Information and Misinformation in Networks

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Very Preliminary Draft

Abstract

We study a non-cooperative game in which agents are connected in a (exogenously given) social network and there exists uncertainty captured through states of the world. Each agent receives a private (partial) signal about the true state of the world. They then pass messages privately (truthfully or misreporting) to their neighbours. Based on the information they receive, they form beliefs about the state of the world and take actions accordingly. In equilibrium an agent over-reports, under-reports or truthfully reports to a neighbour depending on the expected action of the concerned neighbour and how central that neighbour is in the network. We also study how an agent's own bias changes to a change in her neighbour's bias. Finally, we explore the conditions under which it is a Nash Equilibrium for all agents to have no bias.

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1 Introduction

In many real-life situations, people exchange information with others in their social or professional circles. Consider a simple workplace example. Suppose there are two employees, A and B, and two managers, C and D. Employee A reports only to manager C, while employee B reports only to manager D. The two managers do not communicate directly. However, employees A and B need to work together to complete a task successfully. When A shares information with B, he may have an incentive to misreport, since B could compete with him for credit. In contrast, A has no incentive to misreport to her own manager C, because the manager directly evaluates her performance. A similar reasoning applies to employee B. This example shows that in a network of communication, individuals may have incentives to misreport to some of their contacts but not to others. To study such situations more systematically, we consider a setting in which several individuals strategically share information within an exogenously given social network. A link in the network represents a communication channel between two people. At the beginning, each individual receives a private and partial signal about the true state of the world. The true state of the world is determined by the collection of all signals across players. Each person also has a personal bias, which shifts her most preferred outcome away from the true state. In other words, the outcome each person would like to see, their (expected) bliss point, is a combination of what they believe the true state to be and their own bias. Each individual then chooses an action. Several real life examples can be thought of in this scenario. Suppose in a company, each employee gets a private market report and, combined with their personal bias, forms a preferred sales target. They anticipate others' reports and actions, then decide whether to share information truthfully or strategically. In the end, they choose a sales effort close to their own target but also aligned with teammates, since large deviations hurt coordination and performance. Two forces guide this decision. On the one hand, people want their action to be close to their own (expected) bliss point. On the other hand, they also care about aligning their action with the actions of those with whom they communicate directly in the network.

There are several works which study the communication in networks. Notably Hagenbach and Koessler (2009) and Galeotti *et al.*, (2013) highlight the communication networks in which players truthfully communicate with their neighbours. They considered forming the network endogenously and the truthful communication takes place depending on the difference between weighted bias of the concerned players. The weight depends mainly on how many neighbours the concerned player (with whom the communication is taking place) has. There are works in cheap-talk literature which take the bias of the player (specifically bias of the sender in the two following cases) as exogenously given. For example Crawford and Sobel (1982), Chen *et al.* (2008) (though they did not take bias directly, the assumption that for any given type of the sender, sender's optimal action is strictly greater than receiver's optimal action gives a flavour of upward biased sender) etc.

In many standard models, a player's utility is assumed to depend primarily on how close their chosen action is to their individual bliss point(e.g., Tiemann (2019)). While this captures idiosyncratic preferences, it abstracts from two important features of strategic interaction observed in practice. First, individuals often care about the relative positioning of their actions vis-à-vis their peers, since large deviations may generate social costs or coordination failures. Second, utility may also be shaped by the way information is transmitted, as messages influence both beliefs and perceptions within the network. Immorlica *et al.*, (2017) highlight a salient asymmetry in social interactions: observing a neighbour of higher status can impose a negative externality, yet observing a neighbour of lower status does not generate a corresponding positive externality. This pattern reflects status concerns and the normalisation of certain consumption or behavioural choices. For example, in many societies owning a car no longer adds social prestige because it has become the norm. However, the absence of a car generates a negative payoff, since it signals an inability to conform to a basic social norm. In economic terms, conformity to a normalised benchmark yields no marginal benefit, and exceeding the benchmark similarly generates no additional payoff, since the behaviour is already fully normalised. By contrast, falling below the benchmark entails a reputational or social cost. Thus, the externality is one-sided: actions above the norm do not provide positive externalities, while actions below the norm impose negative ones. An analogous mechanism operates in our framework. An agent overstating their type relative to the truth may not experience a significant utility gain. By contrast, being compelled to understate one's type imposes a utility loss, as it conceals true quality or information and alters strategic interaction. These considerations—alignment with one's bliss point, coordination with peers, and the asymmetry in the transmission of type—jointly motivate the formulation of our utility function.

The timeline of the game unfolds as follows. First, each player receives a private signal about the underlying state of the world. Based on this, the player forms an expectation of the state, taking into account the messages he anticipates receiving from her neighbours. Next, he conjectures what other players will infer about the state of the world, given the messages they are likely to receive. From these higher-order beliefs, the player forms expectations about the actions of others. Only then does he decide what messages to send to her neighbours. We focus on a setting with pure private communication, and we assume that the social network is common knowledge. In Crawford and Sobel (1982), the analysis is restricted to two players, and any reported message is commonly observed by both. Ambrus *et al.*, (2013) study hierarchical cheap talk with multiple players, but the final decision is taken by a single agent. Hagenbach and Koessler (2009) and Galeotti *et al.*, (2013) extend the analysis to networked environments where every player acts as both sender and receiver. In their framework, under private communication, players do not observe the content of messages sent between others, but they can rely on the assumption that reporting is truthful. By contrast, in our setting all players act simultaneously as senders and receivers, but unlike the above frameworks, they neither observe what a player reports to others nor can they be sure that any communication is truthful. This introduces strategic uncertainty not only about the content of messages but also about their credibility. We will use first and second order beliefs to tackle the situation.

Results of this paper are as follows. In many economic analyses of this type, an individual's optimal action is typically determined by how close it is to her own bliss point. However, we show that when forming the optimal (expected) action, an individual does not rely solely on her own bliss point. Instead, he considers a weighted average of the bliss points of all players. Assigning equal weights to everyone is not reasonable, and this is where the role of the network becomes important. We demonstrate that these weights are shaped by players' positions in the network, i.e., by their centralities. There are several well-established measures of centrality, such as Bonacich centrality, eigenvector centrality, and degree centrality (see Sadler (2021), Dasaratha (2020)). In our analysis, we adopt a modified version of Bonacich centrality. Specifically, if player j exerts more influence on player i (in the Bonacich centrality sense) relative to player k, then player i attaches a higher weight to j's bliss point compared to that of k. Moreover, we find that the more central a player is, the less weight he assigns to her own bliss point when determining her expected optimal action. Thus, the optimal expected action can be interpreted as a form of bliss-point weighted centrality. The role of Bonacich centrality in shaping actions has also been established in Ballester *et al.* (2006), while Ghiglino *et al.* (2010) introduced the idea of endowment-weighted Bonacich centrality.

