma 1 (iv): We have

$$RD_2^* = \frac{D_2^*}{D_1^*} = \frac{\left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2}{128v^2 - \left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2}.$$

Differentiating with respect to σ we get

$$\frac{\partial RD_2^*}{\partial \sigma} = \frac{256v\left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2}{\bar{\lambda}^d \sqrt{(1-2v)^2 + 32v^2}\left[128v^2 - \left[\sqrt{(1-2v)^2 + 32v^2} - (1-2v)\right]^2\right]^2} > 0.$$

Hence, we have Lemma 1 (iv). *Q.E.D*

The Lemma 1 (i) implies that as the information base increases, the market power of the firm with relatively lower chance of providing high quality product decreases. This is quite intuitive because as size of the information base increases, efficiency of the search engine to provide more perfect signal to the consumer that true chance of providing high quality product by Firm 2 is lower than Firm 1 increases. Therefore, the demand for product of Firm 2 decreases. Hence, Firm

2 charges a lower price to augment its demand.

The Lemma 1 (ii) implies that if the ex-ante asymmetry among the firms is not very high than as the information base increases the market power of the firm with relatively higher chance of providing high quality product varies with the size of public information base non monotonically. If the initial size of public information base is high enough then market power of the Firm 1 increases with size of the public information base and vice-versa. This is a counter intuitive outcome, because the mechanism through which p_2^* decreases with size of information base, by the same mechanism p_1^* should increase with size of information base. However, there is another effect that also determine the market outcome which is the strategic effect, that generates through the decrease in p_2^* due to increases in size of information base. Therefore, when the size of information base increases there are two effects: the information effect that leads to increase in the market power of Firm 1, implying an increase in p_1; and the strategic effect that leads to decrease in p_1 due to the decrease in p_2. Therefore, when the initial information base is high enough then information effect dominates the strategic effect. Hence, p_1 to increase. Also, if the ex-ante asymmetry among the firms is sufficiently high than the information effect always dominates the strategic effect. Hence, p_1 always increases with the size of the public information base. Graphically in Figure 11, p_1 increases (decreases) with information base in green (red) shaded region.

The Lemma 1 (iii) implies that as the information decreases, that is σ increases, the market power of the Firm 2 relative to Firm 1 increases. Given any $v \leq \tilde{v}_1$ it is directly implies by combining the Lemma 1 (i) and (ii). Further, for any $v > \tilde{v}_1$, since the strategic effect works indirectly on p_1 and directly on p_2 therefore the rate at which p_2^* increases due to σ increase will dominates the rate at which p_1^* increases.

The Lemma 1 (iv) implies that as the information decreases that is σ increases a part of the consumer switch from Firm 1 to Firm 2. This is again because of the information effect. Since as σ increases the signal that each consumer received become more noisy and consumer are less able to differentiate between firms. Hence, the demand of firm which has lesser chance of providing

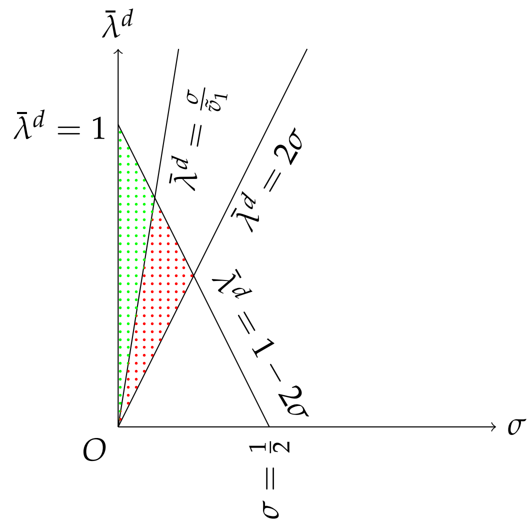

Figure 11: The Parameter Space for Comparative Static for p_1

good quality increases leads to increase in equilibrium quantity of that firm. In other word, when information increases the direct information effect dominates indirect strategic effect that leads to a fall in the price difference following Lemma 1 (iii).

