Multi-task Delegation and Information Transmission

Prerna Khanna   Parikshit Ghosh

Abstract

We study a sender–receiver framework in which a fully informed sender (expert) possesses perfect knowledge of the underlying state, while the receiver must rely on information from sender to make decisions. When multiple tasks depend on the same state, the receiver can, in principle, learn the state exactly by delegating a subset of tasks to the sender. However, the sender's strategic response to the informational value of his delegated actions renders the optimal extent of delegation non-trivial. We provide a comprehensive characterization of optimal delegation under general conditions encompassing convex costs with varying curvature parameters, differential task weights and heterogeneous bias structures.

Keywords: Delegation; Asymmetric Information; Strategic Information Transmission; Signaling; Multi-task.
JEL: D82, D83.

1 Introduction

"I have always thought the actions of men the best interpreters of their thoughts."
— John Locke

Information asymmetries between informed agents and decision-makers constitute a fundamental challenge in economic organizations. When privately informed parties lack decision rights while uninformed principals retain control, suboptimal outcomes may result from the failure to aggregate dispersed information effectively. This information transmission problem is compounded when parties have divergent interests, creating incentives for strategic misrepresentation. The economic literature is replete with models designed to encourage the truthful revelation of information in these settings. Key contributions include the theory of signaling (Rothschild & Stiglitz, 1976; Spence, 1973), mechanism design and contract theory (Laffont & Martimort, 2002; Myerson, 1981). These contributions highlight formal mechanisms of information transmission that rely on enforceable commitments. However, when contracts are incomplete due to unforeseeability of future contingencies—as formalized in the property rights approach of Grossman and Hart (1986) and extended by Hart and Moore (1990)—traditional contracting solutions become infeasible. In such environments, cheap talk communication (Crawford & Sobel, 1982) offers a potential remedy, though the transmitted information is typically coarse and partitioned. Dessein (2002) demonstrates that delegation of decision rights can serve as an alternative to communication, providing a cost-effective mechanism for exploiting local information when the informed party's and principal's interests are sufficiently aligned. Building on this foundation, we argue that delegation possesses an additional signaling value that has been underexplored in the literature: by observing the delegated agent's choices, principals can extract information about the underlying state, which can then be leveraged to optimize decisions retained under their control.

In our model, we consider a two-player game consisting of a decision-maker (hereafter the receiver) who possesses formal authority, and an expert (hereafter the sender) who holds private information regarding the underlying state of nature. We study an environment with $N \geq 2$ tasks that all depend on a single unknown state. Such multi-task, single-state configurations pervade economic and organizational decision-making across diverse domains. In policymaking, for instance, accurate poverty assessment determines optimal resource allocation across multiple social interventions—housing subsidies, nutritional assistance, and employment programs—yet policy experts may exhibit systematic bias toward universally higher funding levels due to benevolent motivations or professional advocacy.

We analyze a partial delegation mechanism where the receiver strategically assigns a subset of tasks to the informed sender. Delegation acquires a signaling role, as the receiver can infer the state of the world from the sender's actions and make fully informed decisions on the remaining tasks. However, this mechanism will not always be optimal for the receiver. The sender, aware that their actions convey information, responds strategically by internalizing this signaling externality, which leads to distortions. We characterize the conditions under which partial delegation constitutes an optimal mechanism for the receiver, despite these signaling distortions. Our analysis identifies the trade-off between the receiver's informational gains from observing the sender's actions and the welfare losses from the sender's strategic response to the signaling incentives embedded in the delegation structure.

Our paper is closest to Gautier and Paolini (n.d.), who also show that delegation can reveal information. They restrict attention to discrete, binary state variables, which they later extended to discrete N-type settings with least-cost separating equilibria, and two tasks with heterogeneous weights. They employ comparative statics to examine delegation outcomes. By contrast, we analyze a continuous state space with multiple tasks, derive explicit conditions under which partial delegation is optimal, and characterize both the scope and extent of delegation. We further allow for heterogeneous weights, biases, and convexities, offering a broader and more general account of the informational role of delegation.

Another related contribution is Alonso (n.d.), who examines how partial control can improve communication when tasks are interdependent within a simultaneous game environment. In contrast, our paper focuses on determining the conditions under which control should be shared, and to what extent, in order to achieve fully informed action at the lowest possible cost, in a sequential game environment.

2 Model Set-up

Consider a model of two-player two-stage sequential game consisting of a Sender (S) and a Receiver (R). The state of the world is characterized by a parameter $x \in \mathbb{R}$, which is drawn from a commonly known, continuous, twice differentiable distribution $F(x)$ with density $f(x)$ and finite support $X \subseteq \mathbb{R}$. The Sender observes the true state x perfectly. The Receiver knows only the prior distribution $F(x)$ but not the realization of x.

There are $N \geq 2$ tasks indexed by $i \in \{1, 2, \ldots, N\}$. Each task requires choosing an action $a_i \in \mathbb{R}$ which depends on same state variable x. The Receiver has control rights over all tasks but may choose to delegate some tasks to the Sender. Let $\lambda \in [0, 1]$ denote the fraction of tasks delegated to the Sender.

Both players have Euclidean state-dependent preferences over actions relative to their ideal points, with quadratic loss functions.

The Receiver's cost is given by:

$$U^R = (a_R - x)^2$$

with bliss point at $a_R = x$.