Our second main result concerns the conditions under which a player reports truthfully, under-reports, or over-reports her private information. This marks a key difference between our framework and that of Hagenbach and Koessler (2009) and Galeotti *et al.* (2013), who restrict attention exclusively to truthful communication. In contrast, we allow for strategic misreporting and show that the cost of deception depends critically on the position of the receiver in the network: specifically, it is more costly to mislead a more central player. Furthermore, we characterize the optimal misreporting strategy for a given type. In particular, we demonstrate (in a specific case) the existence of a one-to-one mapping between the signal received and the message reported. Finally, we establish that the reported message is monotonic in the actual signal, ensuring that higher signals lead to systematically higher reported messages.

The later part of the paper focuses on the role of bias. Once we endogenies the bias parameter, we can analyse how a player's bias adjusts in response to changes in the bias of another player. This adjustment depends on the extent to which the other player influences the neighbours of the concerned player, i.e., her position and importance within the network. We also identify the conditions under which being unbiased constitutes a Nash equilibrium, highlighting when neutrality is a stable outcome of strategic interaction.

2 The Model

Let $N = \{1, 2, \ldots, n\}$ denote the set of players in our game. We assume that these players are connected in a social network represented by a graph g, where g_{ij} denotes the link between players i and j in the network. For any $i, j \in N$, we write $g_{ij} = 1$ if i and j are connected and $g_{ij} = 0$ otherwise. One can also choose $0 \leq g_{ij} \leq 1$ where g_{ij} is interpreted to be the strength of the link between the players i and j. By convention, $g_{ii} = 0, \forall i \in N$. The set of neighbours of player i is denoted by N_i and defined as $N_i = \{j \in N : g_{ij} = 1\}$ and let $n_i = |N_i|$ denote the cardinality of the set N_i and can be interpreted as the total number of neighbours of player i. Let $G = [g_{ij}]_{i,j=1}^{n}$ be the adjacency matrix which represents the network. Here we assume an undirected network, meaning $g_{ij} = g_{ji}, \ \forall i, j \in N$.

Each player chooses an action $a_i \in A_i = \mathbb{R}$ in our model. Let $A = \prod_{i=1}^{n} A_i$ denote the set of all possible actions undertaken by the players and let $a = (a_1, \cdots, a_n) \in A$ denote one particular action vector. Each agent's payoff depends on the action profile, a state of nature θ, and some messages exchanged with her neighbors. Before the game starts, nobody knows the state of nature, but each agent $i \in N$, receives a private signal $\overline{s}_i \in S_i$ about θ, where S_i is a random variable on $[0, 1]$ with distribution function $F_i(\cdot)$ and density function $f_i(\cdot)$. Let $S = \prod_{i=1}^{n} S_i$ denote the space of all possible private signals. The fact that each player has received a private signal and that the signal lies within $[0, 1]$ is common knowledge. Also, $F_i(\cdot)$ is common knowledge $\forall i \in N$ as well. Denote the profile of signals as $\mathbf{s} = (s_i)_{i \in N}$ and let the state of the world be given by $\theta(\mathbf{s}) = \sum_{i \in N} s_i$. This formulation of the state of the world follows Hagenbach and Koessler (2009). The reason for adopting this specific functional form is to abstract from unnecessary complications and focus on the key strategic features of the communication game.

After each player has received her private signal, a communication stage is introduced in which players can send costless and private messages to each other. Each player i after receiving her private signal, sends a message $m_{ij} \in M_{ij}$ to neighbour $j \in N_i$, where M_{ij} denotes the set of messages that can be send by player i to player j and like private signals can be any value between $[0, 1]$. We write $\mathbf{m}_i = (m_{ij})_{j \in Ni}$ to denote a vector of messages sent by player i to her neighbours and $\mathbf{m}i = (m_{ji})_{j \in Ni}$ to denote the vector of messages sent to i by his neighbours. This simply means that communication can occur only between directly connected players. Moreover, because the signal domain is common knowledge, a reported message cannot fall outside the feasible support of the signal. Let $M_i = \prod_{j \in N_i} M_{ij} \in [0, 1]^{N_i}$ denote the message space for player i. The utility function of player i is denoted by $u_i : \prod_{j \in N_i \cup \{i\}} A_j \times S \times M_i \to \mathbb{R}$ and defined as follows:

$$u_i((a_i)_{j \in N_i \cup \{i\}}, \mathbf{s}, \mathbf{m}_i) = -(a_i - \theta(s) - b_i)^2 - \sum_{j \in N_i} \sigma_i (a_i - a_j)^2 + \sum_{j \in N_i} cmin\{m_{ij} - s_i, 0\}, \tag{2.1}$$

where $\sigma_i > 0$ and $c > 0$ $\forall i \in N$. $b_i \in \mathbb{R}$ is the bias of player i. The utility function can be understood as follows. The first term reflects that each player wishes to align her action with her own "bliss point," which is determined by the true state of the world, adjusted for her individual bias. This is a standard quadratic loss formulation widely used in economics to capture deviations from the preferred outcome.

The second component of the utility captures peer effects in the network. Players derive disutility from deviating too much from the actions of their neighbours. Intuitively, individuals prefer to conform to their peers, and the weight σ_i reflects the strength of this conformity motive. This captures the well-studied phenomenon of social influence or strategic complementarities in networks. The third component of the utility captures the cost of misrepresentation in communication. Specifically, it penalises under-reporting: when a player reports a signal lower than the one he actually observed, he incurs a disutility proportional to the magnitude of the understatement. The parameter c reflects the marginal cost of such misreporting. Over-reporting, in contrast, does not carry any direct penalty in this formulation.

Finally, if one wishes to extend the model to allow $0 \leq g_{ij} \leq 1$, this can be interpreted as the strength or intensity of the connection between i and j. In that case, messages are transmitted whenever $g_{ij} > 0$, but the influence of a neighbour depends on the weight of the link. This intensity can be incorporated by scaling the second term (peer effect) and the third term (cost of misreporting) with g_{ij}. In our current formulation, this role is effectively captured by σ_i and c. A more general version could instead use link-specific parameters σ_{ij} and c_{ij} to account for heterogeneous relationships across the network. However, for tractability and to focus on the central questions of the paper, we adopt the simpler specification where $\sigma_i = 1$ and $c_{ij} = c \; \forall i \in N$.