We illustrate the switches of customer after σ changes by Figure 12. Suppose, σ increases from σ_0 to σ_1. Further, if σ is initially at σ_0 then the unit mass of consumer is distributed over the small box $H_1H_2H_3H_4$. Therefore, by definition $H_1H_2 = H_2H_3 = H_3H_4 = H_1H_4 = 2\sigma_0$ and $\square H_1H_2H_3H_4 = 4\sigma_0^2$. Moreover, at $\sigma = \sigma_0$ the line segment A_2A_3 represents the market segmentation line at equilibrium prices. Therefore, given $\sigma = \sigma_0$ at equilibrium, $\triangle A_2A_3H_4/\square H_1H_2H_3H_4$ consumers purchase from Firm 2 and remaining $1 - (\triangle A_2A_3H_4/\square H_1H_2H_3H_4)$ consumers purchase from Firm 1. Now if σ increases to σ_1 the box $H_1H_2H_3H_4$ expands to $J_1J_2J_3J_4$. However, the same unit mass of consumer is now distributed over the large box $J_1J_2J_3J_4$. Suppose, the line segment B_1B_2 is drawn in such a way that $\triangle B_1B_2J_4/\square J_1J_2J_3J_4 = \triangle A_2A_3H_4/\square H_1H_2H_3H_4$. Therefore, if B_1B_2 be the new market segmentation line after σ change then D_2^*/D_1^* remains unchanged due to σ change. Since, $p_1^* - p_2^*$ decreases with σ implies new segmentation line shift above from A_1A_4 but given D_2^*/D_1^* increases with σ implies that new segmentation line do not shift up to the line B_1B_2. Hence, new segmentation line lies between A_1A_4 and B_1B_2. Let C_1C_2 be the new segmentation line. Therefore, $area(B_1C_1C_2B_2B_1)/\square J_1J_2J_3J_4$ be the switching consumers of Firm 2

