

Indian Statistical Institute, Kolkata

Advanced Probability

Fall 2008

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Assignment # 5

Date Given: November 4, 2008 (Tuesday)

Total Points: 20

Date of Submission: November 14, 2008 (Friday)

1. Let $(S_n)_{n \geq 0}$ be an asymmetric simple random walk starting at 0 with $p > 1/2$. Show that [3 + 7 = 10]
 - (a) $X_n := (S_n - (2p - 1)n)^2 - \sigma^2 n$ is a martingale, where $\sigma^2 := 1 - (2p - 1)^2$.
 - (b) $\mathbf{Var}(T) = \sigma^2 / (2p - 1)^3$ where $T := \inf \{ n \geq 0 \mid S_n = 1 \}$.
2. Suppose $(X_n, \mathcal{F}_n)_{n \geq 0}$ is a non-negative super-martingale. Let $N \leq \infty$ is a stopping time with respect to the filtration $(\mathcal{F})_{n \geq 0}$. Then show that X_N is well defined and $\mathbf{E}[X_0] \geq \mathbf{E}[X_N]$. [10]