3 The Notion of Centrality

A brief discussion about the notion of centrality for our paper and some related results are as follows. Let, G^N be the matrix obtained from G by dividing the i'th row of G by $(1 + n_i)$. This can be found in Ghiglino *et al.* (2010) as well. Let,

$$[I - G^N]^{-1} \equiv [c_{ij}]^n_{i,j=1}.$$

Here, c_{ij} is the weighted number of walks from j to i. Intuitively, c_{ij} is the influence (in Bonacich centrality sense) on i from j. One thing to note here is that, G^N is not symmetric (if the graph is not a regular[1] graph). It can be shown that, for each $i \in N$, $c_{ii} \geq max\{c_{ij}, c_{ji}\} \; \forall j \neq i$. Now we introduce the idea where the influence of j on i is discounted by the cardinality of the augmented neighbourhood of j, i.e., $c'_{ij} = \frac{c_{ij}}{1+n_j}$[2]. We observe some properties of the discounted influence. First, $\sum_{j \in N} c'_{ij} = 1, \; \forall i \in N$. Second, for any i, $\lim_{n_i \to \infty} c'_{ii} = 0$. The importance of these observations will be discussed while we will interpret the results of our paper.

We define a set of pair of players N_c as follows.

$$N_c = \{(i, k) \in N \times N \mid c'_{ik} = c'_{ki}\}.$$

Definition 1: A pair of players $(i, k) \in N \times N$ is said to be Degree-Discounted Balanced Pair iff $(i, k) \in N_c$.

[1] Regular graph means $N_i = k \; \forall i$, where $k \in \mathbb{N}$ and $k \leq (n - 1)$.

[2] The augmented neighborhood of player j includes player j along with her neighbours. Hence the cardinality of the augmented neighborhood is $1 + n_j$.

4 Equilibrium

In our model, as already highlighted, a player can only know what message profile he is sending to her neighbours and the message profile he is getting from the neighbours. This is a distinguishing feature of our framework vis-á-vis Crawford and Sobel (1982), Hagenbach and Koessler (2009) and Galeotti *et al.*, (2013), where either the message or the type of the communication (truthful or not) is known to all players in the game. Ours is a private communication game. So a player cannot deterministically know what message is being transmitted between any two other players.

Observe that in our framework actual state of the nature is generated by the addition of individual private signals. Any player $i \in N$, having received her private signal and messages from her neighbours ($(m_{ji})_{j \in N_i}$) forms an expectation about the possible private signal realisation of both neighbours and non-neighbours. Since player i has no communication with non-neighbours, he or she simply takes it to be the expected value of the private signal realisation for each non-neighbour. For her neighbours, player i conjectures the expected private signal realisation given the message he or she has sent to player i. We now define the following expected state of the worlds from perspective of any $i \in N$. For any $j \in N_i$, let $(S_j \mid m_{ji} \in M_{ji})$ be a random variable denoting the possible private signal realisations of player j conditioned on the message sent by him to player i. We assume $(S_j \mid m_{ji} \in M_{ji})$ has the distribution function $G_{ji}(\cdot)$ and the density function $g_{ji}(\cdot)$. Then the expected state of nature for player i given her private signal realistion of $\overline{s}_i$ is given as

$$\theta_i^i(\mathbf{s} \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) = \overline{s}_i + \sum_{j \in N_i} \int_{S_j} s_j g_{ji}(s_j \mid m_{ji})\, ds_j + \sum_{l \notin N_i \cup \{i\}} \int_{S_l} s_l f_l(s_l)\, ds_l \equiv \theta_i(\mathbf{s}). \quad (4.1)$$

The first component reflects her own signal. The second component captures what player i expects about the signals received by her neighbours, even before any messages are transmitted. Finally, player i forms a conjecture about the signals that is received by her non-neighbours, excluding himself. Since the signals received by different players are independent, these terms can be added together. Now we define what player i is conjecturing about the expected state of the world of another player j such that $j \in N_i$ after sending the messages to her ($i's$) neighbours (i.e., $(m_{ij})_{j \in N_i}$). Note that such a conjecture can simply be interpreted as player i's conjecture about player j's conjecture and thus can be thought of as a second-order belief of player i, which is defined as:

$$\theta_j^i(\mathbf{s} \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) = \int_{S_j} s_j g_{ji}(s_j \mid m_{ji})\, ds_j + \sum_{p \in N_j \setminus \{(N_j \cap N_i) \cup i\}} \int_{M_{pj}} \int_{S_p} s_p g_{pj}(s_p \mid m_{pj})\, ds_p\, dm_{pj} \tag{4.2}$$

$$+ \sum_{x \in \{(N_j \cap N_i)\}} \int_{M_{xj}} \int_{S_x} s_x g_{xj}(s_x \mid m_{xj}, m_{xi})\, ds_x\, dm_{xj} + \int_{S_i} s_i g_{ij}(s_i \mid m_{ij})\, ds_i$$

$$+ \sum_{l \notin N_j \cup \{j\}} \int_{S_l} s_l f_l(s_l)\, ds_l \equiv \theta_j^i(\mathbf{s}).$$

The first component is the signal that player i expects player j has received given the message of player j to player i. Second component is player j's expectation about the signals of her neighbours except i. Third component is what j is expecting about i's signal. Finally, j is expecting the signals received by all non-neighbours excluding himself. Similarly, for any $q \notin N_i$,

$$\theta_q^i(\mathbf{s} \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) = \int_{S_q} s_q f_q(s_q)\, ds_q + \sum_{j \in N_q \setminus (N_q \cap N_i)} \int_{M_{jq}} \int_{S_j} s_j g_{jq}(s_j \mid m_{jq})\, ds_j\, dm_{jq} \tag{4.3}$$

$$+ \sum_{x \in (N_q \cap N_i)} \int_{M_{xq}} \int_{S_x} s_x g_{xq}(s_x \mid m_{xq}, m_{xi})\, ds_x\, dm_{xq}$$

$$+ \sum_{l \notin N_q \cup \{q\}} \int_{S_l} s_l f_l(s_l)\, ds_l \equiv \theta_q^i(\mathbf{s}).$$

4.1 Equilibrium Action

The following theorem talks about equilibrium actions of players in our private communication game.

Theorem 4.1. *For any given $\langle N, g, (b_i)_{i \in N} \rangle$ and priors, the unique Bayesian equilibrium action of player i is given by,*

$$a_i'(\boldsymbol{m}_i, \boldsymbol{m}^i) = \sum_{j \in N} c_{ij}'(\theta_j^i(\boldsymbol{s}) + b_j),$$

$$\textit{where} \quad a_i'(\boldsymbol{m}_i, \boldsymbol{m}^i) = E(a_i \mid \bar{s}_i, \boldsymbol{m}_i, \boldsymbol{m}^i).$$

Proof. Given $\bar{s}_i, \mathbf{m}_i$ and $\mathbf{m}^i$, and from equation 2.1, the expected utility of player i be,

$$E_i(u_i(\cdot)) \equiv \int_{S_{-i}} u_i((a_i)_{j\in N_i\cup\{i\}}, \mathbf{s}, \mathbf{m}_i) f_i(\mathbf{s}_{-i} \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) d\mathbf{s}_{-i}$$

where,

$$f_i(\mathbf{s}_{-i} \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) = \prod_{j\in N_i} g_{ji}(s_j \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) \prod_{l\notin N_i\cup\{i\}} f_l(s_l \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i)$$

since, private signals received by the players are independent.

