Designing information about co-workers' successes

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Abstract

Firms consist of teams of workers whose effort often impacts the team as a whole positively, and others' progress, either positively or negatively. Firms can also design how much information should workers have access to when each worker can generate private successes for themselves and the firm. Should workers know when other workers are successful? I study cases when each worker can generate at most one success for themselves and the firm, and where the workers' incentives only depend on the expected number of successful co-workers. This simplifies and reduces the dimensionality of the information-design problem of the firm. I obtain the optimal information policy for the firm for general cross-worker effects. As specific examples, I show that the optimal information policy is to delay full information till a probabilistic time in the case of complementary effort, and provide full information in case of substitutable effort.

1 Introduction

Firms often divide their workers and employees in teams which work on similar tasks or have to achieve related goals. For example, a group of lawyers in a firm work on a case together, a group software developers work on a piece of code together etc. In such cases, the firm, through a team supervisor, may assign separate tasks to each member, and decide the protocol the team would follow in terms of hierarchy, goals, deadlines etc. An important component of this is deciding how often the team members should meet, both individually or together. Another decision may be how often should they report to the manager or their supervisor. Such meetings facilitate information flow between workers or between them and their supervisor.

At the same time, however, each team member is individually ambitious as well. They know that often they are in competition with their co-workers in the same team for rewards in the firm - in the form of pay-raises or promotions. They know that the more their individual contributions to the firm, the better their prospects would be in terms of future rewards.

Workers' tasks and contribution in a team are often interrelated with each other. A group of software developers may work on different sections of a code - each section of which is important for the entire section to work. A group of lawyers working on a case may divide tasks according to each lawyer's specialised skill or competency. In such cases, workers' effort is complementary to each other. In other cases, effort is substitutable. For instance, a group of analysts working together on a report and without clear demarcation of responsibilities may suffer due to free-riders who benefit from others' work.

In these different cases, how should the firm or the team supervisor manage information flow between workers? Should the manager discuss the workers' progress with one another or should she wait until a certain time or goal?

I study a dynamic model of a team of workers, who individually put in effort that influences their own success, as well as their co-workers. They are individually only concerned with their own success and observe it. This success is also observed by the firm who also benefits from it. Once a worker is ensured of a success, they are privately ensured of a payoff - either due to a future promotion or reward that the firm had committed to earlier. Hence, the worker has no more incentive to put in effort. However, the firm may still be able to ensure whether news about this workers' success is spread in the rest of the team or not.

I study a general model, where for different effort levels of the other worker, the effect on one particular worker's success may be positive or negative. I show how to partially solve for the firm's value function - solving the value function for a subset of the workers' beliefs, specifically starting from a point in time when the workers know how many of their peers have achieved success.

Then, I study two cases - one with complementary effort and one with substitutable effort. I show that it is in the firm's interest to commit to withholding information in case of complementary effort and revealing all information fully in case of substitutable effort. In case of complementary effort, other workers' successes imply fewer workers putting in effort in the future. This reduces the incentive of such workers who have still not observed a success. In the case of substitutable effort, the effect is exactly the opposite. This explains the optimal information policy to be followed by the firm in each case. Using the technique of dynamic information design and the concavification result in Ely (2017) and Kamenica and Gentzkow (2011), we can precisely identify the optimal information policy in the each case.

2 Model

The model consists of one firm or principal and a team of $n \in \mathbb{N}$ workers or agents in the firm. Let N denote the set of the workers. Time is continuous and indexed by t. Workers are myopic, but the firm is not and its discount rate is ρ.

The Static Game:

Each agent i can choose effort $a_i \in [0, 1]$ at every t. I will assume that each agent's effort is privately observable only to him.

The worker incurs a flow cost $c(a_i, \bar{a}) > 0$, where $\bar{a} := \sum_{j \in N} a_j$. Assume that c is lower semicontinuous, piecewise C^1 and increasing in a_i. A worker also stochastically achieves a success over time, which is observable only to themselves and the firm, but not to their co-workers or other agents. Each worker earns a payoff of $v > 0$ from a success, and can achieve a maximum of one success. For each worker, this success arrives at an instantaneous rate denoted by $\lambda(a_i) : [0, 1] \longrightarrow [0, \infty)$. The firm earns a payoff normalized to one on every worker's success. λ is upper semicontinuous, piecewise C^1 and strictly increasing in a_i, implying that the firm prefers that the workers put in effort.