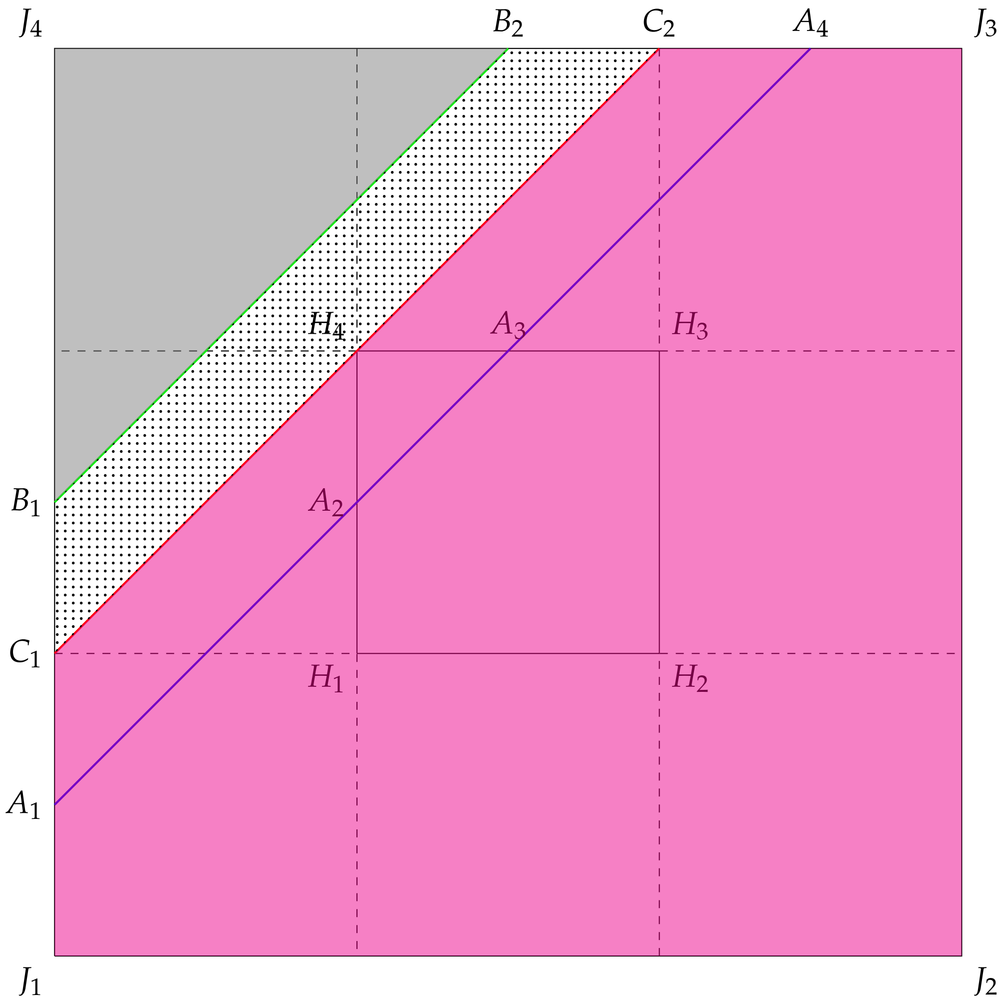

Figure 12: Consumer's switch after σ increase.

after σ change.

Combining Lemma 1 (i) and Lemma 1 (iv) one can conclude that π_2^* increases with σ. However, the behavior of equilibrium profit of the Firm 1 due to change in σ is ambiguous and hence the equilibrium aggregate profit.

Proposition 1 (i) If $\bar{\lambda}^d < 1/(1+2\tilde{v}_2)$ then we have $\partial\pi_1^*/\partial\sigma \gtreqless 0$ iff $v \gtreqless \tilde{v}_2 \approx 0.2998183220$. Otherwise we always have $\partial\pi_1^*/\partial\sigma < 0$.

(ii) If $\bar{\lambda}^d < 1/(1+2\tilde{v}_3)$ then we have $\partial\pi^*/\partial\sigma \gtreqless 0$ iff $v \gtreqless \tilde{v}_3 \approx 0.2411748160$. Otherwise we always have $\partial\pi^*/\partial\sigma < 0$.

Proof: Proof of (i): Differentiating the equilibrium profit of Firm 1 given by the condition 24 with respect to σ we get

$$\frac{\partial \pi_1^*}{\partial \sigma} = -\frac{s^d \Omega_{\pi_1}(v)}{128\sqrt{(-2v+1)^2 + 32v^2}\, v^3}$$

where

$$\Omega_{\pi_1}(v) = \left[\begin{array}{c} \left(\frac{3}{2} - \frac{3v}{2}\right)\left((-2v+1)^2 + 32v^2\right)^{\frac{3}{2}} + 1 - 5v + 22v^2 + 76v^3 - 1368v^4 \\ + \left(-\frac{5}{2} + \frac{21}{2}v - 60v^2 + 218v^3\right)\sqrt{(-2v+1)^2 + 32v^2} \end{array} \right].$$

Given $v \in [0, 1/2]$ therefore $\partial\pi_1^*/\partial\sigma \gtreqless 0$ if and only if $\Omega_{\pi_1}(v) \lesseqgtr 0$, solving we get $v \gtreqless \tilde{v}_2$. Finally, $\tilde{v}_2\bar{\lambda}^d \leq Min\{\bar{\lambda}^d/2, (1-\bar{\lambda}^d)/2\}$ if and only if $\bar{\lambda}^d \leq 1/(1+2\tilde{v}_2)$. Hence, proved.

Proof of (ii): Differentiating the equilibrium joint profit given by the condition (26) with respect to σ we get,

$$\frac{\partial \pi^*}{\partial \sigma} = -\frac{s^d\left(\left(68v^3 + 3v - 1\right)\sqrt{(-2v+1)^2 + 32v^2} - 792v^4 + 44v^3 + 22v^2 - 5v + 1\right)}{64\sqrt{(-2v+1)^2 + 32v^2}\, v^3} \gtreqless 0 \Leftrightarrow v \gtreqless \tilde{v}_3.$$