Differentiating the expected utility of i with respect to a_i and taking the first order condition, we get

$$-2(E(a_i \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) - E(\theta(\mathbf{s}) \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) - b_i) - 2\sum_{j\in N_i}(E(a_i \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) - E(a_j \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i)) = 0.$$

Let $E(a_j \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) \equiv a_j^{i\prime}(\mathbf{m}_i, \mathbf{m}^i)$. Define,

$$\theta_i(\mathbf{s} \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i) \equiv E(\theta(\mathbf{s}) \mid \bar{s}_i, \mathbf{m}_i, \mathbf{m}^i).$$

Thus, we get

$$a_i'(\mathbf{m}_i, \mathbf{m}^i) - \frac{\sum_{j\in N_i} a_j^{i\prime}(\mathbf{m}_i, \mathbf{m}^i)}{(n_i+1)} = \frac{\theta_i(\mathbf{s}) + b_i}{(n_i+1)}$$

Here $M = [m_{ij}]_{i,j\in N}$ is the message matrix where $m_{pq} = 0$ if $g_{pq} = 0$ for $p, q \in N$.

This can be written in matrix notation as follows.

$$\mathbf{a}'(M) - G^N \mathbf{a}'(M) = \begin{bmatrix} \frac{\theta_1(\mathbf{s})+b_1}{(n_1+1)} \\ \frac{\theta_2(\mathbf{s})+b_2}{(n_2+1)} \\ \vdots \\ \frac{\theta_n(\mathbf{s})+b_n}{(n_n+1)} \end{bmatrix}$$

The above system of equations can be further rewritten as

$$\Longrightarrow \mathbf{a}'(M) = (I - G^N)^{-1} \begin{bmatrix} \frac{\theta_1(\mathbf{s})+b_1}{(n_1+1)} \\ \frac{\theta_2(\mathbf{s})+b_2}{(n_2+1)} \\ \vdots \\ \frac{\theta_n(\mathbf{s})+b_n}{(n_n+1)} \end{bmatrix}$$

Using Perron Frobenius theorem, $(I-G^N)^{-1}$ exists since maximum sum across all rows of $[G^N]$ is $\frac{n_{i^*}}{n_{i^*}+1}$, for some $i^* \in N$ and $\frac{n_{i^*}}{n_{i^*}+1} < 1$. Thus equilibrium expected action of player i is given by,

$$a'_i(\mathbf{m}_i, \mathbf{m}^i) = \sum_{j \in N} c'_{ij}(\theta_j(\mathbf{s}) + b_j).$$

Moreover, player i's expectation about the expected $\theta(\mathbf{s})$ of player j is $\theta^i_j(\mathbf{s})$. Suppose player j is a neighbour of i. So i knows m_{ij} and m_{ij} is not known to other players. So for some $k \neq i$, $\theta^i_j(s) \neq \theta^k_j(s)$ may happen. So, we get

$$a'_i(\mathbf{m}_i, \mathbf{m}^i) = \sum_{j \in N} c'_{ij}(\theta^i_j(\mathbf{s}) + b_j).$$

This proves our claim. □

Remark 4.2. In absence of any network, equilibrium action would be just the bliss point the player. But incorporating the network effect, the expected equilibrium action of a player is weighted average of bliss point of all players since $\sum_{j \in N} c'_{ij} = 1, \ \forall i \in N$. Moreover, if a player's neighbours increases, he puts lesser weight to her own bliss point since $\lim_{n_i \to \infty} c'_{ii} = 0$ and c'_{ii} is the weight of player i's bliss point in her equilibrium action.

4.2 Equilibrium Signalling

We divide the discussion for two kinds of players. First we consider a player $k \in N_i$ who is a naive neighbour with respect to k, i.e., he accepts players i's message as truth. Suppose, $m_{ik} = \underline{s}_i$. Therefore, $(m_{ij})_{j \in N_i} = (\underline{s}_i, (m_{ij})_{j \in N_i \setminus \{k\}})$. The following result is for a naive neighbour and tries to answer when truthful reporting is incentivable for player i to a naive neighbour 'k' when the messages relayed by him to other neighbours are held fixed, i.e., $(m_{ij})_{j \in N_i \setminus k}$ is given. We take $(m_{ji})_{j \in N_i}$ as fixed as well. The fact that player k is naive is common knowledge between player i and k only. Let $\overline{a}'_i$ and $\overline{a}'_k$ denote the equilibrium actions of player i and player k respectively when i truthfully reports to k, i.e., $m_{ik} = \overline{s}_i$, and let $\underline{a}'_i$ and $\underline{a}'_k$ denotes the respective actions when i misreports to k, we write $m_{ik} = \underline{s}_i \neq \overline{s}_i$.

Theorem 4.3. *Suppose $\exists\ i, k \in N$ such that $k \in N_i$ is a naive player with respect to player i. Moreover, $(i, k) \in N_c$. Then the following holds:*

- $(\theta_i(s) + b_i) + \sum\limits_{j \in N_i \setminus k} a'_j - n_i \overline{a}'_i > \frac{c_{kk}}{c_{ik}}(\overline{a}'_k - \overline{a}'_i) \implies \underline{s}_i > \overline{s}_i.$

- $\frac{c_{kk}}{c_{ik}}(\bar{a}'_k - \bar{a}'_i) - c \leq (\theta_i(s) + b_i) + \sum_{j \in N_i \setminus \{k\}} a'_j - n_i \bar{a}'_i \leq \frac{c_{kk}}{c_{ik}}(\bar{a}'_k - \bar{a}'_i) \implies \underline{s_i} = \bar{s}_i.$

- $(\theta_i(s) + b_i) + \sum_{j \in N_i \setminus \{k\}} a'_j - n_i \bar{a}'_i < \frac{c_{kk}}{c_{ik}}(\bar{a}'_k - \bar{a}'_i) - c \implies \underline{s_i} < \bar{s}_i.$

Proof. We have,

$$\bar{a}'_i = c'_{ik}\bar{s}_i + A_1, \text{ and } \underline{a'_i} = c'_{ik}\underline{s_i} + A_1. \tag{4.4}$$

Here $A_1 = a'_i - c'_{ik}m_{ik}$, a'_i is as defined in Theorem 4.1.

