Information Provision with Endogenous States*

Sanjari Kalantri†

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Abstract

In this paper, I analyse a dynamic model of information provision. A long-lived sender with commitment power provides information about a state of the world to a receiver, who then chooses an action to maximise his short-term payoff. The players have misaligned preferences that depend both on the state and the action. Importantly, the state evolves over time based on the action taken by the receiver. Thus, the sender faces a trade-off between the short-term gain from disclosing information that maximises her current payoff, and the future benefit from influencing the evolution of the state, via the receiver's action. I characterise the sender's optimal information provision rule when the state evolution process is positively associated, negatively associated and independent of the action taken by the receiver.

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†Nuffield College and Department of Economics, University of Oxford; sanjari.kalantri@nuffield.ox.ac.uk

1 Introduction

In many settings, externalities are intertemporal. For example, a firm's leverage influences its future profitability;[1] the quality of our natural resources depends on the steps taken to preserve them;[2] an individual's health depends on their lifestyle: diet, exercise, etc.;[3] investment in education influences earning prospects;[4] the spread of a contagious infection depends on the degree of interpersonal contact within a population;[5] etc. A common feature in all these cases is that current *actions* influence future *payoffs*.

Additionally, in some of these examples, a decision-maker relies on an expert for information. Consider the following scenarios. A startup owner approaches an investor seeking investment for her business. She prefers receiving investment to not, and also prefers that her firm is a profitable one than not. She expects that receiving investment would improve the firm's future profitability and valuation. Thus, she provides information to the investor about her firm in order to persuade the investor into investing. Based on the current profitability and valuation, the investor chooses to provide funds to the owner or not.

During an epidemic, governments publish data informing their citizens about the spread and intensity of a viral infection, who then choose to go out or stay indoors. The government prefers a healthy population in the long-run, but also prefers economic activity in the short-run, which is only possible when citizens go out. If a large proportion of the population is infected with a deadly infection, the government will dissuade citizens from going out.

[1]Numerous theoretical and empirical studies analyse how a firm's capital structure impacts its profitability and vice-versa; these are cited in the review articles by Harris and Raviv (1991) and Myers (2001).

[2]This led to the notion of "sustainable development"; see the report by the World Commission on Environment and Development (1987).

[3]See, for example, reports by the World Health Organisation (2002, 2003).

[4]See Mincer (1958) for an early empirical analysis; Schultz (1961) and Becker (1962) expanded on his results.

[5]Kermack and McKendrick (1927) is an early mathematical analysis on the relationship between population density and the size of an epidemic. There is a large economics literature on the value of using policies that reduce interpersonal contact to stop the spread of a virus; see, for example, Geoffard and Philipson (1996) and Adda (2016).

In both these cases, the expert provides information about a payoff-relevant *state of the world*,[6] that evolves over time as a result of past states and actions. I apply this to a dynamic model of information provision. Specifically, I ask: how should a Sender design an information provision rule in an endogenously evolving environment?

In this paper, I analyse a dynamic model with a single long-lived Sender with commitment power and a sequence of short-lived Receivers. They have misaligned preferences that depend both on an evolving state and the action taken by the Receiver. Since the Receiver is short-lived, he plays a myopic best response every period. Given this, the problem for the Sender reduces to a stochastic dynamic optimisation problem, who wishes to maximise her long-run average discounted payoff.

Neither player observes the state, but they share a common prior on its distribution. Every period, the Sender publicly chooses an information provision rule, that is, a statistical experiment á la Blackwell, and commits to truthfully revealing its signal realisation. The players observe the signal realisation, update their belief about the true state using Bayes' rule, the Receiver chooses an action, and the game proceeds to the next period.

The key departure from dynamic models of information provision previously explored in the literature is that the state evolution process is *endogenous*: the state and the Receiver's action together determine the probability of transitioning to a given state in the next period. Thus, when the Sender chooses an information provision rule, she indirectly chooses the probability with which an action is taken. This, in turn, influences the distribution over next period's prior. To the best of my knowledge, this is the first paper to analyse information provision when the Receiver's action on the evolution of the state.

To highlight the key trade-off faced by the Sender, I analyse a simple model by making a few assumptions. First, I assume that the state space is binary, i.e. it is either *good* or *bad*, with the Sender preferring that the game is in the good state rather than the bad state.

[6] A state is a random variable capturing any underlying event affecting payoffs.

Second, I assume that the Receiver's action space is binary as well, i.e. it is either *high* or *low*, with the Sender preferring that the high action is chosen. These assumptions together imply that *within* a period, the Sender wishes to maximise the probability with which the high action is taken, and *across* periods, she wishes to maximise the probability with which the game is in the good state.

On the other hand, the Receiver wishes to match action to state: take the high action when the state is good, and the low action when the state is bad. I further assume that the state evolution process only depends on the action taken by the Receiver. Thus, the Sender faces the following trade-off: the short-term gain from disclosing information about the current state and influencing the Receiver's current action, versus the long-term benefit from influencing the evolution of the state, via the Receiver's (current) action.

The Sender's optimal information provision rule depends on the relationship between states and actions. If the state evolution process is independent of the action as well, then the Receiver imposes no externalities on the future through his action. In this case, the best that the Sender can do is to maximise her current (stage-game) payoff. She does so by maximising the probability with which the high action is chosen, since she prefers the high action to the low action.

If taking the high action is more likely to take the game into the good state than the low action, then the complementarity between states and actions implies that the Sender can do no better than using the information provision rule that is myopically optimal. This is because the Sender achieves her twin objectives of maximising the probability of the high action and the maximising the occurrence of the good state.

However, if taking the low action is more likely to take the game into the good state than taking the high action, then for the Sender, inducing the action that is myopically optimal leads to lower payoffs in the future. I show that there are various optimal information provision rules that depend on the Sender's stage-game payoffs:

(a) If the Sender's stage-game payoff from the high action is very large (regardless of the state), then the Sender's preference for the high action outweighs the preference for the good state. Thus, she still finds it optimal to maximise the probability of the high action.

(b) If the Sender gets a large payoff in the stage-game from being in the good state (regardless of the action), then the preference for the good state outweighs the preference for the high action. Thus, she finds it optimal to maximise the probability of the low action as it is more likely to result in the good state in the future.

(c) Finally, if the Sender's stage-game payoffs from a mis-match between state and action is very low, then the Sender's long-run preferences fully align with the Receiver's stage-game preferences. Thus, the Sender fully reveals the state to the Receiver.

1.1 Related Literature

This paper belongs to the literature on dynamic information provision, which builds on the insights in Aumann and Maschler (1995) and Kamenica and Gentzkow (2011). The closest papers to the model analysed in this paper are Renault, Solan, and Vieille (2017), Ely (2017) and Lehrer and Shaiderman (forthcoming). In their setting, a single long-lived Sender communicates with a Receiver who behaves myopically, as here. The state evolves exogenously according to a Markov chain, that is, independently of the Receiver's action. In Renault, Solan, and Vieille (2017), the Sender has state-independent payoffs, so every period, she only wants to maximise the probability that a particular action is taken. The authors identify a sufficient condition under which the information provision rule that is myopically optimal (referred to as the "greedy policy") is also optimal in the dynamic game. Lehrer and Shaiderman (forthcoming) extend this analysis and characterise the necessary and sufficient conditions under which the greedy policy is optimal even when the Sender has state-dependent payoffs.

In the basic model of Ely (2017), the state space is binary, and evolves exogenously, with one of the states being absorbing. Here as well, the Sender has state-independent payoffs. Ball (2023) analyses a model in which both players are long-lived and the state evolves exogenously. The Sender has state-dependent preferences that are misaligned from the Receiver, and therefore, faces a trade-off: he gains from revealing accurate information about the state in the current period, but this reduces his informational advantage in the future. Both authors characterise the optimal policy in their respective models.

In Khantadze, Kremer, and Skrzypacz (2025), a sequence of Receivers take actions about a binary state specific to them, where the states have an arbitrary correlation structure. The authors compare simultaneous persuasion procedures to sequential ones and are interested in learning when the Sender can achieve her first-best payoff. One can interpret sequential procedures as a dynamic persuasion game in which the state evolves in an arbitrary manner, independently of the Receiver's action.

This paper also relates to the literature on Markovian games and stochastic games: Escobar and Toikka (2013), Renault (2006), Renault, Solan, and Vieille (2013), Shapley (1953) and Solan and Vieille (2015).

1.2 Roadmap

In Section 2, I introduce a basic two-state, two-action, infinite-horizon model of information provision with endogenously evolving states. I analyse this model, solving for the optimal information provision rules in section 3. In section 4, I consider a more general model of information provision with finite state and action spaces. Section 5 concludes. All proofs are in the appendix.

2 Basic Model

In this section, I introduce a basic two-state, two-action, infinite-horizon model of information provision with endogenously evolving states, as will be made clearer below. Time, indexed by $t \in \mathbb{N}$, is discrete.

A long-lived Sender (she) and an uninformed, short-lived Receiver (he) interact in a period. Their stage-game payoffs depend on the state of the world $\omega_t \in \Omega = \{G(\text{ood}), B(\text{ad})\}$ and the Receiver's action $a_t \in \mathcal{A} = \{h(\text{igh}), \ell(\text{ow})\}$ in that period, and are given as follows:

$u_S(\omega_t, a_t)$	$\omega_t = G$	$\omega_t = B$
$a_t = h$	1	x
$a_t = \ell$	y	0

$u_R(\omega_t, a_t)$	$\omega_t = G$	$\omega_t = B$
$a_t = h$	1	0
$a_t = \ell$	0	1

where $x \in [0, 1]$ and $y \in [0, 1]$. Clearly, within a period, the Sender weakly prefers being in state G to state B (the preference is strict if $x < 1$ and $y > 0$), and weakly prefers action h to action ℓ (the preference is strict if $x > 0$ and $y < 1$). The Sender's stage-game payoffs are *state-independent* if and only if $x = 1$ and $y = 0$; otherwise, they are *state-dependent*. Likewise, the Sender's stage-game payoffs are *action-independent* if and only if $x = 0$ and $y = 1$; otherwise, they are *action-dependent*. Meanwhile, the Receiver wants to match action to state: he strictly prefers taking action h in state G and action ℓ in state B. Both the players are assumed to know the stage-game payoffs.

Since the Receiver is short-lived, he plays myopic best responses that maximise his stage-game payoff. On the other hand, the Sender maximises her long-run average discounted payoff:

$$(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_S(\omega_t, a_t),$$

where $\delta < 1$ is the Sender's discount factor.

The main departure from previous models of dynamic information provision is that the state evolves *endogenously* over time: the period-t action determines the probability of transitioning to a state ω in period-$t+1$. Formally, the transition probabilities are given by $\mathbb{P}(\omega'_{t+1} \mid a_t)$, such that $\sum_{\omega'_{t+1} \in \Omega} \mathbb{P}(\omega'_{t+1} \mid a_t) = 1$. Since the state and action spaces are binary, I simplify notation by using $p^h := \mathbb{P}(\omega_{t+1} = G \mid a_t = h)$ and $p^\ell := \mathbb{P}(\omega_{t+1} = G \mid a_t = \ell)$ to denote the transition probabilities. These include the following cases:

(i) *Absorbing states:* If $p^h = 1$ and $p^\ell = 1$, then the good state is an absorbing state: irrespective of which action is taken, the game always transitions to the good state with certainty and stays there forever. Similarly, if $p^h = 0$ and $p^\ell = 0$, then the bad state is an absorbing state.

