Pricing, Technology, and Enforcement in Markets with Digital Piracy: A Hotelling Approach

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Abstract

We analyze the strategic interaction between an original content provider and a pirate firm in a spatial Hotelling framework. The model accounts for technological difficulty and probabilistic enforcement as determinants of consumer adoption and firm profits. Under uniform distributions, we derive closed-form equilibrium prices, technological effort, and market shares. Comparative statics reveal how enforcement intensity, technological barriers, and pirate revenue parameters shape equilibrium outcomes. While the original and pirate firms are pure substitutes in market share, their pricing and technological strategies display strategic complementarities. The results clarify how regulatory interventions and technological investments influence market power and profitability. By combining analytical tractability with flexible specifications for pirate costs and revenues, the framework provides clear insights for policymakers and managers addressing digital piracy.

JEL Classification: D43, L13, K42, O34

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1 Introduction

The digital age has democratized access to information and entertainment on an unprecedented scale, yet it has also amplified one of the creative industries' most persistent challenges: digital piracy. Far from being a solved problem, commercial piracy remains a formidable economic force. In 2024 alone, piracy websites received more than 216 billion visits worldwide, with the US economy losing an estimated $29.2 billion in annual revenue to digital video piracy [7, 4]. The landscape is complex and dynamic; while mature legal

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platforms have successfully curbed piracy in sectors such as music and software, the fragmentation of video streaming services and the unmet demand in publishing are driving a resurgence in illicit consumption [7, 3, 9].

Someone can get a movie without spending any money on a pirated website, but finding the website is itself cumbersome and if one finds it, they will have to deal with a lot of popup ads and malwares which is very irritating and inconvenient to the consumer(EY The Rob Report, 2024). Accessing a pirate website is not easy and requires some technical knowledge, like torrents, VPN, IPTVs, etc. Although, with messaging app like Telegram, accessing pirated content has now become a bit easy in India. But in countries like Germany, accessing pirated websites are very difficult due to stricter law enforcement. It is not like the owner of pirated websites have benevolent motive behind running these web sites and providing the content at free of cost (EY The Rob Report, 2024). The pirated websites generate revenue through running ads. Between June 2020 and May 2021, pirated websites generated an estimated 1.34 billion US Dollars through advertisements (Statista Research Department, 2023).

This enduring cat-and-mouse game between legitimate producers and for-profit pirates necessitates a deeper, more nuanced understanding of the strategic forces at play. The economic literature has traditionally modeled this conflict through the lens of strategic competition, often focusing on variables such as price, pirate copy quality degradation, and enforcement penalties [8]. Foundational models, including spatial competition frameworks such as the Hotelling model, have provided tractable and powerful insights by representing consumer choice along a single dimension of heterogeneity, such as technical skill or "tech-savviness" [5]. In these frameworks, a consumer's decision to pirate is primarily a function of their ability to overcome technical barriers and the monetary cost of the legal alternative. Although elegant, this approach implicitly assumes that *ability* to pirate is perfectly correlated with *willingness* to do so, a simplification that overlooks the complex internal and social factors that govern consumer behavior.

In parallel, a rich body of research in behavioral economics, sociology, and psychology has consistently demonstrated that the decision to pirate is not purely transactional. These studies, often grounded in frameworks like the Theory of Planned Behavior (TPB), highlight the critical role of non-economic factors such as moral obligation, ethical judgment, and perceived social norms [6, 12]. Research shows that an individual's ethical orientation and moral beliefs are significant predictors of their intention to engage in piracy, independent of their technical skills or financial constraints [1]. This moral calculus is often shaped by rationalization techniques, where consumers can perceive piracy as a "victimless crime" to resolve the cognitive dissonance between their actions and their knowledge of the law [2]. However, these crucial behavioral insights have rarely been integrated into formal game-theoretic models of strategic interaction between firms. The economic analysis of firm strategy and the behavioral analysis of consumer ethics have largely proceeded on separate tracks, leaving a critical gap in our understanding of the digital marketplace.

This paper develops a model of strategic competition between a legitimate content producer and a for-profit pirate that explicitly accounts for multi-dimensional consumer heterogeneity. In contrast to existing models that conflate technical ability with willingness to pirate, we separate these dimensions by introducing a two-dimensional consumer space defined by both *technical savviness* and *ethical standards*. This distinction makes it possible to capture consumers who are able but unwilling to pirate, as well as those who are willing but lack the necessary skills.

Methodologically, we extend the Hotelling framework to incorporate this richer heterogeneity. This allows us to derive equilibrium outcomes for prices, access costs, and market shares under uniform distributions, while also providing a structure that generalizes to broader consumer distributions and alternative pirate revenue models. Substantively, our results show that enforcement intensity and technological barriers interact with the ethical composition of the consumer base in shaping equilibrium outcomes. This helps explain why similar enforcement policies can have markedly different effects across societies.

Our work contributes to three areas of research: (i) the industrial organization of digital markets, by extending spatial competition models to multi-dimensional heterogeneity; (ii) the economics of piracy, by embedding moral and behavioral factors into firm strategy; and (iii) policy design, by providing a framework to evaluate how interventions targeting technical access and moral suasion can work in tandem. In doing so, we offer a more complete account of the economic and ethical forces that shape the persistence of digital piracy.

The paper proceeds as follows. Section 2 develops the basic piracy model, where consumers are heterogeneous only in their level of technical knowledge. We derive the equilibrium and comparative statics under specific functional forms for utility and profit functions. Section 3 extends this baseline model by incorporating heterogeneity in consumers' moral standards, and conducts the same analysis in this richer setting. Section 4 generalizes the framework to allow for arbitrary probability distributions of consumer characteristics as well as general utility and profit functions. Section 5 concludes. This is followed by the references and an appendix that presents the details of the general model. Proofs for the functional-form cases are omitted, as they are straightforward to derive.

2 Model: A Basic Piracy Model

We consider two firms, referred to as the *original* and the *pirated* firm. The original firm enjoys a monopoly over the production of a digital good and operates under constant marginal cost $c > 0$. It sets a price p for the product. In contrast, the pirated firm reproduces the good and offers it to consumers at zero monetary cost. Nevertheless, obtaining the pirated version is not costless: consumers incur an implicit cost associated with the "difficulty" of accessing the pirated source (e.g., a website). We interpret this as a *transportation cost*[1], denoted by $t \geq 0$.

[1]This captures the "difficulty" a consumer faces in accessing the pirated source, such as navigating to a website or bypassing restrictions.

We will be utilizing the *Hotelling model* with exogenously given locations of both firms. The Hotelling model is a spatial competition framework that describes how firms compete in a linear market. In our case, the two firms are located at opposite ends of a line segment, and consumers are uniformly distributed along the line.

We consider heterogeneous consumers, who differ in a single dimension: their level of technical knowledge. Consumers are assumed to be located along the line segment, indexed by positions $x \in [0, 1]$. At one end of the line segment ($x = 0$), we have *tech noob*, the least technically proficient consumers, who find it very difficult to access pirate firm. At the other end ($x = 1$), we have the most technically proficient, or *tech-savvy*, consumers, who find it easiest to access the pirated firm.

The original firm is located at $x = 0$.[2] The pirated firm is located at $x = 1$.[3] Below, the figure demonstrates the locations of the original and pirated firms along the consumer heterogeneity line:

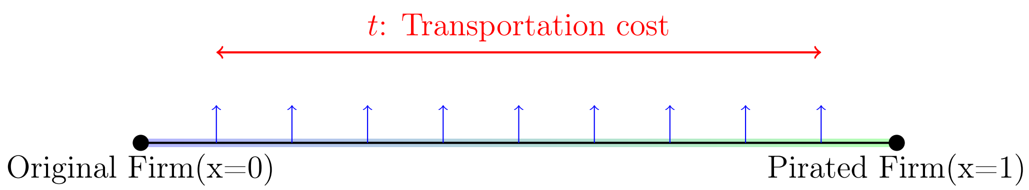

Figure 1: Hotelling line with original and pirated firms and heterogeneous consumers.

The utility of a consumer located at $x \in [0, 1]$ from purchasing the product from the original firm is given by

$$U_O = v - p, \tag{2.1}$$

where v is the intrinsic value of the good and p is the price set by the original firm.

