A duopoly model of open source and proprietary products*

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Abstract

This paper studies a duopoly model of product competition between two firms: a proprietary firm and an open source firm. Firms sell the same product, but of different kinds. There is a probability that the quality of each product can suffer a damage ("a bug"). For the proprietary product, the bug can be only internally fixed by the firm during its next upgrade. In contrast, for the open source product, the bug can be fixed by end users (consumers) and the probability of fixing the problem increases as more cosumers purchase the open source product. There are two segments of consumers: a segment of low-technical consumers who always buys the proprietary product and another segment of high-technical consumers who are distributed in the Hotelling linear city unit interval with two firms at the end points. It is shown that when the damage to the quality caused by the adverse event is minor, in the unique equilibrium outcome the proprietery firm serves both segments, sets a higher price and obtains a higher profit. In contrast, when the damage to the quality caused by the adverse event is of moderate magnitdue, in the unique equilibrium outcome two firms are two local monopolists in two segments. Moreover, the open source firm sets a higher price and obtains a higher profit.

Keywords: proprietary product; open source product; Hotelling duopoly

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1 Introduction

Open source products or more generally products that are given to users through free licenses have become quite common in recent years. In particular, open source softwares are frequently used in information technology. In this context, one specific aspect that is of interest is proprietary products (that is, products that can be used only by paying a price), usually sold by large dominant firms, coexist with such open source products. For example, the Windows operating system is a proprietary product that has a dominant market share. At the same time, the open source operating system of Linux continues to hold to its small but robust market share and have an increasingly devoted community of users over time. Given such examples a natural question is what are the specific considerations of an open source firm that gives it incentive to give its product for free in a situation of strategic interaction with a dominant proprietary firm.

In this paper we consider a duopoly model with two firms selling the same product of different kinds: one proprietary firm and another open source firm. There is a probability that each product might face an adverse event that can cause a damage to the quality of the product ("a bug"). For the proprietary product, this bug cannot be fixed by the end users (the consumers); it can be only fixed internally by the firm when it launches its next upgrade. By contrast, for the open source product, the users can work on the problem and they may be able to fix it; when more users use the open source product, it is more likely that the problem will be fixed.

We consider two different segments of consumers. There is a segment of low-technical consumers who are not capable of fixing any likely problems with the product. These consumers never purchase from the open source firm. The proprietary firm is a local monopolist with this segment of buyers (alternatively, this segment is like a "captive segment" for the proprietary firm, see, e.g., Basu and Bell, 1991). There is another segment of high-technical buyers who are distributed in a Hotelling linear city in a unit interval between the proprietary and open source firms. These buyers pay the prices and transportation costs to purchase the product. However, in addition, the fraction of high-technical buyers who purchase from the open source firm also determines the probability with which the bug of the open source product can be fixed.

While our work is still in progress, our preliminary findings suggest that the relative prices and the profits of the two firms as well as the kinds of segments they serve depend on the magnitude of the adverse event (the bug). If the damage to the quality of the product caused by the bug is relatively small, then in the unique equilibrium outcome of the duopoly, the proprietary firm sets a higher price and obtains a higher profit than the open source firm; moreover, the proprietary firm serves both segments of low-technical and high-technical buyers (Proposition 1). However, if the damage to the quality of the product caused by the bug is of moderate magnitude, then in the unique equilibrium outcome of the duopoly, there is a separation in that the proprietary firm exclusively serves the low-technical buyers and becomes a purely local monopolist of that segment; on the other hand, the open source firm exclusively serves the high-technical buyers and becomes a purely local monopolist of that segment. In this case the open source firm sets a higher price and obtains a higher profit (Proposition 2).

The literature has pointed out different reasons behind open sources or free licenses such as R&D advantages, building reputation and gaining from future services of certifications and installations. Another reason for free licenses can be the long term goal of promoting a new technology that has future potential (see Zeigler et al., 2014 for these different aspects). Another objective can be to extablish a technology standard in the industry by giving incentives of investments in related technologies (see, e.g., Vertinsky, 2018). Although competition between proprietary and open source firms have received some attention (e.g., Sen, 2017), the literature has not adequately looked at different strategic aspects of competition and the important roles of end users. Our paper makes an attempt to present a thorough strategic analysis of these aspects.

