SEQUENTIAL ADOPTION BETWEEN COMPETING PLATFORMS UNDER DIFFERENT REGIMES*

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Abstract

(EXTENDED ABSTRACT): This paper adapts the model by Farrell and Saloner (1985) to theoretically explore the adoption between two rival platforms by two types of agents, each inherently preferring one of the two platforms, when consumption is characterized by positive network externalities. Agents are forward-looking and the search is for subgame perfect Nash equilibria. Platforms are one-sided, for example: email, video conferencing, or instant messaging domains/platforms, languages, etc. The aim is to use deterministic adoption patterns to predict outcomes under different strategic or regulatory regimes - single homing (exclusivity of users) or multihoming, isolated or interoperable platforms, or those offering compatibility adaptations - and their interactions; to understand the trade-offs between multihoming and interoperability, between first mover advantage (incumbency or installed base) and preference loyalty; and to thus aid regulatory insight.

I find that number of preferring or loyal agents is often itself deterministic - regardless of the order of agents' moves - in the adoption outcome between isolated platforms. This occurs when the number of preferring agents of either or both platforms exceeds a 'sufficiency threshold', which is inversely proportional to the degree of differentiation between the platforms (or equivalently, the degree of heterogeneity between agents preferring the two platforms). Only when numbers of neither type of agents is 'sufficient' enough, does the order of moves matter; first mover advantage exists in this case when the number of a type moving first exceeds a 'critical installed base', which is the difference between the size of the population (of agents) and the 'sufficiency threshold'.

The possibility of agents multihoming changes the advantage to moving first: first movers of a type can now more easily induce agents of the other type to adopt the platform preferred by those moving first - either as the only platform adopted or as a second adoption only to connect with agents on it. The latter occurs of course if the cost of adopting a platform is low enough, and occurs

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despite other agents being sufficient in number. Multihoming however disappears in an equitably interoperable world – where order of agents' moves also becomes irrelevant.

From the platform's perspective: unsurprisingly, either the incumbent or first mover, or the platform that alone has sufficient loyal customer base prefers isolation with exclusivity (single homing) restrictions; but both platforms are indifferent between isolation-exclusivity and interoperability only when both have sufficient numbers of loyal customers - which is itself possibly only with large differentiation between platforms (or large preference heterogeneity between both agent types). Other results are still work in progress.

This is important given the recent DMA (Digital Markets Act) in Europe, and in its wake the argument regarding mandating increased compatibility and interoperability between digital platforms. The paper hopefully, fills the gap in the literature of formal modelling of the interconnectedness of compatibility choice by (or regulatory imposition on) platforms and the homing ability and decisions of consumers.

Keywords: communication platforms; compatibility; interoperability; single homing; multihoming.

JEL Categories: L1; D4; L8.

1 INTRODUCTION

Languages are one-sided platforms[1] providing communication between users, each language isolated from another such that speakers of one successfully communicate amongst themselves and need bridges of translation - which can often leak/lose the essence of meanings and interpretations - to communicate with speakers of other languages.

There are network effects in a language such that it benefits a person to speak/learn a language that is spoken by more other people. There are adoption costs of learning a language, unless one is a linguaphile. And there are sometimes, or often, inherent preferences for a certain language, be it one's native language or one's inherited language or a language that one is interested in preserving or keeping alive. In short, languages can be referred to as one-sided communication platforms, that are somewhat incompatible, and such that people are capable of having ex ante preferences over them independent of the size of their networks.

Is the incompatibility of languages alongwith people's inherent preferences for them responsible for the prevalence of multihoming (bilingualism and multilingualism) in languages; and how exactly does the incompatibility of languages affect the social need of people to speak a common language (the emergence of a global language), which in its extreme can be interpreted as the 'natural monopoly' of a language?

These are some of the motivating questions of this paper. Moroever, unlike languages that are inherently incompatible communication platforms, in our time and age communication software platforms - for example, email, instant messaging (IM), video or conference calling providers, etc. - can choose or be regulated to be compatible with rival domains. How does this choice, as well as agents' cost/price of adoption of a platform affect adoption decisions and outcomes?

This paper models a thought experiment to explore the interactions between compatibility and homing choices by platforms and consumers, respectively, and to under-

[1] For more on this analogy refer to Marschak, 1965; Colomer, 1990; Selten & Pool, 1991; Grin, 1992; and Church & King, 1993, to name a few.

stand the resulting market outcomes of monopoly or competition, conditional on the size of adoption cost/price. To focus on this, I blur platform choice of price, unlike much of the literature on one-sided platforms that is concerned with price competition (Fudenberg and Tirole, 2000; Banerji and Dutta, 2009; Bloch and Querou, 2013; Aoyagi, 2018), and take it to be exogenously given. This is also motivated by the observation that the bulk of communication software platforms have zero explicit price for consumers, who face obscure or obfuscated cost/price of adoption, paying with their user data which is hard to quantify or compare between apps/platforms; this lack of easy comparison of platform adoption price by consumers also maps well with the analogy of unknown cost of learning languages.I thus model a constant adoption price/cost across two competing platforms that have varying appeal for different consumers.

The incentives and disincentives of choosing compatibility with a rival platform in this model are thus not related to price. That is, unlike price (choice and) competition models, wherein compatibility raises consumers' valuation because it enables them to connect with users of other platforms as well, and thus enables platforms to raise prices (Farrell and Klemperer, 2007), this incentive to choose compatibility is switched off in this model due to exogenous fixed price.

The literature in one-sided markets characterized with network effects has so far mostly treated compatibility as that resulting from a coincidence of all users choosing the same technology, thus interpreting compatibility as standardization (Arthur, 1989; David, 1985; Farrell and Saloner, 1985; Farrell and Saloner, 1986; Katz and Shapiro, 1986). I diverge from that treatment, and model compatibility as a choice for the platforms or firms or a regulatory regime that can be imposed on them.