The utility from purchasing from the pirated firm is given by

$$U_P = v - (1 - x)\, t, \tag{2.2}$$

where t denotes the transportation cost (difficulty in accessing the pirated website). The effective cost of accessing the pirated firm is $(1 - x)\, t$, which increases as a consumer's technical proficiency x decreases (i.e., as the consumer is "farther" from the pirated firm in terms of technical ability)

Both goods are assumed to be of the same quality, and we abstract from quality differences.[4]

[2]Without loss of generality, we place the original firm at $x = 0$ since it does not incur any transportation cost.

[3]This placement reflects the assumption that the most technically proficient consumers can easily access the pirated website, while less technical consumers face greater difficulty. Hence, it is natural to position the pirated firm at the opposite end of the line.

[4]For a normal consumer, the experience of using pirated goods does not lead to any significant loss in utility. For instance, the human ear can rarely differentiate between a 256 kbps MP3 and a FLAC song, unless the person is a sound engineer or professional. Consequently, we assume that both the original and pirated goods provide the same intrinsic value v to the consumer.

Accordingly, both the original and pirated products provide a utility value of v.

2.1 Analysis

Let $\tilde{x}$ denote the location of the consumer who is indifferent between purchasing from the original firm and the pirated firm. That is, $\tilde{x}$ is the consumer for whom the utility from both firms is equal: $v - p = v - (1 - \tilde{x})\,t.$. This gives us the following lemma.

Lemma 2.1. *Given that consumers are uniformly distributed along the line segment, the demand functions for the original and pirated firms are*

$$D_O(p,t) = \tilde{x} = 1 - \frac{p}{t}, \quad 0 < p < t, \tag{2.3}$$

$$D_P(p,t) = 1 - D_O(p,t) = \frac{p}{t}, \quad 0 < p < t. \tag{2.4}$$

Consumers with $x \leq \tilde{x}$ will buy from the original firm, while those with $x > \tilde{x}$ will buy from the pirated firm.

2.1.1 The Original Firm

The original firm has a monopoly on the production of the digital good and sets a price p for its product. The demand for the original product is given by $D_O(p)$. The original firm incurs a constant marginal cost $c \geq 0$ for producing the digital good. Thus, its profit is given by

$$\pi_O(p;t,c) = (p-c)\,D_O(p) = (p-c)\left(1 - \frac{p}{t}\right). \tag{2.5}$$

The original firm chooses the price p to maximize its profit. The optimization problem is:

$$\max_{0<p<t} \ \pi_O(p;t,c) = (p-c)\left(1 - \frac{p}{t}\right),$$

2.1.2 The Pirate Firm

The pirate firm copies the original firm's product, incurring a fixed production cost($\omega > 0$), and makes it available to consumers at zero price. Although the pirate firm does not charge consumers directly and hence does not include the demand function in its profit function in the usual way, it can still generate revenue through the sale of advertising space on its website.

The revenue for the pirate firm is given by: $R(F) = F \times D_P = (1-x) \times M = (1 - \frac{p}{t})$
The revenue function $R(F)$ captures the idea that the pirate firm earns advertising revenue proportional to its consumer base[5].

[5]Each consumer generates F units of revenue, so if a fraction $D_P = (1-x)$ of consumers use the pirate firm, the total advertising revenue is $R = F\,D_P$. In other words, revenue increases linearly with the number of consumers visiting the pirate website.

However, operating a pirate website entails legal risks. If caught, the pirate firm can be sued by the original firm and face significant penalties. We assume that the pirate firm is caught with probability ρ, and the cost if caught is M per consumer lost to piracy, with $M > 0$.

The probability of being caught is assumed to increase with the number of consumers using the pirate website, since higher usage makes it easier to trace the pirate firm and increases the attention from the original firm. Let $1 - x$ denote the fraction of consumers using the pirate firm. Then, the total expected cost of being caught is

$$C = \rho\,(1-x)\,M - \omega = M\,(1-x)^2 - \omega = M\left(\frac{p}{t}\right)^2 - \omega,$$

since $1 - x = D_P = \frac{p}{t}$.

Hence, the pirate firm's profit is

$$\pi_P(t; p, F, M, \omega) = F\,\frac{p}{t} - M\left(\frac{p}{t}\right)^2 - \omega. \tag{2.6}$$

The pirate firm chooses the effective transportation parameter t to maximize its profit:

$$\max_t \; \pi_P(t; p, F, M, \omega) = F\,\frac{p}{t} - M\left(\frac{p}{t}\right)^2 - \omega.$$

2.1.3 Equilibrium

We determine the equilibrium using the *Nash equilibrium* concept: each firm (the original firm and the pirate) chooses their strategy; price p for the original, transfer t for the pirate,optimally, given the other's choice. Solving the original firm's and the pirate firm's optimization problems, we obtain the following lemma.

Lemma 2.2. *Solving the original firm's and pirate firm's optimization problems, we obtain their best-response functions:*

$$p^*(t) = \frac{t+c}{2}, \qquad \pi_O(p) \textit{ is strictly concave in } p \textit{ since } \frac{\partial^2 \pi_O}{\partial p^2} = -\frac{2}{t} < 0 \textit{ for } t > 0, \tag{2.7}$$

$$t^*(p) = \frac{p}{D_P^*} = \frac{2M}{F}\,p, \qquad \pi_P(D_P) \textit{ is strictly concave in } D_P \textit{ since } \frac{d^2 \pi_P}{dD_P^2} = -2M < 0. \tag{2.8}$$

Solving for the reaction functions of both firms yields the Nash equilibrium. The following proposition shows the equilibrium price p and transportation cost t.

Proposition 2.3. *The equilibrium price p and corresponding t are*

$$p^* = \frac{cF}{2(F-M)}, \tag{2.9}$$

$$t^* = \frac{cM}{F-M}. \tag{2.10}$$

The equilibrium exists and is interior when

$$M < F < 2M, \quad F > 0, \ M > 0, \ p < v, \ c > 0.$$

Proposition 2.4. *At the equilibrium* (p^*, t^*)*, the corresponding profits are:*

$$\pi_O^* = \frac{c\,(2M - F)^2}{4M(F - M)} > 0, \quad \pi_P^* = \frac{F^2}{4M} - \omega. \tag{2.11}$$

The pirate firm earns nonnegative profit, when, $\omega \leq \frac{F^2}{4M}$*. If this fails, the pirate exits* $(D_P = 0)$ *and the interior equilibrium no longer applies.*

The following remarks summarize key boundary outcomes and special cases that arise when the interior equilibrium is infeasible or constrained. They clarify how participation, valuation caps, and cost conditions affect the feasible strategies and payoffs of the pirate and the original firm. While they are not formal propositions or lemmas, they help the reader interpret the corner solutions and provide intuition for the model's limiting cases. These remarks can be skipped on a first reading without affecting the main results.

Remark 2.5. If the interior price implied by the best responses exceeds v, the original firm's demand collapses to zero for $p > v$. In this case, the constrained best response is $p = v$[6]. The pirate still targets its optimal market share, choosing t to implement $D_P^* = F/(2M)$. The constrained equilibrium is therefore

$$p^{c} = v, \quad t^{c} = \frac{2M}{F}v, \quad D_P^{c} = \frac{F}{2M}, \quad D_O^{c} = 1 - \frac{F}{2M}.$$

The corresponding profits are

$$\pi_O^{c} = (v - c)\left(1 - \frac{F}{2M}\right), \quad \pi_P^{c} = \frac{F^2}{4M} - \omega.$$

This equilibrium is valid if $v \geq 0$, $F \in (M, 2M)$, and $p^* > v$ (i.e., $cF/(2(F - M)) > v$). If $(v - c)\left(1 - \frac{F}{2M}\right) < 0$, the original firm can always deviate to $p > v$ and obtain zero profit, so the relevant equilibrium payoff is $\max\{0, \pi_O^{c}\}$.

Remark 2.6. When fixed/enforcement costs are too high, the pirate exits ($\pi_P < 0$) and the original becomes a monopolist facing an outside option. Then

$$D_O = \begin{cases} 1, & p \leq v, \\ 0, & p > v. \end{cases}$$

[6] Since $D_O = 1 - p/t$ only applies when $U_O \geq U_P$ and $U_O \geq 0$; if $p > v$, then $U_O < 0$ and demand is zero.