The model has the following immediate implications: as every worker earns a maximum of one success, this implies that a worker has no incentive to put in effort if she has already achieved a success. Also, as the cost is dependent on $\bar{a}$, each worker's incentive to exert effort depends on others' effort values.

We make the following assumption throughout to simplify the analysis.

Assumption 1. *For all $\bar{a}$, $c(0, \bar{a}) = 0$.*

The following assumption is crucial to solve the firm's problem.

Assumption 2. *$c(a_i, \bar{a})$, is linear in $\bar{a}$.*

Assumptions 1 and 2 imply that the cost function can be expressed as $c(a_i, \bar{a}) = c_1(a_i) + \bar{a}c_2(a_i)$, where c_1 and c_2 are functions such that $c_1(0) = c_2(0) = 0$.

In each period, in case the static game has multiple equilibria, I suppose that the workers, who have not achieved success by that time, play the *symmetric firm-preferred* equilibrium. That is, the as-yet unsuccessful workers play symmetric strategies, and if there are multiple equilibria with symmetric strategies, they play the most-preferred equilibrium for the firm.

Information, beliefs, and timeline of moves:

The workers have a belief over others' successes, which influences each worker's effort choice. As we restrict ourselves to symmetric equilibria, only the number of unsuccessful workers matters for a worker's decision. Hence, the number of workers who have achieved success will be the unobserved state of the world for the workers at any time. Only the incentives of the unsuccessful workers matter and we only need to track their beliefs.

Let J_t denote the set of workers and $j_t = |J_t|$ denote the number of workers at time t who have achieved a success until time t. If the workers could observe each others' successes, j could be treated as the state variable in $\{0, 1, 2, \ldots, n-1\}$, and the firm's discounted payoff represented as a function of j. For one worker and their beliefs, j_t is a Markov chain in continuous time - in fact, it belongs to a class of processes known as *'Pure Birth' processes*, where the Markov chain stochastically jumps in increments of 1 and $j_t = n - 1$ is an absorbing state. (We model the game till at least one worker is unsuccessful. If j_t is n, then the game has already ended.)

Let $\mathbf{p}_t$ denote the belief of a worker at time t. It can be represented as a vector $\left(p_{0t}, p_{1t}, \ldots, p_{(n-1)t}\right)$, where p_{st} is the belief of the worker that exactly s workers have succeeded by time t. For any natural number k, let $[k] \coloneqq \{0, 1, 2, \ldots, k\}$. Then, $p \in \Delta[n-1]$.

The firm can give coarse or fine information about others' successes to all the workers through a signal structure. Following Ely (2017) and Kamenica and Gentzkow (2011), I assume that the firm commits to a signal structure, and the communication involves 'Bayesian Persuasion'. As is well known in the literature, we don't need to concern ourselves with the distribution over signals the firm chooses. We can restrict ourselves to allowing the firm to choose the distribution over posterior beliefs of the workers. We will assume that the firm can only provide publicly observable signals to all workers, and the workers cannot share their information among themselves.

Here, I state the timing of events at every t. Consider the discrete-time analog of this setup, where time periods are of length dt. At any time t, the workers start with the common belief $\mathbf{p}_t$. The firm observes the state and sends a signal to the workers. Since the signals are public and the workers' effort strategies are symmetric, all workers will have the same beliefs on-path. The workers will update to an interim belief $\mathbf{q}_t$, take an action, observe their possible success and generate a possible payoff. Then the workers and the firm move to the next period $t + dt$. We will study the continuous-time version of the above game, as $dt \to 0$.