Similarly, for player k we have

$$\bar{a}'_k = (c'_{ki} + c'_{kk})\bar{s}_i + A_2, \text{ and } \underline{a'_k} = (c'_{ki} + c'_{kk})\underline{s_i} + A_2 \tag{4.5}$$

Here $A_2 = a'_k - c'_{ki}m_{ik}$, a'_k is as defined in Theorem 4.1.
In the neighbourhood of equilibrium action, we get

$$V(\bar{s}_i, \underline{s_i}) \equiv \underline{u_i}((a_i)_{j \in N_i \cup \{i\}}, \mathbf{s}, (m_{ij})_{j \in N_i}) - \bar{u}_i((a_i)_{j \in N_i \cup \{i\}}, \mathbf{s}, (m_{ij})_{j \in N_i})$$

boils down to

$$(\bar{a}'_i - \underline{a'_i})(\bar{a}'_i + \underline{a'_i} - 2\theta(\mathbf{s}) - 2b_i) + [(\bar{a}'_i - \underline{a'_i}) - (\bar{a}'_k - \underline{a'_k})][(\bar{a}'_i + \underline{a'_i}) - (\bar{a}'_k + \underline{a'_k})] +$$

$$\sum_{j \in N_i \setminus k} [\bar{a}'_i - \underline{a'_i}][(\bar{a}'_i + \underline{a'_i}) - 2a'_j] + c\,min\{\underline{s_i} - \bar{s}_i, 0\}.$$

From Equations (4.4) and (4.5), the above expression simplifies to

$$(\bar{s}_i - \underline{s_i})c'_{ik}[c'_{ik}(\bar{s}_i + \underline{s_i}) + 2A_1 - 2\theta(\mathbf{s}) - 2b_i] + (\bar{s}_i - \underline{s_i})(-c'_{kk})[(-c'_{kk})(\bar{s}_i + \underline{s_i}) + 2A_1 - 2A_2]$$

$$+ \sum_{j \in N_i \setminus k} (\bar{s}_i - \underline{s_i})c'_{ik}[c'_{ik}(\bar{s}_i + \underline{s_i}) + 2A_1 - 2a'_j] + c\,min\{\underline{s_i} - \bar{s}_i, 0\}.$$

The above expression then can be rewritten as

$$(\bar{s}_i - \underline{s_i})[(c'^2_{ik}n_i + c'^2_{kk})(\bar{s}_i + \underline{s_i}) + c'_{ik}[2A_1 - 2(\theta(\mathbf{s}) + b_i)] - c'_{kk}(2A_1 - 2A_2)$$

$$+2c'_{ik}(n_i - 1)A_1 - 2c'_{ik}A_3] + c\,min\{\underline{s_i} - \bar{s}_i, 0\}$$

(Here $A_3 \equiv \sum_{j \in N_i \setminus k} a'_j$)
which further simplifies to

$$(\bar{s}_i - \underline{s_i})[(c'^2_{ik}n_i + c'^2_{kk})(\bar{s}_i + \underline{s_i}) + A_4] + c\,min\{\underline{s_i} - \bar{s}_i, 0\} \tag{4.6}$$

(Here, $A_4 \equiv c'_{ik}[2A_1 - 2(\theta(\mathbf{s}) + b_i)] - c'_{kk}(2A_1 - 2A_2) + 2c'_{ik}(n_i - 1)A_1 - 2c'_{ik}A_3]$).

Note that, A_4 is independent of $\underline{s_i}$. Let, $A_4 < -2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i$. So there exists $\varepsilon > 0$, such that, $A_4 = -2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i - \varepsilon$. Further, from Equation (4.6) we get

$$V(\bar{s}_i, \underline{s_i}) = (\bar{s}_i - \underline{s_i})[(c'^2_{ik}n_i + c'^2_{kk})(\bar{s}_i + \underline{s_i}) - 2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i - \epsilon] + cmin\{\underline{s_i} - \bar{s}_i, 0\}$$

which again equals to

$$(\bar{s}_i - \underline{s_i})[(c'^2_{ik}n_i + c'^2_{kk})(\underline{s_i} - \bar{s}_i) - \epsilon] + cmin\{\underline{s_i} - \bar{s}_i, 0\}$$

If $\bar{s}_i = \underline{s_i}$, then $V(\bar{s}_i, \underline{s_i}) = 0$. Also, $V(\bar{s}_i, \underline{s_i}) < 0$ if $\bar{s}_i > \underline{s_i}$. Let $\bar{s}_i < \underline{s_i}$. Then $V(\bar{s}_i, \underline{s_i}) > 0$ if $(c'^2_{ik}n_i + c'^2_{kk})(\underline{s_i} - \bar{s}_i) - \epsilon < 0$. This implies that

$$\bar{s}_i < \underline{s_i} < \bar{s}_i + \frac{\epsilon}{c'^2_{ik}n_i + c'^2_{kk}}.$$

Thus we get, $A_4 < -2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i \implies \bar{s}_i < \underline{s_i}$. Now, let $A_4 > -2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i$. Then there exists $\delta > 0$, such that, $\delta < c$ and $A_4 = -2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i + \delta$. One obtains from Equation (4.6)

$$V(\bar{s}_i, \underline{s_i}) = (\bar{s}_i - \underline{s_i})[(c'^2_{ik}n_i + c'^2_{kk})(\bar{s}_i + \underline{s_i}) - 2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i + \delta] + cmin\{\underline{s_i} - \bar{s}_i, 0\}$$
$$= (\bar{s}_i - \underline{s_i})[(c'^2_{ik}n_i + c'^2_{kk})(\underline{s_i} - \bar{s}_i) + \delta] + cmin\{\underline{s_i} - \bar{s}_i, 0\}$$

Let $\bar{s}_i < \underline{s_i}$. We get, $V(\bar{s}_i, \underline{s_i}) < 0$. Moreover, if $\bar{s}_i > \underline{s_i} \implies V(\bar{s}_i, \underline{s_i}) > 0$, we must have,

$$\underline{s_i} > \bar{s}_i + \frac{c - \delta}{c'^2_{ik}n_i + c'^2_{kk}}.$$

This is a contradiction since we started with $\delta < c$. Thus, best response for i will be $\underline{s_i} = \bar{s}_i$ and $V(\bar{s}_i, \underline{s_i}) = 0$. From $A_4 < -2(c'^2_{ik}n_i + c'^2_{kk})\bar{s}_i$, we get

$$(\theta_i(s) + b_i) + \sum_{j \in N_i \backslash k} a'_j - n_i \bar{a}_i{}' > \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}).$$

Thus,

$$(\theta_i(s) + b_i) + \sum_{j \in N_i \backslash k} a'_j - n_i \bar{a}_i{}' > \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) \implies \underline{s_i} > \bar{s}_i.$$

Moreover, it follows that.

$$(\theta_i(s) + b_i) + \sum_{j \in N_i \backslash k} a'_j - n_i \bar{a}_i{}' \leq \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) \implies \underline{s_i} = \bar{s}_i.$$