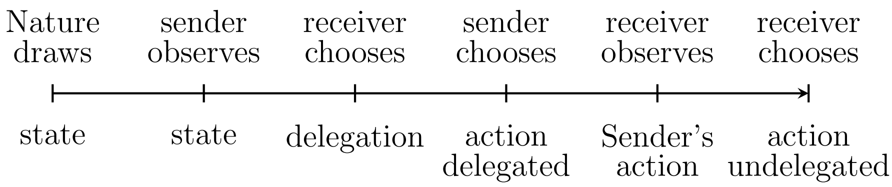
Sender's cost is -

$$U^R = (a_S - x - b)^2$$

The sender exhibits bias $b \in \mathbb{R}$ in their preferred action relative to the receiver, creating an informational friction.

Timing

The game proceeds as follows:



Strategies

The Sender's action a^S serves a dual role:

1. Direct: Affects payoff from the λ fraction of delegated tasks
2. Indirect (signaling value): Conveys information about x to the Receiver, influencing a^R

Recognizing the informational content of their actions, the sender chooses a signaled action $\hat{x}$ to minimize total cost across both delegated and non-delegated tasks. The sender's optimization problem is:

$$\min_{\hat{x}} \left[\lambda \cdot (a^S(\hat{x}) - x - b)^2 + (1 - \lambda) \cdot (\hat{x} - x - b)^2\right],$$

Let $\mu(\hat{x}|a^S) = \hat{x}$ denote the Receiver's posterior belief about the state given observed action a^S. The Receiver's optimal action for non-delegated tasks is:

$$a^R = \hat{x} = \mathbb{E}[x|a^S] = \int x \, d\mu(x|a^S).$$

is determined by the Receiver's Bayesian updating given the signaling strategy.

Equilibrium

We characterize the strategic interaction between Sender and Receiver using the solution concept of Perfect Bayesian Equilibrium. An equilibrium in this delegation game consists of a signaling strategy $a^S(x)$ for the Sender, a response strategy $a^R(a^S)$ for the Receiver, and a belief system $\mu(x|a^S)$ representing the Receiver's posterior distribution over states conditional on observing the Sender's action.

The equilibrium must satisfy three consistency conditions. First, sequential rationality requires that the Sender's signaling strategy be optimal given the Receiver's anticipated response. That is, for each realization of state x, the chosen action $a^S(x)$ must minimize the Sender's cost taking into account how this action influences the Receiver's subsequent decisions on non-delegated tasks.

Second, the Receiver's response strategy must be optimal given the posterior beliefs induced by the observed action. Upon observing action a^S, the Receiver updates beliefs about the underlying state according to $\mu(x|a^S)$ and chooses the action a^R that minimizes cost under this posterior distribution.

Third, belief consistency requires that the Receiver's posterior beliefs be derived from the Sender's equilibrium strategy using Bayes' rule whenever possible. On the equilibrium path, beliefs must be consistent with the signaling strategy, while off-equilibrium beliefs may be specified arbitrarily provided they satisfy the intuitive criterion or other appropriate refinements.

This equilibrium concept ensures that both players' strategies are mutually best responses and that information transmission occurs rationally, accounting for the strategic incentives created by the delegation arrangement and the underlying conflict of interest between the players.

First-Order Condition

Taking the first-order condition with respect to $\hat{x}$ in the Sender's problem (2):

$$\frac{\partial}{\partial \hat{x}} \left[\lambda \cdot (a^S(\hat{x}) - x - b)^2 + (1-\lambda) \cdot (\hat{x} - x - b)^2 \right] = 0.$$

This yields:

$$2a'^S(\hat{x})\lambda(a^S(\hat{x}) - x - b) + 2(1-\lambda)(\hat{x} - x - b) = 0.$$

Under full revelation, the solution yields optimal action:

$$a^S(x) = x + \frac{b}{\lambda}.$$

This expression captures the sender's strategic distortion, which represents an "overshoot" relative to the true state x, with the magnitude depending on the delegation proportion λ. Cost minimizing delegation will be $\lambda = 1$, full delegation.

Proposition 1. *Under any symmetric, strictly convex cost function $C(\cdot)$, the Sender's optimal truthful strategy exhibits overshooting: $a^S(x) > x + b$ for all x and receiver's optimal delegation strategy is full $\lambda = 1$*

Two important findings come out of this proposition First, despite the option of partial delegation receiver might not avail it. Second, if the symmetrical nature of cost function breaks, we can have a different equilibrium, which we will tackle in the next section.

3 Different Curvature of Cost Function

We continue with an environment consisting of N tasks, each dependent on a unidimensional state x known only to the sender. Both sender and receiver possess state-dependent strictly convex Euclidean preferences, with the crucial distinction that they exhibit different degrees of curvature in their cost functions:

$$C_S = (a - x - b)^s, \qquad C_R = (a - x)^r$$

where $s > r$, measures the varied curvature.

