Trade, Liquidity, and Monetary Spillovers under Dollar Dominance

[PRELIMINARY AND INCOMPLETE]*

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Abstract

The US dollar is the dominant currency for international trade invoicing. Additionally, US dollar bonds have low returns reflecting a liquidity premium over other assets traded internationally. Using these facts in a two-country model with international trade and liquidity we find a striking difference in spillovers of monetary policy emanating from the dominant country as compared to its non-dominant trading partner. While firm entry falls in both countries in response to a contractionary monetary policy in the dominant country, it rises in the dominant country in response to the same shock from the non-dominant country. Moreover, there is an amplified fall in firm entry only in the non-dominant country in response to synchronous monetary tightening in both countries as compared to the sum of effects from asynchronous tightening.

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1 Introduction

The United States is at the center of the international monetary and financial system. The US dollar is the dominant currency for international trade invoicing (see Gopinath and Itskhokhi, 2022 for a review). In addition, US dollar bonds typically have low returns reflecting a liquidity premium over other assets traded internationally. Thus, US assets are more liquid in the sense that they are more easily recognized and more widely accepted in exchange as compared to other countries assets of otherwise similar quality. This also makes the United States the world's preferred supplier of safe assets (Krishnamurthy and Lustig, 2019).

Do the above facts imply that the monetary policy of the US Federal Reserve will have large spillovers on macroeconomic and financial outcomes for other countries relative to other major economies? Conversely, will other countries monetary policy not have such effects i.e. will they be muted, absent or reversed? Empirical evidence shows that policy actions by the US Federal Reserve indeed transmits globally and affects international trade, output and commodities prices. Importantly, it also affects the global financial markets and liquidity conditions worldwide (Miranda-Agrippino and Rey, 2022; Dedola et al. 2017; Degasperi et al., 2021). Miranda-Agrippino and Rey (2020) label the former as the Global Trade and Commodity Cycle and the latter is the Global Financial cycle of Rey (2013). In contrast, the European Central Bank (see Ca'Zorzi et al., 2020) plays a less important role in driving the Global Financial Cycle but it is still an important driver of the Global Trade and Commodity Cycle.[1]

The dominance of the United States in the international monetary and financial systems is a key to rationalize the above empirical findings. In this paper we focus on two key aspects of this dominance to examine monetary policy spillovers on trade, prices, unemployment, productivity, interest rates and liquidity supply. First, in the international monetary system the US dollar is the currency of invoicing exports and second, in the international financial system US assets command a higher liquidity premium. The theoretical model also yields some interesting con-

[1]They find that local output and inflation respond in a 'textbook way', but while the ECB policy has no effect on US variables, the Fed policy has a large effect on the EA real activity and financial markets. Even though ECB policy does not affect US variables, the trade spillovers emanating from ECB monetary policy does spread to other countries.

clusions on the effect of synchronous monetary tightening, liquidity changes, trade integration and labor market shocks on unemployment, productivity and interest rates.

We incorporate the international trade framework of Melitz (2003) to succinctly capture the reallocative effects of lower trade costs across producers. In particular, an increase in trade linkages between two countries as captured by a reduction in trade costs results in productivity gains through firm selection. Second, since we focus on the US dollar being the currency of international trade invoicing, we use a class of models with an explicit role for money as medium of exchange, namely Lagos and Wright (2005) and Rocheteau and Wright (2005). Moreover, return on assets including claims to firm's profits and government bonds will reflect a liquidity premium as they provide additional liquidity services as in Rocheteau and Rodriguez-Lopez (2011) and Rodriguez-Lopez (2021). Finally, labor market frictions as in Diamond (1982) and Mortensen and Pissarides (1994) allows us to model unemployment while also being key in linking the monetary and trade channels via labor market tightness.

We find a striking difference in spillovers of monetary policy emanating from the dominant country as compared to its non-dominant trading partner. A contractionary monetary policy in the dominating country leads to a substantial fall in output and asset flows in both countries while that in the non-dominating country only transmits into a marginal fall in output for its trading partner. While firm entry falls in both countries in response to a contractionary monetary policy in the dominant country, it rises in the dominant country in response to the same shock from the non-dominant country.

Moreover, the effects of synchronous monetary tightening in both countries as compared to the sum of the effects from asynchronous tightening are also different. There is an amplified fall in firm entry in the non-dominant country but not so in the dominant country. The amplified response follows from the fact that after a synchronous tightening, demand falls across the board from all households because money is costlier in both countries and there is no way to switch demand. This leads to an amplified slowdown in global liquidity which translates to a greater rise in liquidity premium and interest rates for the dominant country. This in turn makes it more desirable for firms to enter in the dominant country which offsets the initial fall in firm entry. Thus, in the end the effect of synchronous monetary tightening is not felt as strongly in the dominant country as compared to the non-dominant country.

There is some theoretical work to explain asymmetric spillover of monetary policies from the US and other countries. Gopinath and Itskhoki (2021) shows how the asymmetry in invoicing currencies in global trade translates into an asymmetric impacts of shocks originating in the dominant currency issuer versus in other countries. They do not however model the financial system and unemployment effects. The work closest to the current paper is Jiang, Krishnamurthy and Lustig (2023) which assumes a downward sloping world convenience demand curve for safe dollars bonds. They are able to capture asymmetric effect of US monetary policy as it affects the supply of these safe assets. While they do focus on how the Global Financial cycle of Rey (2013) is affected, they assume that dollar bonds are safe. We will explicitly model liquidity to explain why US assets are special and also be explicit on how trades are mediated using currency where US dollar is the dominant currency for payments. Other works have focused on heterogeneity in risk aversion, or weaker financial frictions in the US.

In this paper, to model a dominant currency regime we focus on the currency's role in "payments" or as a medium-of-exchange. This is different from its role in "pricing" or as a unit of account or a store of value. The literature on medium of exchange role of currency typically uses a similar class of search-based theories of money for example Matsuyama, Kiyotaki and Matsui (1993) and Zhang (2014) among others. More recently, Chahrour and Valchev (2022) looks at the interaction of endogenous liquidity premia and the demand for store-of-value assets. These papers focus on the endogenous emergence of currency regimes with or without dominant international currencies. The current paper instead takes the currency regime as given in order to focus on the link between monetary policy and international trade.

The strand of the literature on dominant currencies in the currency invoicing line typically focuses on the interaction of nominal price stickiness with pricing complementarities (Gopinath, Itskhokhi and Rigobon, 2010; Mukhin, 2022) or links between invoicing decisions and currency's role as a store of value (Gopinath and Stein, 2020). The literature in open economy macro that analyzes the role of monetary policy usually takes this approach with the invoicing currency as exogenous and anchors the model on nominal price rigidities typically without money, or with money as a redundant asset (most closely related is Cacciatore and Ghironi, 2021). However, as Lagos and Zhang (2022) shows that the justification to use the moneyless approach can be overturned with a more explicit role of payments.

2 Model Environment

In this section we will describe our two country open economy model where country 1 is the dominant country in a sense to be made precise. In each country there is a measure 1 of households, n_i active firms and e_i traders, $i \in \{1, 2\}$. Households supply labor to firms. Labor is supplied only domestically and since matches in the labor market are between one firm and one worker, the unemployment rate in country $i \in \{1, 2\}$ is $u_i = 1 - n_i$. Traders supply financial services on over-the-counter markets to households in both countries.