(ii) *Action-independent evolution:* If $p^h = p^\ell$, then the state in period-$t+1$ is independent of period-t action.

(iii) *Positive/Negative association:* If $p^h > p^\ell$, then states and actions are positively associated: taking the high action in any state implies a greater probability of being in the good state in the next period, rather than taking the low action. Similarly, if $p^h < p^\ell$, then states and actions are negatively associated.

The Sender and the Receiver don't observe the state; rather, they share a common prior distribution $p_1 \in \Delta(\Omega)$ at the beginning of the game. Every period, the Sender publicly chooses an information provision rule (that is, a Blackwell experiment) and commits to truthfully revealing its signal realisation.[7] Formally, an information provision rule is a pair (S, π_t), where S is a set of signals, and $\{\pi_t(\cdot \mid \omega)\}_{\omega \in \Omega}$ is a set of probability distributions

[7] As mentioned in Renault, Solan, and Vieille (2017), an alternative interpretation would be that the Sender observes the sequence of states, and chooses and commits to an information disclosure policy *ex-ante*, i.e. prior to the beginning of the game. This is because the Receiver is short-lived, so the Sender's problem reduces to a single-agent stochastic dynamic optimisation problem in which she is constrained by the Receiver's best-response function. This has a recursive formulation: see section 3 and equation (3.2) below. In this case, a strategy would map histories (past states and actions and current state) into distributions over signals.

over signals, conditional on the true state ω. The Receiver observes the chosen information provision rule and its signal realisation, updates his belief about the state using Bayes' rule, and chooses an action that maximises his stage-game payoffs. Based on the action chosen by the Receiver, the state evolves for the next period. Since the Sender has no private information in this setup, she updates her belief in the same manner as the Receiver: after observing the signal realisation and after the Receiver takes an action.

To be specific, let $p_t := \mathbb{P}(\omega_t = G)$ denote the common prior belief of the state being G in period-t. The players update to their posterior belief $q_t := \mathbb{P}(\omega_t = G \mid s)$ after observing the signal realisation $s \in S$ and the chosen information provision rule, using Bayes' rule.[8] After the Receiver chooses an action, the state evolves based on the transition probabilities described above. Specifically, the players' prior in period-$t+1$ is given by:

$$p_{t+1}(q_t) = \begin{cases} q_t p^h + (1-q_t)p^h = p^h, & \text{if } a_t = h \\ q_t p^\ell + (1-q_t)p^\ell = p^\ell, & \text{if } a_t = \ell \end{cases} \tag{2.1}$$

Thus, the players' prior in a period is a conditional distribution over states given the action taken in the previous period.

Within a period, the Sender's problem amounts to choosing a distribution over the posterior beliefs the experiment induces. Denote by $\mu \in \Delta(\Delta(\Omega))$ the distribution over the posterior beliefs. It is well known from the literature on repeated games and Bayesian persuasion (Aumann and Maschler (1995); Kamenica and Gentzkow (2011)) that due to Bayesian updating, the feasible set of information provision rules from which the Sender can choose are those in which the distribution over posterior beliefs is such that they equal the prior in expectation, i.e. $\mathbb{E}_\mu(q_t) = p_t$ (Bayes' plausibility).[9]

[8] That is, $q_t(\omega) = \frac{p_t \pi_t(s|G)}{p_t \pi_t(s|G) + (1-p_t)\pi_t(s|B)}$.

[9] A key result in Aumann and Maschler (1995) is that for a given distribution over posteriors such that they equal the prior in expectation, the Sender can always design an experiment that correlates the signals with the state in such a way that it induces the said distribution over posteriors.

However, since the problem is dynamic, by choosing a distribution over posterior beliefs, the Sender indirectly chooses the probability with which an action is taken. This, in turn, influences the distribution of the next period's prior, as can be seen in equation (2.1). Thus, when choosing an information provision rule, the Sender chooses a (stochastic) sequence of players' beliefs.[10] The Sender faces a trade-off: the short-term gain from disclosing information about the current state versus the benefit from influencing the evolution of the state, via the Receiver's action.

The basic model maps into the examples discussed in the Introduction. In the first scenario, the Sender is the firm owner approaching investors (Receivers) for funding. The firm owner prefers investment (high action) and a profitable firm (good state). In this case, states and actions are likely to positively associated: investment is more likely to lead to the firm being profitable, than non-investment.

In the second scenario, the Sender is the government informing its citizens (Receivers) about the epidemic. In the short-run, the government prefers economic activity, which is possible when individuals go out (high action), and a healthy population (good state). In this case, there is likely to be negative association: reduced interpersonal contact is more likely to lead to a healthy population.

3 Analysis of the Basic Model

3.1 The Myopic Provision Rule

I first analyse a static version of the model. In any period, the problem facing the Receiver is simple. Given his period-t posterior belief q_t, the Receiver gets an expected payoff of

[10] Ely (2017) formalised this by proving a *dynamic obfuscation principle*: the Sender's problem of choosing an information provision rule is equivalent to choosing a stochastic process of the Receiver's beliefs. The key difference between his model and the model analysed in this paper is that the state evolved *exogenously* (i.e. independently of Receiver's action) in the former.

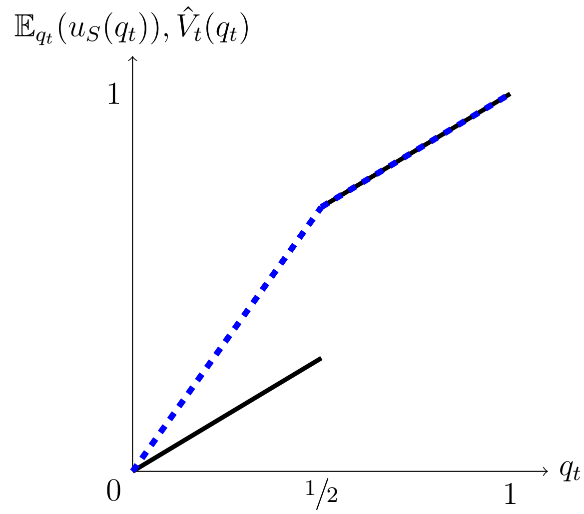

Figure 1: Sender's interim (black) and optimal (blue) expected payoff under the myopic provision rule

$q_t \cdot 1 + (1 - q_t) \cdot 0$ if he chooses action h, and an expected payoff of $q_t \cdot 0 + (1 - q_t) \cdot 1$ if he chooses action ℓ, so that $a(q_t) = h$ if $q_t \geq 1/2$, and $a(q_t) = \ell$ otherwise.[11] Given this, the Sender's interim expected payoffs are:

$$\mathbb{E}_{q_t}(u_S(q_t)) = \begin{cases} q_t \cdot 1 + (1 - q_t) \cdot x, & \text{if } q_t \geq \frac{1}{2} \\ q_t \cdot y, & \text{if } q_t < \frac{1}{2} \end{cases} \tag{3.1}$$

where $\mathbb{E}_{q_t}(u_S(q_t)) := \mathbb{E}_{q_t}(u_S(\omega_t, a(q_t)))$. In Figure 1 above, the Sender's interim expected payoff is plotted by the solid black line.

The concavification[12] of the Sender's interim expected payoff (3.1) gives the Sender's optimal expected payoff, and is given by:

$$\hat{V}_t(q_t) = \min\{(1 + x)q_t, x + q_t(1 - x)\},$$

[11]I assume that the Receiver breaks ties in the Sender's favour, so that the Sender's interim expected payoffs are upper semi-continuous. This is needed to ensure existence of the Sender's optimum.

[12]The concavification of a function f is the smallest concave function greater than or equal to f.

which is plotted by the dashed blue line in Figure 1. Evaluating the above function at the prior, we see that the Sender's optimal payoff from information provision is $x + p_t(1 - x)$ if $p_t \geq 1/2$ and $(1 + x)p_t$ if $p_t < 1/2$. Thus, the optimal information provision rule for the Sender is to not disclose anything if $p_t \geq \frac{1}{2}$; if $p_t < \frac{1}{2}$, she induces a posterior belief of $\frac{1}{2}$ with probability $2p_t$ and a posterior of 0 with complementary probability.[13] This leads to the high action being taken with probability 1 if $p_t \geq 1/2$ and with probability $2p_t$ if $p_t < 1/2$. Indeed, subject to Bayes' plausibility, the Sender designs an information provision rule that maximises the probability of the high action being taken within a period, as she weakly prefers action h to action ℓ. I will refer to the information provision rule described above as the *myopic provision rule.*

Remark 1. The myopic provision rule is uniquely optimal if and only if $x \neq 0$. If $x = 0$, there are a range of optimal policies, including the myopic provision rule and full disclosure.

In the static model, notice that the state plays no role in influencing the Sender's incentives. This leads me to the question: if the Sender also has preferences over the state, how should she communicate to maximise her current and future payoffs?

3.2 Optimal Dynamic Provision Rule

The question at the end of the previous sub-section underscores the key trade-off facing the Sender in any period: maximising short-term payoff versus influencing the evolution of the state (and thus future payoff) via the Receiver's (current period) action. In other words, the Sender has twin objectives: *within* a period, she wishes to maximise the probability of the high action, while across periods, she wishes to maximise the probability of the game being in the good state.

[13] Let H and L denote the signals that induce actions h and ℓ respectively. The signal structure that leads to the beliefs mentioned above are: if $p \geq 1/2$, then $\mathbb{P}(H \mid G) = \mathbb{P}(H \mid B) = 1$; if $p < 1/2$, then $\mathbb{P}(H \mid G) = 1$, $\mathbb{P}(H \mid B) = \frac{p}{1-p}$, $\mathbb{P}(L \mid G) = 0$, and $\mathbb{P}(L \mid B) = \frac{1-2p}{1-p}$.

Recall that the Receiver chooses the high and low actions if $q_t > 1/2$ and $q_t < 1/2$, respectively, with tie-breaking in favour of the Sender when indifferent. This, combined with equation (2.1), implies the following:

Lemma 1. For all $t \geq 2$, the period-t prior is given by:

$$p_{t+1}(q_t) = \begin{cases} p^h, & \text{if } q_t > \frac{1}{2} \\ p^\ell, & \text{if } q_t < \frac{1}{2} \end{cases}.$$

That is, the common prior from the second period onward is either p^h or p^ℓ. This greatly simplifies the analysis.

Let $V(\cdot)$ denote the Sender's maximum payoff from the dynamic optimisation problem. This function is characterised by the solution to the following Bellman equation:

$$\begin{gathered} V(p_t) = \max_{\mu \in \Delta(\Delta(\Omega))} \mathbb{E}_\mu \{(1-\delta)\mathbb{E}_{q_t}(u_S(q_t)) + \delta V(p_{t+1}(q_t))\} \\ \text{subject to } \mathbb{E}_\mu(q_t) = p_t \end{gathered} \tag{3.2}$$

where $p_{t+1}(q_t)$ is as defined in Lemma 1.

In appendix A.1, I show that the value function $V(\cdot)$ is concave over its domain. This implies that for any prior, the Sender induces a distribution over posteriors that has a maximum of two elements in its support. This is because if there are more than two posteriors inducing the same action, due to concavity of $V(\cdot)$, the Sender can weakly improve her payoff by inducing a single posterior that induces the said action. Therefore, it is without loss of generality to restrict the space of signal realisations to two signals, such that each induces a different posterior, and therefore, a different action.