The paper is organized as follows. We present a brief background of open source softwares in Section 2. We present the model in Section in 3 and our main results obtained so far are presented in Section 3.3. We conclude in Section 4.

2 A brief background of open source softwares

An operating system (OS) is the core software that enables users to interact with the hardware devices of a computer. In doing so, an operating system must manage and protect the essential resources of a computer: central processing unit (CPU), memory (RAM), secondary storage media (such as a hard drive). For us to be able to use a computer, an operating system must be loaded into its memory. Thus, an operating system simply is an integral part of any computing device, whether it is a server or a pc or a smart phone.

Unix, the first modern multi-user, multi-tasking operating system, was designed in AT&T and Bell Labs and the first version was released in 1971. MS-DOS, the predecessor of Windows OS, was developed by Microsoft in 1980s for single-user IBM PCs. Both Unix and MS-DOS (later Windows) are proprietary software, for which (i) source code is not publicly available; it is owned, managed and maintained by the vendor and (ii) users need to purchase a licensing fee to be able to use the operating system.

Microsoft provides warranty, tech support for any bug fixing, and regular security updates in exchange for a licensing fee. AT&T, on the other hand, charged a nominal fee to academic institutions to use Unix operating system but offered no official tech support. Since there was no official support, Unix users eventually formed a community to share bug fixes. This initiative eventually led to a new type of license called Copyleft license that enabled all users of a program under this license to use, copy, modify, and distribute their modifications while prohibiting privatization of the program. Software designed under Copyleft license is known as open-source software. This licensing paved the way for designing an efficient and powerful operating system called Linux.

In 1991, Linus Torvald, a Computer Science graduate student at the University of Helsinki, designed a free Unix based operating system with the intention of free distribution of this OS. Linux, being open source, soon gained popularity among the community of software developers, and eventually became a worldwide distributed OS development project. Eventually, over 20,000 unique contributors had been involved in this project over the 29 years since the initial release, as stated in the 2020 Linux Kernel History Report from Carnegie Mellon University in August 2020 (The Linux

Foundation, 2016). According to this report, approximately 14,000 individual developers from over 1,300 different companies have contributed to the development and improvement of the Linux kernel since 2005, making it one of the largest cooperative software projects in the world.

When the Lunux project gained recognition, various companies were founded to distribute variations of Linux using various business models such as the distributor and the software producer models. According to Krishnamurthy (2003), some advantages of open-source business models are their robustness (due to the involvement of large global communities, open-source softwares are more reliable and secure, enabling bugs to be identified and fixed more quickly) and user flexibility (users are not locked into a single vendor and they can customize, modify, or mix software to suit their needs). On the other hand, version proliferation and difficulty of use for non-technical users are some disadvantages of open-source softwares.

3 The model

Consider two firms $1, 2$, who sell two different kinds of the same product. Firm 1 sells a proprietary product, while firm 2 sells an open source product. We assume that any consumer purchases at most one unit of the product. A consumer needs to have a certain level of technical expertise to use the (open source) product of firm 2, while using the (proprietary) product of firm 1 does not require any technical knowledge. Accordingly, we consider two types of consumers. There is a set of casual or low-technical consumers (type L) presented by a unit interval, where each point corresponds to a specific low-technical consumer. There is another set of high-technical consumers (type H) presented by a separate unit interval, whose each point corresponds to a specific high-technical consumer.

No type L consumer is capable of using the product of firm 2, so type L consumers always buy from firm 1, provided they have a net positive utility. On the other hand, type H consumers can buy from either firm 1 or firm 2. Specifically, we assume the unit interval of type H consumers is a Hotelling linear city, wth firms $1, 2$ located at two ends of the unit interval. Consider any type H consumer who is located at point $x \in [0, 1]$. This consumer has distance x from firm 1 and distance $1 - x$ from firm 2. The unit cost of transportation is $t > 0$. Thus, this consumer has to pay transportation cost tx to purchase from firm 1 and cost $t(1 - x)$ to purchase from firm 2.