Households in both countries consume three types of consumption goods. The first is financial services, y which provides utility $\nu(y)$, where ν is increasing and strictly concave, $\nu(0) = 0$, $\nu'(0) \to \infty$, and $\nu'(\infty) = 0$. For a trader, the disutility from providing y units of the service is y. The other homogeneous good, x is produced by firms which gives linear utility x to households when consumed. This good will serve as the numéraire. The third is a differentiated good which gives utility $U(Q_i)$ to households in country-i, $i \in \{1, 2\}$. Here $Q_i = (Q_{i1}^{\rho} + Q_{i2}^{\rho})^{1/\rho}$ and $Q_{ij} = \left(\int_0^{\Omega_{ij}} q_{\omega_j}^{\rho} d\omega_j\right)^{1/\rho}$, $j \in \{1, 2\}$ with elasticity of substitution $\sigma = \frac{1}{(1-\rho)} > 1$. We assume $U(Q_i) = Q_i^{\xi}/\xi$ where $0 < \xi < \rho$. The restriction $\xi < \rho$ ensures that varieties are better substitutes for each other than the numéraire as in Helpman and Itskhoki (2010).[2]

In a given period, there are three stages. There is discounting at the rate β between periods. Stage 1 is when firm and worker matches are formed. Job vacancies are posted at cost k in stage 3 of the previous period. Each such vacancy is filled with probability $q^v(\theta)$ and each unemployed finds a job with probability $q^f(\theta)$. Market tightness, θ is the ratio of vacancies to unemployed. Matched firms $n = (1 - u)$ separate with probability $\delta > 0$. Each matched firm has the technology to produce $A > 0$ units of the numéraire. Each firm can also instead produce

[2] Note that, we have assumed that domestic goods and imports have the same elasticity of substitution. But, the subset of domestically produced goods Ω_{11} is different from that produced in foreign Ω_{21} for country 1 households. Moreover, while the utility function has identical parameters in the foreign country, the subsets of goods available for consumption in country 2 are Ω_{21} and Ω_{22} which may differ from that in country 1.

q units of the differentiated good subject to $c(q) + c(q_x) \leq A$, where the cost of producing the good for domestic households is $c(q) = \frac{q}{\varphi} + f$, and for foreign is $c(q_x) = \tau \frac{q_x}{\varphi} + f_x$. Firms face an additional fixed cost of exporting $f_x > f > 0$, and a variable trade cost $\tau > 1$. Each firm draws φ from a common distribution $g(\varphi)$ with support $(0, \infty)$ and a continuous cumulative distribution $G(\varphi)$.

In stage 2 households desire to consume the differentiated goods produced by a fraction $N_j \leq n_j$ of all firms and financial services, y produced by traders. Firms in each country produce a variety $\omega_j \in (0, \Omega_j)$. But, households cannot meet all sellers in the market due to search frictions. A household in country-i meets $\alpha_{ij}(N_j^o) \leq N_j^o$ firms in the market where $i \in \{1, 2\}$, $j \in \{1, 2\}$, $o \in \{d, x\}$. In other words, $\alpha_{ij}(N_j^o)$ is the measure of country j firms producing for households in country i. Only a fraction of firms export i.e. $N_j^x < N_j^d = N_j$ The measure of country i households serviced by a firm in country-$j, j \in \{1, 2\}$ is $\alpha_{ij}(N_j^o)/N_j^o$.

When a household meets a firm to purchase its good, limited commitment for future repayments makes credit transactions infeasible. This necessitates the use of a medium of exchange to trade. Central banks in respective countries print money at the rate γ_i i.e. $M_{i,+1} = (1 + \gamma_i)M_i$ which households can obtain in stage 3. There is an additional cost $\eta \geq 1$ to obtain foreign currencies. The value of country-i currency in terms of the numéraire is ϕ_i. We will consider stationary monetary equilibria where $\phi_{i,+1}M_{i,+1} = \phi_i M_i$, so $\phi_i/\phi_{i,+1} = 1 + \gamma_i$, which implies that γ_i is the rate of inflation in country i.

Firms recognize and accept money for retail trades but, firms in country 1 do not accept money issued in country 2. This assumption brings about an asymmetry in payments and aims to capture the dominance of county 1. Country 1 households do not obtain currency 2 as they can use currency 1 for both domestic and foreign trades while country 2 households need currency 1 to import. But, we will assume $(1 + \gamma_1) < (1 + \gamma_2) < (1 + \gamma_1)\eta < (1 + \gamma_2)\eta$ to ensure that country 2 households use currency 1 for imports only and currency 2 for domestic consumption.

In this stage, households also desire to consume some financial services produced by traders. They meet these traders in bilateral matches which are again characterized by limited commitment making credit infeasible. So, households must rely on liquid assets which they sell to traders in return for financial services. They can use money, government bonds and/or claims to future revenue streams of private firms. The value of country-i non-money assets in terms of the numéraire is ϕ_i^a. A household in either country meets one trader every period. The trader is from country-i with probability $\zeta(e_i)$ where $\zeta(e_1) + \zeta(e_2) \leq 1$. Traders from the dominant country 1 only recognize assets issued in country 1, while the non-dominant country 2 sellers accept both. Households buy financial services by making take-it-or-leave-it offers to the sellers.

In stage 3 trade for the homogeneous good x takes place costlessly and under perfect competition. Hence it is an integrated international market with free movement of currency and goods.[3] All the matched firms n_j in country $j \in \{1, 2\}$ produce the numéraire x_j given their technology A_j. They pay wages $w_j = w_0 + Aw^f, 0 < w^f < 1$ to employed households. Unemployed households receive unemployment benefits, b. All households pay a lump sum tax T_i to their governments. Households consume x_i and get leisure l_i if unemployed. The per period utility of households is $U(Q_i)+\zeta_1\nu(y_1)+\zeta_2\nu(y_2)+x_i+(1-e)l_i$, where $e = 1$ if employed. Household's budget constraint in stage 3 is: $x_i + \phi_1(\eta_i m_{i1,+1} - m_{i1}) + \phi_2(m_{i2,+1} - m_{i2}) + \phi_1^a(a_{i1,+1} - a_{i1}) + \phi_2^a(a_{i2,+1} - a_{i2}) = ew_i + (1 - e)ub - T_i$. Note that $\eta_1 = 1$ and $\eta_2 = \eta$ which is the cost of obtaining foreign currency. Money and asset holdings decisions i.e. the next period holdings of $m_{i1}, m_{i2}, a_{i1}, a_{i2}$ are taken given prices ϕ_i, ϕ_i^a. Asset holdings a_i here comprise one-period government bonds B_i and claims to the revenue stream of matched firms.

The government consumes G_i, pays the unemployed benefits b and levies a lumpsum tax T_i to finance its expenditures including financing the terminal payment of bonds. The one-period bonds B_i issued by the government pays one unit of the numéraire. The present discounted value of a bond is $1/(1 + r^a)$ where r^a is the real rate of return on liquid assets (both public and private) and $r_i^a = \phi_{i,+1}^a/\phi_i^a - 1$. The consolidated budget of the government and central bank authority in country i is: $G_i + u_i.b = T_i + \phi_i(M_{i,+1} - M_i) + (\phi_i^a/\phi_{i,+1}^a - 1)B_i + \eta_j M_{1,+1}^j$.

We will now elaborate each agent's problem in each period along with their value functions before solving the open economy equilibrium of the model in Section 3.

[3]Like in Rogdriguez-Lopez (2021), Chaney (2008), Helpman and Itskhoki (2010), and Helpman et al. (2004), the assumption of a costlessly traded homogeneous good ensures that trade is balanced: any Home surplus in the trade of differentiated goods is met with an identical home deficit in the trade of the homogeneous good.

2.1 Households

In stage 1, the value function of households in country-$i \in \{1, 2\}$ who enter the stage as unemployed ($e = 0$) is:

$$U_i^{h0}(m_i, a_i) = [1 - q^f(\theta)]V_i^{h0} + q^f(\theta)V_i^{h1}. \tag{1}$$

For an employed household ($e = 1$) the value function is:

$$U_i^{h1}(m_i, a_i) = (1 - \delta)V_i^{h1} + \delta V_i^{h0}. \tag{2}$$

Recall that country 1 has the dominant currency as firms in both countries accept it and hence country 1 households use their currency for both domestic and imported consumption of goods. Households also meet sellers of financial services from country-i in an over-the-counter market with probability ζ_i. If they meet sellers from country 1 they can sell assets issued in country 1 only but if they meet sellers in country 2 they can sell both country's assets.[4] Thus, in stage 2 the value function of a household in country-1 i.e. the dominant country who enters the stage with employment status $e \in \{0, 1\}$ is (given $q_{\omega_1}, q_{\omega_2}$).

$$\begin{aligned} V_1^{he}(m_{11}, m_{12}, a_{11}, a_{12}) = U(Q_1) + \zeta_1\nu(y_1) + \zeta_2\nu(y_2) + \\ W_1^{he}\left(m_{11} - \frac{\int_0^{\Omega_{11}} p_{\omega_1}q_{\omega_1}d\omega_1}{\phi_1} - \frac{\int_0^{\Omega_{12}} p_{\omega_2}q_{\omega_2}d\omega_2}{\phi_1}, m_{12}, a_{11} - d_{11}, a_{12} - d_{12}\right). \end{aligned} \tag{3}$$

Country 2 households use currency 2 for domestic consumption and currency 1 only for imports from foreign firms as discussed before. Matches on the OTC market for financial services is the same as for households in country 1. Thus, in stage 2 the value function of a household in country-2 is:

$$\begin{aligned} V_2^{he}(m_{21}, m_{22}, a_{21}, a_{22}) = U(Q_2) + \zeta_{21}\nu(y_{21}) + \zeta_{22}\nu(y_{22}) + \\ W_2^{he}\left(m_{21} - \frac{\int_0^{\Omega_{21}} p_{\omega_1}q_{\omega_1}d\omega_1}{\phi_1}, m_{22} - \frac{\int_0^{\Omega_{22}} p_{\omega_2}q_{\omega_2}d\omega_2}{\phi_2}, a_{21} - d_{21}, a_{22} - d_{22}\right). \end{aligned} \tag{4}$$

[4] Since money is costly to hold, households only bring money for retail trades with firms and not to purchase financial services.