Let q^h and q^ℓ denote the posteriors that induce actions h and ℓ, respectively. Then, Bayes' plausibility implies that these need to be chosen such that $p = \alpha q^h + (1-\alpha)q^\ell$, where $\alpha := \mu(q^h) \in [0,1]$ is the probability with which the posterior q^h is induced.[14] Then, the Bellman equation (3.2) can be re-written as (suppressing the time index for the current prior):

$$
\begin{aligned}
V(p) = \max_{\alpha, q^h, q^\ell} \{(1-\delta)[\alpha(q^h + (1-q^h)x) + (1-\alpha)q^\ell y] + \delta\alpha V(p^h) + \delta(1-\alpha)V(p^\ell)\} \\
\text{subject to } p = \alpha q^h + (1-\alpha)q^\ell, q^h \in [1/2, 1], q^\ell \in [0, 1/2]
\end{aligned}
\tag{3.3}
$$

The optimal values of α, q^h and q^ℓ depend on the values of p, x, y, and on the relationship between p^h and p^ℓ. Thus, the association between states and actions determines the Sender's optimal policy. In the dynamic game, the Sender can always guarantee herself the payoff obtained by repeatedly playing the myopic provision rule. Under some conditions, I show below that this is not necessarily optimal. There are 3 cases to consider: independence between states and actions, positive association and negative association.

3.2.1 Action-independent evolution

I begin with the following lemma, whose proof is in appendix A.2:

Lemma 2. If $p^h = p^\ell$, then for all $(x, y) \in [0,1]^2$, the myopic provision rule is optimal.

When $p^h = p^\ell$, then the state evolution process is independent of both state and action. Thus, the Receiver imposes no externalities on the future through his choice of action. The Sender, too, is unable to influence future states through her choice of information provision rule. Therefore, the best that the Sender can do is maximise her current payoff, which implies the optimality of the myopic provision rule.

[14] Under the myopic provision rule, if $p < 1/2$, then $\alpha = 2p$, $q^h = 1/2$ and $q^\ell = 0$. On the other hand, if $p \geq 1/2$, then $\alpha = 1$ and $q^h = p$.

3.2.2 Positive Association

The proposition below, proven in appendix A.3, states that the myopic provision rule remains optimal, even under positive association.

Proposition 1. If $p^h > p^\ell$, then for all $(x, y) \in [0,1]^2$, the myopic provision rule is optimal for the Sender.

The intuition for the above result is as follows. Recall that the Sender weakly prefers being in the good state than the bad state, and weakly prefers that the Receiver take the high action than the low action. Thus, within a period, she wants to maximise the probability with which the high action is taken, and in the long-run, she also wants to maximise the probability with which the game is in the good state. Due to positive association between states and actions, both of these are achieved by the myopic provision rule: it maximises the probability of the high action in a period, and also leads to a greater probability of being in the good state in the future. Hence, when persuading an investor to invest in her firm, the firm owner can do no better than treating the problem as a static one.

3.2.3 Negative Association

In this case, the Sender faces a trade-off: inducing the action that is myopically optimal leads to lower payoffs in the future. Consider the following extreme cases:

(i) $x = 1$ and $y = 0$: the Sender has state-independent payoffs, i.e. she is indifferent between states. She only wants to maximise the probability of the high action every period. Since she is also present-biased ($\delta < 1$), it follows that the myopic provision rule remains optimal.

(ii) $x = 0$ and $y = 1$: the Sender has action-independent payoffs, i.e. she is indifferent between actions. She only wants to maximise the probability of the game being in the

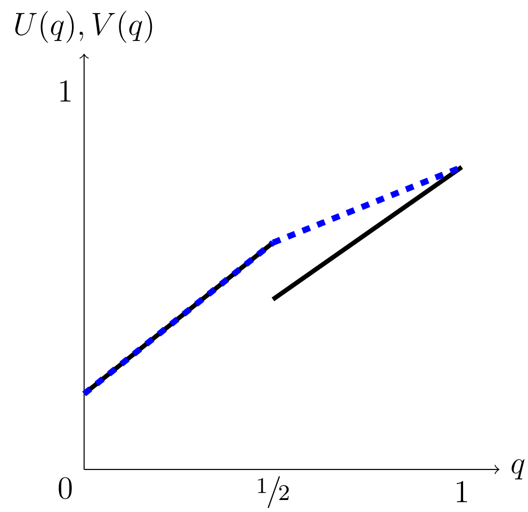

Figure 2: Sender's interim (black) and optimal (blue) expected payoff under the reverse provision rule

good state. This is achieved by maximising the probability with which the low action is taken, since $p^h < p^\ell$.

As seen by point (ii) above, for some values of x and y, it is not optimal for the Sender to use the myopic provision rule. I refer to the information provision rule that maximises the probability of the low action as the *reverse provision rule* (see Figure 2). The Sender doesn't disclose any information if $p \leq 1/2$; if $p > 1/2$, she induces a posterior belief of $1/2$ with probability $2(1-p)$ and a posterior belief of 1 with complementary probability.[15,16]

The following proposition characterises the optimal information provision rules under negative association:

Proposition 2. If $p^h < p^\ell$, there exist thresholds $\bar{x} = \frac{\delta(p^\ell - p^h)}{1+\delta(p^\ell - p^h)}$ and $\bar{y} = \frac{1}{1+\delta(p^\ell - p^h)}$ such that:

[15] The signal structure that leads to the beliefs mentioned above are: if $p \leq 1/2$, then $\mathbb{P}(L \mid G) = \mathbb{P}(L \mid B) = 1$ so that $\alpha = 0$ and $q^\ell = p$; if $p > 1/2$, then $\mathbb{P}(L \mid G) = \frac{1-p}{p}$, $\mathbb{P}(L \mid B) = 1$, $\mathbb{P}(H \mid G) = \frac{2p-1}{p}$, $\mathbb{P}(H \mid B) = 0$ so that $\alpha = 2p-1$, $q^h = 1$ and $q^\ell = 1/2$.

[16] Note that here, the Receiver breaks ties in the Sender's favour by choosing the low action when his posterior belief is $1/2$.

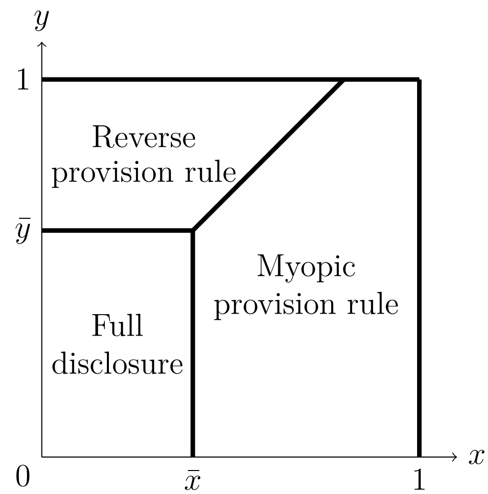

Figure 3: Optimal information provision rules under negative association

(i) If $x \geq \bar{x}$ and $y \leq L(x)$, the myopic provision rule is optimal;

(ii) If $L(x) \leq y$ and $y \geq \bar{y}$, the reverse provision rule is optimal; and

(iii) If $(x, y) \leq (\bar{x}, \bar{y})$, full disclosure is optimal for the Sender.

The function $L(x)$ is linear; its exact functional form depends on magnitudes of p^ℓ and p^h.

The proof is in appendix A.4. Figure 3 above plots the optimal information provision rules for different values of x and y. Recall that x and y are the Sender's payoffs from actions *mis-matching* the state: taking the high action in the bad state and taking the low action in the good state, respectively. Intuitively, for high values of x and low values of y, the Sender's preference for the high action is outweighed by her preference for the good state in the dynamic game. This implies that she still finds it optimal to maximise the probbaility of the high action, and therefore, uses the myopic provision rule. This region includes the extreme of $x = 1$ and $y = 0$ considered at the beginning of this subsection.

Similarly, for low values of x and high values of y, the Sender's preference for the good state is outweighed by her preference for the high action. Thus, the Sender wishes to maximise the probability of that action which leads to a greater probability of being in the good state in the future. Since $p^h < p^\ell$, this is achieved by the reverse provision rule. It also includes the other extreme of $x = 0$ and $y = 1$ discussed at the beginning of this subsection.

For low values of both x and y, the Sender gets a very high payoff of 1 if and only if the game is in the good state and the high action is taken. If she maximises the probability of the high action, she also maximises the probability of the bad state in the future, which would give her very low payoffs. On the other hand, if she maximises the probability of the low action, she maximises the probability of the good state in the future, but gets a very low payoff in the current period. Therefore, the Sender would sometimes like the high action to be taken and sometimes the low action. She resolves this trade-off by fully disclosing the state to the Receiver. I show in appendix A.4 that when x and y are smaller than the thresholds defined in Proposition 2, the Sender's long-run preferences and the Receiver's stage-game preferences are fully aligned. That is, given the state, they both prefer the same action to be taken, which implies the optimality of full disclosure.[17]

Proposition 2 highlights why governments have different (informational) responses to various epidemics. This depends on a variety of factors: their valuations of x and y; the effects of actions on the future, captured through the transition probabilities p^h and p^ℓ; and the discount factor δ.

Recall that in the static model, full disclosure was also an optimal information provision rule if and only if $x = 0$. In comparison, under negative association, there are a range of values for which full disclosure is *uniquely* optimal: specifically, when $(x, y) < (\bar{x}, \bar{y})$. In Figure 3, for the Sender, the myopic provision rule and full disclosure are payoff-equivalent when $x = \bar{x}$ and $y \leq \bar{y}$; the reverse provision rule and full disclosure are payoff-equivalent when $y = \bar{y}$ and $x \leq \bar{x}$; and the myopic and reverse provision rules are payoff-equivalent when $x \geq \bar{x}$, $y \geq \bar{y}$ and $y = L(x)$. Specifically, at $(\bar{x}, \bar{y})$, the Sender is indifferent between all

[17] The associated signal structure is: $\mathbb{P}(H \mid G) = \mathbb{P}(L \mid B) = 1$, $\mathbb{P}(H \mid B) = \mathbb{P}(L \mid G) = 0$, $\forall p \in [0, 1]$, which implies $\alpha = p$, $q^h = 1$ and $q^\ell = 0$.

three rules. The Receiver, on the other hand, prefers more information to less.

Finally, I comment on the asymmetry in Figure 3. Observe that $\bar{x} < 1/2 < \bar{y}$ and that when $x = 1$ and $y = 1$, the myopic provision rule is uniquely optimal. There are two reasons for this asymmetry. The first is that the Sender discounts the future, since $\delta < 1$ by assumption, so she would like to maximise her current payoff, or in other words, maximise the probability of the high action. Secondly, even if $\delta = 1$ so that the Sender weighs the present and the future equally, $\bar{x} < 1/2 < \bar{y}$ and the myopic provision rule is uniquely optimal when $(x, y) = (1, 1)$, except if $p^\ell = 1$ and $p^h = 0$. That is, if the future state post taking an action is uncertain, the Sender's desire for instant gratification means that she still likes to maximise her current payoff. However, if $p^\ell = 1$, $p^h = 0$ and $\delta = 1$, then $\bar{x} = \bar{y} = 1/2$ and the Sender is indifferent between the myopic and reverse provision rules when $(x, y) = (1, 1)$.

3.2.4 Relationship to Renault, Solan, and Vieille (2017)

Renault, Solan, and Vieille (2017) analysed a dynamic model of information provision as well in which the Receiver's action space was binary. However, there are two key differences. First, in their model, the state evolved exogenously, i.e., the process only depended on the current state. Second, the Sender had state-independent payoffs: she received a positive payoff if one of the actions was taken, and a payoff of 0 from the other. Under these conditions, the authors showed that if the state space is binary, the myopic provision rule was optimal for the Sender irrespective of the prior.