3 Analysis

Let $\hat{a}^*(\mathbf{p})$ denote an unsuccessful worker equilibrium effort choice at belief p, and $\bar{a}^*(\mathbf{p})$ denote $\sum_{m \in N} a^*_m(\mathbf{p})$. The first Lemma shows that assumption 2 implies that each worker's choice of effort at belief $\mathbf{p}$ only depends on $\mathbf{p}$ through the value of $\mathbb{E}_{\mathbf{p}}(j)$, i.e. the expected number of other workers who have achieved a success.

Lemma 1. *Suppose* $\mathbf{p}$ *and* $\mathbf{q}$ *are distinct beliefs in* $\Delta[n-1]$. *Then,*

$$\mathbb{E}_{\mathbf{p}}(j) = \mathbb{E}_{\mathbf{q}}(j) \quad \Longrightarrow \quad a^*(\mathbf{p}) = a^*(\mathbf{q}) \tag{1}$$

Proof. Our restriction to symmetric equilibria implies that $\sum_{m \in N} \hat{a}^*_m(\mathbf{p}) = (n - j_t)\hat{a}^*(\mathbf{p})$, and

$\mathbb{E}_{\mathbf{p}}(\bar{a}^*(\mathbf{p})) = (n - \mathbb{E}_{\mathbf{p}}(j))a^*(\mathbf{p})$. Thus, $\hat{a}^*(\mathbf{p})$ solves

$$\hat{a}^*(\mathbf{p}) \in \operatorname{argmax}_{a\in[0,1]} \left(\lambda(a)v - c_1(a) - (n - \mathbb{E}_{\mathbf{p}}(j))\hat{a}^*(\mathbf{p})c_2(a)\right) \tag{2}$$

If there are multiple optima for the objective function on the RHS in the above expression, then we choose $\hat{a}^*(\mathbf{p})$ from this set such that it maximizes the firm's profit, which is $(n - \mathbb{E}_{\mathbf{p}}(j))\lambda(\hat{a}^*(\mathbf{p}))$. There will be a unique such $\hat{a}^*(\mathbf{p})$, because λ is strictly increasing. □

Hence, we define $a^* : [0, n-1] \longrightarrow [0,1]$, such that $a^*(\mathbb{E}_{\mathbf{p}}(j)) \coloneqq \hat{a}^*(\mathbf{p})$. I will define $\hat{j} \coloneqq \mathbb{E}_{\mathbf{p}}(j)$, suppressing the dependence on $\mathbf{p}$, when obvious. Our model implies that $a^*(\hat{j})$ will be piecewise C^1 on $[0, n-1]$.

For instance, if we additionally assume that the functions λ, c_1 and c_2 are differentiable everywhere on the domain, and that the objective function on the RHS has a unique interior optimum, then worker i's best response is a_i such that

$$\lambda'(a_i)v - c_1'(a_i) + \mathbb{E}_{\mathbf{p}}\left(\bar{a}\right)c_2'(a_i) = 0$$

In our equilibrium of interest, this implies that $a^*(\mathbf{p})$ solves

$$\lambda'(a^*(\hat{j}))v - c_1'(a^*(\hat{j})) - (n - \hat{j})a^*(\hat{j})c_2'(a^*(\hat{j})) = 0$$

The profile of actions determines the rate at which they succeed and how their beliefs are updated as they enter the next period. If $\mathbf{p}_{t+dt}$ denotes the prior belief of the workers next period, then the workers update their beliefs according to a linear function denoted by f, i.e. $\mathbf{p}_{t+dt} = f(\mathbf{q}_t)$.[1]

Let $\mathbf{A}(\mathbf{p})$ denote the infinitesimal transition matrix for the Markov chain j_t. The value of j_t is the realized number of successful workers at time t. For ease of notation, define $\lambda^*(j, \hat{j}) \coloneqq (n - j - 1)\lambda(a^*(\hat{j}))$. For a given worker i, this expression denotes the rate at which at least one worker, apart from themselves, achieves success, when the actual number of successful agents is j. Thus, when the state is j and the agents' belief is $\mathbf{p}$, the state changes to $j+1$ at rate

[1]See Lemma 1 of Ely (2017).