Using the linearity of $W_i^{he}(\cdot)$ for either country-i we get,

$$V_i^{he}(m_{i1}, m_{i2}, a_{i1}, a_{i2}) = U(Q_i) - \int_0^{\Omega_{i1}} p_{\omega_1} q_{\omega_1} d\omega_1 - \int_0^{\Omega_{i2}} p_{\omega_2} q_{\omega_2} d\omega_2 + \phi_1 m_{i1} + \phi_2 m_{i2} + \\ + \zeta_{i1}[\nu(y_{i1}) - d_{i1}] + \zeta_{i2}[\nu(y_{i2}) - d_{i2}] + \phi_1^a a_{i1} + \phi_2^a a_{i2} + W_i^{he}(0). \quad (5)$$

In stage 3, the value function of households in country-i becomes:

$$W_i^{he}(m_{i1}, m_{i2}, a_{i1}, a_{i2}) = \max_{m_{ij,+1}, a_{ij,+1}, x_i} [x_i + (1-e)l_i + \beta U_i^{he}(m_{i1,+1}, m_{i2,+1}, a_{i1,+1}, a_{i2,+1})], \quad (6)$$

s.t.

$$x_i + \phi_1(\eta_i m_{i1,+1} - m_{i1}) + \phi_2(m_{i2,+1} - m_{i2}) + \phi_1^a(a_{i1,+1} - a_{i1}) + \phi_2^a(a_{i2,+1} - a_{i2}) \\ = ew_i + (1-e)ub - T.$$

Note that $\eta_1 = 1$ and $\eta_2 = \eta$. Eliminating x_i from the budget constraint, the maximization for $m_{ij,+1}$ is the same for employed or unemployed households because $\partial U_i^{he}/\partial m_{ij,+1}$ is independent of $m_{ij,+1}$ and is given as follows[5].

$$\max_{m_{i1}, m_{i2}} \left\{ -\phi_{1,-1} m_{i1} - \phi_{2,-1} m_{i2} + \beta \left[U(Q_i) - \int_0^{\Omega_{i1}} p_{\omega_1} q_{\omega_1} d\omega_1 - \int_0^{\Omega_{i2}} p_{\omega_2} q_{\omega_2} d\omega_2 + \phi_1 m_{i1} + \phi_2 m_{i2} \right] \right\}. \quad (7)$$

Similarly, the maximization for $a_{ij,+1}$ is independent of the rest the problem due to linearity of $W(\cdot)$ and is given as follows:

$$\max_{a_{i1}, a_{i2}} \left\{ -\phi_{1,-1}^a a_{i1} - \phi_{2,-1}^a a_{i2} + \beta \left\{ \zeta_{i1}[\nu(y_{i1}) - d_{i1}] + \zeta_{i2}[\nu(y_{i2}) - d_{i2}] + \phi_1^a a_{i1} + \phi_2^a a_{i2} \right\} \right\}. \quad (8)$$

2.2 Firms

In stage 1, the value function of a firm in country-$i \in \{1, 2\}$ who enters the stage as an unmatched firm who has paid the entry cost k is:

[5] As we will see later, country-1 households do not carry m_2 since it is costly to hold and it serves no liquidity purpose for them.

$$U_i^{f0}(\varphi) = q^v(\theta_i)EV_i^{f1}(\varphi). \tag{9}$$

For a matched firm the value function is:

$$U_i^{f1}(\varphi) = (1-\delta)EV_i^{f1}(\varphi). \tag{10}$$

In stage 2, unmatched firms value is zero and that of matched firms is given by (given p_i, q_i^f):

$$V_i^{f1}(\varphi \leq \hat{\varphi}) = W_i^{f1}(A, 0, 0, \varphi). \tag{11}$$

$$V_i^{f1}(\hat{\varphi} < \varphi \leq \hat{\varphi}_x) = W_i^{f1}[A - c(q_i^f), m_{ii}^f, 0, \varphi], \tag{12}$$

where $\phi_i m_{ii}^f = p_i(\varphi)q_i^f(\varphi)$. If the firm exports then country 1 (dominant) firms' value is given by,

$$V_1^{f1}(\varphi > \hat{\varphi}_x) = W_1^{f1}[A - c(q_1^f) - c_x(q_{1x}^f), m_{11}^f, 0, \varphi], \tag{13}$$

where $\phi^* m_2^f = p_x(\varphi)q_x^f(\varphi)$, and country 2 firms' value is given by,

$$V_2^{f1}(\varphi > \hat{\varphi}_x) = W_2^{f1}[A - c(q_2^f) - c_x(q_{2x}^f), m_{22}^f, m_{21}^f, \varphi], \tag{14}$$

In stage 3, the values are,

$$W_i^{f0}(\varphi) = \max\{0, -k + \frac{1}{1+r_i^a}U_i^{f0}(\varphi)\}, \tag{15}$$

$$W_i^{f1}(A, 0, 0, \varphi) = A - w + \frac{1}{1+r_i^a}U_i^{f1}(\varphi). \tag{16}$$

$$W_i^{f1}(A - c(q_i^f) - f, m_{ii}^f, 0, \varphi) = A - c(q_i^f) - w + \phi m_{ii}^f + \frac{1}{1+r_i^a}U_i^{f1}(\varphi). \tag{17}$$

For country-1,

$$W_1^{f1}[A - c(q_1^f) - c_x(q_{1x}^f), m_{11}^f, 0, \varphi] = A - c(q^f) - c(q_{1x}^f) - w + \phi_1 m_{11}^f + \frac{1}{1+r_1^a}U_i^{f1}(\varphi). \tag{18}$$

For country-2,

$$W_2^{f1}[A-c(q_2^f)-c_x(q_{2x}^f),m_{22}^f,m_{21}^f,\varphi]=A-c(q^f)-c(q_x^f)-w+\phi_1 m_{21}^f+\phi_2 m_{22}^f+\frac{1}{1+r_2^a}U_2^{f1}(\varphi). \tag{19}$$

The free entry condition for firms in country-i is:

$$k=\frac{1}{1+r_i^a}q^v(\theta_i)EV_i^{f1}.$$

where the expected value of a matched firm EV_i^{f1} will be given by taking the expectation over the following values depending on φ:

$$V_i^{f1}(\varphi\leq\hat{\varphi})=A-w+\frac{1}{1+r_i^a}(1-\delta)EV_i^{f1}(\varphi), \tag{20}$$

$$V_i^{f1}(\hat{\varphi}<\varphi\leq\hat{\varphi}_x)=A-c(q_i^f)-w+\phi_i m_{ii}^f+\frac{1}{1+r_i^a}(1-\delta)EV_i^{f1}(\varphi). \tag{21}$$

Value of exporting firms in country-1 is:

$$V_1^{f1}(\varphi>\hat{\varphi}_x)=A-c(q_1^f)-c(q_{1x}^f)-w+\phi_1 m_{11}^f+\frac{1}{1+r_1^a}(1-\delta)EV_1^{f1}(\varphi). \tag{22}$$

And, in country-2:

$$V_2^{f1}(\varphi>\hat{\varphi}_x)=A-c(q_2^f)-c(q_{2x}^f)-w+\phi_1 m_{21}^f+\phi_2 m_{22}^f+\frac{1}{1+r_2^a}(1-\delta)EV_2^{f1}(\varphi). \tag{23}$$

2.3 Traders

In stage 1, traders of financial services are dormant and in stage 2 trader-i's value function is:

$$V_i^t(\cdot)=\zeta_i[-c^t(y_i)+d_i]+\phi_1^a a_{i1}+\phi_2^a a_{i2}+W_i^t(0). \tag{24}$$

In stage 3, their value function is given by:

$$W_i^t(m_{i1},m_{i2},a_{i1},a_{i2})=\max_{m_{ij,+1},a_{ij,+1},x_i}[x_i+\beta V_i^t(m_{i1,+1},m_{i2,+1},a_{i1,+1},a_{i2,+1})], \tag{25}$$

s.t.

$$x_i+\phi_1(m_{i1,+1}-m_{i1})+\phi_2(m_{i2,+1}-m_{i2})+\phi_1^a(a_{i1,+1}-a_{i1})+\phi_2^a(a_{i2,+1}-a_{i2})=0.$$

This gives us the maximization for a_{i1} (since sellers receive no surplus from trades in stage 2),

$$\max_{a_{i1}} \left\{-\phi^a_{1,-1} a_{i1} + \beta \left\{0 + \phi^a_1 a_{i1}\right\}\right\}. \tag{26}$$

Thus, if $\phi^a_{1,-1}/\beta\phi^a_1 = 1$, then OTC sellers are indifferent between holding a_{i1} or not. If $\phi^a_{1,-1}/\beta\phi^a_1 > 1$ then $a_{i1} = 0$ and if $\phi^a_{1,-1}/\beta\phi^a_1 < 1$ then a solution does not exist. Similarly for other assets.