Consider the case where $x = 1$ and $y = 0$, so that the Sender has state-independent payoffs. The following corollary follows from Lemma 2 and Propositions 1 and 2:

Corollary 1. If the Sender has state-independent payoffs, then irrespective of the association between states and actions, the myopic provision rule is optimal for the Sender.

That is, if the Sender has state-independent payoffs, the myopic provision rule is optimal even when the state evolution process depends only on the action. Intuitively, if the Sender has state-independent payoffs, she only cares about the action taken in a period. Since she prefers action h to action ℓ, she wants to maximise the probability of action h within a period. This is achieved by the myopic provision rule.

4 General Model

I adopt the general model of Ely (2017), and allow for endogenous states. In the general model, the Sender's and Receiver's period-t payoffs depend on a state of the world $\omega_t \in \Omega$ and the Receiver's action $a_t \in \mathcal{A}$ in that period, and are given by $u_S(\omega_t, a_t)$ and $u_R(\omega_t, a_t)$, respectively. I assume that the sets Ω and $\mathcal{A}$ are finite, with $|\Omega| \geq 2$ and $|\mathcal{A}| \geq 2$. As earlier, since the Receiver is short-lived, he plays myopic best responses that maximise his stage-game payoff, while the Sender maximises her long-run average discounted payoff.

The state evolution process depends on both the period-t action and the period-t state, together determining the probability of transitioning to a state ω in period-$t+1$. Formally, the state transition probabilities are given by $\mathbb{P}(\omega' \mid \omega, a)$, such that $\sum_{\omega' \in \Omega} \mathbb{P}(\omega' \mid \omega, a) = 1$. This includes the following cases:

(i) Absorbing states: If, for a given state ω, $\mathbb{P}(\omega \mid \omega, a) = 1 \; \forall a \in \mathcal{A} \iff \mathbb{P}(\omega' \mid \omega, a) = 0$ $\forall \omega' \neq \omega, \forall a \in \mathcal{A}$, then state ω is absorbing: once the game reaches state ω, it stays there forever. This is the case analysed in the basic model in Ely (2017).

(ii) Action-independent evolution: If, for every distinct pair $\omega, \omega' \in \Omega$, $\mathbb{P}(\omega' \mid \omega, a) = \mathbb{P}(\omega' \mid \omega, a')$, $\forall a, a' \in \mathcal{A}$, then the state transition process is independent of action, and only depends on the state. This is the case explored in Renault, Solan, and Vieille (2017)[18] and in the general model of Ely (2017).

[18] Renault, Solan, and Vieille (2017) analyse a special class of games in which the Receiver's action space is binary.

(iii) State-independent evolution: If, for every $a \in \mathcal{A}$, $\mathbb{P}(\omega \mid \omega', a) = \mathbb{P}(\omega \mid \omega'', a)$, $\forall \omega, \omega', \omega'' \in \Omega$, then the state transition process is independent of state, and only depends on the action, as in the basic model of Section 2.

The players' common prior distribution in period-t is represented by a vector $p_t \in \Delta(\Omega)$. Let $p_t(\omega)$ denote the period-t prior probability of being in state ω, so that $\sum_{\omega \in \Omega} p_t(\omega) = 1$. The Sender chooses an information provision rule (S, π_t), defined in the same manner as earlier. Using the same arguments as in the basic model, it is straightforward to show that it is without loss of generality to assume that $|S| \leq |\mathcal{A}|$, with each signal realisation inducing a different action. The players update to their posterior belief $q_t(\omega) := \mathbb{P}(\omega_t = \omega \mid s)$ after observing the signal realisation $s \in S$ and the chosen information provision rule, using Bayes' rule.

After the Receiver chooses an action, the state evolves based on the transition probabilities described above. Given the posterior q_t in period-t and the action $a \in \mathcal{A}$ taken in period-t, the prior in period-$t+1$ is given by:

$$p_{t+1}(\omega') = \sum_{\omega \in \Omega} q_t(\omega) \cdot \mathbb{P}(\omega' \mid \omega, a) \tag{4.1}$$

Thus, the players' prior in a period is a conditional distribution over states given the signal realisations and actions in all periods preceding that period. Updating for every state $\omega \in \Omega$ using equation (4.1), we get the whole vector of period-$t+1$ beliefs. Given q_t and action $a \in \mathcal{A}$, let $\phi^a(q_t)$ denote period-$t+1$'s prior. That is, each action results in a different prior in period-$t+1$.

Remark 2. From equation (4.1), we see that $\phi^a(q) = \phi^{a'}(q)$ $\forall q \in [0,1]$, $\forall a, a' \in \mathcal{A}$ if and only if there is action-independent evolution. That is, the Receiver's prior in period-$t+1$ is the same irrespective of the action taken, which I denote by $\phi(q)$. Thus, we can see that Ely (2017) is a special case of the model presented in this section.

Given the Receiver's best-response function $a(q_t)$, the Sender's stage-game interim expected payoffs are given by $\mathbb{E}_{q_t}(u_S(q_t)) \coloneqq \mathbb{E}_{q_t}(u_S(\omega_t, a(q_t)))$. Let $V(\cdot)$ denote the Sender's maximum payoff from the dynamic optimisation problem. This function is characterised by the solution to the following Bellman equation:

$$\begin{gathered} V(p_t) = \sup_{\mu \in \Delta(\Delta(\Omega))} \mathbb{E}_\mu\{(1-\delta)\mathbb{E}_{q_t}(u_S(q_t)) + \delta V(\phi^a(q_t))\} \\ \text{subject to } \mathbb{E}_\mu(q_t) = p_t \end{gathered} \tag{4.2}$$

In the functional equation above, the expression in the curly parentheses is the Sender's interim expected payoffs in the dynamic game. Concavification of these interim payoffs gives us the Sender's optimal expected payoff, so that:

Theorem 1. The functional equation (4.2) characterising the Sender's optimal expected payoff has a unique fixed point and is given by:

$$V(p) = \text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta V(\phi^a(q))\}.$$

Ely (2017) proved a special case of the theorem above with $\phi(q)$ replacing $\phi^a(q)$. As in the special case, the proof of the more general case (in appendix B.1) uses the same arguments and the contraction mapping theorem to show the existence and uniqueness of a fixed point.

5 Conclusion

In this paper, I analyse a simple model of information provision with endogenous states. I show that depending on the preferences of the Sender and the nature of association between the states and actions, different information provision rules are optimal. In particular, the provision rule that is myopically optimal is sometimes optimal even in the dynamic game, but not always.

I make a few observations about the model. First, in the basic model, I solved for the case where the state transition process depends only on the action in closed-form; the model with the state transition process depending on the state as well is yet to be solved. The analysis in section 3 provides some intuition on what the optimal information provision rule may look like in this slightly more general model. Second, the Sender and the Receiver shared a common prior about the state. This assumption simplified the analysis, but need not necessarily be the case.

These observations provide directions for future research. A direction to analyse is when the Sender and the Receiver have different prior beliefs about the state. Broader results about optimal information provision rules on the general model of section 4 with a finite state space and a finite action space could also be explored.

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Appendix A Proofs of Section 3

A.1 Preliminaries

Lemma 3. $V(\cdot)$ is concave over its domain.

Proof. The proof of the result is in Renault, Solan, and Vieille (2017), re-produced here. To show concavity, we have to show that $V(p) \geq \lambda V(p') + (1-\lambda)V(p'')$, where $p = \lambda p' + (1-\lambda)p''$ for $\lambda \in [0,1]$. At belief p, the Sender can guarantee an expected payoff of at least $\lambda V(p') + (1-\lambda)V(p'')$ by inducing posteriors of p' with probability λ and p'' with complementary probability, and then playing the optimal strategy at the induced posterior. The result then follows since by definition, $V(p)$ denotes the maximum payoff to the Sender at the prior p. ■

Corollary 2. The support of the distribution $\mu \in \Delta(\Delta(\Omega))$ has at most 2 elements.

Proof. Since $V(\cdot)$ is concave, if two or more posteriors induce the same action from the Receiver, the Sender gains by inducing a single (combined) posterior that induces the said action. Since the Receiver's action space is binary, therefore the Sender induces a maximum of two posteriors at any prior p. ■

The Bellman equation (3.3) characterising the Sender's optimal value is reproduced below:

$$V(p) = \max_{\alpha, q^h, q^\ell} \{(1-\delta)[\alpha(q^h + (1-q^h)x) + (1-\alpha)q^\ell y] + \delta\alpha V(p^h) + \delta(1-\alpha)V(p^\ell)\}$$
$$\text{subject to } p = \alpha q^h + (1-\alpha)q^\ell, q^h \in [1/2, 1], q^\ell \in [0, 1/2]$$

Claim 1. In the optimal information provision rule, for a given p, one of the following are true: $\alpha = 0$, $\alpha = 1$, or if $\alpha \in (0,1)$, then $q^\ell \in \{0, \frac{1}{2}\}$ and $q^h \in \{\frac{1}{2}, 1\}$ with $q^\ell \neq q^h$.

Proof. The Sender's interim expected payoffs in the dynamic game are given by:

$$\mathbb{E}_q(u_S(q)) = \begin{cases} (1-\delta)(q+(1-q)x)+\delta V(p^h), & \text{if } q > \frac{1}{2} \\ (1-\delta)qy+\delta V(p^l), & \text{if } q < \frac{1}{2} \end{cases} \tag{A.1}$$

The Receiver breaks ties in the Sender's favour when $q = 1/2$. Equation (A.1) is piecewise linear in q. Therefore, its concavification will also be piecewise linear. The result follows. ∎

By Lemma 1, we saw that the (common) prior from the second period onward is either p^h or p^ℓ. Therefore, in the continuation game starting from the second period, we have the following:

Lemma 4. In the continuation game starting from the second period, $V(p^h)$ and $V(p^\ell)$ are a weighted average of the Sender's per-period expected payoff at priors p^h and p^ℓ. The weights are determined endogenously within the model; they depend on the information provision rule used at priors p^h and p^ℓ.

Proof. From the above, we see that the Sender's optimal choice of α, q^h and q^ℓ depends on the prior p, values of x and y, and on p^h and p^ℓ. Denote by $\alpha^*(p)$, $q^{h^*}(p)$ and $q^{\ell^*}(p)$ the values of α, q^h and q^ℓ that maximise the Bellman equation above at the prior p. Since the value function $V(\cdot)$ holds at all priors, it must hold at p^h and p^ℓ too.

