Lateral and Longitudinal Collision Risk (A Case Study using December 2010 TSD)

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(Joint work with Deepayan Sarkar and BOBASMA Team, AAI)

Workshop on Airspace Safety Monitoring
Indian Statistical Institute, New Delhi
May 7, 2012

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- The goal was to confirm that the *Target Level of Safety (TLS)* $(5 \times 10^{-9}$ accidents per flight hour), was met.
- Note that it was pre-RHS time and so the separation standards were
  - 50 NM lateral separation between all the parallel routes;
  - 10 minutes longitudinal separation between front and behind aircrafts.

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- To help India/AAI establish an *En-route Monitoring Agency (EMA)* for Bay of Bengal and Arabian Sea.
- This was of course prestigious for India/AAI.
- Moreover this it would help in reducing the current separation standards and hence a sharp increase in air traffic volume hopefully leading to positive effect on India's economy.

Achievements

- This joint work and a subsequent analysis were presented in two successive *RASMAG* meetings at the *International Civil Aviation Organization (ICAO)* Asia Pacific Office at Bangkok, Thailand.

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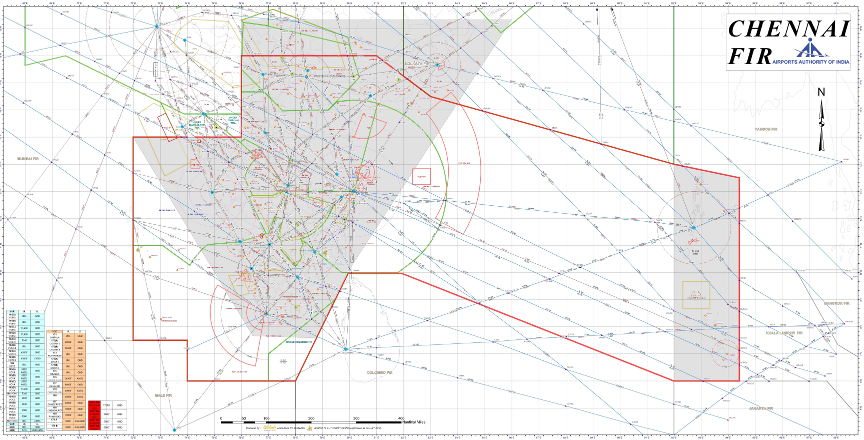
Map of Chennai FIR



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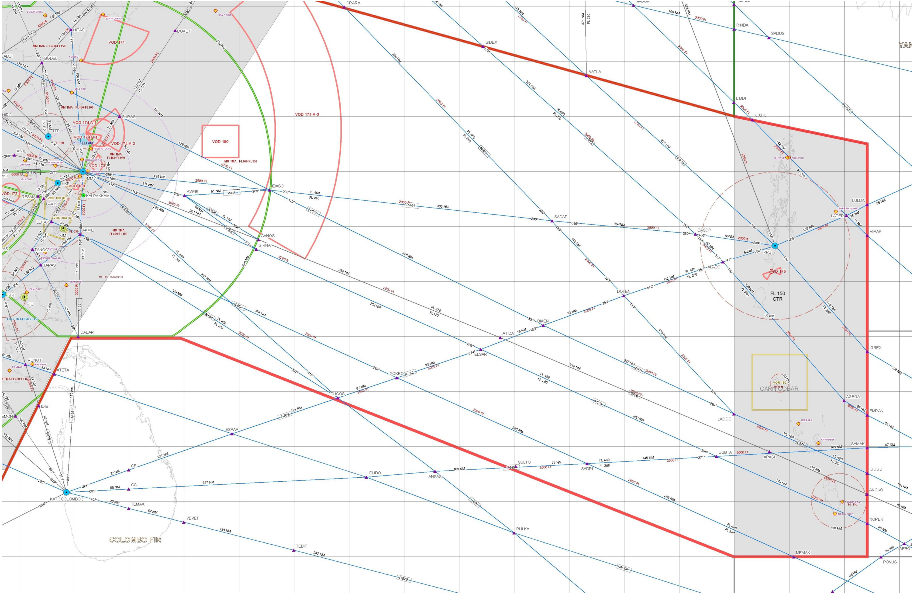

Data Sets Used for this Study

- **Traffic Sample Data (TSD):** Traffic sample data from Chennai FIR for the month of December 2010 was used. Some data pruning was done to remove reporting errors in the data.