3 Open economy equilibrium

3.1 Dominant country Households

Consumption of goods When a household in country-1 desires to consume, given her real balances z_1, she maximizes her consumption of each variety as follows,

$$\max_{q_{\omega_1}, q_{\omega_2}} \left\{U(Q_1) - \int_0^{\alpha_{11}} p_{\omega_1} q_{\omega_1} d\omega_1 - \int_0^{\alpha_{12}} p_{\omega_2} q_{\omega_2} d\omega_2\right\}, \tag{27}$$

s.t.

$$Z_1 = Z_{11} + Z_{12} \equiv \int_0^{\alpha_{11}} p_{\omega_1} q_{\omega_1} d\omega_1 + \int_0^{\alpha_{12}} p_{\omega_2} q_{\omega_2} d\omega_2 \leq z_{11} \equiv \phi_1 m_{11}. \tag{28}$$

Here, p_{ω_i} is the price of variety ω_i produced by a firm in country-i of stage 2 goods in terms of late consumption goods. And, Z_{11} is defined as the total real domestic expenditure on stage 2 goods and Z_{12} is the same for imports. Recall that $Q_1 = (Q_{11}^\rho + Q_{12}^\rho)^{1/\rho}$, $Q_{1j} = \left(\int_0^{\alpha_{1j}} q^\rho_{\omega_j} d\omega_j\right)^{1/\rho}$, $j \in \{1, 2\}$ where $\sigma = \frac{1}{(1-\rho)} > 1$.

The first order condition with respect to q_{ω_i} is:

$$U'(Q_1)\left(\frac{Q_1}{Q_{11}}\right)^{1/\sigma} q_{\omega_i}^{-1/\sigma} \frac{\sigma}{\sigma - 1} = p_{\omega_i}. \tag{29}$$

Consider two varieties, ω_i^1 and ω_i^2 to get, $q_{\omega_i^1}/q_{\omega_i^2} = (p_{\omega_i^1}/p_{\omega_i^2})^\sigma$, where $i \in \{1, 2\}$ is for the two countries. After further manipulation, and defining $P_1 \equiv (P_{11}^{1-\sigma} + P_{12}^{1-\sigma})^{\frac{1}{(1-\sigma)}}$ where $P_{1j} \equiv \left(\int_0^{\alpha_{1j}} p^{1-\sigma}_{\omega_j} d\omega_j\right)^{1/1-\sigma}$ for $j \in \{1, 2\}$, we get: $\frac{Z_1}{P_1} = q_{\omega_i}\left(\frac{p_{\omega_i}}{P_1}\right)^\sigma$. Using this in the definition of Q_1, we get $Q_1 = \frac{Z_1}{P_1}$. We have that $Z_1 = Z_{11} + Z_{12}$ and $P_1 Q_1 = Z_1$. The optimal allocation of any given expenditure between domestic and imported goods is given by:

$$Z_{1i} = Z_1 \left(\frac{P_{1i}}{P_1} \right)^{1-\sigma}. \tag{30}$$

The problem of choice of m_{11} for all households becomes,

$$\max_{m_{11}} \left\{ -\phi_{-1,1} m_{11} + \beta \left\{ \left[U(Q_1) - \phi_1 m_{11} \right] + \phi_1 m_{11} \right\} \right\}. \tag{31}$$

Given that money is costly to hold, households will not carry more money than they need to purchase stage 2 consumption goods. So, we have that $z_{11} = Z_{11} + Z_{12}$. The household's choice of domestic money balances m_{11} which determines its total (including domestic and imports) expenditure on goods $Q_1 = Z_1/P_1$ is given by:

$$\frac{U'(Z_1/P_1)}{P_1} = \frac{1+\gamma_1}{\beta}. \tag{32}$$

Consumption of financial services When a household in country-1 desires to consume financial services, given her asset holdings a_{11}, a_{12}, she maximizes her consumption as follows. If she meets a seller from country 1 with probability ζ_1 then,

$$\max_{y_1, d_1 \leq \phi_1^a a_1} \left\{ \nu(y_1) - d_1 \right\}, \tag{33}$$

s.t.

$$-C(q_1) + d_1 \geq 0. \tag{34}$$

Given that the buyer will take all the surplus from this trade we get $-C(q_1) + d_1 = 0$ which gives us:

$$y_1 = \max\{y^*, C^{-1}(\phi_1^a a_1)\}. \tag{35}$$

Similarly, if she meets a seller from country 2 who accepts both country's assets, we get:

$$y_2 = \max\{y^*, C^{-1}(\phi_1^a a_2 + \phi_2^a a_2)\}. \tag{36}$$

Thus, the problem of choice of a_1, a_2 becomes:

$$\max_{a_1, a_2} \left\{ -\phi_{-1,1}^a a_1 + \phi_{-1,2}^a a_2 + \beta \left\{ \zeta_1 \left[\nu(y_1) - \phi_1^a a_1 \right] + \zeta_2 \left[\nu(y_2) - \phi_1^a a_2 - \phi_2^a a_2 \right] + \phi_1^a a_1 + \phi_2^a a_2 \right\} \right\}. \tag{37}$$

The household's choice of assets gives us the demand for these assets as follows:

$$0 = -\phi^a_{-1,1} + \beta\phi^a_1 \left\{\zeta_1 \left[\nu'(y_1) - 1\right] + \zeta_2 \left[\nu'(y_2) - 1\right] + 1\right\}, \tag{38}$$

$$0 = -\phi^a_{-1,2} + \beta\phi^a_2 \left\{\zeta_2 \left[\nu'(y_2) - 1\right] + 1\right\}. \tag{39}$$

Using $r^a_i = \phi^a_{i,+1}/\phi^a_i - 1$ we get:

$$1 = \beta(1 + r^a_1) \left\{\zeta_1 \left[\nu'(y_1) - 1\right] + \zeta_2 \left[\nu'(y_2) - 1\right] + 1\right\}, \tag{40}$$

$$1 = \beta(1 + r^a_2) \left\{\zeta_2 \left[\nu'(y_2) - 1\right] + 1\right\}. \tag{41}$$

3.2 Non-dominant country Households

Consumption of goods When a household desires to consume, given her real balances z, she maximizes her consumption of each variety as follows,

$$\max_{q_{\omega_1}, q_{\omega_2}} \left\{U(Q_2) - \int_0^{\alpha_{21}} p_{\omega_1} q_{\omega_1} d\omega_1 - \int_0^{\alpha_{22}} p_{\omega_2} q_{\omega_2} d\omega_2\right\}, \tag{42}$$

s.t.

$$Z_{21} \equiv \int_0^{\alpha_{21}} p_{\omega_1} q_{\omega_1} d\omega_1 \leq z_{21} \equiv \phi_1 m_{21} \text{ and } Z_{22} \equiv \int_0^{\alpha_{22}} p_{\omega_2} q_{\omega_2} d\omega_2 \leq z_{22} \equiv \phi_2 m_{22}. \tag{43}$$

Here as described previously, Z_{21} is defined as the total real expenditure on stage 2 goods from firms in country 1 by country 2 households and Z_{22} is the same for imports or consumption from firms in country 2. Since households in country 2 need to pay for imports using country 1's currency we have that $Z_{22} \leq z_{22}$.