Optimal monopoly pricing is

$$p^{\text{mono}} = \begin{cases} v, & v \geq c, \\ \text{any } p > v, & v < c \text{ (no sales)}, \end{cases} \qquad \pi_O^{\text{monopoly}} = \max\{v - c, 0\}, \quad D_O^{\text{mono}} = \mathbb{1}\{v \geq c\}.$$

The pirate's profit is $\pi_P = -\omega < 0$, so exit is payoff-maximizing. This corner outcome is relevant for any $F > 0$, $M > 0$ with $\omega > F^2/(4M)$.

Remark 2.7. If the pirate's unconstrained target $D_P^* = F/(2M) \geq 1$ and the outside option does not block piracy, the equilibrium pushes to the boundary $D_P = 1$ (everyone pirates). This requires

$$p \geq t \quad \text{and} \quad t \leq v,$$

so that the original faces zero demand and the pirate's utility is nonnegative for all $x \in [0, 1]$.

Under these conditions, the corner equilibrium is

$$D_P^\dagger = 1, \quad D_O^\dagger = 0, \quad p^\dagger \geq t^\dagger, \quad t^\dagger \leq v,$$

with profits

$$\pi_O^\dagger = 0, \quad \pi_P^\dagger = F - M - \omega.$$

If $F - M - \omega < 0$, the pirate prefers to exit, reverting to the monopoly outcome. If the valuation cap[7] blocks piracy for some consumers ($t > v$), then $D_P < 1$ and the all-pirate corner is infeasible.

Remark 2.8. If the interior equilibrium is infeasible ($p^* \leq 0$ or $F = M$), the game attains a boundary equilibrium determined by pirate participation and the valuation cap v. Two cases arise:

1. **Pirate exit:** if $\omega > F^2/(4M)$, the pirate chooses not to operate, and the original becomes a monopolist (Subsection 7.2).
2. **Constrained coexistence:** if $\omega \leq F^2/(4M)$ and piracy is viable, the pirate targets $D_P^\star = \frac{F}{2M} \in [0, 1/2]$ by setting $t = \frac{2M}{F} p$, while the original chooses $p \in [0, v]$. Maximizing $(p - c)\left(1 - \frac{p}{t}\right)$ either
   (a) hits the cap $p = v$ if v supports positive profit, yielding the constrained pair of Subsection 7.1, or
   (b) chooses $p > v$ to obtain zero revenue if $(v - c)\left(1 - \frac{F}{2M}\right) \leq 0$.

Hence, when $F \leq M$, the equilibrium is a *boundary* one, determined by the valuation cap and participation rather than interior first-order conditions.

[7]The valuation cap v represents the maximum price a consumer is willing to pay for the good or service. It limits the original firm's pricing and the pirate's effective transfer t. If $t > v$, some consumers cannot participate, preventing the pirate from achieving full market coverage ($D_P < 1$).

2.2 Comparative Statics

Comparative statics is the analysis of how the equilibrium outcomes of a model change in response to variations in the underlying parameters. In our setting, once the equilibrium demand and pricing behavior of firms are characterized, we can ask how these outcomes respond to changes in key parameters such as the transportation cost t or the price p, or penalty per user(which is paid by pirate firm),F or per user ad revenue, M. This exercise allows us to understand the economic intuition of the model more clearly: for instance, whether higher access costs to piracy (a larger t) shift more consumers toward the original firm, or how the original firm's demand reacts to changes in its own price. Since, the profit functions in equilibrium are functions of M and F, so we would also like to see how does change in M and F changes the profits of the firms. Such analysis helps connect the theoretical model to real-world policy or market implications.

We introduce the following proposition:

Proposition 2.9. *We have* $D_O(p,t) = 1 - \dfrac{p}{t}$ *for* $0 < p < t$*, with* $t > 0$ *and the original firm's profit is given by*

$$\pi_O(p; t, F, M) = (p - M)\, D_O(p,t) - F,$$

with $M, F > 0$*. The following comparative–static results hold:*

$$\frac{\partial D_O}{\partial t} = \frac{p}{t^2} \; > \; 0, \tag{2.12}$$

$$\frac{\partial \pi_O}{\partial t} = \frac{(p-c)\,p}{t^2} \; > \; 0, \tag{2.13}$$

$$\frac{\partial \pi_O}{\partial F} = (p-c)\frac{p}{t^2}\left(-\frac{2Mp}{F^2}\right) \; < \; 0, \tag{2.14}$$

$$\frac{\partial \pi_O}{\partial M} = (p-c)\frac{p}{t^2}\frac{2p}{F} \; > \; 0, \tag{2.15}$$

$$\frac{\partial^2 \pi_O}{\partial t\, \partial p} = \frac{2p-c}{t^2}, \tag{2.16}$$

The comparative statics provide the following intuition. The positive effect of transportation cost, $\frac{\partial \pi_O}{\partial t} > 0$, reflects that as it becomes harder for consumers to access the pirate firm, more consumers buy the original, increasing its profit. The negative effect of the penalty per user on the pirate firm, $\frac{\partial \pi_O}{\partial M} < 0$, shows that higher penalties reduce the attractiveness of piracy, which indirectly affects the original firm's strategic positioning. The positive effect of the pirate firm's ad revenue, $\frac{\partial \pi_O}{\partial F} > 0$, indicates that higher revenue from piracy allows the pirate firm to sustain competition, which can make the original firm's pricing more profitable when adjusted. The positive cross-partial $\frac{\partial^2 \pi_O}{\partial t\, \partial p} > 0$ demonstrates strategic complementarity: as piracy becomes more costly, increasing the original firm's price becomes even more profitable. Overall, the demand function confirms that the original and pirated goods are pure substitutes, while the profit derivatives illustrate how changes in the pirate firm's costs and revenues influence the original firm's optimal decisions.

Proposition 2.10. *Given the pirate firm's demand* $D_P = \frac{p}{t}$ *and profit* $\pi_P = F \cdot \frac{p}{t} - M\left(\frac{p}{t}\right)^2$, *the comparative statics are:*

$$\frac{\partial \pi_P}{\partial p} = 0, \quad \frac{\partial D_P}{\partial p} = \frac{1}{t} > 0, \quad \frac{\partial \pi_P}{\partial F} = \frac{F}{2M} > 0, \quad \frac{\partial \pi_P}{\partial M} = -\frac{F^2}{4M^2} < 0.$$

The comparative statics of the pirate firm show how profit and demand respond to key parameters. $\frac{\partial \pi_P}{\partial p} = 0$ indicates that profit is independent of the original firm's price. Demand, $\frac{\partial D_P}{\partial p} = 1/t > 0$, rises as the original good becomes relatively more expensive. Profit increases with advertising revenue, $\frac{\partial \pi_P}{\partial F} = F/(2M) > 0$, and decreases with per-user penalty, $\frac{\partial \pi_P}{\partial M} = -F^2/(4M^2) < 0$. These results show that the pirate firm's demand is sensitive to relative prices, while profit is determined by monetization and legal costs.

2.3 Discussion

The model examines the interaction between an original firm and a pirate firm in a digital goods market, capturing consumer heterogeneity and strategic firm behavior within a simple Hotelling framework. Consumers are distributed along a line representing technical ability, with those facing higher effective costs to access the pirate more likely to purchase the original product. This generates the demand function $D_O(p,t) = 1 - p/t$, directly linking consumer accessibility to legal purchases.

Comparative statics reveal how market conditions and firm strategies interact. As the effective cost of piracy t rises, demand shifts toward the original firm, raising profits. The positive cross-partial $\frac{\partial^2 \pi_O}{\partial t\, \partial p} > 0$ indicates that higher piracy costs amplify the profitability of raising the original product's price, reflecting strategic complementarity between accessibility and pricing. Changes in pirate firm parameters further shape market outcomes: higher per-user penalties M reduce piracy, indirectly benefiting the original firm, while higher ad revenue F enables the pirate to sustain competition, constraining the original firm's pricing yet motivating careful strategic adjustments.

