

Indian Statistical Institute, Delhi Centre

Branching Processes

Fall 2009

Assignment # 4

Date Given: October 8, 2009 (Thursday)
Date Due: October 15, 2009 (Thursday)

Total Points: 20

1. Suppose N is a non-negative integer valued random variable.
 - (a) Show that if $\mathbf{E}[N^p] < \infty$ for some $p > 1$ then $\mathbf{E}[N \log N] < \infty$. [4]
 - (b) Find an example where $\mathbf{E}[N] < \infty$ but $\mathbf{E}[N \log N] = \infty$. [3]

2. Let $(Z_n)_{n \geq 1}$ be a *sub-critical* GW branching process with progeny distribution N . Let $m := \mathbf{E}[N] < 1$ and $\sigma^2 := \mathbf{V}(N) < \infty$. Assume $\mathbf{P}(Z_0 = 1) = 1$. Let S be the total size of the population. Show that S has finite mean and variance. Find the mean and variance of S in terms of m and σ^2 . [2 + 4 + 4 = 10]

3. In the above problem if we assume that the process is critical, that is $m = 1$, then what can you say about $\mathbf{E}[S]$? [3]