Sender's minimization problem is -

$$\min_{x'} \ \lambda \left(a(x') - x - b\right)^s + (1 - \lambda)\left(x' - x - b\right)^s$$

Solving it gives sender's response as -

$$a(x) \ = \ x + \left(\frac{(1-\lambda)^{1/(s-1)} + \lambda^{1/(s-1)}}{\lambda^{1/(s-1)}}\right) b$$

The receiver chooses the delegation proportion λ to minimize their cost, which is incurred only on the delegated portion:

$$\begin{aligned} C_R &= \lambda \left(\left(\frac{(1-\lambda)^{1/(s-1)} + \lambda^{1/(s-1)}}{\lambda^{1/(s-1)}} \right) b \right)^r \\ &= \lambda^{1-r/(s-1)} \left(\left((1-\lambda)^{1/(s-1)} + \lambda^{1/(s-1)} \right) b \right)^r \\ &= \lambda^{(s-1-r)/(s-1)} \left(\left((1-\lambda)^{1/(s-1)} + \lambda^{1/(s-1)} \right) b \right)^r \end{aligned}$$

Since $s > r$ and also since since s and r are both even then $s - r$ is atleast $2 \implies (s-1) > r \implies s - 1 - r > 0$

C_R is minimized when λ tends to 0, indicating that the receiver optimally delegates the minimal feasible proportion of tasks (e.g., $1/N$).

The result emerges from the interaction between differential curvature parameters and strategic distortion incentives:

- Sender's Sensitivity to Deviation (Higher Curvature s): Because the sender has a higher curvature (s), they are more acutely penalized for large deviations from their ideal action. When delegation is minimal λ is small), the signaling burden concentrates on few tasks, but the sender's high curvature constrains extreme overshooting behavior.

- Receiver's Sensitivity to Deviation (Lower Curvature r): The receiver, with a lower curvature (r), is less sensitive to large deviations. When the sender overshoots on delegated tasks, the receiver's lower curvature minimizes the penalty associated with such distortions. Consequently, while increasing delegation (λ) might reduce per-task distortion, the accumulated cost across multiple distorted tasks exceeds the benefit from marginally improved signaling precision.

The combination of high sender curvature (limiting distortion) and low receiver curvature (reducing distortion costs) creates a strategic complementarity favoring minimal delegation. The receiver leverages the sender's natural reluctance to engage in extreme overshooting while minimizing their own exposure to accumulated distortion costs.

For large N, setting $\lambda = \frac{1}{N}$, the receiver's cost becomes:

$$C_R = \lambda^{(s-1-r)/(s-1)} \left(\left((1-\lambda)^{1/(s-1)} + \lambda^{1/(s-1)}\right) b\right)^r$$
$$= \left(\frac{1}{N}\right)^{(s-1-r)/(s-1)} \left(\left(\left(1-\frac{1}{N}\right)^{1/(s-1)} + \left(\frac{1}{N}\right)^{1/(s-1)}\right) b\right)^r$$

$$\lim_{N\to\infty} C_R \;=\; 0$$

As N increases sender's action on the single delegated task become significantly distorted but, since the cost is incurred on a single task the impact of this distortion on the receiver's overall cost is negligible. In the limit of many tasks, the receiver's information extraction is nearly perfect, and the cost is as if the receiver knew the state x directly.

The analysis demonstrates that differential curvature in cost functions fundamentally alters optimal delegation patterns compared to symmetric curvature cases. The receiver's ability to extract information efficiently through minimal delegation represents a form of informational leverage, where strategic complementarities between curvature parameters enable nearly costless learning in large-task environments. This result has implications for organizational design, suggesting that pairing agents with complementary risk attitudes can enhance information transmission efficiency.

4 Different weights

There are N tasks. The sender's task weights are $w_1, \ldots, w_N$ with $\sum_{i=1}^N w_i = 1$. Without loss of generality, order them $w_{(1)} \geq \cdots \geq w_{(N)}$ and define cumulative top-k mass $S_k := \sum_{i=1}^k w_{(i)}$. The receiver values tasks equally, $y_i = 1/N$ for all i, and the sender's bias is a common scalar b.

If the receiver delegates the top k tasks to the sender, the sender's optimal action is

$$a(x) \;=\; x + \frac{b}{S_k}.$$

The receiver's loss (up to the common scaling by $1/N$) is

$$C(k) \;=\; \frac{k\, b^2}{S_k^2}.$$

Characterization

Proposition 2. *For any optimal delegation size given by*

$$k^* = \arg\min_{1\leq k\leq N} \frac{k}{S_k^2},$$

the choice k^* must weakly satisfy the following two inequalities:

1. ***Global Feasibility Condition***

$$S_{k^*} \geq \sqrt{\frac{k^*}{N}}.$$

2. ***Marginal Condition***

$$w_{(k^*)} \geq \left(\sqrt{\tfrac{k^*}{k^*-1}} - 1\right) S_{k^*-1}.$$

An immediate implication of the proposition is the following corollary.

Corollary 1 (Dominance of single-task delegation under a large top weight)**.** *If $w_{(1)} \geq 1/\sqrt{2}$, then the optimal delegation size is $k^* = 1$ for any $N \geq 2$.*

The optimality conditions reveal several important structural properties of the delegation mechanism. The global feasibility condition implies that the distribution of weights across delegated tasks must be sufficiently front-loaded. The marginal condition demonstrates that delegating an additional task is only optimal if its associated weight is large enough to counterbalance and reduce the sender's tendency to overshoot. Thus, the weight distribution critically determines the delegation regime. When weights are uniformly distributed, full delegation emerges as the optimal strategy. By contrast, when the distribution of weights is more dispersed and skewed, the solution is typically corner favoring minimal delegation. Notably, the corner solution exhibits a desirable asymptotic property: as the number of tasks grows large, the receiver's expected loss converges to zero, replicating the benchmark outcome in which the receiver is fully informed and able to act directly.