Finally, $\tilde{v}_3\bar{\lambda}^d \leq Min\{\bar{\lambda}^d/2, (1-\bar{\lambda}^d)/2\}$ if and only if $\bar{\lambda}^d \leq 1/(1+2\tilde{v}_3)$. Hence, proved. *Q.E.D*

Note that $\tilde{v}_1 < \tilde{v}_2$. If $v \leq \tilde{v}_1$ then by Lemma 1 (ii), p_1^* decreases with σ and by Lemma 1 (iv) along with full coverage, D_1^* decreases with σ. Hence, π_1^* also decreases. If $\tilde{v}_1 < v \leq \tilde{v}_2$ then by Lemma 1 (ii), p_1^* increases with σ and by Lemma 1 (iv) along with full coverage, D_1^* decreases with σ. However, the former is dominated by the latter. Hence, π_1^* decreases. Finally, if $v > \tilde{v}_2$ then by Lemma 1 (ii), p_1^* increases with σ and by Lemma 1 (iv) along with full coverage, D_1^* decreases with σ. However, the former dominates the latter. Hence, π_1^* increases. Given π^* is sum the of $\pi_1^* + \pi_2^*$ and π_2^* increases with σ. Therefore, we have $\tilde{v}_3 < \tilde{v}_2$.

Proposition 2 If $\bar{\lambda}^d \leq 1/(1+2\tilde{v}_4)$ then we have $\partial CS^*/\partial\sigma \gtreqless 0$ iff $v \lesseqgtr \tilde{v}_4 \approx 0.1680724574$. Otherwise we always have $\partial CS^*/\partial\sigma > 0$.

Proof: Given point E in the Figure 10 is lies between the line A_1B_1 and A_2B_2 therefore one can easily show that

$$s^d(\bar{\lambda}^d - 2\sigma) < p_1^* - p_2^* = \frac{s^d[3(\bar{\lambda}^d - 2\sigma) + \sqrt{(\bar{\lambda}^d - 2\sigma)^2 + 32\sigma^2}]}{4} < s^d\bar{\lambda}^d.$$

Therefore, we will expand the consumer welfare formula given by the condition (10) only for the prices that satisfy $s^d(\bar{\lambda}^d - 2\sigma) < p_1 - p_2 < s^d\bar{\lambda}^d$. For these price difference range we have market segmentation Figure 5. Hence, we have

$$CS(p_1, p_2) = \left[\begin{array}{c} \int_{\bar{\lambda}_1-\sigma}^{\bar{\lambda}_2+\sigma+\frac{p_1-p_2}{s^d}} \int_{\bar{\lambda}_2-\sigma}^{\lambda_1-\frac{p_1-p_2}{s^d}} (\lambda_1 s^d + s^L - p_1) d\lambda_2 d\lambda_1 \\ + \int_{\bar{\lambda}_1-\sigma}^{\bar{\lambda}_2+\sigma+\frac{p_1-p_2}{s^d}} \int_{\lambda_1-\frac{p_1-p_2}{s^d}}^{\bar{\lambda}_2-\sigma} (\lambda_2 s^d + s^L - p_2) d\lambda_2 d\lambda_1 \\ + \int_{\bar{\lambda}_2+\sigma+\frac{p_1-p_2}{s^d}}^{\bar{\lambda}_1-\sigma} \int_{\bar{\lambda}_2-\sigma}^{\bar{\lambda}_2+\sigma} (\lambda_1 s^d + s^L - p_1) d\lambda_2 d\lambda_1 \end{array} \right]. \quad (29)$$

Substituting $p_1 = p_1^*$ and $p_2 = p_2^*$ we have $CS^* = CS(p_1^*, p_2^*)$ and the differentiating with respect to σ we get $\partial CS^*/\partial\sigma = -\Omega/192\sigma^3\sqrt{(\bar{\lambda}^d - 2\sigma)^2 + 32\sigma^2} \gtreqless 0$ if and only if $\Omega \lesseqgtr 0$ where