Similarly, it can be shown that,

$$\frac{c_{kk}}{c_{ik}}(\bar{a_k'} - \bar{a_i'}) - c \leq (\theta_i(s) + b_i) + \sum_{j \in N_i \setminus k} a_j' - n_i \bar{a_i}' \implies \underline{s_i} = \bar{s}_i$$

and

$$(\theta_i(s) + b_i) + \sum_{j \in N_i \setminus k} a_j' - n_i \bar{a_i}' < \frac{c_{kk}}{c_{ik}}(\bar{a_k'} - \bar{a_i'}) - c \implies \underline{s_i} < \bar{s}_i.$$

Combining the above four equations together, our result follows. □

Remark 4.4. The above theorem does not only tell us about reporting to a neighbour, but it captures the role of the neighbour's centrality as well. Let us focus on the first part of the theorem. Suppose, $(\theta_i(s) + b_i) + \sum_{j \in N_i \setminus \{k\}} a_j' < n_i \bar{a_i}'$. This implies, $\bar{a_k}' < \bar{a_i}'$. By over-reporting to k (so both a_i' and a_k' increases), player i puts her action further away from her own bliss point and the collective actions of her neighbours except k. By over-reporting, action of i increases by $c_{ik}' d\underline{s_i}$ and that of player k increases by $(c_{ik}' + c_{kk}')d\underline{s_i}$. Thus, difference between action of i and k falls. Moreover, in a network with high value of $\frac{c_{kk}}{c_{ik}}$, player i will try to pull action of player k closer to her own action. High value of $\frac{c_{kk}}{c_{ik}}$ means k has become more central. On the other hand, as player k becomes more central, c_{kk}' falls. It becomes difficult for player i to reduce the difference between her own action and that of k. Moreover, $(\theta_i(s) + b_i) + \sum_{j \in N_i \setminus k} a_j' - n_i \bar{a_i}' \equiv [(\theta_i(s) + b_i) - \bar{a_i}'] + \sum_{j \in N_i \setminus k} [a_j' - \bar{a_i}']$. Thus, by misreporting to k, player i increases the distance of her own action with her own expected bliss point and actions of her neighbours. *Thus, a player will try to pull a neighbour's action closer to her own action if that neighbour is more central. Moreover, reducing the difference of actions with a neighbour becomes harder if the neighbour is more central.* This captures the role of the network in the signalling game. Moreover, if the second condition of the statement holds for all players in the network, we get a truthful communication network as in Hagenbach and Koessler (2009) and Galeotti *et al.*, (2013).

This characterisation shows when a player will over-report or under-report or report truthfully. But this does not give the optimal reporting. The following proposition is about this analysis. Here we try to find out what should be optimal reporting. We get a locus of actual signal received by the player i and and her reporting to the player k, where $k \in N_i$ and k is a naive player with respect to i.

Proposition 4.5. *Suppose $\exists i, k \in N$ such that $k \in N_i$ and k is a naive player with respect to i and $(i, k) \in N_c$. Then,* *(a) For any given N, g and $\bar{s}_i$, there exists a unique optimal reporting of player i to k. This is given by, $\underline{s_i} = \bar{s}_i + \frac{c'_{ik}}{c'^2_{ik}+c'^2_{kk}}[\theta_i(s) + b_i + \sum_{j\in N_i\setminus k} a'_j - n_i\bar{a}_i{}' - \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i})]$, if $\underline{s_i} > \bar{s}_i$.*

(b) $\underline{s_i} = \bar{s}_i + \frac{c'_{ik}}{c'^2_{ik}+c'^2_{kk}}[\theta_i(s) + b_i + \sum_{j\in N_i\setminus k} a'_j - n_i\bar{a}_i{}' - \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) + c]$, if $\underline{s_i} < \bar{s}_i$.

b) Reporting is a strict monotonic function of actual signal, i.e., $\frac{\partial \underline{s_i}}{\partial \bar{s_i}} > 0$.

Part (a) of this result directly follows from the utility function of player i and part (b) follows from part (a).

Now we discuss the reporting to a Bayesian neighbour. Suppose $k \in N_i$ is a Bayesian player with respect to i. The additional thing required here is the expectation formed by player i about what player k is thinking about the signal received by i after a message being transmitted from i to k. This is denoted by,

$$E_i(E_k(s_i \mid m_{ik})) = E_i(\int_{S_i} s_i g_{ik}(s_i \mid m_{ik})\, ds_i).$$

Assumption 1 (A1): For some $m'_{ik}, m''_{ik} \in M_{ik}$, if

$m'_{ik} > m''_{ik} \implies E_i(E_k(s_i \mid m'_{ik})) > E_i(E_k(s_i \mid m''_{ik}))$, then

$m^1_{ik} > m^2_{ik} \implies E_i(E_k(s_i \mid m^1_{ik})) > E_i(E_k(s_i \mid m^2_{ik}))\ \forall m^1_{ik}, m^2_{ik} \in M_{ik}$.

Similarly, for some $m'_{ik}, m''_{ik} \in M_{ik}$, if

$m'_{ik} > m''_{ik} \implies E_i(E_k(s_i \mid m'_{ik})) < E_i(E_k(s_i \mid m''_{ik}))$, then

$m^1_{ik} > m^2_{ik} \implies E_i(E_k(s_i \mid m^1_{ik})) < E_i(E_k(s_i \mid m^2_{ik}))\ \forall m^1_{ik}, m^2_{ik} \in M_{ik}$.

Finally, for some $m'_{ik}, m''_{ik} \in M_{ik}$, if

$m'_{ik} > m''_{ik} \implies E_i(E_k(s_i \mid m'_{ik})) = E_i(E_k(s_i \mid m''_{ik}))$, then

$m^1_{ik} > m^2_{ik} \implies E_i(E_k(s_i \mid m^1_{ik})) = E_i(E_k(s_i \mid m^2_{ik}))\ \forall m^1_{ik}, m^2_{ik} \in M_{ik}$.

From **A1**, it is easy to see that if $E_i(E_k(s_i \mid m^1_{ik})) = E_i(E_k(s_i \mid m^2_{ik}))\ \forall m^1_{ik}, m^2_{ik} \in M_{ik}$ such that, $m^1_{ik} > m^2_{ik}$, we get $\underline{s_i} = \bar{s}_i$. Thus, if a player cannot manipulate the action of another player by misreporting, the former player chooses to report truthfully.

If $E_i(E_k(s_i \mid m^1_{ik})) > E_i(E_k(s_i \mid^2_{ik}))\ \forall m^1_{ik}, m^2_{ik} \in M_{ik}$ such that, $m^1_{ik} > m^2_{ik}$, then the conditions of reporting remain same as of a naive neighbour since the direction of change remains same. Thus we conclude this subsection with the following theorem considering the case $E_i(E_k(s_i \mid m^1_{ik})) < E_i(E_k(s_i \mid m^2_{ik}))\ \forall m^1_{ik}, m^2_{ik} \in M_{ik}$ such that, $m^1_{ik} > m^2_{ik}$.