5 Heterogeneous Biases Across Tasks

This section develops a theory of optimal delegation when biases are heterogeneous across tasks, revealing that the structure of bias distribution fundamentally alters delegation incentives and outcomes.

Consider an environment with N tasks, each characterized by a bias parameter b_i for $i = 1, 2, \ldots, N$. Without loss of generality, order these biases such that $b_{(1)} \leq b_{(2)} \leq \cdots \leq b_{(N))}$. The biases can be unidirectional (all biases have the same sign) or bidirectional (biases of both signs are present).

Let $D \subseteq \{1, 2, \ldots, N\}$ denote the delegation set with cardinality $|D| = n$, and let $U = \{1, 2, \ldots, N\} \setminus D$ represent the complementary set of undelegated tasks. Define the aggregate bias measures:

$$S = \sum_{i=1}^{N} b_i, \quad S_D = \sum_{j \in D} b_j, \quad S_U = \sum_{i \in U} b_i$$

When biases differ across tasks, the Sender's optimal strategy cannot treat all delegated tasks identically. The signaling problem requires task-specific actions that balance direct payoffs against informational effects. Under optimal signaling behavior, the Sender's action for each delegated task $j \in D$ takes the form:

$$a_j = x + b_j + t$$

where $t = \frac{S_U}{n}$ represents the common correction term that captures the signaling distortion arising from the Receiver's inference about undelegated tasks.

The Receiver's cost from delegation set D is given by:

$$C(D) = \sum_{j \in D} (b_j + t)^2 = \sum_{j \in D} \left(b_j + \frac{S_U}{n} \right)^2$$

Expanding this expression and utilizing the identity $S_U = S - S_D$, we obtain:

$$\begin{aligned} C(D) &= \sum_{j \in D} b_j^2 + 2\frac{S_U}{n} \sum_{j \in D} b_j + n \left(\frac{S_U}{n} \right)^2 \\ &= \sum_{j \in D} b_j^2 + \frac{2S_U S_D + S_U^2}{n} \\ &= \sum_{j \in D} b_j^2 + \frac{2(S - S_D)S_D + (S - S_D)^2}{n} \\ &= \sum_{j \in D} b_j^2 + \frac{S^2 - S_D^2}{n} \end{aligned}$$

Recognizing that the (population) variance of the delegation set is $\text{Var}(D) = \frac{1}{n} \sum_{j \in D} b_j^2 - \left(\frac{S_D}{n} \right)^2$, we can rewrite the cost function as:

$$\boxed{C(D) = \frac{S^2}{n} + n \cdot \text{Var}(D)}$$

The cost decomposition identifies a fundamental trade-off between two effects. The scale effect, captured by the term $\frac{S^2}{n}$, represents the benefit of delegation. It decreases monotonically with the number of delegated tasks. This effect creates an incentive to delegate extensively, as spreading the aggregate signaling distortion across more tasks reduces the per-task impact.

The composition effect, represented by $n \cdot \text{Var}(D)$, reflects the cost of delegation heterogeneity. As the delegation set expands to include tasks with more diverse biases, the variance increases, generating additional costs. This effect typically grows with delegation size, creating a natural constraint on extensive delegation.

The relative importance of these two effects depends critically on the magnitude of the aggregate bias S. When $|S|$ is large, the scale effect dominates, incentivizing extensive delegation to minimize the squared aggregate bias term. Conversely, when $|S|$ is small, the composition effect becomes paramount, favoring delegation sets with minimal variance.

For any fixed delegation size n, the composition effect is minimized by selecting a consecutive set from the ordered bias sequence. This result follows immediately from the fact that consecutive selection minimizes the variance among all possible subsets of size n. The optimal delegation set therefore takes the form $D^* = \{(i), (i+1), \ldots, (i+n-1)\}$ for some starting index i.

This structural result has important implications for delegation strategy. The location of the optimal consecutive block depends on the interplay between scale and composition effects. The absolute magnitude of individual biases matters only insofar as it affects the aggregate sum S and the variance structure across different contiguous blocks. It is the clustering and relative positioning of biases, not their absolute values, that determines optimal delegation structure.

When biases are bidirectional, the analysis reveals qualitatively different optimal behavior. In the extreme case where $S = 0$ (perfect symmetry in the bias distribution), the scale effect vanishes entirely. The optimal strategy reduces to minimizing the composition effect alone, which is achieved by delegating any single task, yielding zero variance and zero total cost. The identity of the delegated task becomes irrelevant under perfect symmetry.

Another prominent result can be spelled out in the following proposition:

Proposition 3. *Under bidirectional biases, partial delegation ($n < N$) is always optimal.*

This result has profound implications for organizational design. In environments where agent biases are naturally offsetting—such as committees with diverse ideological perspectives or teams with complementary expertise areas—the optimal strategy involves identifying small, homogeneous clusters of tasks for delegation while retaining direct control over the majority of decisions.

We can further characterize the model based on the distribution of biases. For unicluster pattern of biases, the optimal delegation sets for successive delegation sizes form a nested structure. This property implies that the optimal set for size $n + 1$ is always a superset of the optimal set for size n. As a result, the Receiver's problem can be simplified: they can use an incremental marginal analysis (comparing the cost of delegating n tasks to $n + 1$ tasks) to find the optimal delegation size.