$$\Omega = \left[\begin{array}{l} 2376\sigma^4 - 132\bar{\lambda}^d\sigma^3 + 22(\bar{\lambda}^d)^2\sigma^2 - 5(\bar{\lambda}^d)^3\sigma + (\bar{\lambda}^d)^4 \\ - \left((\bar{\lambda}^d)^3 - 3(\bar{\lambda}^d)^2\sigma + 268\sigma^3\right)\sqrt{\{\bar{\lambda}^d - 2\sigma\}^2 + 32\sigma^2} \end{array} \right]$$

$$= (\bar{\lambda}^d)^4 \left[2376v^4 - 132v^3 + 22v^2 - 5v + 1 - \left(1 - 3v + 268v^3\right)\sqrt{\{1 - 2v\}^2 + 32v^2}\right] = (\bar{\lambda}^d)^4\gamma(v).$$

One can show that there exist a unique $\tilde{v}_4 \in (0.167, 0.169)$ such that $\gamma(v) \lesseqgtr 0$ for all $v \lesseqgtr \tilde{v}_4$. Finally, $\tilde{v}_4\bar{\lambda}^d \leq Min\{\bar{\lambda}^d/2, (1 - \bar{\lambda}^d)/2\}$ if and only if $\bar{\lambda}^d \leq 1/(1 + 2\tilde{v}_4)$. Hence, we have the result. *Q.E.D*

Proposition 2 implies if information base is high enough then increasing the size of the information base leads to decrease in the consumer's welfare. We will interpret the Proposition 2 component wise. Note that $\tilde{v}_4 > \tilde{v}_1$. Therefore, we have the following

(a) If $v < \tilde{v}_1$ then following Lemma 1 (iii) An increase in σ changes the market segmentation such that D_1^* decreases and D_2^* increases. Then we have the following:

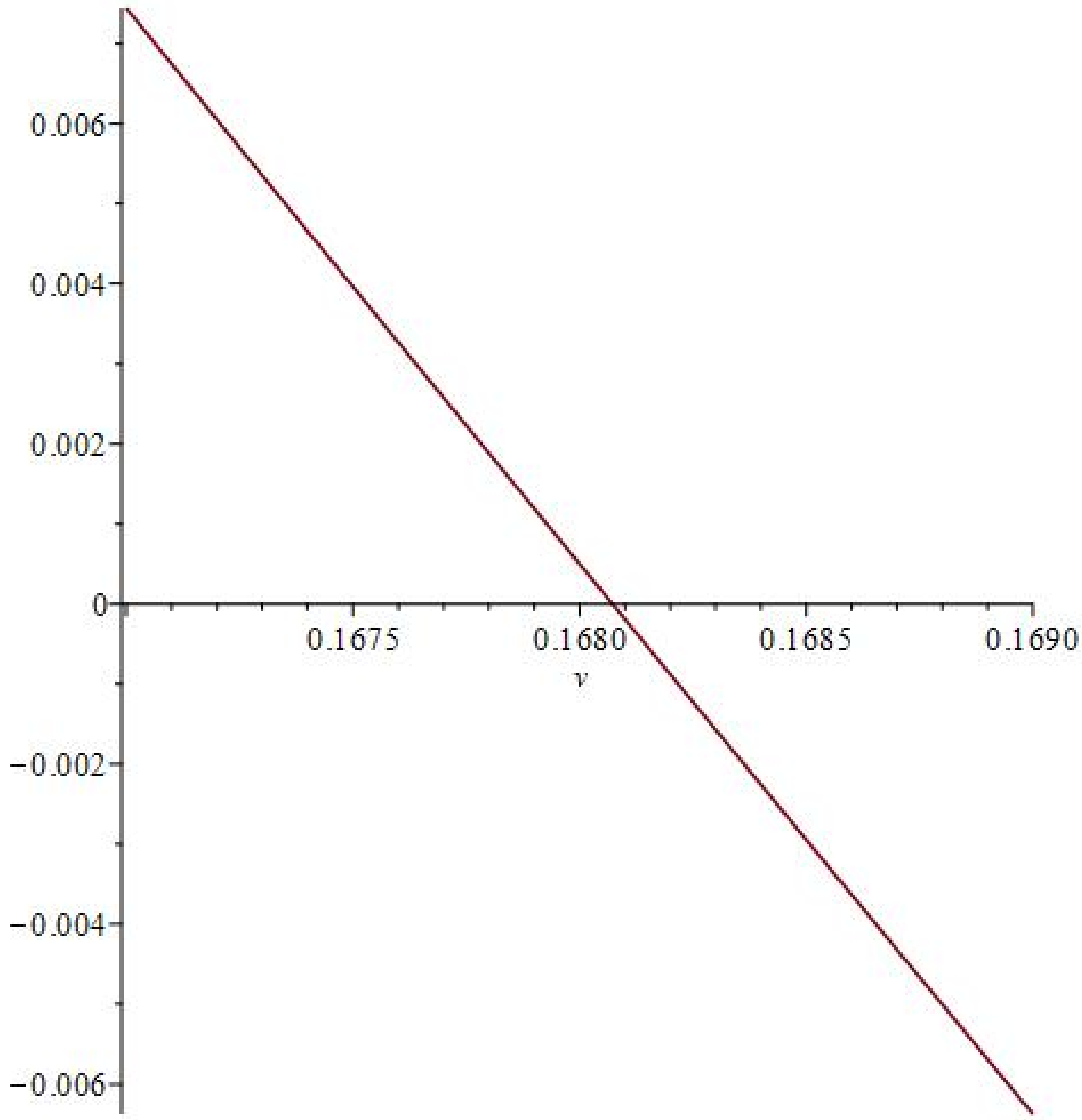

Figure 13: Plot Of $\gamma(v)$

(i) There are two subsets of consumers still buying from Firm 2. One set is getting noises that make them believe, though incorrectly, that the probability of Firm 2 producing high-quality increases. In ex ante sense, they are gaining despite of paying higher p_2 (The part of the gray shaded region shaded by white dots in Figure 14). The other

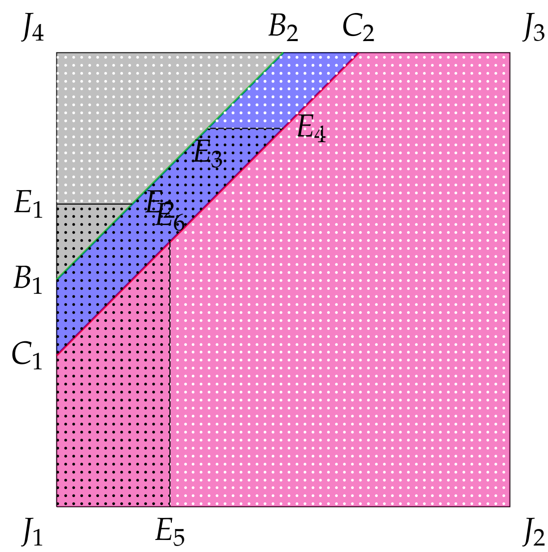

Figure 14: Consumer distribution after σ increase.

subset of consumers are getting noises that make then believe, correctly, that the probability of Firm 2 producing high-quality decreases. So, in ex ante sense, they are losing as they are paying higher p_2 (the part of the gray shaded region shaded by black dots in Figure 14). However, overall the aggregate gain may be subsided by aggregate loss for those who continue to purchase from Firm 2.

(ii) There are two subsets of consumers still buying from Firm 1. One set is getting noises that make them believe, though incorrectly, that the probability of Firm 1 producing high-quality decreases for them. In ex-ante sense, they are losing despite paying lower p_1 (the part of the magenta shaded region shaded by black dots in Figure 14). The other subset of consumers are getting noises that make then believe, correctly, that the probability of Firm 1 producing high-quality increases. So, in ex-ante sense, they are gaining as they are paying lower p_1 (the part of the magenta shaded region shaded by white dots in Figure 14). However, overall the aggregate loss may be subsided by aggregate gain for those who continue to purchase from Firm 1.