Theorem 4.6. *Suppose $E_i(E_k(s_i \mid m^1_{ik})) > E_i(E_k(s_i \mid m^2_{ik}))$ $\forall m^1_{ik}, m^2_{ik} \in M_{ik}$ such that, $m^1_{ik} > m^2_{ik}$ and $(i,k) \in N_c$ holds. Then,*

- $(\theta_i(s) + b_i) + \sum\limits_{j \in N_i \setminus k} a'_j - n_i \bar{a_i}' > \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) \implies \underline{s_i} < \bar{s}_i.$

- $\frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) - c \leq (\theta_i(s) + b_i) + \sum\limits_{j \in N_i \setminus k} a'_j - n_i \bar{a_i}' \leq \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) \implies \underline{s_i} = \bar{s}_i.$

- $(\theta_i(s) + b_i) + \sum\limits_{j \in N_i \setminus k} a'_j - n_i \bar{a_i}' < \frac{c_{kk}}{c_{ik}}(\bar{a'_k} - \bar{a'_i}) - c \implies \underline{s_i} > \bar{s}_i.$

Proof. Without loss of generality, suppose $m^1_{ik} = \bar{s}_i$ and $m^2_{ik} = \underline{s_i}$. Now this theorem can be proved in a similar way as of Theorem 2. □

5 Changing a Network for Truthful Communication

Suppose in a given network, $\exists i, k \in N$ such that $k \in N_i$ and k is a naive player. Except this, Suppose all players are Bayesian players with respect to everyone else. But player i has incentive to misreport to player k. We discuss an example to show how a change in the network architecture can actually make player i report truthfully to player k.

Suppose $N = \{1, 2, 3\}$. The undirected graph is defined as follows. $g_{12} = g_{13} = 1$ and $g_{23} = 0$. Thus this is a star graph where 1 is the star-agent and $2, 3$ are in the periphery. Suppose $s_i \in [0,1]$ $\forall i \in N$ and therefore, $m_{ij} \in [0,1], \forall i$ and j such that $g_{ij} = 1$. Let, $s_i = \frac{i}{10} \forall i$ and $b_1 = 0.05, b_2 = 0.1, b_3 = 0.08$. Suppose, each player i knows $S_j \sim U[0,1]$ $\forall j \neq i$. Everything explained so far except $s'_i s$ is common knowledge. Suppose this is also a common knowledge that if i sends m_{ij} to j, then j thinks $(S_i \mid m_{ij} \in M_{ij}) \sim U[0, m_{ij}]$. Assume that, player i thinks whatever player 1 is telling to i is truthful and this is common knowledge between i and 1 only, and $i = \{2, 3\}$.

Then, in equilibrium, we get,

Suppose, everything explained above remains same only the network changes and becomes a complete graph[3]. Then in equilibrium, we get,

This example highlights the role of signalling in networks. In the star network, Player 1 is the central node connected to players 2 and 3. Both neighbours fully trust whatever player 1 reports. Knowing this, player 1 strategically misreports information

[3] $g_{ij} = 1$ $\forall i \neq j$.

Variable	Equilibrium Value
m_{12}	0.423
m_{13}	0.445
m_{21}	0.654
m_{31}	0.757

Table 1: Equilibrium values of the choice variables in star graph.

Variable	Equilibrium Value
m_{12}	0.1
m_{13}	0.1
m_{21}	0.654
m_{23}	0.6
m_{31}	0.757
m_{32}	0.58

Table 2: Equilibrium values of the choice variables in complete graph.

to them, and the messages m_{12} and m_{13} reflect this misreporting. The network structure allows player 1 to exploit their position of influence.

In contrast, in the complete network, every player is connected to every other player. Here, player 1 cannot misreport to players 2 and 3 because the additional communication channels constrain deviations: any inconsistency can be detected through messages from other players. As a result, player 1's messages to 2 and 3 are truthful, i.e., $m_{12} = m_{13} = s_1$, demonstrating how network structure limits the ability to manipulate information.

For instance, consider a political network where a party leader (player 1) is highly influential. In a star-like structure, two local party members (players 2 and 3) rely solely on the leader for information about policy decisions or strategy. Knowing that the members fully trust him, the leader can misreport or exaggerate facts to influence their actions. In contrast, in a fully connected network where members also communicate with each other, the leader cannot easily misreport because any inconsistency can be verified through peer communication, ensuring that the leader's messages to each member are truthful.

6 Endogenous Bias

Let us assume that players can choose their biases as well. Suppose bias of a player changes. So her action will change. Actions of other players will change as well. Since all players can choose their biases, others players will also adjust their own bias in response to change in the bias of some other player. The following result shows how the change in bias of a player affects other players' biases.

Proposition 6.1. *For any $i, k \in N$, $\frac{db_i}{db_k} > 0$ if $\sum\limits_{j \in N_i} (c_{jk} - c_{ik})(c_{ii} - c_{ji}) \geq 0$.*

Proof. Differentiating the expected utility of player i with respect to b_i and from the first order condition, we get

$$-(a'_i - \theta_i(\mathbf{s}) - b_i)(c'_{ii} - 1) = \sum_{j \in N_i} (a'_i - a'_j)(c'_{ii} - c'_{ji}) \tag{6.1}$$

This gives the optimal choice of b_i. From this we get,

$$-(c'_{ii} - 1)[(c'_{ii} - 1)db_i + c'_{ik}db_k] = \sum_{j \in N_i} [(c'_{ii} - c'_{ji})db_i + (c'_{ik} - c'_{ji})db_k](c'_{ii} - c'_{ji})$$

This implies,

$$\frac{db_i}{db_k} = \frac{c'_{ik}(1-c'_{ii}) + \sum\limits_{j \in N_i} (c'_{jk} - c'_{ik})(c'_{ii} - c'_{ji})}{(c'_{ii}-1)^2 + \sum\limits_{j \in N_i} [(c'_{ii} - c'_{ji})^2}$$

Since, $0 < c'_{ii} < 1$, our result follows. □

This shows the effect of change in bias of a player to change in another player's bias. Player i observes how much player k contributes to her ($i's$) neighbours' centrality compare to k's influence on her own (i). If $k's$ influence is higher on every neighbour (of i), he changes her own bias in same direction with change in $k's$ bias. But if the exact opposite is the case, then i may not go with same direction as k. This may happen for several reasons. Consider a political scenario and the players being political parties. When player k's influence is the strongest on player i (compare to neighbours of i), any change in k's bias affects i's strategic environment more than others. So if k increases bias, i may decrease its bias to avoid appearing too aligned or dependent, especially when k's influence on others is weak, making i's reaction highly visible. This opposite movement helps i preserve independence, relevance, and strategic balance.