Simplifying notation and writing $\alpha^*(p^h) \equiv \alpha^*_h$, $\alpha^*(p^\ell) \equiv \alpha^*_\ell$, $q^{h^*}(p^h) \equiv q^{h^*}_h$, $q^{h^*}(p^\ell) \equiv q^{h^*}_\ell$, $q^{\ell^*}(p^h) \equiv q^{\ell^*}_h$ and $q^{\ell^*}(p^\ell) \equiv q^{\ell^*}_\ell$, we can re-write the Bellman equation above as:

$$V(p^h) = (1-\delta)[\alpha^*_h(q^{h^*}_h + (1-q^{h^*}_h)x) + (1-\alpha^*_h)q^{\ell^*}_h y] + \delta\alpha^*_h V(p^h) + \delta(1-\alpha^*_h)V(p^\ell) \tag{A.2}$$

$$V(p^\ell) = (1-\delta)[\alpha^*_\ell(q^{h^*}_\ell + (1-q^{h^*}_\ell)x) + (1-\alpha^*_\ell)q^{\ell^*}_\ell y] + \delta\alpha^*_\ell V(p^h) + \delta(1-\alpha^*_\ell)V(p^\ell) \tag{A.3}$$

which is a system of two equations in two unknowns $V(p^h)$ and $V(p^\ell)$. Solving equations

(A.2)–(A.3) simultaneously, we get:

$$V(p^h) = \frac{1-\delta+\delta\alpha_\ell^*}{1-\delta\alpha_h^*+\delta\alpha_\ell^*} f_h + \frac{\delta(1-\alpha_h^*)}{1-\delta\alpha_h^*+\delta\alpha_\ell^*} f_\ell \tag{A.4}$$

$$V(p^\ell) = \frac{\delta\alpha_\ell^*}{1-\delta\alpha_h^*+\delta\alpha_\ell^*} f_h + \frac{1-\delta\alpha_h^*}{1-\delta\alpha_h^*+\delta\alpha_\ell^*} f_\ell \tag{A.5}$$

where $f_h := \alpha_h^*(q_h^{h^*} + (1-q_h^{h^*})x) + (1-\alpha_h^*)q_h^{\ell^*}y$ and $f_\ell := \alpha_\ell^*(q_\ell^{h^*} + (1-q_\ell^{h^*})x) + (1-\alpha_\ell^*)q_\ell^{\ell^*}y$. These are the Sender's per-period expected payoffs at priors p^h and p^ℓ, respectively. ■

From equations (A.4)-(A.5), we see that the Sender's optimal values $V(p^h)$ and $V(p^\ell)$ depend on all six choice variables: α_h^*, α_ℓ^*, $q_h^{h^*}$, $q_\ell^{h^*}$, $q_h^{\ell^*}$ and $q_\ell^{\ell^*}$. Moreover, note that:

$$V(p^h) - V(p^\ell) = \frac{(1-\delta)(f_h - f_\ell)}{1-\delta\alpha_h^*+\delta\alpha_\ell^*} \tag{A.6}$$

Since $\delta < 1$ and $\alpha_h^*, \alpha_\ell^* \in [0,1]$, $1-\delta\alpha_h^*+\delta\alpha_\ell^* > 0$. Thus, the sign of $V(p^h) - V(p^\ell)$ depends on the sign of $f_h - f_\ell$.

A.2 Proof of Lemma 2

If $p^h = p^\ell$, then the Bellman equation (3.3) reduces to:

$$V(p) = \max_{\alpha, q^h, q^\ell} \{(1-\delta)[\alpha(q^h + (1-q^h)x) + (1-\alpha)q^\ell y] + \delta V(p^h)\}$$
$$\text{subject to } p = \alpha q^h + (1-\alpha)q^\ell, q^h \in [1/2, 1], q^\ell \in [0, 1/2]$$

Since $\delta V(p^h)$ is independent of α, q^h and q^ℓ, the Sender's problem reduces to maximising her current payoff, which implies the optimality of the myopic provision rule.

A.3 Proof of Proposition 1

The proof relies on the following two lemmata.

Lemma 5. Suppose $p^h > p^\ell$ and $p^h \geq \frac{1}{2}$. Then, if $p \geq \frac{1}{2}$, it is optimal for the Sender to not disclose any information to the Receiver.

Proof. Let $p^h > p^\ell$ and $p^h \geq \frac{1}{2}$. If $p \geq \frac{1}{2}$ and the Sender doesn't disclose any information, the Receiver chooses action h with probability 1 in the current period. The current expected payoff to the Sender is $p + (1-p)x$. Then, with probability 1, the prior in the next period is p^h, which is (weakly) greater than $\frac{1}{2}$ by assumption, so the Receiver chooses action h. This continues *ad infinitum*, so the Sender's long-run payoff is $(1-\delta)(p+(1-p)x)+\delta(p^h+(1-p^h)x)$. Since the Sender weakly prefers action h to action ℓ, this provides an upper bound on the Sender's payoffs.

To prove optimality, consider a one-shot deviation. Suppose that the Sender deviates and discloses some information when $p \geq \frac{1}{2}$. Then, by Corollary 2, the Sender does so in a manner that induces different actions. This means that the Receiver takes the low action with a strictly positive probability now, reducing the Sender's current payoff.[19] Therefore, the next period's prior could be p^h or p^ℓ, both with positive probability.

If the next period's prior is p^h, the Sender reverts back to not disclosing any information, since $p^h \geq \frac{1}{2}$. Thus, in this case, the Sender gets the same continuation payoff as earlier.

If the next period's prior is p^ℓ, there are two cases to consider:

(i) $p^h > p^\ell \geq \frac{1}{2}$: The Sender reverts back to not disclosing any information, since $p^\ell \geq \frac{1}{2}$. Therefore, Sender's continuation payoff is $\delta\{(1-\delta)(p^\ell + (1-p^\ell)x) + \delta(p^h + (1-p^h)x)\}$, which is (strictly) less than $\delta(p^h + (1-p^h)x)$, as $p^h > p^\ell$ by assumption. Thus, in this case, the Sender gets a lower continuation payoff.

(ii) $p^h \geq \frac{1}{2} > p^\ell$: The Sender could use any information provision rule, since now $p^\ell < \frac{1}{2}$. However, notice that by Bayes' plausibility, there is no information provision rule that would induce a posterior (weakly) greater than $\frac{1}{2}$ with probability 1. That is, when the

[19] If the Sender deviates, her current payoff is $\alpha q^h + \alpha(1-q^h)x + (1-\alpha)q^\ell y$, where $\alpha q^h + (1-\alpha)q^\ell = p$, which is weakly less than $p + (1-p)x$.

prior is $p^\ell < \frac{1}{2}$, the Receiver chooses action ℓ with strictly positive probability. This leads to a lower continuation payoff for the Sender, since otherwise she gets the payoff from action h with probability 1 *ad infinitum*, which she (weakly) prefers.

Therefore, the Sender gets a weakly lower current payoff and/or a lower continuation payoff if she discloses any information when $p \geq \frac{1}{2}$. This proves the result. ■

Lemma 6. Suppose $p^h > p^\ell$. Then, if $p < \frac{1}{2}$, it is optimal for the Sender to induce posteriors 0 and $\frac{1}{2}$ such that it satisfies Bayes' plausibility.

Proof. Let $p^h > p^\ell$. If $p < \frac{1}{2}$ and the Sender follows the strategy outlined in the lemma, then she induces action h with probability $2p$ and action ℓ with probability $(1-2p)$, earning an expected payoff of $2p \cdot (\frac{1}{2} + \frac{1}{2}x) + (1-2p) \cdot 0 = p(1+x)$ in the current period. This is weakly greater than $py = \mathbb{E}(u_S(\omega_t, \ell))$, which is the expected payoff from no disclosure. This implies that with probability $2p$ the prior in the continuation game is p^h, which gives her a continuation payoff of $V(p^h)$, and with probability $(1-2p)$ the prior in the continuation game is p^ℓ, which gives her a continuation payoff of $V(p^\ell)$.

In what follows, I show the optimality of the rule described in the lemma in the continuation game, using Lemma 5, and then show that the Sender has no incentive to deviate in the original game as well. There are three cases to consider:

(i) $p^h > p^\ell \geq \frac{1}{2}$: In the continuation game, it is optimal to not disclose any information by Lemma 5 above. As is clear from point (i) in the proof of Lemma 5, the Sender gets a higher continuation payoff if the prior p^h is induced. This means that if $p < \frac{1}{2}$, the Sender's problem reduces to maximising the probability of action h in the current period, since it maximises both current and future payoffs. This is achieved by the provision rule described in the lemma, so the Sender has no incentive to deviate.

(ii) $p^h \geq \frac{1}{2} > p^\ell$: If the prior in the continuation game is p^h, then it is optimal to not disclose any information by Lemma 5 above. If the prior in the continuation game is

p^ℓ, then as is clear from point (*ii*) in the proof of Lemma 5, any information provision rule provides strictly lower payoff than the payoff from action h *ad infinitum*. That is, the Sender wishes to maximise the probability with which the prior is p^h in the next period, and this also maximises her current period payoff since she weakly prefers action h to action ℓ. Again, this is achieved by the provision rule outlined in the lemma, so the Sender has no incentive to deviate. Since this is optimal for any $p^\ell < \frac{1}{2}$ in the continuation game, this is optimal for any $p < \frac{1}{2}$ in the original game as well.

(iii) $\frac{1}{2} > p^h > p^\ell$: The rule described in the lemma maximises the payoff within a period by maximising the probability of action h within that period. It also maximises the probability of the prior being p^h in the next period. Therefore, to prove the optimality of the provision rule, it is sufficient to show that the continuation payoff $\delta V(p^h)$ is greater than the continuation payoff $\delta V(p^\ell)$, under the rule outlined in the lemma. By equation (A.6), $V(p^h) - V(p^\ell) = \frac{(1-\delta)(p^h-p^\ell)(1+x)}{1-2\delta(p^h-p^\ell)} > 0$, since by assumption, $p^h > p^\ell$, $p^h - p^\ell < \frac{1}{2}$ and $\delta < 1$. Therefore, the Sender has no incentive to deviate in the original game, and this concludes the proof. ■

Proof of Proposition 1. If $p \geq \frac{1}{2}$ and $\frac{1}{2} > p^h > p^\ell$, in the continuation game, the Sender uses the myopic provision rule by Lemma 6 above. By not disclosing any information in the first period, the Receiver takes action h with probability 1, maximising the Sender's current and future payoffs (by point (*iii*) in the proof of Lemma 6) so she has no incentive to deviate. This, combined with Lemma 5, together prove that whenever $p^h > p^\ell$, the myopic provision rule is optimal for the Sender. ■

A.4 Proof of Proposition 2

I now consider the case where $p^\ell > p^h$. From point (i) in the proof of Lemma 5, it is easy to see that the myopic provision rule need not necessarily be optimal when $p^\ell > p^h$, as the following lemma shows:

Lemma 7. Suppose $p^\ell > p^h$ and $(x, y) \leq (\bar{x}, \bar{y}) = \left(\frac{\delta(p^\ell - p^h)}{1+\delta(p^\ell - p^h)}, \frac{1}{1+\delta(p^\ell - p^h)}\right)$. Then, full disclosure is optimal for the Sender.

Proof. Given the Bellman equation (3.3) characterising the Sender's optimal value, the Sender's long-run payoffs are given by:

$V(\cdot)$	$\omega = G$	$\omega = B$
$a = h$	$1 - \delta + \delta V(p^h)$	$(1-\delta)x + \delta V(p^h)$
$a = \ell$	$(1-\delta)y + \delta V(p^\ell)$	$\delta V(p^\ell)$

The Sender's (long-run) preferences and the Receiver's (short-run) preferences are aligned[20] if and only if:

$$1 - \delta + \delta V(p^h) \geq (1-\delta)y + \delta V(p^\ell) \iff y \leq 1 - \frac{\delta(V(p^\ell) - V(p^h))}{1-\delta}$$

and

$$(1-\delta)x + \delta V(p^h) \leq \delta V(p^\ell) \iff x \leq \frac{\delta(V(p^\ell) - V(p^h))}{1-\delta}.$$

If the players' preferences are aligned, it is optimal for the Sender to fully disclose the state, i.e. $\alpha^*(p) = p$, $q^{h^*}(p) = 1$ and $q^{\ell^*}(p) = 0$ for all $p \in [0,1]$. In that case, by equation (A.6), $V(p^\ell) - V(p^h) = \frac{(1-\delta)(p^\ell - p^h)}{1+\delta(p^\ell - p^h)}$. Thus, the players' preferences are aligned if and only if:

$$x \leq \frac{\delta(V(p^\ell) - V(p^h))}{1-\delta} = \frac{\delta(p^\ell - p^h)}{1+\delta(p^l - p^h)} = \bar{x}$$

and

$$y \leq 1 - \frac{\delta(V(p^\ell) - V(p^h))}{1-\delta} = \frac{1}{1+\delta(p^l - p^h)} = \bar{y}$$

[20]The players' preferences are aligned if, in every state, the players prefer the same action.