Papers that do model compatibility (or connectivity) as a choice, do so in very different market settings and often without consumers having the choice of single or multihoming: Katz and Shapiro (1985) model firms as choosing compatibility alongwith prices but without consumers being able to multihome; (Regibeau and Rockett, 1996) model firms choosing compatibility and the timing of product introduction in the market; and (Cremer et al., 2000) look at connectivity choice by backbones in the internet.

I take the understanding of multihoming from two-sided and multi-sided platform literature (Caillaud and Jullien, 2001; Rochet and Tirole, 2003; Caillaud and Jullien, 2003; Belleflamme and Peitz, 2019)[2], where it implies adoption of, registration with, usage of, or joining more than one platform (or simply multiple platform registrations), into one-sided platform infrastructure wherein multihoming has not been formally explored by economic theory to the best of my knowledge.

The older literature on the economics of language does discuss multihoming without using that term, but models consumer preference for one language (over another) by allocating an ex ante 'native' language to each consumer (Grenier, 1985; Lang, 1986; Selten and Pool (1991); Church and King, 1993), such that the only choice consumers make is whether to use/learn/adopt another language (i.e. whether to multihome or not). Unlike these papers, consumers in my model do not have any ex ante allocation of platform adoption, and thus can possibly choose to not adopt their most preferred platform. Colomer (1990) models consumers as having different utility levels from using different languages, and thus having a preference between languages; however he models all consumers as knowing both languages such that there is no adoption cost/price (which is important in my model).

This study is important given the recent DMA (Digital Markets Act) in Europe

[2]Often agents on one side multihome to reach divided single homing agents on the other side.

and in its wake the argument in the literature regarding mandating increased compatibility and/or ‘interoperability’ between digital platforms. Scott Morton et al. (2021) argue in favor of interoperability while the CERRE Report (Bourreau et al., 2022) cautions that mandating it will disincentivize multihoming by consumers, with the implication that multihoming increases competition, and reducing it hampers competition. My model illustrates that equitable interoperability precludes consumers’ need to undertake costly multihoming, but consumer welfare is maximum and thus optimal in the interoperable world. My model also illustrates how multihoming bridges the gap between isolated platforms and leads more often to ‘competition in the market’ rather than the contestable market outcomes that result from restricting consumers to single homing, especially when the cost or price of adopting each platform is small. Reducing cost/price of adoption can thus be an alternative means to achieve greater compatibility and competition in such markets.

My model is not fully solved yet; results are preliminary and few so far.

2 THE MODEL

Two competing platforms, X and Y, are ex ante identical and vie for the patronage of N number of agents. Platform i’s profit is $\pi_i(A_i, p)$, where A_i is the number of agents who adopt platform i, and $p > 0$ is the benefit a platform derives from each agent’s adoption. The platform’s profit increases in each of its arguments.

Agent j’s benefit from adopting and using platform i is $B_j(S_i, i)$, where j denotes the type or identity of the agent, with all agents of a type having identical ex ante characteristics and benefits. Here S_i is the number of agents connected with each other on or through platform i when agent j adopts platform i; notice that S_i includes agent j herself. This is inspired by Farrell and Saloner (1985) but differs from their modeling of agent’s type as a continuum and instead creates two discrete types, and extends agents’ benefits into the realm of multihoming (which they do not venture into).

An agent’s adoption of each platform costs her $c > 0$, which can be thought of as a subscription fee or price, or can be interpreted as the cost of having her personal data read (if there is no explicit subscription fee that platforms charge).[3] I will assume this cost, $c > 0$ to be exogenously given, but of course we can extend the discussion to what might happen if platforms or regualtion can influence or choose this cost.

I consider two types of agents, those who prefer platform X, and are denoted with $j = x$; and those who prefer platform Y and are denoted with $j = y$. To keep notation simple, $n_y \in (0, N)$ is the number of Y-preferring agents that exist in the population, and similarly $n_x = N - n_y$ is the number of X-preferring agents that exist in the population.[4]

The number of agents of each kind might differ, but in all other ways the two platforms are ex ante symmetric.

Network externalities exist in the benefits derived from using any platform such that any agent prefers more other agents to be on the platform that she adopts, i.e.

$$B_j(S, i) > B_j(S', i), \forall S > S'; i = \{I, J\}; \tag{1}$$

[3] It is possible to treat $p = c$ such that each platform’s gain from each adopter equals the cost to each adopter of adopting a platform, in which case we call p or c the common market subscription price.

[4] Notice that $n_y = 0$ or $n_y = N$ would be trivial and uninteresting.

with equality whenever $S = S'$, and $B_j(S,i) = 0, \forall S \leq 1$, i.e. an agent gets no benefit unless she is connected with at least one more agent other than herself.

Benefits derived and platform preference can be defined as follows. First, benefits from the two platforms are symmetric, such that X and Y preferring agents derive equal benefits from the same connectedness on their preferred platforms:

$$B_x(S,X) = B_y(S,Y). \tag{2}$$

And second, agent of type j derives greater benefit from platform J than from platform I if both platforms had the same connectedness. Formally,

$$\begin{aligned} &B_x(S,X) > B_x(S,Y), \forall S \leq N; \textit{ and symmetrically,} \\ &B_y(S,Y) > B_y(S,X), \forall S \leq N. \end{aligned} \tag{3}$$

And lastly, if and when agents multihome, I assume that two multihoming agents connect with each other on only one of the two platforms; if both have the same preferred platform, then this connection is formed on that, if not, then it could be on either or might depend on the numbers of such agents.[5] Moreover, moving connections from one's preferred platform to the other platform while keeping other connections unchanged should only reduce one's total benefit.[6] That is,

$$B_j(S+S'',J) + B_j(S',I) > B_j(S,J) + B_j(S'+S'',I);\ S,S' \geq 2. \tag{4}$$

Agents are rational and forward looking, choose sequentially between the two platforms, knowing the order of moves, the chosen adoptions before them, and without any direct communication with other agents. Both n_y and the nature of benefits to all agents from both platforms are also publicly known to both platforms and to all agents.[7] Lastly, to break ties of indifference, I will assume for simplicity that whenever an agent is indifferent between adopting her preferred platform and the other platform, she adopts her preferred platform.