Also, i observes k is not much liked by others. So, going in same direction with k may increase the distance of i from others.

But suppose for some neighbours of i, $k's$ influence is higher only to them than that of to player i. Then player i adjusts the bias using how much player i influences the respective players. Suppose i can influence player j much compare to another player p. Since this is known to i, he understands her own preference already affects j up to some large extent compare to p. But p being less affected by i, p is acting more independently from $i's$ perspective. So, i pays more attention to how k influences p.

Now suppose $\forall i \in N, b_i \in \mathbb{R}_+$. Thus, players can either be unbiased ($b_i = 0$) or upward biased ($b_i > 0$). We try to find a scenario, where $(b_1, b_2, \ldots, b_n) = (0, 0, \ldots, 0)$ is a Nash Equilibrium bias profile. The proposition is as follows.

Proposition 6.2. *For player $i \in N$, assume that $(\frac{dU_i}{db_i})_{b_i \to 0+} \geq 0$. Suppose $b_j = 0\ \forall j \neq i$ and $\theta_i(\boldsymbol{s}) > \sum_{j \in N} c'_{ij}\theta^i_j(\boldsymbol{s})$. Then $\sum_{j \in N_i} [C'_i - C'_j](c_{ii} - c_{ji}) \leq 0 \implies b_i = 0$, where $C'_i = \sum_{j \in N} c'_{ij}\theta^i_j(\boldsymbol{s})\ \forall i \in N$.*

Proof. From (6.1) we get the optimal bias of player i as,

$$b_i = \frac{\sum_{j \in N_i} [C'_j - C'_i](c_{ii} - c_{ji}) - (1 - c'_{ii})(\theta_i(\mathbf{s}) - C'_i)}{\sum_{j \in N_i} (c'_{ii} - c'_{ji})^2 + (c'_{ii} - 1)^2}.$$

Since, $b_i \geq 0$, our result follows. □

Remark 6.3. When $C'_i < C'_j$ for all $j \in N_i$, player i views himself as weaker than all her neighbours in terms of state–weighted centrality. At the same time, her own expected state (bliss point without bias) is already higher than the average expected state (this is also bliss points of all neighbours of i and $i's$ bliss point except bias) of the group(since, $\theta_i(\mathbf{s}) > \sum_{j \in N} c'_{ij}\theta^i_j(\mathbf{s})$). If he adds further bias, he would only move her bliss point even further away from others, increasing the gap. Hence, in this case, it is better for player i to remain unbiased.

When $C'_i > C'_j$ for all $j \in N_i$, player i enjoys the highest state–weighted centrality among her neighbours. Even though all players are unbiased, a positive bias by i *may* pull others upward, since her influence on them is relatively strong. Thus, he has scope to shape the collective outcome.

In intermediate cases, when player i is more central than some neighbours but less central than others, he balances perspectives. The weight $(c_{ii} - c_{ji})$ reflects how much more strongly he relies on her own information compared to that filtered through neighbour j. Intuitively, i pays more attention to neighbours he cannot influence much, because their views add genuinely independent information, while he discounts those he already strongly influences.

7 Conclusion

In this paper, we examined how strategic communication unfolds in networks when players receive private signals about an underlying state of the world and interact with neighbours through unverifiable messages. Our framework departs from the standard information transmission in networks literature by focusing mainly on private rather than public communication and by incorporating an asymmetric misreporting cost. Specifically, individuals incur a penalty for under-reporting but not for over-reporting their signals. This design captures many realistic contexts where exaggeration or optimistic reporting is less costly than suppression or understatement of information. Our analysis shows that equilibrium behaviour is shaped by three interacting forces: (i) the individual's desire to align actions with their own bliss point, which depends on both the true state and personal bias; (ii) the incentive to conform with neighbours, which reflects social or organisational pressures; and (iii) the strategic calculus of misreporting, which depends on both the magnitude of bias and the network position of the receiver. These forces jointly determine whether an individual chooses to truthfully report, under-report, or over-report information to a neighbour. One of our central results is that equilibrium actions can be represented as weighted averages of the bliss points of all players in the network, where the weights are given by a degree-discounted Bonacich centrality measure. This finding establishes a tight link between strategic communication and classical measures of influence in network theory. Importantly, it highlights that more central agents—those with greater structural influence—place relatively less weight on their own preferences, because their actions must balance the pull of multiple neighbours. Conversely, peripheral agents place greater emphasis on their own bliss points, as their influence on the network is weaker. We also identify conditions under which agents misreport to their neighbours. Over-reporting arises when personal bias pushes the player's bliss point above the true state and the influence of the neighbour is not strong enough to discipline exaggeration. Under-reporting, by contrast, is less common because it directly incurs a cost; it occurs only when the comparative weight of neighbour influence makes understatement strategically optimal. Truthful reporting emerges as an equilibrium outcome in intermediate cases, where incentives to misreport in either direction are offset by the costs of doing so. Finally, by allowing bias to be endogenously chosen, we show that neutrality, zero bias, can itself be sustained as a Nash equilibrium under certain conditions. This result underscores that in some environments, strategic pressures and communication frictions can naturally discipline individuals toward unbiasedness, even without external enforcement. Taken together, our results enrich the theory of communication in networks by demonstrating how bias, centrality, and asymmetric costs of misreporting interact to shape equilibrium actions. The model provides insights for organisational design, policymaking, and information aggregation in technologically mediated networks, where private and unverifiable communication is common. Future work can extend this framework along several promising directions. A dynamic version of the model would allow us to study how reputations evolve when players repeatedly interact. Introducing heterogeneity in misreporting costs could capture environments where some individuals face stronger penalties for understatement than others. Another extension could examine settings where messages are partially observable, blending public and private communication. These avenues would broaden the applicability of the model and further illuminate the complex relationship between network structure, incentives, and information sharing.

The findings of this paper carry direct implications for institutions and organisations that rely on internal communication networks. Since central players in a network have disproportionate influence on collective outcomes, mechanisms should be designed to ensure that information reported to them is reliable. One approach is to reduce asymmetry in reporting costs—for instance, by creating monitoring systems that penalise both exaggeration and understatement, thereby discouraging systematic distortion. Additionally, organisations can mitigate the risks of biased communication by introducing redundancy in reporting channels so that central players receive information from multiple, independent sources. This reduces the ability of any single individual to strategically distort outcomes. Finally, the result that neutrality can emerge as an equilibrium suggests that training programs and incentive schemes aimed at reducing bias may be effective, particularly when combined with network-aware policies that recognise the role of centrality. In this way, institutions can better harness communication networks for accurate information aggregation and improved decision-making.

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