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- **Gross Navigational Error (GNE) Data:** This consists of the reports of Gross Navigational Errors were received from India (Chennai, Mumbai, and Kolkata FIRs) and Bangkok for the months of July to December 2010

A Glimpse of the TSD

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Date	Call_sign	Registration	Approval	AC_type	FROM	TO	ETD	ENTRY_POINT	ENTRY_TIME	ENTRY_LEVEL	ROUTE_AFTER_ENTRY	EXIT_POINT	EXIT_TIME	EXIT_LEVEL	ROUTE_BEFORE_EXIT
2	1/12/2010	AIC442	VTSCG	RNP10	A319	VABB	WSSS	1840	IDASO	2037	F370	N571	IGOGU	2222	F370	N571
3	1/12/2010	GIA3209	FHSEA	RNP10	B744	WIMM	OEJN	0100	NOPEK	0217	F380	P574	GIRNA	0347	F380	P574
4	1/12/2010	SIA334	9VSKE	RNP10	A388	WSSS	LFPG	1555	NOPEK	1748	F340	P574	GIRNA	1918	F340	P574
5	1/12/2010	SIA346	9VSKA	RNP10	A388	WSSS	LSZH	1900	NOPEK	1908	F340	P574	GIRNA	2040	**F360**	P574
6	1/12/2010	SIA456	9VSTH	RNP10	A333	WSSS	OMAA	0445	NOPEK	0632	F380	P574	GIRNA	0804	F380	P574
7	1/12/2010	MAS195	9MMRK	RNP10	B772	VABB	WMKK	1820	GIRNA	1703	F350	P574	NOPEK	1958	**F390**	P574
8	1/12/2010	XAX2517	9MXXF	RNP10	A333	VABB	WMKK	1855	GIRNA	2220	F370	P574	NOPEK	2350	**F390**	P574
9	1/12/2010	QFA52	VHQPG	RNP10	A333	VABB	WSSS	0500	GIRNA	0725	F370	P574	NOPEK	0856	**F390**	P574
10	1/12/2010	MAS193	9MMLA	RNP10	B738	VOBL	WMKK	1850	SULTO	2113	F350	N563	MEMAK	2155	F350	N563
11	1/12/2010	AXM212	9MAHY	RNP10	A320	VOBL	WMKK	1305	SULTO	1415	F350	N563	MEMAK	1500	F350	N563
12	1/12/2010	SLK423	9VSBC	RNP10	A319	VOBL	WSSS	1130	IDASO	1305	F390	N571	IGOGU	1423	F390	N571
13	1/12/2010	HDA153	BHWJ	RNP10	A333	VOBL	VHHH	2130	IDASO	2222	F330	P761	LULDA	2352	F330	P762
14	1/12/2010	AXM204	9MAHU	RNP10	A320	VOCI	WMKK	1145	SULTO	1344	F390	N563	MEMAK	1424	F390	N563
15	1/12/2010	BAW15	GYMML	RNP10	B772	EGLL	WSSS	0950	BIDEX	0953	F390	L510	EMRAN	1105	F390	L510
16	1/12/2010	SIA325	9VSWR	RNP10	B77W	EDDF	WSSS	2100	BIDEX	0517	F350	L510	EMRAN	0627	**F370**	L510
17	1/12/2010	QFA6	VHOJD	RNP10	B744	EDDF	WSSS	2220	BIDEX	0650	F350	L510	EMRAN	0800	**F370**	L510
18	12/1/2010	SIA319	9VSWP	RNP10	B77W	EGLL	WSSS	0430	BIDEX	0433	F350	L510	EMRAN	0542	F350	L510
19	12/1/2010	KLM809	PHBQA	RNP10	B772	EHAM	WMKK	1950	BIDEX	0430	F370	L510	EMRAN	0531	F370	L510
20	12/1/2010	MAS1	9MMPB	RNP10	B744	EGLL	WMKK	2200	BIDEX	0817	F370	L510	EMRAN	0927	F370	L510
21	12/1/2010	KLM835	PHBVB	RNP10	B77W	EHAM	WSSS	2000	BIDEX	1801	F370	L510	EMRAN	1910	**F390**	L510
22	12/1/2010	ALK889	4RALA	RNP10	A332	VTBS	VCBI	1415	LULDA	1542	F320	P762	DUGOS	1656	F320	P762
23	12/1/2010	THA307	HSTAZ	RNP10	A306	VTBS	VCBI	1520	LULDA	1630	F320	P762	DUGOS	1744	F320	P762
24	12/1/2010	CPA749	BHOW	RNP10	B744	VHHH	FAJS	1545	LULDA	1900	F320	P762	DUGOS	2004	F320	P762
25	12/1/2010	SQC7342	9VSFD	RNP10	B744	WSSS	VOBL	1200	IGOGU	1344	F340	N571	IDASO	1514	F340	N571
26	12/1/2010	AXM224	9MAHV	RNP10	A320	VOHS	WMKK	0340	GIRNA	0456	F370	P574	NOPEK	0624	F370	P574
27	12/1/2010	SLK477	9VSLK	RNP10	A320	VOHS	WSSS	1850	GIRNA	2017	F330	P574	NOPEK	2147	F330	P574
28	12/1/2010	SIA321	----	RNP10	A388	EGLL	WSSS	0764	BIDEX	0820	F390	L510	EMRAN	0929	F390	L510
29	12/1/2010	XAX2905	9MXXB	RNP10	A333	OIIE	WMKK	1825	BIDEX	2319	F390	L510	EMRAN	0032	F390	L510
30	12/1/2010	ALK886	4RALD	RNP10	A332	VCBI	VTBS	2100	DUGOS	2211	F410	P762	LULDA	2330	**F330**	P762
31	12/1/2010	THA308	HSTAZ	RNP10	A306	VCBI	VTBS	2005	DUGOS	2047	F270	P762	LULDA	2200	**F290**	P762
32	12/1/2010	CSH813	B2566	RNP10	B763	ZSPD	VRMM	1425	LULDA	1447	F320	P762	DUGOS	1600	F320	P762
33	12/1/2010	AXM5701	9MAQB	RNP10	A320	WMKP	VOMM	0015	SAMAK	0138	F360	P574	GIRNA	0305	F360	P574
34	12/1/2010	CSH814	B2566	RNP10	B763	VRMM	ZSPD	1850	DUGOS	2101	F310	P762	LULDA	2215	F310	P762
35	12/1/2010	THA338	HSTAP	RNP10	A306	VOMM	VTBS	1845	IDASO	1923	F310	P761	LULDA	2048	F310	P762
36	12/1/2010	SIA529	9VSQN	RNP10	B772	VOMM	WSSS	1745	GIRNA	1836	F390	P574	NOPEK	2006	F390	P574
37	12/1/2010	SLK435	9VSLF	RNP10	A320	VOMM	WSSS	1045	GIRNA	1427	F390	P574	NOPEK	1557	F390	P574
38	12/1/2010	AXB684	VTAYB	RNP10	B738	VOMM	WSSS	0740	IDASO	1020	F390	N571	IGOGU	1153	F390	N571
39	12/1/2010	JAI16	VTJGU	RNP10	B738	VOMM	WSSS	1945	IDASO	2040	F330	N571	IGOGU	2210	F330	N571
40	12/1/2010	CPA632	BHLU	RNP10	A333	VOMM	VHHH	2145	IDASO	2215	F370	P761	LULDA	2340	F370	P762
41	12/1/2010	CPA018	BHUO	RNP10	B744	VOMM	VHHH	2100	IDASO	2144	F330	P761	LULDA	2314	F330	P762
42	12/1/2010	UAE356	A6EBH	RNP10	B77W	OMDB	WIII	0110	IDASO	0452	F330	N451	IGOGU	0623	F330	N571
43	12/1/2010	SIA326	9VSWJ	RNP10	B77W	WSSS	EDDF	0845	IGOGU	0751	F300	N571	IDASO	0921	F300	N571
44	12/1/2010	AIC443	VTSCM	RNP10	A319	WSSS	VABB	0245	IGOGU	0434	F360	N571	IDASO	0606	F360	N571
45	12/1/2010	XAX2516	9MXXC	RNP10	A333	WMKK	VABB	1000	IGOGU	1450	F380	N571	IDASO	1614	F380	N571
46	12/1/2010	SIA422	9VSQI	RNP10	B772	WSSS	VABB	1830	IGOGU	2016	F360	N571	IDASO	2146	F360	N571

