Indian Statistical Institute, Delhi Centre

Branching Processes

Fall 2009

Assignment # 1

Date Given: September 14, 2009 (Monday) **Total Points: 20**
Date Due: September 21, 2009 (Monday)

1. Suppose f be a *probability generating function (p.g.f.)* of a distribution on $\mathbb{N} \cup \{0\}$ and ϕ be a *characteristic function* of a distribution on $\mathbb{R}$. [3 + 4 = 7]

   (i) Show that $f \circ \phi$ is a characteristic function.

   (ii) Hence or otherwise find the distribution of the following random variable:

   $$\sum_{i=1}^{N} X_i \, ,$$

   where $(X_i)_{i \geq 1}$ are i.i.d. Exponential (λ) random bariables and are independent of N which has Geometric (p) on $\mathbb{N}$ (that is, N counts the number of trials before the first success, where the success probability is p).

2. Let $(Z_n)_{n \geq 0}$ be a *Galton-Watson branching process* with progeny distribution $p_j = e^{-\lambda} \frac{\lambda^j}{j!}$ $j \geq 0$ and starting at $Z_0 \equiv 1$. [(1 + 2) + 4 = 7]

   (i) Find the mean and variance of Z_n for any $n \geq 1$.

   (ii) Show that the extinction probability $q \equiv q(\lambda) \uparrow 1$ as $\lambda \downarrow 1$.

3. Consider a Galton-Watson branching process $(Z_n)_{n \geq 0}$ with some progeny distribution $(p_j)_{j \geq 0}$. Show that as a Markov Chain it has an unique stationary distribution $\pi_0 = 1, \pi_j = 0 \, \forall \, j \geq 1$.

   Can we conclude then that $\mathbb{P}(Z_n = 0) \to 1$ as $n \to \infty$? If not then why ?

   [4 + (1 + 1) = 6]