For a chosen contiguous block D_n of size n let its sum be S_d. The total sum over all N biases is S.

$$C(n) = S^2/n + n * Var(n)$$

$$C(n+1) = S^2/n + 1 + (n+1) * Var(n+1)$$

$$C(n+1) < C(n)$$

The marginal condition is -

$$\text{(S)} \qquad (n+1)v_{n+1} - nv_n \le \frac{S^2}{n(n+1)}.$$

If the size-$(n+1)$ block is formed by adding one element b to the size-n block with sum S_d, the LHS simplifies to a compact closed form:

$$(n+1)v_{n+1} - nv_n \;=\; \frac{(nb - S_d)^2}{n(n+1)}.$$

Plugging into (S) gives the exact one-step test (no approximations):

$$\boxed{(nb - S_d)^2 \le S^2.}$$

Equivalently,

$$-S \le nb - S_d \le S,$$

or

$$\boxed{b \in \left[-\tfrac{S}{n} + \tfrac{S_d}{n},\ \tfrac{S}{n} + \tfrac{S_d}{n} \right].}$$

Because $S = S_d + S_u$ (where S_u is sum of the non-delegated elements), another convenient form is

$$\boxed{b \le \frac{2S_d + S_u}{n} \quad \text{and} \quad b \ge -\frac{S_u}{n}.}$$

When all $b_i \ge 0$ the lower bound is trivial; the useful constraint is the upper bound.

The framework establishes that more delegation will be cost minimizing only when agency cost is less than the benefits of diminutive scale effect. This implies that large scale effect induces larger delegation. This is the thrust of the following proposition.

Proposition 4. *When biases are unidirectional, the optimal delegation size satisfies*

$$n^* \ge \frac{N}{2}.$$

Proof. The proof follows from the idea that with unidirectional biases, the scale effect grows very fast because S^2 is large and increases with N, while the composition cost (variance) grows much more slowly. When fewer than half the tasks are delegated, adding another task always brings enough scale benefit to outweigh any variance increase. Thus, $n^* < N/2$ cannot be optimal delegation. (see Appendix for proof) □

6 Conclusion

This paper demonstrates that when multiple tasks depend on a common underlying state, partial delegation can serve as an effective mechanism for decision makers to acquire complete information about the state. Our analysis reveals that by strategically delegating a subset of tasks to an informed sender, receivers can extract perfect information through observation of the sender's actions on delegated tasks.

However, despite the inherent attractiveness of this information extraction mechanism, partial delegation is not universally optimal due to the sender's strategic distortion incentives. The sender, recognizing the informational value embedded in their actions, systematically distorts their behavior on delegated tasks, creating a fundamental tension between information acquisition and implementation efficiency.

Our theoretical framework identifies three key environmental characteristics that determine when partial delegation remains welfare-enhancing despite these strategic distortions:

Differential Task Valuation: When senders place higher relative value on delegated tasks compared to receivers, the strategic distortion problem is mitigated. This asymmetry in task importance creates conditions where the sender's incentive to distort is counterbalanced by their heightened concern for performance on high-value delegated tasks, while the receiver's welfare loss from suboptimal actions on less critical tasks remains manageable.

Bias Structure and Dispersion: The configuration of bias parameters across tasks fundamentally shapes delegation optimality. Bidirectional bias structures, where sender preferences diverge from receiver objectives in multiple directions, tend to average out distortions across the delegation portfolio. Similarly, when biases are dispersed into distinct clusters with relatively small magnitudes, the overall distortion effect becomes more manageable, allowing receivers to extract information while limiting implementation costs.

Differential Risk Preferences: Perhaps most significantly, environments where senders exhibit greater risk aversion than receivers—manifested through higher curvature parameters in their cost functions—naturally constrain strategic distortion. Risk-averse senders' heightened sensitivity to large deviations from optimal actions limits their willingness to engage in extreme overshooting behavior, even when such distortions would enhance information transmission. This creates a natural check on the distortion problem that makes delegation more attractive to receiver's.

These findings contribute to the broader literature on organizational design and information transmission by demonstrating that the success of delegation mechanisms depends critically on the alignment of agent characteristics with task structures. The results suggest that effective organizational design requires careful matching of agent types with delegation responsibilities, taking into account not only expertise and information endowments but also risk preferences, bias structures, and task valuations.

Appendix

A Proofs

A.1 Proof of Proposition 1

Sender Overshooting Response

The Sender chooses what state $\hat{x}$ to report through their action by minimizing

$$\min_{\hat{x}} \left[\lambda \cdot C(a^S(\hat{x}) - x - b) + (1-\lambda) \cdot C(\hat{x} - x - b)\right]. \tag{1}$$

Taking the derivative with respect to $\hat{x}$:

$$\lambda \cdot C'(a^S(\hat{x}) - x - b) \cdot \frac{da^S(\hat{x})}{d\hat{x}} + (1-\lambda) \cdot C'(\hat{x} - x - b) = 0. \tag{2}$$

In the truthful equilibrium, we set $\hat{x} = x$. Substituting:

$$\lambda \cdot C'(a^S(x) - x - b) \cdot \frac{da^S(x)}{dx} + (1-\lambda) \cdot C'(x - x - b) = 0. \tag{3}$$

Rearranging:

$$C'(a^S(x) - x - b) \cdot a'^S(x) = -\frac{(1-\lambda)}{\lambda} \cdot C'(-b). \tag{4}$$

Let $u(x) = a^S(x) - x - b$. Then $a'^S(x) = u'(x) + 1$.

$$C'(u(x)) \cdot (u'(x) + 1) = -\frac{(1-\lambda)}{\lambda} \cdot C'(-b). \tag{5}$$

$u(x)$ must be constant. Because otherwise, the left-hand side $C'(u(x)) \cdot (u'(x) + 1)$ varies with x, but the right-hand side is constant. This is a contradiction.