The pirate firm's comparative statics provide complementary insights. Its profit is independent of the original firm's price, $\frac{\partial \pi_P}{\partial p} = 0$, while demand rises when the original good becomes more expensive, $\frac{\partial D_P}{\partial p} = 1/t > 0$, reflecting pure substitutability from the consumer perspective. Profit increases with higher ad revenue, $\frac{\partial \pi_P}{\partial F} = F/(2M) > 0$, and decreases with higher per-user penalties, $\frac{\partial \pi_P}{\partial M} = -F^2/(4M^2) < 0$. Together, these results show that the pirate firm's market reach responds to relative prices, while profitability depends on monetization and legal risks, linking consumer behavior, strategic choice, and enforcement.

Boundary scenarios further clarify equilibrium dynamics. When piracy is unprofitable or demand caps bind near $D_O \to 1$ or $D_O \to 0$, the original firm behaves as a monopolist;

widespread piracy can sharply reduce its market share. These limits highlight how even a simple linear model can capture shifts in market power and profitability.

Policy implications follow naturally. Measures that increase piracy costs, such as stronger enforcement or higher legal penalties, shift consumers toward legal channels, raising original firm profits without changing product quality or price. Conversely, policies that enhance piracy profitability, for instance through ad revenue, intensify competition and reduce legal market share. The model's simplicity makes these mechanisms transparent, illustrating the interplay of accessibility, firm strategy, and regulatory incentives.

While the model abstracts from network effects, dynamic interactions, and heterogeneous valuations, its tractability provides a clear baseline for understanding digital goods markets. By linking comparative statics, strategic complementarities, and policy levers, it highlights how accessibility, enforcement, and firm behavior jointly determine equilibrium outcomes.

3 Model: Extension

In this extended framework, consumers differ along two independent dimensions. Each consumer is characterized by a type $x \sim U[0,1]$, capturing their degree of *tech-savviness*, and by a type $m \sim U[0,1]$, representing their *ethical standard.* A higher x implies that technical barriers to accessing pirated content are easier to overcome, while a higher m reflects stronger moral resistance to engaging in piracy. The market mass is normalized to one, so aggregate demand arises directly from the distribution of types across the unit square.

The original producer sets a price p and incurs a marginal cost $c \geq 0$. The pirate, in contrast, has no marginal cost of production but must pay a fixed cost $\omega > 0$. The parameter $t \geq 0$ represents the technical difficulty of accessing pirated content, which enters as a disutility term, while moral disutility is scaled by a factor $\lambda > 0$.

Relative to the baseline model with a representative consumer, this specification introduces heterogeneity in willingness to pirate: those with low x (low tech-savviness) or high m (high ethical resistance) are less likely to choose the pirate option. Equilibrium is still defined as a Nash equilibrium in the strategies of the original (choice of p) and the pirate (choice of t), but the demand each faces is now determined by the joint distribution of (x, m) rather than by a single cutoff type. This allows the equilibrium to reflect both technological and moral barriers to piracy.

The utility from purchasing the original good is

$$U_O = v - p, \tag{3.1}$$

where v denotes the consumer's valuation and p the price set by the original producer. If instead the consumer pirates, the utility is

$$U_P(x,m) = v - (1-x)t - \lambda m, \tag{3.2}$$

where $(1-x)t$ captures the disutility from technical difficulty, which is lower for more tech-savvy consumers (x close to 1), and λm reflects the moral cost of piracy, increasing with the consumer's ethical standard m.

Each consumer compares these payoffs: she buys the original if $U_O \geq \max\{U_P, 0\}$, pirates if $U_P \geq \max\{U_O, 0\}$, and otherwise opts out of the market if both payoffs are negative.

3.1 Analysis

Consider the most ethical consumer, indexed by $m = 1$. For such a consumer, piracy is strictly unattractive whenever

$$U_P(x,1) = v - (1-x)t - \lambda < 0 \iff \lambda > v - (1-x)t.$$

Since the inequality must hold uniformly for all levels of tech-savviness x, a sufficient condition is simply

$$\lambda > v,$$

because $t \geq 0$ reduces the right-hand side further. This requirement can thus be interpreted as a "completely ethical" condition: if the moral cost of piracy is stronger than the consumer's entire valuation, even the most favorable circumstances for piracy cannot make it appealing. We have the following proposition.

Proposition 3.1. *Piracy is strictly unattractive any consumer iff $\lambda > v$, and $t \geq 0$.*

Lemma 3.2. *The moral cutoff for each consumer, defined by*

$$m^*(x) = \frac{p - (1-x)t}{\lambda},$$

represents the minimum ethical standard m required for a consumer of type x to prefer the original over piracy. Consumers with $m \geq m^*(x)$ buy the original, while those with $m < m^*(x)$ turn to the pirate.

Since $m \sim U[0,1]$, the aggregate demands are

$$D_O(p,t) = 1 - \frac{p}{\lambda} + \frac{t}{2\lambda}, \qquad D_P(p,t) = \frac{p}{\lambda} - \frac{t}{2\lambda}.$$

An interior split[8] *occurs when*

$$0 < p - \tfrac{t}{2} < \lambda.$$

[8] That is, both the original and the pirate attract a positive share of consumers, so neither is driven out of the market.

Consumers choose between purchasing the original content or turning to pirated versions based on three key factors: the price of the original, the difficulty of accessing pirated content, and their personal ethical standard. The concept of a moral cutoff captures the minimum ethical threshold a consumer must meet to prefer the original over piracy. In simple terms, more ethical consumers are willing to pay for the original, while less ethical ones are more likely to pirate.

The condition of interior split, both the original and the pirate attract a positive share of consumers. This situation arises when the price of the original is neither too high to push everyone toward piracy nor too low to make piracy unattractive. In such a balanced market, neither the original nor the pirate is completely driven out, and the distribution of consumer choices reflects both ethical considerations and the relative attractiveness of price and access difficulty.

3.1.1 The Original Firm

The original firm has a monopoly on the production of the digital good and sets a price p for its product. The demand for the original product depends on both the price and the *transportation cost*,t, and is given by $D_O(p,t) = 1 - \frac{p}{\lambda} + \frac{t}{2\lambda}$. The original firm incurs a constant marginal cost $c \geq 0$ for producing the digital good. Thus, its profit is

$$\pi_O(p;t,c) = (p-c)\,D_O(p,t) = (p-c)\left(1 - \frac{p}{\lambda} + \frac{t}{2\lambda}\right).$$

The original firm chooses the price p to maximize its profit. The optimization problem is

$$\max_{0<p<\lambda+t/2} \pi_O(p;t,c) = (p-c)\left(1 - \frac{p}{\lambda} + \frac{t}{2\lambda}\right),$$

where the upper bound ensures demand remains positive.

3.1.2 The Pirate Firm

The revenue of the pirate firm is simply the total income it receives from selling the pirated good. Denoting revenue by R, we have

$$R = F \times D_P,$$

where F is revenue generated per consumer visiting the pirate firm website, and D_P is the demand for the pirate product.

The total cost of the pirate firm, denoted C, consists of two components: a variable expected penalty and a fixed cost. The expected penalty arises because there is a probability ρ of being caught pirating, which we assume grows with the demand for pirated content: $\rho = D_P$. If the pirate is caught, they face a penalty M per consumer using the pirated good, so the expected penalty is $\rho\, D_P\, M = D_P^2\, M$. Additionally, the pirate firm incurs a fixed cost ω for operating and maintaining the pirated service. Hence, the total cost is

$$C = \rho\, D_P\, M + \omega = D_P\, M\, D_P + \omega = M D_P^2 + \omega.$$

Thus, the profit for the pirate firm is given by:

$$\pi_P(t) = R(t) - C(t) = F\left(\frac{p}{\lambda} - \frac{t}{2\lambda}\right) - \left[M\left(\frac{p}{\lambda} - \frac{t}{2\lambda}\right) + \omega\right]. \tag{3.3}$$

Intuitively, this setup captures the idea that as more consumers use the pirated product, the pirate firm earns more revenue but also faces a higher probability of being caught and paying penalties, in addition to fixed operational costs.