A Glimpse of the GNE Data

Year	Month	FIR	Flights	LLE	LLD
2010	AUGUST	KOLKATA	443	0	0
2010	SEPTEMBER	KOLKATA	423	0	0
2010	OCTOBER	KOLKATA	432	0	0
2010	NOVEMBER	KOLKATA	427	0	0
2010	DECEMBER	KOLKATA	545	0	0
2010	JULY	CHENNAI	2679	0	0
2010	AUGUST	CHENNAI	5173	0	0
2010	SEPTEMBER	CHENNAI	5196	0	0
2010	OCTOBER	CHENNAI	5478	0	0
2010	NOVEMBER	CHENNAI	5258	0	0
2010	DECEMBER	CHENNAI	5432	0	0
2010	JULY	MUMBAI	1838	0	0
2010	AUGUST	MUMBAI	1812	0	0
2010	SEPTEMBER	MUMBAI	1792	0	0
2010	OCTOBER	MUMBAI	1884	0	0
2010	NOVEMBER	MUMBAI	1068	0	0
2010	DECEMBER	MUMBAI	1426	0	0
2010	JULY	BANGKOK	1865	0	0
2010	AUGUST	BANGKOK	2330	0	0
2010	SEPTEMBER	BANGKOK	2297	0	0
2010	OCTOBER	BANGKOK	2234	0	0
2010	NOVEMBER	BANGKOK	2108	0	0
2010	DECEMBER	BANGKOK	2061	0	0
2011	JANUARY	BANGKOK		0	0
	Total		54201	0	0