Therefore, $u'(x) = 0$, which means $u(x) = k$ for some constant k.

With $u(x) = k$ constant:

$$C'(k) \cdot (0 + 1) = -\frac{(1-\lambda)}{\lambda} \cdot C'(-b). \tag{6}$$

$$C'(k) = -\frac{(1-\lambda)}{\lambda} \cdot C'(-b). \tag{7}$$

Since C is strictly convex, C' is strictly increasing and has a unique inverse:

$$k = (C')^{-1}\left(-\frac{(1-\lambda)}{\lambda} \cdot C'(-b)\right). \tag{8}$$

Since $u(x) = a(x) - x - b = k$:

$$a^S(x) = x + b + k. \tag{9}$$

Given $b > 0$ and assuming C has minimum at 0 (so $C'(0) = 0$):

- $C'(-b) < 0$ since $-b < 0$ and C' is strictly increasing with $C'(0) = 0$,
- $-\frac{(1-\lambda)}{\lambda} \cdot C'(-b) > 0$ since $\frac{(1-\lambda)}{\lambda} > 0$ and $C'(-b) < 0$,
- $k > 0$ since $(C')^{-1}$ is strictly increasing and $k = (C')^{-1}(\text{positive number}) > (C')^{-1}(0) = 0$.

Thus, the constant $k > 0$ represents the signaling distortion through overshooting the bias

Receiver's Delegation

From the signaling analysis, we established that:

- Sender's optimal action: $a^S(x) = x + b + k$
- Signaling distortion: $k = (C')^{-1}\left(-\frac{(1-\lambda)}{\lambda} \cdot C'(-b)\right)$
- With $b > 0$, we have $k > 0$

The Receiver's cost after delegation is:

$$C_R(\lambda) = \lambda \cdot C(a^S(x) - x) = \lambda \cdot C(b + k)$$

From the signaling equilibrium:

$$k(\lambda) = (C')^{-1}\left(-\frac{(1-\lambda)}{\lambda} \cdot C'(-b)\right)$$

Let $z(\lambda) = -\frac{(1-\lambda)}{\lambda} \cdot C'(-b)$.
Since $C'(-b) < 0$ for $b > 0$, we have:

$$z(\lambda) = \frac{(1-\lambda)}{\lambda} \cdot |C'(-b)| > 0$$

Taking the derivative:

$$\frac{dz}{d\lambda} = \frac{d}{d\lambda}\left[\frac{(1-\lambda)}{\lambda}\right] \cdot |C'(-b)| = -\frac{1}{\lambda^2} \cdot |C'(-b)| < 0$$

Therefore, $z(\lambda)$ is strictly decreasing in λ.
Since $(C')^{-1}$ is strictly increasing (inverse of strictly increasing function) and $z(\lambda)$ is strictly decreasing in λ:

$$\frac{dk}{d\lambda} = (C')^{-1\prime}(z(\lambda)) \cdot \frac{dz}{d\lambda} < 0$$

Therefore, $k(\lambda)$ is strictly decreasing in λ.
As $\lambda \to 1$:

$$z(\lambda) = \frac{(1-\lambda)}{\lambda} \cdot |C'(-b)| \to 0$$

Therefore:

$$k(\lambda) = (C')^{-1}(z(\lambda)) \to (C')^{-1}(0) = 0$$

So $\lim_{\lambda \to 1} k(\lambda) = 0$.
The Receiver's cost is:

$$C_R(\lambda) = \lambda \cdot C(b + k(\lambda))$$

Taking the derivative with respect to λ:

$$\frac{dC_R}{d\lambda} = C(b + k(\lambda)) + \lambda \cdot C'(b + k(\lambda)) \cdot \frac{dk}{d\lambda}$$

We need to show that $\frac{dC_R}{d\lambda} < 0$ for all $\lambda \in (0, 1)$, which would imply that $C_R(\lambda)$ is strictly decreasing in λ, making $\lambda = 1$ optimal.
Term 1: $C(b + k(\lambda)) \geq 0$ (since $C(\cdot) \geq 0$).
Term 2: $\lambda \cdot C'(b + k(\lambda)) \cdot \frac{dk}{d\lambda}$.
Since:

- $\lambda > 0$
- $C'(b + k(\lambda)) > 0$ (because $b + k(\lambda) > 0$ and $C'(0) = 0$ with C' strictly increasing)
- $\frac{dk}{d\lambda} < 0$ (shown above)

We have: $\lambda \cdot C'(b + k(\lambda)) \cdot \frac{dk}{d\lambda} < 0$.
The key insight is that as λ increases:

1. The direct effect $C(b + k(\lambda))$ remains positive but bounded,
2. The signaling distortion $k(\lambda)$ decreases toward zero,
3. The indirect effect $\lambda \cdot C'(b + k(\lambda)) \cdot \frac{dk}{d\lambda}$ becomes increasingly negative.