The pirate firm chooses the technical barrier t to maximize its profit:

$$\max_{t>0}\ \pi_P(t) = R(t) - C(t) = F\left(\frac{p}{\lambda} - \frac{t}{2\lambda}\right) - \left[M\left(\frac{p}{\lambda} - \frac{t}{2\lambda}\right) + \omega\right],$$

3.1.3 Equilibrium

The original firm sets its price to maximize profit, taking the pirate's transportation cost t as given. Its profit is

$$\pi_O(p;t) = (p-c)\, D_O(p,t) = (p-c)\left(1 - \frac{p}{\lambda} + \frac{t}{2\lambda}\right),$$

where c is the marginal cost. The first-order condition for profit maximization with respect to p is

$$\frac{\partial \pi_O}{\partial p} = \left(1 - \frac{p}{\lambda} + \frac{t}{2\lambda}\right) - \frac{1}{\lambda}(p-c) = 0,$$

which yields the original firm's best response function

$$p^{\mathrm{BR}}(t) = \frac{\lambda + \frac{t}{2} + c}{2}.$$

The second derivative $\frac{\partial^2 \pi_O}{\partial p^2} = -\frac{2}{\lambda} < 0$ confirms that this choice maximizes profit.

The pirate firm chooses the transportation cost t to maximize its own profit, which depends on the share of consumers using the pirated good:

$$D_P = \frac{p}{\lambda} - \frac{t}{2\lambda}.$$

The pirate's profit is decreasing in expected penalties, which increase with the demand for piracy. Applying the chain rule,

$$\frac{d\pi_P}{dt} = (F - 2MD_P)\,\frac{dD_P}{dt} = 0,$$

we obtain the profit-maximizing demand share for piracy, $D_P = \frac{F}{2M}$, and the corresponding best response transportation cost

$$t^{\mathrm{BR}}(p) = 2p - \frac{\lambda F}{M}.$$

Concavity in t follows from the concavity of π_P in D_P and the linear mapping from t to D_P.

Lemma 3.3 (Best Response Functions)**.** *The best response functions of the original and pirate firms are given by*

$$p^{\mathrm{BR}}(t) = \frac{\lambda + \frac{t}{2} + c}{2}, \tag{3.4}$$

$$t^{\mathrm{BR}}(p) = 2p - \frac{\lambda F}{M}, \tag{3.5}$$

These functions characterize the optimal strategy of each firm given the choice of the other, forming the foundation for equilibrium analysis.

Solving the system of best responses

$$p = p^{\mathrm{BR}}(t) = \frac{\lambda + \frac{t}{2} + c}{2}, \qquad t = t^{\mathrm{BR}}(p) = 2p - \frac{\lambda F}{M},$$

we substitute $t^{\mathrm{BR}}(p)$ into $p^{\mathrm{BR}}(t)$ to obtain

$$p^* = c + \lambda\left(1 - \frac{F}{2M}\right), \qquad t^* = 2c + 2\lambda\left(1 - \frac{F}{M}\right).$$

The corresponding equilibrium demands are

$$D_P^* = \frac{F}{2M}, \qquad D_O^* = 1 - \frac{F}{2M}.$$

At equilibrium, we also have the identity

$$p^* - \frac{t^*}{2} = \lambda\, D_P^* = \frac{\lambda F}{2M},$$

so the interior condition $0 < p^* - \frac{t^*}{2} < \lambda$ is satisfied if and only if $0 < \frac{F}{2M} < 1$.

Proposition 3.4. *The unique interior equilibrium of the model is characterized by the following values:*

$$p^* = c + \lambda\left(1 - \frac{F}{2M}\right), \qquad t^* = 2c + 2\lambda\left(1 - \frac{F}{M}\right),$$

$$D_O^* = 1 - \frac{F}{2M}, \qquad D_P^* = \frac{F}{2M}.$$

This equilibrium exists under the interior condition

$$0 < \frac{F}{2M} < 1,$$

which ensures that both the original and the pirate attract positive shares of consumers.

At equilibrium, the original sets a price that balances its profit margin against the share lost to piracy, while the pirate sets its "transportation cost" to capture a fraction of the market, resulting in stable, positive demands for both products.

Proposition 3.5. *At the interior equilibrium, the profits of the original and pirate firms are*

$$\pi_O^* = \lambda \left(1 - \frac{F}{2M}\right)^2, \qquad \pi_P^* = \frac{F^2}{4M} - \omega.$$

The following remarks are important for understanding the practical relevance of the derived equilibrium. While the interior Nash equilibrium provides analytical solutions for prices, transfers, and market shares, these solutions are only meaningful if they satisfy feasibility and nonnegativity constraints. The remarks highlight the parameter conditions that ensure a valid interior equilibrium and discuss corner cases where the interior solution is no longer feasible, leading to boundary outcomes for the pirate's share and the original's demand.

Remark 3.6 (Interior Nash Validity)**.** The interior Nash equilibrium derived above is valid under the conditions:

$$\lambda > 0, \quad M > 0, \quad F \geq 0, \quad \omega \geq 0, \quad 0 < \frac{F}{M} < 2, \quad t^* \geq 0.$$

Since $t^* = 2c + 2\lambda \left(1 - \frac{F}{M}\right)$, nonnegativity requires

$$\frac{F}{M} \leq 1 + \frac{c}{\lambda}.$$

If $\frac{F}{M}$ exceeds this bound while remaining less than 2, the pirate's best response would imply $t^* < 0$, which is infeasible; the equilibrium then moves to a boundary with $t = 0$.

Remark 3.7 (Corner Cases: Share Caps)**.** Because shares are constrained to $D_P \in [0, 1]$:

- If $\frac{F}{2M} \leq 0$ (e.g., $F = 0$), then $D_P^\diamond = 0$; the pirate exits (t hits the boundary), $D_O = 1$, and the original sets p on the boundary (interior FOC does not apply).
- If $\frac{F}{2M} \geq 1$ (i.e., $\frac{F}{M} \geq 2$), the pirate would like $D_P = 1$ (everyone pirates), which corresponds to $p - \frac{t}{2} = \lambda$ (boundary); the interior FOCs no longer apply and the original's demand collapses ($D_O = 0$).

3.2 Comparative Statics

We examine how the interior equilibrium outcomes (p^*, t^*) and the corresponding profits respond to changes in the model parameters. Here, t^* represents the transportation cost faced by the pirate, and p^* is the original's price.

Proposition 3.8. *Consider the original and pirate firms as defined above. The comparative statics of equilibrium quantities and profits with respect to exogenous parameters are as follows:*

$$\frac{\partial D_O}{\partial t} = \frac{1}{2\lambda} > 0, \qquad \frac{\partial D_P}{\partial t} = \frac{1}{\lambda} > 0,$$
$$\frac{\partial \pi_O}{\partial t} = (p-c)\frac{1}{2\lambda} > 0, \qquad \frac{\partial \pi_O}{\partial F} = 2\lambda\Big(1 - \frac{F}{2M} \cdot \frac{-1}{2M}\Big) < 0,$$
$$\frac{\partial \pi_O}{\partial M} = 2\lambda\Big(1 - \frac{F}{2M}\Big)\frac{F}{2M^2} > 0, \qquad \frac{\partial \pi_P}{\partial F} = \frac{F}{2M} > 0,$$
$$\frac{\partial \pi_P}{\partial M} = -\frac{F^2}{4M^2} < 0, \qquad \frac{\partial \pi_P}{\partial p} = 0.$$

The comparative statics reveal how equilibrium outcomes respond to changes in exogenous parameters. The original firm's market share and profit increase with the pirate's technology level t, reflecting that higher t expands the total market accessible to both firms. However, the original firm's profit decreases with an increase in the pirate's scale F, since a larger pirate presence erodes the original firm's market share, while it rises with the pirate's expected legal cost parameter M, as higher M reduces the pirate's profitability and competitive pressure.

For the pirate, both profits and market share increase with its own scale F, but profits fall with higher legal enforcement M, reflecting the costliness of expanding the user base under stricter enforcement. The pirate's profit is independent of the original firm's price p in this setting, indicating that, at equilibrium, the pirate optimally targets its user share rather than responding to price directly.