Reich's Collision Risk Model (CRM)

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- For lateral separation the formula turns out to be:

Lateral Collision Risk

$$N_{ay} = P_y(S_y)\,P_z(0)\,\frac{\lambda_x}{S_x}\left\{E_y(\text{same})\left[\frac{|\Delta\bar{V}|}{2\lambda_x}+\frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y}+\frac{|\bar{\dot{z}}|}{2\lambda_z}\right]\right.$$
$$\left.+\,E_y(\text{opp})\left[\frac{|2\bar{V}|}{2\lambda_x}+\frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y}+\frac{|\bar{\dot{z}}|}{2\lambda_z}\right]\right\}$$

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- For longitudinal separation the formula turns to be:

Longitudinal Collision Risk

$$\begin{aligned} N_{ax} &= P_y(0)\,P_z(0)\,\frac{2\lambda_x}{|\dot{x}|}\left(\frac{|\bar{\dot{x}}|}{2\lambda_x}+\frac{|\bar{\dot{y}}(0)|}{2\lambda_y}+\frac{|\bar{\dot{z}}|}{2\lambda_z}\right) \\ &\times\left[\sum_{k=m}^{M} 2Q(k)\ \mathbf{P}(K>k)\right] \end{aligned}$$

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- Lateral deviations of aircrafts on adjacent tracks are statistically independent.
- The aircrafts are approximated by rectangular boxes.
- Vertical, longitudinal, and lateral deviations of an aircraft are statistically independent.
- There is no corrective action by pilots or ATC when two aircrafts are about to collide.

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y(S_y)\, P_z(0)\, \frac{\lambda_x}{S_x} \left\{ E_y(\text{same}) \left[\frac{|\Delta\bar{V}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right] + E_y(\text{opp}) \left[\frac{|2\bar{V}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right] \right\}$$

N_{ay} Expected number of accidents (two for every collision) per flight hour due to the loss of lateral separation between aircrafts flying on tracks with planned S_y NM lateral separation

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$$\left.+\,E_y(\text{opp})\left[\frac{\left|2\bar{V}\right|}{2\lambda_x}+\frac{\left|\bar{\dot{y}}(S_y)\right|}{2\lambda_y}+\frac{\left|\bar{\dot{z}}\right|}{2\lambda_z}\right]\right\}$$

S_y Minimum planned lateral separation

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λ_x Average length of an aircraft flying in airspace

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λ_y Average wingspan of an aircraft flying in airspace

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λ_z Average height of an aircraft flying in airspace

Reich's Lateral Collision Risk Model

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$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\left|\Delta \bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right.$$
$$\left. + E_y\left(\text{opp}\right) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right\}$$

$P_y\left(S_y\right)$ Probability that two aircrafts assigned to two parallel routes with S_y NM lateral separation will lose all planned lateral separation

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\left|\Delta \bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] + E_y\left(\text{opp}\right) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right\}$$

$P_z\left(0\right)$ Probability that two aircrafts assigned to same flight level are at same geometric height

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y(S_y)\, P_z(0)\, \frac{\lambda_x}{S_x} \left\{ E_y(\text{same}) \left[\frac{\left|\Delta \bar{V}\right|}{2\lambda_x} + \frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right] \right.$$
$$\left. + E_y(\text{opp}) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right] \right\}$$

S_x Length of half the interval in NM used to count proximate aircraft at adjacent routes

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\left|\Delta \bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] + E_y\left(\text{opp}\right) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right\}$$

$E_y\left(\text{same}\right)$ Same direction lateral occupancy at same flight level

Reich's Lateral Collision Risk Model

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$$\left. + E_y(\text{opp})\left[\frac{|2\bar{V}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(S_y)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z}\right]\right\}$$

E_y (opp) Opposite direction lateral occupancy at same flight level

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\left|\Delta\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] + E_y\left(\text{opp}\right) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right\}$$

$\left|\Delta\bar{V}\right|$ Average relative speed of two aircraft flying on parallel routes in same direction

Reich's Lateral Collision Risk Model

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$$\left.+E_y\left(\text{opp}\right)\left[\frac{\left|2\bar{V}\right|}{2\lambda_x}+\frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y}+\frac{\left|\bar{\dot{z}}\right|}{2\lambda_z}\right]\right\}$$

$\left|\bar{V}\right|$ Average ground speed on an aircraft

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\left|\Delta \bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right.$$
$$\left. + E_y\left(\text{opp}\right) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right\}$$

$\left|\bar{\dot{y}}\left(S_y\right)\right|$ Average relative lateral speed of aircraft pair at loss of planned lateral separation of S_y