To show $\frac{dC_R}{d\lambda} < 0$, we need:

$$C(b + k(\lambda)) < -\lambda \cdot C'(b + k(\lambda)) \cdot \frac{dk}{d\lambda}$$

Note that:

$$\frac{dk}{d\lambda} = -\frac{1}{\lambda^2} \cdot |C'(-b)| \cdot (C')^{-1\prime}(z(\lambda))$$

Since $(C')^{-1\prime}(z) = \frac{1}{C''((C')^{-1}(z))}$, we have:

$$\left|\frac{dk}{d\lambda}\right| = \frac{|C'(-b)|}{\lambda^2 \cdot C''(b + k(\lambda))}$$

For the Receiver's cost to be minimized at $\lambda = 1$, we need to show that the marginal benefit of increased delegation (reduced signaling distortion) outweighs the marginal cost (increased exposure to delegation).

The key insight is that as $\lambda \to 1$, the signaling distortion $k(\lambda) \to 0$, which means:

$$\lim_{\lambda \to 1} C_R(\lambda) = 1 \cdot C(b + 0) = C(b)$$

For any $\lambda < 1$, we have $k(\lambda) > 0$, so:

$$C_R(\lambda) = \lambda \cdot C(b + k(\lambda)) > \lambda \cdot C(b)$$

Since the signaling distortion creates additional costs that vanish only at full delegation, and these costs outweigh the direct delegation costs, we conclude that $C_R(\lambda)$ is minimized at $\lambda = 1$.

Therefore, full delegation ($\lambda = 1$) is optimal for the Receiver when both players have the same cost function.

The results rely crucially on the symmetric nature of sender's and receiver's cost function because without alignment sender creates signaling distortion optimized for their cost function, but Receiver evaluates it using a different metric. □

A.2 Proof of Proposition 3

Let $b_1, \ldots, b_N$ be given and $S = \sum_{i=1}^{N} b_i$. Assume there is at least one positive and at least one negative bias (the bidirectional set). Consider the subset

$$D := \{\, i : b_i \text{ has the same sign as } S \,\}.$$

$S_D := \sum_{j \in D} b_j$ has the same sign as S, and because the contributions from indices not in D have the opposite sign,

$$|S_D| \geq |S|.$$

Equivalently,

$$S_D^2 \geq S^2. \tag{i}$$

$$C(D) = \sum_{j \in D} b_j^2 + \frac{S^2 - S_D^2}{|D|}.$$

Since $S^2 - S_D^2 \leq 0$ by (i), the second term is ≤ 0. Hence

$$C(D) \leq \sum_{j \in D} b_j^2 \leq \sum_{i=1}^{N} b_i^2 = C(N).$$

The first inequality is strict whenever $S_D^2 > S^2$ (i.e. when S is not exactly equal in magnitude to S_D), or when the second term is strictly negative. The second inequality is strict whenever there exists at least one nonzero bias in the complement U. Thus in all nondegenerate cases you get $C(D) < C(N)$. –

A.3 Proof of Proposition 4

Let $b_1 \leq b_2 \leq \cdots \leq b_N$ with $b_i \geq 0$ for all i. Define

$$S = \sum_{i=1}^{N} b_i, \qquad Q = \sum_{i=1}^{N} b_i^2.$$

For each k let v_k be the minimal variance over all size-k consecutive blocks of the sorted list. The receiver's cost function is

$$C(k) = \frac{S^2}{k} + k\, v_k.$$

Lemma (Minimiser is contiguous). Let $b_1 \leq b_2 \leq \cdots \leq b_N$ be sorted real numbers. For a k-element subset $A = \{x_1 < \cdots < x_k\}$, write

$$\bar{x} = \frac{1}{k} \sum_{i=1}^{k} x_i, \qquad F(A) = \sum_{i=1}^{k} (x_i - \bar{x})^2.$$

Then some minimiser of $F(A)$ over all k-subsets is a *contiguous block* $\{b_j, b_{j+1}, \ldots, b_{j+k-1}\}$.

Proof. We first note an identity: for any k-subset A,

$$F(A) = \sum_{i=1}^{k} (x_i - \bar{x})^2 = \frac{1}{k} \sum_{1 \leq i < j \leq k} (x_i - x_j)^2. \tag{1}$$

Hence minimising variance is equivalent to minimising the sum of squared pairwise gaps.

Suppose, for contradiction, that a minimiser A is not contiguous. Then there exist indices $u < v < w$ such that $b_u, b_w \in A$ but $b_v \notin A$. So A "jumps over" the point b_v.