The product sold by either firm gives the same identical benefit $V > 0$ to every consumer. There is an exogenous probability $\lambda_i \in (0, 1)$ such that the product of firm i suffers from an adverse event (for example, a bug) that lowers the valuation of the product from from V to $V - \varepsilon$, where $0 < \varepsilon < V$. We assume that the of adverse events of products for two firms are independent to each other and moreover for simplicity we assume the probabilities are identical, that is, $\lambda_1 = \lambda_2 = \lambda$.

In the case of firm 1, who sells a proprietry product, the bug of the product can be only fixed centrally by firm 1 when the next version of the product is launched in a future period (a period which is currently outside of our static one-shot model). Thus, any consumer who purchases the product of firm 1 has the expected value $\lambda(V - \varepsilon) + (1 - \lambda)V = V - \lambda\varepsilon$ for the product. Denote $v \equiv V - \lambda\varepsilon$. Note that firm 1 is a "local monopolist" in the unit interval of type L buyers. As long as firm 1 sets a price not exceeding v (the expected benefit from the product), all type L buyers purchase from firm 1. We assume that no price discrimination is allowed, that is, firm 1 must set the same price for the two unit intervals of type L and type H buyers.

Now consider firm 2. In the case of firm 2, who sells an open source product, the type H users of the product can work on the bug and try to fix it. The probability that the bug of the product of firm 2 is fixed is an increasing function of the fraction of type H consumers who buy product 2. Specifically, we assume that if fraction $y \in [0, 1]$ of type H consumers buy product 2, then the bug is fixed with probability y. Thus, any consumer who purchases the product of firm 2 has the following expected value for the product:

$$\lambda[yV + (1 - y)(V - \varepsilon)] + (1 - \lambda)V = V - \lambda\varepsilon + y\lambda\varepsilon$$

Denoting $V - \lambda\varepsilon \equiv v$, the expected value for the product of firm 1 is v, while the expected value for the product of firm 2 when fraction y purchases the product of firm 2 is $v + y\lambda\varepsilon$. We assume v (and therefore V) is sufficiently large, which ensures that the market is "covered", that is, all comsumers purchase from either firm 1 or firm 2 and there is no consumer who does not purchase at all.

3.1 Demands received by firms $1, 2$

To find the demands received by firms $1, 2$ for different combinations of prices, first recall that the unit interval of type L consumers always buy from 1. When firm $1, 2$ announces prices $p_1, p_2 \geq 0$, a type H consumer located at x is indifferent between purchasing from 1 and 2 if and only if its utility net of the price and cost of transportation is the same from products $1, 2$. Taking $y = 1 - x$, for this indifferent cosumer we must have

$$v - p_1 - tx = v + (1 - x)\lambda\varepsilon - p_2 - t(1 - x)$$

Solving the equation above and denoting the solution $x^*(p_1, p_2)$, we have

$$x^*(p_1, p_2) = (p_2 - p_1 + t - \lambda\varepsilon)/(2t - \lambda\varepsilon) \tag{1}$$

For $i = 1, 2$, denote by $D_i(p_1, p_2)$ the demand received by firm i from type H consumers (the fraction of type H consumers who buy from firm i). Note that if p_1 is too large relative to p_2, no type H consumer buys from firm 1. On the other hand, if p_2 is too large relative to to p_2, no type H consumer buys from firm 1. If neither price is large relative to the other, we have $D_1(p_1, p_2) = x^*$ and $D_2(p_1, p_2) = 1 - x^*$. Specifically, if $\lambda\varepsilon < 2t$:

$$(D_1(p_1, p_2), D_2(p_1, p_2)) = \begin{cases} (0, 1) & \text{if } p_2 \leq p_1 + \lambda\varepsilon - t \\ (x^*(p_1, p_2), 1 - x^*(p_1, p_2)) & \text{if } p_1 + \lambda\varepsilon - t < p_2 < p_1 + t \\ (1, 0) & \text{if } p_2 \geq p_1 + t \end{cases} \tag{2}$$