Reich's Lateral Collision Risk Model

Lateral Collision Risk

$$N_{ay} = P_y\left(S_y\right) P_z\left(0\right) \frac{\lambda_x}{S_x} \left\{ E_y\left(\text{same}\right) \left[\frac{\left|\Delta \bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] + E_y\left(\text{opp}\right) \left[\frac{\left|2\bar{V}\right|}{2\lambda_x} + \frac{\left|\bar{\dot{y}}\left(S_y\right)\right|}{2\lambda_y} + \frac{\left|\bar{\dot{z}}\right|}{2\lambda_z} \right] \right\}$$

$|\bar{\dot{z}}|$ Average relative vertical speed of a co-altitude aircraft pair assigned to the same route

Estimates of the Parameters

Parameter	Estimate	Source of the Estimate
S_y	50 NM	Current minimum
λ_x	0.0326051 NM	TSD
λ_y	0.02983705 NM	TSD
λ_z	0.009069301 NM	TSD
$P_y(50)$	4.31577×10^{-8}	Mixture model
$P_z(0)$	0.538	Double Exponential model
S_x	80 NM	± 10-mins longitudinal separation
E_y (same)	0.04880429	TSD
E_y (opp)	0	No opposite direction flights at same flight level
$\left\|\Delta \bar{V}\right\|$	36 knots	TSD
$\|\bar{\dot{y}}(50)\|$	75 knots	Conservative (EMA Handbook)
$\|\bar{\dot{z}}\|$	1.5 knots	Conservative (EMA Handbook)

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$\lvert \bar{\dot{y}}(50) \rvert$	75 knots	Conservative (EMA Handbook)
$\lvert \dot{z} \rvert$	1.5 knots	Conservative (EMA Handbook)

$$N_{ay} = 0.895265 \times 10^{-9}$$

Average Aircraft Dimensions

- Estimated using dimensions of each aircraft type weighted by their proportions in TSD

Probability of Lateral Overlap: $P_y\left(S_y\right)$

- $Y_1 =$ lateral deviation of first aircraft

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Probability of Lateral Overlap: $P_y\left(S_y\right)$

- $Y_1 =$ lateral deviation of first aircraft
- $Y_2 =$ lateral deviation of second aircraft
- Then, probability of lateral overlap (with planned separation S_y)

$$P_y\left(S_y\right) = \mathbf{P}\left(\left|S_y + Y_1 - Y_2\right| \leq \lambda_y\right) \ ,$$

Probability of Lateral Overlap: $P_y\left(S_y\right)$

- Assumption:
  Y_1 and Y_2 identically distributed, independent, with distribution F_y

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- The core distribution G_y represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision.
- The non-core distribution H_y, represents *Gross Navigation Errors (GNE)*, that corresponds to what may be viewed as non-nominal performance.

Probability of Lateral Overlap: $P_y\left(S_y\right)$

- Overall lateral deviation distribution is modeled as

$$F_y\left(y\right) = \left(1-\alpha\right) G_y\left(y\right) + \alpha H_y\left(y\right)$$

- The mixing parameter α is the probability of a *gross navigational error*

Probability of Lateral Overlap: $P_y\left(S_y\right)$

- Overall lateral deviation distribution is modeled as

$$F_y\left(y\right) = \left(1-\alpha\right) G_y\left(y\right) + \alpha H_y\left(y\right)$$

- G_y is modeled by a *Double Exponential* distribution with rate β_y. That is, if $Y_1 \sim G_y$ then

$$\mathbf{P}\left(\left|Y_1\right| > y\right) = e^{-\beta_y y}\,.$$

Probability of Lateral Overlap: $P_y\left(S_y\right)$

- Overall lateral deviation distribution is modeled as

$$F_y\left(y\right) = \left(1-\alpha\right) G_y\left(y\right) + \alpha H_y\left(y\right)$$

- H_y is modeled by "*Separated Double Exponential*" distribution with
  - separation parameter μ_y
  - rate parameter γ_y

  That is, if $Y_2 \sim H_y$ then

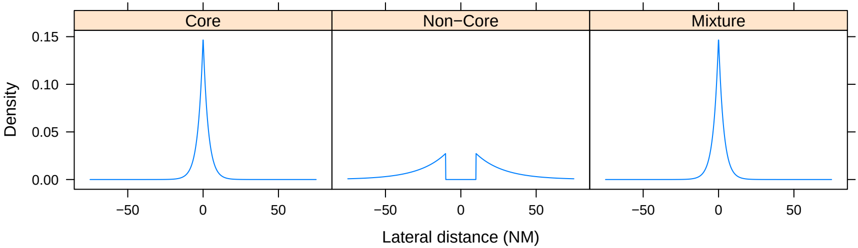
$$\mathbf{P}\left(Y_2 > \mu_y + y\right) = \frac{1}{2} e^{-\gamma_y y} \text{ and}$$