With this minimal structure, what can we predict about adoption choice, and thus about platform competition?

To understand the implications of our assumed structure and benefits, I will use a simple illustration. The assumption of network externalities in (1) and the definition of preferring one platform over the other in (3) can together be illustrated by Figure 1; the two lines need not be straight or have the same slope, what is important and follows from our assumption is that the line or curve of $B_j(S,I)$ lies below that of $B_j(S,J)$ and does not intersect with it.

Symmetric benefits as in (2), combine with (3) to also imply:

$$B_i(S,i) > B_j(S,i), \forall S \leq N; \tag{5}$$

i.e. an X preferring agent enjoys greater benefits from platform X than does a Y preferring agent, for the same number of connected agents on either, and vice versa.

[5]The model automatically simplifies this as only one type of agents will ever multihome.

[6]Of course, the idea of total benefits as the sum of benefits from both platforms is applicable only in case of multihoming.

[7]It will be interesting to see the effects of agents not knowing n_y, not knowing the order of moves, or the chosen adoptions before them, or with communication, etc.; some, if not all of these, are beyond the scope of this paper.

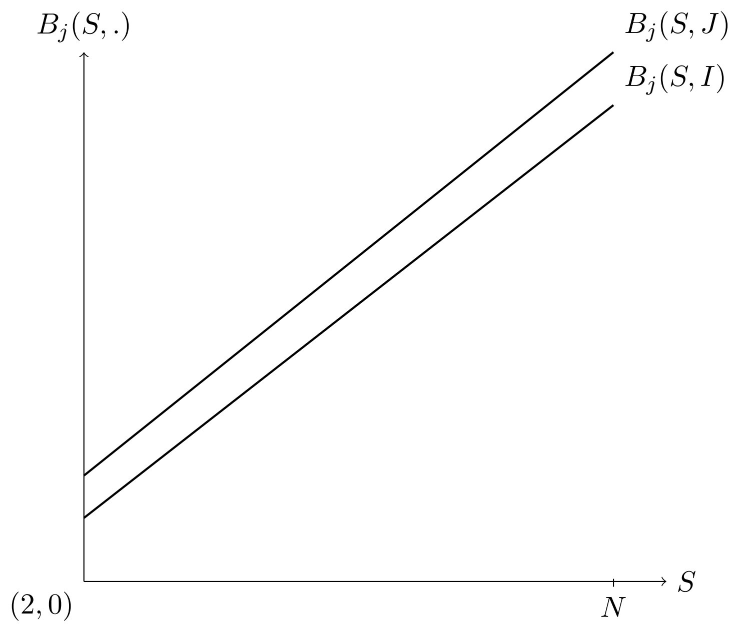

Figure 1: Network Externalities and Platform Preference

3 ISOLATED PLATFORMS

Consider platforms X and Y that are isolated from each other, such that an adopter of either platform can use it to connect with another adopter only on the same platform. What are the possibilities for the resulting Subgame Perfect Equilibria in this game when agents can only single home (adopt at most one platform, or be exclusive to one platform), and when they can multihome (adopting more than one platform each)?

Notice that the assumed structure of benefits, illustrated in figure 1 clearly implies that an agent derives as much benefit from her preferred platform with some smaller connectedness as she does from the other platform with more connectedness. In other words, figure 1 can be redrawn below as figure 2 to point out a level of connectedness on one's preferred platform that gives agents of type j the same benefit as the other platform with all agents on it. Let us denote this level as τ, which is formally defined below.

Definition 1. *Define the sufficiency threshold for i-type agents as τ such that*

$$B_i(\tau, I) = B_i(N, J); i = \{x, y\}, I = \{X, Y\}. \tag{6}$$

And thus, define the number of agents of i-type as being 'sufficient' if $n_i \geq \tau$.

Moreover, then there is a unique level of number of agents on an agent's preferred platform, which is less than $\frac{N}{2}$, at which she is indifferent between her preferred platform and the other platform with remaining agents on it. In other words, if we superimpose the mirror image of $B_j(S, I)$ (which would be $B_j(N - S, I)$) on the graph of $B_j(S, J)$ as in Figure 3, then we have a unique intersection between them, which lies to the left of $\frac{N}{2}$.

Let us define this unique intersection as follows.