Since $p^l > p^h$ and $\delta \in [0, 1)$, $\bar{x} \geq 0$ and $\bar{y} > 0$. We have found a fixed point of the Bellman equation (3.3). This proves a stronger result: full disclosure is optimal if and only if $(x, y) \leq (\bar{x}, \bar{y})$. ■

Claim 2. If $x \geq \bar{x}$ or $y \geq \bar{y}$, it is without loss of generality to restrict ourselves to the myopic and reverse provision rules.

Proof. It follows from the proof of Lemma 7 that full disclosure is not optimal. By Claim 1, in the optimum, we have one of the following: $\alpha = 0$, $\alpha = 1$, or if $\alpha \in (0, 1)$, then either $q^\ell = \frac{1}{2}$ and $q^h = 1$, or $q^\ell = 0$ and $q^h = \frac{1}{2}$. If $p < \frac{1}{2}$, due to Bayes' plausibility, there is no information provision rule that could induce $\alpha = 1$ or $q^\ell = \frac{1}{2}$ and $q^h = 1$. Similarly, if $p > \frac{1}{2}$, there is no information provision rule that could induce $\alpha = 0$ or $q^\ell = 0$ and $q^h = \frac{1}{2}$. There are two cases to consider:

(i) If, when $p > \frac{1}{2}$, the Sender finds it optimal to not disclose any information so that $\alpha = 1$, it implies she wants to maximise the probability of the high action. This, in turn, implies that when $p < \frac{1}{2}$, $\alpha \in (0, 1)$, $q^h = \frac{1}{2}$ and $q^\ell = 0$ is optimal for the Sender. The converse holds as well: if $\alpha \in (0, 1)$, $q^\ell = 0$ and $q^\ell = \frac{1}{2}$ when $p < \frac{1}{2}$, then $\alpha = 1$ when $p > \frac{1}{2}$. This is the myopic provision rule.

(ii) Similarly, if the Sender finds it optimal to not disclose any information when $p < \frac{1}{2}$, it implies she wants to maximise the probability of the low action. This implies that when $p > \frac{1}{2}$, $\alpha \in (0, 1)$, $q^\ell = \frac{1}{2}$ and $q^h = 1$ is optimal for the Sender. As shown above, the converse holds as well. This is the reverse provision rule. ■

Proof of Proposition 2. Lemma 7 proves part (iii) of Proposition 2. Toward proving parts (i) and (ii), by Claim 2, the Sender either uses the myopic provision rule or the reverse provision rule if $x \geq \bar{x}$ or $y \geq \bar{y}$. Observe that in Figures 1 and 2, for given x and y, which of the two rules is optimal depends on the Sender's long-run expected payoff from the two actions when the Receiver's belief is $\frac{1}{2}$. If the payoff from the high action is (strictly) greater than the payoff from the low action at $q = \frac{1}{2}$, then the myopic provision rule is uniquely optimal. If the payoff from the low action is (strictly) greater than the payoff from the high action, then the reverse provision rule is uniquely optimal. And if the payoff from the two actions is equal, then the Sender is indifferent between the two rules.

When the Receiver's belief is $\frac{1}{2}$, the Sender's long-run expected payoff from the high action and the low action is $(1-\delta)(\frac{1}{2}+\frac{1}{2}x)+\delta V(p^h)$ and $(1-\delta)\frac{1}{2}y+\delta V(p^\ell)$. From the discussion above, the Sender is indifferent between the myopic and reverse provision rules if:

$$\begin{aligned}(1-\delta)(\frac{1}{2}+\frac{1}{2}x)+\delta V(p^h) &= (1-\delta)\frac{1}{2}y+\delta V(p^\ell) \\ \iff (1-\delta)(1+x-y) &= 2\delta(V(p^\ell)-V(p^h))\end{aligned} \tag{A.7}$$

There are three cases to consider:

(i) $p^\ell > p^h \geq \frac{1}{2}$: The Sender is indifferent between not disclosing any information under the myopic provision rule ($\alpha = 1$) and disclosing information under the reverse provision rule ($\alpha(p) = 2p-1$, $q^h = 1$, $q^\ell = \frac{1}{2}$) at priors p^h and p^ℓ. By equation (A.6), this gives $V(p^\ell) - V(p^h) = (1-\delta)(p^\ell - p^h)(1-x)$ under the myopic provision rule and $V(p^\ell) - V(p^h) = \frac{(1-\delta)(2-y)(p^\ell-p^h)}{1+2\delta(p^\ell-p^h)}$ under the reverse provision rule. Substituting these values in equation (A.7) above gives the (same) equation of indifference:

$$y = 1 - 2\delta(p^\ell - p^h) + x(1+2\delta(p^\ell - p^h)). \tag{A.8}$$

(ii) $\frac{1}{2} > p^\ell > p^h$: The Sender is indifferent between not disclosing any information under the reverse provision rule ($\alpha = 0$) and disclosing information under the myopic provision rule ($\alpha(p) = 2p$, $q^h = \frac{1}{2}$, $q^\ell = 0$) at priors p^h and p^ℓ. By equation (A.6), this gives $V(p^\ell) - V(p^h) = (1-\delta)(p^\ell - p^h)y$ under the reverse provision rule and $V(p^\ell) - V(p^h) =$

$\frac{(1-\delta)(p^\ell-p^h)(1+x)}{1+2\delta(p^\ell-p^h)}$. Substituting these values in equation (A.7) above gives the (same) equation of indifference:

$$y = \frac{1+x}{1+2\delta(p^\ell - p^h)}. \tag{A.9}$$

(iii) $p^\ell \geq \frac{1}{2} > p^h$: At prior p^ℓ, the Sender is indifferent between the two provision rules as in case (i) above. At prior p^h, the Sender is indifferent between the two provision rules as in case (ii) above. By equation (A.6), this gives $V(p^\ell) - V(p^h) = \frac{(1-\delta)(p^\ell - p^h + x(1-p^\ell-p^h))}{1+\delta-2\delta p^h}$ under the myopic provision rule and $V(p^\ell) - V(p^h) = \frac{(1-\delta)(2p^\ell - 1 + y(1-p^\ell-p^h)}{1-\delta+2\delta p^\ell}$. Substituting these values in equation (A.7) above gives the (same) equation of indifference:

$$y = \frac{1+\delta-2\delta p^\ell}{1+\delta-2\delta p^h} + \frac{1-\delta+2\delta p^\ell}{1+\delta-2\delta p^h}x. \tag{A.10}$$

Thus, the function $L(x)$ in Proposition 2 is the equation of indifference between the myopic and reverse provision rules. It depends on the magnitudes of p^ℓ and p^h, and is given by equations (A.8), (A.9) and (A.10). Observe that all three equations are linear and upward sloping. The point $(\bar{x}, \bar{y})$ lies on each of these lines. On the other hand, when $y = 1$, the value of x derived using each of these equations is strictly less than 1, giving rise to the asymmetry in Figure 3.

Finally, from equation (A.7), if $(1-\delta)(1+x-y) \geq 2\delta(V(p^\ell) - V(p^h))$, then the myopic provision rule is optimal. This is equivalent to $y \leq L(x)$, where the equation of $L(x)$ depends on the values of p^ℓ and p^h, as shown above. Similarly, if $(1-\delta)(1+x-y) \leq 2\delta(V(p^\ell)-V(p^h))$, then the reverse provision rule is optimal, which is equivalent to $y \geq L(x)$. This concludes the proof of Proposition 2. ■

A.5 Proof of Corollary 1

When $x = 1$ and $y = 0$, by equations (A.4)-(A.5), we have that:

$$V(p^h) = \frac{\alpha_h^* - \delta\alpha_h^* + \delta\alpha_\ell^*}{1 - \delta\alpha_h^* + \delta\alpha_\ell^*}, V(p^\ell) = \frac{\alpha_\ell^*}{1 - \delta\alpha_h^* + \delta\alpha_\ell^*}$$

In this case, the choice variables for the Sender are α_h^* and α_ℓ^*, which are the probabilities with which action h is taken at priors p^h and p^ℓ, respectively. Simple algebra reveals that:

$$\frac{\partial V(p^h)}{\partial \alpha_h^*} = \frac{1 - \delta(1 - \alpha_\ell^*)}{(1 - \delta\alpha_h^* + \delta\alpha_\ell^*)^2} > 0, \qquad \frac{\partial V(p^\ell)}{\partial \alpha_h^*} = \frac{\delta\alpha_\ell^*}{(1 - \delta\alpha_h^* + \delta\alpha_\ell^*)^2} \geq 0,$$

$$\frac{\partial V(p^h)}{\partial \alpha_\ell^*} = \frac{\delta(1 - \alpha_h^*)}{(1 - \delta\alpha_h^* + \delta\alpha_\ell^*)^2} \geq 0, \qquad \frac{\partial V(p^\ell)}{\partial \alpha_\ell^*} = \frac{1 - \delta\alpha_h^*}{(1 - \delta\alpha_h^* + \delta\alpha_\ell^*)^2} > 0.$$

That is, in the continuation game starting from the second period, the Sender wishes to maximise the probability with which action h is taken, which is achieved by the myopic provision rule. But this is true for all values of p^h and p^ℓ, so the Sender uses the myopic provision rule even in the original game, and this proves the result. ■

Appendix B Proofs of Section 4

B.1 Proof of Theorem 1

The proof is a general case of Ely (2017). In order to prove the theorem, it is sufficient to show that the operator $TV(q) = \text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta V(\phi^a(q))\}$ is a contraction mapping in the space of bounded real-valued functions on the unit interval with the sup norm. Blackwell's sufficient conditions are satisfied as follows:

(i) Consider two functions $V(\cdot)$ and $W(\cdot)$. If $V(q) \geq W(q)\ \forall q \in [0,1]$, then for given $a \in \mathcal{A}$, $(1-\delta)\mathbb{E}_q(u_S(q)) + \delta V(\phi^a(q)) \geq (1-\delta)\mathbb{E}_q(u_S(q)) + \delta W(\phi^a(q)), \forall q \in [0,1]$.

Since the concavification of a function is the smallest concave function greater than or equal to the said function, this implies that $\text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta V(\phi^a(q))\} \geq \text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta W(\phi^a(q))\}, \forall q \in [0,1]$. Equivalently, $TV \geq TW$, as desired.