(iii) There are again two subsets of consumers who switch from Firm 1 to Firm 2. One set is getting noises that make them believe, though incorrectly, that the probability of Firm 2 producing high-quality relative to Firm 1 increases for them. In ex ante sense, they are gaining despite not only paying higher p_2 but also incurring some switching cost (the part of the blue shaded region, shaded by white dots in Figure 14). The other subset of consumers are getting noises that make then believe, that the probability of Firm 2 producing high-quality decreases relative to Firm 1. So, in ex ante sense, they are losing a even if still incorrectly but with lesser noise and being more informed, they are paying higher p_2 and incurring the switching cost (The part of the blue shaded region shaded by black dots in Figure 14). However, overall the aggregate gain may subsided by aggregate loss for those who switches from Firm 2 to Firm 1.

However, since Firm 1 serve more than 50% of the market implies the overall gain in (ii) will over compensate the combined loss in (i) and (iii) if any. Hence, consumer welfare increases.

(b) If $v > \tilde{v}_1$ then the explanation of those who continue to purchase from Firm 2 and those who switches from Firm 1 to Firm 2 remain unchanged. However, those who continue to purchase from Firm 1, the explanation changes due to the fact that now p_1 increases. Like the explanation of those who continue to purchase from Firm 2, now there are two subsets of consumers who are still buying from Firm 1. One set is getting extreme noises that make them believe, that the probability of Firm 1 producing high-quality increases for them. In ex ante sense, they are gaining despite of paying higher p_1. The remaining consumers in ex ante sense, are losing as they are paying higher p_1. However, overall whether the aggregate gain dominates aggregate loss is depends on the information base. Due to continuity the accumulated gain in (a) is not initially start dominated by the aggregate loss. After a value $\tilde{v}_4$ it will and consequently consumer welfare start falling.

Since the social welfare is sum of the consumer surplus and aggregate profit, then combining Proposition 1 and 2 we can conclude the following about the social welfare.

(i) If $v \in [0, \tilde{v}_4]$, then consumer welfare increases with the increase in public information base but the aggregate profit decreases (the green shaded region in Figure 15). Hence, the consequence of increasing the public information base on the social welfare is ambiguous. This also indicates is a conflict of interest between consumers and producer regarding the increase in the public information base.

(ii) If $v \in (\tilde{v}_4, \tilde{v}_3)$, then the consumer welfare and the aggregate profit both decreases with the increase in public information base (the blue shaded region in Figure 15). Hence, the social welfare decreases with the increase the public information base. There is also no conflict of interest between consumers and producer regarding the increase in the public information base.

(iii) If $v \in [\tilde{v}_3, 1/2]$, then the consumer welfare increases with the decrease in public information base but aggregate profit increases (the red shaded region in Figure 15). Hence, the consequence of increasing the public information base on the social welfare is ambiguous. Therefore, there is again conflict of interest between consumers and producer regarding the increase in the public information base.

Observation 1 We have $\partial W^*/\partial \sigma \geq 0$ if and only if $v > \tilde{v}_5 \approx 0.4842946967$ where $W^* = CS^* + \pi^*$.

6 Discussion

In this section we will discuss what happens when we relax some of our assumptions.

(i) If $|E^b \cap E_i|$ is different for different buyers then also our result qualitatively holds. Since, each $\tilde{v}_i$ is independent of σ and $\bar{\lambda}^d$ is exogenous to every buyer, therefore, if $|E^b|$ differs across buyers then we can segment the unit mass in terms of $|E^b|$. Note that the segment of unit mass assigned by $|E^b| = 0$ have maximum $\sigma = \sigma_0$ and suppose we need to increase $|E^p|$ up to $|E^p|_0$ such that σ_0 will decreases to $\tilde{v}_4\bar{\lambda}^d$. Hence, any increase in $|E^p|$ after $|E^p|_0$ will lead to consumer surplus decreasing for every segment of the buyers.