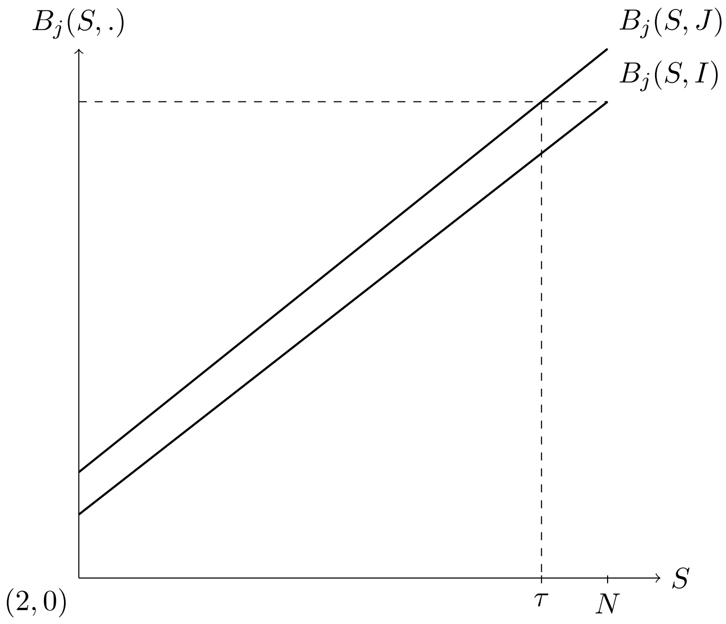

Figure 2: Network Externalities and Platform Preference With τ

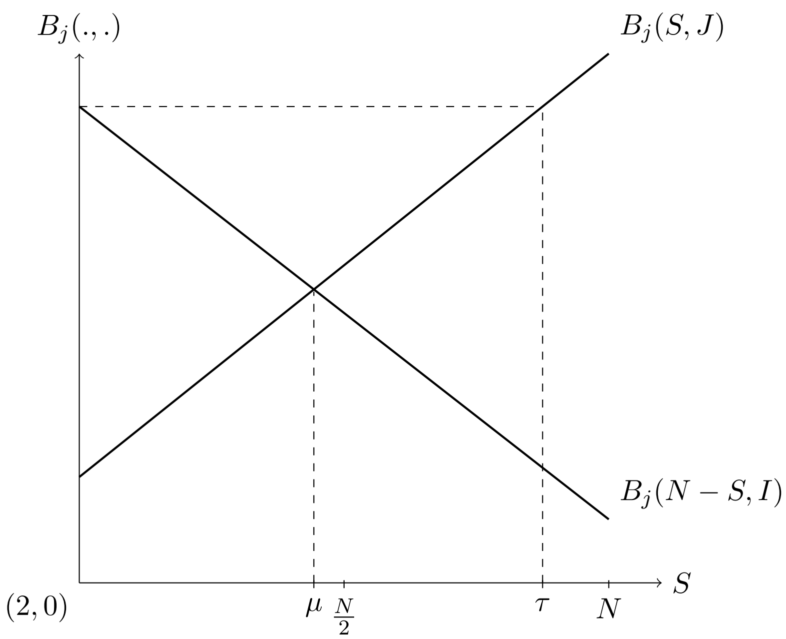

Figure 3: Significance & Sufficiency Thresholds

Definition 2. *Define* $\mu \in (0, \frac{N}{2})$ *as the unique level of number of agents on an agent's preferred platform such that the agent is indifferent between her preferred platform and the other platform with all remaining agents on it.*

$$B_i(\mu, I) = B_i(N - \mu, J); i = \{x, y\}, I = \{X, Y\}. \tag{7}$$

Call this the 'significance threshold', and therefore the number of agents of j*-type as being 'significant' if* $n_j \geq \mu$.

Simply put, an agent of type j derives as much benefit from her preferred platform with μ agents on it, as she does from the other platform with all remaining agents on it, where $\mu < N - \mu$. Because benefits are exogeneously given, so are μ and τ; and symmetric benefits give that both these statistics are the same for x and y type agents.

It is straightforward, from (1) and (7) that sufficient numbers imply strictly significant numbers, but not vice versa. In other words, that $\tau \in (\mu, N)$. Also notice that although we consider agents in discrete numbers, nither μ nor τ need be whole numbers.

It is also worth pointing out that μ and τ should be (intuitively) proportional to each other, and smaller values of these represent greater preference heterogeneity between the agents preferring the two platforms, or equivalently, greater product differentiation between the two platforms (larger gap between the two benefit curves in figures 1 and 2).

3.1 SINGLE HOMING OR EXCLUSIVE ADOPTION

First consider the state of the world in which agents can only adopt one platform, i.e. they can only single home, or that their adoption is exclusive to one platform.

If $B_y(n_y, Y) < B_y(N, X)$ and $B_x(n_x, X) \geq B_x(N, Y)$ both hold, in other words if $n_y < \tau$ but $n_x \geq \tau$ or only x-agents are sufficient in number, then regardless of when the x-preferring agents move, they will all choose X, and foreseeing this and given their own preference then, all y-preferring agents also choose X. Notice that this argument does not need the first mover to be x-preferring, and is thus independent of the preference of the first mover.

Next, consider $B_y(n_y, Y) \geq B_y(N, X)$ and $B_x(n_x, X) \geq B_x(N, Y)$, i.e. both n_y and n_x are sufficient in number (or $n_x, n_y \geq \tau$). This is possible only if $\tau \leq \frac{N}{2}$. Because sufficiency implies strict significance, therefore both types of agents are greater than μ in number, which also implies that *both fall short of* $N - \mu$. Then each agent adopts her preferred platform. Notice also that because the outcome is symmetric in that both platforms are adopted by preferring agents, this outcome is independent of the identity of the first mover.

Third, consider $B_y(n_y, Y) \geq B_y(N, X)$ and $B_x(n_x, X) < B_x(N, Y)$ together; in other words, only $n_y \geq \tau$ and is thus sufficient. Even if the first mover chooses X now, regardless of when the y-preferring agents move, they will all choose Y. Foreseeing this, all x-preferring agents will also prefer to choose Y, and thus all agents adopt Y. This is also independent of the preference-identity of the first mover.

And lastly, the hardest to predict, is if $B_y(n_y, Y) < B_y(N, X)$ and $B_x(n_x, X) < B_x(N, Y)$ both hold together, i.e. neither type is sufficient in number ($n_x, n_y < \tau$). Neither type of agent has the incentive to deviate from a mass adoption of either platform, however, each agent prefers a mass adoption on its own preferred platform and if everyone so far has done that then so will she. This is similar to a sequential n-person version of the Battle of the Sexes game.

Regardless of the order of moves, by backward induction only one platform should be adopted in this case; and the first mover's adoption will be mimicked by all, but it need not be the first mover's preferred platform. First mover advantage determines the outcome only if first movers of a type exceed $N - \tau$ in number. Below are some examples that illustrate

Example 1': Suppose $N = 5, \mu = 2, n_x = 3, n_y = 2$, and the 'sufficient' threshold is 4.

(a) Let the order of moves be x, x, x, y, y. We have $B_y(2, Y) = B_y(3, X)$ from definition 2, but from (1) then $B_y(2, Y) < B_y(4, X)$. All agents will therefore adopt only X.

(b) Let the order of moves be y, y, x, x, x. We have $B_x(5, Y) = B_x(4, X) > B_x(3, X) > B_x(2, X) = B_x(3, Y)$, and therefore everyone will adopt only Y.