(ii) Consider a constant $c \geq 0$. Then, $(V+c)(q) = V(q) + c$ which implies that:

$$\begin{aligned}
T(V+c)(q) &= \text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta(V+c)(\phi^a(q))\} \\
&= \text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta V(\phi^a(q)) + \delta c\} \\
&= \text{cav}\{(1-\delta)\mathbb{E}_q(u_S(q)) + \delta V(\phi^a(q))\} + \delta c \\
&= TV + \delta c,
\end{aligned}$$

as desired. Thus, the operator T is a contraction (with Lipschitz constant $\delta < 1$, by assumption), and by the contraction mapping theorem, $V(\cdot)$ converges to a unique fixed point by iteration. ■

Appendix C Additional Results on the Basic Model

C.1 Sender's first-best decision rule

Suppose that the Sender could take the action. The Sender would never benefit by concealing the state from herself, so here I assume that the Sender knows the state. Thus, the problem reduces to a single-agent decision problem. I refer to the Sender's optimal choice of action as her *first-best decision rule*. I also ask: when does the Sender achieve her first-best payoff in the game with the Receiver?

I first consider the static model. Recall that the Sender weakly prefers the high action to the low action in the stage-game. This means that the Sender's first-best decision rule is to choose the high action, irrespective of the state. Thus, we have the following lemma:

Lemma 8. In the static game against the Receiver, the Sender gets her first-best expected payoff if and only if $x = 0$ or $p \geq \frac{1}{2}$.

Proof. The Sender's first-best expected payoff is $V_{FB}(p) = p + (1-p)x = x + p(1-x)$. The Sender's optimal expected payoff in the static game is $\min\{p(1+x), x + p(1-x)\}$. The payoff deviation from the first-best is:

$$V_{FB}(p) - V(p) = \begin{cases} 0, & \text{if } p \geq 1/2 \\ x(1-2p), & \text{if } p < 1/2 \end{cases}$$

■

Remark 3. In the static game against the Receiver, full disclosure is also an optimal information provision rule for the Sender (see Remark 1). This means that when $x = 0$, the Sender's first-best expected payoff from choosing the high action equals her expected payoff from fully disclosing the state (in both cases, she gets 1 with probability p and 0 otherwise). Thus, $x = 0$ is a sufficient condition for the Sender to get her first-best expected payoff in the static game.

In the case where $p^h = p^\ell$, i.e. there is action-independent evolution, the Sender optimally maximises her stage-game payoff when playing against the Receiver. This implies that the Sender's first-best decision rule is to choose the high action every period. Then, we have the following:

Lemma 9. In the dynamic game against the Receiver, the Sender gets her first-best expected payoff if and only if $x = 0$ or $p \geq \frac{1}{2}$ and $p^h = p^\ell \geq \frac{1}{2}$.

Proof. The Sender's first-best expected payoff is $V_{FB}(p) = (1-\delta)(x + p(1-x)) + \delta(x + p^h(1-x))$. The Sender's optimal expected payoff in the dynamic game depends on the values of p

and p^h, and can be computed using equations (3.3) and (A.4). Thus, the payoff deviation from the first-best is:

$$V_{FB}(p) - V(p) = \begin{cases} 0, & \text{if } p \geq 1/2, p^h \geq 1/2 \\ \delta x(1-2p^h), & \text{if } p \geq 1/2, p^h < 1/2 \\ (1-\delta)x(1-2p), & \text{if } p < 1/2, p^h \geq 1/2 \\ x(1-2p+2\delta(p-p^h)), & \text{if } p < 1/2, p^h < 1/2 \end{cases}$$

■

Note that in the dynamic game under action-independent evolution, if $x \neq 0$, it is not sufficient for $p \geq \frac{1}{2}$ to get her first-best payoff. We also require that $p^h = p^\ell \geq \frac{1}{2}$ so that the high action is taken with probability 1 *ad infinitum*.

Now consider the case where $p^h > p^\ell$. Recall that the Sender weakly prefers the high action and weakly prefers the good state in the stage-game. These imply that under positive association, within a period, the Sender would like to maximise the probability of the high action as it maximises both current and future expected payoffs. Thus, the Sender's first-best decision rule under positive association is to choose the high action irrespective of the state every period. Thus,

Lemma 10. In the dynamic game against the Receiver, if $p^h > p^\ell$, the Sender gets her first-best expected payoff if and only if $p \geq \frac{1}{2}$ and $p^h \geq \frac{1}{2}$.

Proof. The Sender's first-best expected payoff is the same as in action-independence case: $V_{FB}(p) = (1-\delta)(x + p(1-x)) + \delta(x + p^h(1-x))$. The Sender's optimal expected payoff in the dynamic game depends on the values of p, p^ℓ and p^h, and can be computed using equations (3.3), (A.4) and (A.5). Let $\gamma \coloneqq (1-x)\delta(p^h - p^\ell) \geq 0$. The payoff deviation from the first-best is:

$$V_{FB}(p) - V(p) = \begin{cases} 0, & \text{if } p \geq 1/2, p^h \geq 1/2 \\ \frac{\delta(1-2p^h)(x+\gamma)}{1-2\delta(p^h-p^\ell)}, & \text{if } p \geq 1/2, p^h < 1/2 \\ (1-\delta)(1-2p)(x+\gamma), & \text{if } p < 1/2, p^\ell \geq 1/2 \\ \frac{(1-\delta)(1-2p)(x+\gamma)}{1-\delta+2\delta p^\ell}, & \text{if } p < 1/2, p^h \geq 1/2, p^\ell < 1/2 \\ \frac{(1-2p+2\delta(p-p^h))(x+\gamma)}{1-2\delta(p^h-p^\ell)}, & \text{if } p < 1/2, p^h < 1/2 \end{cases}$$

■

Thus, we retrieve the deviation payoffs under action-independence when $p^h = p^\ell$ in above equation. Note that under positive association, $x = 0$ is not a sufficient condition for the Sender to get her first-best payoff. This is because under positive association, full disclosure is not an optimal policy when $x = 0$, unlike in the static model (see Remarks 1 and 3). This is due to the positive dynamic effect of choosing the high action .

Finally, consider the case where $p^h < p^\ell$. This is the less obvious case. Recall that the Sender's long-run payoffs are given by:

$V(\cdot)$	$\omega = G$	$\omega = B$
$a = h$	$1-\delta+\delta V(p^h)$	$(1-\delta)x+\delta V(p^h)$
$a = \ell$	$(1-\delta)y+\delta V(p^\ell)$	$\delta V(p^\ell)$

There are four cases to consider:

(i) If $(1-\delta)+\delta V(p^h) \geq (1-\delta)y+\delta V(p^\ell)$ and $(1-\delta)x+\delta V(p^h) \geq \delta V(p^\ell)$, then the Sender finds it optimal to choose the high action irrespective of the state.

(ii) If $(1-\delta)+\delta V(p^h) \leq (1-\delta)y+\delta V(p^\ell)$ and $(1-\delta)x+\delta V(p^h) \leq \delta V(p^\ell)$, then the Sender finds it optimal to choose the low action irrespective of the state.

(iii) If $(1-\delta)+\delta V(p^h) \geq (1-\delta)y+\delta V(p^\ell)$ and $(1-\delta)x+\delta V(p^h) \leq \delta V(p^\ell)$, then the Sender finds it optimal to match action to state: choose the high action in the good state and the low action in the bad state.

(iv) If $(1-\delta)+\delta V(p^h) \leq (1-\delta)y+\delta V(p^\ell)$ and $(1-\delta)x+\delta V(p^h) \geq \delta V(p^\ell)$, then the Sender finds it optimal to *mis-match* action to state: choose the low action in the good state and the high action in the bad state.

Using equations (A.4) and (A.5), we can compute the values of $V(p^h)$ and $V(p^\ell)$ based on the cases above. Similar to the proof of Lemma 7 and Proposition 2, we can find the values of x and y for which the four rules mentioned above are optimal. This proves the following ($\bar{x}$ and $\bar{y}$ are as defined in Proposition 2):

Proposition 3. If $p^h < p^\ell$, and the Sender could choose an action, then:

(i) If $x \geq \bar{x}$ and $y \leq 1-\delta(p^\ell - p^h)(1-x)$, then choosing the high action is optimal;

(ii) If $y \geq \bar{y}$ and $x \leq \delta(p^\ell - p^h)y$, then choosing the low action is optimal;

(iii) If $(x, y) \leq (\bar{x}, \bar{y})$, then choosing the high and low actions in the good and bad states respectively is optimal; and

(iv) If $x \geq \delta(p^\ell - p^h)y$ and $y \geq 1-\delta(p^\ell - p^h)(1-x)$, the choosing the low and the high actions in the good and bad states respectively is optimal for the Sender.

Figure 4 plots the Sender's first-best decision rules for different values of x and y. The line of indifference, $L(x)$, in the game with the Receiver lies entirely within the Case (iv) region (except the point $(\bar{x}, \bar{y})$). This implies that if the Sender's first-best decision rule is to choose the high (low) action irrespective of the state, the myopic (reverse) provision rule is optimal against the Receiver. The converse, however, is not true.

The following corollary follows from Propositions 2 and 3:

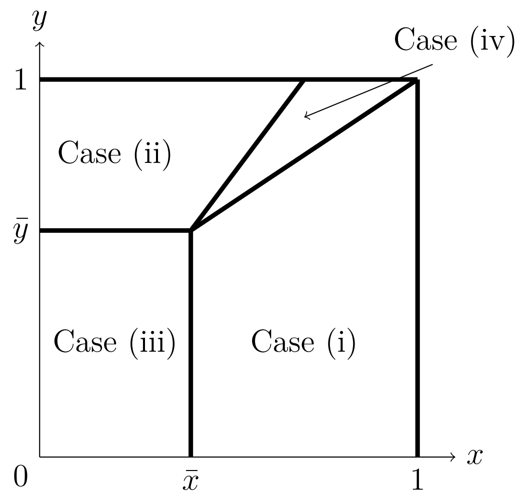

Figure 4: Sender's first-best decision rules

Corollary 3. If $p^h < p^\ell$ and $(x, y) \leq (\bar{x}, \bar{y})$, the Sender gets her first-best expected payoff in the game against the Receiver.

The reason for the above is as follows. When $x \leq \bar{x}$ and $y \leq \bar{y}$, full disclosure is optimal for the Sender in the game against the Receiver. This is because the Sender's and Receiver's preferences align: state-by-state, they both prefer to take the same action. This implies that when the Sender can take the action, she would do the same: match action to state. In this case, the Sender gets her first-best payoff:

$$\begin{aligned} V_{FB}(p) &= (1-\delta)p + \delta p V_{FB}(p^h) + \delta(1-p)V_{FB}(p^\ell) \\ &= \frac{(1-\delta)p + \delta p^\ell}{1 + \delta(p^\ell - p^h)} \text{ [using (A.6)]} \end{aligned}$$

For $(x, y) > (\bar{x}, \bar{y})$, the Sender gets her first-best expected payoff under the conditions stated in the following lemma:

Lemma 11. In the dynamic game against the Receiver, if $p^h < p^\ell$ and $(x, y) > (\bar{x}, \bar{y})$, the Sender gets her first-best expected payoff if and only if:

(i) $p \geq \frac{1}{2}$ and $p^h \geq \frac{1}{2}$ if the Sender's first-best decision rule is to choose the high action; and

(ii) $p \leq \frac{1}{2}$ and $p^\ell \leq \frac{1}{2}$ if the Sender's first-best decision rule is to choose the low action.