$\lambda^*(j, \mathbb{E}_{\mathbf{p}}(j))$ or $\lambda^*(j, \hat{j})$, and when the state is $j = n-1$, it means that only one worker is working and the game ends when this worker achieves a success. Then,

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} -\lambda^*(0,\hat{j}) & 0 & 0 & 0 & \dots & 0 & 0 \\ \lambda^*(0,\hat{j}) & -\lambda^*(1,\hat{j}) & 0 & 0 & \dots & 0 & 0 \\ 0 & \lambda^*(1,\hat{j}) & -\lambda^*(2,\hat{j}) & 0 & \dots & 0 & 0 \\ 0 & 0 & \lambda^*(2,\hat{j}) & -\lambda^*(3,\hat{j}) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \lambda^*(n-2,\hat{j}) & 0 \end{bmatrix}$$

where an element of the matrix $A_{i\hat{i}}$, $i \neq \hat{i}$, is the rate at which the process exits state i and enters state $\hat{i}$. The negative of the element along the diagonal, $-A_{ii}$, is the rate at which the process exits state i.

We see that the transition matrix can be written as

$$\mathbf{A}(\mathbf{p}) = \lambda(a^*(\hat{j}))\hat{\mathbf{A}},$$

where

$$\hat{\mathbf{A}} := \begin{bmatrix} -(n-1) & 0 & 0 & 0 & \dots & 0 & 0 \\ n-1 & -(n-2) & 0 & 0 & \dots & 0 & 0 \\ 0 & n-2 & -(n-3) & 0 & \dots & 0 & 0 \\ 0 & 0 & n-3 & -(n-4) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

Thus, the process by which beliefs are updated as the game moves from time t to time $t + dt$ is specified as

$$\mathbf{p}_{t+dt} = f(\mathbf{q}_t) := \mathbf{q}_t + \mathbf{A}(\mathbf{q}_t) \cdot \mathbf{q}_t dt \tag{3}$$

The Firm's problem

When the workers' belief is $\mathbf{p}$ and the actual state is j, the per-period payoff of the firm is $(n-j)\left(\lambda\left(a^*\left(\hat{j}\right)\right)\right)dt$. Thus, as in Ely (2017), the per-period payoff of the firm depends both on the state j and the workers' belief $\mathbf{p}$. However, appealing to the *'Obfuscation Principle'* - Lemma 2 - in Ely (2017), the firm's problem can be stated only in terms of the worker's beliefs, disregarding the realizations of the state.

Let $[n-1] \coloneqq \{0, 1, 2, \ldots, n-1\}$. For a given value of $\mathbf{p}$, let

$$u(\mathbf{p}) \coloneqq \mathbb{E}_{\mathbf{p}}(n-j)\lambda\left(\mathbb{E}_{\mathbf{p}}\left(\hat{a}^*(\mathbf{p})\right)\right)$$

The firm's problem is to solve

$$\rho V(\mathbf{p}) = \max_{\nu \in \Delta(\Delta[n-1]), E_\nu(\mathbf{q})=\mathbf{p}} E_\nu\left[u(\mathbf{q}) + V'(\mathbf{q}) \cdot \frac{d\mathbf{q}}{dt}\right]$$

From (3), it can be seen that $\frac{d\mathbf{q}}{dt} = \mathbf{A}(\mathbf{q}) \cdot \mathbf{q}$. As shown in Ely (2017), the above problem is equivalent to finding the function V which solves the following,

$$\rho V(\mathbf{p}) = \text{cav}\ \big(u(\mathbf{p}) + V'(\mathbf{p}) \cdot (\mathbf{A}(\mathbf{p}) \cdot \mathbf{p})\big) \tag{4}$$

For any function f, $\text{cav}(f)$ denotes the concave envelope of f. As the value function has to be concave, it must be continuous.

Simplifying the problem

The set of beliefs is $\Delta\{0, 1, 2, \ldots, n-1\}$, and V becomes hard to characterize for a large number of states. However, I utilize the model setup and assumptions to solve for the firm's optimal policy and value function for certain points in the set of beliefs. Specifically, rather than solving for the value function over the entire set of beliefs, we will solve for it over the extreme points on the set, when the starting state is known to the workers.