Intuitively, these results illustrate the strategic interactions between a regulated original firm and a pirate firm: the original firm benefits from higher legal enforcement or weaker pirate scale, while the pirate benefits from its own scale and low enforcement. The positive sign of the cross-derivatives (e.g., $\partial D_O/\partial t > 0$) indicates that the firms' strategies are, in this case, strategic complements with respect to market expansion, as higher pirate technology expands the market for both.

3.3 Discussion

The model illustrates how consumer heterogeneity in technical skills and moral attitudes shapes market outcomes. Consumers with higher technical ability are better equipped to access pirated content, while more ethically conscious consumers are more likely to purchase the original. The cutoff $m^*(x)$ neatly captures this sorting: consumers with $m \geq m^*(x)$ buy the original, and those with $m < m^*(x)$ turn to piracy. This aligns with intuition—ethical and less tech-savvy consumers remain with the original product, while tech-savvy, less moral consumers are more likely to pirate.

From the firms' perspective, the original sets its price to balance profit margins against the erosion of demand from piracy, whereas the pirate chooses the level of technical difficulty to optimize market share given enforcement and cost constraints. In interior equilibrium, both firms can coexist with positive market shares when the parameters satisfy $0 < F/(2M) < 1$. The equilibrium prices and pirate strategies adjust so that the pirate targets a fixed market share, while the original captures the remaining consumers.

The comparative statics provide further insights. The original firm's demand and profit increase with higher pirate-imposed technological barriers (t), reflecting that making piracy harder benefits the original. The original's profit decreases as the pirate's scale (F) grows, but increases with stronger enforcement (M), which reduces the pirate's effective market. For the pirate, profits rise with its own scale (F) but decline with stricter enforcement (M). Interestingly, the pirate's profit does not depend on the original's price at equilibrium, showing that the pirate optimally targets its share independently of the original's pricing.

These results suggest that policies affecting enforcement (M) or the ease of piracy (t) can meaningfully shift market shares and profits. Similarly, factors influencing consumer ethics or technical ability directly shape the equilibrium distribution of original and pirated consumption. The model provides a tractable framework for understanding how firms respond strategically in a mixed market of original and pirated goods.

4 The General Model

4.1 Model Setup

4.1.1 Consumer Space and Distributions

Consumers are described along two independent dimensions. The first is technical savviness, denoted $x \in [0, 1]$, where $x = 0$ represents the least tech-savvy "newbie" and $x = 1$ the most proficient user. We assume that x is distributed according to F_X with density $f_X(x)$ on $[0, 1]$. In earlier models this was taken to be uniform, $f_X(x) = 1$, but here we allow for a more general distribution. The second dimension is ethical standards, denoted $m \in [0, 1]$, where $m = 0$ corresponds to consumers who feel no moral disutility from piracy, while $m = 1$ reflects the strongest moral opposition to it. We assume m is distributed according to F_M with density $f_M(m)$, supported on $[0, 1]$. These two traits jointly determine consumer behavior: more technically savvy consumers face lower costs in accessing pirated goods, while stronger ethical standards discourage piracy even when access is easy. Since x and m are independent, the joint density is given by $f(x, m) = f_X(x)\, f_M(m)$ on the unit square $[0, 1] \times [0, 1]$.

This two-dimensional setup extends the earlier one-dimensional Hotelling-type model, where only accessibility costs determined consumer choice. By incorporating ethical considerations alongside technical ability, it captures a richer spectrum of heterogeneity, allowing the model to distinguish between consumers deterred by ability constraints and those restrained by moral concerns.

4.1.2 Primitives and assumptions

Consumers are characterized by two independent traits: their technical ability $x \in [0,1]$ and their ethical standard $m \in [0,1]$. The joint distribution factorizes into $f(x,m) = f_X(x)\, f_M(m)$, where f_X and f_M are probability densities supported on $[0,1]$. By construction, both integrate to one, and the cumulative distribution of ethical standards is $F_M(m) = \int_0^m f_M(s)\, ds$. Technical ability determines how easily a consumer can access pirated goods. We capture this through a function $d(x)$, which measures the effective *technical friction per unit cost* t faced by type x. Consumers with higher x face lower frictions, so $d(x)$ is nonincreasing, with $d(0) = 1$ for the least tech-savvy consumer and $d(1) = 0$ for the most proficient one.

Assumption 1. For all $x \in [0,1]$, the function $d : [0,1] \to [0,1]$ satisfies $d(0) = 1$, $d(1) = 0$, and $d'(x) < 0$. This formalizes the idea that technical frictions diminish strictly as consumers become more skilled.

We introduce the key parameters of the model. Consumers derive an intrinsic value $v > 0$ from accessing the content. Their moral costs are scaled by a parameter $\lambda > 0$, while the original firm produces at marginal cost $c \geq 0$. The pirate firm collects ad revenue proportional to $F \geq 0$, faces an enforcement-related cost parameter $M > 0$, and incurs a fixed technology cost $\omega \geq 0$. The strategic choices are straightforward: the original firm sets a price $p \geq 0$, while the pirate firm selects a technology parameter $t \geq 0$, which scales the access difficulty. Since consumers differ in technical ability x, the friction they encounter when pirating is given by $d(x)\, t$, where $d(x)$ is nonincreasing.

A consumer of type (x,m) faces two utility options. If she purchases from the original firm, her payoff is

$$U_O(x,m) = v - p. \tag{4.1}$$

If instead she pirates, her payoff is

$$U_P(x,m) = v - d(x)\, t - \lambda m. \tag{4.2}$$

Here m captures the moral cost of piracy: higher values of m reduce the attractiveness of pirated consumption. A fully moral consumer ($m = 1$) may even obtain negative payoff from piracy if

$$v - d(x)\, t - \lambda < 0 \quad \Longleftrightarrow \quad \lambda > v - d(x)\, t. \tag{4.3}$$

Since $d(x)$ decreases in x with $d(0) = 1$ and $d(1) = 0$, a sufficient condition ensuring this inequality for all x is simply $\lambda > v$.

The indifference condition between buying and pirating yields a cutoff in moral standards. For a given x, the consumer is indifferent if $U_O(x,m) = U_P(x,m)$, which implies

$$m^*(x) = \frac{p - d(x)\, t}{\lambda}. \tag{4.4}$$

Consumers with $m \leq m^*(x)$ choose piracy, while those with $m > m^*(x)$ purchase from the original firm. When aggregating across consumers, this cutoff is truncated to the unit interval $[0, 1]$ to reflect the support of m.

4.2 Analysis

Without imposing interior solution, the original and pirate firm's demand shares are

$$D_O(p,t) = \int_0^1 \Big[1 - F_M\big(\max\{0, \min\{1, m^*(x)\}\}\big)\Big]\, f_X(x)\, dx, \tag{4.5}$$

$$D_P(p,t) = \int_0^1 F_M\big(\max\{0, \min\{1, m^*(x)\}\}\big)\, f_X(x)\, dx. \tag{4.6}$$

Remark. The distributions F_X, F_M enter *only* in the aggregation (integration) stage; individual utility depends on the raw (x, m) types through $d(x)$ and m.

For the remainder of the analysis, we impose the *interior condition*

$$0 < m^*(x) < 1 \quad \text{for all } x \in [0, 1],$$

which guarantees that both options—purchasing from the original firm and pirating—are chosen with positive probability in equilibrium.

Under this condition, the demand shares for the original good and for piracy can be expressed as

$$D_O(p,t) = \int_0^1 \left[1 - F_M\!\left(\frac{p - d(x)t}{\lambda}\right)\right] f_X(x)\, dx, \tag{4.7}$$

$$D_P(p,t) = \int_0^1 F_M\!\left(\frac{p - d(x)t}{\lambda}\right) f_X(x)\, dx = 1 - D_O(p,t). \tag{4.8}$$

Here F_M denotes the cumulative distribution function of moral costs m, while f_X is the density of consumer types x.