(c) Let the order of moves be y, x, x, x, y. Because $B_y(2, Y) < B_y(4, X)$, and $\tau = 4$, everyone will adopt only X. There is in fact indifference here for the x-types in folding back their optimal strategy, because $B_x(4, X) = B_x(5, Y)$. If we assume that all such ties of indifference are resolved in favor of agents' preferred platform, then we can say for sure that the first mover expects everyone after him to adopt X and thus adopts X himself. But if the tie-breaking rule is such that in case of indifference an agent chooses what is better for the other agent type, then everyone will adopt platform Y in this case.

Example 2': Suppose $N = 6, \mu = 2, n_x = 4, n_y = 2, \tau = 5$. Let the order of moves be y, x, x, x, x, y. We have $B_y(2, Y) = B_y(4, X)$, and $B_y(5, Y) = B_y(6, X)$. From 1 then we have $B_y(5, X) > B_y(2, Y)$; therefore, if the first five agents each adopts their preferred platform, then the last agent will adopt X. Folding back then, all agents will adopt only X and the preferred platform of the first mover is not adopted in this case!

We can summarize our learning of adoption outcomes on isolated platforms in the following proposition.

Proposition 1. *For agents single homing between isolated platforms:*

If agents preferring both platforms are sufficient in number[8]*, then both will be adopted. If only agents preferring one platform are sufficient in number, then only that platform is adopted. Both these outcomes are independent of agents' order of moves.*

If agents preferring both types are insufficient in number, then only one platform is adopted but the outcome is sensitive to the order of moves relative to the sufficiency threshold. First mover advantage is deterministic only if the number of a type moving first exceeds $N - \tau$.

Figures 4 and 5 show that more often than not, competition is of the nature of contestability or competition for the market. This is depicted as the purple region in Figure 4 where either platform (red or blue) can become the sole to survive in the market, based on the order of moves of agents. It is only when both $n_x, n_y \geq \tau$ - which is possible only when $\tau \leq \frac{N}{2}$ (an unreasonably small τ or an unreasonably high level of differentiation between both platforms) and is illustrated in Figure 5 - does

[8]This is possible only if $\tau \leq \frac{N}{2}$.

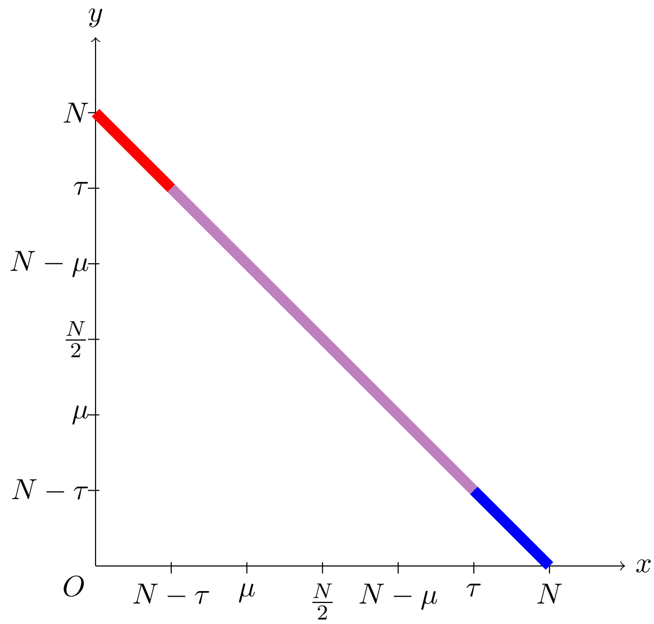

Figure 4: Single Homing on Isolated Platforms; $\tau > N - \mu$

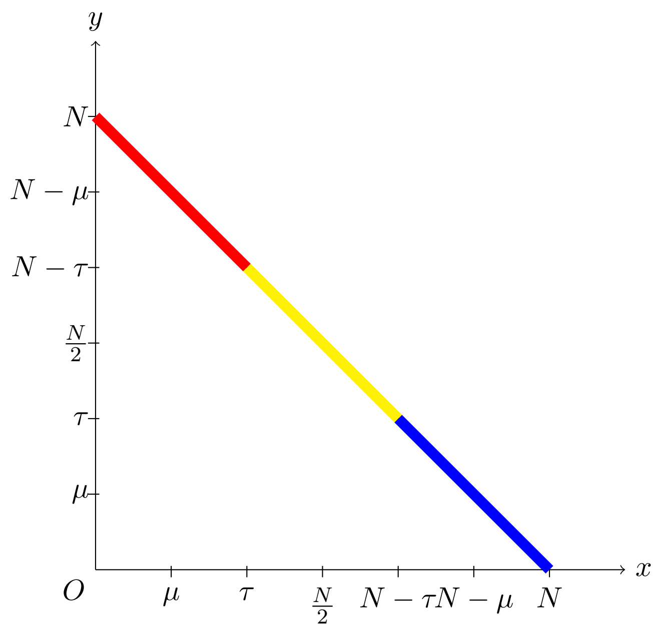

Figure 5: Single Homing on Isolated Platforms; $\tau < \frac{N}{2}$

competition exist in the market between isolated platforms (the yellow region where both n_x and n_y are sufficient and thus both platforms survive).

There is one more configuration possible between μ and τ; given its similarity to Figure 4, this is drawn in the appendix as Figure 8.

Notice the implications from the above results: that it is not necessary for agents preferring a platform to be in the majority (greater than $\frac{N}{2}$) for that platform to exist or be adopted. Nor does it suffice for agents preferring a platform to be in the majority for that platform to be able to wipe out its rival!

Instead, the sufficient number for a platform to be adopted is τ, and therefore on the flip side, the platform whose loyal agents move first en masse in numbers exceeding $N - \tau$ ensures its sole survival. I will refer to $N - \tau$ therefore as the 'critical installed base', although notice that it is important that this base is installed before any agents adopt the other platform.