Consider two varieties, ω^1_i and ω^2_i to get, $q_{\omega^1_i}/q_{\omega^2_i} = (p_{\omega^1_i}/p_{\omega^2_i})^\sigma$, where $i \in \{1, 2\}$ is for the two countries. After further manipulation, we get: $\frac{Z_{2i}}{P_{2i}} = q_{\omega_i} \left(\frac{p_{\omega_i}}{P_{2i}}\right)^\sigma$. Using this in the definition of Q_{2i}, we get:

$$Q_{2i} = \frac{Z_{2i}}{P_{2i}}, \tag{44}$$

Recall that,

$$Q_2 = (Q_{21}^{\rho} + Q_{22}^{\rho})^{1/\rho}. \tag{45}$$

We have that $Z_2 = Z_{21} + Z_{22}$ and $Z_2 = P_{21}Q_{21} + P_{22}Q_{22}$. Given that money is costly to hold, households will not carry more money than they need to purchase stage 2 goods. So, we have that $z_{21} = Z_{21}$ and $z_{22} = Z_{22}$.

The problem of choice of m_{21}, m_{22} for country-2 households becomes,

$$\max_{m_{21},m_{22}} \{-\phi_{1,-1}\eta m_{21} - \phi_{2,-1}m_{22} + \beta \{[U(Q_2) - \phi_1 m_{21} - \phi_2 m_{22}] + \phi_1 m_{21} + \phi_2 m_{22}\}\}. \tag{46}$$

The household's choice of country-2 money balances m_{21} which determines its expenditure on heterogeneous goods Q_{21} is given by:

$$\frac{U'(Q_2)}{P_{21}} \left(\frac{Q_2}{Q_{21}}\right)^{\frac{1}{\sigma}} = \frac{(1+\gamma_1)\eta}{\beta}, \tag{47}$$

and on Q_{22} by,

$$\frac{U'(Q_2)}{P_{22}} \left(\frac{Q_2}{Q_{22}}\right)^{\frac{1}{\sigma}} = \frac{(1+\gamma_2)}{\beta}. \tag{48}$$

Consumption of financial services Country 2's household's choice of assets is exactly the same as country 1's.

3.3 Dominant country Firms

A firm in the dominant country 1 sells q_1^f in return for currency 1 to households in country 1 whose consumption is denoted by q_{11}. Country 1 firms also export goods q_{1x}^f to households in country 2 in return for currency 1 real balances whose consumption is denoted by q_{21}. Since the measure of country 1 households serviced by each firm in country-1 is $\alpha_{11}(N_1)/N_1$ and country 2 households is $\alpha_{21}(N_1^x)/N_1^x$ we have $q_1^f = \alpha_{11}(N_1)/N_1 q_{11}$ and $q_{1x}^f = \alpha_{21}(N_1^x)/N_1^x q_{21}$.

All firms share the same fixed costs $f_x > f > 0$ but have different productivity levels indexed by $\varphi > 0$ as in Melitz (2003).[6] Regardless of productivity each firm faces the same demand

[6] Note that the costs including f, f_x and k are paid in terms of the numéraire. If these costs were in terms of the variety good q, the effect from international trade would be larger.

curve depending on whether it exports or not. Demand for each variety of country 1 firms by country 1 households is ω_1 as $\frac{Z_1}{P_1} = q_{\omega_1}\left(\frac{p_{\omega_1}}{P_1}\right)^{\sigma}$. And, demand for each variety of country 1 firms by country 2 households is, $\frac{Z_{21}}{P_{21}} = q_{\omega_1}\left(\frac{p_{\omega_1}}{P_{21}}\right)^{\sigma}$

Firm's stage 2 profit maximization problem from selling to country 1 households is:

$$\max_{q_1^f, p_1}\left[p_1(\varphi)q_1^f(\varphi) - \frac{q_1^f(\varphi)}{\varphi} - f\right], \tag{49}$$

s.t.

$$\frac{q_1^f(\varphi)}{\frac{\alpha(N_1)}{N_1}} = Q_1\left(\frac{P_1}{p_1(\varphi)}\right)^{\sigma}.$$

This gives us the prices:

$$p_1(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi} = \frac{1}{\rho\varphi}. \tag{50}$$

Firm's stage 2 profit maximization problem from exports:

$$\max_{q_{1x}^f, p_1^x}\left[p_1^x(\varphi)q_{1x}^f(\varphi) - \frac{q_{1x}^f(\varphi)}{\varphi}\tau - f_x\right], \tag{51}$$

s.t.

$$\frac{q_{1x}^f(\varphi)}{\frac{\alpha(N_1^x)}{N_1^x}} = Q_{21}\left(\frac{P_{21}}{p_1^x(\varphi)}\right)^{\sigma}. \tag{52}$$

This gives us:

$$p_1^x(\varphi) = \frac{\sigma}{\sigma-1}\frac{\tau}{\varphi} = \frac{\tau}{\rho\varphi}. \tag{53}$$

Firms do not produce heterogeneous goods if $[p(\varphi)q^f(\varphi) - \frac{q^f(\varphi)}{\varphi} - f] < 0$ and they do not export if $[p_x(\varphi)q_x^f(\varphi) - \frac{q_x^f(\varphi)}{\varphi}\tau - f_x] < 0$. In other words, a firm only produces if its gross profits is no less than its fixed cost of operation f and f_x (for exports). To obtain the cutoff productivity $\hat{\varphi}$ for selling to domestic market, we will use the zero cutoff profit condition, $\pi(\hat{\varphi}) = 0$ and the cutoff productivity $\hat{\varphi}_x$ for selling to foreign market is obtained from $\pi_x(\hat{\varphi}_x) = 0$. For that we need to obtain firm's stage 2 profit after substituting for profit maximizing level of prices and household's demand for firm's variety.

$$\pi_1 = \frac{\alpha(N_1)}{N_1}Q_1\left(\frac{P_1}{p_1(\varphi)}\right)^{\sigma}\left[\frac{1-\rho}{\rho\varphi}\right] - f, \tag{54}$$

$$\pi_1(\varphi) = \frac{\alpha(N_1)}{N_1} Z_1 \left[\frac{P_1}{p_1(\varphi)}\right]^{\sigma-1} \frac{1}{\sigma} - f. \tag{55}$$

Setting $\pi_1 = 0$ i.e. the zero cutoff profit condition gives:

$$P_1 = \left(\frac{\sigma f}{\frac{\alpha(N_1)}{N_1} Z_1}\right)^{\frac{1}{\sigma-1}} \frac{1}{\rho\hat{\varphi}_1}. \tag{56}$$

Similarly, profit from exports:

$$\pi_1^x(\varphi) = \frac{\alpha(N_1^x)}{N_1^x} Z_{21} \left[\frac{P_{21}}{p_1^x(\varphi)}\right]^{\sigma-1} \frac{1}{\sigma} - f_x. \tag{57}$$

The zero cutoff profit condition for exporters will give:

$$P_{21} = \frac{\tau}{\rho\hat{\varphi}_x} \left(\frac{\sigma f_x}{\frac{\alpha(N_1^x)}{N_1^x} Z_{21}}\right)^{\frac{1}{\sigma-1}}. \tag{58}$$

To obtain the average profit plug the respective P_1, P_{21} back into the respective profit functions to first see that profit is increasing in productivity φ and decreasing in the cutoff productivity $\hat{\varphi}$:

$$\pi(\varphi) = f\left[\left(\frac{\varphi}{\hat{\varphi}}\right)^{\sigma-1} - 1\right], \tag{59}$$

$$\pi_x(\varphi) = f_x\left[\left(\frac{\varphi}{\hat{\varphi}_1^x}\right)^{\sigma-1} - 1\right]. \tag{60}$$

Any active firm that draws a productivity level $\varphi < \hat{\varphi}$ will not produce heterogeneous goods in stage 2. The ex-ante probability of successful entry in stage 2 for firms is thus given by $[1-G(\hat{\varphi})]$. This gives us the equilibrium productivity distribution $\mu(\varphi)$ which is the conditional distribution of $g(\varphi)$ on $[\hat{\varphi}, \infty]$ and is given by $\mu(\varphi) = g(\varphi)/[1 - G(\hat{\varphi})]$ if $\varphi \geq \hat{\varphi}$ and zero otherwise. This also defines the average productivity level $\bar{\varphi}$ as a function of the cutoff level $\hat{\varphi}$, $\bar{\varphi}(\hat{\varphi}) = \left[\int_{\hat{\varphi}}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{1/(\sigma-1)}$. Thus, the average profit from stage 2 production is also tied to the cutoff productivity level, which can be written as:

$$\bar{\pi}_1 = \pi(\bar{\varphi}) = \left[\left(\frac{\bar{\varphi}(\hat{\varphi})}{\hat{\varphi}}\right)^{\sigma-1} - 1\right] f. \tag{61}$$

The ex-ante probability of successful entry in stage 2 export markets for firms is given by $[1 - G(\hat{\varphi}_x)]$. The equilibrium productivity distribution of exporting firms is given by $\mu(\varphi) = g(\varphi)/[1 - G(\hat{\varphi}^*)]$ if $\varphi > \hat{\varphi}^*$ and zero otherwise. And, $\left[\int_{\hat{\varphi}_x^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{1/(\sigma-1)} = \bar{\varphi}_x^*(\hat{\varphi}_x^*)$ is the average productivity of exporting foreign firms.