First, the next lemma shows that if no information is provided to the workers by the firm, the evolution of $\hat{j}$ as a function of time is independent of the belief, and only depends on its current value.

Lemma 2. *For distinct* $\mathbf{p}$ *and* $\mathbf{q} \in \Delta[n-1]$, $\mathbb{E}_{\mathbf{p}(t)}(j_t) = \mathbb{E}_{\mathbf{q}(t)}(j_t) \implies \frac{d}{dt}\big(\mathbb{E}_{\mathbf{p}(t)}(j_t)\big) = \frac{d}{dt}\big(\mathbb{E}_{\mathbf{q}(t)}(j_t)\big)$

Proof. Using (3), $\frac{d}{dt}\left(\mathbb{E}_{\mathbf{p}(t)}(j_t)\right)$ can be written as

$$\begin{aligned}
\begin{bmatrix}0 & 1 & 2 & \dots & n-1\end{bmatrix} \cdot \frac{d\mathbf{p}}{dt} &= \begin{bmatrix}0 & 1 & 2 & \dots & n-1\end{bmatrix} \cdot \mathbf{A}(\mathbf{p}) \cdot \mathbf{p} \\
&= \begin{bmatrix}0 & 1 & 2 & \dots & n-1\end{bmatrix} \cdot \begin{bmatrix} -\lambda^*(0,\hat{\jmath})p_0 \\ \lambda^*(0,\hat{\jmath})p_0 - \lambda^*(1,\hat{\jmath})p_1 \\ \lambda^*(1,\hat{\jmath})p_1 - \lambda^*(2,\hat{\jmath})p_2 \\ \vdots \\ \lambda^*(n-2,\hat{\jmath})p_{n-2} \end{bmatrix} \\
&= \lambda^*(0,\hat{\jmath})p_0 + \lambda^*(1,\hat{\jmath})p_1 + \lambda^*(2,\hat{\jmath})p_2 + \dots + \lambda^*(n-2,\hat{\jmath})p_{n-2} \\
&= \mathbb{E}_{\mathbf{p}}(\lambda^*(j,\hat{\jmath}))
\end{aligned}$$

The last line uses the fact that $\lambda^*(n-1,\hat{\jmath})\mathbf{p}_{n-1} = 0$. The above implies that

$$\begin{aligned}
\frac{d}{dt}\left(\mathbb{E}_{\mathbf{p}(t)}(j_t)\right) &= \mathbb{E}_{\mathbf{p}}(\lambda^*(j,\hat{\jmath})) = \mathbb{E}_{\mathbf{p}}\Big((n-j-1)\lambda(a^*(\hat{\jmath}))\Big) \\
&= (n-\hat{\jmath}-1)\lambda(a^*(\hat{\jmath}))\,. \qquad (5)
\end{aligned}$$

Using Lemma 1, we can prove that if $\mathbf{p}$ and $\mathbf{q}$ satisfy the conditions of this Lemma, then $\hat{a}^*(\mathbf{p}) = \hat{a}^*(\mathbf{q}) = a^*(\hat{\jmath})$. The result follows. □

Let $\frac{d}{dt}\mathbb{E}(j)$ be given by the function $\psi(\hat{\jmath}) := (n-\hat{\jmath}-1)\lambda(a^*(\hat{\jmath}))$.

One important implication of Lemma 2 is that it becomes easy to characterize the firm's value for any belief, under the policy that starting at that belief, the firm does not provide any information to the workers. Consider a function $f : [0, n-1] \longrightarrow \mathbb{R}$, that solves the differential equation,

$$\rho f(m) = (n-m)\lambda(a^*(m)) + f'(m)\psi(m)\,, \qquad (6)$$

where $f(n-1)$ is the value when only one worker is working and $(n-1)$ workers have succeeded, i.e. $f(n-1) = \lambda(a^*(n-1))/\rho$.[2]

Corollary 0.1. *If starting at belief* $\mathbf{p} \in \Delta[n-1]$, *the firm provides no information, then*

[2]Our assumptions imply that $f'(m)$ may not be defined for all m. Hence, we consider the solutions of the differential equation in the viscosity sense.