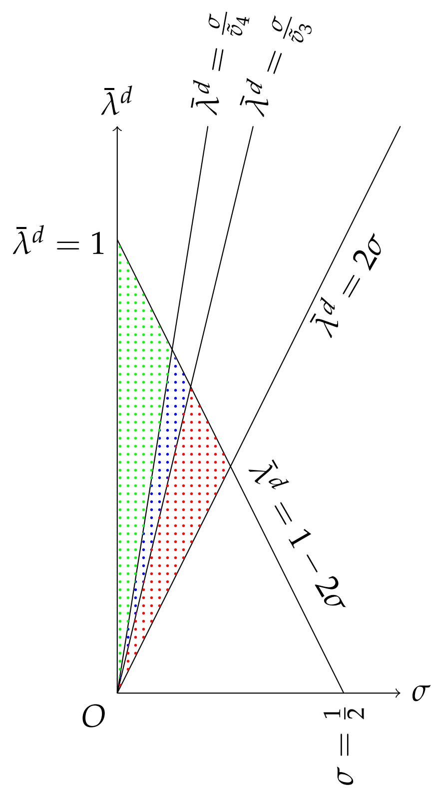

Figure 15: The Parameter Space for Conflict of interest between producer and consumers.

(ii) Now we will discuss about our uniform assumption. Observe for any buyer b, λ_i^b will be generated from the i^th of the experience vector u^e for individual $e \in (E^b \cup E^p) \cap E_i$. Therefore, the distribution of λ_i will be generated by the i^th component of u^e such that $e \in E_i$. Further, given i^th component of any u^e for all $e \in E$ is independent of its j^th component for $i \neq j$ implies λ_i and λ_j is expected to be independent. Note that i^th component of u^e for any $e \in E_i$ is either 0 or 1. Further, $\lambda_i^E = \bar{\lambda}_i$ implies probability that $u_i^e = 1$ for any u^e such that $e \in E_i$ is $\bar{\lambda}_i$. Hence, for any sample $(E^b \cup E^p) \cap E_i$ is from a Bernoulli population E_i with probability of success $\bar{\lambda}_i$ to generate the distribution of λ_i. Therefore, by applying the sampling theory we can conclude that λ_i follows either Binomial distribution (for small sample) or Normal distribution (for large sample) with mean $\bar{\lambda}_i$ and variance $\bar{\lambda}_i(1 - \bar{\lambda}_i)/((E^b \cup E^p) \cap E_i)$. Further, we take bounded noise length that is truncated domain for each λ_i. Hence, λ_i follows either truncated Binomial distribution (for small sample) or truncated Normal distribution (for large sample) with mean $\bar{\lambda}_i$ and variance $\bar{\lambda}_i(1-\bar{\lambda}_i)/((E^b \cup E^p) \cap E_i)$ over $[\bar{\lambda}_i - \sigma, \bar{\lambda}_i + \sigma]$. Now the uniform distribution on a given interval always has a higher or equal entropy compared to any other distribution defined on the same finite interval. Therefore, uniform distribution have maximum uncertainty than any other distribution. Our main result that is Proposition 2 suggest that excessive information can harm consumers even under uniform assumption. Therefore, when we move to any other distribution then due to change in entropy our uncertainty reduces in the buyers belief structure. Thus an average buyer now can differentiate the firms more in terms of there true $\bar{\lambda}_i$. Hence, given any σ if we move from uniform to any other distribution over $[\bar{\lambda}_1 - \sigma, \bar{\lambda}_1 + \sigma] \times [\bar{\lambda}_2 - \sigma, \bar{\lambda}_2 + \sigma]$ Firm 1 will enjoy larger market power than Firm 2. Moreover, if increase in public information base leads to decrease in σ then not only the mean preserving spread of distributions decreases but also variance decreases which leads to shift of the mass from tail of the distribution to its center. However, uniform have equal height for all its support but truncated Binomial or Normal have differ height for its support which is higher near the center than tails. Hence, due to decrease in variance leads to more increase in height near the center for truncated Normal or truncated Binomial than uniform. Hence, after σ decrease on average consumer received more accurate signal under truncated Normal or truncated Binomial than uniform. Hence, firms will be more bifurcated in terms of their true $\bar{\lambda}_i$. Hence, market power gap between Firm 1 and Firm 2 increases due to decrease in σ at a higher rate when we move to uniform to truncated Normal or truncated Binomial. Hence, the channel through which Proposition 2 generated will get larger strength by shifting from uniform to truncated Normal or truncated Binomial.

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