If $\lambda\varepsilon > 2t$:

$$(D_1(p_1,p_2), D_2(p_1,p_2)) = \begin{cases} (0,1) & \text{if } p_2 \leq p_1 + t \\ (x^*(p_1,p_2), 1 - x^*(p_1,p_2)) & \text{if } p_1 + t < p_2 < p_1 + \lambda\varepsilon - t \\ (1,0) & \text{if } p_2 \geq p_1 + \lambda\varepsilon - t \end{cases} \quad (3)$$

3.2 Profit fuctions of firms $1, 2$

For simplicity we assume that each of the two firms $1, 2$ have zero marginal costs. Recall that as long as $p_1 \leq v$, all type L consumers buy from firm 1. Thus, firm 1 always has the demand 1 from these consumers. In addition, firm 1 receives demand $D_1(p_1, p_2)$ from type H consumers. For firm 2, the only demand is the fraction $D_2(p_1, p_2)$ that it receives from type H consumers. Consequently the profit functions of $1, 2$ are given by

$$\pi_1(p_1,p_2) = p_1[1 + D_1(p_1,p_2)] \text{ and } \pi_2(p_1,p_2) = p_2 D_2(p_1,p_2) \quad (4)$$

The strategic interaction between firms $1, 2$ is modeled as the duopoly price competition game Γ, where the objective of firms is to set prices $p_1, p_2 \geq 0$ to maximize the profits given in (4).

3.3 Main results

The next proposition characterizes Nash Equilibrium in pure strategies (NE) of Γ when the magnitude of the adverse event ("the bug") on the product (which is quantified by the term $\lambda\varepsilon$) is not severe. It is shown that in this case there is a unique NE at which firm 1 (the firm with proprietery product) serves both types L and H consumers, firm 1 sets a higher price than firm 2 and obtains a higher profit as well.

Proposition 1 *Suppose $\lambda\varepsilon < t$ and V (the benefit that the product gives) is of intermediate magnitude. Then the game Γ has a unique NE (p_1^*, p_2^*) where*

$$p_1^* = (7t - 4\lambda\varepsilon)/3 \text{ and } p_2^* = (5t - 2\lambda\varepsilon)/3 \quad (5)$$

The NE profits of the firms are

$$\pi_1^* = (7t - 4\lambda\varepsilon)^2/9(2t - \lambda\varepsilon) \text{ and } p_2^* = (5t - 2\lambda\varepsilon)^2/9(2t - \lambda\varepsilon) \quad (6)$$

Firm 1 sets a higher price and obtains a higher profit than firm 2.

Proof If $\lambda\varepsilon < t$, then $\lambda\varepsilon < 2t$ and the profit functions given in (2) is applicable. Taking $D_1(p_1,p_2) = x^*(p_1,p_2)$ and $D_2(p_1,p_2) = 1 - x^*(p_1,p_2)$ from (1) in (2) and (4), we have

$$\pi_1(p_1,p_2) = p_1[1 + (p_2 - p_1 + t - \lambda\varepsilon)/(2t - \lambda\varepsilon)] \text{ and}$$

$$\pi_2(p_1,p_2) = p_2[1 - (p_2 - p_1 + t - \lambda\varepsilon)/(2t - \lambda\varepsilon)]$$

Note that since $\lambda\varepsilon < 2t$, for $i = 1, 2$, the function $\pi_i(p_1, p_2)$ is strictly concave in p_i. The unique solution to the system of equations $\partial\pi_1(p_1, p_2)/\partial p_1 = 0$ and $\partial\pi_2(p_1, p_2)/\partial p_2 = 0$ is given by (5).

Because $\lambda\varepsilon < t$, we have $7t > 4\lambda\varepsilon$ and $5t > 2\lambda\varepsilon$, so p_1^*, p_2^* are both positive. Also observe that $p_2^* - (p_1^* + \lambda\varepsilon - t) = (t - \lambda\varepsilon)/3 > 0$ and $p_1^* + t - p_2^* = (5t - 2\lambda\varepsilon)/3 > 0$, proving that $p_1^* + \lambda\varepsilon - t < p_2^* < p_1^* + t$, so by (2), the fractions of demands from type H consumers are verified to be $x^*(p_1^*, p_2^*)$ and $1 - x^*(p_1^*, p_2^*)$.