The average profit from stage 2 goods exports is,

$$\bar{\pi}_1^x = \pi_x(\bar{\varphi}_x) = \left[\left(\frac{\bar{\varphi}_x(\hat{\varphi}_x)}{\hat{\varphi}_x}\right)^{\sigma-1} - 1\right] f_x. \tag{62}$$

3.4 Non-dominant country Firms

A firm in the non-dominant country 2 sells q_2^f in return for currency 2 to households in country 2 whose consumption is denoted by q_{22}. Country 2 firms also export goods q_{2x}^f to households in country 1 in return for currency 1 real balances whose consumption is denoted by q_{21}. Since the measure of country 2 households serviced by each firm in country-2 is $\alpha_{22}(N_2)/N_2$ and country 1 households is $\alpha_{12}(N_2^x)/N_2^x$ we have $q_{2x}^f = \alpha_{12}(N_2^x)/N_2^x q_{12}$ and $q_2^f = \alpha_{22}(N_2)/N_2 q_{22}$.

All firms again share the same fixed costs $f_x > f > 0$ but have different productivity levels indexed by $\varphi > 0$ as in Melitz (2003). Regardless of productivity each firm faces the same demand curve depending on whether it exports or not. Demand for each variety of country 2 firms by country 2 households is ω_2 as $\frac{Z_{22}}{P_{22}} = q_{\omega_2}\left(\frac{p_{\omega_2}}{P_{22}}\right)^{\sigma}$. And, demand for each variety of country 2 firms by country 1 households is, $\frac{Z_1}{P_1} = q_{\omega_2}\left(\frac{p_{\omega_2}}{P_1}\right)^{\sigma}$.

Firm's stage 2 profit maximization problem from selling to country 2 households is:

$$\max_{q_2^f, p_2}\left[p_2(\varphi)q_2^f(\varphi) - \frac{q_2^f(\varphi)}{\varphi} - f\right], \tag{63}$$

s.t.

$$\frac{q_2^f(\varphi)}{\frac{\alpha(N_2)}{N_2}} = Q_{22}\left(\frac{P_{22}}{p_2(\varphi)}\right)^{\sigma}. \tag{64}$$

This gives us:

$$p_2(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi} = \frac{1}{\rho\varphi}. \tag{65}$$

Firm stage 2 profit maximization problem from exports:

$$\max_{q_{2x}^f, p_2^x}\left[p_2^x(\varphi)q_{2x}^f(\varphi) - \frac{q_{2x}^f(\varphi)}{\varphi}\tau - f_x\right], \tag{66}$$

s.t.

$$\frac{q_{2x}^f(\varphi)}{\frac{\alpha(N_2^x)}{N_2^x}} = Q_1\left(\frac{P_1}{p_2^x(\varphi)}\right)^{\sigma}. \tag{67}$$

This gives us:

$$p_2^x(\varphi) = \frac{\sigma}{\sigma-1}\frac{\tau}{\varphi} = \frac{\tau}{\rho\varphi}. \tag{68}$$

As done earlier, we obtain firm's stage 2 profit,

$$\pi_2 = \frac{\alpha(N_2)}{N_2}Q_{22}\left(\frac{P_{22}}{p_2(\varphi)}\right)^{\sigma}\left[\frac{1-\rho}{\rho\varphi}\right] - f, \tag{69}$$

$$\pi_2(\varphi) = \frac{\alpha(N_2)}{N_2}Z_{22}\left[\frac{P_{22}}{p_2(\varphi)}\right]^{\sigma-1}\frac{1}{\sigma} - f. \tag{70}$$

Setting $\pi_2 = 0$ i.e. the zero cutoff profit condition gives:

$$P_{22} = \left(\frac{\sigma f}{\frac{\alpha(N_2)}{N_2}Z_{22}}\right)^{\frac{1}{\sigma-1}}\frac{1}{\rho\hat{\varphi}_2}. \tag{71}$$

Similarly, profit from exports:

$$\pi_2^x(\varphi) = \frac{\alpha(N_2^x)}{N_2^x}Z_1\left[\frac{P_1}{p_2^x(\varphi)}\right]^{\sigma-1}\frac{1}{\sigma} - f_x. \tag{72}$$

The zero cutoff profit condition for exporters will give:

$$P_1 = \frac{\tau}{\rho\hat{\varphi}_2^x}\left(\frac{\sigma f_x}{\frac{\alpha(N_2^x)}{N_2^x}Z_1}\right)^{\frac{1}{\sigma-1}}, \tag{73}$$

To obtain the average profit plug the respective P_1, P_{22} back into the respective profit functions to get the average profits as before:

$$\bar{\pi}_2 = \pi(\bar{\varphi}) = \left[\left(\frac{\bar{\varphi}(\hat{\varphi}_2)}{\hat{\varphi}_2}\right)^{\sigma-1} - 1\right]f. \tag{74}$$

The average profit from stage 2 goods exports is,

$$\bar{\pi}_2^x = \pi_x(\bar{\varphi}_x) = \left[\left(\frac{\bar{\varphi}_2^x(\hat{\varphi}_2^x)}{\hat{\varphi}_2^x}\right)^{\sigma-1} - 1\right]f_x. \tag{75}$$

3.5 Labor market

The law of motion for unemployment is given by:

$$u_t = u_{t-1} + \delta(1 - u_{t-1}) - q^j(\theta_{t-1})u_{t-1}. \tag{76}$$

In steady state we get the Beveridge curve for country-i,

$$1 - n_i = \frac{\delta}{q^j(\theta_i) + \delta} \text{ OR } n_i = \frac{q^j}{q^j + \delta}, \tag{77}$$

which gives a positive relationship between firm entry ($n = 1 - u$) and market tightness.

The free entry condition for firms in country-i is:

$$k = \frac{1}{1 + r_i^a} q^v(\theta_i) EV_i^{f1}, \tag{78}$$

where the expected value of a filled job in country-1, EV_1^{f1} is:

$$EV_i^{f1} = \{A - w + [1 - G(\hat{\varphi}_i)]\pi_i[\bar{\varphi}(\hat{\varphi}_i)] + [1 - G(\hat{\varphi}_i^x)]\pi_i^x[\bar{\varphi}(\hat{\varphi}_i^x)]\} + \frac{1}{1 + r_i^a}(1 - \delta)EV_i^{f1}(\varphi).$$

The free entry condition becomes:

$$k(r_i^a + \delta) = q^v(\theta_i)\{A - w + [1 - G(\hat{\varphi}_i)]\bar{\pi}_i + [1 - G(\hat{\varphi}_i^x)]\bar{\pi}_i^x\}, \tag{79}$$

where $r_i^a = \phi_{1,i}^a / \phi_{-1,i}^a - 1$. Higher average profits given that the productivity meets the cutoff ($=[1 - G(\hat{\varphi})]\pi(\bar{\varphi})$) implies that fewer firms enter: $q^v(\theta)$ falls or θ rises in response to higher profits and thus n rises.