Notice also that if platforms could influence the numbers of n_x and n_y in the market, and could influence the order of agents' moves (by a platform entering the market earlier than its rival), then a platform could assure itself a market win either through a large loyal customer base that is sufficient or through amassing a critical installed base first. In fact, even a new entrant could turn the market around and contest and claim it if, with its entry, it manages to change customers preferences enough to create a sufficient loyal following!

3.2 MULTIHOMING

If agents can multihome, i.e. adopt more than one platform at the same time, then the outcome would depend, unsurprisingly, on the cost of homing on an additional platform, $c > 0$.

So what can we say about which type of agent, and how many, will multihome in any SPE? To proceed I first assume, hopefully reasonably, that two agents who both multihome, connect with and derive benefits from each other on only one platform. I then try and limit the possibilities below by arguing away their complements; the first is obvious and trivial.

Lemma 1. *All agents in the population cannot be multihoming.*

Suppose all agents did multihome, then at least the very last agent to move would be better off adopting only her preferred platform (and not multihoming) given all other agents are on it.

Lemma 2. *Same-type agents must either all single home or all multihome.*

This lemma is established by the following cases, exploring such a possibility, and each leading to a contradiction.

Let the y-types who multihome be aggregated as a group, numbering n_y^m, and the y-types who do not be aggregated as as a group, numbering n_y^s; similarly collect all x-types who multihome as a group, numbering n_x^m, and those who do not as n_x^s. If all agents of either type are either single homing or multihoming, I simply use n_j for them.

Table 1 makes sense only if R is X such that all n_x^m agents prefer to connect with each other on their preferred platform. Their total net benefits are $B_x(n_x^s + n_y + 1, L) + B_x(n_x^m, R) - 2c$, and would have been $B_x(n_x^m + n_x^s + n_y, L) - c$ if they had only

Table 1: Multi Homing Case 1

Platform L	Platform R
n_x^m	n_x^m
n_x^s	
n_y	

single homed on L. By revealed preference, given they are multihoming, and using $n_x^m + n_x^s = n_x$ it must be that

$$B_x(n_x^s + n_y + 1, L) + B_x(n_x^m, R) > B_x(n_x + n_y, L) + c.$$

But for an agent in n_x^s, single homing on L gives net benefit $B_x(n_x^m + n_x^s + n_y, L) - c$ whereas if one of these agents was to multihome and move their connections with n_x^m onto their preferred platform then their total net benefit would be $B_x(n_x^s + n_y, L) + B_x(n_x^m + 1, R) - 2c$. By their revealed preference, given their choice of single homing, it must be that

$$B_x(n_x^m + n_x^s + n_y, L) + c > B_x(n_x^s + n_y, L) + B_x(n_x^m + 1, R)$$

Both the above inequalities together imply

$$B_x(n_x^s + n_y + 1, L) + B_x(n_x^m, R) > B_x(n_x^s + n_y, L) + B_x(n_x^m + 1, R);$$

but this contradicts (4) if R is their preferred platform.

Table 2: Multi Homing Case 2

Platform L	Platform R
n_x^m	n_x^m
n_x^s	
	n_y

For Table 2, if R is Y, by revealed preference, for any n_x^m agent: $B_x(n_y + 1, R) + B_x(n_x^m + n_x^s, L) > B_x(n_x^m + n_x^s, L) + c$, or simply $B_x(n_y + 1, R) > c$. And for any n_x^s agent: $B_x(n_x^m + n_x^s, L) + c > B_x(n_x^m + n_x^s, L) + B_x(n_y + 1, R)$ or simply that $c > B_x(n_y + 1, R)$, which contradicts the corresponding revealed preference inequality for an n_x^m agent.

However, if R is X, then by revealed preference for any n_x^m agent, $B_x(n_x^m + n_y, R) + B_x(n_x^s + 1, L) > B_x(n_x^m + n_x^s, L) + c$. And for any n_x^s agent: $B_x(n_x^m + n_x^s, L) + c > B_x(n_x^s, L) + B_x(n_x^m + n_y + 1, R)$. Using both agents' revealed preferences we have $B_x(n_x^m + n_y, R) + B_x(n_x^s + 1, L) > B_x(n_x^m + n_x^s, L) + c > B_x(n_x^s, L) + B_x(n_x^m + n_y + 1, R)$, which contradicts (4).

In Table 3, if L is X, then any agent in n_x^m is better off just single homing on L. If R is X, then y-types must be connecting with n_x^m on R, without which they don't have a reason to be there. Then for any n_x^m agent, revealed preference gives $B_x(n_x^s + 1, L) + B_x(n_x^m + n_y, R) > B_x(n_x^m + n_x^s + n_y, L) + c$. But for an n_x^s agent, revealed preference gives $B_x(n_x^m + n_x^s + n_y, L) + c > B_x(n_x^s, L) + B_x(n_x^m + n_y + 1, R)$. Both together contradict 4.

Table 3: Multi Homing Case 3

Platform L	**Platform R**
n_x^m	n_x^m
n_x^s	
n_y	n_y

Table 4: Multi Homing Case 4

Platform L	**Platform R**
n_x^m	n_x^m
n_x^s	
n_y^m	n_y^m
n_y^s	

In Table 4, if L is X, then n_x^m should not multihome on R; if R is X, then n_y^m-types must be connecting with n_x^m on R, without which they don't have a reason to be there. Then for any n_x^m agent, revealed preference gives $B_x(n_x^s + n_y^s + 1, L) + B_x(n_x^m + n_y^m, R) > B_x(n_x^m + n_x^s + n_y^m + n_y^s, L) + c$. But for an n_x^s agent, revealed preference gives $B_x(n_x^m + n_x^s + n_y^m + n_y^s, L) + c > B_x(n_x^s + n_y^s, L) + B_x(n_x^m + n_y^m + 1, R)$. Both together contradict 4.

