Abstract

Max-type Recursive Distributional Equations

by

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In certain problems in a variety of applied probability settings (from probabilistic analysis of algorithms to mean-field statistical physics models), the central requirement is to solve a fixed point equation of the form $X \stackrel{d}{=} g\left(\left(\xi_i, X_i\right), i \geq 1\right)$, where $\left(\xi_i\right)_{i \geq 1}$ and $g\left(\cdot\right)$ are given and $\left(X_i\right)_{i \geq 1}$ are independent copies of X with unknown distribution. We call such an equation a *recursive distributional equation*. Exploiting the natural recursive structure one can associate a tree-indexed process with every solution, and such a process is called a *recursive tree process*. This process in some sense is a solution of an infinite system of recursive distributional equations.

The dissertation is devoted to the study of such fixed point equations and the associated recursive tree process when the given function $g\left(\cdot\right)$ is essentially a "maximum" or a "minimum" function. Such equations arise in the context of optimization problems and branching random walks. The present work mainly concentrates on the theoretical question of *endogeny* : the tree-indexed process being measurable with respect to the given i.i.d *innovations* $(\xi_{\mathbf{i}})$. We outline some basic general theory which is natural from a statistical physics point of view and then specialize to study some concrete examples arising from various different contexts.