To characterize how these demand shares respond to price p and enforcement intensity t, define the auxiliary functions

$$\Phi_0(p,t) := \int_0^1 f_M\!\left(\frac{p - d(x)t}{\lambda}\right) f_X(x)\, dx, \tag{4.9}$$

$$\Phi_1(p,t) := \int_0^1 d(x)\, f_M\!\left(\frac{p - d(x)t}{\lambda}\right) f_X(x)\, dx. \tag{4.10}$$

Both Φ_0 and Φ_1 are nonnegative and bounded. Using the chain rule, and recalling that

$F'_M = f_M$, we obtain

$$\frac{\partial D_O}{\partial p}(p,t) = -\frac{1}{\lambda}\,\Phi_0(p,t), \qquad \frac{\partial D_O}{\partial t}(p,t) = \frac{1}{\lambda}\,\Phi_1(p,t), \tag{4.11}$$

$$\frac{\partial D_P}{\partial p}(p,t) = \frac{1}{\lambda}\,\Phi_0(p,t), \qquad \frac{\partial D_P}{\partial t}(p,t) = -\frac{1}{\lambda}\,\Phi_1(p,t). \tag{4.12}$$

These expressions highlight two comparative statics. First, a higher price p reduces demand for the original good and increases demand for piracy, with the magnitude of adjustment governed by $\Phi_0(p,t)$. Second, stronger enforcement t shifts demand away from piracy toward the original good, provided $\Phi_1(p,t) > 0$. In other words, while prices tilt consumer choices between the two options, enforcement reduces the attractiveness of piracy by increasing its effective cost.

Lemma 4.1. *If* $0 < m^*(x) < 1$ *for all* $x \in [0,1]$*, then the demand functions are*

$$D_O(p,t) = \int_0^1 \left[1 - F_M\!\left(\frac{p - d(x)t}{\lambda}\right)\right] f_X(x)\,dx, \qquad D_P(p,t) = \int_0^1 F_M\!\left(\frac{p - d(x)t}{\lambda}\right) f_X(x)\,dx = 1 - D_O(p$$

4.2.1 Profit Functions

Original Firm

The original firm's profit is given by

$$\pi_O(p,t) = (p - c)\,D_O(p,t).$$

Holding the pirate technology t fixed, the first-order condition (FOC) with respect to the price p is

$$\frac{\partial \pi_O}{\partial p} = D_O(p,t) + (p - c)\,\frac{\partial D_O}{\partial p}(p,t) = 0.$$

Substituting the expression for $\partial D_O/\partial p$ from (4.11) yields

$$D_O(p,t) - \frac{p - c}{\lambda}\,\Phi_0(p,t) = 0, \tag{4.13}$$

which can be rearranged to give the original firm's reaction function:

$$p = c + \frac{\lambda D_O(p,t)}{\Phi_0(p,t)}. \tag{4.14}$$

This equation implicitly defines the optimal price $p^{BR}(t)$ as a function of the pirate's technology t. Conditions for existence and uniqueness of the solution are discussed below.

Pirate Firm

The pirate firm's profit is specified flexibly as

$$\pi_P(p,t) = F\,R(D_P(p,t)) - D_P(p,t)\,M\,C(D_P(p,t)) - \omega,$$

where $R : [0,1] \to \mathbb{R}_{\geq 0}$ is a revenue index and $C : [0,1] \to \mathbb{R}_{\geq 0}$ captures the expected cost or risk. The parameters $F \geq 0$, $M > 0$, and $\omega \geq 0$ represent scale and fixed costs. Revenue depends on the pirate firm's market share, while total cost includes a variable legal cost determined by the probability of detection, $C(D_P)$, scaled by M, and a fixed operational cost ω.

Applying the chain rule together with (4.12) gives

$$\frac{\partial \pi_P}{\partial t} = \Big(FR'(D_P) - MC'(D_P)\,D_P(p,t) - MC(D_P)\Big)\frac{\partial D_P}{\partial t}(p,t) = 0.$$

In an interior solution, $\frac{\partial D_P}{\partial t} = -\Phi_1/\lambda \neq 0$, so the first-order condition reduces to the share-targeting condition

$$F\,R'(D_P^\star) = M\Big(C'(D_P^\star)\,D_P(p^\star,t^\star) - C(D_P^\star)\Big), \tag{4.15}$$

where any interior optimizer $D_P^\star \in (0,1)$ satisfies (4.15). The pirate's reaction is then the set of (p,t) such that

$$D_P(p,t) = D_P^\star. \tag{4.16}$$

Monotonicity, i.e., $\partial D_P/\partial t < 0$ ($\Phi_1 > 0$), ensures that for each p there is at most one $t^{BR}(p)$ solving (4.16).

Condition: (4.15) holds if

$$C'(D_P)\,D_P(p,t) - C(D_P) > 0. \tag{4.17}$$

Lemma 4.2. *Under the interior condition* $0 < m^*(x) < 1$ *for all* $x \in [0,1]$*, the demand shares are*

$$D_O(p,t) = \int_0^1 \Big[1 - F_M\Big(\frac{p - d(x)t}{\lambda}\Big)\Big] f_X(x)\,dx, \tag{4.18}$$

$$D_P(p,t) = \int_0^1 F_M\Big(\frac{p - d(x)t}{\lambda}\Big) f_X(x)\,dx = 1 - D_O(p,t). \tag{4.19}$$

Defining

$$\Phi_0(p,t) := \int_0^1 f_M\Big(\frac{p - d(x)t}{\lambda}\Big) f_X(x)\,dx, \quad \Phi_1(p,t) := \int_0^1 d(x)\,f_M\Big(\frac{p - d(x)t}{\lambda}\Big) f_X(x)\,dx, \tag{4.20}$$

we have

$$\frac{\partial D_O}{\partial p} = -\frac{1}{\lambda}\Phi_0(p,t), \quad \frac{\partial D_O}{\partial t} = \frac{1}{\lambda}\Phi_1(p,t), \tag{4.21}$$

$$\frac{\partial D_P}{\partial p} = \frac{1}{\lambda}\Phi_0(p,t), \quad \frac{\partial D_P}{\partial t} = -\frac{1}{\lambda}\Phi_1(p,t). \tag{4.22}$$

Proposition 4.3. *The pirate firm's profit is*

$$\pi_P(p,t) = F\,R(D_P(p,t)) - D_P(p,t)MC(D_P(p,t)) - \omega.$$

In an interior solution, the first-order condition with respect to t implies the share-targeting rule

$$F\,R'(D_P^\star) = M\Big(C'(D_P^\star)D_P^\star - C(D_P^\star)\Big), \quad D_P^\star \in (0,1),$$

and the reaction correspondence satisfies $D_P(p,t) = D_P^\star$. Monotonicity ensures at most one $t^{BR}(p)$ for each p, provided $C'(D_P)D_P - C(D_P) > 0$.

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4.2.2 Uniform Distribution Hotelling Special Case (Closed-Form Solutions)

For intuition and explicit formulas, consider $f_X \equiv 1$, $f_M \equiv 1$, and $d(x) = 1 - x$. Then

$$D_P(p,t) = \frac{p}{\lambda} - \frac{t}{2\lambda}, \qquad D_O(p,t) = 1 - D_P(p,t) = 1 - \frac{p}{\lambda} + \frac{t}{2\lambda}.$$

Pirate. The share target $D_P^\star$ satisfies $FR'(D_P^\star) = MC'(D_P^\star)$. Given any solution $D_P^\star \in (0,1)$, the reaction correspondence solves $D_P(p,t) = D_P^\star$:

$$t^{BR}(p) = 2p - 2\lambda D_P^\star.$$

Original. Using $D_O = 1 - p/\lambda + t/(2\lambda)$ and $\Phi_0 = 1$ in the FOC:

$$\left(1 - \frac{p}{\lambda} + \frac{t}{2\lambda}\right) - \frac{p-c}{\lambda} = 0 \quad \Longrightarrow \quad p^{BR}(t) = c + \lambda - \frac{t}{2}.$$

Equilibrium. Solving $p = c + \lambda - \frac{t}{2}$ and $t = 2p - 2\lambda D_P^\star$ yields

$$p^* = c + \lambda(1 - D_P^\star), \qquad t^* = 2c + 2\lambda(1 - 2D_P^\star),$$

with profits

$$\pi_O^* = (p^* - c)(1 - D_P^\star) = \lambda(1 - D_P^\star)^2, \qquad \pi_P^* = FR(D_P^\star) - MC(D_P^\star) - \omega.$$

These expressions remain valid for any (R, C) provided $D_P^\star \in (0,1)$ solves $FR'(D) = MC'(D)$.