Table 5: Multi Homing Case 5

Platform L	**Platform R**
n_x^m	n_x^m
n_x^s	
n_y^m	n_y^m
	n_y^s

Table 5 makes sense only if R is Y (because otherwise n_y^s agents have no reason to be there). Notice that the connections between n_x^m and n_y^m are formed on only one platform: if this is on L, then each agent in n_x^s is getting the same benefits on L as an agent in n_x^m, and must then also want to multihome on R to connect with n_y^s just like n_x^m does. However, if this is on R, then each agent in n_x^m should be better off moving this to L and earning the same benefits as n_x^s if that is optimal for n_x^s. That is, either n_x^m is not optimally homing, or n_x^s is not.

In short, in each of the above cases, either n_x^m is not optimally homing, or n_x^s is not. QED.

Both the above lemmas together automatically imply:

Lemma 3. *Both types of agents cannot be multihoming.*

So which type multihomes, based on their numbers, and based on the order of moves?

In general, what multihoming does is it weakens the tradeoff between adopting one's preferred platform and connecting with as many other agents as possible. And in doing so, it weakens the first mover advantage such that late movers can now still adopt their preferred platform (and enjoy better connections between themselves) while still also being able to connect with agents of the other type on their platform.

But now first mover advantage also becomes important in a different manner: because it gives one the opportunity to single home on one's preferred platform and thus commit against multihoming oneself, in order to force the other types of agents to multihome. Suppose some y-types move first to single home on Y such that their number, say n_y^f, satisfies $B_x(n_y^f + 1, Y) \geq c$, then for the x-types following this, the payoffs to consider and compare are as follows:

$$B_x(n_x, X) - c;$$
$$B_x(n_x, X) - c + B_x(n_y + 1, Y) - c;$$
$$B_x(N, Y) - c;$$

where the first is the payoff from single homing on X, the second that from multihoming, and the third that from single homing on Y.

One extreme resolution of the above is x agents are so few in number, or that c is large enough that $B_x(n_x, X) < c$; in this case the first and second payoffs cannot be chosen, and everyone will therefore single home on Y.

The other extreme is $n_x \geq \tau$ in which case the third payoff is dominated by the first, and can thus not be chosen. It should also be reasonable to say that given τ is no small number, these two extremes of n_x cannot coincide, i.e. c is small enough such that if $n_x \geq \tau$, then also $B_x(n_x, X) > c$. With $n_x \geq \tau$, if we have $B_x(n_y^f + 1, Y) \geq c$, then all x-agents (moving after) will then multihome on X and Y both. This outcome is realized regardless of how large n_y itself is.[9]

Notice that we are using payoff of $B_x(n_y + 1, Y)$ on platform Y rather than that of $B_x(n_y^f + 1, Y)$ in our argument above, even though the latter is what compels x-types to adopt Y as well; this is because if such compulsion is successful, then foresight should reveal that all y-agents will single home on their preferred platform and thus the relevant payoff with foresight becomes the former one.

If however, neither of these extreme conditions hold, then the first payoff is dominated by the third, x-agents will multihome as long as $B_x(n_x, X) + B_x(n_y + 1, Y) > B_x(N, Y) + c$, for which it must be that $B_x(n_x, X) > c$ and $B_x(n_y + 1, Y) - c$; otherwise x-types will single home on Y because their own numbers are not sufficient.

A few examples below; notice these are the same as Example 1' in the last subsection, and we have $n_x, n_y < \tau$.

Example 1': Suppose $N = 5, \mu = 2, n_x = 3, n_y = 2$, and the 'sufficient' threshold is 4.

(a) Let the order of moves be x, x, x, y, y. If the first three agents single home on their preferred platform, X, then for the fourth mover, single homing on X will lead to the payoff of $B_y(5, X) - c$; multihoming will lead to the best payoff of $B_y(2, Y) - c + B_y(4, X) - c$; and single homing on Y will lead to the best payoff of $B_y(2, Y) - c$.

Because $\mu = 2$, we have $B_y(2, Y) = B_y(3, X)$; and because $\tau = 4$, we have $B_y(4, Y) = B_y(5, X)$.

If $B_y(2, Y) = B_y(3, X) > c$. This is also the case in this example when enough x agents have moved first to commit against their multihoming to compel y agents

[9] If both types of agents are in sufficient numbers, then both types will adopt their preferred platforms, and the type j whose agents move first such that $B_i(n_j^f + 1, J) - c > 0$, forces the other type to multihome.

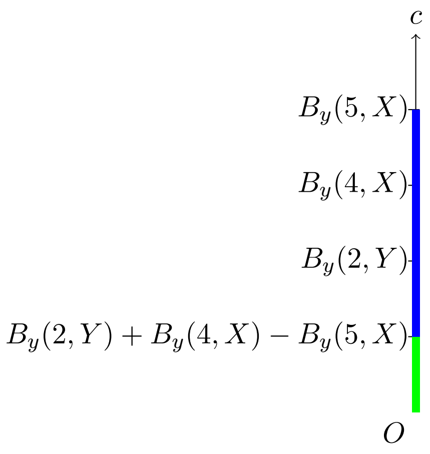

Figure 6: Multi Homing as a function of c; Example 1'a

to adopt X. Multi homing now beats single homing on Y for these following agents. y agents will choose it however only if it also beats single homing on X: i.e. only if $B_y(2,Y) > B_y(5,X) - B_y(4,X) + c$; otherwise they will single home on X.

If $B_y(4,X) > c > B_y(2,Y) = B_y(3,X)$, or if $B_y(4,Y) = B_y(5,X) > c > B_y(4,X)$, then all agents will simply single home on X. And of course, if c is any bigger than $B_y(5,X)$, then y agents do not pariticpate in the market.

Notice also that the first three agents would not profit by deviating to multihoming, as that would incentivize y agents to single home on Y and lower agents x's benefits while adding the cost of another adoption.

Figure 6 illustrates that when c is low (green region), then second moving agents will multihome.

(b) Flipping the order of moves in above: y, y, x, x, x. If the first two agents single home on Y, then for the later movers, the best payoffs are: $B_x(3,X) - c$ from single homing on X; $B_x(5,Y) - c$ from single homing on Y; and $B_x(3,X) - c + B_x(3,Y) - c$ from multihoming.