$$\mathbf{P}\left(Y_2 < -\mu_y - y\right) = \frac{1}{2} e^{\gamma_y y}.$$

Probability of Lateral Overlap: $P_y(S_y)$

- Overall lateral deviation distribution is modeled as

$$F_y(y) = (1-\alpha)\, G_y(y) + \alpha H_y(y)$$



Choice of parameters

- Mixture parameter α:
  - Estimated by taking the 95% *upper confidence limit* from observed GNE data.
  - The estimate is

$$\widehat{\alpha} = 1 - (0.05)^{1/N} = 5.526927 \times 10^{-5}\,,$$

    where $N = 54201$ is the number of flights observed and no gross navigational errors were detected.
  - **Note:** More GNE data with no detected gross navigational error will increase the value of N and hence decrease the value of α which will lead to decrease in the risk.
  - **Note:** This is very conservative estimate compare to the "natural" point estimate.

Choice of parameters

- Core distribution:
  - The parameter β_y is estimated under the RNP10 assumption of ± 10 NM deviation with 95% confidence, this leads to the estimate

$$\widehat{\beta}_y = -\frac{\log 0.05}{10} = 0.299573227\,.$$

Choice of parameters

- Non-core distribution:
  - Separation μ_y is taken to be 10 based on RNP10 consideration
  - Rate γ_y estimated by maximizing the wingspan overlap probability with $S_y = 50$ NM initial separation
  - This is a conservative method similar to what has been used by FAA and also in EUR/SAM.
  - The estimated value of γ_y is 0.05489709

Probability of Lateral Overlap: $P_y\left(S_y\right)$

- $\widehat{\alpha} = 1 - (0.05)^{1/N} = 5.526927 \times 10^{-5}$
- $\widehat{\beta}_y = 0.299573227$
- $\widehat{\mu}_y = 10$
- $\widehat{\gamma}_y = 0.05489709$
- Combining, estimated value of $P_y\left(50\right)$ is 4.31577×10^{-8}.

Probability of Vertical Overlap: $P_z(0)$

- $Z_1 =$ height deviations of first aircraft
- $Z_2 =$ height deviations of second aircraft
- Aircrafts nominally flying at same flight level on adjacent routes
- Then, probability of vertical overlap is

$$P_z(0) = \mathbf{P}\left(|Z_1 - Z_2| \leq \lambda_z\right) ,$$

Probability of Vertical Overlap: $P_z(0)$

- Assumption:
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$$\widehat{\beta}_z = -\frac{\log 0.05}{0.032915} = 91.014196371\,.$$

This is under assumption that a typical aircraft stays within ± 200 ft $= \pm 0.032915$ NM of its assigned flight level 95% of the time.

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  This is under assumption that a typical aircraft stays within ± 200 ft $= \pm 0.032915$ NM of its assigned flight level 95% of the time.
- Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors (ASE) which are not estimable directly. So we use a conservative value of 0.538, as used by MAAR for vertical safety assessment in BOB region.

Lateral Occupancy parameters E_y (same) and E_y (opp)

- *Same direction occupancy*: For a typical aircraft, average number of aircrafts that are "**proximate**"; that is,
  - flying in the same direction as it
  - nominally flying on tracks one lateral separation standard away
  - nominally at the same flight level as it
  - within a longitudinal segment centered on it
- The length of longitudinal segment $(2S_x)$ usually taken to be distance traveled in 20 minutes of flight, giving value of 160 NM.

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  - within a longitudinal segment centered on it
- The length of longitudinal segment $(2S_x)$ usually taken to be distance traveled in 20 minutes of flight, giving value of 160 NM.
- Similar for *opposite direction occupancy*
  - Proximate aircrafts flying in opposite direction, same flight level.
  - Currently flight levels are unidirectional, so taken to be 0.

Estimation of E_y (same) using TSD

- Estimated by computing number of *proximate pairs* in TSD
  - Note time when aircraft on one route passes a waypoint
  - Count number of aircrafts passing homologous waypoint within ± 10 minutes

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  - n is the the total number of aircrafts

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  - n_y is the number of proximate pairs
  - n is the the total number of aircrafts
- Route pairs
  - N877 parallel to (unidirectional) routes L510 (EB) and P628 (WB)
  - N571 parallel to P574
  - P762 has no parallel route

Estimation of E_y (same) using TSD

Count By	Routes	Waypoints	Total	Proximate
Entry	(N877, L510)	(ORARA, BIDEX)	316	2
Entry	(N877, P628)	(IGOGU, IGREX)	389	40
Entry	(P574, N571)	(NOPEK, IGOGU)	1188	80
Entry	(P574, N571)	(GIRNA, IDASO)	1254	38
Exit	(N877, P628)	(ORARA, VATLA)	389	20
Exit	(N877, L510)	(IGOGU, EMRAN)	81	0
Exit	(P574, N571)	(NOPEK, IGOGU)	1276	82
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$$\widehat{E}_y(\text{same}) = \frac{300}{6147} = 0.04880429$$