Consider $A' = (A \setminus \{b_w\}) \cup \{b_v\}$, i.e. replace the extreme element b_w with the skipped element b_v. We compare $F(A')$ and $F(A)$ using (1). Every pairwise distance among elements of A and A' is the same except those involving the swapped element. The contribution of pairs with b_w in A is

$$\sum_{x \in A \setminus \{b_w\}} (b_w - x)^2,$$

while the contribution of pairs with b_v in A' is

$$\sum_{x \in A \setminus \{b_w\}} (b_v - x)^2.$$

Because b_v lies strictly between b_u and b_w, and all other elements of $A \setminus \{b_w\}$ lie between b_u and b_w as well, each term satisfies

$$(b_v - x)^2 < (b_w - x)^2.$$

Summing over $x \in A \setminus \{b_w\}$ gives

$$\sum_{x \in A \setminus \{b_w\}} (b_v - x)^2 < \sum_{x \in A \setminus \{b_w\}} (b_w - x)^2.$$

Therefore $F(A') < F(A)$, contradicting the minimality of A. Hence no minimiser can skip an interior point, and so a minimising set must be contiguous. □

Proposition For every integer n with $1 \leq n < N/2$ we have $C(n) \geq C(N-n)$. Consequently, any strict minimiser n^* satisfies $n^* \geq \lceil N/2 \rceil$.

Proof. Fix n with $1 \leq n < N/2$ and set $m := N - n > n$. Let A be any contiguous block of size n and let $\overline{A}$ denote its complement of size m. Define

$$S_A = \sum_{i \in A} b_i, \quad S_{\overline{A}} = S - S_A, \qquad Q_A = \sum_{i \in A} b_i^2, \quad Q_{\overline{A}} = Q - Q_A.$$

By the variance identity,

$$n\mathrm{Var}(A) = Q_A - \frac{S_A^2}{n}, \qquad m\mathrm{Var}(\overline{A}) = Q_{\overline{A}} - \frac{S_{\overline{A}}^2}{m}. \tag{1}$$

Adding these equalities yields

$$n\mathrm{Var}(A) + m\mathrm{Var}(\overline{A}) = Q - \Big(\frac{S_A^2}{n} + \frac{(S - S_A)^2}{m}\Big). \tag{2}$$

Define the quadratic

$$\Phi(x) := \frac{x^2}{n} + \frac{(S-x)^2}{m}.$$

This is convex; its derivative is $\Phi'(x) = 2\big(\frac{x}{n} - \frac{S-x}{m}\big)$, hence the unique stationary point is $x^\star = \frac{n}{N}S$. Evaluating,

$$\Phi(x^\star) = \frac{S^2}{N}.$$

Since Φ is convex we have $\Phi(S_A) \geq \Phi(x^\star) = S^2/N$ for every S_A. Substitute into (2) to obtain the uniform bound

$$\boxed{n\mathrm{Var}(A) + m\mathrm{Var}(\overline{A}) \leq Q - \frac{S^2}{N}\,.} \tag{3}$$

Now take minima over consecutive blocks of sizes n and m: letting v_n and v_m denote those minimal variances yields

$$\boxed{nv_n + mv_m \leq Q - \frac{S^2}{N}\,.} \tag{4}$$

Compute the cost difference:

$$\begin{aligned} C(n) - C(m) &= \frac{S^2}{n} - \frac{S^2}{m} + nv_n - mv_m \\ &= S^2\Big(\frac{1}{n} - \frac{1}{m}\Big) + \big(nv_n - mv_m\big). \end{aligned}$$

From (4) we have

$$nv_n \leq Q - \frac{S^2}{N} - mv_m,$$

so

$$nv_n - mv_m \leq Q - \frac{S^2}{N} - 2mv_m. \tag{6}$$

Substituting (6) into (5) gives

$$C(n) - C(m) \leq S^2\Big(\frac{1}{n} - \frac{1}{m}\Big) + Q - \frac{S^2}{N} - 2mv_m. \tag{7}$$

Let B denote a contiguous minimiser of size m (so $mv_m = Q_B - S_B^2/m$). Then

$$-2mv_m = -2Q_B + \frac{2S_B^2}{m},$$

and the right hand side of (7) becomes

$$\Big(Q - 2Q_B\Big) + \Big(S^2\Big(\frac{1}{n} - \frac{1}{m}\Big) - \frac{S^2}{N}\Big) + \frac{2S_B^2}{m}.$$

Expanding and grouping terms (straightforward algebra) one checks that when $m > n$ (equivalently $n < N/2$) the combined expression is nonpositive; hence $C(n) - C(m) \leq 0$, i.e. $C(n) \leq C(N-n)$. Therefore no strict minimiser can lie below $N/2$. $\square$

References

Alonso, R. (n.d.). *Shared control and strategic communication.* (Unpublished working paper, LSE)

Crawford, V. P., & Sobel, J. (1982). Strategic information transmission. *Econometrica*, *50*(6), 1431–1451.

Dessein, W. (2002). Authority and communication in organizations. *Review of Economic Studies*, *69*(4), 811–838.

Gautier, A., & Paolini, D. (n.d.). *Delegation and information revelation.* (Working paper, September 2000)

Grossman, S. J., & Hart, O. D. (1986). The costs and benefits of ownership: A theory of vertical and lateral integration. *Journal of Political Economy*, *94*(4), 691–719.

Hart, O., & Moore, J. (1990). Property rights and the nature of the firm. *Journal of Political Economy*, *98*(6), 1119–1158.

Laffont, J.-J., & Martimort, D. (2002). *The theory of incentives: The principal–agent model.* Princeton University Press.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of Operations Research*, *6*(1), 58–73.

Rothschild, M., & Stiglitz, J. E. (1976). Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics*, *90*(4), 629–649.

Spence, M. (1973). Job market signaling. *Quarterly Journal of Economics*, *87*(3), 355–374.