$V(\mathbf{p}) = f(\mathbb{E}_{\mathbf{p}}(j))$.

Proof. $V(\mathbf{p})$ satisfies the HJB equation

$$\rho V(\mathbf{p}) = u(\mathbf{p}) + V'(\mathbf{p}) \cdot (\mathbf{A}(\mathbf{p}) \cdot \mathbf{p})$$

with the boundary condition that $V(\hat{\mathbf{p}}) = \lambda(a^*(n-1))/\rho$, where $\hat{\mathbf{p}}$ is the belief such that $\text{Prob}(j = n-1) = 1$. But, as shown in the proof of Lemma 2, $\mathbf{A}(\mathbf{p}) \cdot \mathbf{p}$ is equal for those $\mathbf{p}$ that have equal values of $\mathbb{E}_{\mathbf{p}}(j)$. Similarly, $u(\mathbf{p})$ is also equal for such $\mathbf{p}$. Thus, the above HJB equation implies that $V(\mathbf{p})$ is also equal for such $\mathbf{p}$. The result follows. □

Under this policy, to calculate the firm's value function, one only needs to track the value of $\hat{j}$ and not the vector of beliefs.

3.1 Partial Characterization of the Value Function

As stated earlier, I obtain a partial characterization of the value function over the extreme points on the set $\Delta[n-1]$, i.e. when for some $i \in [n-1]$, Prob(number of successful workers $= i$)=1. One special case of this is, of course, time 0 when $\text{Prob}(j = 0)$=1.

For $i \in [n-1]$, let $Con(i)$ denote a subset of $\Delta[n-1]$ such that $\mathbf{p} \in Con(i)$ iff for all $\hat{i} < i$, $\text{Prob}_{\mathbf{p}}(j = \hat{i}) = 0$. When a belief is at an extreme point of $\Delta[n-1]$, say where Prob(number of successful workers $= i$)=1 for some $i \in [n-1]$, only beliefs in $Con(i)$ can be induced by the firm in the future. This is because if, at some point in time, the workers believe with probability one that i workers have succeeded, they cannot be made to believe with some positive probability that less than i workers have succeeded, at any point in the future.

For the main proposition, we need to define a functional transformation and its fixed point. Let $i \in [n-1]$. Consider the following set of functions:

$$G[i, n-1] = \{g \in C[i, n-1] \,|\, g \text{ is piecewise } C^1 \text{ and satisfies condition C}\}$$

where condition C is that: $\exists \hat{m} \in [i, n-1]$, such that g is convex over $[i, \hat{m}]$, concave over $[\hat{m}, n-1]$, and $g(n-1) = \lambda(a^*(n-1))/\rho$ ($C[i, n-1]$ is the set of continuous functions from

$[i, n-1]$ onto $\mathbb{R}$). Standard arguments show that this set of functions is closed under the sup-norm.

Additionally, we need to define two functions to represent the how the value evolves under full information or no information. First, I define a candidate value function v for some integer values, supposing that the firm provides full information when j equals such values. Suppose that the firm keeps the workers informed while $j \in (i, \hat{\imath})$, and suppose the terminal value when Prob($j = \hat{\imath}$)=1 is $\bar{v}$. Define $v(m \mid i, \hat{\imath}, \bar{v})$ to be such that for $m \in \{i+1, \ldots, \hat{\imath}-1, \hat{\imath}\}$, it solves the difference equation,

$$\rho v(m) = (n-m)\lambda(a^*(m))(v(m-1) - v(m)) \tag{7}$$

with $v(\hat{\imath}) = \bar{v}$. For any non-integer real number $m \in [i, \hat{\imath}]$, let $v(m)$ be the linear interpolation of the value of v at the closest integers.