Proof. ($\Longrightarrow$):

If the Sender's first-best decision rule is to choose the high action, then her first-best expected payoff is the same as under positive association: $V_{FB}(p) = (1-\delta)(x + p(1-x)) + \delta(x + p^h(1-x))$. The Sender's optimal expected payoff in the dynamic game depends on the values of p, p^ℓ and p^h, and can be computed using equations (3.3), (A.4) and (A.5). The payoff deviation from the first-best expected payoff is: (recall that $\gamma = (1-x)\delta(p^h - p^\ell)$):

$$V_{FB}(p) - V(p) = \begin{cases} 0, & \text{if } p \geq 1/2, p^h \geq 1/2 \\ \frac{\delta(1-2p^h)(x+\gamma)}{1+\delta-2\delta p^h}, & \text{if } p \geq 1/2, p^\ell \geq 1/2, p^h < 1/2 \\ \frac{\delta(1-2p^h)(x+\gamma)}{1+2\delta(p^\ell - p^h)}, & \text{if } p \geq 1/2, p^\ell < 1/2 \\ (1-\delta)(1-2p)(x+\gamma), & \text{if } p < 1/2, p^h \geq 1/2 \\ \frac{[(1-\delta)(1-2p)+\delta(1-2p^h)](x+\gamma)}{1+\delta-2\delta p^h}, & \text{if } p < 1/2, p^\ell \geq 1/2, p^h < 1/2 \\ \frac{[(1-\delta)(1-2p)+\delta(1-2p^h)](x+\gamma)}{1+2\delta(p^\ell-p^h)}, & \text{if } p < 1/2, p^\ell < 1/2 \end{cases} \tag{C.1}$$

Note that since $x \geq \bar{x}$, $x + \gamma > 0$. This proves part (i) of the lemma.

If the Sender's first-best decision rule is to choose the low action, then her first-best expected payoff is: $V_{FB}(p) = (1-\delta)py + \delta p^\ell y$. The Sender's optimal expected payoff in the dynamic game depends on the values of p, p^ℓ and p^h, and can be computed using equations (3.3), (A.4) and (A.5). Let $\beta \coloneqq 1 + \delta(p^\ell - p^h)$. The payoff deviation from the first-best expected payoff is:

$$V_{FB}(p) - V(p) = \begin{cases} \frac{[\beta y-1][(1-\delta)(2p-1)+\delta(2p^\ell-1)]}{1+2\delta(p^\ell-p^h)}, & \text{if } p > 1/2, p^h > 1/2 \\ \frac{[\beta y-1][(1-\delta)(2p-1)+\delta(2p^\ell-1)]}{1-\delta+2\delta p^\ell}, & \text{if } p > 1/2, p^\ell > 1/2, p^h \leq 1/2 \\ (1-\delta)(2p-1)(\beta y-1), & \text{if } p > 1/2, p^\ell \leq 1/2 \\ \frac{\delta(2p^\ell-1)(\beta y-1)}{1+2\delta(p^\ell-p^h)}, & \text{if } p \leq 1/2, p^h > 1/2 \\ \frac{\delta(2p^\ell-1)(\beta y-1)}{1-\delta+2\delta p^\ell}, & \text{if } p \leq 1/2, p^\ell > 1/2, p^h \leq 1/2 \\ 0, & \text{if } p \leq 1/2, p^\ell \leq 1/2 \end{cases} \tag{C.2}$$

Note that since $y \geq \bar{y}$, $\beta y - 1 > 0$. This proves part (ii) of the lemma.

($\Longleftarrow$):

The only case to consider is when $x > \delta y(p^\ell - p^h)$ and $y > 1 - \delta(p^\ell - p^h)(1-x)$, i.e. the Sender's first-best decision rule is *uniquely* to mis-match action to state. Then, her first-best expected payoff is: $V_{FB}(p) = x + \frac{y-x}{1-\delta(p^\ell-p^h)}[(1-\delta)p + \delta p^h]$. There are two cases to consider:

(i) If the myopic provision rule is optimal in the game against the Receiver, then the payoff deviation from the first-best is:

$$\begin{aligned} V_{FB}(p) - V(p) = &\ [V_{FB}(p) - \{(1-\delta)(x + p(1-x)) + \delta(x + p^h(1-x))\}] \\ &+ [\{(1-\delta)(x + p(1-x)) + \delta(x + p^h(1-x))\} - V(p)] \end{aligned}$$

The first term in the square brackets equals $\frac{[(1-\delta)p+\delta p^h][y+\delta(p^\ell-p^h)(1-x)-1]}{1-\delta(p^\ell-p^h)}$ and the second term in the square brackets is given by equation (C.1), so the total is strictly positive.

(ii) If the reverse provision rule is optimal in the game against the Receiver, then the payoff deviation from the first-best is:

$$\begin{aligned} V_{FB}(p) - V(p) = & [V_{FB}(p) - \{(1-\delta)py + \delta p^{\ell}y\}] \\ & +[\{(1-\delta)py + \delta p^{\ell}y\} - V(p)] \end{aligned}$$

The first term in the square brackets equals $\frac{[(1-\delta)(1-p)+\delta(1-p^{\ell})][x-\delta y(p^{\ell}-p^{h})]}{1-\delta(p^{\ell}-p^{h})}$ and the second term in the square brackets is given by equation (C.2), so the total is strictly positive.

■

C.2 Distribution over priors

In Lemma 1, I noted that the common prior from the second period onward can only take on one of two values, p^h or p^{ℓ}. These are the priors following the high action and the low action, respectively. Denote by $k := \mathbb{P}(p = p^h)$ the long-run probability with which the prior is p^h. Equivalently, k measures the probability of the high action being taken in the long-run. Then we have:

Lemma 12. As $\delta \to 1$, the Sender's long-run expected payoff is a weighted average of her per-period expected payoffs at priors p^h and p^{ℓ}, where the weights are the probabilities with which the high action and the low action are respectively induced in equilibrium.

Proof. If the current prior is p^h, then the probability of the next period's prior being p^h is α_h^*. If the current prior is p^{ℓ}, then the probability of the next period's prior being p^h is α_{ℓ}^*. Therefore, k is characterised by the solution to the following:

$$k = k \cdot \alpha_h^* + (1-k) \cdot \alpha_{\ell}^* \iff k = \frac{\alpha_{\ell}^*}{1 - \alpha_h^* + \alpha_{\ell}^*}$$

When $\delta = 1$, then $V(p^h) = V(p^{\ell}) = k \cdot f_h + (1-k) \cdot f_{\ell}$ (see equations (A.4)-(A.5)). Thus, the weights on $V(p_h)$ and $V(p_{\ell})$ in equations are equal to the long-run probability with which the prior is p^h and p^ℓ when $\delta = 1$. This is a function of α_h^* and α_ℓ^*, which are determined by the equilibrium of the game. ■

As noted above, the value of k is a function of α_h^* and α_ℓ^*, which in turn, depend on the values of p^h, p^ℓ, x and y. Let $\hat{k}$ denote the value of k under the various information provision rules used in equilibrium. Let k^* denote the value of k under the Sender's first-best decision rules. Below, I show that if $x \neq 0$, a necessary condition for the Sender to achieve her first-best expected payoff is that she is able to induce the high action from the Receiver with exactly the same probability as her first-best decision rule:

Proposition 4. If $x \neq 0$, the Sender gets her first-best expected payoff only if $\hat{k} = k^*$.

Proof. First, suppose $p^h = p^\ell$. Then $k^* = 1$, $\hat{k} = 1$ if $p^h \geq \frac{1}{2}$, and $\hat{k} = 2p^h$ if $p^h < \frac{1}{2}$. If $x \neq 0$ and if the Sender gets her first-best expected payoff, then $p^h \geq \frac{1}{2}$, by Lemma 9. Thus, if $p^h \geq \frac{1}{2}$, $\hat{k} = k^*$.

Now, suppose $p^h > p^\ell$. Then, $k^* = 1$, $\hat{k} = 1$ if $p^h \geq \frac{1}{2}$ and $\hat{k} = \frac{2p^\ell}{1-2(p^h-p^\ell)}$ if $p^h < \frac{1}{2}$. If the Sender gets her first-best expected payoff, then $p^h \geq \frac{1}{2}$, by Lemma 10. Thus, if $p^h \geq \frac{1}{2}$, $\hat{k} = k^*$.

Finally, suppose $p^h < p^\ell$. There are four cases to consider:

(i) We know that if $(x, y) \leq (\bar{x}, \bar{y})$, the Sender gets her first-best expected payoff, by Corollary 3. We also know that full disclosure is the optimal information provision rule and matching action to state is the Sender's first-best decision rule. Thus, in this case, $\hat{k} = k^* = \frac{p^\ell}{1+p^\ell-p^h}$.

(ii) If the Sender's first-best decision rule is to choose the high action, then the myopic provision rule is optimal against the Receiver. Then, $k^* = 1$, while:

$$\hat{k} = \begin{cases} 1, & \text{if } p^\ell > p^h \geq 1/2 \\ \frac{1}{2(1-p^h)}, & \text{if } p^\ell \geq 1/2 > p^h \\ \frac{2p^\ell}{1+2(p^\ell - p^h)}, & \text{if } 1/2 > p^\ell \end{cases}$$

If the Sender gets her first-best expected payoff, then $p^h \geq \frac{1}{2}$, by Lemma 11. Thus, if $p^h \geq \frac{1}{2}$, $\hat{k} = k^*$.

(iii) If the Sender's first-best decision rule is to choose the low action, then the reverse provision rule is optimal against the Receiver. Then, $k^* = 0$, while:

$$\hat{k} = \begin{cases} \frac{2p^\ell - 1}{1+2(p^\ell - p^h)}, & \text{if } p^\ell > p^h > 1/2 \\ \frac{2p^\ell - 1}{2p^\ell}, & \text{if } p^\ell > 1/2 \geq p^h \\ 0, & \text{if } 1/2 \geq p^\ell \end{cases}$$

If the Sender gets her first-best expected payoff, then $\frac{1}{2} \geq p^\ell$, by Lemma 11. Thus, if $\frac{1}{2} \geq p^\ell$, $\hat{k} = k^*$.

(iv) If the Sender's first-best decision rule is to mis-match action to state, then either the myopic provision rule or the reverse provision rule is optimal against the Receiver. Thus, $k^* = \frac{1-p^\ell}{1-(p^\ell - p^h)}$, while depending on the provision rule, $\hat{k}$ is given by the values mentioned in cases (ii) and (iii). In all of these cases, so the Sender never gets her first-best expected payoff by Lemma 11, and $\hat{k} \neq k^*$. ■

C.3 Perfectly-aligned stage-game payoffs

In this subsection, I consider the case where the Sender and the Receiver have perfectly aligned stage-game payoffs. Consider first the case where both players have the Receiver's preferences in the basic model:

u_S, u_R	$\omega = G$	$\omega = B$
$a = h$	1	0
$a = \ell$	0	1

Obviously, full disclosure is optimal in the stage-game. Moreover, full disclosure is always optimal irrespective of the relationship between p^h and p^ℓ even in the dynamic game. This is because the Sender doesn't have preferences over the state; matching the action to the state gives her a payoff of 1 every period. The Sender gets her first-best expected payoff.

Now, consider the case where both players have the Sender's preferences in the basic model:

u_S, u_R	$\omega = G$	$\omega = B$
$a = h$	1	x
$a = \ell$	y	0

In this case, the Receiver is not persuadable: he would choose the high action every period as it is a (weakly) dominant action for him. This implies that under action-independence and positive association, the Sender would get her first-best expected payoff. Under negative association, however, the Sender gets her first-best expected payoff only when $x > \bar{x}$ and $y < 1 - \delta(p^\ell - p^h)(1 - x)$. These examples highlight that the interesting case to analyse is the one where the Sender has preferences over the state and where the Receiver is persuadable: in the binary state and binary action setting, this is equivalent to the Receiver preferring different actions in different states.