It is immediate from (5) and (6) that firm 1 sets a higher price and obtains a higher profit compared to firm 2. Finally intermediate magnitudes of V ensure that $p_1^* \leq v$ and the NE profit of firm 1 exceeds v (the profit it would obtain by foregoing type H consumers and exclusively serving type L consumers). This completes the proof of the result. ■

In view of Proposition 1, a natural question is if there is an equilibrium outcome at which there is a separation in that firm 1 exclusively serves type L and firm 2 exclusively serves type H consumers.

Proposition 2 *Suppose the magnitude of the adverse event on the product is intermediate, specifically, $7t/4 < \lambda\varepsilon < 2t$. Then the game Γ has a unqiue NE. At the NE two firms are two local monopolies. In particular, firm 1 sets price $p_1 = v$ and exclusively serves type L consumers. Firm 2 sets price $p_2 = v + \lambda\varepsilon - t > v$ and all type H consumers buy from firm 2. Firm 1 obtains profit v and firm 2 obtains profit $v + \lambda\varepsilon - t > v$.*

Proof Note from (2) and (4) that for $p_2 \leq p_1 + \lambda\varepsilon - t$, we have $\pi_2(p_1, p_2) = p_2$, so $\pi_2(p_1, p_2)$ is increasing for $p_2 \leq p_1 + \lambda\varepsilon - t$. Also for $p_2 \geq p_1 + t$, we have $\pi_2(p_1, p_2) = 0$. Finally, for $p_2 \in [p_1 + \lambda\varepsilon - t, p_1 + t]$, we have

$$\pi_2(p_1, p_2) = p_2[1 - (p_2 - p_1 + t - \lambda\varepsilon)/(2t - \lambda\varepsilon)]$$

As $\lambda\varepsilon < 2t$, the function $\pi_2(p_1, p_2)$ is strictly concave in p_i and we note that

$$\partial\pi_2(p_1, p_2)/\partial p_2 \gtreqless 0 \Leftrightarrow p_2 \lesseqgtr (p_1 + t)/2$$

Note that $(p_1 + t)/2 - (p_1 + \lambda\varepsilon - t) = -p_1 + 3t/2 - \lambda\varepsilon \leq 3t/2 - \lambda\varepsilon < 0$ (because $3t/2 = 6t/4 < 7t/4 < \lambda\varepsilon$). Thus in this case $(p_1 + t)/2 < (p_1 + \lambda\varepsilon - t)$, so $\pi_2(p_1, p_2)$ is decreasing for $p_2 \in [p_1 + \lambda\varepsilon - t, p_1 + t]$. This shows that in this case the unique best response of firm 2 to p_1 is $p_2 = p_1 + \lambda\varepsilon - t$.

Observe from (2) that when $p_2 = p_1 + \lambda\varepsilon - t$, we have $D_1(p_1, p_2) = 0$ and $D_2(p_1, p_2) = 1$. Thus, firm 1 has zero demand and firm 2 has the whole market demand 1 of type H consumers. In this case firm 1 has zero profit from type H consumers and it is best for firm 1 to set price $p_1 = v$ to earn the maximu profit $p_1 \times 1 = v \times 1 = v$ from type L consumers. Taking $p_1 = v$, we have $p_2 = p_1 + \lambda\varepsilon - t = v + \lambda\varepsilon - t > v$ (because $t = 4t/4 < 7t/4 < \lambda\varepsilon$). This shows that in this case the unique NE of Γ has $p_1 = v$, $p_2 = v + \lambda\varepsilon - t$. The rest of the conclusions are immediate. ■

4 Concluding remarks

This paper studies a duopoly model to understand the strategic interaction between a proprietary firm and an open source firm. While our work is still in progress, our preliminary findings suggest that the magnitude of "the bug" (the damage to the quality of the product caused by the adverse event) plays an important role to determine which firms will serve what segments of the market and who will price higher. Our work is ongoing and we are still looking at the results for other parametric configurations of the model. Moreover, there is a need to endogenize several aspects such as fixing of the bug internally by the firm, the trade-off of internal expesnes of R&D versus essentially switching part of the R&D to end users (as done by open source firms). Still, our initial findings provide some useful insights to the interactions between proprietary and open source firms.

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