Conditions and Regularity

Demand/Types

- f_X, f_M are continuous on $[0,1]$, nonnegative, and integrate to 1.
- $d : [0,1] \to [0,1]$ is continuous, strictly decreasing, with $d(0) = 1$, $d(1) = 0$.
- **Interiority:** $0 < m^*(x) = \frac{p-d(x)t}{\lambda} < 1$ for all x at candidate (p,t).

Pirate primitives

- $R : [0,1] \to \mathbb{R}_{\geq 0}$, $C : [0,1] \to \mathbb{R}_{\geq 0}$ are continuously differentiable:

$$R'(D) \geq 0, \quad R''(D) \leq 0; \qquad C'(D) \geq 0, \quad C''(D) \geq 0 \text{ for } D \in (0,1).$$

- **Interior target:** unique $D_P^\star \in (0,1)$ solves $FR'(D) = MC'(D)$.
- **SOC at target:** $FR''(D_P^\star) - MC''(D_P^\star) < 0$.

Mapping and best responses

- $D_P(p,t)$ is continuously differentiable with $\partial D_P/\partial t = -\Phi_1/\lambda < 0$ near equilibrium.
- Original FOC has a unique solution $p^{BR}(t)$ for each relevant t.

Feasible parameters

- $v > 0$, $c \geq 0$, $\lambda > 0$, $F \geq 0$, $M > 0$, $\omega \geq 0$.
- Uniform case: $p^* = c + \lambda(1 - D_P^\star) \geq 0$, $t^* = 2c + 2\lambda(1 - 2D_P^\star) \geq 0$.

Remark 4.4. The interior-equilibrium pirate profit depends only on $D_P^\star$ via R and C. Prices and technology shape (p^*, t^*) such that $D_P(p^*, t^*) = D_P^\star$. The uniform Hotelling case yields closed-form (p^*, t^*) once $D_P^\star$ is determined by $FR'(D) = MC'(D)$.

5 Conclusion

This paper develops a model of strategic interaction between a legitimate content producer and a for-profit pirate, explicitly incorporating multi-dimensional consumer heterogeneity. Using tractable functional forms, we derive equilibrium outcomes for prices, market shares, and profits, and perform comparative statics to show how enforcement, technological barriers, and the ethical composition of consumers shape market outcomes. The general model extends these insights to arbitrary distributions and profit functions, although comparative statics are generally not analytically tractable.

Several limitations suggest avenues for future research. Our framework is static and abstracts from the dynamics of enforcement, learning, and repeated interaction. It also considers only a single pirate and does not account for network effects in consumption. Extending the model to multiple pirates, dynamic enforcement, and empirically calibrated distributions of consumer abilities and ethics would enhance both realism and policy relevance.

Despite these limitations, our analysis highlights that digital piracy is not merely a technological problem but a complex interaction of technology, incentives, and ethics. Effective strategies to reduce piracy must therefore account for both technical and moral dimensions of consumer behavior, suggesting that combined interventions; targeting access, enforcement, and ethical awareness, are likely to be more effective than single-faceted approaches.

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A Appendix

A.1 Derivation of General Demand Functions with Ethical Costs

We consider a continuum of consumers indexed by (x, m), where:

- $x \in [0, 1]$ represents the technical proficiency of the consumer, with density $f_X(x)$,
- $m \in [0, 1]$ captures the consumer's moral inclination, with density $f_M(m)$,
- x and m are assumed independent.

Step 1: Utility Functions. Consumers choose between the original product and the pirated version. Their utilities are:

$$U_O = v - p, \qquad U_P(x, m) = v - d(x)\, t - \lambda m,$$

where $d(x)$ is decreasing in x (with $d(0) = 1$ and $d(1) = 0$), capturing the notion that technically skilled consumers find piracy easier.

Step 2: Indifference Condition. A consumer is indifferent between purchasing the original or the pirated copy if:

$$U_O = U_P(x, m) \implies m^*(x) = \frac{p - d(x)\, t}{\lambda}.$$

Step 3: Probability of Purchasing the Original. For a given technical type x, a consumer purchases the original if $m \geq m^*(x)$. Hence:

$$\Pr(\text{buy original} \mid x) = \int_{m^*(x)}^{1} f_M(m)\, dm = 1 - F_M(m^*(x)).$$

Step 4: Corner Cases. To account for extreme values:

$$\Pr(\text{buy original} \mid x) = \begin{cases} 1, & m^*(x) < 0 \\ 0, & m^*(x) > 1 \\ 1 - F_M(m^*(x)), & 0 \leq m^*(x) \leq 1 \end{cases}$$

or equivalently:

$$\Pr(\text{buy original} \mid x) = 1 - F_M\big(\max\{0, \min\{1, m^*(x)\}\}\big).$$

Step 5: Aggregate Demand. Averaging over all technical types, we obtain:

$$\boxed{D_O(p, t) = \int_0^1 \Big[1 - F_M(\max\{0, \min\{1, m^*(x)\}\})\Big] f_X(x)\, dx}$$

$$\boxed{D_P(p, t) = \int_0^1 F_M(\max\{0, \min\{1, m^*(x)\}\}) f_X(x)\, dx}.$$

Step 6: Interior Solution. If the indifference threshold satisfies $0 < m^*(x) < 1$ for all x, truncation is unnecessary:

$$D_O(p,t) = \int_0^1 \Big[1 - F_M\big(\tfrac{p-d(x)t}{\lambda}\big)\Big] f_X(x)\,dx, \quad D_P(p,t) = \int_0^1 F_M\big(\tfrac{p-d(x)t}{\lambda}\big) f_X(x)\,dx.$$

Step 7: Uniform Case. Assume $f_X(x) = f_M(m) = 1$ and $d(x) = 1 - x$. Then $F_M(m) = m$ and:

$$D_O(p,t) = \int_0^1 \left(1 - \frac{p-(1-x)t}{\lambda}\right) dx = 1 - \frac{p}{\lambda} + \frac{t}{2\lambda}, \quad D_P(p,t) = \frac{p}{\lambda} - \frac{t}{2\lambda}.$$

These linear forms correspond to the simplified case analyzed in the main text.

For completeness, define the fraction of consumers buying each option:

$$D_O(p,t) = \int_0^1 \Big[1 - F_M(\max\{0, \min\{1, m^*(x)\}\})\Big] f_X(x)\,dx, \tag{A.1}$$

$$D_P(p,t) = \int_0^1 F_M(\max\{0, \min\{1, m^*(x)\}\}) f_X(x)\,dx, \tag{A.2}$$

$$D_0(p,t) = 1 - D_O(p,t) - D_P(p,t), \tag{A.3}$$

where D_0 is the mass of consumers choosing the outside option.

Interior case. When $0 < m^*(x) < 1$ for all x, the truncation is unnecessary:

$$D_O(p,t) = \int_0^1 \big[1 - F_M((p - d(x)t)/\lambda)\big] f_X(x)\,dx, \tag{A.4}$$

$$D_P(p,t) = \int_0^1 F_M((p - d(x)t)/\lambda) f_X(x)\,dx. \tag{A.5}$$

Uniform case. Assuming $f_X(x) = f_M(m) = 1$ and $d(x) = 1 - x$, we get

$$D_O(p,t) = \int_0^1 \left(1 - \frac{p-(1-x)t}{\lambda}\right) dx = 1 - \frac{p}{\lambda} + \frac{t}{2\lambda}, \tag{A.6}$$

$$D_P(p,t) = \frac{p}{\lambda} - \frac{t}{2\lambda}. \tag{A.7}$$

A.2 Remarks on the General vs. Special Case

- In the general model, reaction functions are implicit: the original solves $D_O + (p - c)\partial_p D_O = 0$, while the pirate solves $D_P = F/(2M)$. The equilibrium piracy cost t must generally be found by solving the integral equation $D_P(p,t) = F/(2M)$.
- The uniform specialization with $d(x) = 1 - x$ yields closed-form expressions, offering intuition and facilitating calibration.
- Independence between x and m simplifies the double integrals. If x and m are correlated, the integrals must be computed with the joint density $f_{X,M}(x,m)$.

- These derivations form the basis for comparative statics and policy analysis in the main text, while the simplified linear case allows transparent analytical insights.