3.6 Aggregation

Finally, we need to get some aggregate variables to complete the equilibrium description of the model. Recall that aggregate price is $P_{2j} \equiv \left(\int_0^{\alpha_{2j}} p_{\omega_j}^{1-\sigma} d\omega_j\right)^{1/1-\sigma}$, $j \in \{1, 2\}$. So we get that the aggregate price households in country 2 face on their imports is the sum of the price charged by exporting firms in country 1,

$$P_{21} = \left(\int_{\hat{\varphi}_1^x}^{\infty} p_1^x(\varphi)^{1-\sigma} \alpha_{21}(N_1^x)\mu(\varphi)d\varphi\right)^{1/(1-\sigma)}, \tag{80}$$

substituting for p_1^x we get,

$$P_{21} = \frac{\tau\alpha(N_1^x)^{\frac{1}{1-\sigma}}}{\rho}\left[\int_{\hat{\varphi}_1^x}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{1/(1-\sigma)}, \tag{81}$$

where $\mu(\varphi)$ is the conditional distribution of $g(\varphi)$ on $[\hat{\varphi}^*, \infty]$ as discussed before. Thus,

$$P_{21} = \frac{\tau\alpha(N_1^x)^{\frac{1}{1-\sigma}}}{\rho\bar{\varphi}_1^x(\hat{\varphi}_1^x)}. \tag{82}$$

Using the expression for P_{21} from the ZCP we get the mass of exporting firms in stage 2 N_1^x:

$$N_1^x = \frac{Z_{21}}{\sigma f_x}\left(\frac{\hat{\varphi}_1^x}{\bar{\varphi}_1^x}\right)^{\sigma-1}. \tag{83}$$

Next, we get that the aggregate price households in country 2 face on their domestic goods as sum of the price charged by firms in country 2,

$$P_{22} = \left(\int_{\hat{\varphi}_2}^{\infty}p_2(\varphi)^{1-\sigma}\alpha(N_2)\mu(\varphi)d\varphi\right)^{1/(1-\sigma)}, \tag{84}$$

substituting for p_2,

$$P_{22} = \frac{\alpha(N_2)^{\frac{1}{1-\sigma}}}{\rho}\left[\int_{\hat{\varphi}_2}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{1/(1-\sigma)}, \tag{85}$$

where $\mu(\varphi)$ is the conditional distribution of $g(\varphi)$ on $[\hat{\varphi}^*, \infty]$ as discussed before. Thus,

$$P_{22} = \frac{\alpha(N_2)^{\frac{1}{1-\sigma}}}{\rho\bar{\varphi}_2(\hat{\varphi}_2)}. \tag{86}$$

Using the expression for P_{22} from the ZCP we get the mass of firms in stage 2 N_2:

$$N_2 = \frac{Z_{22}}{\sigma f}\left(\frac{\hat{\varphi}_2}{\bar{\varphi}_2}\right)^{\sigma-1}. \tag{87}$$

Similarly, get P_{11} and P_{12} and then P_1. Recall that $P_1 \equiv (P_{11}^{1-\sigma} + P_{12}^{1-\sigma})^{\frac{1}{(1-\sigma)}}$ where $P_{1j} \equiv \left(\int_0^{\alpha_{1j}} p_{\omega_j}^{1-\sigma}d\omega_j\right)^{1/1-\sigma}$, $j \in \{1, 2\}$,

$$P_{11} = \frac{\alpha(N_1)^{\frac{1}{1-\sigma}}}{\rho\bar{\varphi}_1(\hat{\varphi}_1)}, \tag{88}$$

and,

$$P_{12} = \frac{\tau\alpha(N_2^x)^{\frac{1}{1-\sigma}}}{\rho\bar{\varphi}_2^x(\hat{\varphi}_2^x)}. \tag{89}$$

$$P_1 = \left[\alpha(N_1)\left(\frac{1}{\rho\bar{\varphi}_1(\hat{\varphi}_1)}\right)^{1-\sigma} + \alpha(N_2^x)\left(\frac{\tau}{\rho\bar{\varphi}_2^x(\hat{\varphi}_2^x)}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{90}$$

Finally, the share of exporting firms is given by :

$$N_j^x = [1 - G(\hat{\varphi}_j^x)]n_j. \tag{91}$$

And, the measure of active firms is given by n_j which is derived from the free entry condition and Beveridge curve, while N_j gives a subset of those firms that are also active in stage 2. The relation between them is given below:

$$N_j = [1 - G(\hat{\varphi}_j)]n_j. \tag{92}$$

Thus, after entry and matching, a fraction $[1-G(\hat{\varphi})]$ of matched firms produce in stage 2. Only matched firms draw a productivity, not all posted vacancies.

Aggregate household demand for financial assets (given that there is a measure 1 of households in both countries) will be equal to aggregate supply of these assets. This gives the market clearing conditions as follows:

$$n_1(1 + r_1^a)\frac{k}{q^v(\theta_1)} + B_1 = \phi_1^a 2a_1, \tag{93}$$

$$n_2(1 + r_2^a)\frac{k}{q^v(\theta_2)} + B_2 = \phi_2^a 2a_2. \tag{94}$$

Under a linear cost function and given that asset supply is scarce[7], we get:

$$n_1(1 + r_1^a)\frac{k}{q^v(\theta_1)} + B_1 = 2y_1, \tag{95}$$

$$n_2(1 + r_2^a)\frac{k}{q^v(\theta_2)} + B_2 = 2(y_2 - y_1). \tag{96}$$

[7]If asset supply is abundant i.e. $(\phi_1^a a_1) \geq y^*$ then $y_1 = y^*$.

Market clearing for money demand and supply gives us:

$$\phi_1 M_1 = Z_1 + Z_{21}, \tag{97}$$

and

$$\phi_2 M_2 = Z_{22}. \tag{98}$$

The exchange rate between the two country's currencies is given by ϕ_2/ϕ_1.

3.7 Trade flows in the heterogeneous goods sector

Each country's output value in the heterogeneous goods sector from selling domestically is $Y_i = N_i \bar{v}_i$, where $\bar{v}_i = \left(\int_{\hat{\varphi}_i}^{\infty} p_i(\varphi) q_i(\varphi) g(\varphi|\varphi > \hat{\varphi}_i) d\varphi\right)^{1/(1-\sigma)}$ is the average output value of country-i firms that sell domestically. The output value from exporting is $Y_i^x = N_i^x \bar{v}_i{}^x$, where $\bar{v}_i{}^x$ can be defined similarly. Given that $\pi_i(\varphi) = p_i q_i/\sigma - f$, we can rewrite Y_i as follows: $Y_i = \sigma(\pi_i + f)N_i$ and $Y_i^x = \sigma(\pi^x + f_x)N_i^x$.

Thus, the trade balance $\mathbb{T}_i$ for country-i in the heterogeneous goods sector is given by $\mathbb{T}_i = Y_i^x - Y_j^x$. As mentioned before, costless trade of the homogeneous good ensures balanced trade. Thus, if country-1 has a trade surplus of $\mathbb{T}_1 > 0$ in the heterogeneous goods sector, it will have a trade deficit in the homogeneous good sector.

3.8 Equilibrium

Definition 1. *The open economy steady-state equilibrium is a 25-tuple* $(Z_1, P_1, Z_{21}, Z_{22}, P_{21}, P_{22}, Q_{21}, Q_{22}, Q_2, r_1^a, r_2^a, y_1, y_2, N_1, N_1^x, N_2, N_2^x, n_1, n_2, \theta_1, \theta_2, \hat{\varphi}_1, \hat{\varphi}_2, \hat{\varphi}_1^x, \hat{\varphi}_2^x)$ *that solves the following system of 25 equations:* (32), (40), (41), $(44)_1$, $(44)_2$, (45), (47), (48), (56), (58), (71), (73), $(77)_1$, $(77)_2$, $(78)_1$, $(78)_2$, (83), (87), (90), $(91)_1$, $(91)_2$, $(92)_1$, $(92)_2$, (95), (96).

4 Results

For now we will consider a numerical example to analyze some results of the model. We assume the utility from consuming financial services takes the form: $\nu(y) = A_y y^{a_y}/a_y$. Recall that the

Parameters	**Value**
discount factor β	0.125
utility ρ, ξ	0.4, 0.3
financial services utility A_y, a_y	2.5,0.5
fraction of firms met α	0.5
fraction of traders met ζ_1, ζ_2	0.5,0.5
A_l, a_l	0.86, 0.5
flexible wage proportion	0.7
cost of entry k_e (match $n = 0.97$)	2
separation probability δ	0.1
fixed productivities A_1, A_2	1,1
productivity distribution $G(\cdot)$: Pareto k_G, φ_{min}	1,1
production costs f, f_x, τ	0.2, 0.22, 1.02
inflation γ_1, γ_2	0.02, 0.02

Table 1: Baseline parameter values

utility from differentiated good consumption is $U(Q_i) = Q_i^{\xi}/\xi$ where $Q_i = (Q_{i1}^{\rho} + Q_{i2}^{\rho})^{1/\rho}$ and $Q_{ij} = \left(\int_0^{\Omega_i} q_{\omega_j}^{\rho} d\omega_j\right)^{1/\rho}$, $j \in \{1, 2\}$ with elasticity of substitution $\sigma = \frac{1}{(1-\rho)} > 1$ and we assume that: $0 < \xi < \rho$. In the labor market, $q^v(\theta) = A_l \theta^{-a_l}$ and $q^j(\theta) = \theta q^v(\theta)$. The productivity distribution is Pareto with the CDF given by $G(\varphi) = 1 - (\varphi_{min}/\varphi)^{k_G}$, where $k_G > \sigma - 1$ is a parameter of productivity dispersion – higher k_G means lower heterogeneity. The probability of matching a trader i for any household is $\zeta(e_i) = e_i < 1$ because we assume that this matching function is the minimum of 1 (measure of country -i households) and $e_1 + e_2 < 1$ (measure of all traders). And, households meet $\alpha(N) = \alpha N$ firms. The chosen parameters are reported in Table 1.[8]

In the baseline case the two countries are symmetric except that assets issued by the dominant country (government bonds and private claims to firms) are more liquid than those issued by the non-dominant country. This will imply that the returns on these assets will reflect a

[8]Note that $\gamma_1 = \gamma_2$ violates our assumption $(1+\gamma_1) < (1+\gamma_2) < (1+\gamma_1)\eta < (1+\gamma_2)\eta$ or $\eta > (1+\gamma_2)/(1+\gamma_1) > 1$, so this needs to be changed and updated. But, it will not affect the qualitative results.

