Indian Statistical Institute, Delhi Centre

Branching Processes

Fall 2009

Assignment # 2

Date Given: September 21, 2009 (Monday) **Total Points: 20**
Date Due: September 28, 2009 (Monday)

1. Let q_n be the probability of extinction of a GW branching process starting with one individual and offspring distribution Binomial $\left(n, \frac{c}{n}\right)$ with $c > 1$. Then show that as $\{q_n\}_{n \geq 1}$ has a limit q which is given by the unique solution in $[0, 1]$ of the following equation

$$q = e^{-(1-q)c} .$$

Give a probabilistic interpretation of this q. [8 + 2 = 10]

2. Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space and $\mathcal{G} \subseteq \mathcal{F}$ be a sub-σ-algebra. Let X be an integrable random variable defined on $(\Omega, \mathcal{F}, \mathbf{P})$. Show from first principle that if X and $\mathcal{G}$ are independent then $\mathbf{E}\left[X \,\middle|\, \mathcal{G}\right] = \mathbf{E}\left[X\right]$. [5]

3. Suppose $(\mu_n)_{n \geq 1}$ be a sequence of probability measures on a measurable space $(\Omega, \mathcal{F})$. Prove of disprove with a suitable counter example that there is always a probability $\mathbb{P}$ on $(\Omega, \mathcal{F})$ such that $\mu_n \ll \mathbb{P}$ for all $n \geq 1$. [5]