Estimate of Average Ground Speed

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- $\text{Speed} = \dfrac{\text{distance between entry and exit waypoints}}{\text{traversal time}}$

Estimate of average relative speed $\left|\Delta\bar{V}\right|$

- $\left|\Delta\bar{V}\right|$ = average absolute relative speed of two aircrafts flying on parallel routes in same direction

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- Average absolute speed difference $= 35.13632$.
- We use conservative value 36.

Estimate of Average Relative Lateral Speed: $|\bar{\dot{y}}\,(S_y)|$

- Average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation.

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- The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study.
- We use conservative value 75 knots as per EMA Handbook.

Estimate of Average Relative Vertical Speed: $|\bar{\dot{z}}|$

- Average absolute relative vertical speed for pair of aircrafts on the same flight level of adjacent tracks that has lost lateral separation.

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- Generally assumed that $|\bar{\dot{z}}|$ is independent of amount of lateral separation as well as vertical separation between the aircraft.
- Data on $|\bar{\dot{z}}|$ relatively scarce.
- Estimate typically taken as 1.5 knots which is considered to be conservative (EMA Handbook).

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$\begin{aligned} N_{ax} &= P_y(0)\, P_z(0)\, \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right) \\ &\quad \times \left[\sum_{k=m}^{M} 2Q(k)\ \mathbf{P}(K > k) \right] \end{aligned}$$

N_{ax} Expected number of accidents (two for every collision) per flight hour due to collision between two co-altitude aircraft with planned minimum m NM longitudinal separation.

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$\begin{aligned} N_{ax} &= P_y(0)\,P_z(0)\,\frac{2\lambda_x}{|\dot{x}|}\left(\frac{|\bar{\dot{x}}|}{2\lambda_x}+\frac{|\bar{\dot{y}}(0)|}{2\lambda_y}+\frac{|\bar{\dot{z}}|}{2\lambda_z}\right) \\ &\quad\times\left[\sum_{k=m}^{M} 2Q(k)\ \mathbf{P}(K>k)\right] \end{aligned}$$

m Minimum longitudinal separation in NM.

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M Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$
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N_{ax} \quad = \quad & P_y(0)\, P_z(0)\, \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right) \\
& \times \left[\sum_{k=m}^{M} 2Q(k)\ \mathbf{P}(K > k) \right]
\end{aligned}
$$

λ_x Average length of an aircraft flying in airspace

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λ_y Average wingspan of an aircraft flying in airspace

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

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λ_z Average height of an aircraft flying in airspace

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$
\begin{aligned}
N_{ax} &= P_y(0)\, P_z(0)\, \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right) \\
&\quad \times \left[\sum_{k=m}^{M} 2Q(k)\; \mathbf{P}(K > k) \right]
\end{aligned}
$$

$P_y(0)$ Probability that two aircraft assigned at the same route will be at same across-track position.

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$
\begin{aligned}
N_{ax} \quad = \quad & P_y(0)\, P_z(0)\, \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right) \\
& \times \left[\sum_{k=m}^{M} 2Q(k)\, \mathbf{P}(K > k) \right]
\end{aligned}
$$

$P_z(0)$ Probability that two aircraft assigned to same flight level are at same geometric height.

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$N_{ax} = P_y(0)\, P_z(0)\, \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right) \times \left[\sum_{k=m}^{M} 2Q(k)\ \mathbf{P}(K > k) \right]$$

$|\dot{x}|$ Minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point.

Reich's Longitudinal Collision Risk Model

Longitudinal Collision Risk

$$N_{ax} = P_y(0)\, P_z(0)\, \frac{2\lambda_x}{|\dot{x}|} \left(\frac{|\bar{\dot{x}}|}{2\lambda_x} + \frac{|\bar{\dot{y}}(0)|}{2\lambda_y} + \frac{|\bar{\dot{z}}|}{2\lambda_z} \right) \times \left[\sum_{k=m}^{M} 2Q(k)\ \mathbf{P}(K > k) \right]$$

$|\bar{\dot{y}}(0)|$ Relative across-track speed of same route aircraft pair.