Similarly, define a function to keep track of the value when no information is provided. For $g \in G[i, n-1]$ and $\tilde{m}, m_1 \in [i, n-1]$, let $\underline{v}_g(\tilde{m}, m_1) := f(\tilde{m})$, where f solves the differential equation in (6), with the boundary condition that $f(m_1) = g(m_1)$. An intuitive explanation for this definition is as follows: if no information is provided while $\hat{j} \in (\tilde{m}, m_1)$, and if the value for $\hat{j} = m_1$ equals $g(m_1)$, then the value at $\tilde{m}$ is $\underline{v}_g(\tilde{m}, m_1)$.

For any number m, let $\lfloor m \rfloor$ denote the greatest integer less than m. Define the following functional transformation for any $i \in [n-1]$:

Definition. *Define $T : G[i, n-1] \longrightarrow G[i, n-1]$, according to the following algorithm: First, for g in the domain, define $\hat{m} \in [i, n-1]$, such that:*

- *if for some m_1, for all $\tilde{m} \in [i, m_1]$, $\underline{v}_g(\tilde{m}, m_1)$ is concave in $\tilde{m}$, $\hat{m} := i$,*
- *if for all $\tilde{m} \in [i, n-1]$, $\underline{v}_g(\tilde{m}, n-1)$ is convex in $\tilde{m}$, $\hat{m} := n-1$,*
- *otherwise, in all other cases, $\hat{m} := \max\{m_1 \in [i, n-1] \mid \underline{v}_g(\tilde{m}, m_1)$ is convex in $\tilde{m}$ over $[i, m_1]\}$.*

Then,

$$Tg(m) = \begin{cases} v(m \mid i, \lfloor \hat{m} \rfloor, g(\lfloor \hat{m} \rfloor)) & i \leq m \leq \hat{m} \\ cav((n-m)\lambda(a^*(m)) + g'(m)\psi(m)) & \hat{m} \leq m \leq n-1 \end{cases} \tag{8}$$

Standard arguments can show that T is a contraction on the set $G[i, n-1]$, and hence, by the Banach Fixed Point Theorem, a fixed point exists. Denote this fixed point by $g_i^* \in G[i, n-1]$. Moreover, g_i^* is also guaranteed to be locally Lipschitz continuous over $[i, n-1]$.

Now we are ready to state the main proposition.

Proposition 1. *Suppose $i \in [n-1]$ and an extreme point $\hat{\mathbf{p}}$ of $\Delta[n-1]$, such that $Prob_{\hat{\mathbf{p}}}$(number of successful workers = i)=1. Then, $V(\hat{\mathbf{p}}) = g_i^*(i)$. Additionally, the optimal information policy of the firm is such that for any $\mathbf{p} \in Con(i)$ which is induced on-path (with positive probability) after $\hat{\mathbf{p}}$, $V(\mathbf{p}) = g_i^*(\mathbb{E}_{\mathbf{p}}(j))$.*

We can understand the value function and the associated information policy as follows. Suppose, we begin with the workers' belief with probability equal to one that $j = i$. Then the value at this belief is equal to $g_i^*(i)$. We know that there may exist $\hat{m} \in [i, n-1]$, such that at this value g^* switches from being convex to concave. Then, the associated information policy is for the firm to fully inform the workers about the change of j till $j = \lfloor \hat{m} \rfloor$.

After this point in time, the process enters the concave part of g_i^* and the firm does not fully inform the workers. The worker's beliefs are tracked by the process $\hat{j}$ and the firm's value at a particular $\hat{j}$ is given by $g_i^*(\hat{j})$. If for some m, $g_i^*(m)$ is strictly concave, the firm does not provide any information to the workers when $\hat{j} = m$. If g_i^* is linear over $[m_1, m_2]$, the firm provides information such that when $\hat{j}$ equals m_1, the firm provides just enough information to keep $\hat{j}$ fixed at m_1, or have it jump to m_2.

I will provide a sketch of the proof, which is similar to a proof by backward induction. I will show that the information policy associated with g_i^* and as described above is optimal by showing that the Value function as given in the proposition satisfies (4). Consider first $i = n-2$ - and the associated g_i^*. This case is a sub-case of Ely (2017), one can verify that the value function satisfies (4).