Variable	Baseline	$\gamma^1 \uparrow$ **0.04**	$\gamma^2 \uparrow$ **0.04**
domestic expenditure shares Z_{11}/Z_1	0.52	0.01%	0.03%
Z_{22}/Z_2	0.51	2.34%	-2.39%
output Q_1	1.63	-5.19%	-0.04%
Q_2	1.63	-2.55%	-2.71%
terms of trade P_{21}/P_{12}	0.98	0.30%	-0.42%
firm entry n_1	0.97	-0.002%	0.01%
n_2	0.96	-0.03%	-0.05%
share of exporters N_1^x/N_1	0.85	- 0.68%	0.68%
N_2^x/N_2	0.85	-4.08%	4.27%
global asset flows y	6.16	-0.41%	-0.15%

Table 2: Baseline values and % changes

higher liquidity premium, so we get $-\gamma/(1+\gamma) < r_1^a \leq r_2^a \leq \rho_d$. The nominal interest on the dominant country asset is $\iota_1^a = (1+r_1^a)(1+\gamma_1) - 1$, and if we define spreads as $s_1 = \iota_1^a - \iota$ we get: $s_1 > s_2 \geq 0$. The difference in liquidity properties of assets between the two countries by itself implies that $n_1 \geq n_2$ as seen in Table 2. The average productivity in country 1 is higher than in country 2, but country 2 exporters are on average more productive than exporters in country 1 i.e., $\hat{\varphi}_2 < \hat{\varphi}_1 < \hat{\varphi}_1^x < \hat{\varphi}_2^x$.

Result 1 (Liquidity differences): *If liquidity is scarce in all financial markets and dominant country 1 assets are more liquid than the other, the lower interest rate in country 1 leads to (i) higher firm entry, (ii) larger market capitalization (iii) higher average productivity and (iv) higher share of domestic expenditure in total consumption for country 1.*

Next, we analyze the asymmetric effects of contractionary monetary policies in the two countries as reported in Table 2. An increase in the nominal interest rates on an illiquid asset ι implies an increase in γ through the Fischer equation. Also note that an increase in γ is akin to an increase in inflation since we are considering stationary monetary equilibrium. An increase in the dominant country's nominal interest rate ι_1 i.e. a contractionary monetary policy makes money costlier to hold and it leads to a decline in currency 1 holdings by both households in country 1 and 2. While an increase in the non-dominant country's nominal interest rate ι_2 only leads to a decline in money holdings for households in country 2 as the other households do not carry currency 2.

Result 2 (Asymmetric monetary policy spillovers): *An increase in the dominant country's nominal interest rate ι_1 leads to a simultaneous economic slowdown and a rise in unemployment in countries 1 and 2. While an increase in the non-dominant country's ι_2 leads to a redistribution of production and consumption as u_2 rises but u_1 falls. The consumption spillover from a rise in ι_1 is greater than an equal increase in ι_2. Moreover, there is a larger slowdown in global asset flows or world market capitalization on account an increase in ι_1 than ι_2.*

If the dominant currency 1 becomes costlier to hold, country 1 households reduce their domestic as well as import expenditures (both Z_{11} and Z_{12} fall) so their overall consumption Q_1 falls, while country 2 households shift their expenditure to domestic (Z_{22} rises). But, they also witness an overall decline in their consumption Q_2. This leads to a decline in both domestic and exporting firms in country 1 and only exporting firms in country 2. Country 2 firms selling domestically witness an increase and their average productivity declines. However, overall firm entry in country 2 falls owing to a decline in demand for goods.

As fewer firms enter, private and total liquidity in the financial goods market declines and interest rate falls reflecting a larger liquidity premium on country 1 assets.[9] Country 2 assets do not exhibit a liquidity premium for our baseline example and so r_2^a does not change. Owing to a fall in r_1^a, the fall in n_1 is lower than it would be when r_1^a were fixed. Since $r_2^a = \rho_d$, the fall in n_2 ends up being larger than n_1.

The spillover effects from a change in country 2's monetary policy are quite different. The total consumption Q_i in both countries falls, but Q_2 falls much more. Expenditures of both country households shift to country 1 (Z_{11} and Z_{21} rise, the magnitude of fall in Z_{22} is much larger overall) and as a result more firms enter in country 1 and u_1 falls. Country 2 witnesses a

[9]Note that an increase in the nominal interest rate on an illiquid asset, also leads to increase in nominal rates on liquid assets $\iota_i^a = (1+\gamma_i)(1+r_i^a) - 1$. First, the pass through is directly via γ increases but this effect is dampened via changes in real interest rates on liquid assets as fewer firms enter so r^a decreases.

fall in firm entry and u_2 increases. This implies that private liquidity in country 1 rises but that in country 2 falls. Global asset flows $y = 2(y_1\zeta_1 + y_2\zeta_2)$ decline on account of lower liquidity in country 2 but the extent of the fall is much lower compared to when country 1's monetary policy changes.

Another interesting experiment is to compare the effects of synchronous and asynchronous monetary policy tightening. If all countries tighten simultaneously there is a chance that there is over-tightening globally, while if a country waits for others to tighten there might be failed coordination leading to a slower global policy response. There is an amplified fall in firm entry in the non-dominant country but not so in the dominant country. The amplified response follows from the fact that after a synchronous tightening, demand falls across the board from all households because money is costlier in both countries and there is no way to switch demand. This leads to an amplified slowdown in global liquidity which translates to a greater rise in liquidity premium and interest rates for the dominant country. This in turn makes it more desirable for firms to enter in the dominant country which offsets the initial fall in firm entry. Thus, in the end the effect of synchronous monetary tightening is not felt as strongly in the dominant country as compared to the non-dominant country.

Result 3 (Effects of synchronous versus asynchronous monetary tightening): *An increase in ι_1 and ι_2 simultaneously leads to an amplified increase in u_2 and a similar effect on u_1 as compared to the sum of effect from only ι_1 rise and ι_2 rise. There is also an amplified slowdown in global liquidity flows, y and a greater rise in liquidity premium following synchronous monetary tightening as compared to the sum of asynchronous tightening in the two countries.*

Now, instead we change the supply of government bonds which affects the interest rates on these liquid bonds, ι_i^a. An increase in B_1 leads to a rise in r_1^a and hence fewer firms in country 1 enter, i.e. there is crowing out of private liquidity, but total liquidity is still higher. Lower firms in country 1 means that households in both countries now carry less currency to spend on country 1 goods, which leads to a small increase in demand for country 2 goods and there is a small positive spillover to country 2 as n_2 rises. Note however that total consumption Q_i in both countries declines. On the other hand, an increase in B_2 has no effect because $r_2^a = \rho$

which is its maximum value.

The above results somewhat reflect the empirical findings as documented in Miranda-Agrippino and Rey (2022) where US monetary policy disproportionately affects the global financial cycle. European and Chinese monetary policy while not affecting the global financial cycle as much does affect the global trade and commodities cycle.

Next, consider the effects of a real match productivity shock on the two countries i.e. changes to A_i. As A_i falls, the expected value of a filled vacancy EV_i falls and thus unemployment in the country-i rises. Trade linkages between countries implies that demand shifts to country $j \neq i$, and trading partner country's unemployment u_j falls. With financial linkages, the effects on u_1 will be dampened as r_1^a adjusts.

Result 4 (Match productivity shocks): *An increase in match productivity in country-i leads to a fall in unemployment in country-i and a rise in unemployment in country-j. Given financial linkages, the effect on u_1 is dampened as r_1^a adjusts. If there are no trade linkages but only Financial linkages then there is no effect of A_1 on u_2, but A_2 still leads to a fall in u_1 via a fall in r_1^a.*

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