Because $\mu = 2$, we have $B_x(2,X) = B_x(3,Y)$; and because $\tau = 4$, we have $B_x(4,X) = B_x(5,Y)$.

If $B_x(2,X) = B_x(3,Y) > c$, all three possible payoffs are positive; multihoming beats single homing on X, and will also beat single homing on Y iff $B_x(3,X) + B_x(3,Y) > B_x(5,Y) + c$. If not, then everyone will single home on Y.

If $B_x(3,X) > c > B_x(3,Y) = B_x(2,X)$, then multihoming is ruled out, and because x agents are insufficient in number, their best strategy is to single home on Y.

If $B_x(5,Y) > c > B_x(3,X)$, the only positive payoff is from single homing on Y.

Figure 7 illustrates that when c is low (green region), then second moving agents will multihome.

(c) And the last order of moves: y, x, x, x, y. If only the first agent single homes on Y, and $B_y(2,Y) > B_x(2,Y) > c$, then for the following x movers, the best payoffs are: $B_x(4,X) - c$ from single homing on X; $B_x(5,Y) - c$ from single homing on Y; and $B_x(3,X) - c + B_x(3,Y) - c$ from multihoming.

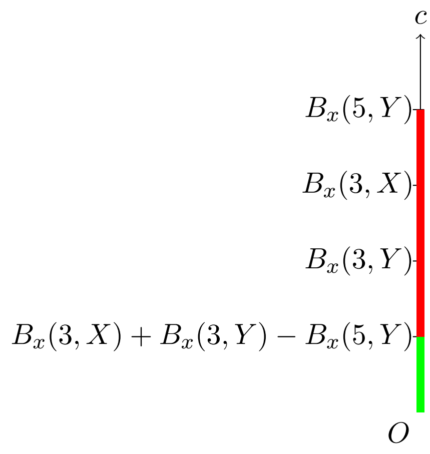

Figure 7: Multi Homing as a function of c; Example 1'b

Because $\mu = 2$, we have $B_x(2, X) = B_x(3, Y)$; and because $\tau = 4$, we have $B_x(4, X) = B_x(5, Y)$; the latter condition gives an indifference for x-types between single homing on X and single homing on Y. Assume this is resolved in favor of X. Multi homing will occur by x-types if $B_x(3, X) + B_x(3, Y) > B_x(4, X) + c$, or that $B_x(3, Y) - [B_x(4, X) - B_x(3, X)] > c$.

4 INTEROPERABLE PLATFORMS

If both platforms are interoperable - in other words, fully and equitably compatible - then an agent adopting either platform derives from it the benefit of connecting with all adopting agents on both platforms. In particular, we are interesting in equitable interoprability in case of direct network effects.

In this case, there is no doubt that each agent adopts her preferred platform, and both platforms are adopted and survive. Interoperability also renders the sequential nature of the game meaningless, and the order of agents' moves do not determine the outcome!

This same outcome is realized with isolated platforms only when $n_x, n_y \geq \tau$, which is itself possible only if $\tau \leq \frac{N}{2}$. In other words, only under these conditions - large preference heterogeneity between the two types of agents, and both having sufficient numbers - are both platforms indifferent between isolation and interoperability.

It is also straightforward that no agent has the need or the incentive to multihome under full compatibility/interoperability.

Whenever $\tau > \frac{N}{2}$ such that both n_x and n_y cannot simultaneously be sufficient, either the platform with larger and sufficient loyal customers prefers isolation (to interoperability) or the first entrant platform prefers isolation such that its early entry influences the order of moves and achieves installed base exceeding $N - \tau$ first. The other platform, or potential entrants unable to make themselves attractive to customers would prefer interoperability. The preference for isolation is therefore an incumbent preference.

5 PLATFORMS CHOOSING DIFFERENTIATION

So far, we have treated μ as exogeneously given. But notice that a larger μ (tending to $\frac{N}{2}$) indicates greater homogeneity between the two platforms, and its opposite indicates greater product differentiation or stronger loyalty for agents' preferred platforms. Because we started off assuming preference for one or the other platform, we forced $\mu < \frac{N}{2}$. But if $\mu = \frac{N}{2}$, then from equation (7), agents should not care which platform they are on.

If $\mu = \frac{N}{2}$ then $\tau = N$, and as a result in isolated platforms (refer to Figure 4) only one will be adopted, and it will be the one the first moving agent chooses given every agent is indifferent between the two platforms. The first mover is then herself indifferent between choosing either platform. This is extreme contestability or competition for the market.

6 APPENDIX

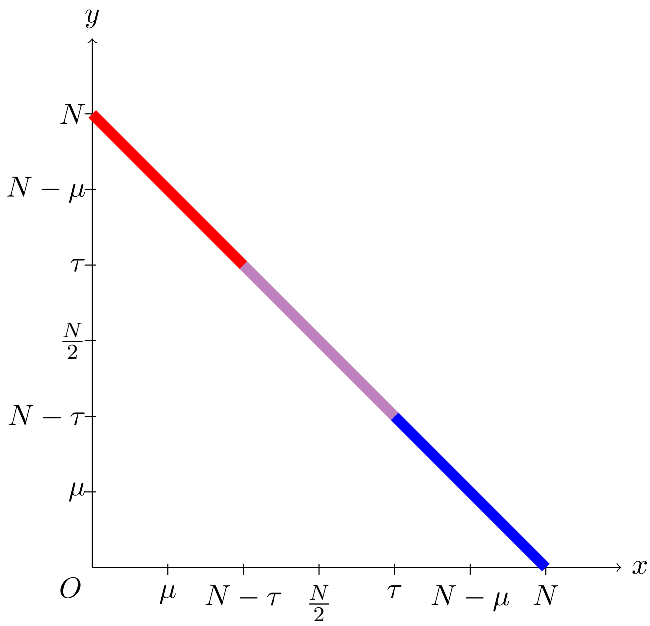

Figure 8: Single Homing on Isolated Platforms; $\tau \in (\frac{N}{2}, N - \mu)$

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