Now, when we consider $i < n-2$, we first suppose that for g_i^*, $\hat{m} = n-1$, i.e. $\underline{v}_{g_i^*}$ is convex throughout. This implies that that in the space of the beliefs $Con(i)$, the solution of the HJB equation (6) is also convex. The concave envelope of this function will be a hyperplane, which implies that at each belief in $Con(i)$, the value function is just the average of the value at the extreme points. This corresponds to full information.

If $\hat{m} < n-1$, then consider the sub-domains of beliefs where $\mathbb{E}_{\mathbf{p}}(\hat{j}) \geq \hat{m}$ and $\mathbb{E}_{\mathbf{p}}(\hat{j}) < \hat{m}$. The value function as defined can be shown to be concave - first over these subdomains separately and then jointly - and is also the concave envelope satisfying (4).

3.2 Benchmark model: Binary effort

Suppose that workers could choose effort $a_i \in \{0, 1\}$. If the workers could freely observe other workers' efforts, each worker will only choose effort 1 iff

$$\lambda(1) - \lambda(0) \geq c\left(1, \bar{a}\right)/v$$

Let $\bar{\lambda} := \lambda(1) - \lambda(0)$. If the workers have belief $\mathbf{p}$, the worker's best response satisfies

$$a_i(\mathbf{p}) = \begin{cases} 1 & \text{if } \bar{\lambda} \geq \mathbb{E}_{\mathbf{p}}\left(c(1, \bar{a})\right)/v \\ 0 & \text{otherwise} \end{cases}$$

for all i.

As stated earlier, assumption 2 implies that the cost function can be written as $c_1(a_i) + c_2(a_i)\bar{a}$. Also, we consider only symmetric strategies. Together, these imply that

$$a^*(\mathbf{p}) = \begin{cases} 1 & \text{if } \bar{\lambda} \geq \left(c_1(1) + c_2(1)\left(n - \mathbb{E}_{\mathbf{p}}(j) - 1\right)a^*(\mathbf{p})\right)/v \\ 0 & \text{otherwise} \end{cases}$$

for all i.

Our restriction to the firm-preferred symmetric equilibrium implies that $a^*(\mathbf{p}) = 1$ if and only if $\bar{\lambda}v - c_1(1) \geq c_2(1)(n - \mathbb{E}_{\mathbf{p}}(j) - 1))$. If the workers' efforts are complements, then $c_2(1)$ is negative. In this case, $a^*(\mathbf{p}) = 1$ if and only if

$$\frac{1}{c_2(1)}\left(\bar{\lambda} - c_1(1)\right) \leq n - \mathbb{E}_{\mathbf{p}}(j) - 1 \qquad \Longleftrightarrow$$

$$E_{\mathbf{p}}(j) \leq n - 1 - \frac{1}{c_2(1)}\left(\bar{\lambda} - c_1(1)\right)$$

If the efforts are substitutes, then $c_2(1)$ is positive, and $a^*(\mathbf{p}) = 1$ with probability 1 if

$$E_{\mathbf{p}}(j) \geq n - 1 - \frac{1}{c_2(1)} \left(\bar{\lambda} - c_1(1)\right)$$

4 Conclusion

We have shown that when employees work in a team and their work impacts their peers', then the firm may be able to incentivise them to work by optimally designing the reporting structure of the team members. In case of substitutable effort, it is optimal if the team members can declare their success in public meetings so that all team members are fully aware of others' progress. However, if the workers' effort is complementary to each other, then it may be better for the team manager to discuss the progress of individual team members separately and not in a public meeting. Our results can be extended to settings when the arrival rate of success may be complementary over some values of the domain, but substitutable over others.

It is important to note that we have made several assumptions in our setting which may alter the result significantly. Firstly, we have assumed that the effort of the workers is unobservable to the firm and that agents are myopic. Further study is needed to gauge if one or both of these assumptions can be relaxed. Even if effort is observable, perhaps it may not be possible to design information revelation policies conditional on workers' effort, or reveal their effort to other workers.

Secondly, with more complicated success arrival functions, the firm may be able to do even better with privately observable signals, rather than public signals as we have assumed.

References

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