Reich's Longitudinal Collision Risk Model

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$|\bar{\dot{z}}|$ Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

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$\mathbf{P}(K > k)$ Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Estimates of the Parameters

Parameter	**Estimate**	**Source of the Estimate**
m	80 NM	Current minimum
M	160 NM	20 minutes longitudinal separation
λ_x	0.0326051 NM	TSD
λ_y	0.02983705 NM	TSD
λ_z	0.009069301 NM	TSD
$P_y(0)$	0.2	Conservative estimate
$P_z(0)$	0.3617939	Double exponential model
$\lvert\bar{\dot{x}}\rvert$	90 knots	Conservative estimate using speed and distance between way points
$\lvert\bar{\dot{y}}(0)\rvert$	1 knot	RASMAG/9 safety assessment
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$$N_{ax} = 0.743608 \times 10^{-9}$$

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- Conservative estimates
  - $v_0 =$ minimum speed observed in TSD $= 315$ knots
  - $d =$ maximum distance between two waypoints $= 338$ NM
  - $|\bar{\dot{x}}| = 97.67442$ knots
  - We use even more conservative value 90 knots

Estimation of $|\bar{\dot{y}}(0)|$

- Relative cross-track speed of same route aircraft pair
- No data is available for estimation of this parameter
- We use conservative value of 1 knot (EMA Handbook)

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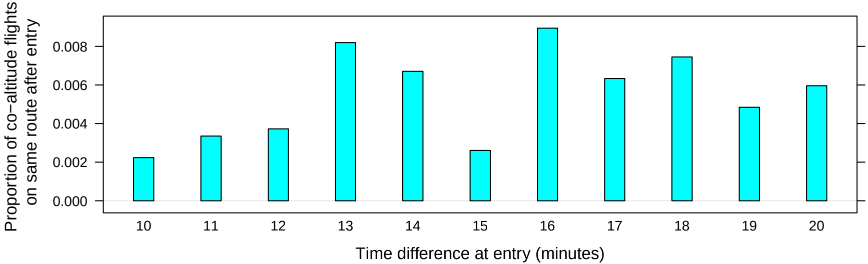

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  - Then

$$\mathbf{P}(K > k) = \mathbf{P}\left(0 < \frac{k}{V' - V} < T_0\right) = \mathbf{P}\left(V' - V > \frac{k}{T_0}\right).$$

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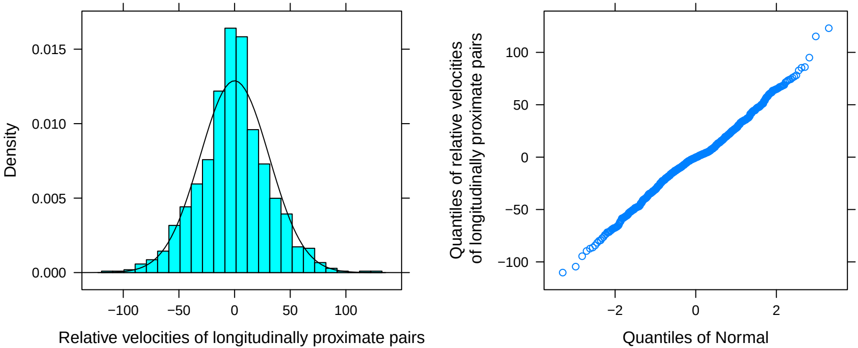

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- Not necessarily Normally distributed
- Conservatively take Normal and Double exponential mixture model

$$f_v\left(v\right) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + \left(1-p\right)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{v^2}{2\sigma^2}}$$

- Parameters (MLE) estimated using EM algorithm, rounded conservatively

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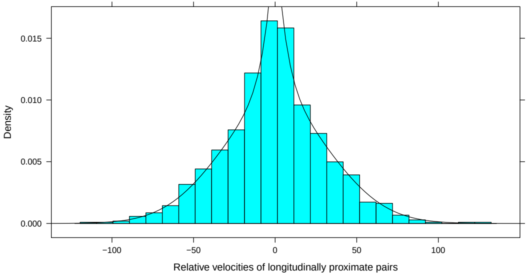

Summary of overtake probability calculations

k (mins)	k (NM)	$Q(k)$	$P(K > k)$
10	80	0.002235469	1.83061×10^{-6}
11	88	0.003353204	1.88145×10^{-7}
12	96	0.003725782	1.6016×10^{-8}
13	104	0.008196721	1.16613×10^{-9}
14	112	0.006706408	8.16394×10^{-11}
15	120	0.002608048	7.35331×10^{-12}
16	128	0.008941878	1.04974×10^{-12}
17	136	0.006333830	1.95268×10^{-13}
18	144	0.007451565	3.89188×10^{-14}
19	152	0.004843517	7.84075×10^{-15}
20	160	0.005961252	1.58302×10^{-15}

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- We also make risk analysis by assuming few GNEs.
- We considered cross-routes which are typically not amenable to standard CRM.
- In our latest analysis we have also incorporated internal way-points and not just the entry and exit points. This gives better estimation of the parameters.

Statistical Challenges which are Yet to be Addressed

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- Answer to this is probably through *re-sampling methods*.

Thank You