1. Suppose a box contains 70 red balls and 30 green balls. You do sampling with replacement of 100 balls. Let $X$ be the number of green balls in the first 40 draws, and $Y$ be the number of green balls in the next 60 draws. Let $T = X + Y$ be the total number of green balls in the sample.

(a) What are the distributions of $X$, $Y$ and $T$? [2 points]

(b) Are $X$ and $Y$ independent? [1 point]

(c) Find $P(X = k | T = t)$ for $0 \leq k \leq t$. [5 points]

(d) Are $X$ and $T$ independent? Why? [2 points]

2. Cards are dealt one after the other from a well shuffled deck of 52 cards until the first King appears. Find a formula for $p(n) :=$ probability that exactly $n$ cards are dealt. [5 points]

3. There are $n$ balls labeled $1, 2, 3, \ldots, n$, and $n$ boxes also labeled $1, 2, 3, \ldots, n$. Balls are being placed in the boxes at random so that, any ball can go into any box, and a box may contain more than one ball.

(a) Find $P(\text{none of the } n \text{ boxes are empty}).$ [2 points]

(b) Say that $i^{\text{th}}$ ball is correctly placed if it goes to the $i^{\text{th}}$ box. Let $A_i$ be the event that $i^{\text{th}}$ ball is correctly placed. Find $P(A_i)$ for $1 \leq i \leq n.$ [2 points]

(c) Let $X$ be the total number of correct placement of balls, find the distribution of $X.$ [5 points]

(d) Find $E[X]$ and $\text{Var}(X).$ [2 points]

(e) If $n = 10,000$ calculate approximately $P(X < 2), P(X = 2)$ and $P(X > 2).$ [4 points]